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A quantitative case for leaning against the wind†

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Abstract

Should a monetary authority lean against the build-up of financial imbalances? We study this policy question in an environment in which there are recurring cycles of financial imbalances that develop over time and eventually collapse in a costly manner. The optimal policy reflects the trade-off between the short-run macroeconomic costs of leaning against the wind and the longer-run benefits of stabilising the financial cycle. We model the financial cycle as a nonlinear Markov regime-switching process, calibrate the model to US data and characterise the optimal monetary policy. Leaning systematically over the whole financial cycle is found to outperform policies of “benign neglect” and “late-in-the-cycle” discretionary interventions. This conclusion is robust to a wide range of alternative assumptions and supports an orientation shift in monetary policy frameworks away from narrow price stability to a joint consideration of price and financial stability.

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1. Introduction

The Great Financial Crisis (GFC) rejuvenated the debate concerning whether and how a central bank should trade off price and financial stability. This renewed interest stands in stark contrast to the pre-crisis consensus, which focused narrowly on price stability. Back then, considerations of private sector indebtedness, financial fragility and asset price bubbles were generally thought to be the rightful purview of prudential regulators and supervisors.

For monetary policy, the pre-crisis approach to financial stability is best characterised as “benign neglect”. That is, it was believed that central banks should focus solely on macroeconomic developments and largely ignore financial booms. Monetary policy, however, would stand ready to clean up the mess during and after the bust. Experience with equity price booms in past decades provided broad empirical support for this view. When such booms went bust, there was little lasting impact on the economy. Therefore, as the logic went, why depress the economy and hold inflation below target over time if a once-in-a-lifetime crisis were to materialise randomly and the central bank could clean up at little cost?

Faith in this view was shattered by the GFC for several key reasons. First, the cleaning-up approach did not work. The losses brought about by the GFC were large and very persistent. Second, the GFC appeared to be far from a random event, at least in retrospect. The crisis was preceded by a housing market boom which, together with the widespread use of securitisation, exposed the build-up of systemic risks in the financial sector. Once viewed through the lens of a “boom-gone-bust” process, the GFC was not simply a once-in-a-lifetime event that stood out in economic history. The GFC was simply the latest and more extreme episode.

Since the GFC, our understanding of financial crisis dynamics has evolved in several important ways as interest in financial crisis research was reinvigorated. First, several empirical studies that looked back in history found that costly financial downturns were preceded by significant credit and asset price booms (Schularick and Taylor (2012) who build on Borio and Lowe (2002)). Having more comprehensive data on indebtedness at our disposal has opened up the possibility that financial cycles can now be better measured and tracked than in the past. Second, empirical studies have also documented that various financial variables can be used to predict financial crises. Financial imbalances, in particular, have been identified as key predictors of financial busts and subpar economic performance (Borio and Drehmann (2009), Drehmann et al (2012), Jorda et al (2013) and Mian et al (2016)). Third, recent theoretical research has put a spotlight on the role of banks and financial intermediaries in macroeconomic models. This line of research has highlighted various types of financial frictions that amplify economic shocks and exacerbate business cycles (see Brunnermeier Oehmke (2012) for a review).

Of particular note, the risk-taking channel of monetary policy has taken on greater prominence recently in both empirical and theoretical research. This channel can be activated in several ways. First, easy monetary policy can encourage banks to seek higher returns and take more risks on their loan books (Borio and Zhu (2008), Jimenez et al (2012) and Dell’Ariccia et al (2013)). Banks may also increase their
reliance on shorter-term funding to take advantage of lower funding costs (Adrian and Shin (2010)). As a result, this excessive risk-taking behaviour leaves the banking sector more vulnerable to shocks. Second, an easier policy can compress risk premia and push asset prices above levels justified by fundamentals. This raises the risks of an asset price bubble that can correct itself abruptly and damage the balance sheets of investors. In extreme situations, the correction can trigger a fire sale of collateralised assets and intensify an economic downturn. Third, easier monetary policy can accelerate the growth of shadow banking activity and liquidity creation outside of the regulatory umbrella. This increases the fragility of the financial system as higher-risk financial intermediaries are subject to costly runs (Moreira and Savov (2016)).

Our evolving understanding of financial crisis dynamics has given rise to several perspectives on the debate. On one side, some have called for greater reliance on macroprudential tools. Under this view, monetary policy should take a backseat. Others have suggested that macroprudential tools should be used as the first line of defence and monetary policy only as a last resort.

However, there is uncertainty about the effectiveness of macroprudential tools, not least owing to the fact that risk-taking channels appear to be so diverse and not fully understood yet. This uncertainty suggests that the targeted approach of macroprudential tools may at best be an incomplete safeguard. And, over time, macroprudential tools can potentially be circumvented via regulatory arbitrage and creative financial engineering.

The shortcomings of macroprudential tools have left open an important role for monetary policy to lean against the wind. The potential benefit from the use of monetary policy is that the policy rate is not only a powerful macroeconomic tool but also the universal price of leverage and risk. In this view, macroprudential tools and the policy rate should be seen as complements, not as substitutes. Taken together, the limits of macroprudential tools and the potential of the policy rate to influence the financial cycle provide a prima facie case for monetary policy to be used in the pursuit of financial stability.

However, the feasibility of using the policy rate does not guarantee its desirability. A number of recent studies have begun exploring the costs and benefits of using monetary policy in the pursuit of financial stability. The emphasis has been on quantifying the costs and benefits with explicit models. One influential strand of recent research has been pioneered by Svensson (2016), Riksbank (2013) and IMF (2015). That strand considers explicit cost-benefit calculations in monetary policy models that feature financial crises. It identifies three key considerations when calibrating the costs and benefits: (i) how much leaning is needed to curb credit growth (those studies’ measure of financial imbalances)?; (ii) how do changes in credit growth affect the likelihood of a future financial crisis?; and (iii) how costly is pre-emptive policy in terms of short-term macroeconomic costs (for example, on unemployment and output)? The authors of those studies find evidence against leaning and argue that this is a robust finding.

This no-leaning prescription has been challenged, however. Adrian and Liang (2016), for example, find that the result is not robust to the relaxation of some key
assumptions. In particular, if leaning helps lower the eventual cost of a crisis sufficiently, then it is beneficial. This alternative assumption is consistent with the historical evidence that strong financial imbalances tend to intensify economic downturns (Jorda et al (2013) and Juselius et al (2016)).

More fundamentally, the benefit of leaning against the wind may be better appreciated if one recognises (i) the endogenous process governing the slow build-up of financial imbalances, which culminates in a crisis if there is sufficient momentum, and (ii) the systematic influence of policy over the entire financial cycle. Indeed, one could argue that leaning is less about averting an imminent crisis and more about fostering financial stability at all times even when a crisis remains a very remote possibility (Borio and Lowe (2002), BIS (2016) and Juselius et al (2016)). In this case, a leaning policy might be better thought of as an integral part of the monetary policy framework, rather than as an occasional deviation from a conventional inflation-targeting approach. This paper attempts to address this gap in the literature.

The paper proposes a dynamic model for evaluating leaning-against-the-wind policies in the presence of recurring financial cycles. The model consists of the conventional macroeconomic block, augmented with a financial cycle block that describes the evolution of financial imbalances and their impact on the economy. In this paper, the financial cycle is persistent and endogenous. In a boom phase, financial imbalances grow over time. Once they reach a sufficiently high level, there is a progressively rising probability that the economy will switch into a financial downturn phase, during which imbalances shrink. The pace at which financial imbalances build up is influenced by a leaning policy, not least reflecting the basic features of the risk-taking channel of monetary policy. Monetary policy can then play a role in constraining the accumulation of imbalances, and consequently lessen the duration (and thus the total cost) of a crisis.

A number of other studies have also used regime-switching models to introduce a financial crisis module to monetary policy modelling. Like ours, these studies posit that the probability of a crisis event depends on some financial variable (leverage in Woodford (2012), credit growth in Ajello et al (2015), the debt level in Alpanda and Ueberfeldt (2016), and credit in a model of a small, open economy in Gerdrup et al (2016)).

Our paper distinguishes itself from these prior studies by considering a persistent dynamic process governing the evolution of the financial imbalance variable consistent with the data. These studies assume that any unexpected opening up of financial imbalances naturally unwind without any policy intervention, so that the risks of a crisis are always expected to diminish over time. It is only when an unlikely sequence of consecutive shocks pushes financial imbalances to a level sufficiently

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2 Our characterisation of the financial cycle is consistent with the recent theoretical literature. It highlights a variety of market imperfections that could give rise to a persistent build-up of financial imbalances, followed by highly nonlinear adjustments (for a review, see Brunnermeier and Oehmke (2012)).

3 The unavoidable cost of a crisis captures the cyclical fall in output that cannot be offset by easing policy. This may be due to financial frictions during downturns. The argument for leaning would be strengthened if a bust led to a stronger and longer-lasting disruption. Gourio et al (2016), for example, allow a crisis to permanently impact technology, thereby strengthening the case for leaning.
high that a crisis becomes possible and a leaning policy might be justified. In our model, the financial cycle exhibits strong persistence from both *ex post* and *ex ante* perspectives. We show that, in this environment, there is a role for monetary policy to react early on and pre-empt large imbalances from building up in the first place. The optimal policy prescribes systematic leaning rather than occasional tightening when crisis risks are deemed to be high. In our model, crises may be unlikely and infrequent, but they are not random events.

