The use of DSGE models at central banks has triggered a strong interest in their forecast performance.

The subsequent material draws from the article “DSGE Model-Based Forecasting” which is slated to appear as a chapter in the *Handbook of Economic Forecasting*: “DSGE Model-Based Forecasting”
Generating Forecasts with a DSGE Model

- DSGE Model = State Space Model
  - Measurement Eq:
    \[
    y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_2(\theta)s_t
    \]
  - State Transition Eq:
    \[
    s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t
    \]
- Posterior distribution of DSGE model parameters:
  \[
p(\theta|Y_{1:T}) = \frac{p(Y_{1:T}|\theta)p(\theta)}{p(Y_{1:T})}, \quad p(Y_{1:T}) = \int p(Y_{1:T}|\theta)p(\theta)d\theta.
  \]
- Objective of interest is predictive distribution:
  \[
p(Y_{T+1:T+H}|Y_{1:T}) = \int p(Y_{T+1:T+H}|\theta, Y_{1:T})p(\theta|Y_{1:T})d\theta.
  \]
- Use numerical methods to generate draws from predictive distribution.
Generating Draws from Predictive Distribution

For $j = 1$ to $n_{sim}$, select the $j$'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$. Generate a draw $s_T^{(j)}$ from this distribution.

2. A draw from $S_{T+1:T+H}|(s_T, \theta, Y_{1:T})$ is obtained by generating a sequence of innovations $\epsilon_{T+1:T+H}^{(j)}$. Then, starting from $s_T^{(j)}$, iterate the state transition equation with $\theta$ replaced by the draw $\theta^{(j)}$ forward to obtain a sequence $S_{T+1:T+H}^{(j)}$:

$$s_t^{(j)} = \Phi_1(\theta^{(j)})s_{t-1}^{(j)} + \Phi_\epsilon(\theta^{(j)})\epsilon_t^{(j)}, \quad t = T + 1, \ldots, T + H.$$  

3. Use the measurement equation to obtain $Y_{T+1:T+H}^{(j)}$:

$$y_t^{(j)} = \Psi_0(\theta^{(j)}) + \Psi_1(\theta^{(j)})t + \Psi_2(\theta^{(j)})s_t^{(j)}, \quad t = T + 1, \ldots, T + H.$$  

$\square$
Algorithm generates \( n_{\text{sim}} \) trajectories \( Y_{T+1:T+H}^{(j)} \) from the predictive distribution.

Possible modification: execute Steps 2 and 3 \( m \) times for each \( j \). Leads to \( m \cdot n_{\text{sim}} \) draws from the predictive distribution.

A point forecast \( \hat{y}_{T+h} \) of \( y_{T+h} \): (i) specify loss function \( L(y_{T+h}, \hat{y}_{T+h}) \); determine prediction that minimizes posterior expected loss:

\[
\hat{y}_{T+h}|T = \arg\min_{\delta \in \mathbb{R}^n} \int_{Y_{T+h}} L(y_{T+h}, \delta) p(y_{T+h}|Y_{1:T}) dy_{T+h}.
\]

Under the quadratic forecast error loss function

\[
L(y, \delta) = \text{tr}[W(y - \delta)'(y - \delta)],
\]

the optimal predictor is the posterior mean

\[
\hat{y}_{T+h}|T = \int_{y_{T+h}} y_{T+h} p(y_{T+h}|Y_{1:T}) dy_{T+h} \approx \frac{1}{n_{\text{sim}}} \sum_{j=1}^{n_{\text{sim}}} y_{T+h}^{(j)},
\]

which can be approximated by a Monte Carlo average.
Smets and Wouters (2007) model, modified to absorb real time information as specified below.

- Observables: output, consumption, investment, real wage growth, hours worked, inflation, Federal Funds rate.

- Real time data, following Edge and Gürkaynak (2010):
  - Recursive out-of-sample forecasting.
  - All estimation samples start in 1964.
  - Blue Chip samples: forecast origins aligned with Blue Chip survey publication dates. We consider January, April, July, and October, ending April 2011
  - Greenbook samples: forecast origins aligned with Greenbook dates. We consider March, June, September, and December, ending Sept. 2004
DSGE Model versus Blue Chip (1992-2011)

- $h = 1$ is current quarter nowcast.
- Growth rates, inflation rates, interest rates are QoQ %
- $h = 1$ is current quarter nowcast.
- Growth rates, inflation rates, interest rates are QoQ %
Figure depicts RMSE ratios: DSGE (reported in various papers) / AR(2) (authors calculation).
In DSGE models the evolution of the endogenous variables is due to exogenous shocks.

