Is Monetary Policy Always Effective? Incomplete Interest Rate Pass-through in a DSGE Model*

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PRELIMINARY VERSION

Abstract

We estimate a regime-switching DSGE model with a banking sector to explain incomplete and asymmetric interest rate pass-through, especially in the presence of a binding zero lower bound (ZLB) constraint. The model is estimated using Bayesian techniques on US data between 1985 and 2016. The framework allows us to explain the time-varying interest rate spreads and pass-through observed in the data. We find that pass-through tends to be delayed in the short run, and incomplete in the long run. All these effects naturally impact the dynamics of the other macroeconomic variables in the model as well.

JEL-codes: C68, E52, F41

Keywords: Banking sector, incomplete or asymmetric interest rate pass-through, DSGE

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1 Introduction

Understanding the transmission mechanism is vitally important for gaining insight into how monetary policy affects the macro-economy. A key link in this chain is the translation of central bank policy rates into the market interest rates faced by borrowers and savers. Delayed and incomplete interest rate pass-through is a “bottleneck” that reduces the impact/effectiveness of monetary policy on the rest of the economy. The problem is of particular interest when the economy is operating in the vicinity of the zero lower bound (ZLB) and there are questions of how much and how fast interest rate cuts will be passed on. In this paper we investigate interest rate pass-through through the lens of a DSGE model with a banking sector and an occasionally binding ZLB constraint.

Interest rate pass-through has been studied in econometric time series models, see for instance de Bondt (2002) and Kok and Werner (2006) for single equation ECM/ARDL models, Frisancho-Mariscal and Howells (2010), Akosah (2015) for VECM models, Sander and Kleimeier (2004) for VAR model, and Zheng (2013), de Haan and Poghosyan (2007), and Apergis and Cooray (2015) for various non-linear econometric time series models. While these contributions are important, they abstract from critical issues that would affect the measure of pass through itself. Those issues pertain, for instance, to the endogeneity of the policy rate, which is usually assumed exogenous in the measure of interest rate pass through. That endogeneity naturally calls for the measurement of pass-through in a structural framework. This is why more than acknowledging the endogeneity of the policy rate, this paper proceeds to estimating pass-through in a Dynamic Stochastic General Equilibrium (DSGE) model.

Little work has been done to seriously address the issue of interest rate pass through in DSGE models. The typical route taken by DSGE modelers, like Beneš and Lees (2010) and Gerali et al. (2010), has been merely to match market interest rates by including various frictions in the interest rate setting process, but without investigating the implications of incomplete interest rate pass through for policy and for the dynamics of macroeconomic variables. The present study aims to fill that gap.

Our goal is to quantify incomplete pass through, try to better understand some of the factors that affect interest rate pass-through and investigate its implications for the effectiveness of monetary policy in normal times but also at the zero-lower bound. To this end we embed the banking structure introduced by Gerali et al. (2010) into a simple regime-switching DSGE model, which we estimate using Bayesian techniques on US data between 1985 and 2016. Our focus on structural DSGE models allows us to highlight the economic channels through which shocks affect the economy, which is important in assessing the transmission from interest rates to the economy. In particular, with such
a strategy we will be able to analyze the consequences of incomplete interest rate pass through for the economy and policy for a wide array of specific shocks.

The regime switching strategy embedded in the approach adds further benefits. The model we study allows for multiple steady states. In particular, we account for the zero lower bound on interest rates through a separate zero-lower bound monetary policy regime. Hence, in contrast to standard DSGE/multivariate models, in which asymmetry, time variation and non-linearities are killed by linearization, our modeling approach allows us to investigate (i) how policy rates affect market rates, especially in the vicinity of the lower bound; (ii) the impact of delayed and incomplete pass-through on the macro-economy and for policy, and finally, (iii) the cost of incomplete interest rate pass-through.

Having moved away from the problematic measure of rate pass-through used in simple econometric models, we need to redefine a measure of pass-through. A further contribution of this paper is to propose two new measures of pass-through in a multivariate system.

Putting all those elements together, we are able to explain the time-varying interest rate spreads and pass-through observed in the data. In particular, we find evidence that

- pass-through tends to be delayed in the short run and incomplete in the long run
- the magnitude of pass-through depends on the shocks that hit the economy: for some shocks pass-through is fast but for some others pass-through is slow
- retail banks tend to adjust their markups to absorb some of the shocks
- the behavior of pass-through in the loan rate is different from that of the deposit rate
- shocks create an asymmetric dynamics at the the zero-lower bound and incomplete pass-through exacerbates that asymmetry
- policy is less effective under incomplete pass through
- interest rate setting rigidities are costly compared to a world with complete pass-through.

Finally we cross-check the robustness of our findings by investigating some alternative ways of modeling the interest rate setting process by retail banks. In particular we look at using Taylor-contracts and Linex adjustment costs. We adapt the Taylor-contracting specification of Coenen et al. (2007) used to model the price setting process, to model the interest rate setting process by retail banks. Taylor-contracts mirror loan and term deposit contracts where bank customers are bound to a given interest rate for a set period of time. In a second model we include Linex adjustment costs in retail banks’ interest
rate setting problem. Linex adjustment costs have been used by Kim and Ruge-Murcia (2011) to model downward nominal wage rigidities and will provide us with an alternative method for capturing any asymmetries in the interest rate setting process.

The remainder of the paper is structured as follows. Section 2 describes a model with a banking sector, while Section 3 defines ways to measure interest rate pass-through. Section 4 discusses estimation and parameterization, while we present the main results in Section 5. We calculate the cost of incomplete pass-through in Section 6, and report results using some alternative specification of banking models in Section 7. Finally we conclude in Section 8.

2 A model with banking

In this section we develop a simple DSGE model with a banking sector. The need for a banking sector arises through a loan-in-advance constraint on intermediate goods producers. More specifically intermediated goods producers are required to finance a portion of their investment goods purchases through a one period loan. Our representation of the banking sector is simple, avoiding the introduction of multiple types of agents as required by the Bernanke et al. (1998) and Iacoviello (2005) frameworks. As a consequence this allows us to focus more attention on the complex mechanisms involved in interest rate setting in the banking sector and interest rate pass-through. However, it also means the model does not have a financial accelerator, which will likely affect interest rate pass-through. The setup of the rest of the model; households, firms and the government sector, is standard. For this reason we only focus on the banking sector and monetary policy in this section, and their relationship to regime switching. A full derivation of the model can be found in Appendix A.

2.1 An Overview of the States

We assume the model economy’s dynamics are conditional on four discrete states of nature. At any given time the model economy can be in one of two monetary policy states and one of two markup states. This is reflected by introducing separate Markov chains for the monetary policy and markup states. The monetary policy state determines whether policy is set according to a Taylor type rule which occurs in the normal state (N), or the economy is at the zero lower bound state (Z) where policy follows an exogenous process, so that \( s_{1,t} = N, Z \). The monetary policy state also affects the markups and markdowns charged by retail banks and the degree of rigidity they face when adjusting market interest rates. The markup state affects whether markups and markdowns on market interest rates are high (H) or low (L) and the degree of rigidity in adjusting market interest rates, when
the economy is away from the lower bound, so that \( s_{2,t} = H, L \). We introduce two regime-switching parameters, \( z(s_{1,t}) \) which is conditional on the monetary policy regime and \( m(s_{2,t}) \) which is conditional on the markup regime. We assume

\[
z(Z) = 1 \quad \text{and} \quad z(N) = 0,
\]

with the states \( Z \) and \( N \) are governed by the following Markov transition matrix

\[
Q_Z = \begin{bmatrix} 1 - p_{N,Z} & p_{N,Z} \\ p_{Z,N} & 1 - p_{Z,N} \end{bmatrix}.
\]

We assume the regime-specific markup parameter takes the values

\[
m(H) = 1 \quad \text{and} \quad m(L) = 0.
\]

The states \( H \) and \( L \) are governed by the Markov transition matrix

\[
Q_m = \begin{bmatrix} 1 - q_{H,L} & q_{H,L} \\ q_{L,H} & 1 - q_{L,H} \end{bmatrix}.
\]

2.2 The Banking Sector

Following Gerali et al. (2010) the banking sector is divided into two types of banks, wholesale banks and retail banks. Wholesale banks collect deposits from retail banks, and produce loans using deposits and bank equity, which they in turn supply to retail banks. The exact setup for this sector can be found in Appendix A. The retail banking sector is comprised of loan-making and deposit taking branches. As a means of representing retail loan and deposit rates as a markup and markdown, respectively, over policy rates, Gerali et al. (2010) treat intermediate loans and deposits issued by retail banks as differentiated. As a consequence of this assumption there is a continuum of loan-making and deposit-taking banks, normalized to unit mass, each producing a differentiated loan or deposit. We let \( z \) index retail banks.

The \( z \)th loan-making bank sets the interest rate on loans to maximize the sum of the expected present value of its profits, subject to a quadratic cost of changing interest rates. This can be represented by

\[
\Psi_{L,0}(z) = E_t \left\{ \sum_{\tau=0}^{\infty} \mathcal{M}_{0,\tau} \left( \frac{P_0}{P_t} \right) \left[ R_{L,t}(z) L_t(z) - \exp (\varepsilon_{L,t}) \mathbb{R}_{L,t} L_t(z) - \ldots - \frac{\phi_L(r_t)}{2} R_{L,t} L_t \left[ \frac{R_{L,t}(z)}{R_{L,t-10}(z)} - 1 \right]^2 \right] \right\},
\]

5
where $\mathcal{M}_{t,t+1}^*$ is the real stochastic discount factor, $P_t$ is the price level, $R_{L,t}(z)$ is the interest rate charged for loans issued by the $z$th bank, $L_t(z)$ is loans issued by the $z$th bank, $\varepsilon_{L,t}$ is a markup shock, $R_{L,t}$ is the aggregate interest rate on loans, $L_t$ is aggregate loans and $R_{L,t}$ the wholesale interest rate charged on loans. Note that the degree of rigidity $\phi_L(r_t)$ is a function of the regime. The $z$th loan-making bank chooses the interest rate on loans to maximize profits. Assuming a symmetric equilibrium leads to the following behavioral rule for the aggregate loan interest rate

$$
\left( \frac{v_L(r_t)}{v_L(r_t)} - 1 \right) \exp (\varepsilon_{L,t}) \frac{R_{L,t}}{R_{L,t} - 1} - 1 - \tilde{\phi}_L(r_t) \left[ \frac{R_{L,t}}{R_{L,t-1}} - 1 \right] + \ldots
\ldots + E_t \left\lbrace \tilde{\phi}_L(r_{t+1}) \mathcal{M}_{t,t+1}^* \left( \frac{1}{\tau_{t+1}} \right) \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \frac{L_{t+1}}{L_t} \left[ \frac{R_{L,t+1}}{R_{L,t}} - 1 \right] \right\rbrace = 0. \tag{6}
$$

This resembles a New Keynesian Phillips curve for the interest rate on loans where the marginal cost term is the interest rate charged on loans by the wholesale bank. The reduced form persistence parameter, $\tilde{\phi}_L(r_t) \equiv \frac{\phi_L(r_t)}{v_L(r_t) - 1}$, and elasticity of substitution between differentiated loans, $v_L(r_t)$, are functions of the regime. We make this relationship more explicit by assuming

$$
\tilde{\phi}_L(r_t) = z(s_{1,t}) \tilde{\phi}_{Z,L} + (1 - z(s_{1,t})) (m(s_{2,t}) \tilde{\phi}_{H,L} + (1 - m(s_{2,t})) \tilde{\phi}_{L,L}). \tag{7}
$$

The loan mark-up is determined according to

$$
\mu_L(r_t) = z(s_{1,t}) \mu_{Z,L} + (1 - z(s_{1,t})) (m(s_{2,t}) \mu_{H,L} + (1 - m(s_{2,t})) \mu_{L,L}), \tag{8}
$$

where the elasticity of substitution between differentiated loans is related to the markup through

$$
v_L(r_t) = \frac{\mu_L(r_t)}{\mu_L(r_t) - 1}. \tag{9}
$$

The $z$th deposit-taking bank sets interest rates to maximize its expected discounted future stream of profits, subject to a quadratic adjustment cost on changing interest rates so that

$$
\Psi_{D_0}(z) = E_t \left\lbrace \sum_{t=0}^{\infty} \mathcal{M}_{t,t+1}^* \left( \frac{P_0}{P_t} \right) \left[ \exp (\varepsilon_{D,t}) \mathcal{R}_{D,t} D_t(z) - R_{D,t}(z)D_t(z) - \ldots - \phi_D(r_t) \frac{R_{D,t}D_t(z)}{2} \right] \right\rbrace, \tag{10}
$$

where $R_{D,t}(z)$ is the deposit interest rate for loans issued by the $z$th bank, $D_t(z)$ is deposits issued by the $z$th bank, $\varepsilon_{D,t}$ is a markup shock, $R_{D,t}$ is the aggregate deposit interest rate, $D_t$ is aggregate deposits and $\mathcal{R}_{D,t}$ the wholesale interest rate charged on deposits, as was
the case for loan-making banks. $\phi_D(r_t)$ is a function of the regime. The $z$th deposit-taking bank chooses deposit interest rates to maximize their lifetime profits. Assuming a symmetric equilibrium leads to the following behavioral rule for aggregate deposit interest rates

$$1 - \left( \frac{\nu_D(r_t)}{\nu_D(r_t) - 1} \right) \exp(\varepsilon_{D,t}) \frac{R_{D,t}}{R_{D,t-1}} \tilde{\phi}_D(r_t) \left[ \frac{R_{D,t}}{R_{D,t-1}} - 1 \right] + \ldots$$

$$\ldots + E_t \left\{ \tilde{\phi}_D(r_{t+1}) \nu_{t+1}^* \left( \frac{1}{\pi_{t+1}} \right) \left( \frac{R_{D,t+1}}{R_{D,t}} \right)^2 \frac{D_{t+1}}{D_t} \left[ \frac{R_{D,t+1}}{R_{D,t}} - 1 \right] \right\} = 0. \quad (11)$$

Just as was the case for loan-making banks, the reduced form rigidity parameter, $\tilde{\phi}_D(r_t) \equiv \frac{\phi_D(r_t)}{\nu_D(r_t) - 1}$, and the elasticity of substitution between differentiated deposits, $\nu_D(r_t)$, are functions of the regime. Furthermore we assume that

$$\tilde{\phi}_D(r_t) = z(s_{1,t}) \tilde{\phi}_{Z,D} + (1 - z(s_{1,t})) (m(s_{2,t}) \tilde{\phi}_{H,D} + (1 - m(s_{2,t})) \tilde{\phi}_{L,D}), \quad (12)$$

and the markdown on deposits is determined by

$$\mu_D(r_t) = z(s_{1,t}) \mu_{Z,D} + (1 - z(s_{1,t})) (m(s_{2,t}) \mu_{H,D} + (1 - m(s_{2,t})) \mu_{L,D}), \quad (13)$$

where the markdown is related to the elasticity of substitution through

$$\nu_D(r_t) = \frac{\mu_D(r_t)}{\mu_D(r_t) - 1}. \quad (14)$$

### 2.3 Monetary Policy

The monetary authority sets interest rates according to

$$R_t = \max (R_{ZLB,t}, R_t^*), \quad (15)$$

where $R_t^*$ is the interest rate set during normal times, which is determined according to a Taylor-type rule

$$R_t^* = R_t^{\rho_R} \left( R^* \left( \frac{\pi_t}{\pi} \right)^{\kappa_\pi} \left( \hat{Y}_t \right)^{\kappa_Y} \right)^{1-\rho_R} \exp(\varepsilon_{R,t}), \quad (16)$$

where $\hat{Y}_t$ is the output gap. $R_{ZLB,t}$ is the interest rate set when the economy is at the zero lower bound, which we assume evolves according to the exogenous process

$$R_{ZLB,t} = K + \varepsilon_{ZLB,t}, \quad (17)$$
**K** is a parameter set equal to the effective lower bound and \( \varepsilon_{ZLB,t} \) is a small shock added to avoid a stochastic singularity. In order to model the lower bound constraint on interest rates using regime-switching, we replace (15) with

\[
R_t = z(s_{1,t})R_{ZLB,t} + (1 - z(s_{1,t}))R^*_t.
\]  

(18)

3 Measuring Pass Through

Measuring interest rate pass-through in single linear equation models is a trivial exercise. In such models the policy interest rate is assumed to be exogenous and long-run interest rate pass-through can be determined by inspecting the estimated coefficients of the model. In multivariate models, however, the task is more complicated, as both the policy interest rate and the market interest rate are usually assumed to be endogenous. A simple approach to measuring pass-through could involve shocking the system with a monetary policy shock and then calculating interest rate pass-through from the resulting impulse response function. While this is a useful exercise in itself, it does not reflect the data generating process as there are a multitude of shocks that can affect the variables in the system.\(^1\)

In this paper we propose two general methods of measuring interest rate pass-through in multivariate models. Our measures reflect the endogenous determination of both the policy and market interest rates and the variety of shocks that can affect the policy interest rate. The nature of multivariate models means that we do not assign a causal interpretation to our measures of pass-through, but instead treat pass-through as a correlation. We investigate our measures of pass-through using a DSGE model, but we note they can easily be applied to other multivariate models like VAR models for example, and used to measure exchange rate pass-through in multivariate models.

Our first method measures pass-through using the impulse responses to all the structural shocks from a multivariate model. As a consequence the degree of pass-through will depend on the shock hitting the economy.\(^2\) Similar methods have been suggested by Shambaugh (2008) and Rincón-Castro and Rodríguez-Niño (2016) to investigate exchange rate pass-through in multivariate models.

