Quantitative easing and the price-liquidity trade-off

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Abstract

We consider the effects of quantitative easing on liquidity and prices of bonds in a theoretical model. Asset purchases initially improve liquidity, but subsequently as the central bank buys and holds bonds to maturity, the bonds become scarcer. The purchases also crowd out other buyers, eventually leading to lower liquidity of the bonds. The effect depends on the share of preferred habitat investors holding the bonds. Lower share of preferred habitat holdings is associated with more elastic demand so that liquidity improves more initially, but then falls more than with high share of preferred habitat holdings. Price impact is larger in case of high share of preferred habitat bond holdings. We calibrate the model to the Eurozone and construct an index measuring the prevalence of preferred habitat investors in each Eurozone country.

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1 Introduction

One of the open questions in the QE literature is in which way do central bank purchases affect liquidity. Investors have voiced concerns of reduced liquidity in the government bond markets due to the central bank asset purchases. This is paradoxical given that liquidity is defined as deep markets where it is possible for investors to purchase or sell an asset without causing a significant change in the asset’s price. When a central bank enters the market with a QE programme, it adds a deep-pocketed buyer to the market. This should a priori improve market liquidity instead of reducing it. This is also true in a theoretical model of over-the-counter market liquidity such as (Duffie, Garleanu and Pedersen 2005; De Pooter, Martin and Pruitt 2015).

Our paper provides a theory of the impact of asset purchases by central banks on market prices, and liquidity of these assets. We show how, in the early phase of an asset purchase programme, an increase in demand for bonds by the central bank leads to an increase in the number of transactions, and hence liquidity. Once the central bank has purchased a significant amount of bonds, the reduction in the supply of bonds on the secondary market leads to a decline in liquidity, reducing the number of transactions as it becomes harder for buyers to meet a seller. Finally, when the central bank tapers the purchases but holds on to the balance of bonds, liquidity falls below the initial level before the purchases as other buyers are crowded out. We show that the results depend on the share of bonds held by preferred habitat investors.

Section 2 lays out a search theoretic framework of over-the-counter debt with arbitrageurs and preferred habitat investors based on (Duffie et al., 2005). The impact of quantitative easing depends on the demand and supply of the bond purchased, or the ‘tightness’ of the market. Prices of bonds are determined analogous to wages in a standard labour market search model. That is, rigidity is introduced into the financial market by means of search friction. As the number of traders in the market increases, it becomes easier for buyers and sellers to find each other and liquidity improves.

(De Pooter et al., 2015) use a search model to study the ECB’s Securities Market Programme (SMP). They model asset purchases as an exogenous reduction in the supply of the assets in the hands of impatient sellers. This alleviates the search friction and leads

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to a reduction in liquidity premium. (De Pooter, Martin and Pruitt, 2015) is, to the best of our knowledge, the only other study of QE to employ a search theoretic framework. The paper finds that the central bank purchases led to a decrease in the bond liquidity premia, by reducing the stock of bonds in the hands of impatient sellers. Because endogenous response to demand is not taken into account for, the result is positive for liquidity. There are also no preferred habitat investors in the model, which means that the impact of QE depends only on the amounts purchased. We append this framework by modelling the central bank purchases explicitly. This allows us to consider also the buyers’ side of the transaction.

In our model, buyers enter the market endogenously, as in (Afonso, 2011). The central bank asset purchases then crowd out other buyers from the market by increasing the price, and reducing the expected payoff to the potential buyers. Liquidity can remain higher than in the baseline case of no purchases as long as the central bank demand remains high. However, once the central bank tapers the purchases, but holds on to the stock of bonds, liquidity falls below the baseline level due to crowding out.

The magnitude of central bank asset purchases on prices is determined by the share of preferred habitat investors holding the bonds. In our framework, the initial share of preferred habitat investors is exogenously determined. Given a fixed issuance of bonds, a larger share of preferred habitat investors implies a smaller share of sellers on the secondary market. Given fewer sellers, it is relatively harder for a buyer like the central bank to acquire bonds. As a result, the impact of a QE programme on prices will be higher.

Preferred habitat investors are also less likely to be crowded out by the central bank purchases. The crowding out effect on liquidity is therefore expected to be smaller with a larger presence of preferred habitat investors. However, those bonds are likely to become scarcer as their supply is reduced when the central bank purchases the bonds and holds them to maturity.

In Section 3 we calibrate our model to the euro area countries. The model shows that ECB asset purchases are likely to have had a higher price impact in countries with a higher share of preferred habitat investors. These bonds are less likely to suffer from reduced liquidity, but become scarcer as the central bank absorbs these assets. It is this effect that is likely to have concerned investors reflected in surveys on sovereign bond market liquidity. The simulation also shows that liquidity can fall to low levels when ECB winds down purchases in countries with a low share of preferred habitat investors.
Empirical studies of the effects of quantitative easing on liquidity have not been conclusive. For example, (Christensen and Gillan, 2013) find that liquidity improved as a result of the Federal Reserve’s purchases of Treasury Inflation Protected Securities (TIPS). (Kandrac, 2013) studies the MBS market during the period of Federal Reserve purchases. He finds that purchases negatively affected volumes, trade-sizes, and implied financing rates in dollar roll transactions, though not bid-ask spreads.

These inconclusive results are at least partly due to the fact that asset purchases initially improve liquidity through the demand effect. Only as bonds are withdrawn from the market, they lead to scarcity. (D’Amico and King, 2013) analyse the repo market in the US and shows that there is a considerable and highly persistent scarcity premium on the government bonds, especially at short maturities, traced back partly to the Fed’s QE programme. (Coroneo, 2015) finds that TIPS improved liquidity, but this effect was dominated by increased scarcity in the market. The Bank of England observed that the government bond market became ‘dislodged’ during its QE programme, and began to lend back a proportion of the gilts it had bought (Paul Fisher, 2010). In the case of the euro area, (Corradin and Maddaloni, 2015) show that during the ECB securities market programme (SMP), the government bonds that were purchased became ‘special’, meaning that their price contained a scarcity premium. This effect was reversed as the ECB began selling purchased bonds back to on the repo market (European Securities and Market Authority, 2016). The (International Capital Markets Association, 2016) released a study warning of reduced liquidity in the European repo market due to regulation and QE. Analysing high frequency German Bund data, (Schlepper et al., 2017) show that ECB’s asset purchases led to an increased scarcity of Bunds and this effect has increased over time.

Section 3.1 develops a preferred habitat index (PHI), a measure of the share of preferred habitat investors. This index is required for the calibration of our model in Section 3. We measure the prevalence of preferred habitat investors from the ECB’s securities holding statistics. To our knowledge, there are no comparable measures for preferred habitat investors at this level of detail. We show that there are large differences in the shares of preferred habitat investor holdings, ranging from less than 2% to just under 50%. We calibrate the model to core and periphery euro area countries with 42% and 21% of bonds held by preferred habitat investors, respectively. The model matches the empirical result that yields moved more as a response of the first PSPP announcement in countries with a large share of preferred habitat investors holding the bonds.

The share of preferred habitat investors is a key to results also in New Keynesian models
with QE. For example, Chen et al. (2012) estimate the share of preferred habitat investor holdings of US treasuries to be only 3%. As a result, they get very small effects from central bank asset purchase programme on yields. (De Graeve and Iversen, 2016) show that the results in this type of model are highly sensitive to the share of preferred habitat investors. Given our finding of around 40% preferred habitat investors in a country with a similar rating to the US, the effects of QE would likely increase substantially in the model.

2 2. A micro-founded model of over-the-counter debt markets

Model set-up

The model is based on a search theoretic model of over-the-counter debt by Duffie, Garleanu and Pedersen (2005) that first showed how over-the-counter market could be modelled through search frictions. Lack of centralised exchange for bonds leads agents to search for a counterparty for their trade. For sovereign bonds, it often means going sequentially from trader to trader to request quotes for the bonds. Organised trading platforms exist for government bonds, especially for large countries, however, investors tend to scout the market to get a view on order books across dealers, which adds to the time to transaction.

The bond prices are dependent on this search friction, which can be decomposed to supply and demand effects, both of which quantitative easing affects. A larger supply of bonds leads to improved search alternatives for the buyers, leading to a decline in the price of the bond. Similarly, higher demand for bonds leads to an increase in the price. The central bank purchases reduce the supply of bonds for a given stock supplied by the government, since the central bank buys the bonds with the aim of holding them to maturity. The demand (flow) effects, come from the increase in demand for bonds by the central bank.