The rest of the paper is organised as follows. The next section lays out the general optimal control problem in the presence of financial stability concerns. Section 3 contrasts the optimal policy under the “random crisis” and “endogenous financial cycle” models, to show why the latter type of models offers a new way of characterising and assessing the benefits of leaning. Section 4 describes the empirical procedure for estimating the financial cycle model using US data. The calibrated model is then used to compute an optimal policy rule under alternative sets of assumptions. The final section concludes that the case for policies that systematically lean against the wind is strong. The main benefit of such policies lies not in averting an impending crisis but in systematically lessening the severity of the financial cycle itself.

2. General statement of the monetary policy problem

A general statement of the monetary policy problem is one in which a central bank attempts to stabilise the macroeconomy by setting a time path for the policy interest rate. Conventionally, the policymaker’s objective is to maximise the discounted sum of the period loss,

\[ -\frac{1}{2} \sum_{t=0}^{\infty} \delta^t \left( \pi_t^2 + \lambda y_t^2 \right), \tag{2.1} \]

where \( y_t \) and \( \pi_t \) denote output and inflation gaps, respectively, and \( \delta \) is the discount factor. The essence of the macroeconomic trade-offs is modelled as a Phillips curve, which takes the form

\[ \pi_t = \kappa y_t + v_t, \tag{2.2} \]

where the aggregate supply shock \( v_t \) gives rise to a trade-off between output and inflation. Here we assume without loss of generality that policy rate can be freely set to achieve a given level of the output gap, \( y_t \). In this setting, the optimal policy calls for choosing \( y_t \) such that

\[ \lambda y_t = -\kappa \pi_t. \tag{2.3} \]

Equation (2.3) is the first-order condition of the problem described in Equations (2.1) and (2.2). It indicates that when a shock pushes inflation away from the target, the central bank adjusts the output gap to offset it.

For pedagogical reasons, we re-cast this optimal control problem as a dynamic programme to highlight the possible ways in which the financial cycle may matter for
policy. Starting with the conventional model as a baseline, the problem can be re-written in the following way:

\[
V(y, \pi, v) = \max_{r_1(y, \pi) \in \Gamma(y, \pi)} \left\{ -\frac{1}{2} \left( \pi^2 + \lambda y^2 \right) + \delta EV(y', \pi', v') \right\}, \quad (2.4)
\]

subject to the law of motion of the macroeconomy and where the choice of the interest rate rule \( r_1(y, \pi) \) is a Taylor-type rule that maximises the objective of the central bank. In this conventional formulation, the role of the financial cycle is absent owing to the assumption that financial cycles play at best a background role and is consistent with many pre-crisis models where finance and money are treated as acting largely as a veil.

Explicitly adding consideration of the financial cycle into the monetary policy can be done in two ways. First, we can re-write the value function as

\[
V(y, \pi, f, s, v) = \max_{r_2(y, \pi) \in \Gamma(y, \pi)} \left\{ -\frac{1}{2} \left( \pi^2 + \lambda y^2 \right) + \delta EV(y', \pi', f', s', v') \right\}, \quad (2.5)
\]

where central bank chooses a policy rule as a function of output and inflation \((y, \pi)\). The financial cycle variable \( f \) influences the probability of entering into a crisis, whose realisation is captured by the crisis indicator \( s \in \{ \text{boom, crisis} \} \). The central bank cares about the financial cycle only to the extent that it influences future outcomes for output and inflation and hence shows up in the value function. Note, in particular, that the monetary policy reaction function in this case is restricted; it only responds directly to output and inflation, but not to the financial cycle itself. This does not mean that the policy settings in equations (2.4) and (2.5) will be similar. For the latter, the macroeconomy is assumed to be influenced by the financial cycle and, therefore, the central bank responds indirectly to the financial cycle to the extent that the financial cycle affects expected output and inflation.

Second, the central bank considers a set of policy rules which depend directly on the state of the financial cycle \( f \) and regime \( s \). The monetary policy problem can be recast, yielding the form,

\[
V(y, \pi, f, s, v) = \max_{r_3(y, \pi, f, s) \in \Gamma(y, \pi, f, s)} \left\{ -\frac{1}{2} \left( \pi^2 + \lambda y^2 \right) + \delta EV(y', \pi', f', s', v') \right\}, \quad (2.6)
\]

where the central bank responds directly to the financial cycle and whether the economy is experiencing a boom or a bust, ie \( f \) as well as \( s \).

The key difference is the form of the policy rules over which the central bank optimises. In Equation (2.5), the central bank chooses a Taylor-type policy rule of the form \( r_2(y, \pi) \in \Gamma(y, \pi) \), while in Equation (2.6) it chooses an extended Taylor-type policy rule of the form \( r_3(y, \pi, f, s) \in \Gamma(y, \pi, f, s) \) which includes a direct response to the financial cycle. Note that if the financial cycle does not matter for welfare, the optimal rule in Equation (2.6) would be equivalent to the rule in Equation (2.5). The extended Taylor-type rule is the focus of our paper.
Recent modelling efforts

Svensson (2016) and the IMF (2015) explore the trade-offs associated with leaning in the context of a restricted version of the general problem. Benefits of their approach are the ease of computation and the innovative feature of modelling financial stability risks as a potential costly bust following an unsustainable credit boom. Leaning in this approach reduces the probability of a crisis but at the short-term cost of lower economic activity and inflation.

There are questions about whether this approach captures all relevant costs and benefits. For example, in the context of a one-off policy intervention, this cost-benefit analysis approach essentially boils down to a comparative static exercise where the central bank weighs the benefit of avoiding a potential boom-bust at some point in the future by raising the policy rate, against the short-term macroeconomic cost. The implementation of the marginal cost-benefit approach is typically performed via a perturbation analysis, i.e., the central bank leans at time $t$. This type of leaning is justified only when the marginal benefit outweighs the marginal cost. We label this as the marginal cost-benefit approach. Note that this type of leaning is a one-time, short-term deviation from a given future path of policy rates. In practice, this type of leaning is often considered as a policy option late in a financial cycle. As such, the policy option would be intended to have only a transitory impact on the shape of the financial cycle.

But does the marginal cost-benefit approach capture the full benefits of leaning compared to the general statement of the problem based on systematically following a policy rule? In terms of the general statement of the problem, the marginal cost-benefit approach could be thought of as a discretionary policy rate deviation from a Taylor-type policy rule (i.e., $r_t(y, \pi)$), a rule that does not take any account of the financial cycle's effect on the economy. In principle, there is no guarantee that such a deviation is welfare enhancing. The costs and benefits need to be assessed quantitatively.

In addition, the implied restrictions of the marginal cost-benefit approach may lead to an underestimation of the full benefits of leaning. In terms of Equations (2.5) and (2.6), the marginal cost-benefit approach imposes the following restrictions on the expected value function,

$$EV(y', \pi', f', s', v') = \int \int V^1(y', \pi', v') g(y', \pi', v') W^2(f', s') h(f', s')$$

$$= \int \left[ p(\text{crisis}, f') V^1(y', \pi', v') g(v', \pi', v') W^2(f', \text{crisis}) h(f') + (1 - p(\text{crisis}, f')) V^1(y', \pi', v') g(v', \pi', v') W^2(f', \text{boom}) h(f') \right]$$ (2.7)

Recasting the policy problem in this way highlights the impact of the restrictions of the marginal cost-benefit approach in three key ways.

First, the marginal cost-benefit modelling approach implies that a central bank considers a policy of leaning at time $t$ that accounts only the costs of the next crisis. This is equivalent to assuming that once a crisis occurs, the economy is crisis-free forever afterwards. In terms of the second line in Equation (2.7), the probability of imminent crisis, $p(\text{crisis}, f')$, becomes zero after the realisation of a crisis and $1 - p(\text{crisis}, f')$ becomes 1. In addition, the central bank is assumed to ignore the possible impact of policy rates on the financial cycle after the crisis; this is equivalent
of dropping $V^2(f',s')\ln(f',s')$ out of the Equation (2.7). These restrictions may be a reasonable assumption if a crisis is truly a once in a lifetime event. But this may not be reasonable if crises are recurring events.

Second, the marginal cost-benefit approach may not fully capture the notion that the probability of a crisis is influenced by a systematic policy rule via its impact of reshaping the dynamics of the financial cycle, $f$. The marginal cost-benefit approach implies that the shape of the future value function $\delta EV(y, \pi, f, s, v)$ is taken as given when assessing the impact of a policy intervention. In other words, it is assumed that policy actions do not change this function. In the general statement of the problem, a shift from a non-leaning policy rule to a leaning policy rule can alter the shape of $\delta EV(y, \pi, f, s, v)$ and hence the likelihood of future crises by influencing the amplitude and duration of the financial cycle over time. By ruling out this systematic policy rule effect on the overall shape of the financial cycle, the marginal cost-benefit modelling approach may underestimate the potential benefits of leaning.

Finally, the marginal cost-benefit approach may also underestimate the benefits of leaning because the total costs of a crisis is typically assumed to not vary with the size of the financial boom. Technically, this implies restrictions on Equation (2.7). In particular, the future value function $\delta EV(y, \pi, f, s, v)$ is separable into the sub-value functions $V^1(y', \pi', v')g(y', \pi', v')$ and $V^2(f', s')\ln(f', s')$. The first term is the sub-value function associated with the conventional monetary policy problem in which the financial cycle does not matter (or finance is a veil). The second term captures the notion that costly crises depend on the state of the financial cycle. Separability implies the financial cycle does not directly influence the state of the macroeconomy. In addition, the marginal cost-benefit approach typically assumes that the crisis cost materialises at the time of the crisis and is fixed in size. It is sometimes suggested that the fixed cost reflects the present value of the costs associated with a crisis. However, such restrictions rule out the possibility that longer and more pronounced financial cycles lead to larger and more costly financial crises. This raises the possible benefits of leaning if the leaning can influence the size of financial booms.