Our second method involves simulating artificial data from the multivariate model, then estimating univariate measures of pass-through on the simulated data. Using this approach we treat the model as a laboratory and test how different assumptions affect the

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1This is a point that has been made by Shambaugh (2008) and Rincón-Castro and Rodríguez-Niño (2016) in the context of measuring exchange rate pass-through.

2In non-linear models, the size and sign of the shock could have an impact on the degree of interest rate pass-through.
degree of pass-through. Moreover, we can calculate an aggregate measure of pass-through using this method, something we cannot easily obtain using our first measure.

3.1 An IRF Based Measure

Exchange rate pass-through has been investigated by Shambaugh (2008) and Rincón-Castro and Rodríguez-Niño (2016) in multivariate models. They recognize that the correlation between the exchange rate and the price of imported goods is a function of not only the parameters of the model, but also the types of shocks hitting the economy. Moreover it is not useful to treat all movements in the exchange rate as exogenous, especially in a multivariate setting where the exchange rate can respond to a number of different shocks and variables. Instead they look at exchange rate pass-through using the impulse responses for a number of different structural shocks. We adopt a similar approach to Shambaugh (2008) and Rincón-Castro and Rodríguez-Niño (2016) when measuring interest rate pass-through, and evaluate it for a set of structural shocks using the impulse responses from the model. Our measure of pass-through \( \tau \) periods after the shock is given by

\[
PT_{M,\tau}(\pm \varepsilon_{j,t}) = \frac{\sum_{t=0}^{\tau} |\hat{R}_{M,t}(\pm \varepsilon_{j,0})|}{\sum_{t=0}^{\tau} |\hat{R}_{t}(\pm \varepsilon_{j,0})|}
\]

where \( M = D, L \)

where \( \hat{R}_{M,t}(\pm \varepsilon_{j,t}) \) is the impulse response for the market interest rate \( t \) periods after the \( j \)th shock has hit the economy. \( \hat{R}_{t}(\pm \varepsilon_{j,t}) \) is the impulse response for the policy interest rate \( t \) periods after the \( j \)th shock has hit the economy. Our IRF-based measure of interest pass-through uses the absolute value of the impulse response functions as secondary cycles in the impulse response function could switch sign. We also allow for differences in pass-through depending on the sign of the shock. This is important if the model exhibits asymmetric impulse response functions.

3.2 Reduced Form Measures

Our second approach involves simulating artificial data from the DSGE model, treating the policy interest rate as exogenous, and then estimating an autoregressive distributed lag (ARDL) model on the simulated data. The ARDL model is chosen because the data are stationary and it is a reasonably common model for estimating interest rate pass-through in the literature. Our ARDL models of the market rate are estimated on 10 lags
of the market rate, the contemporaneous policy rate, and 10 lags of the policy rate. The ARDL model we estimate takes the general form

$$\Delta R_{M,t} (M, \theta) = \sum_{i=1}^{p} \alpha_{M,i} \Delta R_{M,t-i} (M, \theta) + \sum_{j=0}^{p} \alpha_{R,j} \Delta R_{t-i} (M, \theta) + u_t$$

where $M$ refers to the data being generated by a structural model, and $\theta$ represents the parameter vector used to generate the data in the structural model.

Our reduced form single equation measure of pass-through is useful because we can calculate an overall measure of interest rate pass-through. We can also produce counterfactual measure of interest pass-through by changing the parameterization of the DSGE model, to better understand the factors that affect interest rate pass-through.

4 Estimation/Parametrisation

We estimate the model on US data for per capita GDP growth ($\Delta \log Y_t$), per capita consumption growth ($\Delta \log C_t$), per capital investment growth ($\Delta \log I_t$), price inflation ($\pi_t$), wage inflation ($\pi_{W,t}$), the fed funds rate ($R_t$), the loan interest rate ($R_{L,t}$) and the deposit interest rate ($R_{D,t}$). The model is stochastically detrended so we do not need to demean or detrend the data. The estimation sample runs from 1985Q1 to 2016Q3 and includes the great moderation, the financial crisis and the ZLB period. We choose this period because the loan rate does not go back much further although the regime-switching framework can handle longer samples with the possible addition of extra regimes.

We estimate the model using Bayesian methods. More specifically we use the Metropolis Hastings algorithm with 200,000 draws. All computations carried out in RISE.

Note: because the model is non-linear and potentially asymmetric we could estimate nonlinear (threshold) ARDL models, but we do not do so here, to keep things simple.
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<tr>
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<td>$\sigma_N$</td>
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**Table 2.** Parameters description

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<tr>
<td>$\kappa$</td>
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<td>Elasticity of Substitution Between Differentiated Intermediate Goods</td>
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<td>Share of Bank Profits Paid as Dividends</td>
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<td>$\delta_b$</td>
<td>Depreciation Rate of Bank Capital</td>
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<tr>
<td>$\psi$</td>
<td>Fraction of Investment Goods Bought Using Loans</td>
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<td>$\delta$</td>
<td>Depreciation Rate of Physical Capital</td>
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<td>Weight on Habit</td>
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<tr>
<td>$\eta$</td>
<td>Inverse of the Frisch Elasticity of Labor Supply</td>
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<td>Weight on Rotemberg Adjustment Costs for Changing Prices</td>
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<td>$\phi_W$</td>
<td>Weight on Rotemberg Adjustment Costs for Changing Wages</td>
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<td>Degree of Wage Indexation</td>
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<td>$\mu_{H,D}$</td>
<td>Markdown of Deposit Interest Rates in the High State</td>
</tr>
<tr>
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<td>Markup on Loan Interest Rates in the ZLB State</td>
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<td>Markup on Loan Interest Rates in the High State</td>
</tr>
<tr>
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<td>Markup on Loan Interest Rates in the Low State</td>
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<td>$\pi$</td>
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<td>$p_{L,N}$</td>
<td>Transition Probability From ZLB to Normal State</td>
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<td>$g_{Z_t}$</td>
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<td>Persistence Consumption Preference Shocks</td>
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<td>$\rho_G$</td>
<td>Persistence Government Spending Shock</td>
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<td>$\sigma_{AZ}$</td>
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<td>Std. Monetary Policy Shock</td>
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<td>Std. Loan Markup Shock</td>
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<td>$\sigma_D$</td>
<td>Std. Deposit Markup Shock</td>
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<tr>
<td>$\sigma_N$</td>
<td>Std. Labor Shock</td>
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Figure 1. State Probabilities

Probability of High Markup

Probability of ZLB
5 Results/Policy analysis

We now present in Section 5.1 results using our two methods of measuring interest rate pass-through in multivariate models; examining impulse responses to all the structural shocks from a multivariate model, and estimating ARDL models on simulated data. We examine overall pass-through and also the corresponding shocks specific measures of interest rate pass-through. Finally, we examine to what extent interest rate pass-through also depends on the key monetary policy parameters. Section 5.2 then perform some simulations, where we examine interest rate pass through at the zero lower bound on the policy rate.

5.1 Incomplete and nonlinear pass through

We start out, in Figure 2, by analyzing the impulse responses to a contractionary monetary policy shock. The figure compares the responses of the estimated model to the responses from a parameterization of the model with lower pass-through. It can be seen that for the same size of the monetary policy shock, the response of the variables is smaller in the lower-pass-through model than in the estimated model. This also implies that in the lower-pass-through scenario, policy would have to do more in order to achieve the type of adjustment implied by the estimated model. Hence, policy is less effective under incomplete or low interest rate pass-through.

Figure 3 plots the overall pass-through for the deposit rate (left frame) and the loan rate (right frame) using the ARDL measure, discussed in section 3, alongside their 95% probability bands. In this exercise, the simulations used to estimate ARDL models are done using all the shocks in the DSGE model.

We note that for both rates, pass-through is incomplete both in the short term and the long term. Furthermore, in the short term, pass through for the loan rate is smaller than for the deposit rate, while in the long run, the opposite holds i.e. in the long run pass-through is smaller for deposit rate than for loan rate.
In Figures 4 and 5 we graph the corresponding shocks specific measures of interest rate pass-through for the deposit rate and the loan rate respectively. In both figures, we plot in the left frame the pass-through from each shock in turn assuming all the other shocks are zero. In the right frame we do the opposite exercise. That is we turn off each shock in turn letting all the others be active.
Starting with the left frame in Figure 4, we see that for all shocks displayed, pass-through is incomplete.\(^4\) Of these, government spending and loan rate markup shocks have the highest pass-through, followed by labor preference, monetary policy and neutral technology, that have roughly the same pass-through, and then investment specific technology shocks. Finally, the lowest pass-through is observed for the cost push shocks.

The right frame, which analyzes the effect of turning off one shock at the time, confirm the picture from above. All shocks contribute to reduce pass-through. Still, interest pass-through is lower in the absence of investment specific shocks, and marginally higher in the absence of cost-push shocks.

\(^4\)Note that we were unable to estimate an ARDL model with the consumption shock because the correlation between the policy rate and market rate was too high (approx 0.99), implying complete or near complete pass-through on impact for that shock.
Figure 4. Deposit rate pass-through

Figure 5. Loan rate pass-through
Turning to the loan rate, the left frame in Figure 5 suggests that government spending and technology shocks have the highest pass-through, followed by deposit rate, neutral technology, labor preferences and monetary policy shocks. The lowest pass-through is observed for the cost push shocks, as was also the case for the deposit rate. Finally, the right frame shows that interest pass-through is lower in the absence of investment specific shocks, and higher in the absence of cost-specific shocks.

Taken together, Figures 4 and 5 suggest that the degree of interest rate pass-through crucially depends on the shock. The figures also suggest that the pass-through behavior of the loan rate is different from that of the deposit rate.

The pass through measures computed using the ARDL technique where one shock is active at a time turn out to be remarkably similar to those generated using our other pass-through measure based on a more direct computation of the impulse responses. This
can be seen in Figure 6 for the deposit rate pass-through and in Figure 7 for the loan rate pass-through.

We now turn to analyzing how pass-through is affected by key monetary policy parameters. To that end, we measure the long run interest rate pass-through on a grid over the reaction of the policy rate to the output gap ($\kappa_y$), the reaction to inflation ($\kappa_\pi$) and the interest rate smoothing ($\rho_R$). We plot the results for the loan rate and for the deposit rate in Figures 8 and 9 respectively. The message that can be read from the two figures is that everything else equal, the degree of interest rate pass-through is a highly nonlinear function of the policy parameters: changing the value of the interest rate smoothing dramatically changes the profile of the interest rate pass-through with respect to the other policy parameters. Here too, it is seen that the pass-through behavior for the deposit rate is different from that of the loan rate.
Figure 8. Loan Rate Pass-Through Varying Monetary Policy Parameters

\[
\text{Inflation} \quad \rho = 0.222222 \\
\text{Inflation} \quad \rho = 0.444444 \\
\text{Inflation} \quad \rho = 0.666667 \\
\text{Inflation} \quad \rho = 0.888889
\]
Figure 9. Deposit Rate Pass-Through Varying Monetary Policy Parameters

\[ \rho = 0.222222 \]

\[ \rho = 0.444444 \]

\[ \rho = 0.666667 \]

\[ \rho = 0.888889 \]
Another way to look at the relationship between policy parameters and the degree of pass through is to look at each parameter separately. This is what Figure 10 does. We can now more clearly see the important role of the smoothing parameter. For the deposit rate, pass through tends to increase with the degree of interest rate smoothing. The behavior is quite different for the pass through to the loan rate. Originally the degree of pass through increases with the smoothing parameter. But at some point, interest rate pass through starts decreasing just to change course again as the smoothing parameter approaches unity.
5.2 Dynamics at the zero-lower bound

So far we have looked at interest rate pass through without any reference to the lower bound on the policy rate. With the ZLB one should expect the dynamics of the system to change. But before delving into various counterfactual analysis it is important to see how well our model represents the data over the ZLB period. Figure 11 presents in its top panels the simulated series on interest rates (left panel) and on spreads (right panel). The figure also plots in its lower panels the actual counterparts of the those variables zooming in on the period in which the ZLB was active. As can be seen the simulated data compare well with the actual series both in terms of patterns and in terms of magnitudes.

To gain more insight into the workings of the model, we compare the dynamics induced by one sequence of adverse cost-push shocks and the exact same sequence of shocks but with opposite signs. In Figure 12, we see that with the ZLB the dynamics of the system becomes asymmetric. In particular, the adjustment in consumption, investment, output and inflation is smaller in the ZLB scenario than in the opposite scenario where the interest rate has to increase. The combination of this result with the insights from Figure 2 suggest that with lower pass-through the responses of the different variables would be even smaller. Conversely, complete pass through would assuage the effects induced by the
When the US hit the ZLB, loan rates held up, increasing margins. Deposit rates on the other hand were forced to zero, so that deposit margins were compressed. Two natural questions come to mind: first, what would have happened to the economy if the deposit margins had not compressed? second, what would have happened if the loan rate margins had compressed? To shed light on those questions we conduct two counterfactual exercises. In the first exercise we compare the dynamics implied by the estimated model to a parameterization of the model in which the deposit rate margins do not compress. In this latter/counterfactual scenario, as suggested by Figure 13, investment and output would have been smaller.
In the second counterfactual exercise, again we compare the dynamics implied by the estimated model but this time against a parameterization of the model in which loan rate margins are compressed at the ZLB. This second exercise, illustrated in Figure 14, suggests that in that case investments and output would have been higher.
Figure 14. Dynamics at the ZLB. Counterfactual: Compressed Loan Rate Margins at ZLB
6 The cost of incomplete pass-through

What does the economy lose when pass-through is incomplete? To analyse this we investigate the economic costs of incomplete interest rate pass-through by calculating the loss, under various parameterizations of the model.

\[ L_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \gamma_Y \hat{Y}_t^2 + \gamma_R (\Delta \hat{R}_t)^2 \right] \right\} \]

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<tr>
<th>Simulation</th>
<th>Loss</th>
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<td>Base line</td>
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</tr>
<tr>
<td>No Rigidities</td>
<td>621.3524</td>
</tr>
<tr>
<td>No Markups/downs</td>
<td>634.6294</td>
</tr>
<tr>
<td>No Rigidities/Markups/downs</td>
<td>622.7232</td>
</tr>
</tbody>
</table>

- Baseline = estimated model
- No Rigidities = interest rate markups, but no Rotemberg adjustment costs
- No Markups/downs = the model without markups but with rigidities
- No Rigidities/Markups/downs = the model without markups and without rigidities

7 Alternative representation of the banking sector

7.1 Taylor Contracts

In this section we modify the baseline model by replacing the quadratic adjustment costs in the retail banks’ profit functions with a Taylor contracting setup. More precisely, retail banks issue loans and deposits, where the retail interest rate is fixed for \( n \) quarters. The general setup is as before, the banking sector is split into wholesale and retail banks, where the perfectly competitive wholesale bank takes deposits from retail banks while at the same time they issue loans to retail banks. As before, the economy is populated by a continuum of retail banks, normalized to unit mass, that produce differentiated loans and deposits.

The \( z \)th loan-taking bank chooses interest rates on their particular variety of loans to maximize the expected net present value of future profits on an \( n \) period contract. We assume the following form for the loan-taking bank’s profit function

\[ \Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \left[ R_{L,0}(z) L_{0,t}(z) - \mathbb{R}_{L,t} L_{0,t}(z) \right] \right\} , \quad (20) \]
where $R_{L,0}(z)$ is the loan interest rate set by the $z$th loan-taking bank on an $n$ period contract at the start of the loan contract, $L_{0,t}(z)$ is the quantity of loans demanded at time $t$ for a contract signed in period 0 and

$$a_{0,t} = \prod_{j=0}^{t} a_{j,t}(r_t), \text{ where } a_{0,t}(r_t) = 1,$$

allows for the possibility that a contract could be broken before expiration date. This feature was introduced into the Taylor contracting problem by Coenen et al. (2007) and allows for both smoother dynamics from the model and a more realistic setup where some contracts can be broken. Differentiating 20 with respect to $R_{L,0}(z)$ gives the following rule for setting loan interest rates

$$R_{L,0}(z) = \left( \frac{\nu_L}{\nu_L - 1} \right) \frac{E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} M_{0,t}^* \left( \frac{P_t}{P} \right) R_{L,t} R_{L,t}^e L_t \right\}}{E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} M_{0,t}^* \left( \frac{P_t}{P} \right) R_{L,t}^e L_t \right\}}. \quad (21)$$

CES aggregation and perfectly competitive cost minimization by loan packers implies the following index for aggregate loan interest rates

$$R_{L,t} = \left( \frac{1}{\sum_{k=0}^{n-1} a_{t-k,t} (R_{L,t-k}(z))^{1-\nu_L}} \right)^\frac{1}{1-\nu_L}. \quad (22)$$

The $z$th deposit-taking bank chooses deposit interest rates for their variety of deposits to maximize the expected net present value of their profits on an $n$ period deposit contract. This can be represented by the following profit function

$$\Psi_{D,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} \theta_{0,t} M_{0,t}^* \left( \frac{P_0}{P_t} \right) [R_{D,t} D_{0,t}(z) - R_{D,0}(z) D_{0,t}(z)] \right\}. \quad (23)$$

where $R_{D,0}(z)$ is the interest rate set in period 0 by the $z$th deposit taking bank on an $n$ period contract, and $D_{0,t}$ is the corresponding quantity of deposits demanded in period $t$ for a deposit contract entered in period 0. We also have

$$\theta_{0,t} = \prod_{j=0}^{t} b_{j,t}(r_t), \text{ where } b_{0,t}(r_t) = 1,$$

which mirrors the setup used by loan taking banks, which allows some contracts to be broken prematurely. From the $z$th deposit taking bank’s first order conditions we get the
following rule for setting deposit interest rates

\[ R_{D,0}(z) = \left( \frac{v_D}{v_D - 1} \right) E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mu_{0,t}^* \left( \frac{P_t}{P_t^*} \right) R_{D,t}^D D_t \right\} . \]  

(24)

The aggregate deposit interest rate is the CES function of the different contract cohort interest rates

\[ R_{D,t} = \left( \frac{1}{\sum_{k=0}^{n-1} b_{t-k,t} (R_{D,t-k}(z))^{1-v_D}} \right)^{\frac{1}{1-v_D}} . \]  

(25)

The reduced form persistence parameters in the banks’ interest rate setting rules are determined by the markup/pass-through, and monetary policy regimes, according to the following equations

\[ a_{j,t}(r_t) = z(r_t) a_{j,ZLB} + (1 - z(r_t)) (m(r_t) a_{j,N} + (1 - m(r_t)) a_{j,L}) , \]  

(26)

\[ b_{j,t}(r_t) = z(r_t) b_{j,ZLB} + (1 - z(r_t)) (m(r_t) b_{j,N} + (1 - m(r_t)) b_{j,L}) , \]  

(27)

where \( z(ZLB) = 1, z(N) = 0, m(H) = 1 \) and \( m(L) = 0. \)

7.2 Linex Adjustment Costs

In this section we modify the baseline model to include linex adjustment costs. We begin with the same setup used in the baseline model, however the profit function for the \( z \)th loan-taking bank becomes

\[ \Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mu_{0,t}^* \left( \frac{P_0}{P_t} \right) \left[ R_{L,t}(z) L_t(z) - R_{L,t} L_t(z) - \ldots \right. \right. \]

\[ \left. \left. \ldots - \phi_{L2} R_{L,t} L_t \left[ \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right]^2 - \ldots \right. \right. \]

\[ \ldots - \phi_{L2} \psi R_{L,t} L_t \left\{ \exp \left( -\psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) \right) + \ldots \right\} + \ldots \]

\[ \ldots + \psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) - 1 \right\} . \]  

(28)

While the linex adjustment cost nests the quadratic adjustment cost in the limit, as \( \psi \to 0 \), this can also cause numerical instabilities, so we include a quadratic adjustment cost term in addition, which allows for numerical stability in the case that we want quadratic adjustment costs. A similar strategy is adopted by Fahr and Smets (2010).

