The Housing Cost Disease *

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Abstract

Wealth-to-income ratios, housing wealth and wealth inequality have been increasing in most advanced countries at least since the ’70s. We use a simple two-sector life cycle economy with bequests to explain these facts as a consequence of a rising labor efficiency in manufacturing (housing cost disease). When consumption inequality across households is sufficiently large, the housing cost disease has adverse effects on a measure of social welfare based on an egalitarian principle: the higher the housing’s value appreciation, the lower the welfare benefit of a rising labor efficiency in manufacturing.

Keywords: Housing Wealth, Cost Disease, Overlapping Generations, Wealth Inequality.

JEL Codes: D91, O11, H2, G1.

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1 Introduction

Wealth-to-income ratios have been increasing in most advanced economies since the end of the second World War. Since wealth is unevenly distributed, this phenomenon is raising a concern for a widening inequality gap. In this paper, we propose a frictionless two-sector life-cycle model with bequests able to replicate most of the stylized facts concerning the dynamic of total wealth, housing wealth and wealth inequality, as a consequence of an improvement in the efficiency of labor in the general economy, relative to the construction sector, that took place in most advanced countries at least since the 70s.

Piketty (2014), Piketty and Zucman (2014) and Piketty and Saez (2014) attribute the rising wealth-to-income ratios to the falling GDP growth rates and the long-run stability of the saving rate\(^1\), and claim that these trends are responsible for a rising income and wealth inequality. Piketty (2014) has famously recommended a worldwide tax on capital as a way to curb this phenomenon. This interpretation of the historical trends in wealth and inequality is problematic for at least two reasons. First, the joint behavior of the savings and growth rates in advanced countries since 1970 does not explain the rising wealth-to-income ratios on the basis of a simple application of the Solow model (i.e., on average, net saving rates fell by as much, if not more, than income growth rates). Second, the existing trends in wealth-to-income ratios and income shares are strongly determined by the dynamic of housing wealth and capital gains, as documented in recent research by Bonnet et al. (2014), Rognlie (2014) and Weil (2015).

In this paper we show that a sort of Baumol’s cost disease may be responsible for an increase in the housing share of wealth, wealth-to-income ratios and wealth inequality. In the seminal work by Baumol (1967), a market economy has two sectors

\(^1\)However, the theoretical underpinning of this theory has been challenged by Krusell and Smith (2015).
producing two goods using labor as the only input and enjoying different patterns of technological progress. Under perfect labor mobility and wage equalization, a rising labor productivity in the dynamic sector generates a higher production cost and, then, a rising relative output price in the stagnant sector (for example, to use one of Baumol’s examples, music played by a horn quintet). If the demand of the stagnant sector output is sufficiently inelastic, labor will move to this sector and aggregate output growth may decline. We extend Baumol (1967)’s analysis to a model with capital and land, where construction plays the role of the stagnant sector, while manufacturing experiences labor-augmenting technological progress, and we provide a set of conditions under which a rise in the efficiency of labor in manufacturing generates a strong housing price appreciation, a rise in the wealth-to-income ratio (mostly driven by higher housing wealth and a weak dynamics in average labor productivity) and in the size of bequests \((i.e., \text{a rising wealth inequality})\). We refer to this set of phenomena as the \textit{housing cost disease}. 

The empirical evidence supports these findings. We document the existence of a positive and large correlation between a labor-augmenting productivity residual in the manufacturing sector, relative to the one in construction, and the total and housing wealth-to-income ratios for a sample of the 8 largest economies over the 1970-2007 period. On the other hand, and contrary to the prediction of the Solow growth model and to Piketty’s own interpretation, total and housing wealth-to-income ratios appear to be poorly correlated with GDP growth and saving rates since 1970. In particular, the fall in saving rates experienced by many advanced countries in the last forty years more than compensate for the fall in growth rates, so that the Solow-model interpretation has little explanatory power over the 1970-2007 period for the countries in our sample.

In this respect, this paper builds a bridge between two different strands of the literature. The first, attributes most of the the long-run increasing trend in wealth and inequality to saving induced growth (for example, Piketty (2014), Piketty and Zucman
(2014) and Piketty and Saez (2014)). The second, explains movements in housing prices as a result of sectoral changes in productivity (for example, Ngai and Pissarides (2007), Kahn (2008), Iacoviello and Neri (2010) and Moro and Nuno (2012)). We are the first, to our knowledge, to develop a general equilibrium model that relates long-run trends in wealth and inequality to relative growth in labor efficiency and housing prices.

We consider an economy with overlapping generations of heterogeneous altruistic households living for two periods, supplying their labor inelastically when young, and deriving utility from housing services. The only source of heterogeneity between individuals is the degree of parental altruism, represented by a discount rate applied to the next generation’s utility. Consistently with the assumption of one-sided altruism we assume that parents cannot force gifts on their children. Hence, this heterogeneity generates a partition of the set of households at steady states into a subset of rich individuals receiving bequests from their parents and a subset of poor individuals receiving (and giving) no bequests. A key assumption is that housing is a produced good (with labor, capital and land as inputs) and the economy has two sectors: manufacturing and construction. A multi-sector approach is important to study the evolution of wealth’s composition in advanced economies (vs. economies in the early stage of the development process) since housing replaced land in households’ assets and because factor price equalization across sectors generates interesting linkages between the dynamics of productivity and asset prices.

We show that the housing cost disease is most likely under the assumptions that manufacturing is more capital intensive than construction, the land share of income is sufficiently small, housing demand is sufficiently inelastic with respect to its own price, and the elasticity of substitution between capital and labor in the construction sector is close to one. These two elasticities play an important role in our model. In particular, with unitary elasticity of substitution between capital and labor in construction and
between goods (consumption and housing services) in a CES representation of preferences, a rising productivity in manufacturing is allocation neutral, in the sense that total and housing wealth-to-income ratios, as well as the shares of labor across sectors, remain unchanged, whereas bequests (of rich households) increase on a one to one basis. When, instead, housing demand is sufficiently inelastic, the housing cost disease holds and bequests increase more than proportionally with manufacturing productivity.

The robustness of these results is studied numerically simulating the model for a CES specification of preferences and technology. We show that, for any 100 percent increase in the (exogenous) labor efficiency in manufacturing, total wealth-to-income increases by about 30 percent, while housing wealth by approximately 60 percent. The strong increase in housing wealth is driven by a 70 percent increase in the price of new houses, and the effect is stronger the larger the income share going to land owners. Since the interest rate is greater than the population growth rate at equilibrium, steady state net bequests (i.e., the difference between the present value of bequests received from the previous generation and left to the next one) are a positive component of the rich households’ present value of income. Then, assuming that housing is a normal good, bequests and housing wealth are strongly correlated, and this dependence is stronger the higher the equilibrium level of the interest rate. In fact, bequests respond approximately proportionally to an increase in relative labor efficiency. Note that our simulations also show a reallocation of inputs after an increase in relative labor efficiency: the share of labor in construction increases by approximately 30 percent and average productivity increases less then proportionally. As a consequence of the housing cost disease, the dynamics of output per worker is relatively sluggish and, then, a proper definition of the capital share, i.e., a measure of the income accruing to non-labor factors that does not include imputed rents, decreases along with relative labor efficiency. However, if capital intensities in the two sectors are not too far apart, the capital share becomes an increasing function of labor efficiency when imputed rents
are included in the definition of output. This provide some theoretical support to the empirical findings provided in Rognlie (2014).