All these implied restrictions highlight the fact that the marginal cost-benefit approach may not capture the full benefits of leaning. And it also suggests that systematic leaning may outperform discretionary, one-time policy actions. The size of benefits is an empirical question, and we will show that the value function approach calibrated to the historical data yields evidence that favours the case for systematic leaning.

3. Two approaches to incorporating financial stability risks

Whether a leaning policy is justified crucially depends on the nature of the financial stability risks. Two classes of models are described in this section: (i) a random crisis

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4 It is useful to note that if $V^2(f', s')\ln(f', s')$ is a constant, e.g. the policy rate does not affect the financial cycle, then there would be no reason to lean against the wind. Another special case of this assumption is benign neglect; if the central bank can costlessly clean up a crisis, then the optimisation problem simplifies to a standard monetary policy problem (i.e. without a financial cycle).
model; (ii) an endogenous financial cycle model, where recurring waves of financial stability risk wax and wane over time and result in crises at times.

3.1 Random crisis model

Consider the same optimal policy problem as in Equations (2.1) and (2.2), except that we assume a financial crisis takes place randomly with some probability, which can be influenced by the policy rate. A monetary easing leads to a higher $y$, but has a side-effect of exacerbating financial risk-taking and increasing the probability of a crisis, $p(y)$. In a crisis state, the central bank’s loss is exogenously given by $\tilde{m} < 0$, i.e., the crisis cost. Once in a crisis, the economy recovers to a normal state with a fixed probability, $q$.

This version of the problem can be written as the following Bellman equations (in the spirit of the full optimisation problem in (2.6) where we suppress the time subscript):

\[
V = \max_y \left[ -\frac{1}{2} \left( \pi^2 + \lambda y^2 \right) + \delta \left( p(y) (W - V) + V \right) \right] \tag{3.1}
\]

\[
W = \tilde{m} + \delta [q (V - W) + W] \tag{3.2}
\]

where $V$ and $W$ are the values of being in a non-crisis and a crisis state respectively. The first-order condition for this problem is

\[
\lambda y = -\kappa \pi - \delta p'(y) (V - W) \tag{3.3}
\]

The difference between the optimal policy in this case and the rule in Equation (2.3) is the second term on the right-hand side of the equation, i.e., $V - W$ and $p'(y)$. When a crisis is particularly costly ($V - W$ is large) and a change in monetary policy has a significant impact on the probability of a crisis (large $p'(y)$), then it is optimal to lean against the wind by choosing a smaller output gap than would otherwise be prescribed by the standard rule.

This optimality condition echoes that in the exercises in Svensson (2016) and IMF (2015), which equate the marginal benefit of leaning to the sensitivity of the crisis probability to a one-off change in the policy rate. They find that the impact of a policy rate increase on the crisis probability is simply too small to offset the associated short-run macroeconomic cost of leaning. In terms of Equation (3.3), if $p'(y)$ is near zero, the optimal policy is not to raise rates in a discretionary fashion. Note that in these models, the empirical estimates from Schularick and Taylor (2012) are used to calibrate the cost-benefit calculations.

The random crisis model can take a more general form with state-contingent crisis probabilities. For example, the crisis probabilities can depend on the degree of financial imbalances, $f$, which could be seen to represent leverage, the stock of

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5 For simplicity we suppress the role of the IS curve, and focus on the crisis probability as an increasing function of the output gap.
6 The key assumption is that, absent any policy intervention or shocks, \( f_t \) is *strongly mean-reverting so that imbalances tend to unwind naturally rather than build up over time*. Of course, a sequence of adverse shocks can lead to an increase in the level of financial imbalances and therefore raise the likelihood of a crisis. This might be thought of as an ex ante tail risk event interpretation of crisis dynamics. But the model has the property that, once a shock to the financial imbalance variable occurs, *in expectation* (ie no new shocks), the crisis probabilities will decline over time because imbalances are assumed to unwind on their own.7

We cannot rule out this type of financial cycle dynamic in general but it is significantly at odds with the type of financial cycle we outline in the next section and the empirical regularities that we are trying to capture.

### 3.2 The endogenous financial cycle model – a stripped down version

The endogenous financial cycle, \( f_t \), is modelled as recurring financial booms and busts. This captures the notion that financial imbalances gradually build up over time, represented by a persistent increase in \( f_t \), and eventually become so large that a downturn ensues with the economy sliding into a costly crisis. Unlike in the random crisis model, swings in financial imbalances are assumed to be recurring and subject to periods of momentum, ie they are very persistent and not strongly self-correcting in expectation at each point in time.

To capture this intuition, \( f_t \) is assumed to be a Markov chain, taking on values from the discrete countable set \( \{-1, 0, 1, \ldots, \bar{f}\} \). The Markovian assumption offers the ability to capture persistent, recurring swings in a financial cycle. Financial booms and busts can gather momentum over time and this momentum can vary with the state of the economy and the influence of policy. Technically, we model the financial cycle as transitioning through boom and bust states with explicit transition probabilities. For \( f_t \) equal to zero, the boom \((f_t = 1)\) can begin with a transition probability of

\[
P(f_{t+1} = 1 | f_t = 0) = p(\gamma_t)
\]

or \( f_t \) remains in state 0 with probability

\[
P(f_{t+1} = 0 | f_t = 0) = 1 - p(\gamma_t).
\]

The non-negative states of \( f_t \) represent different stages of a financial boom, with a higher value of \( f_t \) indicating a higher degree of imbalances. In a boom, \( f_t \) evolves according to transition probabilities of the form

\[
P(f_{t+1} = f + 1 | f_t = f) = (1 - p_0) p(\gamma_t)
\]

\[
P(f_{t+1} = f | f_t = f) = p_0
\]

\[
P(f_{t+1} = f - 1 | f_t = f) = (1 - p_0)(1 - p(\gamma_t))
\]

for values of \( f \) from 1 to \( \bar{f} - 1 \) and where \( p_0 \) is the unconditional transition probability of staying in state \( f \). When \( f_t \) reaches the value of \( \bar{f} \), the transition probability to the crisis state is \((1 - p_0)p(\gamma_t)\). And in the crisis state, \( f_t \) takes on the value \(-1\). With the transition probability \( q \), the crisis ends and \( f_t \) is reset to zero. One

6 This is also the case in Woodford (2012) and Ajello et al (2016).

7 In Appendix A, we show that the implications of optimal policy in the state-contingent random crisis environment are similar to those of the more basic random crisis model.
property of the specification is that the crisis state materialises via a transition through the $f$ state. It implies that when $f_i$ is low, the likelihood of a future crisis is low; as $f_i$ increases, the likelihood of entering a crisis rises. In the case of a persistent financial cycle, the financial imbalances, $f_i$, will not naturally decline in the absence of shocks but rather can be expected to rise.

Note that the transition probabilities which govern the evolution of the financial cycle from one state of the financial cycle to another are conditional on $y_i$. The dependence of transition probabilities on $y_i$ provides a channel through which monetary policy can influence the financial cycle dynamics. A policy tightening (ie negative output gap) is assumed to help rein in the financial cycle, and lower the probability of the financial imbalances transitioning to a worse stage. Thus, we assume $p'(y_i) > 0$. Once the economy falls into a financial crisis, the central bank incurs a period loss of $\varpi < 0$. This crisis cost is exogenous and cannot be offset by the policymaker. The schematics of the financial cycle model is illustrated in Figure 1 showing how the transition probability scheme differs from the stochastic assumptions of the random crisis model.

The optimal policy problem with an endogenous financial cycle can be written as a system of $F + 2$ Bellman equations

$$V_0 = \max_y \{ L(y) + \delta (1 - p(y)V_0 + p(y)V_1) \}$$  \hspace{1cm} (3.5)

$$V_f = \max_y \{ L(y) + \delta [p_0 V_f + (1 - p_0)[p(y)V_{f+1} + (1 - p(y)V_{f-1})]] \}$$  \hspace{1cm} (3.6)

$$V_{-f} = \max_y \{ L(y) + \delta [p_0 V_{-f} + (1 - p_0)[p(y)W + (1 - p(y)V_{-f-1})]] \}$$  \hspace{1cm} (3.7)

$$W = \varpi + \delta [q V_0 + (1 - q)W]$$  \hspace{1cm} (3.8)
where \( L(y) \equiv -\frac{1}{2}(\pi^2 + \lambda y^2) = -\frac{1}{2}(\kappa^2 + \lambda)y^2 \) and Equation (3.6) holds for \( f = \{1, 2, ..., \bar{f} - 1\} \).

Denoting the optimal policy at state \( f \) by \( y_f \), the first-order conditions are

\[
V_1 - V_0 = \frac{(\kappa^2 + \lambda)y_0}{\delta p'(y_0)}
\]

(3.9)

\[
V_{f+1} - V_{f-1} = \frac{(\kappa^2 + \lambda)y_f}{\delta(1 - p_0)p'(y_f)}
\]

(3.10)

\[
W - V_{\bar{f}-1} = \frac{(\kappa^2 + \lambda)y_{\bar{f}}}{\delta(1 - p_0)p'(y_{\bar{f}})}
\]

(3.11)

where Equation (3.10) holds for \( f = \{1, 2, ..., \bar{f} - 1\} \). There are \( 2\bar{f} + 3 \) equations, to solve for \( 2\bar{f} + 3 \) unknowns (\( \bar{f} + 1 \) output gap choices, and \( \bar{f} + 2 \) optimal values).