The \( z \)th loan taking bank chooses interest rates to maximize their expected profits. In a symmetric equilibrium we obtain the following behavioral relationship for setting loan
interest rates

\[
\left( \frac{v_L}{u_L - 1} \right) \frac{R_{L,t}}{R_{L,t-1}} - 1 - \left( \frac{\phi_L}{u_L - 1} \right) \frac{R_{L,t}}{R_{L,t-1}} \left[ \frac{R_{L,t}}{R_{L,t-1}} - 1 \right] + \ldots \\
\ldots + \left( \frac{1}{u_L - 1} \right) \frac{\phi_{L,2}}{\psi} \left( \frac{R_{L,t}}{R_{L,t-1}} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t}}{R_{L,t-1}} - 1 \right) \right) - 1 \right] + \ldots \\
\ldots + E_t \left\{ \left( \frac{1}{u_L - 1} \right) \frac{M_{t+1}}{\psi_{t+1}} \left( \frac{R_{L,t+1}}{R_{L,t}} \right) \frac{D_{t+1}}{D_t} \times \\
\ldots - \left( \frac{\phi_{L,2}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t+1}}{R_{L,t}} - 1 \right) \right) - 1 \right] \right\} = 0, \tag{29}
\]

The setup for deposit taking banks mirrors that of loan taking banks. Profits for the \( z \)th deposit taking bank, subject to both quadratic and linear adjustment costs, are given by

\[
\Psi_{D,t}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \right\} \\
\left[ \left[ \frac{R_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots}{2 R_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots} \right] - \ldots \right] \\
\ldots - \frac{\phi_D}{\psi} \frac{R_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots}{2 R_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots} \left[ \exp \left( -\psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right) - 1 \right] + \ldots \\
\ldots + E_t \left\{ \left( \frac{1}{u_D - 1} \right) \frac{M_{t+1}}{\psi_{t+1}} \left( \frac{R_{D,t+1}}{R_{D,t}} \right) \frac{D_{t+1}}{D_t} \times \\
\ldots - \left( \frac{\phi_{D,2}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{D,t+1}}{R_{D,t}} - 1 \right) \right) - 1 \right] \right\} = 0, \tag{30}
\]

In a symmetric equilibrium we obtain the following interest rate setting rule for deposit taking banks

\[
1 - \left( \frac{v_D}{u_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} - \left( \frac{\phi_D}{u_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} \left[ \frac{R_{D,t}}{R_{D,t-1}} - 1 \right] + \ldots \\
\ldots + \left( \frac{1}{u_D - 1} \right) \frac{\phi_{D,2}}{\psi} \left( \frac{R_{D,t}}{R_{D,t-1}} \right) \left[ \exp \left( -\psi \left( \frac{R_{D,t}}{R_{D,t-1}} - 1 \right) \right) - 1 \right] + \ldots \\
\ldots + E_t \left\{ \left( \frac{1}{u_D - 1} \right) \frac{M_{t+1}}{\psi_{t+1}} \left( \frac{R_{D,t+1}}{R_{D,t}} \right) \frac{D_{t+1}}{D_t} \times \\
\ldots - \left( \frac{\phi_{D,2}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{D,t+1}}{R_{D,t}} - 1 \right) \right) - 1 \right] \right\} = 0. \tag{31}
\]
8 Conclusion

We use a medium scale regime-switching DSGE model with a banking sector to analyze the effects of incomplete and asymmetric interest rate pass-through. The model is estimated using Bayesian techniques on US data between 1985 and 2016. We find interest rate pass-through to be mostly incomplete, but with the magnitude of the pass through depending on the shocks that hit the economy. Shocks also create asymmetric dynamics at the ZLB and incomplete pass-through exacerbates that asymmetry. We further note that pass-through is nonlinear with respect to policy parameters. In particular, the value of the interest rate smoothing dramatically changes the profile of the interest rate pass-through with respect to the other policy parameters. In all cases we find the behavior of pass-through in the loan rate to be different from that of the deposit rate. Putting all this together, we show that policy is less effective under incomplete pass-through.
References


Appendices

Appendix A  Model

In this section we describe the model economy we use to investigate interest rate pass-through. Our setup is reasonably standard; the model is comprised of households, firms, banks, a fiscal authority and a monetary authority. Each household consumes the final good, supplies its own variety of labor in return for labor income and receives dividends from firms and banks, which they own. Households hold deposits with a retail bank. Labor is differentiated which gives each household a degree of market power and the ability to choose wages, subject to quadratic adjustment costs, to minimize their disutility of working. Firms produce a differentiated intermediate good using a common neutral technology, labor and capital which they own. They choose quantities of labor, capital, investment and prices to maximize the expected present value of their profits, subject to quadratic adjustment costs on changing investment and prices and a loan-in-advance constraint. Final goods are produced by a perfectly competitive “packing” firm that aggregates intermediate goods according to a CES production technology.

In the absence of any frictions or imperfections, conventional DSGE models do not require a banking sector. Following Edwards and Vegh (1997) and Christiano et al. (2005) we introduce a banking sector via a loan-in-advance (LIA) constraint. More specifically firms have to take out a loan at the beginning of the period to pay for a fixed fraction of their investment good purchases each period. Firms repay the loan at the end of the period. Following Gerali et al. (2010), the banking sector is divided into retail and wholesale banks, where retail banks are further divided into deposit-taking and loan-making banks. Gerali et al. (2010) introduces differentiated deposits and loans as a means of introducing markups (and markdowns) of the loan and deposit interest rates over the policy rate.

In the baseline model, loan and deposit taking banks choose loan and deposit interest rates to maximize the present value of their profits, subject to a quadratic adjustment cost on changing interest rates. This results in interest rate setting rules that resemble the Rotemberg Philips curves for price and wage setting. In the Taylor contracting case, retail interest rates are fixed for a set number of quarters. Each quarter the contracts for a fraction of retail banks expire allowing these banks to reset their loan and deposit rates. These banks choose deposit and loan interest rates to maximize the present value of their profits for the duration of the contract. Following Coenen et al. (2007), each period we allow a fraction of loan and deposit takers to break their contracts, without any costs or repercussions. This deviation from the conventional Taylor contracting setup allows for
smoother dynamics, an improved representation of reality as well as nesting the original Taylor contracting specification where contracts cannot be broken. Finally in the Linex case, retail banks choose interest rates to maximize profits subject to Linex adjustment costs on changing interest rates, where the cost of adjustment is asymmetric.

Final loans and deposits are produced by a perfectly competitive aggregator firm that aggregates loans and deposits from the retail banks according to a CES production technology.

**Appendix B  Households**

The economy is populated by a continuum of households, normalized to unit mass. Each household derives positive utility from consumption, relative to the previous periods level of aggregate consumption, and disutility from working. Utility for the \( i \)th household takes the form

\[
U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( \prod_{j=0}^{t} d_{t+j} \right) \left[ A_t \left( \frac{C_t}{Z_{Y,t}} \right)^{1-\sigma} - \kappa N_t(i)^{1+\eta} \right] \right\},
\]

where

\[
\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t},
\]

is a consumption preference-shifter, \( C_t = C_t - \chi \bar{C}_{t-1} \) is a consumption index, \( C_t \) is consumption, \( Z_{Y,t} \) is a composite technology process that grows at the same rate as consumption on the balanced growth path, and \( N_t(i) \) is the labor variety supplied by the \( i \)th household. \( d_{t+j} \) is a preference shifter term where we assume \( d_0 = 1 \). The \( i \)th household faces the following budget constraint

\[
C_t + D_t = \frac{D_{t-1} R_{D,t-1}}{\pi_t} + \frac{W_t(i)}{P_t} N_t(i) - \frac{\phi_W W_t}{P_t} N_t \left[ \frac{W_t(i)}{W_{t-1}(i)} - \bar{\pi}_{W,t} \right]^2 + (1 - \omega) \frac{J_{t-1}}{\pi_t} + T_t + \Psi_t + \Phi_t,
\]

where \( D_t \) is deposits, \( R_{D,t} \) is the interest rate paid on deposits, \( W_t \) is the nominal wage, \( P_t \) is the price level for final goods, \( \pi_{W,t} \) is wage inflation, \( J_{t-1} \) is total profits from the banking sector, \( T_t \) is lump sum taxes, \( \Psi_t \) is profits from intermediate goods producers and \( \Phi_t \) is the price, wage and interest rate adjustment costs that are rebated to households.

The term \( \bar{\pi}_{W,t} \equiv \pi_{W,t}^{\xi_W} \pi_{W}^{1-\xi_W} \) captures wage indexation behavior from wage setters. Perfect competition and cost minimization by the labor packing or aggregating firm leads to the following demand schedule for the \( i \)th household’s variety of labor

\[
N_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon} N_t.
\]
Households choose allocations of date \( t \) consumption, deposits and wages to maximize the sum of their current and expected discounted stream of future period utilities, subject to the budget constraint (equation B.1). Setting this up as the Lagrangean:

\[
\mathcal{L}_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( \prod_{j=0}^{t} d_{t+j} \right) \right\} - \lambda_t \left[ \frac{A_t^t(C_t/Z_{Y,t})^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t(i)^{1+\eta}}{1+\eta} - C_t + D_t - \frac{D_{t-1}R_{D,t-1}}{\pi_t} N_t(i) + \ldots 
+ \frac{\phi W_t}{\pi_t} d_t \left[ \frac{W_t(i)}{W_{t-1}(i)} - \bar{\pi}_{t}\right]^2 - (1-\omega) \frac{J_{t-1}}{\pi_t} - \ldots 
\right]
\]

(B.3)

Substituting B.2 into B.3 gives

\[
\mathcal{L}_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( \prod_{j=0}^{t} d_{t+j} \right) \right\} - \lambda_t \left[ \frac{A_t^t(C_t/Z_{Y,t})^{1-\sigma}}{1-\sigma} - \kappa \frac{N_t(i)^{1+\eta}}{1+\eta} - C_t + D_t - \frac{D_{t-1}R_{D,t-1}}{\pi_t} N_t(i) + \ldots 
+ \frac{\phi W_t}{\pi_t} d_t \left[ \frac{W_t(i)}{W_{t-1}(i)} - \bar{\pi}_{t}\right]^2 - (1-\omega) \frac{J_{t-1}}{\pi_t} - \ldots 
\right]
\]

Optimization results in the following first order conditions. The first order condition for consumption:

\[
\frac{\partial \mathcal{L}_t}{\partial C_t} = A_t (C_t - \chi C_{t-1})^{-\sigma} Z_{Y,t}^{-1} - \lambda_t = 0. \quad \text{(B.4)}
\]

The first order condition for deposits:

\[
\frac{\partial \mathcal{L}_t}{\partial D_t} = -\lambda_t + E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1} R_{D,t+1}}{\pi_{t+1}} \right\} = 0. \quad \text{(B.5)}
\]

The first order condition for wages:

\[
\frac{\partial \mathcal{L}_t}{\partial W_{t}(i)} = \nu \kappa \frac{N_t(i)^{1+\eta}}{W_t(i)} + \lambda_t (1-\nu) \frac{N_t(i)}{P_t} - \lambda_t \phi_W \frac{W_t N_t}{P_t W_{t-1}(i)} \left[ \frac{W_t(i)}{W_{t-1}(i)} - \bar{\pi}_{t} \right] + \ldots 
+ E_t \left\{ \beta d_{t+1} \lambda_{t+1} \phi_W \frac{W_{t+1}(i) W_{t+1} N_{t+1}}{P_{t+1} W_t(i)^2} \left[ \frac{W_{t+1}(i)}{W_t(i)} - \bar{\pi}_{t+1} \right] \right\} = 0. \quad \text{(B.6)}
\]

From B.4 we get the marginal utility of consumption:

\[
\lambda_t = A_t (C_t - \chi C_{t-1})^{-\sigma} Z_{Y,t}^{-1}. \quad \text{(B.7)}
\]
From B.5 we get the consumption Euler equation:

$$\lambda_t = E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1} R_{D,t}}{\pi_{t+1}} \right\}, \quad \text{(B.8)}$$

which we use to construct the real stochastic discount factor:

$$\mathcal{M}_{t,t+1} = E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \right\}. \quad \text{(B.9)}$$

Finally we obtain the wage Phillips curve from equation B.6:

$$\left( \frac{v}{v-1} \right) \kappa N_t^\eta P_t \frac{\pi_{W,t}}{\lambda_t W_t} - 1 - \left( \frac{\phi_W}{v-1} \right) \pi_{W,t} [\pi_{W,t} - \tilde{\pi}_{W,t}] + \ldots$$

$$\ldots + E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\phi_W}{v} \right) \pi_{W,t+1}^2 \frac{N_{t+1}}{\pi_{t+1}} \left( \frac{N_t}{N_{t+1}} \right) [\pi_{W,t+1} - \tilde{\pi}_{W,t+1}] \right\} = 0, \quad \text{(B.10)}$$

where we have assumed a symmetric equilibrium with $W_t(i) = W_t$ and $N_t(i) = N_t$.