An additional contribution of the paper is to clarify the effect of the housing cost disease on social welfare. This may be important in order to assess the merits of public policy in the face of a rising wealth-to-income ratio. By assuming that this is not a desirable outcome because of the implications in terms of possible unequal distribution of wealth across households, Piketty (2014) advocates the institution of a wealth tax. However, if the increase in housing prices depends on the rising relative productivity of non-construction sectors, then policies targeting specifically the housing sector are probably more appropriate, as noted for example by Auerbach and Hassett (2015). We leave to future research the evaluation of such policies, and we instead use the model to analyze whether a change in the composition of wealth toward housing, following a rise of efficiency in manufacturing, may not be desirable from a welfare point of view. Deaton and Laroque (2001) investigated a similar question and found that the presence of a market for housing determines a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the Golden Rule level. Our welfare criterion is based on an egalitarian welfare function that takes into account the households heterogeneous degrees of altruism with respect to the next generations and allows for unrestricted transfers across generations. We conclude that, when housing appreciation is sufficiently strong and consumption inequality sufficiently large, the steady state welfare benefit of a rising labor efficiency in manufacturing is lower than it would be in the planning optimum. In principle, a housing appreciation has two opposite effects on welfare. On the one hand, it raises the wealth of the poor old households so as to relax the non-negativity constraint on bequest values. On the other hand, it makes housing less affordable. The last effect appears to be stronger

\footnote{Note that housing taxation is, in any case, very controversial, since housing is a consumption good, as well as an asset, and home ownership is much more evenly distributed across individuals than stocks and other financial assets.}
than the former and detrimental to an egalitarian social welfare measure when poor households’ consumption is too low. This conclusion casts some doubts on the benefits of a wealth tax when we adopt a (non-paternalistic) egalitarian perspective, as a wealth tax is likely to raise housing prices.

The remainder of the paper is organized as follows. Section 2 presents some stylized facts on wealth ratios trends and inequality for eight developed economies. Section 3 introduces the model. Section 4 characterizes steady states with bequests. Section 5 shows the effects of improvements in relative labor productivity and the conditions for a housing cost disease. Section 6 discusses the welfare implications. Section 7 concludes.

2 Stylized Facts

In this section, we combine the empirical evidence from two sets of stylized facts supporting the housing cost disease. First, we look at the evolution of national wealth and of one of its main components, housing wealth, using data from Piketty and Zucman (2014). Second, we look at the evolution of wealth inequality, using data from Piketty and Saez (2014). Piketty and Zucman (2014) have put together an incredibly rich dataset, starting from national accounts data, for the period 1970-2010, for the largest eight developed economies: the United States, Germany, the United Kingdom, Canada, Japan, France, Italy and Australia. All assets and liabilities are valued at prevailing market prices and are not estimated starting from the sums of previous investment flows. Private wealth is net wealth of households, and assets include all non-financial and financial assets. Public wealth is net wealth of public administrations and government agencies. National wealth is the sum of private and public wealth. Note that the financial component of private wealth includes households’ holdings of domestic

\footnote{For a smaller subset of countries, Piketty and Zucman (2014) provide longer time-series. However, we choose to restrict our focus on a time-period for which we could maximize the number of countries in the sample and with more reliable data. For a detailed description of the data refer to section A in the Appendix or directly to Piketty and Zucman (2014).}
public debt. At the national level, holdings of domestic debt washed out, as they are a liability of the public sector. Housing wealth is one of the components of total wealth, and it measures the net value of households’ real-estate holdings. In this section we consider national, rather than private wealth, while we present data on households’ private and housing wealth in a separate appendix, as public debt should not be part of individuals’ net (of the present value of future taxes) wealth over the long run.

Figure 1 considers the evolution of wealth, for the period 1970-2010. Total national wealth, as a fraction of income, increased tremendously over the period 1970-2010 (top panel). The average wealth to income ratio was equal to 3.3 in 1970 and 5.2 in 2010, for an increase of 57 percent. Italy, France and Japan are the countries where wealth ratios increased the most (135, 72 and 71 percent respectively) reaching values close to 6 in 2010. On the contrary, the US is the country with the smallest increase in this ratio: only 6.8 percent, with a value in 2010 of about 4.3. The pattern for private wealth ratios is qualitatively very similar and shows an even larger increase of about 80 percent in the sample. Housing wealth increased along with total wealth in all countries, with the exception of the US (middle panel). The average housing wealth was equal to 0.8 in 1970 and 1.9 in 2010, for a whopping 137.5 percent increase. A simple back-of-the-envelope estimate puts at about 58 percent the contribution of housing wealth growth to the increase in the national wealth-to-income ratio, leaving a modest 42% to the growth in the business capital component of wealth. Interestingly, the evolution of housing wealth is not uniform across countries. For example, it increased by about 219 percent in Italy and 260 percent in France, but only by 87 percent in Canada and 37 percent in Japan\(^4\). The United States are the only country where the housing wealth to income dropped (by about 19 percent) over the period 1970 to 2010. However, the United States, in 1970, had the second highest housing wealth to income ratio and the latter increased by about 52 percent if the sample ends in 2007, at the onset of the

\(^4\)It is worth recalling that Japan experienced a housing boom followed by a crash in the sample. At the housing peak, in 1990, the housing wealth to income in Japan reached the value of 2.5.
Great Recession, when housing prices collapsed. The bottom panel of figure 1 plots housing as a share of national wealth. In 1970, the average housing share of wealth was about 25.8 percent. In 2010, the same share was about 35.4 percent, and numbers are not much different if we end the sample in 2007. Over the period 1970 to 2010, the evolution of the housing share of wealth was very different across the countries in the sample: for example, it increased by 108 percent in France and by 67 percent in the UK. On the other hand, it dropped by 24 percent in the United States and by 20 percent in Japan. Italy is the country with the largest housing wealth share both at the beginning (38 percent) and at the end (52 percent) of the sample.