We can study the contribution of each shock to historical fluctuations as well as forecasts.

This produces a “story” that goes along with the forecast.
For \( j = 1 \) to \( n_{\text{sim}} \), select the \( j \)'th draw from the posterior \( p(\theta|Y_{1:T}) \):

1. Use the simulation smoother to generate a draw \( S^{(j)}_{0:T} \) from the distribution \( p(S^{(j)}_{0:T} | Y_{1:T}, \theta^{(j)}) \).

2. For each structural shock \( i = 1, \ldots, m \) (which is an element of the vector \( s_t \)):
   1. Compute the sequence of shock innovations \( \epsilon^{(j)}_{i,1:T} \), for instance, by solving \( s^{(j)}_{i,t} = \rho_i^{(j)} s^{(j)}_{i,t-1} + \sigma_i^{(j)} \epsilon^{(j)}_{i,t} \) for \( \epsilon^{(j)}_{i,t} \).
   2. Define a new sequence of innovations \( e_{1:T} \) (\( e_t \) is of the same dimension as \( \epsilon_t \)) by setting the \( i \)'th element \( e_{i,t} = \epsilon^{(j)}_{i,t} \) for \( t = 1, \ldots, T \) and \( e_{i,t} \sim N(0, \sigma_i^2) \) for \( t = T + 1, \ldots, T + H \). All other elements of \( e_t \), \( t = 1, \ldots, T + H \), are set equal to zero.
   3. Starting from \( \tilde{s}_0 = s^{(j)}_0 \), iterate the state transition equation forward using the innovations \( e_{1:T+H} \) to obtain the sequence \( \tilde{S}^{(j)}_{1:T+H} \).
   4. Use the measurement equation to compute \( \tilde{Y}^{(j)}_{1:T+H} \) based on \( \tilde{S}^{(j)}_{1:T+H} \).
Notes: The shock decompositions for output growth (top) and inflation (bottom) are computed using model SW\pi estimated on the last available vintage (May 2011). The black and red lines represent the data and the forecasts, both in deviation from the steady state. The colored bars represent the contribution of each shock to the evolution of the variables.
The shock decompositions for output growth (top) and inflation (bottom) are computed using model SW\(\pi\) estimated on the last available vintage (May 2011). The black and red lines represent the data and the forecasts, both in deviation from the steady state. The colored bars represent the contribution of each shock to the evolution of the variables.
Quality of forecasts is constrained by quality of model and quality of observations. Let’s focus on the latter.

Nowcasts: Professional forecasts beat DSGE forecasts in the short-run with respect to variables for which real time information is available.

Potential solution? Augment the set of observables.

We consider the following external information...

- Inflation Expectations
- External Nowcasts
- Interest Rate Expectations

→ variables that may convey information about the state of the economy not contained in “usual” data set.
Incorporating Inflation Expectations

High-inflation rates from 1970-1982 lead to fairly large estimate of steady-state inflation rate (4 % annualized). ⇒ Upward bias in current inflation forecasts.

Remedy: anchor target inflation rate using long-run inflation expectations. Augment measurement equations:

\[
\pi_t^{O,40} = \pi^* + \mathbb{E}_t \left[ \frac{1}{40} \sum_{k=1}^{40} \pi_{t+k} \right].
\]

Modify policy rule:

\[
R_t = \rho_R R_{t-1} + (1 - \rho_R) \psi_1 (\pi_t - \pi^*) + \ldots
\]

Time-varying inflation target evolves according to:

\[
\pi^*_t = \rho_{\pi^*} \pi^*_{t-1} + \sigma_{\pi^*} \epsilon_{\pi^*,t}.
\]
With and Without Inflation Expectations

- Smets-Wouters: red
- Smets-Wouters with loose prior on $\pi^*$: salmon
- DSGE model with inflation expectations: magenta
“Standard” DSGE model forecasts ignore information from current quarter.