### Appendix C  Investment Goods Producers

A continuum of perfectly competitive investment goods producers produce an identical final investment good. We drop the firms’ subscripts and consider a representative final investment goods producer. Final investment goods $(I_t)$ are produced using a production process that combines investment specific (embodied) technology with raw investment goods $(X_t$, which comes from final goods producers) according to the production function

$$I_t = Z_{I,t} X_t,$$

where embodied (investment specific) technology evolves according to the following process

$$Z_{I,t} = Z_{I,0} \exp \left( g_{Z_t} \cdot t + \mathcal{A}_{Z_t,t} \right), \quad \mathcal{A}_{Z_t,t} = \rho \mathcal{A}_{Z_t,t-1} + \varepsilon_{Z_t,t}. \quad \text{(C.1)}$$

Producers of final investment goods maximize their period profits by choosing the quantity of raw investment goods to use in production, where period profits are given by:

$$\Psi_{I,t} = P_{I,t} I_t - P_t X_t,$$

$$= P_{I,t} Z_{I,t,t} X_t - P_t X_t.$$
We obtain the first order condition for the investment goods producer

\[
\frac{\partial \Psi_{I,t}}{\partial X_t} = P_{I,t} Z_{I,t} - P_t = 0,
\]

which implies

\[
\frac{P_{I,t}}{P_t} = \frac{1}{Z_{I,t}}, \text{ and } P_{I,t} I_t = P_t X_t.
\]

### Appendix D Intermediate Goods Producers

Differentiated intermediate goods are produced by a continuum of firms, normalized to unit mass. The \(h\)th firm produces intermediate goods by combining capital and labor inputs with a common (neutral) technology according to the Cobb-Douglas production technology

\[
Y_t(h) = Z_t K_{t-1}(h)^\alpha N_t(h)^{1-\alpha}.
\]  \(\text{(D.1)}\)

The common neutral technology evolves according to the process

\[
Z_t = Z_0 \exp (g Z \cdot t + \mathcal{A}_{Z,t}), \quad \mathcal{A}_{Z,t} = \rho_{\mathcal{A}_Z} \mathcal{A}_{Z,t-1} + \varepsilon_{Z,t}.
\]  \(\text{(D.2)}\)

Dixit-Stiglitz aggregation and cost minimization by the perfectly competitive final goods producer implies producers of the \(h\)th intermediate good face the following demand schedule

\[
Y_t(h) = \left( \frac{P_t(h)}{P_t} \right)^{-\epsilon} Y_t.
\]  \(\text{(D.3)}\)

Intermediate goods producers own the capital they use in the production process. Firm \(h\)'s capital stock evolves according to the process

\[
K_t(h) = I_t(h) \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_I \right)^2 \right) - (1 - \delta) K_{t-1}(h).
\]  \(\text{(D.4)}\)

Each intermediate goods producer is subject to a loan-in-advance (LIA) constraint when purchasing investment goods. As a consequence each firm must fund a portion of their investment goods through a one period loan. Firm \(h\)'s LIA constraint can be summarized as follows

\[
L_t(h) \geq \psi \frac{P_{I,t}}{P_t} I_t(h).
\]  \(\text{(D.5)}\)

Firms maximize their expected discounted stream of period profits by choosing allocations of date \(t\) investment, capital, labor, loans and date \(t\) prices, subject to constraints D.1, D.4 and D.5 and a quadratic cost on adjusting prices.\(^5\) This can be represented by

\(^5\)Note that we assume all constraints bind with equality in equilibrium.
the Lagrangean:

$$
\Psi_0(h) = E_0 \sum_{t=0}^{\infty} M_{0,t}^* \left\{ \begin{array}{l}
\exp \left( \mathbb{P}_t \right) \frac{P_t(h)}{P_t} Y_t(h) - \frac{W_t}{P_t} N_t(h) - \frac{P_t}{P_t} I_t(h) + L_t(h) - d_t \frac{R_{L,t-1} L_{t-1}(h)}{\pi_t} - \ldots \\
\ldots - \frac{\phi_t}{2} Y_t \left[ \frac{P_t(h)}{P_{t-1}(h)} - \tilde{\pi}_t \right]^2 - \ldots \\
K_t(h) - I_t(h) \left( 1 - \frac{\phi_t}{2} \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_t \right)^2 \right) - \ldots \\
\ldots (1 - \delta) K_{t-1}(h) \\
\ldots - \Phi_t(h) \left[ Y_t(h) - Z_t K_{t-1}(h) \right] - \ldots \\
\ldots - \Upsilon_{L,t}(h) \left[ L_t(h) - \psi \frac{P_t}{P_t} I_t(h) \right]
\end{array} \right\} 
$$

(D.6)

Where $\tilde{\pi}_t = \pi_{t-1}^{\xi} \pi^{1-\xi}$ is the inflation index firms index prices to when adjusting prices and $M_{t,t+1} = E_t \left\{ \beta \lambda_{t+1} \lambda \right\}$ is the modified real stochastic discount factor. We use this stochastic discount factor in place of the household’s stochastic discount because the household’s stochastic discount factor causes implausibly large swings in investment and capital when we switch between the normal and ZLB (steady-) states. To ensure a degree of symmetry between the household’s consumption Euler equation and the firm’s intertemporal borrowing decision, we augment the firm’s repayment decision with the preference shifter term $d_t$ so that the first order condition resembles what we would observe if the firm were using the household’s stochastic discount factor. This also prevents implausibly large swings in the price of new capital goods when switches between the normal and ZLB (steady-) states.

Substituting D.3 into D.6 gives:

$$
\Psi_0(h) = E_0 \sum_{t=0}^{\infty} M_{0,t}^* \left\{ \begin{array}{l}
\exp \left( \mathbb{P}_t \right) \left( \frac{P_t(h)}{P_t} \right)^{1-\epsilon} Y_t - \frac{W_t}{P_t} N_t(h) - \frac{P_t}{P_t} I_t(h) + L_t(h) - d_t \frac{R_{L,t-1} L_{t-1}(h)}{\pi_t} - \ldots \\
\ldots - \frac{\phi_t}{2} Y_t \left[ \frac{P_t(h)}{P_{t-1}(h)} - \tilde{\pi}_t \right]^2 - \ldots \\
K_t(h) - I_t(h) \left( 1 - \frac{\phi_t}{2} \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_t \right)^2 \right) - \ldots \\
\ldots (1 - \delta) K_{t-1}(h) \\
\ldots - \Phi_t(h) \left[ \left( \frac{P_t(h)}{P_t} \right)^{-\epsilon} Y_t - Z_t K_{t-1}(h) \right] - \ldots \\
\ldots - \Upsilon_{L,t}(h) \left[ L_t(h) - \psi \frac{P_t}{P_t} I_t(h) \right]
\end{array} \right\}
$$

Optimization by the firm results in the following set of first order conditions. The first
order conditions for investment:

\[
\frac{\partial \Psi_I(h)}{\partial I_t(h)} = -\frac{P_{t,t}}{P_t} + Q_t(h) \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_I \right)^2 - \phi_I \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_I \right) \frac{I_t(h)}{I_{t-1}(h)} \right] + \ldots
\]

\[
\ldots + \psi \Upsilon_{L,t}(h) \frac{P_{t,t}}{P_t} + E_t \left\{ \mathcal{M}_{t,t+1}^* \phi_I Q_{t+1}(h) \left( \frac{I_{t+1}(h)}{I_t(h)} - \mu_I \right) \left( \frac{I_{t+1}(h)}{I_t(h)} \right)^2 \right\} = 0. \quad \text{(D.7)}
\]

From equation (D.9) we get the firm’s demand for labor and prices.

\[
\frac{\partial \Psi_t(h)}{\partial K_t(h)} = -Q_t(h) + E_t \left\{ \mathcal{M}_{t,t+1}^* \left( \alpha \Phi_t + 1 \left( -\delta \right) Q_{t+1}(h) \right) \right\} = 0. \quad \text{(D.8)}
\]

hours worked

\[
\frac{\partial \Psi_t(h)}{\partial N_t(h)} = -W_t \frac{P_t}{P_t} + \left( 1 - \alpha \right) \Phi_t(h) \frac{Y_t(h)}{N_t(h)} = 0. \quad \text{(D.9)}
\]

loans

\[
\frac{\partial \Psi_t(h)}{\partial L_t(h)} = 1 - \Upsilon_{L,t}(h) - E_t \left\{ \mathcal{M}_{t,t+1}^* d_{t+1} \frac{R_{L,t}}{\pi_{t+1}} \right\} = 0. \quad \text{(D.10)}
\]

and prices

\[
\frac{\partial \Psi_t(h)}{\partial P_t(h)} = (1 - \varepsilon) \exp \left( \frac{\varepsilon I_t(h)}{P_t(h)} \right) + \varepsilon \Phi_t(h) \frac{Y_t(h)}{P_t(h)} - \phi_P \frac{Y_t(h)}{P_t(h)} \left[ \frac{P_t(h)}{P_{t-1}(h)} - \bar{\pi}_t \right] + \ldots
\]

\[
\ldots + E_t \left\{ \phi_P \mathcal{M}_{t,t+1}^* Y_{t+1} \frac{P_{t+1}(h)}{P_t(h)}^2 \left[ \frac{P_{t+1}(h)}{P_t(h)} - \bar{\pi}_{t+1} \right] \right\} = 0. \quad \text{(D.11)}
\]

Rearranging D.7 gives:

\[
\frac{P_{t,t}}{P_t} = Q_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_I \right)^2 - \phi_I \left( \frac{I_t(h)}{I_{t-1}(h)} - \mu_I \right) \frac{I_t(h)}{I_{t-1}(h)} \right] + \psi \Upsilon_{L,t} \frac{P_{t,t}}{P_t} + \ldots
\]

\[
\ldots + E_t \left\{ \mathcal{M}_{t,t+1}^* \phi_I Q_{t+1} \left( \frac{I_{t+1}(h)}{I_t(h)} - \mu_I \right) \left( \frac{I_{t+1}(h)}{I_t(h)} \right)^2 \right\} = 0.
\]

From equation (D.8) we get the standard Tobin’s Q relationship:

\[
Q_t = E_t \left\{ \mathcal{M}_{t,t+1}^* \left( \alpha \Phi_t + 1 \left( -\delta \right) Q_{t+1} \right) \right\}.
\]

From D.9 we get the firm’s demand for labor

\[
\frac{W_t}{P_t} = (1 - \alpha) \Phi_t \frac{Y_t}{N_t}.
\]
From D.10 we get the firm’s demand for loans

\[ 1 - \Upsilon_{L,t} = E_t \left\{ \Upsilon_{t+1}^* \frac{R_{L,t}}{\pi_{t+1}} \right\} \]

From D.11 we get the Price Phillips curve

\[
\left( \frac{\varepsilon}{\varepsilon - 1} \right) \Phi_t - \exp (P_t) - \left( \frac{\phi_P}{\varepsilon - 1} \right) \pi_t [\pi_t - \tilde{\pi}_t] + \ldots \\
\ldots + E_t \left\{ \left( \frac{\phi_P}{\varepsilon - 1} \right) \Upsilon_{t+1}^* \frac{Y_{t+1}}{Y_t} \pi_{t+1} [\pi_{t+1} - \tilde{\pi}_{t+1}] \right\} = 0.
\]

where we assume a symmetric equilibrium so that \( P_t(i) = P_t \) and \( Y_t(i) = Y_t \).

**Appendix E  Monetary Policy and Markup Regimes**

We assume the model economy’s dynamics are conditional on four discrete states of nature. At any given time the model economy can be in one of two monetary policy states and one of two markup states. This is reflected by introducing separate Markov chains for the monetary policy and markup states. The monetary policy state determines whether policy is set according to a Taylor type rule which occurs in the normal state (N), or the economy is at the zero lower bound state (Z) where policy follows an exogenous process, so that \( s_{1,t} = N, Z \). The monetary policy state also affects the markups and markdowns charged by retail banks and the degree of rigidity they face when adjusting market interest rates. The markup state affects whether markups and markdowns on market interest rates are high (H) or low (L) and the degree of rigidity in adjusting market interest rates, when the economy is away from the lower bound, so that \( s_{2,t} = H, L \). We introduce two regime-switching parameters, \( z(s_{1,t}) \) which is conditional on the monetary policy regime and \( m(s_{2,t}) \) which is conditional on the markup regime. We assume

\( z(Z) = 1 \) and \( z(N) = 0 \), \hspace{1cm} (E.1)

with the states \( Z \) and \( N \) are governed by the following Markov transition matrix

\[
Q_Z = \begin{bmatrix}
1 - p_{N,Z} & p_{N,Z} \\
p_{Z,N} & 1 - p_{Z,N}
\end{bmatrix}.
\] \hspace{1cm} (E.2)

We assume the regime-specific markup parameter takes the values

\( m(H) = 1 \) and \( m(L) = 0 \).

\( \hspace{1cm} (E.3) \)

44
The states $H$ and $L$ are governed by the Markov transition matrix

$$Q_m = \begin{bmatrix} 1 - q_{H,L} & q_{H,L} \\ q_{L,H} & 1 - q_{L,H} \end{bmatrix}.$$ (E.4)

### Appendix F The Banking Sector

Following Gerali et al. (2010) the banking sector is divided into three different types of banks: wholesale banks, deposit taking banks and loan making banks. Wholesale banks take deposits from deposit taking banks and combine them with bank equity to supply loans to loan making banks. Deposit taking banks supply deposits to aggregators, who in turn supply them to households. Loan making banks supply deposits to aggregators, who in turn bundle them and supply them to intermediate goods producers.

#### F.1 Loan and Deposit Demand

##### F.1.1 Deposits

There is a continuum of deposit taking banks normalized to unit mass. Each bank supplies a differentiated stock of deposits. Deposits supplied to the $i$th household are bundled by an aggregator according to the CES technology

$$D_t(i) = \left[ \int_0^1 D_t(i, z)^{1 - \frac{1}{\nu D(r_t)}} dz \right]^{\frac{\nu D(r_t)}{\nu D(r_t) - 1}}.$$

Cost minimization by the perfectly competitive aggregator implies the following demand for deposits by the $i$th household for deposits from the $z$th bank

$$D_t(i, z) = \left( \frac{R_{D,t}(z)}{R_{D,t}} \right)^{-\nu D(r_t)} D_t(i).$$

Aggregating over households

$$D_t = \int_0^1 D_t(i) di = \int_0^1 \left[ \int_0^1 D_t(i, z)^{1 - \frac{1}{\nu D(r_t)}} dz \right]^{\frac{\nu D(r_t)}{\nu D(r_t) - 1}} di = \int_0^1 D_t(z)^{1 - \frac{1}{\nu D(r_t)}} dz^{\frac{\nu D(r_t)}{\nu D(r_t) - 1}},$$

which implies the aggregate demand function for deposits from the $z$th bank

$$D_t(z) = \left( \frac{R_{D,t}(z)}{R_{D,t}} \right)^{-\nu D(r_t)} D_t.$$
F.1.2 Loans

There is also a continuum of banks, each supplying a differentiated loan product, normalized to unit mass. Loans supplied to the $h$th firm are produced according the CES aggregation technology

$$L_t(h) = \left[ \int_0^1 L_t(h, z)^{1 - \frac{1}{\nu_L(r_t)}} dz \right]^\frac{\nu_L(r_t)}{\nu_L(r_t)-1}.$$

Cost minimization by the perfectly competitive loan aggregators, implies the demand schedule for the $h$th intermediated goods producer for loans produced by the $z$th retail bank

$$L_t(h, z) = \left( \frac{R_{L,t}(z)}{R_{L,t}} \right)^{-\nu_L(r_t)} L_t(h).$$

Aggregating over firms

$$L_t = \int_0^1 L_t(h) dh = \int_0^1 \left[ \int_0^1 L_t(h, z)^{1 - \frac{1}{\nu_L(r_t)}} dz \right]^\frac{\nu_L(r_t)}{\nu_L(r_t)-1} dh = \left[ \int_0^1 L_t(z)^{1 - \frac{1}{\nu_L(r_t)}} dz \right]^\frac{\nu_L(r_t)}{\nu_L(r_t)-1},$$

which also leads to the aggregate demand function for loans from the $z$th retail bank

$$L_t(z) = \left( \frac{R_{L,t}(z)}{R_{L,t}} \right)^{-\nu_L(r_t)} L_t.$$

F.2 Wholesale Banks

Wholesale banks are constrained to obey the following balance sheet identity

$$L_t(w) = D_t(w) + K_{B,t}(w),$$

so that the $w$th bank must fund its loans ($L_t(w)$) through deposits ($D_t(w)$) or bank equity ($K_{B,t}(w)$). Bank equity expands and contracts according to the following process

$$K_{B,t}(w) = (1 - \delta_B) K_{B,t-1}(w) + \omega J_{t-1}(w).$$

The $w$th wholesale bank maximizes the expected present value of their future profit streams by choosing the quantity of deposits and loans, subject to the balance sheet
\begin{equation}
\Psi_0(w) = E_0 \left\{ \sum_{t=0}^{\infty} \mathcal{M}^*_0,0 \left( \frac{P_0}{P_t} \right) \left[ \mathbb{R}_{L,t} L_t(w) - \mathbb{R}_{D,t} D_t(w) - K_{B,t}(w) - \ldots - \Theta_t [L_t(w) - D_t(w) - K_{B,t}(w)] \right] \right\},
\end{equation}

from the wholesaler’s first order conditions we get the following relationships between loan rates, deposit rates and the policy rate

\begin{align*}
\mathbb{R}_{L,t} &= R_t, \\
\mathbb{R}_{D,t} &= R_t.
\end{align*}

F.3 Retail Banks

Retail banks produce differentiated loans and deposits. They are also subject to frictions that prevent them from adjusting retail interest rates one for one with wholesale interest rates. We consider two types of interest rate setting frictions. Following Gerali et al. (2010), we assume that retail banks are subject to quadratic costs of adjustment.