The model is set in continuous time with a continuous flow of meetings subject to the search friction. The agents meet each other randomly, with uniform probability of meeting a certain type of agent. The search intensity is represented by the Poisson parameter $\lambda$, such that the mean time to meet a new agent is $1/\lambda$. We study the steady state of the model to draw conclusions about final prices, and liquidity, abstracting from the dynamics to arrive at those equilibria.
In order to study the effects of quantitative easing, we add a central bank as an additional buyer, debt that matures stochastically, and preferred habitat investors who hold debt to maturity. The share of preferred habitat investors holding the bonds is crucial to the results. The supply and demand effects are non-linear and depend on the initial share of the bonds held by the preferred habitat investors. Large holdings by preferred habitat investors, whom we assume to be hold-to-maturity investors imply that the number of sellers active on the market is smaller. That reduces the buyers’ probability of meeting an active sellers, inducing the buyers to pay a higher price for the bonds.

There is a continuum of six types of agents: high, and low type sellers, high type buyers, central bankers, preferred habitat investors, and outside investors. Low type denotes low liquidity, those agents want to liquidate their holdings for cash to finance consumption. In the model those agents have a low discount rate.

Each bond holder holds just one bond. Once those bonds mature, or the investors sell the bond, they consume a unique good available to them, which is used as a numeraire. On the buyer side, the agents: buyers, central bankers, and outside investors all hold an endowment of 1 unit of an asset which they can use to purchase the bond from the sellers. We now go through each of the agents in turn.

**Sellers, low type** with mass $\alpha_{sl}$ are impatient, wanting to liquidate their bond holding in order to consume. They hold one bond, which they would like to sell either to a buyer, or a central banker. The probability that they meet a buyer depends on the mass of buyers are on the market, i.e. the ease of meeting them. Once a low-type seller finds a buyer, he receives a price $P$ for the bond, exits the market, and consumes 1 unit of the consumption good.

**Sellers, high type**, with mass $\alpha_{sh}$, each hold one bond. Because they are patient, they do not trade when they meet buyers. They do however, receive a liquidity shock with probability $\theta$, and become low type sellers, at which point they trade with a buyer when they meet them. The liquidity shock in this case is a funding liquidity shock, as opposed to a market liquidity shock.

**Buyers, high type**, whose measure is $\alpha_b$ hold a transaction asset in value of 1, which they would like to use to buy a bond from a seller. Buyers are patient, with discount factor zero. For this reason, they become high rather than low type sellers after the transaction.

**Preferred habitat investors**, with mass $\alpha_{phi}$ are hold-to-maturity investors, holding the bonds at quantity of one each. They withdraw the mass of bonds they hold from the
secondary market, and do not participate in the search process.

**Central bank** is represented by a measure $\alpha_{cb}$. They buy bonds off the secondary market, and add to that stock of preferred habitat investors by holding the bonds to maturity. Therefore, central bank purchases reduce the number of bonds on the market, with implications on yields and liquidity. For simplification, we model the central bank as a many central bankers each holding one bond, rather than one central bank holding many bonds. This does not change the results of the paper.

Investor flows are shown in figure 1. Investors in the model are high and low type sellers, preferred habitat investors, buyers, and central bankers with measures $\alpha_{sh}$, $\alpha_{sl}$, $\alpha_{phi}$, $\alpha_b$, $\alpha_{cb}$ respectively. They meet each other randomly and trade if there are benefits to trade for both. Investor flows are shown in figure 1.

Governments are passive in the model, having supplied a stock $D$ of bonds to the secondary market. Those bonds mature stochastically at rate $\delta$. That means that the bonds held by impatient, low-type sellers may mature before they find a buyer, while the bonds of high type sellers might mature before they receive a liquidity shock. When a
bond matures, the government will have to pay 1 to the investor holding the maturing bond. With a default probability $q$, the government does not honour its repayments and investors receive a recovery value $\gamma < 1$ for the bond.

Matching on the market depends on the relative measures of investors. The probability that any of the agents meets another, depends on the measures of those investors on the market. Larger presence makes meetings more likely. This search friction makes supply and demand for bonds relevant for price. The price no longer depends only on the return, but also on the ease of trading it.

Market tightness is equivalent to the ratio of active buyers to active, low-type sellers, or equivalently demand to supply, $\alpha_{b} + \alpha_{cb}$. Tightness is two-sided in this model. If the tightness ratio is low, the market is tight due to low liquidity, while if the ratio is large, bonds are scarce.

2.1 Model solution with exogenous entry of buyers

We now set up the expected utilities, and the bargaining process required to solve the model and to reach the first results. We show that the price in the market is affected by QE in two ways. Firstly, the central bank purchases add a buyer to the market, increasing demand. Secondly, the central bank buys bonds from active, low-type sellers, and by holding them to maturity, reduce the supply of bonds on the secondary market.

The expected utility of a low-type seller is shown in equation 1. These low-type sellers are the only impatient agents in the model, and have a discount factor $\rho$. The other agents in the model have a discount factor of zero.

The first two terms of the low-type sellers’ value function inside the brackets show returns from the bond maturing. The bonds mature stochastically with probability $\delta$, paying 1 as long as the government does not default on its obligations. The government defaults with probability $q$, in which case the bondholders recover $\gamma$.

The probability of meeting a counterparty depends on the mass of those counterparties on the market. With probabilities $\lambda \alpha_{b}$, and $\lambda \alpha_{cb}$, the seller meets a buyer, and a central bank respectively, and gets a price $P$ for the bond when the transaction succeeds. $\lambda$ is the Poisson probability of meeting a counterparty, so that $1/\lambda$ reflects the time it takes to find one.
\[ V_{sl} = \frac{1}{(1 + \rho)} [\delta(1 - q) + \delta\gamma q + (\lambda\alpha_b + \lambda\alpha_{cb})P + (1 - \delta - \lambda\alpha_b - \lambda\alpha_{cb})V_{sl}] \] (1)

Buyers expected utility is shown in equation (2). They pay a flow search cost of \( e \) while they are actively searching for a seller. They meet a seller with probability \( \lambda\alpha_{sl} \), in which case they purchase a bond for a price \( P \), and become high-type sellers with expected return of \( V_{sh} \).

\[ V_b = -e + \lambda\alpha_{sl}(V_{sh} - P) + (1 - \lambda\alpha_{sl})V_b \] (2)

High type sellers’ expected return is shown in equation (3). Their bond matures with the same probability \( \delta \) as low-type sellers’. Similarly, the repayments in case of a default and non-default are also the same. The high-type seller can be hit by a funding liquidity shock that arrives with probability \( \theta \), after which they switch type to impatient sellers.

\[ V_{sh} = \delta(1 - q) + \delta\gamma q + \theta V_{sl} + (1 - \delta - \theta)V_{sh} \] (3)

We describe the expected returns of the central bank, and the preferred habitat investors in the next section.

### 2.2 Bargaining over price

When low type sellers meet a buyer, or a central banker, they bargain through Nash bargaining, and trade. The bargaining process is set up to match the process for government bond purchases by the European central bank. The key features are that the purchases are designed to be market neutral, and that the bonds are bought over the counter. Some countries purchase the assets in auctions, in which case the price formation process would need to be modelled differently.

Market neutrality in this model means that the central bank is paying the same price for the bond as any other buyer. In practice this requires making an assumption in the value function of the central bank to ensure that the expected return of the central banker does not differ from the expected return of the buyer.

With Nash bargaining, we solve for the price using the expected surpluses of each of the bargaining party. The expression for price is in equation (4) for the bargaining between a low-type seller and a buyer, and in equation (5) for bargaining between a low-type seller
and a central banker. We denote the bargaining power of a buyer, or a central banker by $\beta$. In order to ensure that the price does not differ between a buyer and a central banker, $V_{sh}$ has to equal $V_{phi}$ and $V_b$ has to equal $V_{cb}$.

$$P = \beta V_{sl} + (1 - \beta)(V_{sh} - V_b)$$  \hspace{1cm} (4)
$$P = \beta V_{sl} + (1 - \beta)(V_{phi} - V_{cb})$$  \hspace{1cm} (5)

We start by writing out the value functions of a buyer and a central bank explicitly:

$$V_b = -e + \lambda \alpha_{sl}(V_{sh} - P) + (1 - \lambda \alpha_{sl})V_b$$
$$V_{cb} = -e + \lambda \alpha_{sl}(V_{phi} - P) + (1 - \lambda \alpha_{sl})V_{cb}$$

A central banker pays the same search cost as the buyer, $e$. If it meets a seller, it pays a price $P$ for the bond. After trading, the central banker becomes a hold-to-maturity investor, which in this case is equivalent to being a preferred habitat investor. We therefore denote their continuation utility by that of the preferred habitat investors, $V_{phi}$. The only difference between these two agents is in the continuation utilities they receive following a trade. For the trading price to not differ, those utilities will have to be equal. The expressions for them are shown in equations 6 and 7.

$$V_{sh} = \delta(1 - q) + \delta \gamma q + \theta V_{sl} + (1 - \delta - \theta)V_{sh}$$  \hspace{1cm} (6)
$$V_{phi} = \delta(1 - q) + \delta \gamma q + \theta V'_{phi} + (1 - \delta - \theta)V_{phi}$$  \hspace{1cm} (7)

High-type sellers have a probability $\theta$ of being hit by a funding liquidity shock, and becoming a low-type seller with a lower return of $V_{sl}$. While the preferred habitat investors do not change type, we assume that they are subject to similar shocks that lower their return by the same amount, such that $V_{sl} = V'_{phi}$. One way to think about these shocks is in terms of aggregate negative demand shocks for example, which make the patient sellers want to liquidate their holdings. Simultaneously, while the preferred habitat investors do not change type, and are less responsive to price changes, they never-the-less bare the cost
of these reductions in the valuation of the assets.