**Approach 1 (News):** true $Y_{T+1} = \text{external info } Z_T + \text{noise}$

**Approach 2 (Noise):** $\text{external info } Z_T = \text{true } Y_{T+1} + \text{noise}$

Approaches are the same for hard conditioning: $\text{noise} = 0$.

Under both approaches the forecaster essentially obtains information about the shocks $\epsilon_{T+1}$ as well as the state $s_T$ at forecast origin.

The subsequent results are generated under **Approach 2**, using nowcasts for output growth, inflation, and interest rates.
For $j = 1$ to $n_{sim}$, select the $j$’th draw from the posterior $p(\theta | Y_1:T)$:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T | \theta^{(j)}, Y_1:T)$.
2. In period $T + 1$ use

$$z_{T+1} = y_{1,T+1} - \eta_{T+1}, \quad y_{1,T+1} \perp \eta_{T+1}$$

as measurement equation for the nowcast $z_{T+1}$ assuming $\eta_{T+1} \sim N(0, \sigma_\eta^2)$.
3. Use the Kalman filter updating to compute $p(s_{T+1} | \theta^{(j)}, Y_1:T, z_{T+1})$ and generate a draw $s_{T+1}^{(j)}$ from this distribution.
4. Draw a sequence of innovations $\epsilon_{T+2:T+H}^{(j)}$ and, starting from $s_{T+1}^{(j)}$, iterate the state transition equation forward to obtain the sequence $S_{T+2:T+H}^{(j)}$.
5. Use the measurement equation to compute $Y_{T+1:T+H}^{(j)}$ based on $S_{T+1:T+H}^{(j)}$. \(\square\)
For \( j = 1 \) to \( n_{\text{sim}} \), select the \( j \)'th draw from the posterior \( p(\theta|Y_{1:T}) \):

1. Use the Kalman filter to compute mean and variance of the distribution \( p(s_T|\theta^{(j)}, Y_{1:T}) \).

2. Generate a draw \( \tilde{y}^{(j)}_{1,T+1} \) from the distribution \( p(\tilde{y}^{(j)}_{1,T+1}|Y_{1:T}, z_{T+1}) \) using

\[
y_{1,T+1} = z_{T+1} + \eta_{T+1}, \quad z_{T+1} \independent \eta_{T+1}, \quad \eta_{T+1} \sim N(0, \sigma^2_{\eta}).
\]

3. Treating \( \tilde{y}^{(j)}_{1,T+1} \) as observation for \( y_{1,T+1} \) use the Kalman filter updating step to compute \( p(s_{T+1}|\theta^{(j)}, Y_{1:T}, \tilde{y}^{(j)}_{1,T+1}) \) and generate a draw \( s^{(j)}_{T+1} \) from this distribution.

4. Draw a sequence of innovations \( \epsilon^{(j)}_{T+2:T+H} \) and, starting from \( s^{(j)}_{T+1} \), iterate the state transition equation forward to obtain the sequence \( S^{(j)}_{T+2:T+H} \).

5. Use the measurement equation to obtain \( Y^{(j)}_{T+1:T+H} \) based on \( S^{(j)}_{T+2:T+H} \).
<table>
<thead>
<tr>
<th>Forecast Origin</th>
<th>End of Est. Sample $T$</th>
<th>External Nowcast $T+1$</th>
<th>Forecast $h=1$</th>
<th>Forecast $h=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apr 92</td>
<td>91:Q4</td>
<td>92:Q1 from Apr 92 BC</td>
<td>92:Q1</td>
<td>92:Q2</td>
</tr>
<tr>
<td>Jul 92</td>
<td>92:Q1</td>
<td>92:Q2 from Jul 92 BC</td>
<td>92:Q2</td>
<td>92:Q3</td>
</tr>
<tr>
<td>Jan 93</td>
<td>92:Q3</td>
<td>92:Q4 from Jan 93 BC</td>
<td>92:Q4</td>
<td>93:Q1</td>
</tr>
</tbody>
</table>
With and Without Blue Chip Nowcasts

- DSGE model with inflation expectations: magenta
- DSGE model with inflation expectations and nowcasts: salmon
With and Without Blue Chip Nowcasts

- DSGE model with inflation expectations: magenta
- DSGE model with inflation expectations and nowcasts: salmon
We are utilizing multi-step interest-rate forecasts from Blue Chip.

