TIPS Liquidity Premium and Quantitative Easing

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Abstract

We assess the effect of the QE2 program on the TIPS liquidity premium using a latent factor approach and a counterfactual exercise. In the context of a state-space model for nominal and TIPS yields, we identify the TIPS liquidity premium as the common component in TIPS yields that is unspanned by nominal yields. We then construct a counterfactual TIPS liquidity premium that exploits suitable conditioning information. Results indicate that the QE2 program had only limited effect on the TIPS liquidity premium.

JEL classification codes: C33, C53, E43, G12.

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1 Introduction

Following the 2008 financial crisis, the Federal Reserve conducted large-scale asset purchases, known as quantitative easing (QE), in order to lower long-term interest rates and spur economic growth. A large literature focusses on the impact of these large-scale asset purchases on nominal Treasuries, and there is a consensus that QE successfully reduced yields on long-maturity nominal Treasuries, see among others Krishnamurthy and Vissing-Jorgensen (2011), Gagnon, Raskin, Remache, Sack et al. (2011) and Joyce, Lasaosa, Stevens and Tong (2011). However, little attention has been paid to the effect of the QE program on the Treasury Inflation-Protected Securities (TIPS) liquidity premium.

TIPS are fixed-income securities with coupons and principal payments indexed to the non-seasonally-adjusted CPI for all urban consumers. They were introduced in 1997 and now constitute 10% of the outstanding U.S. Treasury debt. Given that TIPS have a smaller and less liquid market than US nominal Treasury bonds, they pay a liquidity premium, representing the compensation required by investors to hold a security that is less liquid than its nominal counterpart, see Gürkaynak, Sack and Wright (2010), D'Amico, Kim and Wei (2018) and Campbell, Shiller and Viceira (2009), among others. The QE program included purchases of large amounts of TIPS, and assessing the effects of TIPS purchases on the TIPS liquidity premium may allow to further understand the benefits and risks of QE programs.

Large-scale TIPS purchases may have affected the TIPS liquidity premium through two opposite channels: the liquidity channel and the scarcity channel. The liquidity channel implies that large-scale asset purchases may have reduced the liquidity premium required by investors to buy TIPS, as the Federal Reserve's purchases may have made it less costly for investors to sell TIPS. However, this effect of the QE program on the TIPS liquidity premium depends on the Federal Reserve's flow of purchases and, therefore, it should have been limited to the duration of the QE program. On the other hand, the scarcity channel implies that the Federal Reserve's TIPS purchases may have reduced the already scarce stock of TIPS available to investors and this instead may have increased the liquidity premium in the TIPS market.

The liquidity channel is closely related to the market functioning channel described by Gagnon et al. (2011) and it implies that QE can improve market functioning. The scarcity channel instead is related to the local supply channel of Vayanos and Vila (2009) and D'Amico, English, López-Salido and Nelson (2012), and it implies that the purchase by the Federal Reserve of assets with a specific maturity leads to lower yields of assets with similar maturities. However, in the presence of a liquidity premium component, this mechanism implies a higher liquidity premium for the purchased assets. This trade-off with liquidity is not considered in the local supply channel, as noted by Ferdinandusse, Freier and Ristiniemi (2017), and it could represent an important risk of asset purchase policies. For example, Kandrac (2013) finds evidence of market disruption caused by the Federal Reserve purchases of Mortgage Backed Securities, and Schlepper, Hofer, Riordan and Schrimpf (2017) find an adverse impact of asset purchases on Bund market liquidity.

In this paper, we assess the effect of the QE2 program on the TIPS liquidity premium using a latent factor approach to measure the TIPS liquidity premium, and a counterfactual exercise to quantify the effect of the QE2 program on the TIPS liquidity premium. In the framework of a state-space model for nominal and TIPS yields, we identify the relative mispricing of TIPS with respect to nominal Treasury bonds as the common component in TIPS yields that is unspanned by nominal yields. This approach is in line with D'Amico et al. (2018) and allows us to get an estimate of the TIPS liquidity premium that does not depend on a specific TIPS maturity, and that does not require preselecting a liquidity proxy or using inflation swaps, that can also be subject to trading frictions, see Fleming and Sporn (2013).

We estimate a joint state-space model for nominal and TIPS yields that treats the liquidity premium in the TIPS market as an unobservable component that we extract simultaneously with the yield curve factors. Our empirical model is a Dynamic Factor Model for nominal and TIPS yields with zero restrictions on the factor loadings of nominal yields on the TIPS liquidity premium factor. These zero restrictions capture the fact that the TIPS liquidity premium factor is unspanned by nominal yields. Using daily US data from January, 2 2005 to December, 31 2014, we obtain estimates of the TIPS liquidity premium factor by Quasi-Maximum Likelihood using the Kalman filter and the EM algorithm following the approach proposed by Coroneo, Giannone and Modugno (2016).

We then analyze the effect on the TIPS liquidity premium of the QE2 program announced on November, 3 2010 and implemented from November, 23 2010 to June, 17 2011. This program involved \$600 billion purchases of Treasury securities, of which \$26 billions were TIPS purchases, implying that the Federal Reserve's purchases of Treasuries within the QE2 program have been larger and concentrated in a shorter time spam with respect to the other large-scale asset purchases. We assess the effect of the QE2 programme on the TIPS liquidity premium by performing a counterfactual analysis. This is easily implementable in our framework regardless of the dimensionality of the conditioning variables, thanks to the use of the state-space representation and Kalman filtering, see Bańbura, Giannone and Lenza (2015).

Our objective is to construct a counterfactual path for the TIPS liquidity premium factor that does not incorporate the QE2 program, but that exploits suitable conditional information. In our framework, a conditioning variable is suitable for the construction of a counterfactual TIPS liquidity premium if it has predictive ability for the TIPS liquidity premium, but it is not directly affected by the QE nor by the TIPS liquidity premium itself. We find that measures of financial stress, such as the market-wide illiquidity measure of Hu, Pan and Wang (2013) and the corporate spreads, satisfy these conditions. The resulting counterfactual TIPS liquidity factor is on average higher than the realized TIPS liquidity factor, however the difference is only marginally significant when taking into account the accuracy of the counterfactual. We therefore conclude that the QE2 program had only a marginal effect on the TIPS liquidity premium, and that the liquidity channel was only marginally stronger than scarcity channel.

This paper is related to Christensen and Gillan (2013) and Abrahams, Adrian, Crump, Moench and Yu (2016), that also analyze the impact on the QE2 on the TIPS liquidity premium. Christensen and Gillan (2013) find that the QE2 reduced the TIPS liquidity premium, implying that the liquidity channel has been stronger than the scarcity channel, while Abrahams et al. (2016) do not find any effect of the QE2 program on the TIPS liquidity premium. The main difference with our work is how the TIPS liquidity premium is estimated. Abrahams et al. (2016) use an observable liquidity proxy, while Christensen and Gillan (2013) use replicating portfolios. Our counterfactual results reconcile the results in Abrahams et al. (2016) with the ones is Christensen and Gillan (2013). In particular, as Abrahams et al. (2016) we find that overall the QE2 program did not affect the TIPS liquidity premium, however as Christensen and Gillan (2013) we also document that the effect of the QE2 program on the TIPS liquidity premium peaked during the middle of the program in April 2011, when the realized TIPS liquidity premium was significantly smaller than the counterfactuals. In in this short time period, the liquidity channel was significant, and stronger than the scarcity channel.

The paper is organized as follows. In Section 2, we define and identify the liquidity premium in the TIPS market. Section 3 introduces the data set and presents some preliminary evidence. Section 4 outlines the estimation procedure. In Section 5 we report estimation results. Section 6 describes the Quantitative Easing programme. Section 7 shows the counterfactual results and Section 8 contains robustness results using an alternative data set and model specification. Finally, Section 9 concludes.

2 Identifying the TIPS liquidity premium

2.1 Decomposing nominal and TIPS yields

The yield of a nominal zero-coupon Treasury bond of any maturity can be decomposed into the underlying real yield, the inflation expectation over the remaining life of the bond and the inflation risk premium. Denoting by $y^N_{t,\tau}$ the nominal yield with maturity τ, we can write

$$y^N_{t,\tau} = y^R_{t,\tau} + \pi^e_{t,t+\tau} + IP_{t,t+\tau} \tag{1}$$

where $y^R_{t,\tau}$ denotes the real yield with maturity τ, $\pi^e_{t,t+\tau}$ is the expected rate of inflation for the remaining life of the bond and $IP_{t,t+\tau}$ is the inflation risk premium.

The real yield $y^R_{t,\tau}$ can, in principle, be proxied by a Treasury Inflation-Protected Security (TIPS) with the same maturity. However, while the market for nominal U.S. Treasuries is the most liquid financial market, TIPS only represent 10% of the outstanding U.S. Treasury debt. This implies that TIPS investors face liquidity risk due to the possibility that they may need to make portfolio adjustments after the initial auction or before maturity, being forced to buy or sell TIPS in the secondary market. For this reason, TIPS investors may demand a liquidity premium for holding an instrument that is less liquid than nominal Treasury securities. If we denote by $y^T_{t,\tau}$ the TIPS yield with maturity τ, we have

$$y^T_{t,\tau} = y^R_{t,\tau} + LP_{t,\tau} \qquad (2)$$

where $LP_{t,\tau}$ is the liquidity premium at time t for the TIPS with maturity τ.