Since wealth is typically unevenly distributed, the strong increase in wealth ratios should be associated with an increase in inequality. Figure 2 plots the wealth shares of the top 10 and 1 percent of the wealth distribution and shows to simple facts: first, wealth is very concentrated in the upper tail of the distribution; second, the wealth shares of the richest households have been gradually increasing since the ’70s. With respect to figure 1, the sample includes different countries, or geographical areas (i.e., France, the UK, the United States, Sweden and Europe), and for a longer time-period that goes from 1950 to 2010. Over the full sample, wealth inequality decreased: the average wealth share of the top 10 (top 1) percent was 73.4 (36.1) percent in 1950 and 65.4 (26.3) percent in 2010. However, if we split this time interval, we see that wealth inequality declined steeply from 1950 to 1970 in all countries (with the exception of the United States, where it did not change much), and has been increasing at a slower pace since 1980, or earlier. In particular, the average wealth shares of both the top 10 and 1 percent of the wealth distribution increased by about 5 percent since 1970. Using a different set of data, Cragg and Ghayad (2015) find similar results for the United States where median wealth is below mean wealth by a factor of 5, and mean wealth is above the wealth of the 80th percentile of the wealth distribution. Note that the reliability of wealth data is often questionable, and for this reason, for robustness, in
Figure 1: Households wealth

Notes: This figure plots the evolution over time of national wealth to national income (top panel); housing wealth to national income (middle panel); housing as a share of national wealth (bottom panel). National wealth is the sum of private and public wealth and is evaluated at market prices. The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia. Data are annual for the period 1970-2010 from Piketty and Zucman (2014). Additional details on the series are available in the Appendix (section A).

section A of the appendix we show how income, and not just wealth, inequality has been increasing over the period we consider.

The main conjecture investigated in this paper is the positive effect of relative labor productivity in manufacturing on the wealth ratios, with a strong effect on housing wealth. To provide some initial empirical support to our claim, we combine data on wealth ratios with data on relative productivity estimated from the EU KLEMS Growth and Productivity Accounts (O’Mahony and Timmer, 2009), which provide sectoral data on gross value added, capital and labor inputs and capital and labor shares, that we
use to build a measure of relative productivity in manufacturing with respect to the
construction sector. In particular, for each country in the Piketty and Zucman (2014)’s
sample, we follow the assumptions on technology of the model (discussed in section
5.5) and generate annual time-series of labor-augmenting productivity in manufacturing
relative to construction based on Cobb-Douglas production function in manufacturing,
and CES production function for construction\(^5\).

In the classic Solow model, the long run (or steady state) wealth-to-income ratio

\(^5\)KLEMS data are currently available up to 2007. Therefore, we consider the sample period 1970-
2007. For some countries, the sample period is shorter due to lack of data. Section A in the appendix
presents additional details on the data and on the construction of the indices of relative productivity.
Table 1: Long-run growth rates

<table>
<thead>
<tr>
<th>%Δ</th>
<th>US</th>
<th>DE</th>
<th>UK</th>
<th>CA</th>
<th>JP</th>
<th>FR</th>
<th>IT</th>
<th>AU</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>NWI</td>
<td>39.15</td>
<td>22.82</td>
<td>75.11</td>
<td>19.15</td>
<td>29.35</td>
<td>56.46</td>
<td>128.14</td>
<td>42.76</td>
<td>51.62</td>
</tr>
<tr>
<td>PWI</td>
<td>51.68</td>
<td>68.44</td>
<td>71.04</td>
<td>45.74</td>
<td>44.55</td>
<td>72.32</td>
<td>168.58</td>
<td>60.09</td>
<td>72.81</td>
</tr>
<tr>
<td>HWI</td>
<td>28.79</td>
<td>71.22</td>
<td>193.26</td>
<td>47.76</td>
<td>-21.26</td>
<td>134.71</td>
<td>187.91</td>
<td>39.18</td>
<td>85.20</td>
</tr>
<tr>
<td>s</td>
<td>-78.39</td>
<td>-31.32</td>
<td>-58.08</td>
<td>4.04</td>
<td>-75.74</td>
<td>-19.87</td>
<td>-61.68</td>
<td>-31.91</td>
<td>-44.12</td>
</tr>
<tr>
<td>g</td>
<td>-38.88</td>
<td>-92.84</td>
<td>-2.85</td>
<td>-58.16</td>
<td>-99.06</td>
<td>-70.46</td>
<td>-98.89</td>
<td>-22.33</td>
<td>-60.43</td>
</tr>
<tr>
<td>a</td>
<td>392.69</td>
<td>119.87</td>
<td>82.56</td>
<td>166.00</td>
<td>356.65</td>
<td>61.76</td>
<td>641.85</td>
<td>14.49</td>
<td>229.48</td>
</tr>
</tbody>
</table>

Notes: This table reports the cumulated percentage changes, over the period 1970-2007, for the countries in the sample, of the following variables: national wealth-to-income (NWI), private wealth-to-income (PWI), housing wealth-to-income (HWI), housing share of national wealth (HW/W), national saving as a fraction of income (s), income growth rate (g), and relative labor efficiency (a). The cumulated change for g is computed as the percentage change between last and first sample values of the linear trend extracted from the income growth series in a longer sample starting in 1950. The last column reports the cross-country average total change. Data are annual from Piketty and Zucman (2014) and O’Mahony and Timmer (2009) for the period 1970-2007. Data for the US start in 1977, for Japan in 1973, for France in 1980 and for Australia in 1982.

(renoted as β in Piketty and Zucman (2014)) should be equal to the ratio between the (net of depreciation) saving rate and real income growth rate. In table 1, we report the cumulated growth rates, over the sample 1970-2007, for the main variables we consider. Note that national wealth ratios increased on average by about 51 percent, while private and housing wealth increased even more. Income growth rates declined over time, on average by about 60 percent, while saving rates declined, on average by about 44 percent. To compute the cumulated changes in income growth rates we extracted linear trends from the time-series of annual growth rates for the longer time-sample 1950-2010 (and 1961-2010 for Canada, Italy and Australia) and then computed the cumulated percentage changes, over the 1970-2010 period, using the trend series. Therefore, the percentage change in g gives an idea of change in average income growth over the long-run. Italy, the US and Japan are the countries with the sharpest increase in relative productivity: it increased sixfold in Italy, and threefold in Canada and the US. On average, relative productivity increased by about 229 percent.

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6Note how these figures are slightly different than those discussed in the previous paragraphs since we compute cumulated changes up to 2007, and not 2010, and we limit the samples to periods with data on relative productivity.
Since, on average, the slow-down in income growth is larger than for the saving rates, Solow’s explanation could be valid as the numerator of the $beta$ ratio declined less than the denominator. We test more formally the Solow hypothesis by running cross-sectional regressions of long-run changes in the wealth ratios on the changes in saving and income growth rates. In particular, we regress the cumulated percentage changes in the wealth ratios, over the 1970-2007 period, on a constant and the cumulated percentage changes in national saving and income growth rates. The first column of table 2 reports the main results from these regressions. Note that the regressions assume a linear structure, rather than considering directly the $beta$ ratio. The $R^2$ is not impressive (12%) and the coefficients on $s$ and $g$ are not statistically different from zero. The point estimate for $g$ has the right sign (i.e., negative), while that for $s$ has the wrong sign and is negative so that an increase in the national saving rate is associated to a drop in national wealth. In the second column of table 2 we consider a regression specification that includes only the changes in relative labor productivity (and a constant). The $R^2$ jumps to 0.3, and $a$ is positive and significant at the 90% confidence interval. Therefore, the housing cost disease’s interpretation provides a better fit to the data. In the last column of the table we consider a specification which includes both $s, g$ and $a$. Since the number of observations is just equal to 8, when looking at these results we need to consider the extremely small number of degrees of freedom. Under this latter specification, the $R^2$ increases marginally with respect to the specification that includes only $a$. The coefficients on $a$ and $g$ are borderline significant (the p-values are, respectively, equal to 12.3% and 10.7%). However, while $a$ has the correct sign, $g$ is now positive so that an increase in $g$ is associated to an increase in the wealth ratios.