Following this assumption, we return to the bargaining process. The bargaining process collapses to solving for the price in a trade between a low-type seller, and buyer. The price paid by a low-type seller, and a central banker will be equal to that.

With the pricing equation 4, we can solve for all the value functions, and the price:

\[
V_b = -\frac{e}{\lambda \alpha_{sl}} + \frac{(\delta(1-q) + \delta \gamma q)\rho - \theta k(\rho + \delta) - \delta k(\rho + \delta + \lambda \alpha_b + \lambda \alpha_{cb})}{(\delta + \theta)(\rho + \delta)} \quad (8a)
\]

\[
V_{sl} = \frac{(\delta(1-q) + \delta \gamma q) + k(\lambda \alpha_b + \lambda \alpha_{cb})}{\rho + \delta} \quad (8b)
\]

\[
V_{sh} = \frac{(\delta(1-q) + \delta \gamma q)(\rho + \delta + \theta) + \theta k(\lambda \alpha_b + \lambda \alpha_{cb})}{(\delta + \theta)(\rho + \delta)} \quad (8c)
\]

\[
P = \frac{\delta(1-q) + \delta \gamma q}{\rho + \delta} + \frac{(1 - \beta) e(\lambda \alpha_b + \lambda \alpha_{cb} + \rho + \delta)}{\beta \lambda \alpha_{sl}(\rho + \delta)} \quad (8d)
\]

where \( k = \frac{(1 - \beta) e}{\lambda \alpha_{sl}} \)

Price is a sum of two parts, \textit{fundamental value}, and \textit{liquidity premium}. The fundamental value is a function of bond characteristics: maturity \( \delta \), default probability \( q \), recovery rate \( \gamma \), and the discount factor \( \rho \). These are factors that enter a typical bond pricing equation, where quantitative easing has no effect on the price.

The second part of the pricing equation contains the liquidity, or scarcity premium that is a function of the supply, \( \alpha_{sl} \), and demand, \( \alpha_b \), and \( \alpha_{cb} \) for the asset. A higher value implies that investors pay a higher premium for the bond.

Tightness here is a broader concept than liquidity. Liquidity is about having deep enough markets, with enough buyers so that an investor is able to sell a bond without affecting its price. In terms of the measure of tightness, it means that the ratio of buyers to sellers is large enough. The measure captures also tightness on the sellers side, that is, scarcity of bonds, when the ratio of buyers to sellers is large.
Bond purchases can affect the market tightness from both sides. On one hand, they increase the demand for bonds, by increasing the measure of central bankers, while on the other hand it crowds out sellers as the central bank holds the bonds to maturity. When we endogenise the model, we also show that central bank purchases can crowd out other buyers, lowering $\alpha_b$.

### 2.3 Results with exogenous entry of buyers

We explore the key results of the model below in a simple version of the model where we assume that both $\alpha_{sl}$ and $\alpha_b$ are exogenous. We show that the price of the bond depends on demand and supply, and that central bank bond purchases increase liquidity of the bonds.

#### 2.3.1 Impact of asset purchases on yields

**Proposition 1.** Price increases with increasing demand, and declines with increasing supply.

**Proof.** A partial derivative of price in equation 8d in terms of demand is:

$$\frac{\partial P}{\partial \alpha_b} = \frac{(1 - \beta)e}{\beta(\rho + \delta)\lambda\alpha_{sl}}$$

Since $\beta < 1$, the derivative is positive and price increases with demand.

A partial derivative of price in equation 8d in terms of supply of bonds on the secondary market is:

$$\frac{\partial P}{\partial \alpha_{sl}} = -\frac{(1 - \beta) e(\lambda\alpha_b\lambda\alpha_{cb} + \rho + \delta)}{\beta \lambda^2\alpha_{sl}(\rho + \delta)}$$

which is negative for $\beta < 1$. Therefore, price falls with an increase supply.

The impact furthermore depends on the share of active sellers in the economy. Both of the partial derivatives above have $\alpha_{sl}$ in the denominator, which implies that the effect of increasing demand or supply on price is muted for a large number of sellers on the market. This is the main result in Afonso (2011) who show that markets get congested as the share
of sellers increases.

**Proposition 2. Central bank purchases increase price**

*Proof.* Central bank purchases increase price through both demand and supply channels. As a central bank announces its intention to purchase the bonds, it in effect decides to enter the market as a buyer, increasing demand through an increase in $\alpha_{cb}$. This is the flow effect of asset purchases. Price increases as a result of increasing central bank demand, as is seen in equation (11).

$$\frac{\partial P}{\partial \alpha_{cb}} = \frac{\beta \rho (\beta + \delta) \alpha_{sl}}{\beta (\rho + \delta) \alpha_{cb}}$$

(11)

Asset purchases also affect price through a reduction in the stock of bonds. Given our assumption of a fixed stock of bonds, when central bank purchases the bonds, its holdings of bonds increase and the holdings by active sellers fall. A reduction in the measure of sellers leads to an increase in price given the derivative in equation (10).

We can also see this below in equation (12). The total amount of debt in the economy is the sum of bonds held by all the investors: $D = \alpha_{sl} + \alpha_{sh} + \alpha_{ph}$. We can replace the measure of sellers in the equation for price (8d) to see that the increase in the mass of preferred habitat investors (central bankers become hold-to-maturity investors) leads to a higher price.

$$P = \frac{(\delta (1 - \rho) + \delta \gamma q)}{\rho + \delta} + \frac{(1 - \beta) e}{\beta \lambda (D - \alpha_{sh} - \alpha_{ph})} \frac{(\rho + \delta + \lambda \alpha_{kb} + \lambda \alpha_{cb})}{\rho + \delta}$$

(12)

This supply effect is equivalent to the portfolio balance channel of quantitative easing. In term structure models such as (Hamilton and Wu 2012), it is the purchases from preferred habitat investors that drive the price movements. These investors do not want to move away from their preferred maturity, and rating class of bonds and therefore require a premium to move. Here the mechanism is more directly related to bond supply and demand. Even without any preferred habitat investors, bond prices move simply due to changes in the relative mass of buyers and sellers. QE here works purely through the
demand and supply mechanism, even in presence of only arbitrageurs on the market. However, the mass of preferred habitat investors influences the scale of the price impact as we show in the next section.

2.3.2 Impact of asset purchases on liquidity

We now extend the analysis to the impact of asset purchases on liquidity of the bonds.

**Proposition 3.** Liquidity improves initially as the central bank increases demand for bonds. It worsens subsequently when the central bank withdraws bonds off the secondary market.

*Proof.* Liquidity is modelled as a measure of transactions, or meetings on the market:

\[
L = \lambda \alpha_{sl} \alpha_b + \lambda \alpha_{sl} \alpha_{cb}
\]  

(13)

When the central bank increases demand for bonds, increasing \( \alpha_{cb} \), it becomes easier for sellers to match with a buyer, increasing the number of transactions on the market. Therefore liquidity improves at the start of the purchases. Subsequently, as the central bank purchases bonds and withdraws them off the secondary market, the mass of active sellers shrinks and liquidity declines.

2.3.3 Impact depends on the mass of preferred habitat investors

The results above hold, but the magnitude of them depends on the share of preferred habitat investors holding the bonds

**Proposition 4.** Price increases more as a result of bond purchases when the share of preferred habitat investors holding the bonds is larger.