If the TIPS secondary market is sufficiently liquid, the liquidity premium is zero and Equations (1)-(2) imply that, all else equal, any change in TIPS yields should be reflected one-to-one into a change in nominal yields. The TIPS liquidity premium creates a wedge between TIPS yields and real interest rates. In particular, Equations (1)-(2) imply that any component that affects TIPS yields but not nominal Treasury yields is related to the TIPS liquidity premium.

2.2 Liquidity premium in the TIPS market

The liquidity premium in the TIPS market measures the liquidity premium in the entire TIPS market. As such, it can be measured as the common factor in the liquidity premia of TIPS with different maturities. If we denote by L_t the liquidity premium in the TIPS market, then

$$LP_{t,\tau} = a^L_\tau + B^L_\tau L_t + \varepsilon^L_{t,\tau} \qquad (3)$$

where a_τ^L is the maturity-specific intercept, B_τ^L contains the factor loadings of the liquidity premium of the TIPS with maturity τ on the liquidity premium factor and $\varepsilon_{t,\tau}^L$ represents the maturity specific component of the liquidity premium of the TIPS with maturity τ.

In order to extract the TIPS liquidity premium factor L_t in Equation (3), we model nominal and TIPS yields using a dynamic factor model. We assume that the yield curve of nominal Treasuries is described by a few latent yield curve factors, while the TIPS yield curve is driven by both the yield curve factors and the TIPS liquidity factor. In practice, following Equations (1)–(2), we identify the liquidity premium in the TIPS market L_t as the common component in TIPS yields that is unspanned by nominal yields.

Formally, we assume that nominal yields with different time to maturities are driven by a vector of $r_X \times 1$ of common latent factors X_t as follows

$$y_{t,\tau}^N = a_\tau^N + B_\tau^N X_t + \varepsilon_{t,\tau}^N \tag{4}$$

where a_τ^N is the maturity-specific intercept, B_τ^N contains the factor loadings of the nominal yield with maturity τ on the latent factors (common across maturities) and $\varepsilon_{t,\tau}^N$ represent the maturity-specific component of the nominal yield with maturity τ. In the same way, for real yields we have

$$y_{t,\tau}^R = a_\tau^R + B_\tau^R X_t + \varepsilon_{t,\tau}^R \tag{5}$$

where a_τ^R is the maturity-specific intercept, B_τ^R contains the factor loadings and $\varepsilon_{t,\tau}^R$ is the maturity-specific component of the real yield with maturity τ.

Let the vector $y_t^N = (y_{t,\tau_1^N}^N, \ldots, y_{t,\tau_n^N}^N)'$ collect the nominal yields with maturities $(\tau_1^N, , \ldots, \tau_n^N)$ and the vector $y_t^T = (y_{t,\tau_1^T}^T, \ldots, y_{t,\tau_m^T}^T)'$ collect the TIPS yields with maturities $(\tau_1^T, \ldots, \tau_m^T)$. Following equations (2)–(5), the joint model for nominal and TIPS yields can be written as

$$\begin{pmatrix} y_t^N \\ y_t^T \end{pmatrix} = \begin{pmatrix} a^N \\ a^T \end{pmatrix} + \begin{bmatrix} B^N & 0 \\ B^R & B^L \end{bmatrix} \begin{pmatrix} X_t \\ L_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix} \tag{6}$$

where $a^T = a^R + a^L$ and $\varepsilon_t^T = \varepsilon_t^R + \varepsilon_t^L$.

Equation (6) identifies two sets of latent factors: the yield curve factors X_t and the TIPS liquidity factor L_t. The yield curve factors are in line with the literature that exploits the high level of comovement of yields with different maturities to provide a parsimonious representation of the yield curve. This literature has proven that yield curve factor models are very successful in fitting the yield curve of nominal interest rates, see Litterman and Scheinkman (1991), Duffee (2002) and Coroneo, Nyholm and Vidova-Koleva (2011). As for the liquidity factor, by noticing that TIPS are less liquid than nominal Treasuries, see Gürkaynak and Wright (2012) and Fleckenstein, Longstaff and Lustig (2014), we are able to identify the TIPS liquidity premium factor L_t as the driver of the wedge between real and TIPS yields. We implement this identification condition through the zero factor loading restrictions in Equation (6), which imply that the liquidity factor does not affect the current nominal yield curve, as shown in Equations (1)–(2). This approach is in line with D'Amico et al. (2018), that jointly model nominal and TIPS yields in the framework of an affine term structure model, and allow TIPS yields to be driven by a factor that is unspanned by nominal Treasuries.

We allow the $(n+m)$ idiosyncratic components collected in $\varepsilon_t = \left[(\varepsilon_t^N)', (\varepsilon_t^T)'\right]'$ to follow independent univariate AR(1) processes

$$\varepsilon_t = A\varepsilon_{t-1} + v_t,\ v_t \sim N(0, R) \tag{7}$$

where A and R are diagonal matrices, implying that the common factors fully account for the joint correlation of the observations.

The $(r_X+1) \times 1$ vector of zero mean latent factors follow a VAR(1)

$$\begin{pmatrix} X_t \\ L_t \end{pmatrix} = \begin{bmatrix} \Phi_{XX} & \Phi_{XL} \\ \Phi_{LX} & \Phi_{LL} \end{bmatrix} \begin{pmatrix} X_{t-1} \\ L_{t-1} \end{pmatrix} + \begin{pmatrix} u_t^X \\ u_t^L \end{pmatrix}, \tag{8}$$

where the innovations $u_t = \left[(u_t^X)', (u_t^L)'\right]'$ are normally distributed with zero mean and variance

Table 1: Variances explained by principal components

	Nominal	Nom+TIPS
PC1	0.941	0.902
PC2	0.996	0.955
PC3	1.000	0.995
PC4	1.000	0.998
PC5	1.000	0.999

Note: this table report the percentage of variance of nominal yields (first column) and of nominal and TIPS yields jointly (second column) explained by the first four principal components extracted from nominal yields (first column) and from nominal and TIPS yields jointly (second column).

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Finally, the innovations driving the common factors, u_t in Equation (8), and the residuals to the idiosyncratic components of the individual variables, v_t in Equation (7), are normally distributed and mutually independent. This implies that the common factors are not allowed to react to variable specific shocks.

3 Data

We use end-of-day yield curve data spanning the period January, 2 2005 to July, 21 2014, for a total of 2,339 observations. Nominal yields and TIPS yields with maturities 3, 5, 8, 10, 15 and 20 years are based on zero-coupon yield curves fitted at the Federal Reserve Board, see Gürkaynak, Sack and Wright (2007) and Gürkaynak et al. (2010) for details.

We exclude the short end of the nominal yield curve to avoid the possible non-linearities associated with short maturities reaching the zero lower bound in the last part of the sample. Also, in order identify the effect on the TIPS liquidity premium around the days of the QE2 announcement and operations, we use a daily frequency. In Section 8, we report robustness results using weekly data and a selection of short, medium and long maturities for the nominal yields.

In Table 1 we report the percentage of variance of nominal yields and of jointly nominal and TIPS yields explained by the first five principal components extracted, respectively, from nominal yields and jointly from nominal and TIPS yields. Results in Table 1 show that two factors fully explain the cross-section of nominal yields, but when considering nominal and TIPS yields jointly an additional factor should be included in the analysis. This indicates that TIPS yields are driven by a factor that is unspanned by nominal yields which accounts for the liquidity premium in the TIPS market. Accordingly, in our analysis we use two factors to explain the cross-section of nominal yields, i.e. $r_X = 2$, and one factor to explain the liquidity premium in the TIPS market. This choice is due to the peculiarity of our data set, namely daily yields from mid to very long maturities. In Section 8, we report results on weekly data with short to long maturities for which we estimate a model with three yield curve factors, in line with Litterman and Scheinkman (1991).

To compare our estimates of the TIPS liquidity premium, we construct a liquidity proxy from inflation swap rates. We use mid-quotes of inflation swap rates with maturity 3, 5, 8, 10, 15 and 20 years from Datastream converted to continuously compounded basis. Following Haubrich, Pennacchi and Ritchken (2012), we compute real rates as the difference between equivalent maturity nominal Treasury yields and inflation swap rates. We then construct a liquidity proxy as the average, across maturities, of the difference between equivalent maturity TIPS yields and real rates constructed using inflation swaps.