F.4 Interest Rate Setting Frictions: Rotemberg Adjustment Costs

F.4.1 Loan Branch

The \( z \)th loan-making bank sets the interest rate on loans to maximize the sum of the expected present value of their profits, subject to a quadratic cost of changing interest rates.

\begin{equation}
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t}^* \left( \frac{P_0}{P_t} \right) \left[ R_{L,t}(z) L_t(z) - \exp(\varepsilon_{L,t}) \mathbb{R}_{L,t} L_t(z) - \ldots - \phi_t \left( \frac{R_t(z)}{R_{L,t-1}(z)} - 1 \right)^2 \right] \right\}.
\end{equation}

Substituting in the demand for loans

\begin{equation}
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t}^* \left( \frac{P_0}{P_t} \right) \left[ \frac{1}{1-v_{L,(r_t)}} R_{L,t} L_t(z) - \ldots - \exp(\varepsilon_{L,t}) \mathbb{R}_{L,t} R_{L,t}(z)^{-v_{L,(r_t)}} R_{L,t} L_t(z) - \ldots - \phi_t \left( \frac{R_t(z)}{R_{L,t-1}(z)} - 1 \right)^2 \right] \right\}.
\end{equation}

Note that we have dropped the quadratic adjustment cost on changing loans that is present in the Gerali et al. (2010) model because it only had a very minimal impact of the dynamics of loans and loan interest rates in our model.
The first order condition for the \( z \)th loan-making bank

\[
\frac{\partial \Psi_{L,t}(z)}{\partial R_{L,t}(z)} = (1 - v_L(r_t)) L_t(z) + v_L(r_t) \exp(\varepsilon_{L,t}) \frac{R_{L,t}}{R_{L,t-1}} L_t(z) - \ldots
\]

\[
\ldots - \phi_L(r_t) R_{L,t-1} \left[ \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right] + \ldots
\]

\[
\ldots + E_t \left\{ \phi_L(r_{t+1}) \mathcal{M}^*_t \right\} \frac{R_{L,t+1} R_{L,t+1}(z) L_{t+1}}{\pi_t R_{L,t}(z)^2} \left[ \frac{R_{L,t+1}(z)}{R_{L,t}(z)} - 1 \right] = 0.
\]

Which gives the following Phillips curve relationship for interest rate setting

\[
\left( \frac{v_L(r_t)}{v_L(r_t) - 1} \right) \exp(\varepsilon_{L,t}) \frac{R_{L,t}}{R_{L,t-1}} - 1 - \left( \frac{\phi_L(r_t)}{v_L(r_t) - 1} \right) R_{L,t} \left[ \frac{R_{L,t}}{R_{L,t-1}} - 1 \right] + \ldots
\]

\[
\ldots + E_t \left\{ \left( \frac{\phi_L(r_{t+1})}{v_L(r_t) - 1} \right) \mathcal{M}^*_t \right\} \left( \frac{1}{\pi_t} \right) \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \frac{L_{t+1}}{L_t} \left[ \frac{R_{L,t+1}}{R_{L,t}} - 1 \right] = 0, \quad (F.1)
\]

where we assume a symmetric equilibrium so that \( R_{L,t}(z) = R_{L,t} \) and \( L_t(z) = L_t \). We further simplify this as follows

\[
\left( \frac{v_L(r_t)}{v_L(r_t) - 1} \right) \exp(\varepsilon_{L,t}) \frac{R_{L,t}}{R_{L,t-1}} - 1 - \tilde{\phi}_L(r_t) R_{L,t} \left[ \frac{R_{L,t}}{R_{L,t-1}} - 1 \right] + \ldots
\]

\[
\ldots + E_t \left\{ \tilde{\phi}_L(r_{t+1}) \mathcal{M}^*_t \right\} \left( \frac{1}{\pi_t} \right) \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \frac{L_{t+1}}{L_t} \left[ \frac{R_{L,t+1}}{R_{L,t}} - 1 \right] = 0. \quad (F.2)
\]

where \( \tilde{\phi}_L(r_t) = \frac{\phi_L(r_t)}{v_L(r_t) - 1} \) and

\[
\tilde{\phi}_L(r_t) = z(s_{1,t}) \tilde{\phi}_{ZLB} + (1 - z(s_{1,t})) (m(s_{2,t}) \tilde{\phi}_{H} + (1 - m(s_{2,t})) \tilde{\phi}_{L}). \quad (F.3)
\]

Likewise, the markup on loans and the markdown on deposits are determined by

\[
\mu_L(r_t) = z(s_{1,t}) \mu_{ZLB} + (1 - z(s_{1,t})) (m(s_{2,t}) \mu_{H} + (1 - m(s_{2,t})) \mu_{L}), \quad (F.4)
\]

where the markup is related to the elasticity of substitution through

\[
v_L(r_t) = \frac{\mu_L(r_t)}{\mu_L(r_t) - 1}. \quad (F.5)
\]
F.4.2 Deposit Branch

The zth deposit taking bank sets interest rates to maximize their expected discounted future stream of profits, subject to a quadratic adjustment cost on changing interest rates

\[ \Psi_{D,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} M_{t+1}^* \left( \frac{P_0}{P_t} \right) \left[ \frac{\exp(\varepsilon_{D,t}) R_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots}{2} \right] \right\}. \]

Substituting in the demand function for deposits

\[ \Psi_{D,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} M_{t+1}^* \left( \frac{P_0}{P_t} \right) \left[ \frac{\exp(\varepsilon_{D,t}) R_{D,t}(z)^{-v_D(r_t)} R_{D,t}^{v_D(r_t)} D_t - \ldots}{2} \right] \right\}. \]

The first order condition for deposits

\[ \frac{\partial \Psi_{D,t}(z)}{\partial R_{D,t}(z)} = -v_D(r_t) \exp(\varepsilon_{D,t}) \frac{R_{D,t}}{R_{D,t}(z)} D_t(z) - (1 - v_D(r_t)) D_t(z) - \ldots \]

\[ \ldots - \phi_D(r_t) \frac{R_{D,t} D_t}{R_{D,t-1}(z)} \left[ \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right] + \ldots \]

\[ \ldots + E_t \left\{ \phi_D(r_{t+1}) M_{t+1}^* \frac{R_{D,t+1} R_{D,t+1}(z) D_{t+1}}{\pi_{t+1} R_{D,t}(z)^2} \left[ \frac{R_{D,t+1}(z)}{R_{D,t}(z)} - 1 \right] \right\} = 0, \]

which gives the following Phillips curve type relationship for interest rates set by deposit taking banks

\[ 1 - \left( \frac{v_D(r_t)}{v_D(r_t) - 1} \right) \exp(\varepsilon_{D,t}) \frac{R_{D,t}}{R_{D,t}(z)} - \left( \frac{\phi_D(r_t)}{v_D(r_t) - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} \left[ \frac{R_{D,t}}{R_{D,t-1} - 1} \right] + \ldots \]

\[ \ldots + E_t \left\{ \left( \frac{\phi_D(r_{t+1})}{v_D(r_t) - 1} \right) M_{t+1}^* \left( \frac{1}{\pi_{t+1}} \right) \frac{R_{D,t+1}}{R_{D,t}} \right\}^2 \frac{D_{t+1}}{D_t} \left[ \frac{R_{D,t+1}}{R_{D,t} - 1} \right] = 0, \] (F.6)

where we assume a symmetric equilibrium so that \( R_{D,t}(z) = R_{D,t} \) and \( D_t(z) = D_t \). We further simplify this as follows

\[ 1 - \left( \frac{v_D(r_t)}{v_D(r_t) - 1} \right) \exp(\varepsilon_{D,t}) \frac{R_{D,t}}{R_{D,t}(z)} - \tilde{\phi}_D(r_t) \frac{R_{D,t}}{R_{D,t-1}} \left[ \frac{R_{D,t}}{R_{D,t-1} - 1} \right] + \ldots \]

\[ \ldots + E_t \left\{ \tilde{\phi}_D(r_{t+1}) M_{t+1}^* \left( \frac{1}{\pi_{t+1}} \right) \frac{R_{D,t+1}}{R_{D,t}} \right\}^2 \frac{D_{t+1}}{D_t} \left[ \frac{R_{D,t+1}}{R_{D,t} - 1} \right] = 0. \] (F.7)
where \( \tilde{\phi}_D(r_t) = \frac{\phi_L(r_t)}{\nu_L(r_t) - 1} \) and

\[
\tilde{\phi}_D(r_t) = z(s_{1,t}) \phi_{ZLB,D} + (1 - z(s_{1,t}))(m(s_{2,t}) \phi_{H,D} + (1 - m(s_{2,t})) \phi_{L,D}).
\] (F.8)

Likewise, the markup on loans and the markdown on deposits are determined by

\[
\mu_D(r_t) = z(s_{1,t}) \mu_{ZLB,D} + (1 - z(s_{1,t}))(m(s_{2,t}) \mu_{H,D} + (1 - m(s_{2,t})) \mu_{L,D}),
\] (F.9)

where the markdown is related to the elasticity of substitution through

\[
u_D(r_t) = \frac{\mu_D(r_t)}{\mu_D(r_t) - 1}.
\] (F.10)

### Appendix G  Monetary Policy

The monetary authority sets policy according to

\[
R_t = \max \left( R_{ZLB,t}, R^*_t \right),
\] (G.1)

where \( R^*_t \) is the interest rate set in normal times according to the Taylor-type rule

\[
R^*_t = R^* \rho_{R} F_{t-1} \left( \frac{\pi_t}{\pi^*} \right)^{\kappa_{\pi}} \left( \tilde{Y}_t \right)^{\kappa_{\gamma}} 1 - \rho_{R} \exp(\varepsilon_{R,t}),
\] (G.2)

where \( \tilde{Y}_t \) is the output gap. We use a measure of the output gap in the Taylor-type rule because growth measures of GDP do not result in a negative shadow interest rate when at the lower bound. We use the CBO’s output gap as the measure that policy responds to because Primiceri and Justiniano (2009) show that measures of potential output from a flex price DSGE model closely resemble official measures like the CBO’s output gap. We assume that the CBO output gap can be described by the following process

\[
\tilde{Y}_t = \kappa_P \left( \frac{Y_t}{Y^P_t} \right) \exp(F_t),
\] (G.3)

where \( Y^P_t \) is potential output and \( F_t \) is an exogenous process with the following law of motion

\[
F_t = \rho_F F_{t-1} + \varepsilon_{F,t}.
\] (G.4)

We set the priors on \( \rho_F \) and \( \sigma_F \) to ensure that \( F_t \) only plays a limited role in explaining the output gap. We define potential output as the level of output in an economy without pricing, wage and interest rate frictions, without the monopolist competition in the goods, labor and banking markets and without the loan in advance constraint. In other words
potential output is the level that would reign in the real business cycle representation of a frictionless economy. When the economy is at the lower bound, interest rates evolve according to

$$R_{ZLB,t} = K + \varepsilon_{ZLB,t} \quad \text{(G.5)}$$

where $K$ is the effective lower bound on interest rates and $\varepsilon_{ZLB}$ is a small shock that prevents a stochastic singularity at the lower bound. Which in the regime-switching setup we replace (G.1) with

$$R_t = z(s_{1,t})R_{ZLB,t} + (1 - z(s_{1,t}))R_t^* \quad \text{(G.6)}$$

### Appendix H  Fiscal Policy

Government spending follows a very simple autoregressive process

$$\frac{G_t}{Z_{Y,t}} = \left( \frac{G_{t-1}}{Z_{Y,t-1}} \right)^{\rho_G} \left( \frac{G}{Z_Y} \right)^{1-\rho_G} \exp(\varepsilon_{G,t}) \quad \text{(H.1)}$$

while the government runs balanced budgets, setting lump sum taxes equal to government expenditures

$$T_t = G_t \quad \text{(H.2)}$$

### Appendix I  Market Clearing and Equilibrium

We assume a symmetric equilibrium so that:

- $N_t = \int_0^1 N(t)di$, $Y_t = \int_0^1 Y(t)dh$, $D_t = \int_0^1 D_t(z)dz$, $L_t = \int_0^1 L_t(z)dz$, $J_t = \int_0^1 J_t(z)dz$. Substituting the profit and cost functions into the budget constraint gives:

$$C_t + \frac{P_{I,t}}{P_t} I_t + G_t + D_t = \frac{D_{t-1}R_{D,t-1}}{\pi_t} + L_t - \frac{L_{t-1}R_{L,t-1}}{\pi_t} + Y_t + (1 - \omega) \frac{J_{t-1}}{\pi_t} \quad \text{(I.1)}$$

### Appendix J  Potential Output

The following set of equations describe the frictionless economy:

$$Y_t^P = C_t^P + I_t^P + G_t^P \quad \text{(J.1)}$$

$$K_t^P = I_t^P \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t^P}{I_{t-1}^P} - \mu_I \right)^2 \right) + (1 - \delta) K_{t-1}^P \quad \text{(J.2)}$$

51
\[ 1 = Q_{t}^P \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_{t}^P}{I_{t-1}^P} - \mu_I \right)^2 - \frac{\phi_I}{2} \left( \frac{I_{t}^P}{I_{t-1}^P} - \mu_I \right) \frac{I_{t+1}^P}{I_{t+1}^P} \right] + \ldots \]
\[ \ldots + E_t \left\{ \mathcal{M}_{t,t+1} \phi_I Q_{t+1}^P \left( \frac{I_{t+1}^P}{I_{t}^P} - \mu_I \right) \left( \frac{I_{t+1}^P}{I_{t}^P} \right)^2 \right\}, \quad (J.3) \]

\[ Q_{t}^P = E_t \left\{ \mathcal{M}_{t,t+1} \left( \alpha \frac{Y_{t+1}^P}{K_{t+1}^P} + (1 - \delta) Q_{t+1}^P \right) \right\}, \quad (J.4) \]

\[ (1 - \alpha) \frac{Y_{t}^P}{N_{t}^P} = \kappa_{P,t} \frac{(N_{t}^P)^{\eta}}{\lambda_{t}^P}, \quad (J.5) \]

\[ \lambda_{t}^P = A_t \left( C_{t}^P - \chi C_{t-1}^P \right)^{-\sigma}, \quad (J.6) \]

\[ \mathcal{M}_{t,t+1}^P = E_t \left\{ \beta \frac{\lambda_{t+1}^P}{\lambda_{t}^P} \right\}, \quad (J.7) \]

\[ Y_{t}^P = Z_t \left( K_{t-1}^P \right)^{\alpha} \left( N_{t}^P \right)^{1-\alpha}, \quad (J.8) \]

\[ W_{t}^P = (1 - \alpha) \frac{Y_{t}^P}{N_{t}^P}. \quad (J.9) \]

**Appendix K  Trends**

The Cobb Douglas production functions in the intermediate goods and investment goods producing sectors imply the following composite technology processes. The effective investment technology

\[ Z_{I,t} = Z_{I,t} Z_{Y,t}, \quad (K.1) \]

where \( Z_{Y,t} \) is the as yet unknown composite technology for intermediate goods. The intermediate investment production function implies the following relationship for the effective technology in that sector

\[ Z_{Y,t} = Z_t \left( Z_{I,t} Z_{Y,t} \right)^{\alpha}, \quad (K.2) \]

\[ Z_{Y,t} = Z_t^{\frac{1}{1-\alpha}} Z_{I,t}^{\frac{\alpha}{1-\alpha}}. \quad (K.3) \]
Combining K.1 and K.3 allows us to write the effective investment technology in terms of neutral and investment specific technology

\[ Z_{t,t} = Z_t^{\frac{1}{\gamma}} Z_t^{\frac{1}{\gamma}}. \] 

(K.4)

**Appendix L  Model Equations**

For the set of model variables: \( C_t, I_t, K_t, Q_t, P_{t,t}/P_t, D_t, R_{D,t}, \pi_t, W_t/P_t, \pi_{W,t}, N_t, \lambda_t, Y_{L,t}, \Phi_t, \mathcal{M}_{t,t+1}, R_{L,t}, Y_t, \mathbb{R}_{D,t}, \mathbb{R}_{L,t}, L_t, K_{B,t}, J_t, R_t, A_t, Z_t, Z_{t,t}, Z_{Y,t}, \mathcal{S}_{Z,t}, \mathcal{A}_{Z,t}, \mathcal{N}_t, \mathcal{I}_t, \mathcal{R}_t, \mathcal{S}^t, R_{ZLB,t}, \tilde{G}_t, \Delta \log Y_t, \Delta \log I_t, \Delta \log \hat{Y}_t, \hat{F}_t, N_t^P, Q_t^P, Y_t^P, K_t^P, I_t^P, C_t^P, W_t^P, \mathcal{M}_t^P, \lambda_t^P, \kappa_t, \) and \( \mathbb{P}_t, \) the model is described by the following set of equations:

\[ C_t + \frac{P_{t,t}}{P_t} I_t + G_t + D_t = \frac{D_{t-1} R_{D,t-1}}{\pi_t} + L_t - \frac{L_{t-1} R_{L,t-1}}{\pi_t} + Y_t + (1 - \omega) \frac{J_{t-1}}{\pi_t}, \] 

(L.1)

\[ \psi I_t = L_t, \] 

(L.2)

\[ K_t = I_t \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - \mu_I \right)^2 \right) + (1 - \delta) K_{t-1}, \] 

(L.3)

\[ \frac{P_{t,t}}{P_t} = Q_t \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t}{I_{t-1}} - \mu_I \right)^2 - \phi_I \left( \frac{I_t}{I_{t-1}} - \mu_I \right) \frac{I_t}{I_{t-1}} \right) + \psi Y_{L,t} \frac{P_{t,t}}{P_t} + \ldots \]

\[ \ldots + E_t \left\{ \mathcal{M}_{t,t+1} \phi_I Q_{t+1} \left( \frac{I_{t+1}}{I_t} - \mu_I \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\}, \] 

(L.4)

\[ Q_t = E_t \left\{ \mathcal{M}_{t,t+1} \left( \alpha \Phi_{t+1} \frac{Y_{t+1}}{K_t} + (1 - \delta) Q_{t+1} \right) \right\}, \] 

(L.5)

\[ \left( \frac{\phi_W}{\nu - 1} \right) \pi_{W,t} \left[ \pi_{W,t} - \pi_{W,t} \right] = \left( \frac{v}{\nu - 1} \right) \kappa N_t^P \frac{P_t}{\lambda_t W_t} - 1 + \ldots \]

\[ \ldots + E_t \left\{ \left( \frac{\phi_W}{\nu - 1} \right) \mathcal{M}_{t,t+1} \frac{\pi_{W,t+1}^*}{\pi_{t+1}} \left( \frac{N_{t+1}}{N_t} \right) \left[ \pi_{W,t+1} - \pi_{W,t} \right] \right\}, \] 

(L.6)

\[ \frac{W_t}{P_t} = \frac{\pi_{W,t} W_{t-1}}{\pi_t P_{t-1}}, \] 

(L.7)

\[ \lambda_t = A_t (C_t - \chi C_{t-1})^{-\sigma} Z_{Y,t}^{\sigma-1}, \] 