*Proof.* The partial derivatives in equations 9 and 10 depend on the share of bonds held by preferred habitat investors, \( \alpha_{ph} \). Since the amount of bonds issued by the government, \( D \) is held fixed, we can write that \( D = \alpha_{sl} + \alpha_{sh} + \alpha_{ph} \). Using this relation, we can write the partial derivatives as:

\[
\frac{\partial P}{\partial \alpha_{b}} = \frac{(1 - \beta)e}{\beta(\rho + \delta)\lambda(D - \alpha_{ph} - \alpha_{sh})}
\]  

(14)

\[
\frac{\partial P}{\partial \alpha_{sl}} = -\frac{(1 - \beta)}{\beta} \frac{e(\lambda \alpha_{b} \lambda \alpha_{cb} + \rho + \delta)}{\lambda(D - \alpha_{ph} - \alpha_{sh})^2(\rho + \delta)}
\]  

(15)
Equation 14 shows that the impact of increase in demand for bonds by the central bank is larger, for a larger share of the bonds held by preferred habitat investors $\alpha_{ph}$.

Purchasing bonds from active sellers, and holding them to maturity reduces the mass of active sellers $\alpha_{sl}$. This reduction leads to an increase in the price, and that increase is larger for a larger mass of preferred habitat investors on the market, as can be seen in equation 15.

Finally, the impact on liquidity is also affected. A partial derivative of 13 in equation 16 depends on the share of sellers, and substituting that by $D - \alpha_{ph} - \alpha_{sh}$ we can see that the improvement in liquidity from increase in central bank demand is larger the fewer preferred habitat investors there are.

$$\frac{\partial L}{\partial \alpha_{cb}} = \lambda \alpha_{sl}$$ (16)

Bonds with a large share of preferred habitat investors are characterised by high demand and low supply. Central bank purchases exacerbate the situation by further increasing demand and reducing supply. Therefore search frictions become more binding and price moves more as a result.

In the case of low share of preferred habitat holdings, there are few buyers, but many sellers, and when a central bank increases demand, the search friction is instead alleviated. It is easy to match with the sellers, and liquidity improves as the number of buyers increases.

### 2.4 Model solution with endogenous entry of buyers

Making the entry of buyers endogenous adds another dimension. As was shown in the previous section, when a central bank purchases bonds, it reduces the supply of bonds on the secondary market, and increases the demand for those bonds. Both of these actions lead to a higher equilibrium price.

We now add outside investors to the model, who compare the value function of their outside option to the value function of a buyer to decide whether to enter the market as a buyer. When the value function of a buyer declines, fewer outside investors find it profitable to enter the market. The value function of buyers is in equation 8a. It depends negatively on the mass of central bankers demanding bonds, and positively on the supply
of bonds, $\alpha_{sl}$. Therefore, when the central bank increases demand, and reduces supply, the value function of the buyers declines, leading outside investors to reduce entry to the market. Through this mechanism, quantitative easing crowds out other buyers.

In the next step, we endogenise the entry of buyers in the model\[2\] The entry flows of outside investors are denoted by $g$. Those outside investors compare the value of their outside option $K$, to the value of becoming a buyer, $V_b$. If the value of his outside option $V_K$ is lower than the value of a buyer, the investor decides to enter the market and becomes a buyer. The outside investors are heterogeneous in their outside option $K_i$. For simplicity, we assume that the value of the outside option $V_{Ki}$ of each outside investor equals $K_i$. The value of the outside option of a marginal investor, the one that is indifferent to entering, is denoted by $K_m$. Every outside investor with a value of the outside option less than or equal to $K_m$ enters, and every outside investor with a value of the outside option greater than $K_m$ does not enter. Therefore we get that:

$$g = \int_{K_m}^{K} f(K)dK = F(K_m)$$

(17)

At equilibrium, $V_K = V_b$, and given our assumption that $V_K = K_m$, it follows that $K_m = V_b$. We can write the above condition therefore as $g = F(V_b)$. This is the equilibrium condition. We call $g$ the entry flows and $F(V_b)$ the buyer value.

Equilibrium solution involves solving both the function $g$, and $V_b$ for the share of active sellers $\alpha_{sl}$ and looking for the $\alpha_{sl}$ that solves the system. In order to do that, we need to specify the investor flows. In steady state the inflows of outside investors to the economy $g$, must equal the outflows, the matches between sellers and buyers, and the central bank, i.e.

$$g = \lambda \alpha_{sl} \alpha_b + \lambda \alpha_{sl} \alpha_{cb}$$

(18)

The flows of patient, high-type sellers, $\alpha_{sh}$ can be written out explicitly. The first term of this flow equation in [19] has the inflows of buyers who are matched a seller, $\lambda \alpha_{sl} \alpha_b$. A share $\lambda \alpha_{sl}$ of the buyers meet a seller and become a high-type seller. With probability $\theta$, the high-type sellers receive a liquidity shock, and with probability $\delta$, their debt matures and they exit the market:

\[2\] We follow closely Afonso (2011)
\[ \dot{\alpha}_{sh} = \lambda \alpha_{sl} \alpha_b - \theta \alpha_{sh} - \delta \alpha_{sh} \] (19)

At equilibrium \( \dot{\alpha}_{sh} = 0 \), allowing us to get that \( \lambda \alpha_{sl} \alpha_b = (\theta + \delta) \alpha_{sh} \). We can substitute this into the equation for inflows of outside investors \( g \) above and get that \( g = (\theta + \delta) \alpha_{sh} + \lambda \alpha_{sl} \alpha_{cb} \).

The flows of preferred habitat investors are similar to the flows of high-type sellers. Inflows consist of central bankers that have met a low-type seller and become now a hold-to-maturity investor. The only way these investors leave their position is through their bond maturing, which happens with probability \( \delta \):

\[ \dot{\alpha}_{ph} = \lambda \alpha_{sl} \alpha_{cb} - \delta \alpha_{ph} \] (20)

Setting the preferred habitat investor flows to zero, the equation can again be substituted to the equation for \( g \), together with the condition that the total amount of debt in the economy consists of bonds held by the high-, and low-type sellers, and preferred habitat investors, \( D = \alpha_{sl} + \alpha_{sh} + \alpha_{ph} \). Finally, we get:

\[ g = (\theta + \delta)(D - \alpha_{sl}) - \theta \alpha_{ph} \] (21)

Solving this system of flow equations, we get \( \alpha_b \) as a function of \( \alpha_{sl} \), exogenously determined \( \alpha_{ph} \), and parameters only:

\[ \alpha_b = \frac{(\theta + \delta)(D - \alpha_{sl} - \alpha_{ph})}{\lambda \alpha_{sl}} \] (22)

Substituting \( \alpha_b \) into the equation for \( V_b \) in we get \( V_b \) as a function of \( \alpha_{sl} \) and parameters only:

\[ V_b = \frac{\rho(\delta - (1 - \gamma)\delta q)}{(\delta + \theta)(\rho + \delta)} \left( 1 - \frac{\beta}{\lambda \alpha_{sl}} \right) e \left[ 2 + \frac{\delta \lambda \alpha_{cb}}{(\delta + \theta)(\rho + \delta)} + \frac{\delta(D - \alpha_{sl} - \alpha_{ph})}{\alpha_{sl}(\rho + \delta)} \right] \] (23)

\[ \text{[Later on we calibrate the initial share of preferred habitat investors. The share of bonds purchased by the central bank are similarly calibrated and we know the probability with which the debt held by the preferred habitat investors matures. Hence \( \alpha_{ph} \) is predetermined.]} \]
We can now search for the $\alpha_{sl}$ that solves for the intersection of the entry flows in equation (21) and buyers value function $F(V_b)$ where $V_b$ is described in equation (23). The intersection of $F(V_b)$ and $g$ give us the equilibrium $\alpha_{sl}$ with which we can derive all the other variables of the model.

Rest of the results follow from the solution to these two key equations. The buyers’ value function is upward sloping in $\alpha_{sl}$. An increasing share of active sellers makes it easier for buyers to be matched and thereby alleviates the search friction from the buyer’s side. This improves the expected return of becoming a buyer.

The entry flow condition, $g$ in (21) is downward sloping for $\alpha_{sl}$. The result comes from solving the investor flows in equilibrium. More active sellers in the market implies, for a given amount of bonds, that the mass of high-type sellers is smaller. This means that fewer buyers are entering the market and becoming high-type sellers.

Since the buyer value function is upward sloping for $\alpha_{sl}$, and the entry condition of buyers is downward sloping, we can solve the equilibrium in the model by searching for the $\alpha_{sl}$ where the two curves intersect.

There is no closed form solution to the model. For this reason we firstly explore some of the properties of the solution and then simulate it with a calibration to the Eurozone.

2.5 Results with endogenous entry of buyers

Impact of asset purchases on yields

Below we show some of the results that are different from those presented in the above section with exogenous entry of buyers. The results are mostly dampened through the effect the crowding out has on the price. We start by analysing the impact of central bank bond purchases on the price of bonds.