BC interest-rate forecasts are treated as observations of agents’ expectations in the model.

Particularly useful for forecasting when interest rates are near ZLB.

SW model allows for a serially correlated monetary policy disturbances:

\[ r_t^m = \rho r_{t-1}^m + \sigma r^m \epsilon_t^m. \]

Augment \( r_t^m \) by anticipated policy shocks that capture future expected deviations from the systematic part of the monetary policy rule:

\[ r_t^m = \rho r_{t-1}^m + \sigma r^m \epsilon_t^m + \sum_{k=1}^{K} \sigma r_m,k \epsilon_{k,t-k}^m. \]

Policy shocks \( \epsilon_{k,t-k}^m, k = 1, \ldots, K \), are known to agents at time \( t - k \), but affect the policy rule with a \( k \) period delay in period \( t \).
| Forecast Origin | End of Est. Sample $T$ | External Nowcast $T + 1$ | Interest Rate Exp $R^e_{T+2|T+1}, \ldots, R^e_{T+5|T+1}$ | Forecast $h = 1$ |
|----------------|-----------------------|--------------------------|---------------------------------|----------------|
| Apr 92         | 91:Q4                 | 92:Q1 from Apr 92 BC     | 92:Q2 - 93:Q1                   | 92:Q1         |
| Jul 92         | 92:Q1                 | 92:Q2 from Jul 92 BC     | 92:Q3 - 93:Q2                   | 92:Q2         |
| Jan 93         | 92:Q3                 | 92:Q4 from Jan 93 BC     | 93:Q1 - 93:Q4                   | 92:Q4         |
DSGE model with inflation expectations and nowcasts: salmon

DSGE model with inflation expectations and nowcasts and interest rate expectations: turquoise
With and Without Interest Rate Expectations

DSGE model with inflation expectations and nowcasts: salmon
DSGE model with inflation expectations and nowcasts and interest rate expectations: turquoise
Main Findings Re Usefulness of External Information

Positive:

- A time-varying inflation target combined with long-run inflation expectations worked well in a number of models.
- Using external nowcasts improves DSGE model forecasts in the short-run. For some series improvement carries over to medium-run.

Mixed:

- Interest-rate expectations in conjunction with anticipated policy results improve interest rate forecasts in particular near ZLB.
- Anticipated policy shocks tend to trigger large output and inflation movements which lead to forecast deterioration.
Central banks are often interest in forecasts conditional on an interest rate path. In DSGE model, we can control the interest rate path with

- unanticipated shocks;
- anticipated shocks.
The Effects of Monetary Policy Shocks

- Euler Equation
  \[ y_t = \mathbb{E}[y_{t+1}] - (R_t - \mathbb{E}[\pi_{t+1}]) \]

- Phillips curve
  \[ \pi_t = \beta \mathbb{E}[\pi_{t+1}] + \kappa y_t \]

- Monetary policy rule with unanticipated and anticipated monetary policy shocks:
  \[ R_t = \frac{1}{\beta} \pi_t + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R \]
Impulse Responses to Anticipated Shock

- **Output:**

\[
\frac{\partial y_{t+h}}{\partial \epsilon_{R,K,t}} = -\psi^{1+K-h}, \quad h = 0, \ldots, K
\]

where \( \psi = (1 + \kappa/\beta)^{-1} \).

- **Inflation:**

\[
\frac{\partial \pi_{t+h}}{\partial \epsilon_{R,K,t}} = -\kappa \psi \left( \psi^{K-h} + \beta^{K-h} + \psi^{K-h} \sum_{i=1}^{K-1-h} \left( \frac{\beta}{\psi} \right)^i \right), \quad h = 0, \ldots, K,
\]

where \( \beta/\psi = \beta + \kappa \).
The Effects of Policy Anticipation

**Unanticipated policy shocks**

**Interest Rates**

**Output Growth**

**Inflation**

**Six-periods ahead anticipated policy shocks**

**Interest Rates**

**Output Growth**

**Inflation**
The Effects of Policy Anticipation ... and Forecasts Conditional on an FFR Path

Interest Rates

Output Growth

Inflation