As possible conditioning variables for the counterfactual analysis, we consider the TED spread (defined as the spread between the three month LIBOR and the three-month Tbill rates), the Chicago Board Options Exchange Volatility Index (VIX), the Cleveland Financial Stress Index, the corporate spread (defined as the spread between the Baa corporate and the ten-year Treasury rates), the bid-ask spread on the three-month Tbill and the illiquidity measure of Hu et al. (2013) which is a market-wide measure of illiquidity. Data for all variables is obtained from the FRED database, except for the illiquidity measure of Hu et al. (2013), available from the authors.[1]

[1] The illiquidity measure of Hu et al. (2013) is available at http://www.mit.edu/ junpan/

4 Estimation

The joint model for nominal and TIPS yields in Equations (6)–(8) is a restricted dynamic factor model with autocorrelated idiosyncratic components. In order to cast the model in a state-space form, we augment the vector of state variables with the vector of idiosyncratic components ε_t and an additional state variable c_t restricted to one at every period (by fixing its initial value to one and the variance of its innovations to zero). We then rewrite the measurement equation as

$$\begin{pmatrix} y_t^N \\ y_t^T \end{pmatrix} = \begin{bmatrix} B^N & 0 & a^N & I_n & 0 \\ B^R & B^L & a^T & 0 & I_m \end{bmatrix} \begin{pmatrix} X_t \\ L_t \\ c_t \\ \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix} + \begin{pmatrix} v_t^N \\ v_t^T \end{pmatrix} \tag{9}$$

where $X_t = (X_{1,t}, X_{2,t})'$, $\left((v_t^N)', (v_t^T)'\right)' \sim N(0, \epsilon I_{n+m})$ and ϵ is a coefficient that we fix to 1^{-12}. In the same way, we write the state equation as

$$\begin{pmatrix} X_t \\ L_t \\ c_t \\ \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix} = \begin{bmatrix} \Phi_{XX} & \Phi_{XL} & 0 & 0 & 0 \\ \Phi_{LX} & \Phi_{LL} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & A^N & 0 \\ 0 & 0 & 0 & 0 & A^T \end{bmatrix} \begin{pmatrix} X_{t-1} \\ L_{t-1} \\ c_{t-1} \\ \varepsilon_{t-1}^N \\ \varepsilon_{t-1}^T \end{pmatrix} + \begin{pmatrix} u_t^X \\ u_t^L \\ \nu_t \\ v_t^N \\ v_t^T \end{pmatrix} \tag{10}$$

with $\left((u_t^X)', (u_t^L)', \nu_t, (v_t^N)', (v_t^T)'\right)' \sim N\left(0, blkdiag(Q, \epsilon, R)\right)$ and $A = diag(A^N, A^T)$.

The model in (9)–(10) is a restricted state-space model for which maximum likelihood estimators of the parameters are not available in closed form. Conditionally on the factors, the model reduces to a set of linear regressions. As consequence, we compute Maximum Likelihood estimates using the Expectation Maximization (EM) algorithm introduced by Shumway and Stoffer (1982) and Watson and Engle (1983). This estimator is feasible when the number of variables is large, and robust to non Gaussianity and to the presence of weak cross-sectional correlation among the idiosyncratic terms, see Doz, Giannone and Reichlin (2012). In addition, as shown in Coroneo et al. (2016), using the Kalman filter and the EM algorithm allows us to easily impose linear restrictions on the parameters. For more details see Appendix A.

4.1 Counterfactual analysis

Let's now assume that at time t_0 a policy is implemented and we are interested in assessing the effect of this particular policy on the liquidity premium in the TIPS market. This can be done by comparing the realized path of the TIPS liquidity premium factor with a counterfactual TIPS liquidity premium factor that does not incorporate the policy.

The counterfactual TIPS liquidity premium factor is the forecast of the TIPS liquidity factor for $t \geq t_0$ conditional on past TIPS and nominal yields, and possibly past and future values of suitable conditioning variables. A conditioning variable is suitable for the construction of a counterfactual path for the liquidity premium if it is informative about the liquidity premium in the TIPS market and it is invariant to the policy. In our framework, the first condition is satisfied if the conditioning variable Granger causes the TIPS liquidity premium. The invariance condition instead is satisfied if the policy does not directly affect the conditioning variables, and the TIPS liquidity premium (that in principle can be affected by the policy) does not feedback into the conditioning variables, i.e. if the TIPS liquidity premium does not Granger cause the conditioning variable.

We collect the set of zero mean conditioning variables in the $w \times 1$ vector W_t and add them to the state equation (10), see Appendix A for the full state-space representation. We then denote the counterfactual TIPS liquidity factor as

$$L^*_{t|T} \equiv E[L_t | y^N_1, \dots, y^N_{t_0-1}, y^T_1, \dots, y^T_{t_0-1}, W_1, \dots, W_T], \ t \geq t_0, \tag{11}$$

and the associated mean squared error as

$$V^*_{t|T} = E[(L_t - L^*_{t|T})(L_t - L^*_{t|T})'], \ t \geq t_0. \tag{12}$$

In practice, we are interested in extracting the TIPS liquidity factor for $t \geq t_0$ from a data set of TIPS, nominal yields and conditioning variables, where TIPS and nominal yields are unobserved from t_0. Both the counterfactual TIPS liquidity factor in (11) and its accuracy in (12) can be easily computed in our framework regardless of the dimensionality of the conditioning variables, thanks to the use of the state-space representation and Kalman filtering, see Bańbura et al. (2015). Following Durbin and Koopman (2012), we use a modified state-space model where the dimensionality of the vector of observable variables varies over time. In practice, after t_0 only the rows that refer to the conditioning variables will enter into the measurement equation.

We construct the $(1-\alpha)$ confidence interval for the counterfactual TIPS liquidity premium factor as

$$CI(L^*_{t|T})_{1-\alpha} = \left(L^*_{t|T} - \Phi^{-1}(1-\alpha/2)\sqrt{V^*_{t|T}},\ L^*_{t|T} + \Phi^{-1}(1-\alpha/2)\sqrt{V^*_{t|T}}\right),\ t \geq t_0 \qquad (13)$$

where $\Phi^{-1}(1-\alpha/2)$ denotes the $(1-\alpha/2)$ quantile of the standard normal distribution.

5 Estimated TIPS liquidity factor

In this section, we report estimation results for the joint model for nominal and TIPS yields in (9)-(10) using the full sample of data, i.e. from January, 2 2005 to December, 31 2014, with a particular focus on the estimated TIPS liquidity factor.

Table 2 reports the percentage of variance of nominal and TIPS yields explained by the estimated latent factors. The table shows that the model has a good fit for both nominal and TIPS yields, with at least 96% of the variance of yields explained by the latent factors. The first latent factor explains the bulk of the variation in both nominal and TIPS yields, while the second yield curve factor explains up to 14% of the variance of nominal and TIPS yields.

The TIPS liquidity factor by construction does not affect nominal yields but it has substantial explanatory power for TIPS yields. It explains up to 22.9% of the variance of TIPS yields and its explanatory power is higher for shorter maturities, implying that investors require higher compen-

Table 2: Variance explained by the latent factors

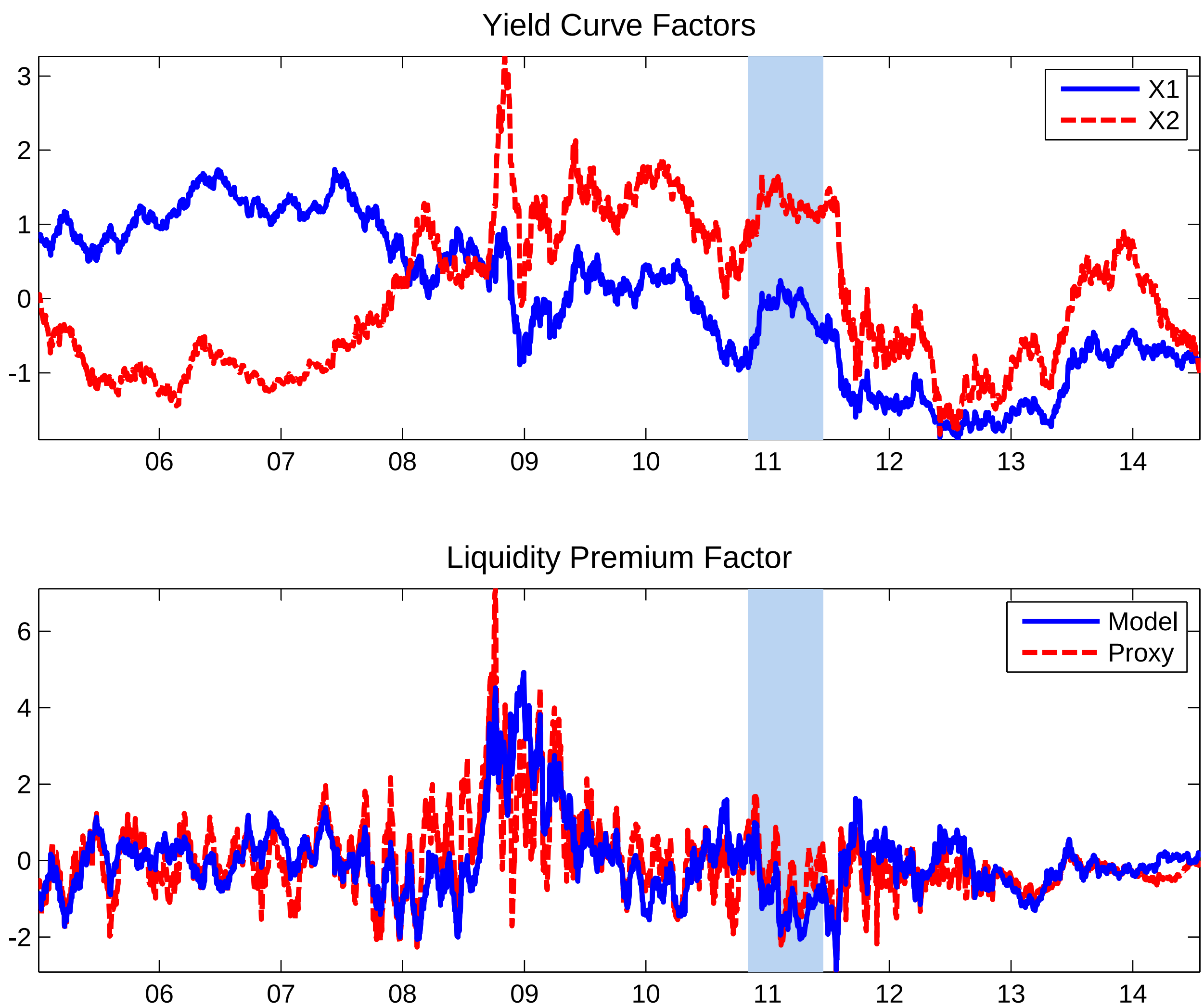
	Nominal Yields			TIPS Yields			
Mat	X_1	X_2	Total	X_1	X_2	L	Total
3	0.850	0.146	0.996	0.722	0.005	0.229	0.956
5	0.947	0.053	1.000	0.810	0.001	0.179	0.991
8	0.999	0.000	0.999	0.841	0.017	0.141	1.000
10	0.991	0.009	1.000	0.840	0.032	0.125	1.000
15	0.945	0.048	0.993	0.809	0.078	0.098	0.988
20	0.901	0.058	0.960	0.744	0.135	0.088	0.969