A few salient features of the model can be inferred from the value functions and the first-order conditions. First, the value function achieves a lower bound during a crisis state, reflecting unavoidable costs of crises; that is, in the crisis state with period loss \( \bar{w} < 0 \), the value function \( W \) is lower than any values associated with the non-crisis states.\(^9\) Second, the higher the financial imbalances, the higher the probability of an eventual crisis (i.e., the value function \( V_f \) is strictly decreasing in \( f \)). Given these two properties, the first-order conditions imply that it is optimal to set \( y < 0 \) in all non-crisis states (in a crisis state, it is assumed without loss of generality that there is no policy trade-off given the fixed transition probability \( q \)). In other words, the central bank is willing to tolerate economic slack when leaning systematically against the financial cycle.\(^{10, 11}\)

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\(^8\) We assume no supply shocks for simplicity.

\(^9\) Technically, for non-crisis states, the central bank can set the output gap to zero and therefore guarantee a current period payoff of zero. Because the continuing value can be negative for non-crisis states due to the possibility of entering a crisis state in the future (which entails a negative period loss \( \bar{w} \)), the value function in non-crisis states is negative by design. Because the likelihood of a crisis increases with the value of \( f \), the value function in the crisis state is the lower bound of the state-dependent value functions.

\(^10\) One can consider the extent of economic slack as the insurance premium against the risk of a costly crisis.

\(^11\) In the earlier monetary policy research on asset price bubbles, Bernanke and Gertler (1999, 2001) and Cecchetti et al (2000) examine the costs and benefits over shorter policy horizons. Using a model closer to the spirit of the endogenous financial cycle model but with a more restricted set of monetary policy rules, Filardo (2001, 2004, 2006, 2011) finds support for the leaning hypothesis. The endogenous financial cycle approach highlights the general optimal control result that costly financial cycles or asset price bubbles provide a reason for monetary policy to lean. In this paper, nonlinear policy rules are considered, which enables us also to consider policy trade-offs of the type hinted at in Blanchard (2000) and Gruen et al (2003).
To gain quantitative insights, we parameterise the baseline model and characterise the optimal monetary policy numerically. For the transition probabilities, we assume a logistic functional form,

$$p(y) = \frac{\exp(y + \Delta)}{1 + \exp(y + \Delta)}, \quad (3.12)$$

and the following parameterisation: $\bar{f} = 4$, $\kappa^2 + \lambda = 1.5$, $\delta = 0.9$, $q = 0.2$, $\bar{\sigma} = -1$, $p_0 = 0.3$ and $\Delta = 0$. Note that when $\Delta > 0$, the financial cycle exhibits greater momentum, ie $f$ has a higher probability of rising than falling.

Under this parameterisation, the optimal policy is indeed to lean by setting $y < 0$ for all states of the world. The optimal policy and the associated value function are shown in Figure 2, left-hand panel. A few properties of the solution are worth highlighting. First, the policymaker is worse off with higher degree of financial imbalances (the value function is decreasing in $f$); this justifies systematic leaning in order to restrain the likelihood of an expansion of $f$.12 Second, the degree of leaning grows with $f$, suggesting that the benefit of leaning increases as a crisis becomes more imminent. Third, even though a lower $f$ implies a lower crisis risk and a higher value function, the optimal policy always results in some leaning throughout the boom period.

Figure 2, right-hand panel, provides a schematic that summarises intuitively the justifications for leaning against the wind. The optimal policy in essence seeks to equate the marginal cost of output gap forgone with the marginal benefit of securing a low $f$. In the financial cycle model, both are non-zero, and the optimal policy would entail some leaning.

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12 Not shown is the optimal value in crisis, $W$, which is equal to –3.8 and strictly lower than all non-crisis values.
Figure 3 shows the optimal policy for this case, as well as other values of $\Delta$. Each curve represents the optimal output gap at each stage of the financial cycle $f$, as a function of $\Delta$. For $\Delta$ marginally above zero, the optimal policy prescribes more front-loading of leaning at earlier stages of the financial cycle. It pays to lean earlier before imbalances grow too large.

With a higher crisis cost, the benefits from leaning correspondingly rise and the case for leaning early is more favourable. Figure 4 plots the optimal policy under the assumption $\bar{w} = -15$ (versus $\bar{w} = -1$ in Figure 3). The optimal choice of the output gap is more negative for each $f$, compared to Figure 3. Moreover, the modes of the curves are shifted to the right compared to Figure 3. This indicates that not only does...
the central bank lean more, but a stronger leaning is justified over a wider range of values of $\Delta$. A higher crisis cost also has implications for the early leaning result – for a sufficiently high $\Delta$, it is optimal to lean more forcefully at the earlier stages of the cycle, e.g. more when $f = 3$ than at $f = 4$.\(^{13}\)

The exercises highlight a key policy takeaway. Assumptions of a nonzero crisis cost and an endogenous financial cycle together imply that the value function is declining with the degree of financial imbalances – the central bank is worse off whenever the imbalances grow. Faced with such prospects, the central bank has an incentive to open up an output gap, to counter the upward movement in financial imbalances and hence to lower the likelihood of a future crisis.

4. Optimal policy in the endogenous financial cycle model

The stripped down version of the financial cycle model provides the intuition for why systematic leaning may yield benefits, and also serve as a motivation for a more full-fledged model. We now construct a flexible dynamic model of the financial cycle, calibrate it using US data and characterise the optimal degree of leaning.

First, we recast the crisis dynamics by spelling out the dynamics in terms of a financial booms and busts. This allows us to capture different types of financial cycles: severe financial cycles that result in very costly crises and less extreme financial deleveraging episodes that do not but are nonetheless costly.

Second, we model the financial cycle as a stochastic process which can be estimated with data on the financial cycle. The financial cycle is modelled as a 2-regime Markov process with a boom regime and a bust regime. The transition probabilities are now time-varying, depend on the degree of financial imbalances and imply an endogenously persistent financial cycle. It is important to note that an endogenously persistent cycle is very different from a conventional persistent-shock process. The endogenous cycle is one in which the ups and downs in the financial cycle can occur without shocks. A conventional persistent-shock model in contrast relies on sustained, correlated shocks to generate financial ups and downs. Without the shocks, this type of process is strongly self-correcting.

Third, during financial busts, we assume an output loss that cannot be avoided by easing monetary policy. One can think of this as a shorthand for the assumption that during financial deleveraging, monetary policy faces limits in the ability to cleaning-up-after, e.g. due to the balance sheet effects of debt overhang and persistent effects on confidence. There is then a loss associated with a downturn, creating incentives to prevent large financial imbalances from occurring in the first place.\(^{14}\)

\(^{13}\) In the limits of $|\Delta| \to \infty$, leaning disappears for technical reasons, because output gap has a vanishing effect on the transition probabilities, making leaning increasingly expensive.

\(^{14}\) An alternative formulation could stress a lower bound on the nominal interest rate that prevents the central bank from closing the output gap completely.
In this section, we lay out the model specification, describe how to calibrate the model using the US data as an example, and study optimal policy.

4.1 Model specification

Financial cycle block

The financial cycle, $f_t$, follows a random walk process with a drift that depends on the regime $s_t \in \{1, 2\}$, corresponding to boom and bust phases respectively. The nominal policy rate $r_t$ is assumed to influence the financial cycle, with a sensitivity that is regime-dependent and takes the form

$$
    f_t = f_{t-1} + \alpha s_t + \gamma s_t r_{t-1} + e^{f_t},
$$

(4.1)

where $\alpha_1 > 0$ and $\alpha_2 < 0$ by definition of booms and busts and $e^{f_t} \sim N\left(0, \sigma^2\right)$. We assume that $\gamma_1, \gamma_2 \leq 0$, consistent with a risk-taking channel.

The regime $s_t$ follows a (non-homogeneous) Markov process with logistic transition probabilities,

$$
    P_t\left(s_{t+1} = 2 \mid s_t = 1\right) = \zeta_1 \frac{\exp\left(\psi_1 \left(f_t - f^H\right)\right)}{1 + \exp\left(\psi_1 \left(f_t - f^H\right)\right)}
$$

(4.2)

$$
    P_t\left(s_{t+1} = 1 \mid s_t = 2\right) = \zeta_2 \frac{\exp\left(\psi_2 \left(f_t - f^L\right)\right)}{1 + \exp\left(\psi_2 \left(f_t - f^L\right)\right)}
$$

(4.3)

The ‘sensitivity’ parameters $\psi_1 > 0$ and $\psi_2 < 0$ capture the sensitivity of the transition probabilities to developments in the financial cycle. The sign restrictions are such that a probability of transitioning from a boom to a bust grows as financial imbalances build, and the probability of transitioning from a bust to a recovery rises the lower the imbalances. These assumptions capture the endogenous financial cycle idea: a high degree of financial imbalances is a pre-condition for a crisis, a recovery from which is only possible once deleveraging has reduced the stock of imbalances sufficiently.

Around the ‘tipping points’ $f^H > 0$ and $f^L < 0$, the transition probabilities are most sensitive to changes in $f_t$. The parameters $\zeta_1$ and $\zeta_2$, with $0 < \zeta_1, \zeta_2 < 1$, scale the transition probabilities in a multiplicative fashion, and are the limiting transition probabilities as $f_t \to \infty$ and $-\infty$ respectively. In sum, eleven parameters define the dynamics of the financial cycle block.