**Proposition 5.** Central bank purchases increase the price, but the effect is partially muted by changes in the measure of sellers and buyers.

**Proof.** As was shown in the case with exogenous $\alpha_{sl}$ and $\alpha_b$, the demand from central bank, i.e. an increase in $\alpha_{cb}$ leads to an increase in price. However, now also the measure of both sellers and buyers adjust.

The increase in central bank demand affects only the buyer value function $F(V_b)$. The derivative of $V_b$ with respect to the purchases is:
\[
\frac{\partial V_b}{\partial \alpha_{cb}} = -\frac{1 - \beta}{\beta} \frac{e}{\lambda \alpha_{sl} (\delta + \theta)(\rho + \delta)} \delta
\]  
(24)

As a result, the buyer value function \( F(V_b) \) shifts down as \( \alpha_{cb} \) increases, since the price increase from the central bank purchases lowers the value function of the buyers. When \( F(V_b) \) shifts down, the intersection of the equilibrium shifts to the right, meaning that equilibrium values of both \( g \) and \( F(V_b) \) are lower, and that the equilibrium mass of sellers is higher. By equation 22, \( \alpha_b \) then decreases. These two effects put a counteracting downward pressure on price (equation 8d). Therefore, price rises by less when the entry of buyers is endogenous.

\[\square\]

**Proposition 6.** Central bank asset purchases crowd out other buyers.

**Proof.** As was shown above, the central bank demand for bonds lowers the equilibrium inflows of outside investors \( g \). This is the crowding out effect. With endogenous entry of buyers, the outside investors can now decide whether to enter the market, depending on the central bank purchases’ effect on their entry condition. Central bank purchases lead to a higher price, which reduces the value of becoming a buyer. Since the value of the outside option does not change, there are more investors for whom the value of their outside option is higher than the value of becoming a buyer. Fewer investors therefore enter the market.

Because the measure of buyers is also now lower, the price impact of the purchases is more muted. The central bank purchases increase the price impact, while the decline in the number of other buyers reduces it.

\[\square\]

**Impact of asset purchases on liquidity**

**Proposition 7.** Response of liquidity to central bank purchases is smaller with endogenous entry of buyers.

**Proof.** The mechanism is otherwise the same as with exogenous entry of buyers, except that the decline in the measure of buyers dampens the response.

\[\square\]
3 Model simulations with a euro area Preferred Habitat Index

We simulate the model numerically by looking for the measure of sellers, $\alpha_{sl}$ that solves the equilibrium condition presented in Section 2.5. That is, the point where the entry flows of outside investors equal the expected return of becoming a buyer. The model is calibrated to the Eurozone.

3.1 A Preferred Habitat Index for the euro area

We construct an index of preferred habitat investors from the ESCB securities holdings statistics (SHS).\footnote{For more information on the SHS database, see Rousov and Caloca (2015), European Central Bank (2015), Boermans and Vermeulen (2016).} This relatively new database contains quarterly data on the holdings of securities, including government debt securities, at a security-by-security level. Compared to the more standard aggregate data, it allows for an overview of the holders and issuers of securities by economic sectors at a very granular level of detail (excluding Eurosystem holdings), including their interdependencies. Previously, this kind of detailed data was in the Euro area only available for deposits and loans, or more recently only at the macro level in the who-to-whom tables in National Accounts statistics. We describe the database in more detail in Appendix A.

Our index of preferred habitat investors is a composite indicator, consisting of the holdings of investors that are likely to be preferred habitat investors, as a share of the total government debt securities issued by euro area countries (excluding Eurosystem holdings). In particular, we consider central banks, general governments outside the euro area, insurance companies, and pension funds (both in and outside the Euro area) to be more likely to preferred habitat investors than other investors in Euro area sovereign bonds.\footnote{For examples of papers that model preferred habitat investors in a macro model, see Andrés, López-Salido and Nelson (2004), De Graeve and Iversen (2016).}

Our euro area index of preferred habitat investors is new. To our knowledge, there exists no comparable cross-country comparable data on the holders of government debt nor a measure for preferred habitat investors at this level of detail. Blattner and Joyce (2016) is one of the few papers to construct a proxy measure of preferred habitat investors by estimating the amount of free floating debt (i.e. excluding foreign official holdings) on the basis of IMF data of official holdings. The method is based on Arslanalp and Poghosyan (2015).
Given public data limitations, their measure contains information on foreign official holdings of debt and does not consider holdings by pension funds, and insurance companies. Andritzky (2012) develops a measure of institutional investors from public sources for the G20 countries, which includes a breakdown to domestic banks, pension funds and insurance companies, and domestic central bank, but excludes foreign central bank holdings.

There are a few other papers that use the security holdings database, but with a different approach to ours. Boermans, Frost and Steins Bisschop (2016) use the security holdings database at a security-by-security level to study the effect of market liquidity and ownership on bond price volatility, but focus on concentration of ownership rather than investor characteristics. Koijen, Koulishcer, Nguyen and Yogo (2017) do focus on investor characteristics and use security level holdings data to construct a measure of risks exposures across major investor sectors and countries. Studying portfolio flows and the dynamics of risk exposures during the PSPP programme from 2015Q2 to 2015Q4, they find that foreign investors, banks and mutual funds rebalance their portfolios, whereas euro area insurers and pension funds purchase the same bonds as the ECB.

In Figure 2 the index is presented by (unweighted) country groupings, separating between the larger and higher rated countries and the other Eurozone countries, and the (weighted) Eurozone average, while showing the three components of the index. It is clear that the large difference in the index between the two sets of countries is particularly due to the differences in holdings by central banks, and general governments outside the Euro area, whereas the distribution is less dispersed for insurance companies and pension funds. However, at the individual country level, there is more dispersion that is partly evened out in the country groupings.
Figure 2: Preferred habitat investors index per sector, 2014 average, for (unweighted) country groupings and the (weighted) euro area average.
3.2 Calibration of the model

Calibration of the model is shown below in table 3. Bargaining power of the buyers, $\beta$ is set to 0.5. Correspondingly, the bargaining power of the sellers, $(1 - \beta)$ is also 0.5. The average sovereign debt maturity in the Eurozone is 7 years and the value is quite similar for most of the countries. We therefore set $\delta$, the probability of debt maturing in any given year to 0.14. The high-type sellers are the only agents in the model with a discount factor, and that is set to 0.05. As is common in literature, we set the recovery value to 0.4. The Poisson intensity of the search process, $\lambda$ is a constant in this model. We set it to 600, which means that if the measure of sellers is one, it takes about a half of a business day on average to find a seller. The probability of liquidity shock is harder to calibrate, and we set it to 0.10. In each year there is a 10% probability of getting hit by a liquidity shock. It should not be too unreasonable given that it is an annual probability. The buyer search cost is set very low at 0.001. We assume a beta distribution for the entry condition, and set the parameters of the distribution $\alpha$ and $\beta$ to 1 and 2 respectively.

<table>
<thead>
<tr>
<th>Buyers bargaining power</th>
<th>$\beta$</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of debt maturing</td>
<td>$\delta$</td>
<td>0.14</td>
</tr>
<tr>
<td>Sellers’ discount factor</td>
<td>$\rho$</td>
<td>0.05</td>
</tr>
<tr>
<td>Recovery rate</td>
<td>$\gamma$</td>
<td>0.4</td>
</tr>
<tr>
<td>Search intensity</td>
<td>$\lambda$</td>
<td>600</td>
</tr>
<tr>
<td>Probability of a liquidity shock</td>
<td>$\theta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Buyers’ search cost</td>
<td>$\epsilon$</td>
<td>0.001</td>
</tr>
<tr>
<td>alpha of the beta distribution</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>beta of the beta distribution</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

We calibrate the model to the Eurozone to see how the impact of QE on yields and liquidity differs among the Eurozone countries. The difference in response in the model comes from the different shares of preferred habitat investors in each country, therefore this information is essential for the calibration. We construct an index of preferred habitat investors from the ECB’s securities holding statistics, which allows us to differentiate investor categories in the sovereign bond market. Our index measures the share of investors that are likely to be preferred habitat investors (pension funds, insurers and central banks outside the Eurozone) as a share of the total sovereign bond holding of each Eurozone Member State. As seen in figure 5 on page 36, this index differs widely among Eurozone countries, varying from 2% to more than 50%. It’s value is both correlated with the size of the bond sovereign bond market and the rating of the country.

Ideally we would calibrate the model to each country separately, but due to confidentiality of the data, we cannot reveal the preferred habitat index for each country. We therefore split the sample in two groups of larger Eurozone countries with high and low...
share of preferred habitat bondholders. The group of countries with a higher share of preferred habitat investors consists of Austria, Belgium, Germany, Finland, France, and Netherlands, while the group with a lower share consists of Ireland, Italy, Portugal, and Spain. The shares of preferred habitat investors in these countries can be seen in figure 3. Inside the box are the countries in the high preferred habitat holdings group and outside those in the low preferred habitat group.