In the next section, we present a simple overlapping generation life-cycle growth model with two sectors that qualitatively accounts for the stylized facts outlined in this section.
### Table 2: Cross-sectional regressions

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td>cost</td>
<td>0.31</td>
<td>0.31</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.06)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>a</td>
<td>0.09</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>-0.42</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.67)</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>-0.04</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.24)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.12</td>
<td>0.30</td>
<td>0.38</td>
</tr>
<tr>
<td>N</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Notes: This table reports results from OLS cross-sectional regressions. The dependent variable are the cumulated changes in national wealth-to-income ratios. We consider three different specifications for the regressors: the first, includes the cumulated changes in the saving ($s$) and income growth ($g$) rates; the second, includes the cumulated changes in relative labor productivity in manufacturing ($a$); the third, includes $s, g$ and $a$. All regressions includes a constant. HAC standard errors are reported in brackets. The last two rows report the R-square and the number of observations. Country-level cumulated changes are for the period 1970-2007 with the exception of the US (1977-2007), Japan (1973-2007), France (1980-2007) and Australia (1982-2007). Data are annual from Piketty and Zucman (2014) and O’Mahony and Timmer (2009).

### 3 The Model

In this section we present a simple life-cycle model of a competitive economy with two sectors, construction and manufacturing, three assets, business capital, housing and land, and exogenous technical progress. Labor and capital are used in both sectors whereas land is used in the construction sector only. The amount of land available for construction is assumed to be time-variant and regulated by the government (for example, the government can use zoning regulation to adjust the quantity of land available to construction). Capital and labor are totally mobile across sectors and firms are perfectly competitive. We show that, under factor price equalization, a rise in labor efficiency in manufacturing generates, *ceteris paribus*, a rise in the relative price of construction. However, to evaluate whether this leads to higher housing values and wealth to income ratios, we have to consider the response of housing demand and the reallocation of inputs across the two sectors in equilibrium. For example, the rise in housing prices may reduce the demand for construction so that labor may shift back to manufacturing. In addition, a higher housing price and a reallocation of capital and
labor may reduce business capital and affect the wage rate and the interest rate. We use the model to identify conditions on technologies and households’ preferences compatible with the stylized facts characterizing the largest developed economies presented in section 2.

3.1 Households

A set \( L_t \) of households, growing at a rate \( n \geq 0 \), is born every period \( t = 0, 1, 2, \ldots \). Households live for two periods, supply labor time inelastically, in young age only, and have identical time-invariant preferences for manufacturing consumption and housing services, the latter being measured by the housing stock. Households are characterized by some degree of altruism with respect to their offsprings defined by an individual specific discount rate of the next generation’s utility. In particular, households born at time \( t \) belong to different types, indexed by \( i \), with \( i \) in a finite set \( \mathcal{I} \), and each type \( i \) composed of a mass \( m_i \) of individuals (i.e., a collection of positive numbers, \((m_i)_{i \in \mathcal{I}}\), such that \( \sum_{i \in \mathcal{I}} m_i = 1 \)), with life-time utility defined by:

\[
V_{t,i}^t = u(c_{t,i}^t, c_{t+1,i}^t, h_{t+1,i}^i) + \theta_i(1 + n)V_{t+1,i}^t,
\]

for all \( t \geq 0 \), where \((c_{t,i}^t, c_{t+1,i}^t)\) are age-contingent consumptions, \( h_{t+1,i}^i \) is the housing stock acquired by the household in young age. Preferences satisfy the following assumption.

**Assumption 1.** The (inter-generational) discount factors satisfy \( \theta_i(1 + n) < 1 \) for all \( i \in \mathcal{I} \) and the utility function belongs to the CES class, i.e., for some positive parameters, \((\chi^y, \chi^o, \chi^h, \gamma)\), with \( \sum_{j=y,o,h} \chi^j = 1 \),

\[
U(c^{y,i}, c^{o,i}, h^i) = \begin{cases} 
\left[ \chi^y(c^{y,i})^{\frac{\gamma-1}{\gamma}} + \chi^o(c^{o,i})^{\frac{\gamma-1}{\gamma}} + \chi^h(h^i)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma-1}} & \text{if } \gamma \neq 1, \\
\chi^y \log c^{y,i} + \chi^o \log c^{o,i} + \chi^h \log h^i & \text{otherwise.} 
\end{cases}
\]

The upper bound on the discount rates, \( \theta_i \), insures convergence of each dynasty’s
long-run utility function. Importantly, the CES hypothesis guarantees normality and the "law of demand" (in particular, the demand for housing is decreasing in its own price). Notice that the parameter $\gamma$ represents the elasticity of substitution between each pair of goods.

We assume perfect financial markets allowing for unlimited lending and borrowing and, for simplicity, we ignore the housing rental market\(^7\). Any household born at time $t$ acquires land and residential property when young, enjoys the housing services generated by it, resells the property when old, leaves some bequests to the offsprings and receive a lump-sum transfer (tax) from the government. The latter represents the rebate to the households of the revenue that the government obtains by selling the new land made available for construction. Due to the absence of financial frictions, we can write the households' inter-temporal budget constraints as:

$$c_t^i + \frac{c_{t+1}^i}{1 + r_{t+1}} + \pi_t h_{t+1}^i + \frac{(1 + n)b_{t+1}^i}{1 + r_{t+1}} = W_t + b_t^i + \frac{\tau_{t+1}}{1 + r_{t+1}}.$$  \hfill (2)

where $b_t^i$ denotes bequests, $W_t$ is the time-$t$ real wage, $\tau_{t+1}$ is the net transfer from the government (assumed to be individual-independent), and

$$\pi_t = q_t^h - (1 - \delta)q_{t+1}^h/(1 + r_{t+1})$$

denotes the user cost of housing, \textit{i.e.}, the cost of a unit of housing net of the present value of selling the same un-depreciated unit the next period and $\delta \in (0, 1)$ is the housing depreciation rate. Notice that the above representation of the inter temporal budget constraint takes into account the absence of arbitrage opportunities related to the assets included in households' portfolios, such as bonds, housing and land. Since