Monetary policy plays a role in stabilising the financial cycle, by counteracting the upward momentum of $f_t$ in booms (Equation (4.1)). And by making large imbalances unlikely to occur on average, leaning also helps limit the expected duration of a bust. In our model, monetary policy can lessen the total cost of a downturn.
Macroeconomic block

The macroeconomic block is an IS curve describing the behaviour of the output gap, \( y_t \), which for simplicity is assumed to take a backward-looking form as in Rudebusch and Svensson (1999)

\[
y_t = \beta y_{t-1} + \phi r_{t-1} + e^y_t
\]

(4.4)

with \( 0 < \beta < 1, \phi < 0, \) and \( e^y_t \sim N\left(0, \sigma_y^2\right) \). The lagged output term allows for output persistence. A forward-looking IS curve may be less appropriate in the current setting, as it would imply spending reduction in anticipation of a crisis, a feature at odds with anecdotal observations that exuberance tends to precede crises (see Ajello et al (2016) for the motivation of a similar assumption).

We abstract from inflation considerations for simplicity and without loss of generality, as our focus is not on the policy trade-offs arising from supply-side shocks. Note that the central bank in our formulation already has an implicit long-run inflation objective, since it aims to stabilise output deviations around the trend, through the business and financial cycles. In principle, a Philips curve can be introduced, though at an added computational cost given an extra state variable.

Central bank objective function

The IS equation describes the normal part of output under the influence of monetary policy. The central bank’s objective function is to set monetary policy \( r_t \) to maximise the objective function

\[
U = E \sum_{t=1}^{\infty} \delta^{t-1} \left( -\frac{1}{2} y_t^2 - c I\left(s_t = 2\right) \right)
\]

(4.5)

where \( \delta \) is the discount factor and \( I\left(s = 2\right) \) is an indicator function equal to one when \( s = 2 \) and zero otherwise. During the bust phase, we assume the central bank incurs an exogenous output loss \( c > 0 \) that cannot be offset or mitigated by easier monetary policy. The optimisation is subject to the constraints posed by the IS curve Equation (4.4) and the financial cycle dynamics Equations (4.1)–(4.3).

The assumption that monetary policy easing cannot absorb the bust cost \( c \) is crucial. If monetary policy were effective in ‘cleaning up after’ and capable of offsetting any output loss associated with a crisis, then trivially there would be no gain from leaning to prevent a bust.15 In the context of the leaning debate, we are ruling out benign neglect monetary policies as being feasible. We do consider an extension where the bust cost \( c > 0 \) is endogenous. This allows us to address the possibility that a weaker economy is more heavily hit by a bust than a stronger economy. We will show that, as long as there exists a lower bound to the cost \( c \) (ie monetary policy cannot fully offset the impact of a financial bust), the optimality of a leaning policy carries over.

15 For example, the loss function used in Svensson (2016) has the property that a policy easing can increase output (pre-crisis or during a crisis) and lower the impact of the crisis on welfare. See Section 4.3 below for the implications of such a loss function in our setting.
4.2 Calibrating the model

Data

The financial cycle block is calibrated using the quarterly data for United States over the period 1960 Q1 to 2015 Q1, whereas the output equation is estimated over 1954 Q3 to 2015 Q3. The output gap is from the Congressional Budget Office. The interest rate gap is the federal funds rate minus the sample average. The financial cycle is an updated series based on the methodology of Drehmann et al (2012). Figure 5 plots the resulting financial cycle series $f_t$.

Estimating the financial cycle block

We estimate the parameters of the financial cycle block using a maximum likelihood procedure for regime-switching models, a la Hamilton (2008), but generalised to allow time-varying transition probabilities (also see Filardo (1994)). The financial cycle $f_t$ is the observed variable that provides inferences about the latent regime $s_t$. Let $\Omega_{t-1}$ denote the information set at time $t-1$ and let $\theta \equiv \{\sigma_f, \alpha_1, \gamma_1, \alpha_2, \gamma_2, \zeta_1, \zeta_2, \psi_1, \psi_2, f^H, f^L\}$ be the set of the financial cycle parameters. At time $t-1$, the likelihood function of observing $f_t$ conditional on regime $s_t = s$ is given by

$$g(f_t \mid s, \Omega_{t-1}, \theta) = \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp \left\{ -\frac{(f_t - f_{t-1} - \alpha_s - \gamma_s' s_{t-1})^2}{2\sigma_f^2} \right\}. \tag{4.6}$$
Denoting the conditional probability of being in regime $s$ by $\xi_{s,t-1}$. The unconditional likelihood function is then given by

$$g\left( f_t \mid \Omega_{t-1}, \theta \right) = \sum_s \sum_s \xi_{s,t-1} p_{t-1}(s) g\left( f_t \mid \xi_{t-1}, \omega_{t-1}, \theta \right).$$

To complete the algorithm and generate the likelihood for all $t$, the conditional regime probability is recursively updated according to

$$\xi_{s,t} = \frac{\sum_s \xi_{s,t-1} p_{t-1}(s) g\left( f_t \mid \xi_{t-1}, \omega_{t-1}, \theta \right)}{g\left( f_t \mid \Omega_{t-1}, \theta \right)}.$$  

The maximum likelihood estimates are calculated numerically as the solution to

$$\hat{\theta}_{MLE} = \arg\max_{\theta} \left\{ \sum_{t=1}^{T} \log g\left( f_t \mid \Omega_{t-1}, \theta \right) \right\},$$

subject to the restrictions $\alpha_1 \geq 0$, $\alpha_2 \leq 0$, $\gamma_1 \leq 0$, $\gamma_2 \leq 0$, $\psi_1 > 0$, $\psi_2 < 0$, $0 < \zeta_1 < 1$, $0 < \zeta_2 < 1$, $f^H > 0$ and $f^L < 0$.

The model's flexibility allows a good fit to the financial cycle data, but raises potential identification issues. The amplitude, duration and turning points of the financial cycle are all time-varying, and observing the time-series $f_t$ alone may not be sufficient to jointly identify all the transition probability parameters precisely. For example, the observed distribution of the cycle's turning points (peaks and troughs) could be explained by different combinations of scale and sensitivity parameters. To address this identification issue, we add constraints with the strategy to identify a number of models with reasonable properties and investigate the robustness of optimal monetary policy.

With this strategy in mind, three restriction schemes are considered. In scheme A, we calibrate the tipping points $f^H$ and $f^L$ to match historical averages of the financial cycle peaks and troughs in the United States. In scheme B, we place bounds on the sensitivity parameters, $\psi_1 \leq 10$ and $\psi_2 \geq -10$, and estimate all other parameters. This restriction implies that it takes some time for the transition probability to pick up over the cycle. Finally in scheme C, we assume bounds $\psi_1 \leq 10$ and $\psi_2 \geq -10$, and impose constraints on the expected turning points to match historical averages. We implement this by maximising a weighted sum of a standard likelihood function and a loss function that penalises the deviations of the implied turning points from the targets (with the weight on the latter being set arbitrarily high at 100).

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16 Technically, the likelihood function is relatively flat with respect to the scale and sensitivity parameters. Intuitively, the observed turning points in the financial cycle are consistent with different parametric assumptions. For example, the probability of entering a downturn may rise rapidly with the degree of financial imbalances, corresponding to a high sensitivity parameter and a low scale parameter. Or, the regime-switching probability may rise gradually, with a high scale parameter. Both cases are consistent with stochastic turning points of the type seen in the data.
Estimation results

The estimates are reported in Table 1. Estimates of all the ‘drift’ parameters in Equations (4.4) and $\sigma_f$ are robust to different estimation schemes. On average, $f_t$ expands slowly during booms, and falls more quickly during busts. Thus, booms tend to last longer than busts on average, in line with what was documented by Drehmann et al (2012) and Claessens et al (2011) for moderate cycles.

Tighter monetary policy helps rein in the boom - a one percentage point increase in the policy rate gap lowers the expected increase in financial imbalances by about 20 percent per quarter ($-0.0087/0.0424$). Interestingly, a more accommodative monetary policy does not help slow down contractions associated with the bust according to our estimates, so we set the impact of policy during busts to zero. The impact of monetary policy on the financial cycles is therefore restricted to booms. The results are also robust for many other estimation schemes (not shown).\(^{17}\)

\(^{17}\) We explore the possibility of nonlinear drifts and policy effect on the financial cycle and confirm that linearity is a good approximation. We also investigate a special case of the policy effect being subject to diminishing return, so even extreme policy intervention cannot eliminate cycles. This special case is rejected by the data. In our estimated model, it is feasible for the central bank to stabilise the financial cycle.
All three estimation schemes enjoy similarly good fit to the data, but imply different shapes of transition probabilities as shown in Figure 6. In scheme A, the transition probabilities change relatively slowly with the stage of the financial cycle, $f$, and the model allows for time-varying peaks and troughs by admitting higher unconditional switching probabilities due to the scale parameters. The probability of entering a bust approaches the maximum of 0.21 per quarter as imbalances become extreme (the probability of exiting a bust approaches 0.26 per period). In schemes B and C, the bounds on the sensitivity parameters $\psi$ are binding, and the transition probabilities are more sensitive to $f$ around the tipping points, $f_H$ and $f_L$. The limiting transition probabilities in these cases are lower at around 10 percent per period.

Are the restrictions on $\psi$ too stringent? Simple calculations suggest not. With $\psi = 10$, $f$ needs to move by about 0.45 around the tipping points for the transition probability to increase from a near-minimal level of $0.1 \times \zeta$ to a near-maximal level of $0.9 \times \zeta$. In a boom, $f$ rises on average by $\alpha_1 = 0.04$ per quarter when the rate gap is zero, implying that the probability of entering a bust would grow from nearly zero to the maximum level in about 10 quarters. In a bust, the associated length is only four quarters.