We then compute the shares of bonds purchased in each country by the Eurozone central banks. There is some heterogeneity because the purchases are divided according to the capital key while debt levels vary in each country. The initial purchases, on which our study concentrates on, amounted to 912 billion euros over 19 months (March 2015 until September 2016) with approximately 48 billion government bonds bought each month. The 912 billion euros of purchases was divided among the Eurozone countries according to the capital key.

Table 2 shows the results of our calculations of shares of outstanding long-term debt securities purchased in each Eurozone country. Some small Eurozone countries in particular stand out here. Those countries have very small debt markets, especially Estonia that hardly has any outstanding long-term debt. Similarly the government of Luxembourg only issues bonds occasionally and the market for those bonds is small and fairly illiquid. As a result, the central banks of Estonia and Luxembourg are mainly buying bonds of international institutions in Europe.

There is not a very large difference in the shares of long-term bonds purchased across the high- and low-rating countries used in the calibration, and therefore the difference in impact in yields in those countries cannot be explained by the reduction in the supply of bonds alone.

Statistics used in the calibration for these country groups are shown below in table 3. The difference in the share of preferred habitat investors between the groups is quite large. In the high group, 42% of debt is held by preferred habitat investors, while in the low group the figure is 21%. The amounts purchased as a share of long-term bonds are very similar in both groups.

The default probabilities are computed from benchmark 10 year sovereign yields on 1st of December 2014, 3 days before the ECB press conference where Draghi hinted about the upcoming asset purchases programme. An approximation for a risk-neutral default intensity that we use is \( \frac{y - r}{1 - RR} \) where \( y \) is the yield on 1st of December 2014, \( r \) is the risk
Table 2: Purchases as a percentage of outstanding long-term debt securities

<table>
<thead>
<tr>
<th>Country</th>
<th>Capital Key</th>
<th>Debt to GDP</th>
<th>% Purchases</th>
<th>% Purchases</th>
<th>% Purchases</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK</td>
<td>1.10</td>
<td>54</td>
<td>26.86</td>
<td>24.84</td>
<td>24.84</td>
</tr>
<tr>
<td>LV</td>
<td>0.40</td>
<td>40</td>
<td>61.76</td>
<td>37.94</td>
<td>20.27</td>
</tr>
<tr>
<td>LU</td>
<td>0.29</td>
<td>24</td>
<td>43.47</td>
<td>23.64</td>
<td>17.95</td>
</tr>
<tr>
<td>MT</td>
<td>0.09</td>
<td>68</td>
<td>16.57</td>
<td>15.50</td>
<td>15.50</td>
</tr>
<tr>
<td>SI</td>
<td>0.49</td>
<td>81</td>
<td>17.59</td>
<td>14.86</td>
<td>14.86</td>
</tr>
<tr>
<td>FI</td>
<td>1.78</td>
<td>59</td>
<td>16.81</td>
<td>13.45</td>
<td>13.45</td>
</tr>
<tr>
<td>NL</td>
<td>5.69</td>
<td>69</td>
<td>15.19</td>
<td>11.50</td>
<td>11.50</td>
</tr>
<tr>
<td>ES</td>
<td>12.56</td>
<td>98</td>
<td>14.15</td>
<td>11.08</td>
<td>11.08</td>
</tr>
<tr>
<td>DE</td>
<td>25.57</td>
<td>75</td>
<td>15.07</td>
<td>10.75</td>
<td>10.75</td>
</tr>
<tr>
<td>CY</td>
<td>0.21</td>
<td>108</td>
<td>31.52</td>
<td>10.42</td>
<td>10.42</td>
</tr>
<tr>
<td>PO</td>
<td>2.48</td>
<td>130</td>
<td>19.71</td>
<td>10.03</td>
<td>10.03</td>
</tr>
<tr>
<td>AT</td>
<td>2.79</td>
<td>85</td>
<td>11.09</td>
<td>9.15</td>
<td>9.15</td>
</tr>
<tr>
<td>FR</td>
<td>20.14</td>
<td>95</td>
<td>11.84</td>
<td>9.02</td>
<td>9.02</td>
</tr>
<tr>
<td>BE</td>
<td>3.52</td>
<td>107</td>
<td>9.73</td>
<td>7.49</td>
<td>7.49</td>
</tr>
<tr>
<td>IT</td>
<td>17.49</td>
<td>132</td>
<td>9.17</td>
<td>7.47</td>
<td>7.47</td>
</tr>
<tr>
<td>IE</td>
<td>1.65</td>
<td>110</td>
<td>12.21</td>
<td>7.40</td>
<td>7.40</td>
</tr>
<tr>
<td>GR</td>
<td>2.89</td>
<td>177</td>
<td>38.47</td>
<td>8.31</td>
<td>7.13</td>
</tr>
<tr>
<td>EE</td>
<td>0.27</td>
<td>11</td>
<td>1471.52</td>
<td>120.53</td>
<td>2.70</td>
</tr>
</tbody>
</table>

In the fourth column are computed the shares of purchases as a percentage of long-term securities. The purchases themselves are the capital key times the total size of the asset purchases programme. In the fifth column the purchases are taken as a share of total government debt. In the last column are the same figures as in fifth column, except that they are adjusted to the amount the central banks are able to buy. The central banks are able to purchase only 33% of each country’s bonds without becoming a senior debt holder. For example in case of Latvia, the purchases would amount to over 60% of outstanding long-term securities. The maximum central banks can purchase is 33% of the securities, which is 20.27% of total debt.

Figure 3: Preferred habitat investor index per Eurozone country, 2014. Countries used in calibration.
Table 3: Calibration of groups

<table>
<thead>
<tr>
<th></th>
<th>Preferred habitat</th>
<th>Default probability</th>
<th>Purchases as a share of long-term bonds</th>
<th>Average maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>High preferred habitat</td>
<td>0.42</td>
<td>0.23</td>
<td>13.29</td>
<td>6.68</td>
</tr>
<tr>
<td>Low preferred habitat</td>
<td>0.21</td>
<td>2.14</td>
<td>13.81</td>
<td>7.48</td>
</tr>
</tbody>
</table>

free rate, German benchmark yield in this case and RR is the recovery rate that we set to 40% as in the calibration.

The average maturity of loans in both groups is very similar, and it is even longer in the low rating group. This is mostly due to the low rating groups having official loans with very long maturities. We calibrate the average maturity to be 7 years for both of these groups.

3.3 Event study of the first ECB PSPP announcement

We can now compute the announcement effect of quantitative easing on 10-year benchmark yields in various Eurozone countries. The first public sector purchase programme was announced in a press conference on 22 January 2015. The announcement was well anticipated but the expectations should have been the same for all of the concerned countries and should not affect the results.\footnote{For an assessment of expectations of QE, see Altavilla, Carboni and Motto (2015); De Santis (2016)} We control for the change in the local stock market index to ensure that the impact is not tainted by concomitant local events. The Greek debt negotiations were happening at the same time and for this reason we control also for the Greek stock index.

We estimate equation 25 where the yield is the 10-year benchmark yield for each country, QE is a dummy that is zero on non-announcement days, and one on the announcement day, 22 January 2015. Note that the yields are computed from end of day prices so that the difference on the announcement day is the difference in yield at the end of 22 January 2015 and end of the day 21 January 2015. Stocks variable is the local stock market index and GRstocks is the Greek stock market index. All variables are added contemporaneously.

\[
D.log(yield)_t = QE_t + D.log(bidask_t) + D.log(stocks_t) + D.log(GRstocks_t) + \epsilon_t \tag{25}
\]

The results are shown in table 5. Looking at the log basis point (percentage) changes
in the yields, the ordering by the impact is identical to ordering the countries by their credit ratings. The higher the share of preferred habitat investors (correlated with the credit rating), the larger is the impact of quantitative easing, as predicted by our search theoretical model.