\(^7\)Households are assumed to derive some, however small, satisfaction from ownership, so that, in the absence of market frictions, ownership is a dominant choice relative to renting. The average home ownership rate across OECD countries is approximately 67%. At the top of the distribution are countries like Greece (87%) and Spain with high rates of more than 80%; while at the bottom countries like Germany (43%), Japan (36%) and Switzerland (35%). Data are from the OECD (2012).
parental altruism is one-sided, we rule out forced gifts from children to parents, and impose the non-negativity constraint

\[ b_{t+1}^i \geq 0. \]

Denoting with \( u_{j,t}^i \) the partial derivative of \( u(c_{t}^{i,t}, c_{t+1}^{i,t}, h_{t+1}^{i,t}) \) with respect to the \( j \)-th argument and with \( q^z \) the price of a unit of land, the first order conditions characterizing the young household’s optimal consumption choice subject to the budget constraint, (2), are

\[
\begin{align*}
    u_{1,t}^i &= (1 + r_{t+1})u_{2,t}^i, \\
    u_{1,t}^i \pi_t &= u_{3,t}^i, \\
    u_{2,t}^i &\geq \theta_t u_{1,t+1}^i,
\end{align*}
\]

(3)
together with the complementary slackness condition

\[ b_{t+1}^i (u_{2,t}^i - \theta_t u_{1,t+1}^i) = 0 \]

(4)
and the no-arbitrage condition related to land investment,

\[
q_t^z = (q_h^h f_{h,z}(k_{t+1}^h, z_{t+1}) + q_{t+1}^z)/(1 + r_{t+1}),
\]

(5)
where \( q^z \) denotes the unit price of land. From the above first order conditions we derive the time-\( t \) households saving, \( s_t^i \), the demand for housing, \( h_{t+1}^i \), and the supply of bequests, \( b_{t+1}^i \). These are specified as:

\[
\begin{align*}
    s_t^i &= S^i(\pi_t, r_{t+1}, W_t + b_t^i + \tau_{t+1}/(1 + r_{t+1})), \\
    h_{t+1}^i &= H_{d,i}^h(\pi_t, r_{t+1}, W_t + b_t^i + \tau_{t+1}/(1 + r_{t+1})), \\
    b_{t+1}^i &= B^i(\pi_t, r_{t+1}, W_t + b_t^i + \tau_{t+1}/(1 + r_{t+1})).
\end{align*}
\]
3.2 Production

Manufacturing output \( (Y^m) \) can be consumed or used as capital in both sectors and construction output \( (Y^h) \) corresponds to investment in new housing. Technology is represented by the production functions

\[
Y^h_t = F_h(K^h_t, A^h_t L^h_t, Z_t), \quad Y^m_t = F_m(K^m_t, A^m_t L^m_t),
\]

where, for \( j = h, m \), \( K^j \) and \( L^j \) are the amounts of capital and labor employed in the two sectors, \( A^j \) is a labor-augmenting technological level and \( Z \) represents the amount of land available for construction. We let this factor be time dependent because we assume that the government, or some other public authority, may decide to change the amount of land available for construction to enact specific policies or respond to demographic variables.

For analytical convenience, we restrict \( F_h \) in the class of CES productions functions and we impose more general standard restrictions on \( F_m \).

**Assumption 2.** Both production functions display constant returns to scale, are strictly increasing, strictly concave, continuously differentiable and such that

\[
\lim_{K^j/A^jL^j \to 0} F_{j,2}/F_{j,1} = 0, \quad \lim_{K^j/A^jL^j \to \infty} F_{j,2}/F_{j,1} = \infty,
\]

\[
\lim_{K^m \to 0} F_{m,1} > \min_{i \in \mathcal{I}} 1/\theta_i > \lim_{K^m \to \infty} F_{m,1},
\]

where \( F_{j,i} \) denote the partial derivatives of \( F_j \) with respect to the \( i \)-th argument. More specifically, \( F_h \) belongs to the class of CES productions functions, i.e., for given positive values, \( (\alpha, \eta, \sigma^h) \), such that \( \alpha \in (0, 1) \), \( \eta \in [0, 1) \) and \( \alpha + \eta < 1 \),

\[
F_h(\cdot) = \begin{cases} 
\left( \alpha(K^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \alpha - \eta)(A^h L^h)^{\frac{\sigma^h - 1}{\sigma^h}} + \eta Z^{\frac{\sigma^h - 1}{1 - \sigma^h}} \right)^{\frac{\sigma^h}{1 - \sigma^h}} & \text{if } \sigma^h \neq 1, \\
(K^h)^\alpha (A^h L^h)^{1 - \alpha - \eta} Z^\eta & \text{otherwise},
\end{cases}
\]
The boundary restrictions on $F_m$ simplify the exposition and guarantee the existence of a meaningful steady state. Importantly, the CES representation for $F_h$ implies that the profit maximizing values of the capital-labor ratios under factor price equalization are independent of the amount of land, $Z$, which is a very useful simplification.

It is convenient to provide a more compact notation by normalizing variables with respect to the level of labor efficiency. In particular, for $j = h, m$, let $k^j = K^j/A^jL^j$ be the sector-specific capital intensities, $z = Z/A^hL^h$ the available land per unit of labor efficiency in construction and $y^j$ the sector-specific labor productivities in efficiency units. By constant returns to scale, we can write

$$y^h = F_h(k^h, 1, z) \equiv f_h(k^h, z) \quad y^m = F_m(k^m, 1) \equiv f_m(k^m),$$

where $f_j$ denotes the intensive-form production functions.

We assume that firms in construction and manufacturing are price-takers and labor and capital are fully mobile across the two sectors. Now let $a = A^m/A^h$ be the labor augmenting efficiency in manufacturing relative to construction (henceforth relative productivity); $w = W/A^m$ the wage rate per units of efficiency in the manufacturing sector; and denote with $f_{h,k}$ and $f_{h,z}$ the partial derivatives of $f_h(k^k, z)$ with respect to $k^h$ and $z$, respectively. Then, letting $q^h$ be the price of a unit of new housing, profit maximization at any interior solution (i.e., strictly positive $(k^h_t, L^h_t, y^h_t)$ for $j = h, m$) implies

$$1 + r = f_{m,k} = q^h f_{h,k}, \quad (9)$$

$$w = f_m - k^m f_{m,k} = (q^h/a)[f_h - k^h f_{h,k} - z f_{h,z}]. \quad (10)$$

By the properties of the production functions, the above two equations provide a well defined map from $(r, a)$ into $(w, k^h, k^m)$ for all $a > 0$ and $r$ in a suitable interval. In particular, we can state the following.
Proposition 1. For any given strictly positive \((r, a)\), with \(r \in \mathcal{A} = [r, \bar{r}]\), there is a unique solution, \((w(r), k^h(r, a), k^m(r))\), to equations (9), (10), as a differentiable function of \((r, a)\), such that \(w, k^h\) and \(k^m\) are all decreasing in \(r\) and

\[
\frac{\partial k^h}{\partial a} = \sigma^h. \tag{11}
\]