Note: this table reports the percentage of variance of nominal yields (left panel) and of TIPS yields (right panel) explained by the estimated latent factors of the model in (9)-(10) for each observed maturity.

sation for holding shorter-term TIPS rather than longer-term ones. This might be due to the fact that growth of the TIPS markets has not occurred uniformly, see Shen (2006). For example, the Treasury has issued 10-year TIPS every year since the TIPS program began in 1997. On the contrary, the 5-year TIPS, were issued in 1997 and 1998, but then not again until 2005. An additional explanation could be due to the presence of different types of investors in different segments of the TIPS yield curve, i.e. preferred-habitat investors as in Vayanos and Vila (2009). In particular, investors in the long end of the TIPS yield curve, e.g. pension funds, are more likely to buy and hold and, thus, do not require a liquidity premium in order to invest in TIPS since they will rarely need to turn over their positions.

Figure 1 reports the estimated yield curve factors (top panel) and the TIPS liquidity premium factor (bottom panel). The first yield curve factor has a decreasing pattern in our sample due to a general decline in interest rates in this period. The second yield curve factor is more volatile than the first and is higher in the middle of the sample. The bottom plot of Figure 1 reports the TIPS liquidity factor and a proxy for the average liquidity in the TIPS market, constructed as the standardized average, across maturities, of the difference between TIPS yields and real rates computed using inflation swaps and nominal yields of the same maturity, as described in Section 3. The figure shows that the TIPS liquidity factor is highly correlated with this empirical

Figure 1: Estimated yield curve and TIPS liquidity factor



The figure displays the estimated latent factors from model (9)-(10). The top panel displays the estimated yield curve factors X_1 and X_2. The bottom panel displays the estimated TIPS liquidity premium factor L (continuous blue line) and a proxy for the TIPS liquidity premium factor constructed using inflation swaps (red dashed line). The blue shaded areas indicate the QE2 operation period.

proxy. The pairwise correlation coefficient is 75%. We can also notice that the proxy for the average liquidity in the TIPS market constructed using inflation swaps is more volatile than the estimated TIPS liquidity premium factor. This may be due to the fact that inflation swaps are also subject to liquidity frictions, see Fleming and Sporn (2013), and therefore the liquidity proxy constructed using inflation swaps measures the liquidity premium in both TIPS and inflation swaps, see Christensen and Gillan (2012). Figure 1 also shows that during the subprime crisis, the TIPS liquidity premium became more volatile and, in September 2008, following the Lehman Brothers collapse, the liquidity premium in the TIPS market increased substantially. This indicates that in this period investors required a higher compensation in order to invest in an instrument that is less liquid than its nominal counterpart. The TIPS liquidity premium remained at this higher level until mid-2009, when it returned to pre-2008 levels.

6 Quantitative Easing

Following the 2008 financial crisis, the Federal Reserve conducted massive asset purchases know as quantitative easing (QE) to lower long-term interest rates and spur economic growth.

On November 25, 2009 the Federal Open Market Committee (FOMC) announced the first quantitative easing program (QE1) which would involve purchases in government-sponsored enterprises (GSEs) debt and in mortgage-backed securities (MBS). On March 18, 2009 the FOMC announced that the QE1 program would involve additional purchases in GSEs and MBS. It was also announced that the program would involve purchases of $300 billion in long-term Treasury securities. The Treasury purchases ended on October 29, 2009 and involved $6.1 billion in TIPS purchases.

The second quantitative easing program (QE2) was announced on November 3, 2010 with the target of expanding the Federal Reserve's balance sheet by $600 billions through Treasury security purchases over an eight-month period. The gross purchases of Treasury securities from November 3, 2010 until June 30, 2011 amounted to nearly $750 billion, of which about $26 billion were TIPS purchases.

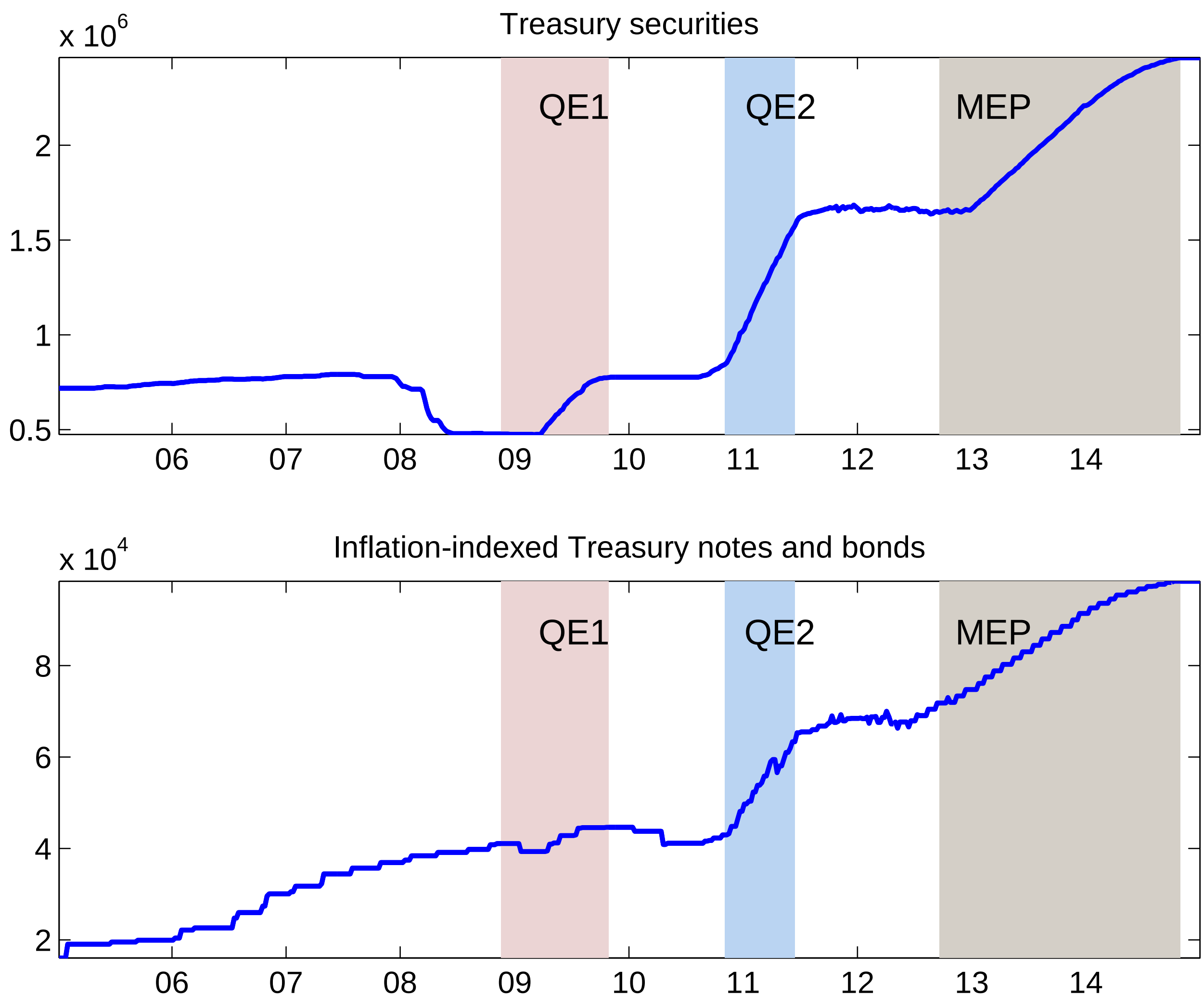
On September 21, 2011 the FOMC announced the third QE program, know as maturity extension program (MEP), which would involve Fed purchases of $400 billion in long-term Treasuries and equivalent sales in short-term Treasuries. On June 20, 2012 the FOMC announced that purchases of long-term bonds and the sales of short-term bonds would continue through 2012 and would involve a total of $600 billion in purchases and sales of securities. On December 12, 2012 the Fed announced that it would continue to purchase $45 billion in long-term Treasuries per month but without the sale of short-term Treasuries to sterilize purchases. The MEP involved TIPS purchases for a total of $27.1 billion, all in TIPS with more than 6 years to maturity. The TIPS sales within the MEP totaled $13.4 billion and only included TIPS with less than 3.25 years to maturity.