Figure 5 depicts the financial cycle, its fitted value, and the filtered probability of being in a boom, $\xi_t$, based on estimation scheme C (other schemes yield obtain almost identical results in terms of the fit and regime probability). The estimated model can explain the United States financial cycle well, and the estimated regime probabilities are broadly in line with how one might visually identify boom and bust episodes, including the latest global financial crisis and the recovery. The model interprets smaller swings in $f$ during 1970s and 1980s as driven by shocks, and not boom-bust cycles.
Parameterising the macro block and the crisis cost

We assume the following simple IS equation

\[ y_t = 0.9y_{t-1} - 0.4r_{t-1} + e_t^y, \quad \sigma_t^y = 1.5. \]  \hspace{1cm} (4.10)

The marginal impact of monetary policy on output of –0.4 is calibrated to roughly match the maximum effect used in the FRB/US model. This parameterisation implies that closing a 1 percent output gap requires a policy rate gap of 2.25 percent. Later, we will consider IS curve specifications with a lower sensitivity to policy rates.18

The cost of financial downturn is set to –10 per quarter as the baseline case. In the loss function in Equation (4.5), this is equivalent to an output gap of roughly –4.5 percent.19 Finally, the discount factor is set at 0.99. In Section 4.4, we will consider how variations in all these parameters influence the optimal monetary policy.

4.3 Optimal monetary policy

Given the Markovian structure, the monetary policy problem can be expressed in the dynamic programming form where the value function is determined by the state variables \((f, y, s)\):

\[
V(f_t, y_t, s_t) = \max_{r(f_t, y_t, s_t)} \left\{ -\frac{1}{2}y_t^2 - cI(s_t = 2) \right\} + \delta E_tV(f_{t+1}, y_{t+1}, s_{t+1})
\]  \hspace{1cm} (4.11)

subject to the estimated laws of motion for \(f_t, y_t,\) and \(s_t\). The expectations at time \(t\) on the right hand side of Equation (4.11) are taken with respect to the joint probability distribution of these state variables.

The optimal policy function \(r(f, y, s)\) and associated value function \(V(f, y, s)\) are numerically computed using the collocation method, which entails solving Equation (4.11) for a chosen set of discrete coordinates over the state space. We approximate the value function using a cubic spline basis function, construct quadratures to capture shocks, and choose a relatively fine grid along the \(f\) dimension to ensure a good approximation to a potentially highly nonlinear optimal policy.20

Figures 7–9 show the 3-dimensional plots of the results, corresponding to estimation schemes A, B and C respectively. A common feature across all schemes is that the boom values \(V(f, y, 1)\) strictly exceed the bust values \(V(f, y, 2)\) for any state \(f\) and \(y\) – the two value surfaces are shown in the top left panels, while the difference

18 For example, an OLS estimate of Equation (4.10) yields \(y_t = 0.96y_{t-1} - 0.05r_{t-1} + e_t^y\) with significant coefficients.

19 This assumption is somewhat more conservative than in Ajello et al (2016) which assumes 10 percent output loss for 8 quarters (implying a crisis cost of –50 per period for a quadratic loss function). Assuming that the same cost is incurred for 6 years, which is the expected duration of a bust regime based on our transition probability estimates, their assumption is equivalent to 6.9 percent output loss per period in discounted terms. Our lower cost assumption may be justified by the fact that it is meant to capture the cost of an average financial downturn, including both full-blown crises and less severe deleveraging episodes. It might be thought of as an expected cost of a financial downturn.

20 See Judd (1999) and Miranda and Fackler (2002) for details on the numerical optimisation algorithm.
Optimal policy under Scheme A

between the two surfaces is plotted in the top right panels. The difference is uniformly positive as long as there is a non-zero downturn cost. A consequence of this positive gap is that \( V(f, y, s) \) always decreases with \( f \) for any \( y \) and \( s \). This is because a higher \( f \) during booms weakly raises the probability of entering a downturn within any horizon, which reduces the value function, while in busts, a lower \( f \) raises the likelihood of improving the value function associated with a recovery. In addition, the policymaker is always worse off under a higher degree of financial imbalance, and prefers to be in a boom than a bust. Along the output gap axis, the value function is concave with maxima at \( y = 0 \).

The optimal monetary policy during booms and busts is shown in the lower two panels of Figures 7–9. For a given regime \( s \) and the financial cycle, \( f \), the optimal rate gap declines linearly with the output gap, as is standard for a quadratic loss.
function (lower left panels). But the optimal interest rate varies with $f$ for any given level of the output gap. To see this clearly, we compute the difference between this optimal policy rate and the optimal policy obtained under the assumption of no downturn cost where the central bank simply sets the policy rate to target $y = 0$. In all estimation schemes, it is optimal to set the policy rate gap higher than otherwise during booms – i.e., leaning is optimal. This is so even at low levels of $f$ when the likelihood of a downturn is extremely remote. The degree of leaning rises with $f$ as the probability of a bust increases, but tapers off as the degree of imbalances reaches an extreme level.

In Figure 10, we show the optimal policy when the output gap is fixed at zero for the three estimation schemes (effectively a cross-sectional slice of the lower-right panels in Figures 7–9). Here, a policymaker who ignores the financial cycle would have set the policy rate gap uniformly at zero given that the output gap is already closed.
The left panel of Figure 10 shows optimal policy during booms, which involves tightening at all stages of the financial cycle. At lower levels of financial imbalances, less leaning is required. But as imbalances build and \( f \) approaches the tipping point, it is optimal to lean progressively more. The nonlinear policy response is due to the endogenous regime-switching probabilities. In schemes B and C, the transition probabilities rise rapidly around the tipping points, and hence the optimal policy is to respond more aggressively in these states. The lower tipping points in schemes B and C also suggest that policy needs to be tightened earlier at lower values of \( f \).

The result supporting the case for early leaning is robust to alternative estimation schemes. At lower levels of financial imbalances, the size of optimal leaning policy is very similar across all estimation schemes – just below 20 basis points at \( f = -3 \), and about 25 basis points on average for \( f \leq 0 \). The maximum amount of leaning is between 70–80 basis points, which is optimal as financial imbalances near the tipping...
But even there, an 80 basis points tightening of policy does not appear unusually large – it implies 0.7 x 0.4=0.32 percent lower output. As policy leans systematically throughout the financial cycle, the results indicate that the amount of leaning in any given quarter need not be large.

One interesting feature of the optimal policy function is its bell-shape around tipping points. As $f$ exceeds a tipping point, the transition probabilities become progressively less sensitive to $f$ as the probabilities asymptote to 1. In this case, monetary policy becomes less effective in pre-emptively preventing a bust. As a consequence, it is optimal to lower the degree of leaning at extreme levels of $f$.

The estimates for $\gamma$ suggest that, during busts, monetary policy has little impact on the financial cycle dynamics. Thus when the economy just enters a bust and large imbalances remain, an optimising policymaker focuses primarily on macro stability and targets a zero output gap (hence implementing a zero rate gap if the output gap is already zero; right-hand panel of Figure 10). As the degree of financial imbalances declines, the transition back to a boom regime becomes more likely and the central bank eases policy. Easing is not intended to influence the financial cycle in this bust state, but to stabilise output in anticipation of future leaning.

### 4.4 Robustness

In this section, we consider various scenarios that could affect the optimal degree of leaning, including alternative assumptions about (i) downturn costs $c$, (ii) IS curve parameters, (iii) the degree of central banker’s patience, (iv) persistence of the financial cycle, $f$ and (v) endogeneity of downturn costs with respect to output.
Downturn costs

The implications of different exogenous downturn costs is straightforward as shown Figure 11, where we consider the alternatives $c = 5$ and $c = 20$. We find that the optimal policy is proportional to the downturn costs at all stages of the financial cycle – when the cost doubles, the optimal degree of leaning is twice as high, and so on. This result generalises the principle that some leaning is optimal as long as the cost of a bust is positive.

IS curve parameters

Greater output persistence implies more long-lasting effects of leaning on output, raising the marginal cost of leaning. To shed some light on the quantitative implications of this dependence on the extent to which a central bank would want to lean, we compare the optimal policy rules under three alternative IS curve specifications:

- Lower persistence: $y_t = 0.7y_{t-1} - 0.4r_{t-1} + e_t^y$ (4.12)
- Flatter IS curve: $y_t = 0.9y_{t-1} - 0.1r_{t-1} + e_t^y$ (4.13)
- Low output impact: $y_t = 0.7y_{t-1} - 0.1r_{t-1} + e_t^y$ (4.14)

In the first alternative, output is less persistent ('lower persistence' case) than the baseline IS curve. In the second, output is less sensitive to the policy rate ('flatter IS' case). In the third, output has lower persistence and is less sensitive to the policy rate.

Optimal policies under the alternatives are shown in Figure 12 (using estimation scheme A). Lower output persistence indeed leads the central bank to lean more.
The optimal degree of leaning is about twice as large as in the baseline. The flatter IS curve has a significantly larger quantitative impact on optimal policy. The impact is many times larger than that coming from a reduction in the policy sensitivity parameter. Combining lower persistence with a flatter slope raises the optimal degree of leaning even further. In other words, leaning yields benefits heavily on the financial cycle at every stage, including earlier in the cycle. The optimal degree of leaning is about twice as large as in the baseline.

But the flatter IS curve has a significantly larger quantitative impact on optimal policy. The impact is many times larger than that coming from a reduction in the policy sensitivity parameter. Combining lower persistence with a flatter slope raises the optimal degree of leaning even further. In other words, leaning yields benefits
that grows nonlinearly when IS curves are flatter and less sensitive to the policy rate, as shown in Figure 13.