(L.8)
\[ \lambda_t = E_t \left\{ \beta d_{t+1} \frac{\lambda_{t+1} R_{D,t}}{\pi_{t+1}} \right\}, \]  
\text{(L.9)}

\[ 1 - \Upsilon_{L,t} = E_t \left\{ \mathcal{M}_{t,t+1}^* \right\}, \]  
\text{(L.10)}

\[ \mathcal{M}_{t,t+1}^* = E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\}, \]  
\text{(L.11)}

\[ \left( \frac{\phi_P}{\varepsilon - 1} \right) \pi_t [\pi_t - \tilde{\pi}_t] = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \Phi_t - \exp (\mathbb{P}_t) + \ldots \]  
\[ \ldots + E_t \left\{ \left( \frac{\phi_P}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1}^* \frac{Y_{t+1}}{Y_t} [\pi_{t+1} - \tilde{\pi}_{t+1}] \right\}, \]  
\text{(L.12)}

\[ K_{B,t} = (1 - \delta_B) K_{B,t-1} + \omega J_{t-1}, \]  
\text{(L.13)}

\[ J_t = R_{L,t} L_t - R_{D,t} D_t - R_t K_{B,t}, \]  
\text{(L.14)}

\[ L_t = D_t + K_{B,t}, \]  
\text{(L.15)}

\[ \mathbb{R}_{D,t} = R_t, \]  
\text{(L.16)}

\[ \mathbb{R}_{L,t} = R_t, \]  
\text{(L.17)}

\[ Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha}, \]  
\text{(L.18)}

\[ \frac{W_t}{P_t} = (1 - \alpha) \Phi_t \frac{Y_t}{N_t}, \]  
\text{(L.19)}

\[ R_t^* = R_{t-1}^{*\rho_R} \left( R^* \left( \frac{\pi_t}{\pi} \right)^{\kappa_n} \left( \frac{Y_t}{\bar{Y}} \right)^{\kappa_Y} \right)^{1-\rho_R} \exp (\varepsilon_{R,t}), \]  
\text{(L.20)}

\[ \hat{Y}_t = \kappa_P \left( \frac{Y_t}{\bar{Y}_t} \right) \exp (F_t), \]  
\text{(L.21)}

\[ F_t = \rho_F F_{t-1} + \varepsilon_{F,t}, \]  
\text{(L.22)}
\[ R_{ZLB,t} = K + \varepsilon_{ZLB,t}, \quad \text{(L.23)} \]

\[ R_t = z(s_{1,t})R_{ZLB,t} + (1 - z(s_{1,t}))R_t^*, \quad \text{(L.24)} \]

\[ \frac{G_t}{Z_{Y,t}} = \left( \frac{G_{t-1}}{Z_{Y,t-1}} \right)^{\rho_G} \left( \frac{G}{Z_Y} \right)^{1-\rho_G} \exp(\varepsilon_{G,t}), \quad \text{(L.25)} \]

\[ \frac{P_{I,t}}{P_t} = \frac{1}{Z_{I,t}}, \quad \text{(L.26)} \]

\[ \log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \quad \text{(L.27)} \]

\[ Z_t = Z_0 \exp(g_{Zt} + A_{Z,t}), \quad \text{(L.28)} \]

\[ Z_{I,t} = Z_{I,0} \exp(g_{Zt} + A_{Z,t}), \quad \text{(L.29)} \]

\[ \left( \frac{v_L(r_t)}{v_L(r_t) - 1} \right) \exp(\varepsilon_{L,t}) \left( \frac{R_{L,t}}{R_{L,t+1}} \right) - \phi_L(r_t) \left( \frac{R_{L,t}}{R_{L,t+1}} \right)^2 \left( \frac{R_{L,t+1}}{R_{L,t}} \right) - 1 + \ldots \]

\[ \ldots + E_t \left\{ \phi_L(r_{t+1}) \mathcal{M}_{t+1}^{-1} \left( \frac{1}{\pi_{t+1}} \right) \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \left( \frac{R_{L,t+1}}{R_{L,t}} \right) - 1 \right\} = 0, \quad \text{(L.30)} \]

\[ 1 - \left( \frac{v_D(r_t)}{v_D(r_t) - 1} \right) \exp(\varepsilon_{D,t}) \left( \frac{R_{D,t}}{R_{D,t+1}} \right) - \phi_D(r_t) \left( \frac{R_{D,t}}{R_{D,t+1}} \right)^2 \left( \frac{R_{D,t+1}}{R_{D,t}} \right) - 1 + \ldots \]

\[ \ldots + E_t \left\{ \phi_D(r_{t+1}) \mathcal{M}_{t+1}^{-1} \left( \frac{1}{\pi_{t+1}} \right) \left( \frac{R_{D,t+1}}{R_{D,t}} \right)^2 \left( \frac{R_{D,t+1}}{R_{D,t}} \right) - 1 \right\} = 0, \quad \text{(L.31)} \]

\[ Y_t^P = C_t^P + I_t^P + G_t^P, \quad \text{(L.32)} \]

\[ K_t^P = I_t^P \left( 1 - \frac{\phi_I}{2} \left( \frac{I_t^P}{I_{t-1}^P} - \mu_t \right)^2 \right) + (1 - \delta) K_{t-1}^P, \quad \text{(L.33)} \]
\[ 1 = Q_t^P \left[ 1 - \frac{\phi_I}{2} \left( \frac{I_{t}^P}{I_{t-1}^P} - \mu_I \right)^2 - \phi_I \left( \frac{I_{t}^P}{I_{t-1}^P} - \mu_I \right) \frac{I_{t}^P}{I_{t-1}^P} \right] + \ldots \]

\[ \ldots + E_t \left\{ \mathcal{M}_{t,t+1}^P \phi_I Q_{t+1}^P \left( \frac{I_{t+1}^P}{I_t^P} - \mu_I \right) \left( \frac{I_{t+1}^P}{I_t^P} \right)^2 \right\}, \quad (L.34) \]

\[ Q_t^P = E_t \left\{ \mathcal{M}_{t,t+1}^P \left( \alpha \frac{Y_{t+1}^P}{K_t^P} + (1 - \delta) Q_{t+1}^P \right) \right\}, \quad (L.35) \]

\[ (1 - \alpha) \frac{Y_t^P}{N_t^P} = \kappa_{P,t} \left( \frac{N_t^P}{\lambda_t^P} \right)^\eta, \quad (L.36) \]

\[ \lambda_t^P = A_t \left( C_t^P - \chi C_{t-1}^P \right)^{-\sigma}, \quad (L.37) \]

\[ \mathcal{M}_{t,t+1}^P = E_t \left\{ \beta \frac{\lambda_t^{P+1}}{\lambda_t^P} \right\}, \quad (L.38) \]

\[ Y_t^P = Z_t \left( K_{t-1}^P \right)^\alpha \left( N_t^P \right)^{1-\alpha}, \quad (L.39) \]

\[ W_t^P = (1 - \alpha) \frac{Y_t^P}{N_t^P}, \quad (L.40) \]

\[ \Pi_t = \rho \Pi_{t-1} + \varepsilon_{\Pi,t}, \quad (L.41) \]

\[ \kappa_t = \rho_k \kappa_{t-1} + \varepsilon_{\kappa,t}, \quad (L.42) \]

\[ \tilde{\pi}_t = \pi_{t-1}^{\xi} \pi_t^{1-\xi}, \quad (L.43) \]

\[ \tilde{\pi}_{w,t} = \pi_{t-1}^{\xi_w} \pi_t^{1-\xi_w}, \quad (L.44) \]

\[ \hat{N}_t = \frac{N_t}{N} \exp \left( \varepsilon_{N,t} \right), \quad (L.45) \]

\[ \Delta \log Y_t = \log Y_t - \log Y_{t-1}, \quad (L.46) \]

\[ \Delta \log C_t = \log C_t - \log C_{t-1}, \quad (L.47) \]
\[ \Delta \log I_t = \log I_t - \log I_{t-1}, \quad (L.48) \]

\[ A_{Z,t} = \rho A_{Z,t} + \varepsilon_{Z,t}, \quad (L.49) \]

\[ A_{Z,t} = \rho A_{Z,t} + \varepsilon_{Z,t}, \quad (L.50) \]

\[ Z_{Y,t} = Z_{Y}^{1} - \alpha \pi_{t} + \beta \pi_{t}^{1} - \alpha I_{Z}, \quad (L.51) \]

\[ Z_{I,t} = Z_{I}^{1} - \alpha \pi_{t} + \beta \pi_{t}^{1} - \alpha I_{Z}, \quad (L.52) \]

**Appendix M Stochastically Detrended Model**

The model is non-stationary. To make the model stationary we rewrite the set of model equations in terms of the following stochastically detrended variables: 

\[ \tilde{C}_{t} = C_{t}/Z_{Y,t}, \]
\[ \tilde{I}_{t} = I_{t}/Z_{I,t}, \]
\[ \tilde{K}_{t} = K_{t}/Z_{I,t}, \]
\[ \tilde{Q}_{t} = Q_{t}/Z_{I,t}, \]
\[ \tilde{D}_{t} = D_{t}/Z_{Y,t}, \]
\[ \tilde{RD}_{t} = R_{D,t}/Z_{Y,t}, \]
\[ \tilde{L}_{t} = L_{t}/Z_{Y,t}, \]
\[ \tilde{W}_{t} = W_{t}/Z_{Y,t}, \]
\[ \tilde{W}_{P} = W_{P,t}/Z_{Y,t}, \]
\[ \tilde{L}_{t}^{P} = L_{t}^{P}/Z_{I,t}, \]
\[ \tilde{K}_{t}^{P} = K_{t}^{P}/Z_{I,t}, \]
\[ \tilde{I}_{t}^{P} = I_{t}^{P}/Z_{I,t}, \]
\[ \tilde{C}_{t}^{P} = C_{t}^{P}/Z_{Y,t}, \]
\[ \tilde{W}_{P}^{P} = W_{P,t}^{P}/Z_{Y,t}, \]
\[ \tilde{L}_{t}^{P} = L_{t}^{P}/Z_{I,t}, \]
\[ \tilde{K}_{t}^{P} = K_{t}^{P}/Z_{I,t}, \]
\[ \tilde{I}_{t}^{P} = I_{t}^{P}/Z_{I,t}, \]
\[ \tilde{C}_{t}^{P} = C_{t}^{P}/Z_{Y,t}, \]
\[ \tilde{W}_{P}^{P} = W_{P,t}^{P}/Z_{Y,t}, \]

The transformed set of model equations:

\[ \tilde{C}_{t} + \tilde{I}_{t} + \tilde{G}_{t} + \tilde{D}_{t} = \frac{\tilde{D}_{t-1} - \tilde{R}_{D,t-1}}{\mu_{Y,t} \pi_{t}} + \tilde{L}_{t} - \frac{\tilde{L}_{t-1} - \tilde{R}_{L,t-1}}{\mu_{Y,t} \pi_{t}} + \tilde{Y}_{t} + (1 - \omega) \frac{\tilde{J}_{t-1}}{\mu_{Y,t} \pi_{t}}, \quad (M.1) \]

\[ \psi \tilde{I}_{t} = \tilde{L}_{t}, \quad (M.2) \]

\[ \tilde{K}_{t} = \tilde{I}_{t} \left( 1 - \frac{\phi_{I}}{2} \left( \frac{\tilde{I}_{t}}{\mu_{I,t}} - \mu_{I} \right) \right)^{2} + (1 - \delta) \frac{\tilde{K}_{t-1}}{\mu_{I,t}}, \quad (M.3) \]
\[
\hat{P}_{t,t} = \hat{Q}_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{\hat{I}_t}{I_{t-1}} \mu_{I,t} - \mu_I \right) \right]^2 - \phi_I \left( \frac{\hat{I}_t}{I_{t-1}} \mu_{I,t} - \mu_I \right) \hat{P}_{I,t} + \psi \gamma_{L,t} \hat{P}_{I,t} + \ldots
\]

\[
\ldots + E_t \left\{ \mathcal{M}_{t,t+1}^* \Phi_{t+1} \left( \frac{\hat{I}_{t+1}}{I_t} \mu_{I,t+1} - \mu_I \right) \left( \frac{\hat{I}_{t+1}}{I_t} \mu_{I,t+1} \right) \right\}, \quad (M.4)
\]

\[
\hat{Q}_t = E_t \left\{ \mathcal{M}_{t,t+1}^* \left( \alpha \Phi_{t+1} \hat{Y}_{t+1} + (1 - \delta) \frac{\hat{Q}_{t+1}}{\mu_{Z_t,t+1}} \right) \right\}, \quad (M.5)
\]

\[
\left( \frac{\phi_W}{v - 1} \right) \pi_{W,t} [\pi_{W,t} - \tilde{\pi}_{W,t}] = \left( \frac{v}{v - 1} \right) \kappa \frac{N_i^n}{\hat{\lambda}_W} - 1 + \ldots
\]

\[
\ldots + E_t \left\{ \left( \frac{\phi_W}{v - 1} \right) \mathcal{M}_{t,t+1}^* \left( \frac{\hat{\pi}_{W,t+1}^2}{\pi_{t+1}} \right) \left( \frac{N_{t+1}}{N_t} \right) [\pi_{W,t+1} - \tilde{\pi}_{W,t+1}] \right\}, \quad (M.6)
\]

\[
\tilde{W}_t = \frac{\pi_{W,t}}{\mu_{Y,t} \tilde{\pi}_t} W_{t-1},
\]

\[
\tilde{\lambda}_t = A_t \left( \tilde{C}_t - \chi \frac{\hat{C}_{t-1}}{\mu_{Y,t}} \right)^{-\sigma},
\]

\[
\tilde{\lambda}_t = E_t \left\{ \beta d_{t+1} \frac{\hat{\lambda}_{t+1} R_{D,t}}{\mu_{Y,t+1} \pi_{t+1}} \right\}, \quad (M.9)
\]

\[
1 - \gamma_{L,t} = E_t \left\{ \mathcal{M}_{t,t+1}^* d_{t+1} \frac{R_{L,t}}{\pi_{t+1}} \right\}, \quad (M.10)
\]

\[
\mathcal{M}_{t,t+1}^* = E_t \left\{ \beta \frac{\hat{\lambda}_{t+1}}{\mu_{Y,t+1} \hat{\lambda}_t} \right\}, \quad (M.11)
\]

\[
\left( \frac{\phi_P}{\varepsilon - 1} \right) \pi_t [\pi_t - \tilde{\pi}_t] = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \Phi_t - \exp (\hat{P}_t) + \ldots
\]

\[
\ldots + E_t \left\{ \left( \frac{\phi_P}{\varepsilon - 1} \right) \mathcal{M}_{t,t+1}^* \frac{\hat{Y}_{t+1}}{\hat{Y}_t} \mu_{Y,t+1} \pi_{t+1} [\pi_{t+1} - \tilde{\pi}_{t+1}] \right\}, \quad (M.12)
\]
\[
\left( \frac{v_L(r_t)}{v_L(r_t) - 1} \right) \frac{R_{L,t}}{R_{L,t}} - 1 - \tilde{\phi}_L(r_t) \frac{R_{L,t}}{R_{L,t-1}} \left[ \frac{R_{L,t}}{R_{L,t-1}} - 1 \right] + \ldots \\
\ldots + E_t \left\{ \tilde{\phi}_L(r_{t+1}) \mathcal{M}^*_{t,t+1} \left( \frac{1}{\pi_{t+1}} \right) \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \frac{\tilde{L}_{t+1}}{L_t} \mu_{Y,t+1} \left[ \frac{R_{L,t+1}}{R_{L,t}} - 1 \right] \right\} = 0, \quad (M.13)
\]

\[
1 - \left( \frac{v_D(r_t)}{v_D(r_t) - 1} \right) \frac{R_{D,t}}{R_{D,t}} - \tilde{\phi}_D(r_t) \frac{R_{D,t}}{R_{D,t-1}} \left[ \frac{R_{D,t}}{R_{D,t-1}} - 1 \right] + \ldots \\
\ldots + E_t \left\{ \tilde{\phi}_D(r_{t+1}) \mathcal{M}^*_{t,t+1} \left( \frac{1}{\pi_{t+1}} \right) \left( \frac{R_{D,t+1}}{R_{D,t}} \right)^2 \frac{\tilde{D}_{t+1}}{D_t} \mu_{Y,t+1} \left[ \frac{R_{D,t+1}}{R_{D,t}} - 1 \right] \right\} = 0, \quad (M.14)
\]

\[
\tilde{L}_t = \tilde{D}_t + \tilde{K}_{B,t}, \quad (M.15)
\]

\[
\tilde{K}_{B,t} = (1 - \delta_B) \frac{\tilde{K}_{B,t-1}}{\mu_{Y,t}} + \omega \frac{\tilde{J}_{t-1}}{\mu_{Y,t}}, \quad (M.16)
\]

\[
\tilde{J}_t = R_{L,t} \tilde{L}_t - R_{D,t} \tilde{D}_t - R_t \tilde{K}_{B,t}, \quad (M.17)
\]

\[
R_{D,t} = R_t, \quad (M.18)
\]

\[
R_{L,t} = R_t, \quad (M.19)
\]

\[
\tilde{Y}_t = \left( \frac{\tilde{K}_{t-1}}{\mu_{Y,t}} \right)^\alpha N_t^{1-\alpha}, \quad (M.20)
\]

\[
\tilde{W}_t = (1 - \alpha) \Phi_t \frac{\tilde{Y}_t}{N_t}, \quad (M.21)
\]

\[
R_t^* = R_{t-1}^{\rho_R} \left( R^* \left( \frac{\pi_t}{\pi} \right)^{\kappa_R} \left( \tilde{Y}_t \right)^{\kappa_R} \right)^{1-\rho_R} \exp (\varepsilon_{R,t}), \quad (M.22)
\]

\[
\tilde{Y}_t = \kappa_P \left( \frac{Y_t}{\tilde{Y}_t^p} \right) \exp (F_t), \quad (M.23)
\]