In Figure 2 we report the Federal Reserve's outright holdings of Treasury securities and TIPS. The figure shows that the Federal Reserve's outright holdings of both nominal Treasuries and TIPS sharply increased during the QE2 program. This because the Fed purchases of Treasuries within the QE2 program have been larger and concentrated in a shorter time spam with respect to the QE1 and the MEP programs.

Such massive purchases of TIPS may have affected the TIPS liquidity premium through two opposite channels. First, the liquidity channel implies that the QE2 programme may have reduced the liquidity premia required by investors to buy TIPS, as the Federal Reserve's purchases may have made it less costly for investors to sell TIPS. However, this effect of the QE2 programme on the TIPS liquidity premium depends on the Federal Reserve's flow of purchases and, therefore, it should have been limited to the duration of the large-scale asset purchases programme. Second, the scarcity channel implies that the Federal Reserve's TIPS purchases may have reduced the already scarce stock of TIPS available to investors, and this instead may have increased the liquidity premium in the TIPS market.

The liquidity channel is closely related to the market functioning channel described by Gagnon et al. (2011) and it implies that QE can improve market functioning. The scarcity channel instead is related to the local supply channel of Vayanos and Vila (2009) and D'Amico et al. (2012), and it

Figure 2: Federal Reserve's outright holdings of Treasuries



The figure displays the total face value of the Federal Reserve's outright holdings of Treasury securities (top graph) and inflation-indexed Treasury notes and bonds (bottom plot). The shaded areas indicates the QE1, QE2 and the MEP operation periods. Data are weekly.

Table 3: QE2 TIPS purchase

	Dates	Amount (Mill.)	Average Maturity
0	03-Nov-10		
1	23-Nov-10	$1,821	9.43
2	08-Dec-10	$1,778	8.88
3	21-Dec-10	$1,725	16.09
4	04-Jan-11	$1,729	16.98
5	18-Jan-11	$1,812	14.64
6	01-Feb-11	$1,831	13.58
7	14-Feb-11	$1,589	14.16
8	04-Mar-11	$1,589	11.37
9	18-Mar-11	$1,653	17.77
10	29-Mar-11	$1,640	18.29
11	20-Apr-11	$1,729	23.17
12	04-May-11	$1,679	13.62
13	16-May-11	$1,660	20.49
14	07-Jun-11	$1,589	14.3
15	17-Jun-11	$2,129	5.98
	Average	$1,730	14.58

Note: this table reports the QE2 TIPS purchase operations dates along with the amount (in millions) and the (weighted) average maturity.

implies that the purchase by the Federal Reserve of assets with a specific maturity leads to lower yields of assets with similar maturities. However, in the presence of a liquidity premium component, this mechanism implies a higher liquidity premium for the purchased assets. This trade-off with liquidity is not considered in the local supply channel, as noted by Ferdinandusse et al. (2017), and it could represent an important risk of asset purchase policies.

Table 3 contains the exact dates of the TIPS purchases along with the amount and the average maturity. As a preliminary assessment of the effect of the QE2 program on the liquidity in the TIPS market, we report in Table 4 the cumulative responses of the estimated liquidity premium in the TIPS market around the days of the QE2 announcement (Nov. 3, 2010) and operations (from Nov. 23, 2010 to Jun. 17, 2011) over different time windows (from one day to five days change). The table shows that on the day of the QE2 program announcement the estimated TIPS

Table 4: Cumulative responses of the estimated TIPS liquidity premium around QE2 events

		1	2	3	4	5
0	03-Nov-10	-0.262	0.082	0.172	0.108	-0.231
1	23-Nov-10	0.191	-0.306	0.163	0.146	0.120
2	08-Dec-10	-0.272	-0.006	-0.420	-0.287	-0.864
3	21-Dec-10	-0.081	-0.193	-0.297	-0.092	-0.163
4	04-Jan-11	0.168	-0.199	-0.234	0.057	0.099
5	18-Jan-11	0.069	0.298	0.083	0.317	0.314
6	01-Feb-11	-0.235	-0.457	-0.856	-1.141	-1.255
7	14-Feb-11	0.223	0.087	0.037	0.119	0.022
8	04-Mar-11	0.284	0.310	0.199	0.505	0.728
9	18-Mar-11	-0.042	0.021	-0.020	-0.126	-0.291
10	29-Mar-11	-0.053	-0.101	-0.174	-0.227	-0.069
11	20-Apr-11	-0.137	-0.144	0.180	0.145	0.261
12	04-May-11	0.146	0.337	0.294	0.241	0.092
13	16-May-11	-0.079	0.044	-0.137	-0.121	-0.088
14	07-Jun-11	0.043	-0.039	-0.305	-0.163	-0.066
15	17-Jun-11	-0.096	-0.214	-0.264	-0.475	-0.438
	Mean	-0.008	-0.030	-0.099	-0.062	-0.114
	Median	-0.047	-0.022	-0.078	-0.017	-0.068

Note: this table reports the cumulative changes in the estimated TIPS liquidity premium factor around the days of the QE2 announcement (Nov. 3, 2010) and operations (from Nov. 23, 2010 to Jun. 17, 2011) over different window sizes (from one day change to five days change). The changes in the estimated TIPS liquidity factor are computed from the day before the event, i.e. the two day change for the QE2 announcement is the difference in the estimated TIPS liquidity premium factor between November 4, 2010 and November 2, 2010.

liquidity premium declined. In addition, on eleven out of fifteen TIPS operation dates of the QE2 program, the estimated liquidity premium in the TIPS market declined either the same day or the following day. The average and median of the changes of the liquidity factor on the QE2 dates are negative, indicating that the program may have lowered the liquidity premium required by market participants in order to invest in TIPS. To formally assess the impact of the QE2 program on the liquidity premium in the TIPS market, in the next section we perform a counterfactual analysis.

Table 5: Granger causality tests

Null: the variable does not Granger cause L

	TED	VIX	CFSI	IL	CS	BA	$\widehat{X}_1$	$\widehat{X}_2$
Fstat	2.375	4.331	2.295	11.262	6.266	0.005	2.199	0.804
pval	(0.12)	(0.04)	(0.13)	(0.00)	(0.01)	(0.94)	(0.14)	(0.37)

Null: L does not Granger cause the variable

	TED	VIX	CFSI	IL	CS	BA	$\widehat{X}_1$	$\widehat{X}_2$
Fstat	0.001	0.297	0.071	0.430	1.170	6.313	24.599	9.490
pval	(0.98)	(0.59)	(0.79)	(0.51)	(0.28)	(0.01)	(0.00)	(0.00)

Note: this table report likelihood ratio test statistics and p-values. All variance-covariance matrix are robust to autocorrelation and heteroscedasticity. All statistics refer to a univariate VAR(1) of the listed variable and the estimated TIPS liquidity factor L. TED refers to the TED spread (spread between the three month LIBOR and the three month Tbill rates), VIX refers to the Chicago Board Options Exchange Volatility Index, CFSI refers to the Cleveland Financial Stress Index, IL refers to the illiquidity measure of Hu et al. (2013), CS refers to the corporate spread (spread between the Baa corporate and the ten year Treasury rates), BA denotes the bid-ask spread on the three month Tbill, $\widehat{X}_1$ and $\widehat{X}_2$ are the estimated yield curve factors from (9)–(10).

7 Counterfactual results

Our objective is to assess the effect of the QE2 program on the liquidity premium in the TIPS market. We do this by comparing the realized path of the TIPS liquidity premium factor with a counterfactual TIPS liquidity premium factor that does not incorporate the QE2 program, but that exploits suitable conditional information.

As explained in Section 4.1, conditioning variables can be used in the constructions of the counterfactual TIPS liquidity premium if they Granger cause the TIPS liquidity premium, they are not Granger-caused by the TIPS liquidity premium, and they are not directly affected by the policy. As for the first two conditions, in Table 5 we report Granger causality tests for a set of potential conditioning variables that include the TED spread, the VIX index, the Cleveland Financial Stress Index, the illiquidity measure of Hu et al. (2013), the corporate spread, the bid-ask spread on the three month nominal Treasury bill and the estimated yield curve factors.