A sketch of the proof is the following. Since \(f_{m,k}(k^m)\) is decreasing in \(k^m\), the marginal productivity of capital in manufacturing, \(f_{m,k}(k^m)\), is locally invertible in some interval \(\mathcal{A} = [r, \bar{r}]\). Then, by the profit maximization condition, \(1 + r = f_{m,k}(k^m)\), we obtain \(k^m = k^m(r)\), with \(k^m(.)\) decreasing in \(r\) and such that \(k^m(r) = \infty, k^m(\bar{r}) = 0\). Existence of the function \(w(r)\) follows from (9) and \(k^m = k^m(r)\), which proves also \(w'(r) < 0\). Now observe that, since the CES production function is homothetic, the ratio between the marginal product of labor and the marginal product of capital in the construction sector depends on \(k^h\) only and

\[
\frac{f_h(k^h, z) - k^h f_{h,k}(k^h, z) - z f_{h,z}(k^h, z)}{f_{h,k}(k^h, z)} = \omega_h(k^h),
\]

where \(\omega_h(k^h)\) is increasing and invertible in \(\mathbb{R}_+\). By factor price equalization,

\[
\omega_h(k^h) = a w(r)/r.
\]

and, then, \(k^h = \omega_h^{-1}(aw(r)/(1+r)) \equiv k^h(r, a)\). By taking the derivative, we obtain (11). Notice that \((w, k^h, k^m)\) are independent of \(z\), a consequence of the CES specification of \(F_h\). By equations (9), (10) and the above findings we can write the housing price as

\[
q^h = \frac{(1 + r)}{f_{h,k}(k^h(r, a), z)} \tag{12}
\]

for all \((r, z, a)\) in \(\mathcal{A} \times \mathbb{R}^2_+\). The way in which \(r, z\) and \(a\) affect \(q^h\) will be discussed in a
3.3 Equilibrium

For simplicity, we take \( A^h = 1 \) (this restriction is clearly a harmless normalization with nomothetic preference) and assume that the government adjusts the amount of land available for construction to the changing population in such a way as to generate a time-invariant land per unit of labor. In particular, we restrict the land policy as follows

\[
Z_t = \xi L_t. \tag{13}
\]

This assumption allows us to normalize the equilibrium variables and have a dynamic system that is neutral with respect to the level of population. Although this implies that the government is unable to change the rate at which land grows or decays relative to the population level, there is still room for examining the effects of changing the relative size of available land.

Let \( K, L \) and \( H \) be the total stock of business capital, labor and housing. Full employment implies

\[
L_t = L_t^h + L_t^m, \tag{14}
\]

\[
K_t = K_t^h + K_t^m, \tag{15}
\]

\[
H_{t+1} = Y_t^h + (1 - \delta)H_t. \tag{16}
\]

Now we express all equilibrium restrictions and the relevant variables in per-capita units. In particular, letting

\[
k_t = K_t / L_t, \quad h_t = H_t / L_t, \quad \lambda_t = L_t^h / L_t,
\]
we replace the full employment conditions (14), (15) and (16) with

\[ k_t = \lambda_t k_t^h + (1 - \lambda_t) a_t k_t^m, \]  
\[ (1 + n) h_{t+1} = \lambda_t y_t^h + (1 - \delta) h_t \]

and \( \lambda_t \in [0, 1] \). To close the model we define the per capita aggregate saving and housing demand

\[ s_t \equiv \sum_i m_i S_i^t(\pi_t, r_{t+1}, W_t + b_t^i + \tau_{t+1}/(1 + r_{t+1})), \]
\[ h_{t+1}^d \equiv \sum_i m_i H_{t+1}^d(\pi_t, r_{t+1}, W_t + b_t^i + \tau_{t+1}/(1 + r_{t+1}))/ (1 + n), \]

and impose market clearing in the housing and capital markets, i.e.,

\[ h_{t+1} = h_{t+1}^d, \]
\[ s_t = (1 + n)(k_{t+1} + q_t^h h_{t+1} + q_t^\xi). \]

By profit maximization and factor price equalization,

\[ k_t^m = k^m(r_t), \quad k_t^h = k^h(r_t, a_t), \quad W_t = a_t w(r_t), \quad q_t^h = (1 + r_t)/f_{h,k}(k_t^h, z_t), \]

where, because of the land policy (13),

\[ z_t = \xi/\lambda_t. \]

Finally, by the government balanced budget condition

\[ \tau_t = q_t^\xi (1 + n)n\xi. \]

Remember that the functions in (21) are positive and continuous and guarantee an
interior allocation of factors across sectors for \( r \) in \( \mathcal{A} = [r, \bar{r}] \) and all \( a > 0 \). Then, an interior competitive equilibrium is a positive sequence,

\[
\{k_t, k^h_t, k^m_t, \lambda_t, h_t, b_{t+1}, r_{t+1}, w_t, q^h_t, q^z_t \}_{t=0}^{\infty},
\]

with \( r_{t+1} \in \mathcal{A} \) and \( \pi_t = q^h_t - (1 - \delta)q^h_{t+1}/(1 + r_{t+1}) > 0 \) for all \( t \geq 0 \), satisfying the optimality conditions (3)-(5), the market clearing conditions (17), (18), (20), (19), the factor price equalization conditions (21), the land policy (22) and the balanced budget condition (23), for all \( t \geq 0 \), for a given sequence of relative productivities, \( \{a_t\}_{t=0}^{\infty} \), and some initial conditions, \((k_0, h_0, b_0, r_0) > 0\).

4 Positive Bequests Steady States

From now on we concentrate on a steady state equilibrium with two types of households. In particular, let \( \mathcal{I} = \{p, r\} \) and \( \theta_r > \theta_p \). We say that household type \( r \) is rich and household type \( p \) is poor, although we could as well say that the former is more altruistic than the latter with respect to their own children.