\[
F_t = \rho_F F_{t-1} + \varepsilon_{F,t}, \quad (M.24)
\]
\[ R_{ZLB,t} = K + \varepsilon_{ZLB,t}, \]  
(M.25)

\[ R_t = z(s_{1,t})R_{ZLB,t} + (1 - z(s_{1,t}))R_t^*, \]  
(M.26)

\[ \frac{G_t}{Z_Y,t} = \left( \frac{G_{t-1}}{Z_Y,t-1} \right)^{\rho_G} \left( \frac{G}{Z_Y} \right)^{1-\rho_G} \exp(\varepsilon_{G,t}), \]  
(M.27)

\[ \log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \]  
(M.28)

\[ \log \mu_{Z,t} = g_Z + \Delta\Delta_{Z,t}, \]  
(M.29)

\[ \log \mu_{Z,t} = g_{Z_I} + \Delta\Delta_{Z,t}, \]  
(M.30)

\[ \log \mu_{Z,t} = \left( \frac{1}{1-\alpha} \right)(\log \mu_{Z,t} + \alpha \log \mu_{Z,t}), \]  
(M.31)

\[ \log \mu_{I,t} = \left( \frac{1}{1-\alpha} \right)(\log \mu_{Z,t} + \log \mu_{Z,t}), \]  
(M.32)

\[ \tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t, \]  
(M.33)

\[ \tilde{K}_t = I_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{\tilde{I}_t}{I_{t-1}} - \mu_I \right)^2 + \left( 1 - \delta \right) \frac{\tilde{K}_{t-1}}{\mu_{I,t}} \right], \]  
(M.34)

\[ 1 = \tilde{Q}_t \left[ 1 - \frac{\phi_I}{2} \left( \frac{\tilde{I}_t}{I_{t-1}} - \mu_I \right)^2 - \phi_I \left( \frac{\tilde{I}_t}{I_{t-1}} - \mu_I \right) \frac{\tilde{I}_t}{I_{t-1}} - \mu_I \right] + \ldots \]

\[ \ldots + E_t \left\{ \mathcal{M}_{t,t+1} \phi_{I_{t+1}} \tilde{Q}_{t+1} \frac{\tilde{I}_{t+1}}{I_{t+1}} - \mu_I \left( \frac{\tilde{I}_{t+1}}{I_{t+1}} - \mu_I \right) - \mu_{I,t+1} \right\}, \]  
(M.35)

\[ \tilde{Q}_t = E_t \left\{ \mathcal{M}_{t,t+1} \left( \alpha \frac{\tilde{Y}_{t+1}}{\tilde{K}_t} \mu_{Y,t+1} + (1 - \delta) \frac{\tilde{Q}_{t+1}}{\mu_{Z,t+1}} \right) \right\}, \]  
(M.36)

\[ (1 - \alpha) \frac{\tilde{Y}_t}{N_t} = \kappa \left( \frac{N_t^P}{N_t^P} \right)^{\eta}, \]  
(M.37)
\[ \tilde{X}_t^P = A_t \left( \tilde{C}_t^P - \chi \frac{\tilde{C}_t^{P-1}}{\mu_{Y,t}} \right)^{-\sigma}, \]  
(M.38)

\[ \mathcal{M}_{t,t+1}^P = E_t \left\{ \beta \frac{\tilde{X}_{t+1}^P}{\mu_{Y,t+1}^P} \right\}, \]  
(M.39)

\[ \tilde{Y}_t^P = \left( \frac{\tilde{K}_{t-1}^P}{\mu_{I,t}} \right)^{\alpha} (N_t^P)^{1-\alpha}, \]  
(M.40)

\[ \tilde{W}_t^P = (1 - \alpha) \frac{\tilde{Y}_t^P}{N_t^P}, \]  
(M.41)

\[ \tilde{P}_t = \rho \tilde{P}_{t-1} + \varepsilon_{\tilde{P},t}, \]  
(M.42)

\[ \kappa_t = \rho \kappa_{t-1} + \varepsilon_{\kappa,t}, \]  
(M.43)

\[ \tilde{\pi}_t = \pi_{t-1}^{\xi} \pi_1^{1-\xi}, \]  
(M.44)

\[ \tilde{\pi}_{W,t} = \pi_{t-1}^{\xi_w} \pi_1^{1-\xi_w}, \]  
(M.45)

\[ \tilde{N}_t = \frac{N_t}{N} \exp (\varepsilon_{N,t}), \]  
(M.46)

\[ \Delta \log Y_t = \log \tilde{Y}_t - \log \tilde{Y}_{t-1} + \log \mu_{Y,t}, \]  
(M.47)

\[ \Delta \log C_t = \log \tilde{C}_t - \log \tilde{C}_{t-1} + \log \mu_{Y,t}, \]  
(M.48)

\[ \Delta \log I_t = \log \tilde{I}_t - \log \tilde{I}_{t-1} + \log \mu_{I,t}, \]  
(M.49)

\[ sA_{Z,t} = \rho sA_{Z,t} + \varepsilon_{Z,t}, \]  
(M.50)

\[ sA_{Z,t} = \rho sA_{Z,t} + \varepsilon_{Z,t}, \]  
(M.51)

\[ \tilde{Z}_{Y,t}^{\frac{1}{\alpha}} Z_{Y,t}^{\frac{\alpha}{\alpha}}, \]  
(M.52)
\[ Z_{l,t} = Z_i^\frac{1}{t} Z_{l,t}^{\frac{1}{t}}. \] (M.53)

**Appendix N  Steady State Model**

\[ \phi_L(r_t) = z(s_{1,t})\phi_{ZLB,L} + (1 - z(s_{1,t})) (m(s_{2,t})\phi_{H,L} + (1 - m(s_{2,t})) \phi_{L,L}), \] (N.1)

\[ \phi_D(r_t) = z(s_{1,t})\phi_{ZLB,D} + (1 - z(s_{1,t})) (m(s_{2,t})\phi_{H,D} + (1 - m(s_{2,t})) \phi_{L,D}), \] (N.2)

\[ \mu_D(r_t) = z(s_{1,t})\mu_{ZLB,D} + (1 - z(s_{1,t})) (m(s_{2,t})\mu_{H,D} + (1 - m(s_{2,t})) \mu_{L,D}), \] (N.3)

\[ \mu_L(r_t) = z(s_{1,t})\mu_{ZLB,L} + (1 - z(s_{1,t})) (m(s_{2,t})\mu_{H,L} + (1 - m(s_{2,t})) \mu_{L,L}), \] (N.4)

\[ \frac{\mu_D(r_t)}{\mu_D(r_t)} - 1 = \mu_D(r_t), \] (N.5)

\[ \frac{\mu_L(r_t)}{\mu_L(r_t)} - 1 = \mu_L(r_t), \] (N.6)

\[ \mathcal{A}_Z,t = 0, \] (N.7)

\[ \mathcal{A}_{Z,t} = 0, \] (N.8)

\[ A_t = 1, \] (N.9)

\[ \pi_t = \pi, \] (N.10)

\[ \tilde{\pi}_t = \pi, \] (N.11)

\[ \log \mu_{Z,t} = g_Z + \Delta \mathcal{A}_{Z,t}, \] (N.12)
\[
\log \mu_{Z,t} = g_Z + \Delta A_{Z,t}, \quad \text{(N.13)}
\]
\[
\log \mu_{Y,t} = \left( \frac{1}{1-\alpha} \right) \left( \log \mu_{Z,t} + \alpha \log \mu_{Z,t} \right), \quad \text{(N.14)}
\]
\[
\log \mu_{I,t} = \left( \frac{1}{1-\alpha} \right) \left( \log \mu_{Z,t} + \log \mu_{Z,t} \right), \quad \text{(N.15)}
\]
\[
\pi_{W,t} = \mu_{Y,t} \pi_t, \quad \text{(N.16)}
\]
\[
\tilde{\pi}_{W,t} = \pi_{W,t}, \quad \text{(N.17)}
\]
\[
\tilde{P}_{I,t} = 1, \quad \text{(N.18)}
\]
\[
R_{ZLB,t} = \frac{R_{ZLB,D}}{\mu_{ZLB,D}}, \quad \text{(N.19)}
\]
\[
R_t^* = \left( \frac{\pi_t}{\beta \mu_{N,D}} \right) \exp \left( \left( \frac{1}{1-\alpha} \right) (g_Z + \alpha g_{Z_I}) \right), \quad \text{(N.20)}
\]
\[
R_t = z(s_{1,t}) R_{ZLB,t} + (1 - z(s_{1,t})) R_t^*, \quad \text{(N.21)}
\]
\[
R_{D,t} = \mu_D(r_t) R_t, \quad \text{(N.22)}
\]
\[
d_t = \pi_t \exp \left( \left( \frac{1}{1-\alpha} \right) (g_Z + \alpha g_{Z_I}) \right) \frac{1}{R_{D,t} \beta}, \quad \text{(N.23)}
\]
\[
\mathbb{R}_{L,t} = R_t, \quad \text{(N.24)}
\]
\[
\mathbb{R}_{D,t} = R_t, \quad \text{(N.25)}
\]
\[
\mathcal{M}_{t,t+1}^* = \frac{\beta}{\mu_{Y,t}}, \quad \text{(N.26)}
\]
\[
\Upsilon_{L,t} = 1 - \frac{\beta d_t R_{L,t}}{\mu_{Y,t} \pi_t}, \quad \text{(N.27)}
\]
\[
\begin{align*}
\bar{q} &= \left( \frac{1}{1-i_y-g_y} \right) \left( \frac{(R_{L,t} - R_{D,t}) \psi i_y \left( 1 - \frac{R_t - 1 + \delta_B}{\mu_{Y,t} \pi_t} \right) - \frac{1}{\pi_t}}{R_t - R_{D,t} + \frac{1-(1-\delta_B)\mu_{Y,t}}{\omega/\mu_{Y,t}}} - i_y - g_y + 1 \right), \\
& \quad \text{(N.32)} \\
\bar{c}_y &= \bar{q} (1 - i_y - g_y), \\
& \quad \text{(N.33)} \\
n_y &= \left( \frac{k_y}{\mu_{I,t}} \right)^{\frac{\omega}{\sigma}}, \\
& \quad \text{(N.34)} \\
\bar{W}_t &= \left( 1 - \alpha \right) \Phi_t, \\
& \quad \text{(N.35)} \\
l_y &= \psi i_y, \\
& \quad \text{(N.36)} \\
k_b_y &= \frac{(c_y + i_y + g_y - 1)}{(1 - ((R_t - 1 + \delta_B)/(\mu_{Y,t} \pi_t)) - 1/\pi_t)}, \\
& \quad \text{(N.37)} \\
d_y &= l_y - k_b_y, \\
& \quad \text{(N.38)} \\
j_y &= R_{L,t} l_y - R_{D,t} d_y - R_t k_b_y, \\
& \quad \text{(N.39)} \\
\bar{Y}_t &= \left( \frac{v - 1}{v} \right) \left( \frac{A \bar{W}_t}{\kappa(c_y - \chi c_y/\mu_{Y,t})^{\sigma n_y^{\eta}}} \right)^{\frac{1}{\sigma + \eta}}, \\
& \quad \text{(N.40)} \\
N_t &= n_y \bar{Y}_t, \\
& \quad \text{(N.41)}
\end{align*}
\]
\[ \tilde{K}_t = k_{-y} \tilde{Y}_t, \quad (N.42) \]
\[ \tilde{I}_t = i_{-y} \tilde{Y}_t, \quad (N.43) \]
\[ \tilde{C}_t = c_{-y} \tilde{Y}_t, \quad (N.44) \]
\[ \tilde{G}_t = g_{-y} \tilde{Y}_t, \quad (N.45) \]
\[ \tilde{L}_t = l_{-y} \tilde{Y}_t, \quad (N.46) \]
\[ \tilde{K}_{B,t} = k_{b_{-y}} \tilde{Y}_t, \quad (N.47) \]
\[ \tilde{D}_t = d_{-y} \tilde{Y}_t, \quad (N.48) \]
\[ \tilde{J}_t = j_{-y} \tilde{Y}_t, \quad (N.49) \]
\[ \tilde{\lambda}_t = A_t (\tilde{C}_t - \chi \tilde{C}_t / \mu_{Y,t})^{-\sigma}, \quad (N.50) \]
\[ \Delta \log Y_t = \log \mu_{Y,t}, \quad (N.51) \]
\[ \Delta \log C_t = \log \mu_{Y,t}, \quad (N.52) \]
\[ \Delta \log I_t = \log \mu_{I,t}, \quad (N.53) \]
\[ \hat{N}_t = 1, \quad (N.54) \]
\[ \hat{Y}_t = 1, \quad (N.55) \]
\[ F_t = 0, \quad (N.56) \]
\[ \mathbb{P}_t = 0, \quad (N.57) \]
\[ \kappa_t = 0, \quad (N.58) \]
\[ \hat{Q}_t^P = 1, \quad (N.59) \]
\[ \mathcal{M}_t^P = \beta / \mu_{Y,t}, \quad (N.60) \]
\[ kp_{-y} = \frac{\alpha \mu_{Y,t}}{\hat{Q}_t^P (1 / \mathcal{M}_t^P - (1 - \delta) / \mu_{Z,t})}, \quad (N.61) \]
\[ ip_{-y} = kp_{-y} (1 - (1 - \delta) / \mu_{I,t}), \quad (N.62) \]
\[ np_{-y} = \left( \frac{k_{p_{-y}}}{\mu_{I,t}} \right)^{\frac{\alpha}{1-\alpha}}, \quad (N.63) \]
\[ w_t^P = \frac{(1 - \alpha)}{np_{-y}}, \quad (N.64) \]
\[ Y_t^P = \left( \frac{A_t \tilde{Y}_t^P}{(\kappa_P(cP_y - \chi(cP_y / \mu_{Y,t})^\sigma nP_y^n))^{1/\sigma + \eta}} \right), \]  
(N.65)

\[ N_t^P = nP_y \tilde{Y}_t^P, \]  
(N.66)

\[ \tilde{K}_t^P = kP_y \tilde{Y}_t^P, \]  
(N.67)

\[ \tilde{I}_t^P = iP_y \tilde{Y}_t^P, \]  
(N.68)

\[ \tilde{C}_t^P = \tilde{Y}_t^P - \tilde{I}_t^P - g_y \tilde{Y}_t^P, \]  
(N.69)

\[ \tilde{\lambda}_t^P = A_t(\tilde{C}_t^P - \chi(\tilde{C}_t^P / \mu_{Y,t})^{-\sigma}). \]  
(N.70)

**N.1 Omega**

\[ \tilde{K}_{B,t} = \frac{\tilde{C}_t + \tilde{I}_t + \tilde{G}_t - \tilde{Y}_t}{1 - \left( \frac{R_t - 1 + \delta_B}{\mu_{Y,t} \pi_t} \right) - \frac{1}{\pi_t}}, \]  
(N.71)

\[ \tilde{K}_{B,t} = \frac{\varrho \left( \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t \right) + \tilde{I}_t + \tilde{G}_t - \tilde{Y}_t}{1 - \left( \frac{R_t - 1 + \delta_B}{\mu_{Y,t} \pi_t} \right) - \frac{1}{\pi_t}}, \]  
(N.72)

\[ \tilde{J}_t = R_{L,t} \psi \tilde{I}_t - R_{D,t} \left( \psi \tilde{I}_t - \tilde{K}_{B,t} \right) - R_t \tilde{K}_{B,t}, \]  
(N.73)

\[ \omega = \tilde{K}_{B,t} \left( \frac{1 - (1 - \delta_B) / \mu_{Y,t}}{J_{B,t} / \mu_{Y,t}} \right), \]  
(N.74)

\[ R_{L,t} \psi \tilde{I}_t - R_{D,t} \left( \psi \tilde{I}_t - \tilde{K}_{B,t} \right) - R_t \tilde{K}_{B,t} = \tilde{K}_{B,t} \left( \frac{1 - (1 - \delta_B) / \mu_{Y,t}}{\omega / \mu_{Y,t}} \right), \]  
(N.75)

\[ (R_{L,t} - R_{D,t}) \psi \tilde{I}_t = \tilde{K}_{B,t} \left( R_t - R_{D,t} + \frac{1 - (1 - \delta_B) / \mu_{Y,t}}{\omega / \mu_{Y,t}} \right), \]  
(N.76)

\[ \frac{(R_{L,t} - R_{D,t}) \psi \tilde{I}_t}{(R_t - R_{D,t} + \frac{1-(1-\delta_B)/\mu_{Y,t}}{\omega / \mu_{Y,t}})} = \varrho \left( \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t \right) + \tilde{I}_t + \tilde{G}_t - \tilde{Y}_t \left( \frac{1 - (1 - \delta_B) / \mu_{Y,t}}{\omega / \mu_{Y,t}} \right), \]  
(N.77)

\[ \left( \frac{(R_{L,t} - R_{D,t}) \psi \tilde{I}_t}{(R_t - R_{D,t} + \frac{1-(1-\delta_B)/\mu_{Y,t}}{\omega / \mu_{Y,t}})} - \tilde{I}_t - \tilde{G}_t + \tilde{Y}_t \right) \frac{1}{(\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t)} = \varrho. \]  
(N.78)
N.2 Interest Rate Setting Frictions: Taylor Contracts

N.2.1 Loan Branch

Loan taking banks, when subject to a Taylor-contracting friction, set interest rate contracts for a maximum duration of \( n \) quarters. In the general representation we allowing for stochastic contract duration by banks. Stochastic duration follows a similar idea to Calvo pricing, namely there is some probability each period that the loan contract could be broken, in which case both parties would have to renegotiate a new contract. In the general setup there exists a separate probability of separation for contracts that have lasted for different lengths. Indexation follows the pattern used in price and wage setting, namely that banks can index to the differential between last periods interest rate and the initial periods interest rate.