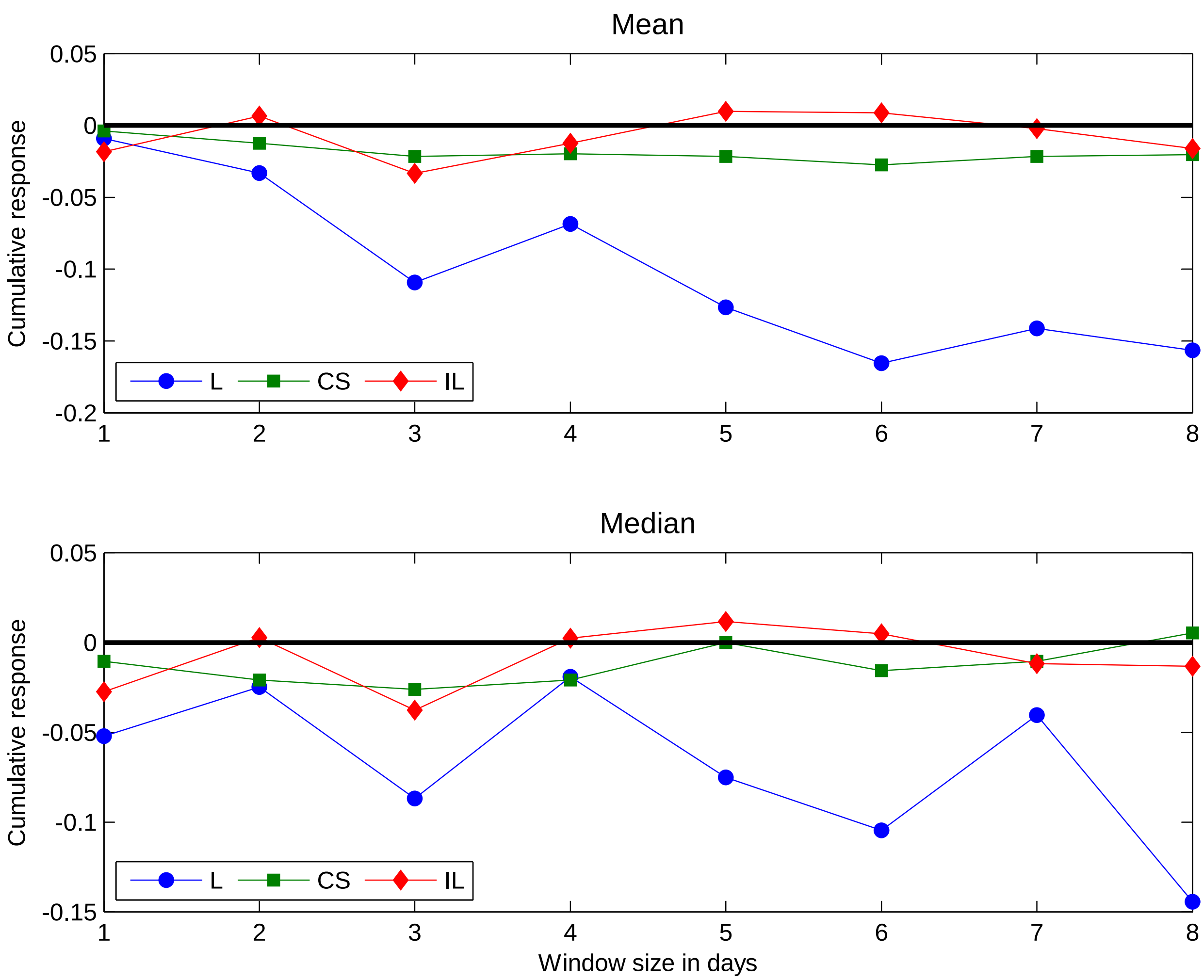
Results in Table 5 indicate that only the illiquidity measure of Hu et al. (2013) and the corporate spread have significant predictive ability for the TIPS liquidity premium at 1% significance level and are not significantly predicted by the TIPS liquidity premium. The illiquidity measure of Hu et al. (2013) is a market-wide illiquidity measure that exploits the connection between the amount of arbitrage capital in the market and observed noise in U.S. Treasury bonds. Hu et al. (2013) show that this measure captures episodes of liquidity crises of different origins across financial markets. The corporate bond spread measures default risk premium and liquidity premium in the corporate bond market, see Longstaff, Mithal and Neis (2005). Overall results in Table 5 indicate that the TIPS liquidity premium is Granger-caused by measures of financial stress but, given the size of the TIPS market, events in this market have limited ability to spread across financial markets.

Table 5 also indicates that variables related to the yield curve of nominal Treasuries, i.e. the bid-ask spread on the three month Tbill and the yield curve factors, do not have any predictive ability for the TIPS liquidity premium factor. On the contrary, they are Granger-caused by the TIPS liquidity premium, indicating that changes in the TIPS liquidity premium affect nominal Treasuries.

The last condition that a conditioning variable should satisfy in order to be included in a counterfactual exercise is that it should not be directly affected by the policy. In Figure 3 we report the mean and the median cumulative responses of the estimated TIPS liquidity premium, the standardized corporate spread and the standardized illiquidity measure of Hu et al. (2013) around the dates of QE2 events, over different time windows. The figure indicates that both the corporate spread and the illiquidity measure were not directly affected by the QE2 operations, as opposed to the TIPS liquidity premium that instead shows large declines for any time window. Therefore, we conclude that both the illiquidity measure and the corporate spread are suitable conditioning variables for the construction of the counterfactual TIPS liquidity premium factor.

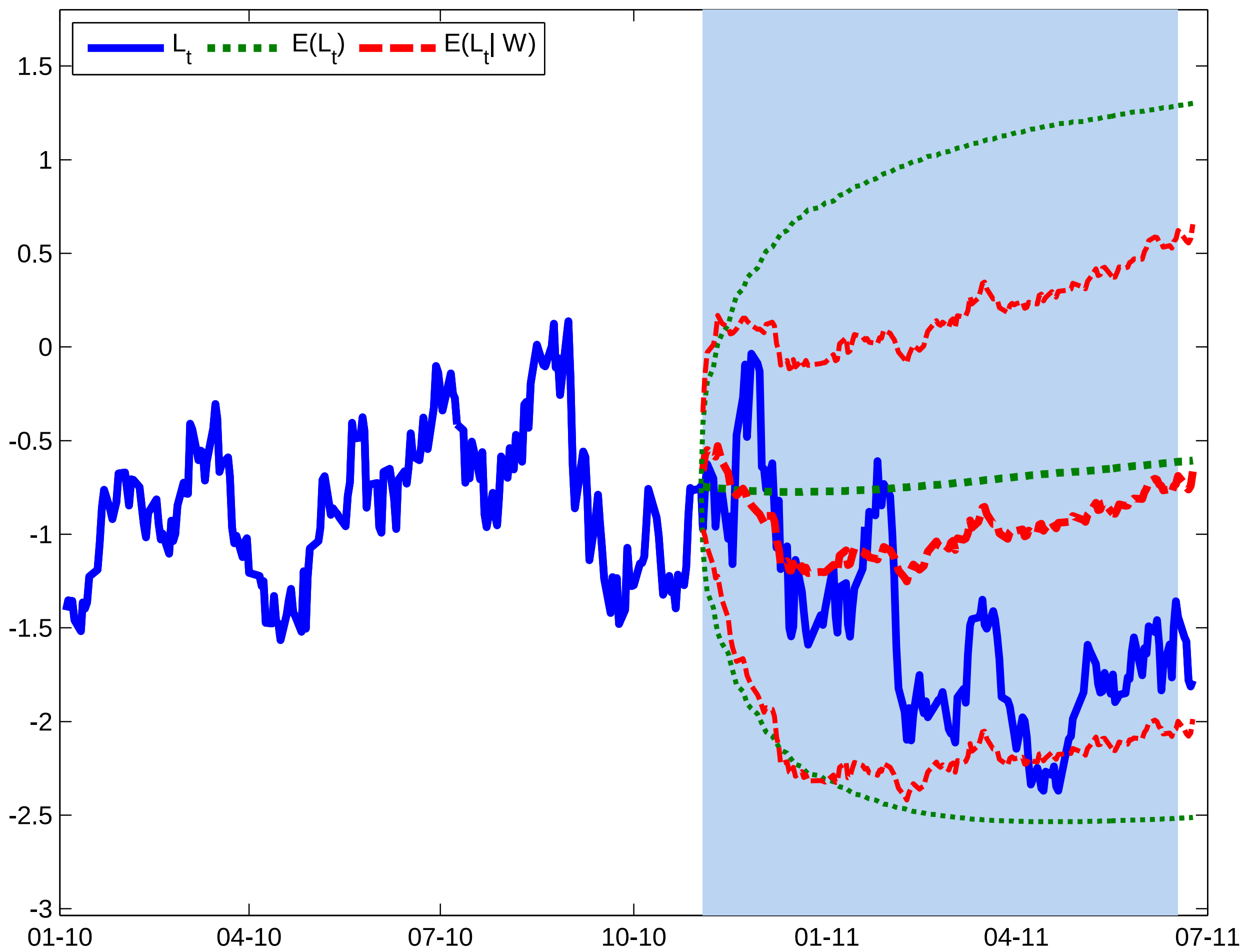
We construct the counterfactual TIPS liquidity premium factor by estimating the joint model for TIPS and nominal treasuries in (9)-(10) using observations up to November, 2 2010, i.e. the day before the announcement of the QE2 purchases. This implies that the counterfactual keeps historical pre-QE2 relations among the variables. We then assume that nominal and TIPS yields are only observed until November, 2 2010, and that conditioning variables are always observed.

Figure 3: Cumulative responses around QE2 events



The figure displays the mean and median cumulative responses of the estimated TIPS liquidity premium factor (L), the standardized corporate spread (CS) and the standardized illiquidity measure of Hu et al. (2013) (IL) around the days of the QE2 announcement and operations over different time windows (from one day to eight days).

Figure 4: TIPS liquidity premium factor: realized vs counterfactuals



The figure displays the estimated TIPS liquidity premium (continuous blue line) and two counterfactuals in which nominal and TIPS yields are observed only until November, 2 2010, as defined in (11). The thick green dotted line is the counterfactual TIPS liquidity premium factor that does not use any conditioning information, i.e. $W = \emptyset$. The thick dashed red line refers to the counterfactual TIPS liquidity premium factor that conditions on the illiquidity measure of Hu et al. (2013) and the corporate spread. The thin lines delimitate the 95% confidence intervals for the counterfactuals with conditioning information (dashed) and without (dotted), computed as in (13). The blue shaded area indicates the QE2 operation period.

