The Dark Corners of the Labor Market

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The main lesson of the crisis is that we were much closer to those dark corners than we thought—and the corners were even darker than we had thought too. – Olivier Blanchard (2014), in “Where Danger Lurks”.
Single vs multiple steady states

Single steady state

Multiple steady states

**This paper:** look for evidence in U.S. labor market data:

i) Estimate reduced-form model of the labor market and infer steady state(s).

ii) Quantitative horse race between standard DMP model and extension with multiple steady states.

**Result:** at least 3 steady states:

\[ A : u \approx 5\% \text{ (stable)} \]
\[ B : u \approx 10\% \text{ (unstable)} \]
\[ C : u > 10\% \text{ (stable)} \]
Part I: reduced-form model

\[ u_t = \left(1 - \rho_{f,t}\right) u_{t-1} + \rho_{x,t} \left(1 - \rho_{f,t}\right) (1 - u_{t-1}) \]

\[ \rho_{x,t} = \rho_x(S_t) \]
\[ \rho_{f,t} = \rho_f(S_t) \]

\( u_t \): unemployment rate
\( \rho_{f,t} \): job finding rate
\( \rho_{x,t} \): job loss rate
\( S_t \): vector containing \( m \) aggregate state variables
Steady states: example

Example: \( \rho_x (S_t) = \bar{\rho}_x, \quad \rho_f (S_t) = \gamma_0 + \gamma_1 u_{t-1} \)

\[ \bar{u} = (1 - \gamma_0 - \gamma_1 \bar{u}) \bar{u} + \bar{\rho}_x (1 - \gamma_0 - \gamma_1 \bar{u}) (1 - \bar{u}) \]

quadratic equation, two solutions for \( \bar{u} \). Only one state variable \((S_t = u_{t-1})\).
Estimating steady states

Let $x_t \in \mathbb{R}^n$ be a vector of observed outcomes (including $\rho_{x,t}$, $\rho_{f,t}$ and possibly other variables, but not $u_t$).

Assumption underlying third equality: equilibrium mapping from $[s_1,t; s_2,t]$ to $[s_1,t; x_t]$ is invertible. Satisfied by linearized DSGE models. May need more than $m$ observables if outcomes are non-monotonic functions of the state variables. Check that forecasts condition on enough variables (residual autocorrelation, more observables, more lags, etc.).
Estimating steady states

Let $\mathbf{x}_t \in \mathbb{R}^n$ be a vector of observed outcomes (including $\rho_{x,t}, \rho_{f,t}$ and possibly other variables, but not $u_t$).

Estimate direct $k-$step ahead forecasting function:

$$E_t \mathbf{x}_{t+k} = E \left[ \mathbf{x}_{t+k} \mid S_t \right]$$
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Estimate direct \( k \)-step ahead forecasting function:

\[
E_t x_{t+k} = E \left[ x_{t+k} | S_t \right] \\
= E \left[ x_{t+k} | s_{1,t} ; s_{2,t} \right] \\
\text{observed unobserved}
\]
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\[
E_t x_{t+k} = E [ x_{t+k} | S_t ] = E [ x_{t+k} | s_{1,t}, s_{2,t} ]
\]

where \( s_{1,t} \in \mathbb{R}^{m-n} \) contains lags of variables in \( [x_t; u_t] \), and \( s_{2,t} \in \mathbb{R}^n \).
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where $s_{1,t} \in \mathbb{R}^{m-n}$ contains lags of variables in $[x_t ; u_t]$, and $s_{2,t} \in \mathbb{R}^n$.

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- Satisfied by linearized DSGE models. May need more than $m$ observables if outcomes are non-monotonic functions of the state variables.
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$$\mathbb{E}_t x_{t+k} = \mathbb{E} [x_{t+k} | S_t]$$

$$= \mathbb{E} [x_{t+k} | s_{1,t}; s_{2,t}]$$

where $s_{1,t} \in \mathbb{R}^{m-n}$ contains lags of variables in $[x_t; u_t]$, and $s_{2,t} \in \mathbb{R}^n$.

Assumption underlying third equality: equilibrium mapping from $[s_{1,t}; s_{2,t}]$ to $[s_{1,t}; x_t]$ is invertible.

- Satisfied by linearized DSGE models. May need more than $m$ observables if outcomes are non-monotonic functions of the state variables.

- Check that forecasts condition on enough variables (residual autocorrelation, more observables, more lags, etc.).
Estimating steady states

Steady state(s) satisfy:

\[
\begin{align*}
\bar{x} &= \mathbb{E}[x_{t+k} | \bar{s}_1; \bar{x}] \\
\bar{u} &= \bar{\rho}_x (1 - \bar{\rho}_f) / \left( \bar{\rho}_x (1 - \bar{\rho}_f) + \bar{\rho}_f \right)
\end{align*}
\]

which is a system of \( n + 1 \) equations in \( n + 1 \) unknowns.
Data


- CPS data on unemployment rate and flow rate from $U$ to $E$ (gross-flows).