It is clear from the first order conditions (3)-(4) that we may have two type of steady states. One is such that \( r \leq (1 - \theta_r)/\theta_r \) and no individual leaves any bequests, so that the resulting equilibrium is equivalent to the one that would take place in a canonical overlapping generations economy. The other is such that the rich individuals leave positive bequests, whereas the poor leave zero bequests at any time. In this case we have

\[
r = (1 - \theta_r)/\theta_r \equiv r^* < (1 - \theta_p)/\theta_p.
\]

We refer to the first type of equilibrium as a zero bequests steady state (ZBSS) and the second type a positive bequests steady state (PBSS). In what follows, we focus exclusively on PBSS, mostly because this type of equilibrium allows for a sharp char-
acterization of intra-generational inequality. It is understood that a PBSS is assumed to be interior, i.e., to imply that both sectors are active. Notice that the existence of such steady state is guaranteed by assumption 2. In particular, by the boundary restrictions on $f_m$, there exists a unique value of the wage rate per unit of efficiency, $w^* \equiv w(r^*)$, and the wage rate per unit of efficiency in manufacturing is $w = w^*$ at a PBSS, whereas $w \geq w^*$ at a ZBSS. Hence, in a PBSS, the real interest rate and the wage rate in units of efficiency do not depend on the relative productivity parameter, $a$, and, since $r$, $w$, $k^m$ and $k^h$ are set by the parameters $r^*$ and $a$, the remaining equilibrium variables to be determined by the steady state equilibrium conditions are the average capital stock, $k$, the share of labor in construction, $\lambda$, the housing stock, $h$, the asset prices, $q^h$, $q^z$, and the steady state bequest, $b$. Later on in this section we will show that these variables can be derived from a reduced-form characterization in terms of just two equations and two unknowns. The two equations are the market clearing condition in the housing market and the capital market equilibrium; the two unknowns are the rich households’ bequests, $b$, and the value of housing wealth, $v = q^h h$. In particular, the next two sections will be devoted to the derivation of a mapping from $(v, a)$ into $(\lambda, q^h, q^z, h^d, s, k)$, where $h^d$ and $s$ are the aggregate demand for housing and the aggregate saving.

4.1 Asset Prices

By the land policy (22), the asset prices and the user cost of housing are a function of $a$ and $\lambda$ through the equations

$$q^h = \frac{1 + r^*}{f_{h,k}(k^h, z)}, \quad q^z = \frac{q^h f_{h,z}(k^h, z)}{r^*}, \quad \pi = q^h \left( \frac{\delta + r^*}{1 + r^*} \right), \quad (24)$$

where $k^h = k^h(r^*, a)$ and $z = \xi/\lambda$. Notice that, since $r^* > n \geq -\delta$, the user cost of housing is strictly positive.
A first step in the reduced form characterization is to derive the share of labor in construction, \( \lambda \), and the average capital-labor ratio, \( k \), as functions of \((v,a)\). In particular, by the land policy, (22), and the definition of \( q^h \) provided in (24), \( \lambda \) is determined implicitly by \( v \) (and \( a \)) through the following equation

\[
\frac{\lambda f_h(k^h(r^*,a),\xi/\lambda)}{f_{h,k}(k^h(r^*,a),\xi/\lambda)} = \left( \frac{\delta + n}{1 + r^*} \right) v.
\]

(25)

It can be shown that the left hand side of the above equation is increasing in \( \lambda \), so that we can make the following claim.

**Proposition 2.** For all \( v \in [0,v^m(a)] \), with

\[
v^m(a) = \frac{f_{h,k}(k^h(r^*,a),\xi)}{f_h(k^h(r^*,a),\xi)} \left( \frac{1 + r^*}{\delta + n} \right),
\]

there exists a value \( \lambda(v,a) \in [0,1] \) solving equation (25), such that \( \lambda_v(v,a) > 0 \), \( \lambda_a(v,a) < 0 \) and \( \lambda(0,a) = 0 \), \( \lambda(v^m(a),a) = 1 \).

Quite intuitively, a rise in housing wealth generates a reallocation of labor toward the construction sector and a rise in \( a \) a reallocation of labor away from this sector. It is worth noticing that these are only partial effects. In equilibrium, housing wealth, \( v \), is affected by \( a \), so that a rise in productivity in the manufacturing sector may shift labor to the construction sector if this has a positive and large enough effect on \( v \).

Having defined \( \lambda \) as a function of \((v,a)\), we can derive asset prices as functions of \((v,a)\) as well, i.e.,

\[
q^h(v,a) \equiv (1 + r^*)/f_{h,k}(k^h(r^*,a),\xi/\lambda(v,a)),
\]

(26)

\[
q^*(v,a) \equiv q^h(v,a)f_{h,z}(k^h(r^*,a),\xi/\lambda(v,a))/r^*.
\]

(27)

In order to study the comparative statics of the model, it is important to understand how \( v \) and \( a \) affect the price functions, \( q^j(v,a) \), for \( j = h, z \). To this end, we first observe
that the effect of a higher housing wealth on asset prices is unambiguously positive. In fact, since \( \lambda_v(v, a) > 0 \), a rise in housing wealth generates a larger labor share in construction, reduces the land-to-labor ratio in the construction sector and, then, it reduces the marginal product of capital and it increases the marginal product of land. Since \( r \) is pinned down by \( \theta_r \), all asset prices increase.

On the other hand, evaluating the impact of \( a \) on asset prices for given \( v \) is more complicated. When \( \eta = 0 \) (unproductive land) \( q^h \) is independent of \( v \) and, by differentiation of \( q^h \) with respect to \( a \), we derive \( \partial q^h / \partial a > 0 \). Hence, similarly to the Baumol prediction, a rise in productivity in manufacturing would generate a rise in the relative price of the stagnant output price. In more general cases, \( a \) affects \( \lambda \) (for any given \( v \)) and the latter affects the land-labor ratio, \( z \). In particular, since \( z = \xi / \lambda(v, a) \), \( \lambda_a < 0 \) and the CES specification implies \( f_{h,k,z} > 0 \), \( q^h \) and \( q^z \) are increasing in \( a \) for given \( z \) and decreasing in \( z \) for given \( a \). Hence, for given \( v \), a rise in \( a \) has a positive direct effect on asset prices because of a higher capital labor ratio in the construction sector, but it has a negative effect through a higher land-labor ratio induced by a lower share of labor in this sector. In particular, define the sector-specific factor shares

\[
S^j_k = f_{j,k}k^j / f_j, \quad S^h_z = f_{h,z}z / f_h, \quad S^h_l = 1 - S^h_k - S^h_z.
\]

The following proposition gives a more precise statement about these effects in terms of elasticities of asset prices with respect to \( v \) and \( a \). To simplify the notation, from now on we use a "hat" to denote partial elasticities, i.e., letting \( h(x) \) be any differentiable function in \( \mathbb{R}^n \), we let \( \hat{h}_{x_i} = \partial \log h(x) / \partial \log x_i \).