Table 4: Impact of asset purchases in the Eurozone on yields

<table>
<thead>
<tr>
<th>Country</th>
<th>Basis points</th>
<th>Log(basis points)</th>
<th>Rating</th>
<th>Abs mean change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-9.3</td>
<td>-15.5***</td>
<td>AAA</td>
<td>3.2</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-9.9</td>
<td>-15.3***</td>
<td>AA+</td>
<td>3.1</td>
</tr>
<tr>
<td>Finland</td>
<td>-7.7</td>
<td>-14.8***</td>
<td>AA+</td>
<td>3.7</td>
</tr>
<tr>
<td>Austria</td>
<td>-9.3</td>
<td>-14.3***</td>
<td>AA+</td>
<td>3.3</td>
</tr>
<tr>
<td>France</td>
<td>-7.6</td>
<td>-11.0***</td>
<td>AA</td>
<td>3.5</td>
</tr>
<tr>
<td>Belgium</td>
<td>-7.0</td>
<td>-9.5***</td>
<td>AA</td>
<td>4</td>
</tr>
<tr>
<td>Ireland</td>
<td>-9.4</td>
<td>-9.0***</td>
<td>A-</td>
<td>4.1</td>
</tr>
<tr>
<td>Spain</td>
<td>-10.7</td>
<td>-8.2***</td>
<td>BBB</td>
<td>3.9</td>
</tr>
<tr>
<td>Italy</td>
<td>-8.3</td>
<td>-7.1***</td>
<td>BBB</td>
<td>4</td>
</tr>
<tr>
<td>Portugal</td>
<td>-17.6*</td>
<td>-6.3**</td>
<td>BB</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Change in 10-year benchmark yields of Eurozone sovereign bonds following announcement of QE on 22 Jan 2015, controlling for bid-ask spreads, local stock index, and Greek stock index. Ratings are S&P’s long-term sovereign ratings. Absolute mean change is the average daily change in basis points. The sample is from 3 January 1995 to 29 March 2015.

The results can be compared to typical absolute mean changes in ten year yields in the sample period running from early 1995 to 2015. In basis points, which are listed in the first column, the impact of QE on the announcement day is approximately 2-3 times the typical movement in yields.

The differences in the impact of QE on yields measured in basis points between most countries are small, even though the volatility of the yields differs, as shown in last column. The change in yields is also not significant in basis points for any of the countries except Portugal.

In order to assess the announcement impact on yields across eurozone countries, we prefer to smooth the volatility, which is typically done by using logs of the variables. As an example, a 0.5 percentage point movement might not be very large when yields are at 10%, while it is large when yields are at 1%. Cox, Ingersoll and Ross (1985) model short rates with the zero lower bound explicitly, showing that the volatility of the short rate depends on its level.

A simple regression of the ten year yield on the QE shock interacted with the preferred habitat investor indicator confirms the results of the model, that the impact of QE is larger,
the more preferred habitat investors hold the bonds. When the preferred habitat investor index is zero, yields fall by 6.7%. For each percentage point increase in the preferred habitat holdings yields fall by an additional 0.14%. The difference between the country with lowest and highest shares of preferred habitat holdings is nearly 40 percentage points (See figure 3).

Table 5: Impact of QE on yields

<table>
<thead>
<tr>
<th></th>
<th>log(basis points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QExPH</td>
<td>-.14*</td>
</tr>
<tr>
<td></td>
<td>(-2.11)</td>
</tr>
<tr>
<td>QE</td>
<td>-6.7***</td>
</tr>
<tr>
<td></td>
<td>(-3.31)</td>
</tr>
<tr>
<td>PH</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(-1.61)</td>
</tr>
</tbody>
</table>

QE is the announcement dummy that is one on 22 January 2015. PH is the share of long-term bond holdings by preferred habitat investors. QExPH interacts these two variables. t-statistics are given below the coefficients in parentheses.

We also assess the announcement impact on liquidity. The results are in table 6 which shows the impact of bid-ask spreads to a QE dummy shock on 22 January 2015. The model suggested that liquidity improves when the central bank announces that it is entering as a buyer, and the improvement is larger in countries with least preferred habitat investors. Although the responses of the variables have the right sign (except in Netherlands), and the responses are generally larger in countries with lower rating, none of the responses are significant.

As shown in the model, the impact on liquidity is multifaceted. Initially, liquidity improves, after which it starts declining. As liquidity declines, bonds that are in high demand by preferred habitat investors can become scarce. Deeper analysis of the liquidity and scarcity effects is outside the scope of this paper. We therefore refer to some of the existing studies and leave the rest to future work.

3.4 Simulation of the model

We simulate the model using the calibration in section 3.2. The model was solved for the bond price, but it is more convenient to look at the impact on yields. We therefore solve for the yield with the following bond pricing formula, where \( y \) is the yield, and maturity is \( \frac{1}{\delta} \):
Table 6: Impact of asset purchases in the Eurozone on liquidity

<table>
<thead>
<tr>
<th>Country</th>
<th>Basis points</th>
<th>Abs mean change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>1.4</td>
<td>8.5</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-1.0</td>
<td>9.1</td>
</tr>
<tr>
<td>Finland</td>
<td>2.1</td>
<td>10.2</td>
</tr>
<tr>
<td>Austria</td>
<td>1.3</td>
<td>11.9</td>
</tr>
<tr>
<td>France</td>
<td>1.6</td>
<td>13.8</td>
</tr>
<tr>
<td>Belgium</td>
<td>1.0</td>
<td>12.8</td>
</tr>
<tr>
<td>Ireland</td>
<td></td>
<td>15.2</td>
</tr>
<tr>
<td>Spain</td>
<td>1.5</td>
<td>14.1</td>
</tr>
<tr>
<td>Italy</td>
<td>1.6</td>
<td>11.1</td>
</tr>
<tr>
<td>Portugal</td>
<td>4.6</td>
<td>9.8</td>
</tr>
</tbody>
</table>

Change in bid-ask spreads of Eurozone sovereign bonds following announcement of QE on 22 Jan 2015, controlling for a change in 10-year yield, local stock index, and Greek stock index. Absolute mean change is the average daily change in basis points. The sample is from 3 January 1995 to 29 March 2015.

\[
y = (1/P)\delta - 1
\]  \hspace{1cm} (26)

Graph 4 shows the results of the simulation. We show four artificial periods of simulation. In the first period purchases are zero and the share of preferred habitat investors is set to the initial levels we find in data for each group. The demand and supply side of purchases are analysed separately in the following two periods, such that in period two demand increases, and in period three demand remains at a level demanded by the central bank, and supply falls. More specifically, in period two, the central bank purchases 13% of the bonds in each group. In period three those purchases are added to the stock of preferred habitat investors and while the central bank wants to buy 13% of the bonds. The period three value can be understood as the combined supply and demand effect on yields and liquidity. Note that in both periods we keep the measure of buyers constant in order to show the effect of crowding out separately. In period four the central bank stops the purchases, while it holds the share of bonds it purchased in the previous period on its balance sheet. In this last period we allow the measure of buyers adjust endogenously in order to show the crowding out effect. The period four effect is therefore the effect of ending the asset purchases that leads to an increase in yields due to the fall in central bank demand.

In period two, as central bank demand for the bonds increases, yields fall, 6.6% in the group with a low share of preferred habitat bond holdings. This can be compared to the
close to the 9.7% decline in yield in the group with a high share of preferred habitat holdings of the bonds. The demand element of the model explains approximately two thirds of the decline in yields observed on the announcement day of the first PSPP programme. On average, the 10 year benchmark yields declined 13.4% in the group with a high share of preferred habitat holdings of bonds, while they decline 7.7% in the group with low share of preferred habitat holdings of bonds.

In the model, the yields decline further in period three as the supply effects of the central bank purchases are added. Period three decline in the yield is therefore the combined effect of increase in demand, and reduction in supply. The combined effect in the group with high share of preferred habitat holdings is 10.8 compared to 9.7 in the group with low share of preferred habitat holdings, both closer to the impact in data.

In this model the effects of increase in demand by the central bank are larger than the effects of reduction in supply. It is interesting because typically studies concentrate on the supply, or stock effects of quantitative easing, and these are thought to be more important than the demand effects. On the other than, the supply effects in this model are permanent, they last until central bank sells off the balance sheet, while the demand effects last only while the central bank purchases the bonds.

In the last, fourth period of the simulation, the central banks tapers the purchases, i.e. it stops buying the bonds but holds the balance sheet. We also allow the number of buyers to vary now, which means that some of the outside investors are allowed to determine whether to enter the market. Yields increase in this case, but especially in the countries with a low share of preferred habitat holdings. This is because demand falls due to the crowding out effect.

Liquidity improves more in the countries with fewer preferred habitat investors as the central bank increases demand where it is scarce. Eventually, as the central bank stops the purchases but keeps holding the purchased bonds, liquidity falls to a lower level than it was initially. This is because we allow the entry of buyers to adjust endogenously in the last period, where the central bank holdings then crowd out potential buyers.