- Construct job loss rate to be consistent with transition identity.

- IV estimator to account for noise in observations, using lagged values as instruments.
where \( u_t^* = \frac{\rho_x, t \left( 1 - \rho_f, t \right)}{\rho_x, t \left( 1 - \rho_f, t \right) + \rho_f, t} \); Hall (2005).
Model specifications

Three specifications turn out to summarize the main results:

(I) \( \mathbb{E}_{t} \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \varepsilon_{t+k} \)

(II) \( \mathbb{E}_{t} \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \gamma_3 u_t + \varepsilon_{t+k} \)

(III) \( \mathbb{E}_{t} \rho_{f,t+k} = \gamma_0 + \gamma_1 \rho_{x,t} + \gamma_2 \rho_{f,t} + \gamma_3 u_t + \gamma_3 u_t^2 + \varepsilon_{t+k} \)

plus AR(1) for \( \rho_{x,t} \).
Model selection

Diagnostics statistics

Model (I):
\[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \epsilon_{t+k} \]

Model (II):
\[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \beta_3 u_t + \epsilon_{t+k} \]

Model (III):
\[ \rho_{f,t+k} = \beta_0 + \beta_1 \rho_{f,t} + \beta_2 \rho_{x,t} + \beta_3 u_t + \beta_4 u_t^2 + \epsilon_{t+k} \]

Correlation \( \epsilon_t, \epsilon_{t+k+1} \) vs. forecast horizon in months (k)

R\(^2\) statistic vs. forecast horizon in months (k)
Implied steady states

Steady state curve for $\rho_{x,t}^* = \rho_x$. Shaded area’s denote 90 percent (bootstrapped) confidence bands.
Robustness

1. Additional higher-order terms
   ▶ similar results to model (III);

2. AR(1) specification
   ▶ similar to model (I) → invalidated

3. Alternative data source (duration-based CPS data)
   ▶ similar results;

4. Longer data sample (1960-2014)
   ▶ similar results;

5. Additional macro variables / unobserved states (Industrial Production, Consumer Price Inflation, Federal Funds Rate)
   ▶ similar results;

6. Alternative estimator (OLS)
   ▶ similar results;

7. Additional lags
   ▶ similar results;
Phase diagram

DARK CORNER

2009:7
\( \Delta u = 0 \)
\( \Delta \rho_f = 0 \)

2000:9
2009:11

unemployment rate (u)
job finding rate (\( \rho_f \))

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Part II: horse race between search and matching models

Feed job loss rates observed in the data through:

1. standard Diamond-Mortensen-Pissarides model $\rightarrow$ single steady state

2. extension with skill losses à la Pissarides (1992) $\rightarrow$ multiple steady states...
   - ...but unique dynamic equilibrium
Model

- Risk neutral agents
- Random search
- Exogenous but stochastic rate of job loss

-Timing within period:

1. Rate of job loss is revealed and job losses take place.

2. Job losers and previously unemployed workers find a job with an endogenous probability $\rho_{f,t}$. Vacancies ($v_t \geq 0$) are posted at a cost $\kappa > 0$ per unit and filled with an endogenous probability $q_t$.

3. Production and consumption take place. Employed workers produce $\bar{A}$ units of goods and receive a wage. Unemployed workers receive $b < \bar{A}$ units of goods.
Model: skill losses

- Job losers who immediately find a new job retain their productivity.

- Job losers who become unemployed need to be re-trained upon re-employment, at a cost $\chi \geq 0$ to the employer. Basic DMP model is obtained by setting $\chi = 0$.

- The fraction of job searchers with reduced skills, $p_t$, is given by:

$$p_t = \frac{u_{t-1}}{u_{t-1} + \rho_{x,t} (1 - u_{t-1})}.$$
Vacancy posting (free-entry) condition

\[
\frac{\kappa}{q_t} + p_t \chi = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^t s_{t,t+k} \left( A - w_{t+j} \right) + \xi_t
\]

where

- \( s_{t,t+k} \equiv \prod_{j=1}^{k} (1 - \rho_{x,t+j}) \) is the probability that the match survives until period \( t + k \)
- \( \xi_t \) is the Lagrange multiplier on the constraint \( \nu_t \geq 0 \).
Labor market

- Matching function:
  \[ m_t = s_t^{\alpha} v_t^{1-\alpha}, \]
  where \( s_t \equiv u_{t-1} + \rho_{x,t} (1 - u_{t-1}) \) is the number of searchers ⇒ \( \rho_{f,t} = \frac{m_t}{s_t} \)
  and \( q_t = \frac{m_t}{v_t} = \rho_{f,t}^{\frac{\alpha}{\alpha-1}} \).