The counterfactual TIPS liquidity premium and the corresponding accuracy are then computed, respectively, as in (11) and (12), where $t_0 =$ November, 3 2010.

We perform two counterfactual exercises. In the first, we do not use any conditioning variable, i.e. $W_t = \emptyset$. In the second, we use as conditioning variables the illiquidity measure of Hu et al. (2013) and the corporate spread, i.e. $W_t = [IL_t, CS_t]$. Results in Figure 4 show that the two counterfactual liquidity premia are on average higher than the realized TIPS liquidity premium. The figure also shows that using conditioning variables has two effects on the counterfactual. First, when using conditioning information, the counterfactual path for the TIPS liquidity factor gets on average closer to the realized liquidity premium. This indicates that part of the observed reduction of the TIPS liquidity premium in this period is due to a general reduction in illiquidity and risk premia in financial markets. Second, the accuracy of the counterfactual increases as more conditioning information is used. This because the conditioning variables provide additional information.

As for the comparison of the realized TIPS liquidity premium with the counterfactuals, Figure 4 shows that the realized TIPS liquidity premium is within the 95% confidence bands of both counterfactuals most of the times. The only exception is in April 2011, when the realized TIPS liquidity premium factor is significantly lower than the counterfactual constructed using the illiquidity measure of Hu et al. (2013) and the corporate spread. This indicates that in this period the liquidity channel was significantly stronger than the scarcity channel. However, this effect is only temporary and the realized TIPS liquidity premium factor reverts back within the 95% confidence bands in May 2011. We therefore conclude that the QE2 program had only a marginal effect on the liquidity premium in the TIPS market, and that the liquidity channel was only marginally stronger than scarcity channel.

8 Alternative data and specification

Results in Sections 5 and 7 are obtained using a daily data set for nominal and TIPS yields with maturities 3, 5, 8, 10, 15 and 20 years. Our choice of a daily frequency is motivated by the possibility of identifying daily changes in the TIPS liquidity premium around the dates of the QE2 announcement and operations. Also, we select mid to very long maturities to avoid the possible non-linearities associated with short maturities reaching the zero lower bound in the last part of the sample.

However, usually joint models for nominal and inflation-indexed bonds are estimated using weekly or monthly data, and a selection of short, medium and long maturities for the nominal yields, see Abrahams et al. (2016), Christensen, Lopez and Rudebusch (2010), Joyce, Lildholdt and Sorensen (2010), Haubrich et al. (2012) and Carriero, Mouabbi and Vangelista (2016). For this reasons, one may wonder how the results in Sections 5 and 7 are sensitive to our data choices. To address this concern, in this section we re-estimate the TIPS liquidity premium and perform the counterfactual exercise using end-of-week data for nominal yields with maturities 3 months, 6 months and 1, 3, 5, 8 and 10 years, and TIPS yields with maturities 3, 5, 8 and 10 years. One advantage of using the short end of the nominal yield curve is that a third factor for the nominal yield curve can be estimated. Accordingly, we re-estimate our model in (9)-(10) using three yield curve factors, as common in the literature since the work of Litterman and Scheinkman (1991).

Table 6 reports the percentage of variance of nominal and TIPS yields explained by the estimated latent factors when using the weekly data set and three yield curve factors. Results are very similar to the ones in Table 2. The additional yield curve factor X_3 explains very little of nominal yields and slightly more of TIPS yields, but in any case less than 2%. The TIPS liquidity factor explains a similar proportion of the variance of TIPS as in Table 2. Also, the estimated TIPS liquidity factor reported in Figure 5 does not look sensitive to the data set and the inclusion of a third yield curve factor.

The counterfactual results using weekly data reported in Figure 6 support our conclusion that the liquidity channel was only marginally stronger than the scarcity channel, and that overall the QE2 program did not significantly affect the TIPS liquidity premium. In addition, Figure 6 confirms that the effect of the QE2 program on the TIPS liquidity premium peaked during the middle of the program in April 2011, when the realized TIPS liquidity premium was significantly smaller than

Table 6: Variance explained by the latent factors (weekly data set)

Nominal Yields

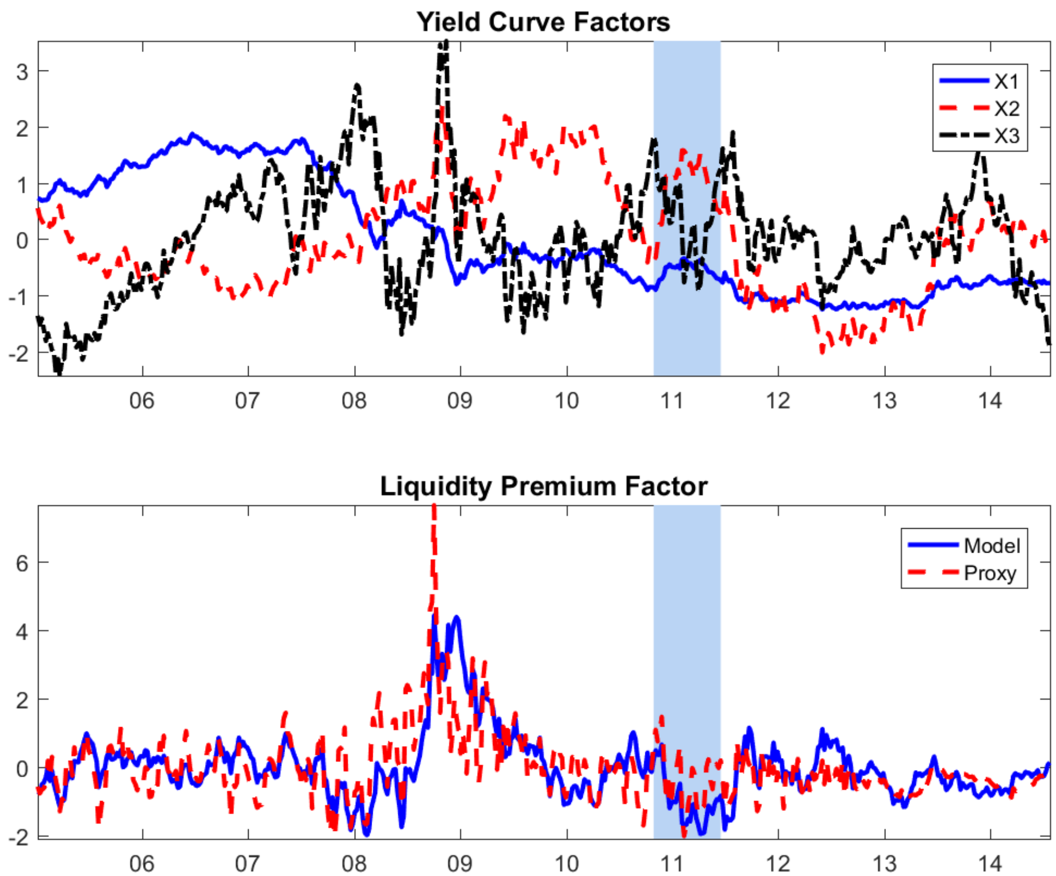
Maturity (years)	X_1	X_2	X_3	L	Total
0.25	0.926	0.070	0.002	0.000	0.996
0.5	0.940	0.060	0.001	0.000	1.000
1	0.954	0.046	0.000	0.000	1.000
3	0.992	0.004	0.005	0.000	1.000
5	0.987	0.011	0.003	0.000	1.000
8	0.905	0.096	0.000	0.000	1.000
10	0.839	0.159	0.002	0.000	0.996

TIPS Yields

Maturity (years)	X_1	X_2	X_3	L	Total
3	0.707	0.029	0.007	0.246	0.991
5	0.730	0.084	0.005	0.180	1.000
8	0.697	0.159	0.008	0.130	0.994
10	0.671	0.198	0.011	0.110	0.987