The magnitude of the decline in liquidity depends on how much the measure of buyers is allowed to vary. Since demand by preferred habitat investors is typically less elastic to price, countries with high share of preferred habitat investor holdings of bonds can be thought to have a higher remaining demand, in which case liquidity would not fall as much. Therefore, the liquidity decline is mostly a concern for countries with low preferred habitat
Figure 4: Price and liquidity impact from the calibrated model
4 Conclusion

We presented a search-theoretic model of over-the-counter debt that allows us to analyse the impact of central bank purchases on yields. The impact is predominantly determined by tightness on the bond market, the ratio between sellers and buyers. In turn, the tightness of the market is influenced by the share of preferred habitat investors. These investors are unwilling to sell their bonds to the central banks and for this reason, prices and yields move more in markets with a higher share of preferred habitat investors, i.e. markets that are tighter.

With data from the ECB securities and holding statistics, we construct a new index for the share of preferred habitat investors in Eurozone countries. This index varies strongly across Eurozone countries, and is positively correlated with sovereign debt ratings and the size of the bond market. We calibrate the model to the share of preferred habitat investors for two groups of higher and lower rated larger Eurozone countries, and match a part of the observed impact on sovereign yields from the announcement of the ECB asset purchase programme.

The model also predicts a liquidity trade-off effect. The impact on liquidity depends also on the tightness of the bond market. Asset purchases by the central bank improve liquidity initially, as they represent the addition of another large buyer to the market. However, as the central bank reduces the stock of bonds on the secondary market available for sale by holding the bonds to maturity, it subsequently reduces liquidity. In countries with fewer preferred habitat investors, liquidity at the end of purchases is lower than before the start of the purchases.
References


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Appendix A : Preferred habitat investor index

Securities Holding Statistics (SHS)

Securities Holding Statistics data are collected on a security by security level (based on Regulation ECB/2012/24, as amended by ECB/2015/18) for four security types: short- and long-term debt securities, quoted shares and investment funds shares/units, and subsequently linked with reference data on individual securities from the Centralised Securities Database (CSDB) with additional attributes referring to individual securities and their issuers. The data cover holdings of securities aggregated by selected investor sectors of each Euro area country, excluding the holdings by the eurosystem. The main holding sectors available are (i) deposit-taking corporations, (ii) money market funds, (iii) investment funds, (iv) financial vehicle corporations, (v) insurance corporations, (vi) pension funds, (vii) other financial corporations, (viii) general government, (ix) non-financial corporations, (x) households and (xi) non-profit institutions serving households. For holdings by non-Euro area investors, the mandatory sector breakdown is more restricted and distinguishes only between holdings by General Government and NCBs and the remaining investors.

For our purpose, we focus on the debt securities issues by Eurozone general governments that are held by (i) central banks and governments outside the Eurozone; (ii) insurance companies, both inside and outside the Eurozone, and (iii) pension funds, both inside and outside the Eurozone.

A caveat to be taken into account concerns the collection of data of the holdings of Euro area securities by non-euro area investors, which is to a large extent collected indirectly via custodians and thus may not capture the country of the final investor (i.e., the data suffer from custodial bias). This custodial bias presents a potential risk of double-counting with euro area holdings, where they are held by euro area financial investors in custody outside the euro area (or of double counting euro area holdings, in case of chains of custodians).

Custodial bias would not be expected to significantly influence the data on the holdings of non-euro area central bank and general government, insurance companies and pension funds. If at all, there could be a potential undercounting of the holdings of euro area securities by these sectors, in particular those by insurance corporations and pension funds. Given the larger than average contribution of holdings outside the euro area to the index of the countries with the highest share of preferred habitat investors in our index, this would likely imply an even larger dispersion across countries.
Through the potential double counting, custodial bias could influence the total amount of securities held, which is covered in the data base. Since we express our index as a share of total securities issued, we investigate this potential bias by comparing the total amount of securities included with the amount of general government debt issued by EA countries according to a different data source, the Euro Area Accounts (EEA). This check also allows to address the lack of Eurosystem data in SHSS. While the total amount of debt covered by both databases is very similar (close to 100% for the euro area), there are some differences across countries. In particular, the SHS data base includes smaller amounts held of securities issued by smaller countries than the debt issued according to the EEA, whereas the amount attributed to larger countries with larger financial sectors is higher. Figure 5 shows the preferred habitat index calculated with denominator based on the SHS and on the EEA database. For most countries, the differences are limited, but if there are differences they increase the dispersion of the index across countries. Since the EEA database provides a full coverage of the issued securities, we base the denominator of our index on this database, with the numerator based on the SHS database.

**Preferred habitat investor index**

Our index of preferred habitat investors is a composite indicator, consisting of the holdings of economic sectors that are likely to be preferred habitat investors, as a share of
the total government debt securities issued by euro area countries (excluding Eurosystem holdings). In particular, we consider central banks and general government outside the Euro area, insurance companies, and pension funds (both in and outside the Euro area) to be more likely to preferred habitat investors than other investors in Euro area sovereign bonds.

Central banks hold government bonds of other countries as foreign reserves, assets that can be easily sold in distress. This gives them a special preference for liquid and safe assets and is considered as a particular form of preferred habitat investment (see for instance in Krishnamurthy and Vissing-Jorgensen (2011) for the US Treasuries). While there are few detailed statistics about the holdings of central banks, the ones that do publish show a clear preference for higher-rated and more liquid sovereigns. See table 7. For example, the Riksbank mostly holds German bonds, and more Austrian than Italian bonds. Likewise, the Swiss National Bank, which does not publish a country breakdown, holds most of its foreign currency fixed income assets in securities of AAA-rated countries.

Table 7: Fixed income assets in foreign reserves, end 2014

<table>
<thead>
<tr>
<th>Riksbank</th>
<th>Swiss National Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>68%</td>
</tr>
<tr>
<td>France</td>
<td>12%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>9%</td>
</tr>
<tr>
<td>Belgium</td>
<td>5%</td>
</tr>
<tr>
<td>Austria</td>
<td>4%</td>
</tr>
<tr>
<td>Italy</td>
<td>2%</td>
</tr>
</tbody>
</table>


Swiss National Bank: holdings of foreign currency fixed income assets: [http://www.snb.ch/en/iabout/assets/id/assets_reserves](http://www.snb.ch/en/iabout/assets/id/assets_reserves)

General government holdings outside the Euro are aggregated together with central banks in the SHS database. However, we consider it likely that the entities in general government that hold foreign sovereign bonds, such as social security funds or sovereign wealth funds, display the same preferred habitat investor characteristics as pension funds and insurers.

According to the preferred habitat theory, institutional factors and regulations influences the behaviour of certain investors, which determines the maturity and asset classes in which they will invest. We consider this to primarily be the case for insurers and pension funds, which both have long-term obligations and are subject to supervision and regula-
tions, including sometimes restrictions on the geographical area or rating of instruments to invest in. For example, held to maturity accounting rules discourage insurance companies, and other long-term investors of selling bonds on the secondary market. These rules state that if an entity sells and therefore marks to market more than an insignificant amount of bonds it holds, it will not be able to account any financial assets as held to maturity in the current and the following two financial years, including all assets in its portfolio. (International Accounting Standards 39 [n.d.])

Our index is a proxy index, based on the characteristics of the investor, rather than the actual behaviour. It is of course possible that for example pension funds act as arbitrageurs with all or part of their sovereign debt holdings, or that other investor sectors act as preferred habitat investors. It is also a broad proxy as the SHS database limits the level of disaggregation of investor sectors that can be considered. In particular, the holding of insurers cannot be broken down in different types of insurers (e.g. life insurers), which might be relevant for the type of maturity that is preferred.

Due to confidentiality of the data we are unable to identify individual countries. However, we can mention some broad characteristics and present country groupings. First, there is a strong correlation between the size of the country and the preferred habitat index. For example, the nine Eurozone countries with the lowest preferred habitat index represent cumulatively less than 10% of the ECB capital key (which reflects the respective country’s share in the total population and gross domestic product, and is the basis for the distribution of the ECB asset purchase programme). Second, there is a strong correlation between the rating of the sovereign and the preferred habitat index, with higher
rated countries having a higher share of preferred habitat investors. Thirdly, when we consider the different components of the index, it is noteworthy that countries with a large second-pillar pension system or a large insurance sector also have a high share of sovereign holdings by these sectors.

Our preferred habitat investor index is relatively stable over time. In figure 7, the quarterly evolution of the index in 2014 and 2015 is shown for the Euro area average and selected country groupings, as well as the annual averages. While there has been some convergence in this period between higher and lower rated sovereigns, the different score on the index remain pronounced, both before and after the start of QE. It should be noted that the index might be influenced by various factors, e.g. the sale of foreign reserves by central banks outside the Euro area, the emergence of some Euro area countries out of EU/IMF financial adjustment programmes, etc.

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7The SHS data base contains only experimental data before 2013-Q4.