\[
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \left[ R_{L,0}(z)L_{0,t}(z) - \mathbb{R}_{L,t}L_{0,t}(z) \right] \right\}. \tag{N.79}
\]

where

\[
a_{0,t} = \prod_{j=0}^{t} a_{j,t}(r_t), \text{ where } a_{0,t}(r_t) = 1,
\]

are the probabilities of continuing the contract into the next period. In the classical Taylor contracting setup \( a_{j,t} = 1 \) for \( j = 0, \ldots, t \). Demand for loans for banks that set contracts in period 0 obey the following schedule

\[
L_{0,t}(z) = \left( \frac{R_{L,0}(z)}{R_{L,t}} \right)^{-\nu_L} L_t. \tag{N.80}
\]

Substituting the demand function N.80 into N.79 gives

\[
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \left[ (R_{L,0}(z))^{1-\nu_L} R_{L,t}^{\nu_L} L_t - \mathbb{R}_{L,t} L_{0,t}(z) \right] \right\}.
\]

The first order condition

\[
\frac{\partial \Psi_{L,0}(z)}{\partial R_{L,0}(z)} = E_t \left\{ \sum_{t=0}^{n-1} a_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \left[ (1 - \nu_L) R_{L,0}(z)^{-\nu_L} R_{L,t}^{\nu_L} L_t + \ldots + \nu_L \mathbb{R}_{L,t} R_{L,0}(z)^{-\nu_L-1} R_{L,t}^{\nu_L} L_t \right] \right\} = 0.
\]
From the first order condition we get the relationship for interest rates set in period 0

\[ R_{L,0}(z) = \left( \frac{v_L}{v_L - 1} \right) \frac{E_t \left\{ \sum_{t=0}^{n-1} \alpha_{0,t} \mathcal{M}_{0,t} \left( \frac{P_t}{P_t^0} \right) [R_{L,t}^{0,t} R_{L,t}^{1,t}] \right\}}{E_t \left\{ \sum_{t=0}^{n-1} \alpha_{0,t} \mathcal{M}_{0,t} \left( \frac{P_t}{P_t^0} \right) R_{L,t}^{1,t} \right\}}. \]

Which we can rewrite in stochastically detrended form as follows

\[ R_{L,0}(z) = \left( \frac{v_L}{v_L - 1} \right) \frac{E_t \left\{ \sum_{t=0}^{n-1} \alpha_{0,t} \mathcal{M}_{0,t} \left( \frac{P_t}{P_t^0} \right) [R_{L,t}^{0,t} R_{L,t}^{1,t} \tilde{L}_t \prod_{k=0}^{t} \mu_{Y,k}] \right\}}{E_t \left\{ \sum_{t=0}^{n-1} \alpha_{0,t} \mathcal{M}_{0,t} \left( \frac{P_t}{P_t^0} \right) R_{L,t}^{1,t} \tilde{L}_t \prod_{k=0}^{t} \mu_{Y,k} \right\}}. \]

where \( \mu_{Y,0} = 1 \). Using the aggregate interest rate index \( x \), we can construct an index for aggregate loan interest rates using the interest rates set by the different cohorts of banks

\[ R_{L,t} = \left( \frac{1}{\sum_{k=0}^{t} \alpha_{t-k,t} (R_{L,t-k}(z))^{1-v_L}} \right)^{1-v_L}. \]

N.2.2 Deposit Branch

Likewise, deposit taking banks subject to a Taylor-contracting friction, choose interest rates to maximize the expected present value of profits for the \( n \) quarter duration of the contract. In this more general setup we allow for stochastic contract durations

\[ \Psi_{D,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} \theta_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) [R_{D,t} D_{0,t}(z) \Psi_{D,0}(z) - R_{D,0}(z) D_{0,t}(z)] \right\}. \]  

(N.81)

where

\[ \theta_{0,t} = \prod_{j=0}^{t} b_{j,t}(r_t), \text{ where } b_{0,t}(r_t) = 1, \]

Demand for deposits with interest rates set in period \( t \) are given by

\[ D_{0,t}(z) = \left( \frac{R_{D,0}(z)}{R_{D,t}} \right)^{-v_D} D_t. \]  

(N.82)

Substituting N.82 into N.81

\[ \Psi_{D,0}(z) = E_t \left\{ \sum_{t=0}^{n-1} \theta_{0,t} \mathcal{M}_{t,t+1} \left( \frac{P_0}{P_t} \right) \left[ R_{D,t} R_{D,0}(z)^{-v_D} R_{D,t}^{v_D} D_t \cdots \right. \right. \]

\[ \left. \left. \cdots R_{D,0}(z)^{-v_D} R_{D,t}^{v_D} D_t \right] \right\}. \]
The first order condition for deposits is given by
\[
\frac{\partial \Psi_{D,0}(z)}{\partial R_{D,0}(z)} = E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \left[ -v_D R_{D,t} R_{D,0}(z)^{-v_D} R_{D,t}^{v_D} D_t - \ldots - (1 - v_D) R_{D,0}(z)^{-v_D} R_{D,t}^{v_D} D_t \right] \right\} = 0.
\]

Rearranging
\[
R_{D,0}(z) = \left( \frac{v_D}{v_D - 1} \right) \frac{E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \mathbb{R}_{D,t} R_{D,t}^{v_D} D_t \right\}}{E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) R_{D,t}^{v_D} D_t \right\}}.
\]

Or in stochastically detrended form
\[
R_{D,0}(z) = \left( \frac{v_D}{v_D - 1} \right) \frac{E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \mathbb{R}_{D,t} R_{D,t}^{v_D} \tilde{D}_t \prod_{k=0}^{t} \mu_{Y,k} \right\}}{E_t \left\{ \sum_{t=0}^{n-1} b_{0,t} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) R_{D,t}^{v_D} \tilde{D}_t \prod_{k=0}^{t} \mu_{Y,k} \right\}}.
\]

The aggregate deposit interest rate is the CES function of the different contract cohort interest rates
\[
R_{D,t} = \left( \frac{1}{\sum_{k=0}^{n-1} b_{t-k,t} (R_{D,t-k}(z))^{1-v_D}} \right)^\frac{1}{1-v_D}.
\]

**Appendix O Markup and Pass-through Regimes**

We add an additional regime switching parameter \( m(r_t) \) which is governed by the following Markov transition matrix.
\[
Q_m = \begin{bmatrix}
1 - q_{H,L} & q_{H,L} \\
q_{L,H} & 1 - q_{L,H}
\end{bmatrix},
\]

where \( m(H) = 1 \) and \( m(L) = 0 \). The reduced form persistence parameters in the banks’ interest rate setting rules are determined according to the following equations:

**O.1**
\[
a_{j,t}(r_t) = z(s_{1,t}) a_{j,ZLB} + (1 - z(s_{1,t})) (m(s_{2,t}) a_{j,N} + (1 - m(s_{2,t})) a_{j,L}),
\]

**O.2**
\[
b_{j,t}(r_t) = z(s_{1,t}) b_{j,ZLB} + (1 - z(s_{1,t})) (m(s_{2,t}) b_{j,N} + (1 - m(s_{2,t})) b_{j,L}).
\]
Appendix P  Linex Adjustment Costs

P.1 Loan Making Banks

There is a continuum of loan making banks, normalized to unit mass. The $z$th loan making bank chooses a loan interest rate for their variety of loans, that maximizes their expected discounted sum of current and future profits, subject to a quadratic adjustment cost and a linex adjustment cost. We write this more formally as the profit function

$$
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \right\} \left[ \begin{array}{c}
R_{L,t}(z)L_t(z) - R_{L,t}L_t(z) - \ldots \\
\ldots - \frac{\phi_L}{2} R_{L,t}L_t \left[ \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right]^2 - \ldots \\
\ldots - \frac{\phi_{L^2}}{\psi^2} R_{L,t}L_t \left\{ \exp \left( -\psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) \right) + \ldots \right\} \\
\ldots + \psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) - 1 \end{array} \right].
$$

While the linex adjustment cost nests the quadratic adjustment cost in the limit as $\psi \to 0$, this can also cause numerical instabilities, so we include a quadratic adjustment cost term in addition, which allows for numerical stability in the case that we want quadratic adjustment costs.

Substituting in the demand function for the $z$th variety of deposits gives

$$
\Psi_{L,0}(z) = E_t \left\{ \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \left( \frac{P_0}{P_t} \right) \right\} \left[ \begin{array}{c}
R_{L,t}(z)^{1-v_L} R_{L,t}^{v_L} L_t - R_{L,t} R_{L,t}(z)^{-v_L} R_{L,t}^{v_L} L_t - \ldots \\
\ldots - \frac{\phi_L}{2} R_{L,t}L_t \left[ \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right]^2 - \ldots \\
\ldots - \frac{\phi_{L^2}}{\psi^2} R_{L,t}L_t \left\{ \exp \left( -\psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) \right) + \ldots \right\} \\
\ldots + \psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) - 1 \end{array} \right].
$$
The first order condition with respect to the loan interest rate gives

\[
\frac{\partial \Psi_{L,t}(z)}{\partial R_{L,t}(z)} = (1 - u_L) L_t(z) + u_L R_{L,t} R_{L,t}(z) - v_L R_{L,L} R_{L,t} + \phi_L \frac{R_{L,L} L_t}{R_{L,t-1}(z)} \left[ \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right] + \ldots
\]

\[
+ \frac{\phi_{L,t}}{\psi} \left( \frac{R_{L,L} L_t}{R_{L,t-1}(z)} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t}(z)}{R_{L,t-1}(z)} - 1 \right) \right) - 1 \right] + \ldots
\]

\[
+ \left\{ \left( \frac{1}{v_L - 1} \right) \mu_{t+1}^{*} \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \left[ \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right] - \ldots
\]

\[
- \left( \frac{\phi_{L,t}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t+1}}{R_{L,t}} - 1 \right) \right) - 1 \right] \right\} = 0.
\]

With some rearranging this gives us loan makers interest rate setting rule

\[
\left( \frac{v_L}{v_L - 1} \right) \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 - \frac{\phi_L}{v_L - 1} \frac{R_{L,t}}{R_{L,t-1}(z)} \left[ \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right] + \ldots
\]

\[
+ \frac{1}{v_L - 1} \left( \frac{\phi_{L,t}}{\psi} \right) \left( \frac{R_{L,t}}{R_{L,t-1}(z)} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right) \right) - 1 \right] + \ldots
\]

\[
+ \left\{ \left( \frac{1}{v_L - 1} \right) \mu_{t+1}^{*} \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \left[ \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right] - \ldots
\]

\[
- \left( \frac{\phi_{L,t}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t+1}}{R_{L,t}} - 1 \right) \right) - 1 \right] \right\} = 0.
\]

When we stochastically detrend we get

\[
\left( \frac{v_L}{v_L - 1} \right) \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 - \frac{\phi_L}{v_L - 1} \frac{R_{L,t}}{R_{L,t-1}(z)} \left[ \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right] + \ldots
\]

\[
+ \frac{1}{v_L - 1} \left( \frac{\phi_{L,t}}{\psi} \right) \left( \frac{R_{L,t}}{R_{L,t-1}(z)} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right) \right) - 1 \right] + \ldots
\]

\[
+ \left\{ \left( \frac{1}{v_L - 1} \right) \mu_{t+1}^{*} \left( \frac{R_{L,t+1}}{R_{L,t}} \right)^2 \left[ \frac{R_{L,t}}{R_{L,t-1}(z)} - 1 \right] - \ldots
\]

\[
- \left( \frac{\phi_{L,t}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{L,t+1}}{R_{L,t}} - 1 \right) \right) - 1 \right] \right\} = 0.
\]

**P.2 Deposit Taking Banks**

There is a continuum of deposit taking banks normalized to unit mass. Each bank offers their own variety of deposit. The zth deposit taking bank chooses interest rates for their variety of deposit to maximize the expected sum of present and future profits. We can
write the profit function for the \( z \)th deposit taking bank as follows

\[
\Psi_{D,t}(z) = E_t \left\{ \sum_{i=0}^{\infty} \mathcal{M}_{0,t}^* \left( \frac{P_0}{P_t} \right) \right\}
\]

\[
\begin{align*}
\Psi_{D,t}(z) &= E_t \left\{ \sum_{i=0}^{\infty} \mathcal{M}_{0,t}^* \left( \frac{P_0}{P_t} \right) \right\}
\begin{bmatrix}
\mathbb{R}_{D,t} D_t(z) - R_{D,t}(z) D_t(z) - \ldots \\
\ldots - \frac{\phi_D}{2} R_{D,t} D_t \left[ \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right] \ldots \\
\ldots - \frac{\phi_D^2}{\psi^2} R_{D,t} D_t \left\{ \exp \left( -\psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right) + \ldots \\
\ldots + \psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right] \\
\end{bmatrix}.
\end{align*}
\]

Substituting in the demand function for deposits for the \( z \)th variety of bank deposits gives

\[
\Psi_{D,t}(z) = E_t \left\{ \sum_{i=0}^{\infty} \mathcal{M}_{0,t}^* \left( \frac{P_0}{P_t} \right) \right\}
\begin{align*}
\begin{bmatrix}
\mathbb{R}_{D,t} R_{D,t}(z)^{-v_D} R_{D,t}^{v_D} D_t - R_{D,t}(z)^{-v_D} R_{D,t}^{v_D} D_t - \ldots \\
\ldots - \frac{\phi_D}{2} R_{D,t} D_t \left[ \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right] \ldots \\
\ldots - \frac{\phi_D^2}{\psi^2} R_{D,t} D_t \left\{ \exp \left( -\psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right) + \ldots \\
\ldots + \psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right] \\
\end{bmatrix}.
\end{align*}
\]

The first order condition with respect to deposit interest rates

\[
\frac{\partial \Psi_{D,t}(z)}{\partial R_{D,t}(z)} = -v_D \mathbb{R}_{D,t} R_{D,t}(z)^{-v_D-1} R_{D,t}^{v_D} D_t - (1 - v_D) D_t(z) - \phi_D \frac{R_{D,t} D_t}{R_{D,t-1}(z)} \left[ \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right] + \ldots
\]

\[
\begin{bmatrix}
\ldots + \frac{\phi_D}{\psi} \frac{R_{D,t} D_t}{R_{D,t-1}(z)} \left[ \exp \left( -\psi \left( \frac{R_{D,t}(z)}{R_{D,t-1}(z)} - 1 \right) \right) - 1 \right] + \ldots \\
\ldots + E_t \left\{ \frac{R_{D,t+1}(z)}{R_{D,t}(z)} \right\} \left[ \frac{R_{D,t+1}(z)}{R_{D,t}(z)} - 1 \right] - \ldots
\end{bmatrix}
\]

\[
\begin{bmatrix}
\ldots - \frac{\phi_D}{\psi} \left[ \exp \left( -\psi \left( \frac{R_{D,t+1}(z)}{R_{D,t}(z)} - 1 \right) \right) - 1 \right] \right] = 0,
\]

after some rearranging we get the interest rate setting rule for deposit taking banks.

\[
1 - \left( \frac{v_D}{v_D - 1} \right) \mathbb{R}_{D,t} - \left( \frac{\phi_D}{v_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} \left[ \frac{R_{D,t}}{R_{D,t-1}} - 1 \right] + \ldots
\]

\[
\begin{bmatrix}
\ldots + \left( \frac{1}{v_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} \left[ \exp \left( -\psi \left( \frac{R_{D,t}}{R_{D,t-1}} - 1 \right) \right) - 1 \right] + \ldots \\
\ldots + E_t \left\{ \frac{1}{v_D - 1} \mathcal{M}^*_{t,t+1} \pi_{t+1} \left( \frac{R_{D,t+1}(z)}{R_{D,t}(z)} \right)^2 \right\} \left[ \frac{R_{D,t+1}(z)}{R_{D,t}(z)} - 1 \right] - \ldots
\end{bmatrix}
\]

\[
\begin{bmatrix}
\ldots - \frac{\phi_D}{\psi} \left[ \exp \left( -\psi \left( \frac{R_{D,t+1}(z)}{R_{D,t}(z)} - 1 \right) \right) - 1 \right] \right] = 0.
\]
When we stochastically detrend we get

\[
1 - \left( \frac{v_D}{v_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} - \left( \frac{\phi_D}{v_D - 1} \right) \frac{R_{D,t}}{R_{D,t-1}} [R_{D,t} - 1] + \ldots
\]

\[
+ \left( \frac{1}{v_D - 1} \right) \frac{\phi_{D2}}{\psi} \left( \frac{R_{D,t}}{R_{D,t-1}} \right) \left[ \exp \left( -\psi \left( \frac{R_{D,t}}{R_{D,t-1}} - 1 \right) \right) - 1 \right] + \ldots
\]

\[
+ \left( \frac{1}{v_D - 1} \right) \mathcal{M}_{t,t+1}^* \mu_{Y,t+1} \pi_{t+1}^{-1} \left( \frac{R_{D,t+1}}{R_{D,t}} \right)^2 \frac{\hat{D}_{t+1}}{\hat{D}_t} \times \frac{\phi_D \left( \frac{R_{D,t+1}}{R_{D,t}} - 1 \right) - \ldots}{\ldots - \left( \frac{\phi_{D2}}{\psi} \right) \left[ \exp \left( -\psi \left( \frac{R_{D,t+1}}{R_{D,t}} - 1 \right) \right) - 1 \right]} \right)
\]

\[= 0.\]