Degree of central bank patience

A greater weight placed on future outcomes should reinforce the case for leaning, as a central bank would at the margin have an incentive to sacrifice more today to avoid future downturns. We evaluate this argument in the context of our model by considering alternative discount factors.

In Figure 14, the baseline optimal policy with the discount factor $\delta = 0.99$ is plotted together with the optimal policy under $\delta = 0.95$ and $\delta = 0.999$ (all based on scheme A and original IS curve). Greater patience significantly strengthens the case for front-loading leaning in the cycle. For example when $\delta = 0.999$, the rate gap is set to about 35 basis points in a boom, even at very low levels of financial imbalances. Moreover, the central bank also leans more forcefully at very high $f$, even if the marginal influence on the transition probability may be declining. This is because a more forward-looking central bank internalises more fully the endogenous downturn duration. Allowing $f$ to increase implies a longer period of time it will take to unwind the stock of imbalances, leading to a more prolonged period spent in the downturn. Greater forward-lookingness places a higher penalty on a longer downturn, leading to stronger leaning at later stages of the financial cycle.

Persistence of the financial cycle

We consider the optimal policy under a less persistent financial cycle than in the baseline. The financial cycle is now assumed to follow a first-order autoregressive process of the form,
where \( \gamma < 0 \) and \( \rho < 1 \). In this case, financial imbalances do not build up momentum in expectation. Rather financial imbalances are mean-reverting and, in expectation, gravitate towards zero over time. Note, that in this specification, a boom-bust cycle is only possible when there is a correlated sequence of positive shocks that push up \( f \), as in the random crisis model.\(^{21}\)

Figure 15 shows the optimal policies when \( \rho = 0.9 \) and \( \rho = 0.99 \). In both cases, there is little incentive for early leaning, because the central bank faces a much less favourable trade-off between the short-term macroeconomic costs of leaning and the impact on the likelihood of a crisis. The more persistent process with \( \rho = 0.99 \) entails earlier leaning, as it takes longer for financial imbalances to unwind. A higher persistence implies that the likelihood of a future crisis is larger at any given \( f \). However, when \( f \) becomes very high, the looming crisis provides an incentive for leaning more strongly in either cases.\(^{22}\)

\[^{21}\] Once a downturn occurs, we assume as before that \( f \) declines, until eventually there is a regime switch back to a boom.

\[^{22}\] The case of vanishing persistence \( \rho \to 0 \) (not shown) implies less leaning than any of the cases presented. There is still a gain from leaning in such a case, even if the financial cycle variable is close to being an i.i.d variable. This is because the central bank wants to increase the probability of entering a bust with low imbalances, which would entail a less prolonged bust phase.
Endogenous downturn costs

One insight from Svensson (2016) is that the potential benefit of leaning can be fully offset if the cost of a crisis depends on the state of the economy. Intuitively, a weak economy is likely to suffer more than a strong economy for a given downturn. One way to incorporate this idea into the modelling framework is to alter the period loss function in the baseline in the following way

\[ L_t = (y_t - cI(s_t = 2))^2. \]  

(4.16)

Under this specification, \( c \) can be interpreted as the output loss associated with a crisis. With this loss function, Svensson (2016) shows that leaning with the wind is the optimal policy. By leaning with the wind, a central bank can bolster the economy when a crisis is imminent so as to cushion the negative impact of a bust of a given cost.

The period loss function (4.16), however, has less appeal in our multi-period setting. Rewriting the baseline model with this loss function yields a fundamentally different optimal policy. In this case, the central bank can essentially offset the total impact of the bust on the loss function without a macroeconomic trade-off. In this case, the central bank lowers the policy rate until the output gap \( y \) is equal to \( c \). Therefore, the period loss is exactly equal to zero in the bust, ie cleaning up is costless. With zero loss once the bust occurs, there is no incentive to lean against the wind during the boom. Figure 16 shows the optimal policy when using the period loss function (4.16), which entails substantial leaning with the wind as well as cleaning up during a bust.

Note that a cleaning-up-after strategy is a first best strategy if it is feasible. This is the premise behind the benign neglect view – that the unwinding of financial
imbalances can be costless in a macroeconomic sense. But the feasibility of such a strategy has been challenged by financial boom-bust experiences of the type associated with the GFC and others earlier in history.

Endogenous downturn costs are nonetheless an interesting issue. One can consider alternative loss functions that allow for costlier crises when the economy is weaker without necessarily implying a benign neglect strategy. For example, consider

\[
L = -\frac{1}{2} y_t^2 - c I \left( s_t = 2 \right) I \left( y_t < 0 \right) g \left( y_t \right)
\]

where \( g \left( y_t \right) \) is a decreasing function and \( g \left( 0 \right) = 1 \). This loss function still implies that downturn costs are higher when the output gap is negative, but the costs are not reduced when the output gap is positive. In other words, the downturn costs have a lower bound of \( c \).

The optimal policy associated with this case is shown in Figure 16, denoted by ‘Cost with lower bound’. The function \( g \left( y \right) \) is assumed to be linear. The early leaning result is again established, and is qualitatively similar to the baseline case. The inference we draw from this exercise is that as long as a central bank cannot completely eliminate the cost of a bust and the financial cycle is sufficiently persistent, then optimal policy is to lean and to do so early in the cycle.

4.5 Assessing the bias with the marginal cost-benefit approach

As noted in the introduction, Svensson (2016) and IMF (2015) report evidence against leaning using the marginal cost-benefit approach. In other words, they find that the marginal cost of a one-time tightening of the policy rate to rein in a financial boom exceeds the marginal benefits. It appears that their conclusion contradicts the findings in our model. This raises the question of whether the difference arises solely from some calibration detail or from the different approaches. The latter could indicate that the full benefits of systematic leaning (that stabilises the endogenous financial cycle) are not captured in the marginal cost-benefit approach.

To address this possibility, it is instructive to first consider how the cost-benefit calculations differ between our approach and theirs. The optimal policy in our model is characterised by the first-order condition of Equation (4.11),

\[
\frac{\partial}{\partial r_t} \left( \frac{1}{2} y_t^2 \right) = \frac{\partial}{\partial r_t} E_t V \left( f_{t+1}, y_{t+1}, s_{t+1} \right)
\]

\[
= \frac{\partial}{\partial r_t} \left( P_t \left( \text{crisis} \mid \text{boom} \right) \int V \left( \tilde{f}, \tilde{y}, \text{crisis} \right) dF_t \left( \tilde{f}, \tilde{y} \right) \right) + \left( 1 - P_t \left( \text{crisis} \mid \text{boom} \right) \right) \int V \left( \tilde{f}, \tilde{y}, \text{boom} \right) dF_t \left( \tilde{f}, \tilde{y} \right) \right),
\]

which must hold for all \( t \). The left-hand side is the marginal cost of leaning and the right-hand side is the marginal benefit of leaning.

The characterisation of the marginal cost is the same under the marginal cost-benefit approach. The difference is the way in which the marginal benefit is measured. In the marginal cost-benefit approach, it is defined as
\[
\frac{\partial P_t(\text{crisis} \mid \text{boom})}{\partial t} \cdot (\hat{V}(\text{crisis}) - \hat{V}(\text{boom})),
\]

where \( P_t \) is the transition probability function from a boom to a crisis at a given time \( t \). Note that the value function in Equation (4.19), \( \hat{V} \), is an estimate of the welfare cost of a crisis, based on the assumption of a no-leaning baseline. In general, an estimate of \( \hat{V} \) necessarily differs from \( V \) in Equation (4.18), which is a function of the optimal leaning policy rule. The latter takes into account the full effect of a systematic leaning rule on the value function. In the case of a very persistent, endogenous financial cycle, the difference between \( \hat{V} \) and \( V \) is likely to be substantial, making the marginal benefit in (4.19) a poor guide for the optimal policy. 23

To get a sense of the quantitative bias of the marginal cost-benefit approach, we apply the marginal cost-benefit approach to our model. Suppose that both the output gap and interest rate gap are initially zero and the degree of imbalances is assumed to be \( f = 2 \), so that the risk of entering a downturn is already relatively high. The central bank then raises the policy rate gap for 4 quarters by 20 basis points, before reverting back to the output-targeting policy rule thereafter. Figure 17 shows the paths of interest rate and output gaps, as well as the implied marginal cost and benefit of such an action. The marginal cost reflects the impact on output. The marginal benefit is the decline in the expected cost of a crisis, which is a product of (i) how much the rate increase curbs the rise in \( f \), (ii) how much the transition probability is lowered and (iii) the downturn cost \( c \).

This one-time 20-basis-points tightening results in the marginal cost that is substantially larger than the marginal benefit. This result based on applying the marginal cost-benefit approach to our model is therefore consistent with that in the recent literature: a one-time tightening of monetary policy is not welfare-enhancing from the marginal cost-benefit analysis perspective! The type of leaning in this experiment is a one-time, discretionary use of the policy rate in the midst of a financial boom. In contrast, the type of leaning emphasised earlier in the paper is a policy of systematic rule-based leaning over the whole financial cycle.24

In sum, this quantitative assessment suggests that only by taking the full endogenous financial cycle dynamics into account and by considering optimal policy

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23 The value function approach and the marginal cost-benefit approach will yield the same leaning results if the policy rate does not materially influence the future value function. This would occur under two different assumptions. First, in our model, if the monetary authority can completely clean a financial bust’s effect on the macroeconomy, the future value function is not affected by monetary policy actions during the boom. Then the marginal cost-benefit approach would yield accurate calculations. Also, if the financial cycle were immaterial or completely exogenous, the future value function would not be affected by leaning and the marginal cost-benefit approach would be accurate. But such assumptions about the financial cycle are at odds with the assumption of the exercise in this paper, ie that the financial cycle is costly, endogenous and unavoidable.