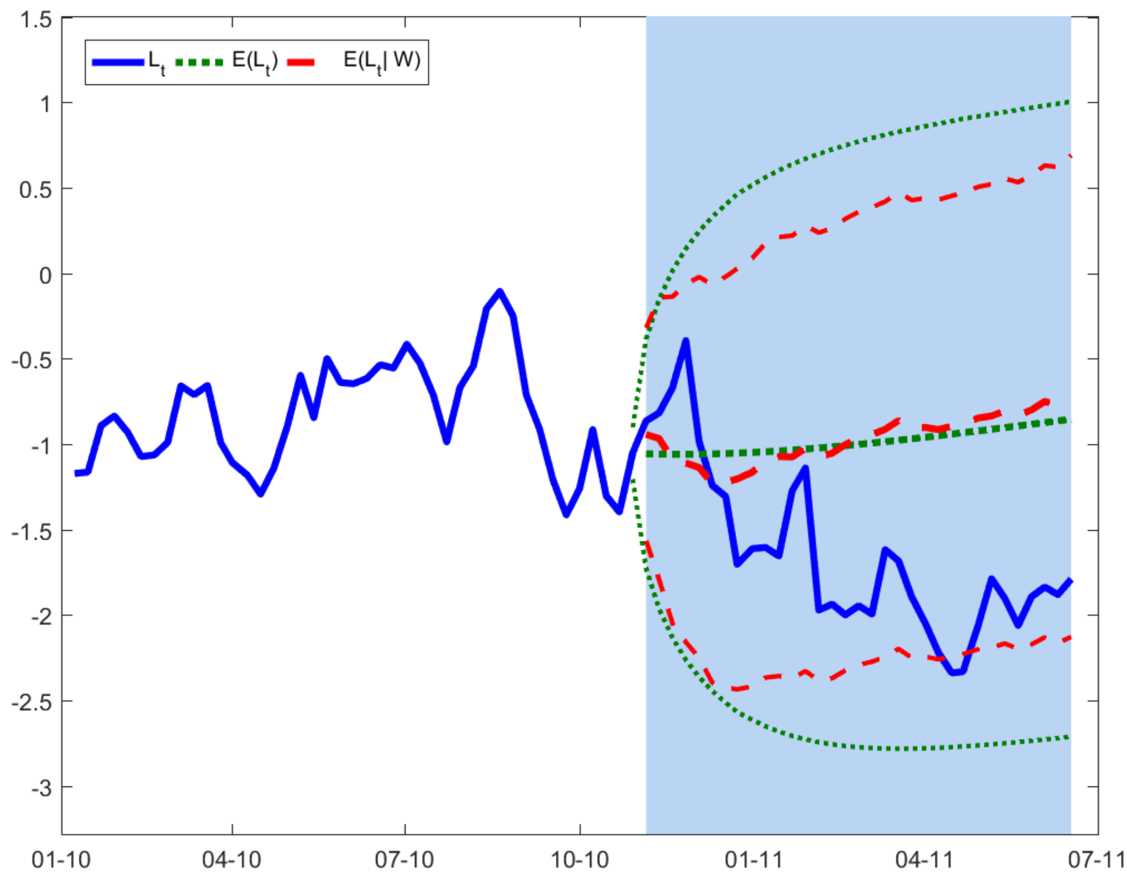
Note: this table reports the percentage of variance of nominal yields (top panel) and of TIPS yields (bottom panel) explained by the estimated latent factors of the model in (9)-(10) for each observed maturity. Results are obtained using the weekly data set and 3 yield curve factors.

Figure 5: Estimated yield curve and TIPS liquidity factor (weekly data set)



The figure displays the estimated latent factors from model (9)-(10) using the weekly data set. The top panel displays the estimated yield curve factors X_1, X_2 and X_3. The bottom panel displays the estimated TIPS liquidity premium factor L (continuous blue line) and a proxy for the TIPS liquidity premium factor constructed using inflation swaps (red dashed line). The blue shaded areas indicate the QE2 operation period.

Figure 6: TIPS liquidity premium factor: realized vs counterfactuals (weekly data set)



The figure displays the estimated TIPS liquidity premium (continuous blue line) and two counterfactuals in which nominal and TIPS yields are observed only until October, 29 2010, as defined in (11). The thick green dotted line is the counterfactual TIPS liquidity premium factor that does not use any conditioning information, i.e. $W = \emptyset$. The thick dashed red line refers to the counterfactual TIPS liquidity premium factor that conditions on the illiquidity measure of Hu et al. (2013) and the corporate spread. The thin lines delimitate the 95% confidence intervals for the counterfactuals with conditioning information (dashed) and without (dotted), computed as in (13). The blue shaded area indicates the QE2 operation period. All results are for the weekly data set and using 3 yield curve factors.

the counterfactuals. The only difference between Figure 6 and Figure 4 lies in the starting point of the counterfactual, which mainly affects the unconditional counterfactual. This happens because using weekly data we need to construct the counterfactual assuming that nominal and TIPS yields are observed only until October, 29 2010 (the week before the QE2 announcement), while using daily data we can use all the information up to the day before the actual announcement of the QE2 program.

9 Conclusion

In this paper, we assess the effect of the QE2 program on the TIPS liquidity premium using a latent factor approach and a counterfactual exercise.

We construct a counterfactual path for the TIPS liquidity premium factor that does not incorporate the QE2 program but that uses conditioning information. We define suitable conditioning variables for a counterfactual as variables that are invariant to the policy but informative about the variable on which we want to construct the counterfactual. Empirical results indicate that the measure of market-wide illiquidity of Hu et al. (2013) and the corporate spread are suitable conditioning variables for the construction of a counterfactual TIPS liquidity premium.

The resulting counterfactual TIPS liquidity factor is on average higher than the realized TIPS liquidity factor and the confidence bands shrink as conditioning information is used. However, we find that the effect of the QE2 program on the TIPS liquidity premium was significantly smaller than the counterfactuals only during the middle of the program. In in this short time period, the liquidity channel was significant, and stronger than the scarcity channel. Overall, we find only mild evidence for a significant effect of the QE2 program on the liquidity premium in the TIPS market.

A Estimation

The joint model for nominal yields (y_t^N), TIPS yields (y_t^T) and conditioning variables (W_t) can be written in compact notation as

$$\begin{aligned} z_t &= B^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*) \\ F_t^* &= \Phi^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q^*) \end{aligned}$$

where

- $z_t = \begin{pmatrix} y_t^N \\ y_t^T \\ W_t \end{pmatrix}$, $B^* = \begin{bmatrix} B^N & 0 & 0 & a^N & I_n & 0 \\ B^R & B^L & 0 & a^T & 0 & I_m \\ 0 & 0 & I_w & 0 & 0 & 0 \end{bmatrix}$, $F_t^* = \begin{pmatrix} X_t \\ L_t \\ W_t \\ c_t \\ \varepsilon_t^N \\ \varepsilon_t^T \end{pmatrix}$, $R^* = \epsilon I_{n+m+w}$;
- $\Phi^* = \begin{bmatrix} \Phi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & A \end{bmatrix}$, $u_t^* = \begin{bmatrix} u_t \\ \nu_t \\ v_t \end{bmatrix}$, $Q^* = \begin{bmatrix} Q & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & R \end{bmatrix}$
- ϵ is a coefficient that we fix to 1^{-12}.
- If $W_t = \emptyset$ we have a model without conditioning variables, as in (9)-(10)

We can write the restrictions on the factor loadings B^* and on the transition matrix Φ^* as

$$H_1 \ \text{vec}(B^*) = q_1, \qquad H_2 \ \text{vec}(\Phi^*) = q_2, \tag{A.1}$$

where H_1 and H_2 are selection matrices, and q_1 and q_2 contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$, and define $z = [z_1, \ldots, z_T]$ and $F^* = [F_1^*, \ldots, F_T^*]$. Then denoting the parameters by $\theta = \{B^*, \Phi^*, Q^*, \pi_1, V_1\}$, we write the joint loglikelihood of z_t and F_t, for $t = 1, \ldots, T$, as

$$
\begin{aligned}
L(z, F^*; \theta) \;\; = \;\; & -\sum_{t=1}^{T} \left(\frac{1}{2} \left[z_t - B^* F_t^* \right]' (R^*)^{-1} \left[z_t - B^* F_t^* \right] \right) + \\
& -\frac{T}{2} \log |R^*| - \sum_{t=2}^{T} \left(\frac{1}{2} [F_t^* - \Phi^* F_{t-1}^*]' (Q^*)^{-1} [F_t^* - \Phi^* F_{t-1}^*] \right) + \\
& -\frac{T-1}{2} \log |Q^*| + \frac{1}{2} [F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1] + \\
& -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda_1' \left(H_1 \operatorname{vec}(B^*) - q_1 \right) + \\
& + \lambda_2' \left(H_2 \operatorname{vec}(\Phi^*) - q_2 \right)
\end{aligned}
$$

where λ_1 and λ_2 contain the lagrangian multipliers associate with the constraints in (A.1).

The EM algorithm allows to obtain maximum likelihood estimates of the parameters and the latent factors by alternating Kalman filter extraction of the factors to the maximization of the likelihood. In practice, at the j-th iteration the algorithm we perform two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and the estimates from the previous iteration

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)})|z]$$

   which depends on three expectations

$$
\begin{aligned}
\hat{F}_t^* &\equiv E[F_t^*; \theta^{(j-1)}|z] \\
P_t &\equiv E[F_t^* (F_t^*)'; \theta^{(j-1)}|z] \\
P_{t,t-1} &\equiv E[F_t^* (F_{t-1}^*)'; \theta^{(j-1)}|z]
\end{aligned}
$$

   These expectations can be computed, for given parameters of the model, using the Kalman filter.

2. In the Maximization-step, we update the parameters maximizing the expected log-likelihood

with respect to the parameters θ:

$$\theta^{(j)} = \arg\max_{\theta} \mathcal{L}(\theta)$$

Given that the restrictions in (A.1) are linear. This step can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving.

In order to start the algorithm, we initialize the yield curve factors with the first two normalized the principal components extracted from nominal yields. We then project the TIPS yields on these initial yield curve factors and use the first normalized principal component of the residuals of this regression to initialize the liquidity factor. Initial values for all parameters are then computed by OLS using the initial guesses of yield and liquidity factors.

Notice that the TIPS liquidity premium factor L_t is uniquely identified by the zero restrictions in the factor loadings. On the contrary, the yield curve factors X_1 and X_2 are only identified up to a rotation. To uniquely identify the yield curve factors we impose that the nominal yield with the shortest maturity has a positive factor loading on X_1 and a negative factor loading on X_2. This can be easily imposed by rotating the principal components in the initialization of the EM algorithm.

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