- Assume firms have all bargaining power ⇒ \( w_t = \bar{w} = b \) (rigid real wage). Could be relaxed.
Model summary

\begin{align}
    u_t &= \left(1 - \rho_{f,t}\right) u_{t-1} + \rho_{x,t} \left(1 - \rho_{f,t}\right) (1 - u_{t-1}) \tag{1} \\
    p_t &= \frac{u_{t-1}}{u_{t-1} + \rho_{x,t} (1 - u_{t-1})} \tag{2} \\
    \rho_{x,t} &= (1 - \lambda_x) \bar{\rho}_x + \lambda_x \rho_{x,t-1} + \varepsilon_{x,t} \tag{3} \\
    \beta \mathbb{E}_t \left(1 - \rho_{x,t+1}\right) \left(\chi p_{t+1} - \zeta_{t+1} + \kappa \rho_{f,t+1}^{\frac{\alpha}{1-\alpha}}\right) &= \chi p_t - \zeta_t + \kappa \rho_{f,t}^{\frac{\alpha}{1-\alpha}} - \bar{A} + \bar{W} \tag{4}
\end{align}

An equilibrium is characterized by laws of motion for $u_t, \rho_{f,t}, \rho_{f,t}, p_t$ and $\zeta_t$ that satisfy the above four equations, and the complementary slackness condition $\rho_{f,t} \zeta_t = 0$. The state of the aggregate economy can be summarized as $S_t = \left\{\rho_{x,t}, u_{t-1}\right\}$.
Phase diagram: no skill losses

\[ \rho_f \]

\[ \Delta u = 0 \]

\[ \Delta \rho_f = 0 \]
Phase diagram: skill losses

\[ \Delta \rho_f = 0 \]

\[ \rho_f \]

\[ \Delta u = 0 \]

\[ u \]
Parameter values

- Model period: 1 month
- Steady-state targets:

<table>
<thead>
<tr>
<th>target</th>
<th>no skill losses</th>
<th>skill losses</th>
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<tbody>
<tr>
<td>$u^A$</td>
<td>0.055</td>
<td>0.055</td>
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<tr>
<td>$u^B$</td>
<td>—</td>
<td>0.095</td>
</tr>
<tr>
<td>parameter</td>
<td>description</td>
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<tr>
<td>-----------</td>
<td>------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>1.04$^{-\frac{1}{12}}$</td>
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<tr>
<td>$\alpha$</td>
<td>matching function elast.</td>
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</tr>
<tr>
<td>$\kappa$</td>
<td>vacancy cost</td>
<td>0.989</td>
</tr>
<tr>
<td>$\overline{A}$</td>
<td>worker productivity</td>
<td>1</td>
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<tr>
<td>$\overline{\rho}_x$</td>
<td>s.s. job loss rate</td>
<td>0.021</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>persistence job loss rate shocks</td>
<td>0.896</td>
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<tr>
<td>$\overline{\sigma}_x$</td>
<td>s.t. deviation job loss shocks</td>
<td>$7.91e^{-4}$</td>
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<tr>
<td>$\chi$</td>
<td>re-training cost</td>
<td>0</td>
</tr>
<tr>
<td>$b$</td>
<td>flow from unemployment</td>
<td>0.997</td>
</tr>
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Propagation
deterministic simulation
### Simulation

**job loss rate** ($\rho_{x,t}$)

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<tr>
<td>Rate</td>
<td>0.016</td>
<td>0.018</td>
<td>0.020</td>
<td>0.022</td>
<td>0.024</td>
<td>0.028</td>
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**job finding rate** ($\rho_{f,t}$)

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<tr>
<td>Rate</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
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</table>

**unemployment rate** ($u_t$)

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<tbody>
<tr>
<td>Rate</td>
<td>0.2</td>
<td>0.25</td>
<td>0.3</td>
<td>0.35</td>
<td></td>
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</tbody>
</table>

Data

- DMP model with skill losses
- Basic DMP model
- Constant job finding rate

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Simulation

job finding rate ($\rho_{f,t}$)

- Data
- DMP model with skill losses
- Basic DMP model
- Constant job finding rate

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Simulation

Data

DMP model with skill losses

Basic DMP model

Constant job finding rate
Conclusion

- Multiple-steady state model provides superior description of data.
- Threshold at around 10% unemployment.
- Possibly large and non-linear policy implications.
Appendix: firm decision problem

Large firms with constant returns-to-scale technologies decide on number of vacancies \((v_t)\), hires \((h_t)\) and employment \((n_t)\). Decision problem:

\[
V(n_{t-1}, S_t) = \max_{h_t, n_t, v_t} \left( \bar{A} - w_t \right) n_t - \left( (\chi - d_t) \rho_t + \frac{\kappa}{q_t} \right) h_t + \beta \mathbb{E}_t V(n_t, S_{t+1}),
\]

subject to
\[
\begin{align*}
n_t &= \left( 1 - \rho_{x,t} \right) n_{t-1} + h_t, \\
h_t &= q_t v_t, \\
h_t &\geq 0,
\end{align*}
\]

where \(w_t\) is the wage and \(d_t\) is a possible wage deduction for newly hired workers with reduced skills.