24 In fact, using this marginal cost-benefit approach, there is a case for leaning only when the magnitude is no greater than 5 basis points. This result is striking considering that the rule-based optimal policy derived earlier prescribes leaning as much as 80 basis points for the same initial conditions (see Figure 7). The benefits from leaning therefore are measured to be much lower quantitatively based on the one-time marginal cost-benefit calculation than based on the value function approach.
rules can the full benefit of a leaning policy be accurately measured. It also highlights the fact that even in a model where systematic leaning is optimal, it still may not be welfare-enhancing to act in a discretionary fashion late in a financial cycle boom.

5. Conclusion

This paper establishes the case for a systematic leaning-against-the-wind policy in an environment where there is a recurring financial cycle with costly crises. The policy’s main benefit arises from a dampening of the whole financial cycle – in terms of the frequency and severity of financial downturns. By dampening the cycle, a central bank reduces overall volatility, even though from a short-run perspective there may be somewhat higher macroeconomic conditional volatility.

The conclusion from our model stands in contrast to those models which downplay the dampening effect of monetary policy by assuming financial cycles that have a strong inherent tendency to self-correct. From our model’s perspective, the

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1 The policy interest rate is assumed to be tightened by 20 basis points for 4 quarters, starting from the initial conditions of, \( r = 0 \), \( y = 0 \) and \( f = 2 \). The policy rate is then re-adjusted to target zero output gap subsequently. The marginal benefit is the change in the expected period downturn cost, based on estimation Scheme A. The marginal cost is the increase in quadratic loss as a result of lower output gap.

25 For example, central banks that apply the marginal cost-benefit approach in assessing their own policy forecasting models may be at risk of concluding erroneously that there are no net benefits to leaning when in fact there may be. Central banks would have to check whether the full set of dynamics of the crisis are modelled in order to evaluate whether the equivalence between the marginal cost-benefit approach and the full value-function approach is satisfied.
existing analytical approach tends to underestimate the benefits of leaning by focusing on: i) strongly self-correcting financial cycles, and ii) a one-off policy action to address an imminent financial crisis. As a consequence, the policy assessment of a one-time discretionary tightening during a financial boom only gives a lower bound of the potential benefits from leaning.

In many respects, the modelling debate today harkens back to the one during the high and volatile inflation period in the 1970s and early 1980s. Model-wise, the debate then centred on the policy advice from short-term Keynesian models and from policy rule-based dynamic models (Lucas (1981)). Which model provided reliable guidance for policymakers? By the end of the debate, few believed that short-term discretionary policy actions to counter a rise in inflation was the best approach to achieve lasting price stability. Indeed, stop-go monetary policies in the 1970s proved to be ineffective and destabilising. Rather, strong price stability-oriented monetary policy frameworks, that were loosely “rule”-based and transparent, offered a more fruitful approach to achieving the desired results. In the same way, if financial cycles are considered an inherent part of the fabric of financially liberalised systems – which seems to be the case – models based on systematic policy responses may offer more reliable guidance that those based on one-off discretionary actions. And, if so, the debate should focus on understanding and assessing different policy rules. Of course, operationalising the basic thrust of such models and demonstrating the accuracy of the policy advice remain a challenge.

There are a number of practical reasons for qualifying the strength of our modelling conclusion. Given the difficulty of accurately tracking the financial cycle in real time, the net benefit from systematic leaning will depend on a consideration of type 1 and type 2 errors – that is, the errors of acting as if financial imbalances were growing when in fact they were not, and of not acting when financial imbalances were in fact growing. If a central bank cannot accurately track the extent of financial imbalances, additional costs need to be factored into the assessment of the net benefit. The consequences of such imperfect information conditions for cost-benefit assessment is left for future research.

Further analysis is also called for as our understanding of costly financial cycles evolves. In this paper, the financial cycle has been treated as being captured sufficiently well by a single variable, “\( f \)”. Recent research, however, has emphasised a more nuanced relationship between different aspects of the financial cycle such as leverage, stocks versus flows of debt, debt service burdens, cross-border gross financial flows and bubbly prices. The dynamic interaction of these variables with the strength of the financial cycle and the economy is important. In addition, even though the trade-offs with inflation dynamics are implicit to our baseline model, layering on an explicit Philips curve relationship would allow us to explore an additional set of relevant policy issues. For example, uncertainties about the slope of the Philips curve can be analysed. As well, one could explore how policy credibility would be maintained even if leaning implied allowing inflation to run below target for a prolonged period of time. One possibility is to extend the notion of policy credibility beyond a fixed numerical target, and towards a more general through-the-cycle sustainability criterion.
In addition, an exploration of the micro mechanisms that generate endogenous financial cycles could yield important insights into the incentives that different actors in the financial system face and, as a consequence, the effectiveness of policy rates when leaning. Incorporating such features into our framework would enrich the analysis and shed new light on the nature of optimal policy.

Finally, even though we considered different ways of reflecting the costs of financial cycles, other important considerations may have been missing from our model. During the boom phase of a financial cycle, for example, there may be a rapid rise in the capital stock. On the one hand, as shown in recent studies for advanced economies (e.g., Gopinath et al. (2015) and Adalet McGowan et al. (2016)), the increase in the capital stock associated with booms has resulted in unproductive capital overhangs or inefficient capital allocation across economic sectors. By contrast, financial booms in our model do not have long-run negative consequences for productivity and potential growth. Such considerations could raise the benefits of systematic leaning (Juselius et al. (2016) and Gourio et al. (2016)).

On the other hand, financial booms may, under certain circumstances, contribute to future growth prospects. This may be the case in some emerging market economies. By accelerating the transition from a low to a high capital economy, a financial boom may yield additional benefits even if an eventual financial bust reverses some of them. Building in different types of growth effects may clarify some of the policy trade-offs facing central banks. In addition, the degree of substitutability or complementarity of macroprudential and monetary policy tools in managing the financial cycle is also key to any full calibration of the cost-benefit analysis. In this paper, we implicitly assumed that macroprudential tools could not fully stabilise the financial cycle (BIS (2016)).

Overall, our dynamic modelling approach is certainly not the last word on the issue of leaning against the wind. But the approach and the calibration of the financial cycle shows that our class of models provides a rich environment in which to assess the various costs and benefits associated with leaning.
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A. Appendix

In this appendix, we discuss an extension of the random crisis model which allows for a state-contingent crisis probability function. Suppose as in Woodford (2012) and Ajello et al (2016) that the crisis likelihood is increasing in the degree of financial risk-taking or imbalances $f_i$ (which again could be leverage, stock of credit, asset prices and so on), and is given by $p(f_i)$ where $p'(f_i) > 0$. Monetary policy then influences the evolution of leverage, which, crucially, is assumed to be a mean-reverting process:

$$f_i = \rho f_{i-1} + \gamma y_i + \epsilon_i$$

(A.1)

with $\rho < 1$ and $\gamma < 0$.

The Bellman equations in this case are:

$$V_t(f_{i-1}) = \max_{y_t} \left\{ \frac{1}{2} \left( \frac{\pi_t^2}{\rho^2} + \lambda y_t^2 \right) + \beta \left( p(f_i)(W_{t+1} - V_{t+1}(f_i)) + V_{t+1}(f_i) \right) \right\}$$

(A.2)

$$W_t = \bar{w} + \beta \left[ q(V_{t+1}(f_i) - W_{t+1}) + W_{t+1} \right]$$

(A.3)

Solving for the first-order conditions subject to the Phillips curve and Equation (A.1) gives

$$\lambda y_i = -\kappa \pi_i + \beta \gamma p'(f_i)(W_{t+1} - V_{t+1}(f_i)) + \beta \gamma (1 - p(f_i))V'_{t+1}(f_i)$$

(A.4)

which is almost identical to (3.3) except for the last term, which takes into account the effect of changes in the state variable on the optimal value. Applying the envelope theorem, the term $V'_{t+1}(f_i)$ is given by a recursive formula

$$V'_{t+1}(f_i) = \frac{\rho}{\gamma} \left( \kappa \pi_{t+1} + \lambda y_{t+1} \right) + \beta \rho p'(f_i+1)(W_{t+2} - V_{t+2}(f_{i+1}))$$

$$+ \beta \rho (1 - p(f_{i+1}))V'_{t+2}(f_{i+1})$$

(A.5)

The optimality conditions (A.4) and (A.5) now include current as well as all future marginal crisis probabilities $p'(f_i)$. However, the expected crisis cost $T$ periods ahead are discounted by a factor $\rho^{2T}$ (this can be readily checked by solving (A.5) forward). The rule of engagement is a close variation of the stripped-down random crisis model. It is optimal to lean only when the policy instrument is effective in reducing the probability of the crisis today and within a relatively short horizon ahead.

The intuition for this result is simple. Given a mean-reverting process in (A.1), the risk of entering a crisis is always expected to decline over time along with a reversion of $f$ back to its steady state. Indeed, in the neighbourhood of the steady state, the policymaker may expect to never lean when looking ahead. Only after successive positive shocks $\epsilon_i > 0$ have pushed $f_i$ up sufficiently that $p'(f_i)$ is no longer negligible, will leaning policy be justified. One can thus wait until a crisis becomes a clear and present danger before acting. Because a crisis ultimately occurs due to a particular realisation of shocks, this model may be thought of as a random crisis type.
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