**Proposition 3.** The partial elasticities of \( q^h(v, a) \) and \( q^z(v, a) \) are

\[
\hat{q}^h_v(v, a) = \frac{S^h_z(q^h y^h / (1 + r^*))^{\sigma_h-1}}{\sigma^h(1 - S^h_z) + S^h_z(q^h y^h / (1 + r^*))^{\sigma_h-1}}, \quad \hat{q}^z_v(v, a) = \frac{1 - (1 - \sigma^h)\hat{q}^h_v}{\sigma^h} \tag{28}
\]
\begin{equation}
\hat{q}^h(a, v) = (1 - S^h_k) - ((1 - S^h_k) + \sigma^h S^h_k) \hat{q}^h_v, \quad \hat{q}^z(a, v) = - \left( \frac{1 - \sigma^h}{\sigma^h} \right) \hat{q}^h_v. \tag{29}
\end{equation}

The basic conclusions that we derive from the above proposition are that the partial elasticities of $q^h$ and $q^z$ with respect to $v$ are positive, with $\hat{q}^h_v$ in $[0, 1]$ and $\hat{q}^z_v \geq \hat{q}^h_v$, that $q^z$ is increasing in $a$ if and only if $\sigma^h \geq 1$ and that

$$\hat{q}^h_a \geq 0 \iff \hat{q}^h_v \leq \frac{(1 - S^h_k)}{(1 - S^h_k) + \sigma^h S^h_k}.$$ 

Evidently, the above condition holds when $S^h_z$ is not too large or $S^h_k$ is sufficiently close to zero. However, more general conditions may also generate a positive relation between $q^h$ and $a$. In the specific case of a Cobb-Douglas production function in the construction sector,

$$\hat{q}^h(v, a) = S^h_z, \quad \hat{q}^z(v, a) = 1, \quad \hat{q}^h(v, a) = S^h_i, \quad \hat{q}^z(v, a) = 0. \tag{30}$$

### 4.2 Housing Demand and Savings Functions

The age-contingent consumptions and housing demand at steady states are the solutions to the maximization of $u(c^{y,i}, c^{o,i}, h^i)$ subject to the (present value) budget constraint

\begin{equation}
c^{y,i} + \frac{c^{o,i}}{1 + r^*} + \pi h^i = aw^* + \frac{(r^* - n)b^i}{1 + r^*} + \frac{\tau}{1 + r^*} \equiv I^i, \tag{31}
\end{equation}

where

\begin{equation}
\pi \equiv q^h(v, a)(\delta + r^*)/(1 + r^*) \equiv \pi(v, a), \tag{32}
\end{equation}

\begin{equation}
\tau = q^z(1 + n)n_\xi \equiv \tau(v, a). \tag{33}
\end{equation}
Notice that the CES utility specification has the important implication that individuals’ demand are homothetic, they satisfy the law of demand \((i.e.,\ housing\ and\ age-contingent\ consumptions\ are\ decreasing\ in\ their\ own\ price)\) and the elasticities of consumption and housing demand with respect to the user cost of housing, \(i.e.,\ \hat{c}_y^i, \hat{c}_o^i, \hat{h}_\pi^i\), are independent of individuals’ wealth. In particular, we can write

\[c_y^i = \phi_y^i I_i^i, \quad \frac{1}{1 + r^*} c_o^i = \phi_o^i I_i^i, \quad \pi h_i^i = \phi_h^i I_i^i,\]

for all \(i = p, r\), where, for \(j = y, o, h\),

\[
\phi_y^i = \frac{(\chi_y^y)^\gamma}{(\chi_y^y)^\gamma + (\chi_o^o)^\gamma(1 + r^*)^{-1} + (\chi_h^h)^\gamma \pi^{1-\gamma}}, \\
\phi_o^i = \frac{(\chi_o^o)^\gamma (1 + r^*)^{-1}}{\chi_y^y)^\gamma + (\chi_o^o)^\gamma(1 + r^*)^{-1} + (\chi_h^h)^\gamma \pi^{1-\gamma}}, \\
\phi_h^i = \frac{(\chi_h^h)^\gamma \pi^{1-\gamma}}{\chi_y^y)^\gamma + (\chi_o^o)^\gamma(1 + r^*)^{-1} + (\chi_h^h)^\gamma \pi^{1-\gamma}}.
\]

define the expenditure shares, all positive and continuous functions of \(\pi\), and such that \(\sum_{j=y,o,h} \phi_j^i = 1\). Notice that the parameter \(\gamma\) represents the elasticity of substitution between each pair of goods. From now on we will concentrate on the housing demand and the saving functions,

\[h_i^i = h_i^i(\pi, aw^*, b_i^i, \tau), \quad s_i^i(\pi, aw^*, b_i^i, \tau). \tag{34}\]

Recalling that \(r^* > n\) and that, by assumption 1, consumption and housing are normal goods, we conclude that \(h^r\) and \(s^r\) are increasing in bequests and that the bequest motive provides more wealth \((i.e.,\ more\ consumption\ opportunities)\) to the \(r\)-type individual relative to the \(p\)-type at the PBSS. Since \(b^r > b^p = 0\), we drop the index \(r\) on the rich households’ bequests, and identify \(b^r\) with the variable \(b\). An important property of the model that will be exploited in the next section is that, given \(v\) and \(a\),
the impact of a rising $b$ on saving is greater than the impact on the value of housing demand.

**Proposition 4.** For all $b > 0$, and for given $(v,a)$, $\partial s^r/\partial b > \partial q^h h^r / \partial b > 0$.

Notice that this property holds more generally than for CES utility representations and it just requires normality and $r^* > n \geq 0$.

A final observation is that the CES specification allows for an easy aggregation of individuals’ elasticities. Namely, let

\[
\begin{align*}
    h^d(b,v,a) & \equiv \sum_i m_i h^i(\pi(v,a), aw^* b^i, \tau(v,a))/(1 + n), \\
    s(b,v,a) & \equiv \sum_i m_i s^i(\pi(v,a), aw^* b^i, \tau(v,a))
\end{align*}
\] (35)

be the aggregate demand for housing and aggregate saving. Then, the elasticities of these functions with respect to $\pi$ are

\[
\begin{align*}
    \hat{h}^d_{\pi} = \frac{1}{1 + n} \sum_i \left( \frac{m_i h^i}{h^d} \right) \hat{h}^i_{\pi}, \\
    \hat{s}_{\pi} = \sum_i \left( \frac{m_i s^i}{s} \right) \hat{s}^i_{\pi},
\end{align*}
\]

and, from the CES specification we derive,

\[
1 + \hat{h}^d_{\pi} = (1 - \gamma)(1 - \phi^h), \quad \hat{s}_{\pi} = (1 - \gamma)\phi^h c^y/s,
\] (37)

where $\gamma$ is the elasticity of substitution between goods and $c^y = \sum_i m_i c^{y,i}$ is the aggregate young age consumption. Hence, if $\gamma < 1$, a rise in the user cost of housing generates, *ceteris paribus*, a rise in the demand for housing wealth saving and $\gamma = 1$ implies $\hat{h}^d_{\pi} = -1, \hat{s}_{\pi} = 0$. Observe that this implies a straightforward characterization of the aggregate housing demand elasticities with respect to $v$ and $a$. In particular, ignoring the impact of these variables on government transfers, $\tau$ (a small impact when
\[ n \text{ is close to zero), we have } \]
\[ \hat{h}_{j}^{d} = \hat{h}_{\pi}^{d} q_{j}, \quad \hat{s}_{j} = \hat{s}_{\pi} q_{j} \]
for \( j = v, a \). Then, recalling (37), the above imply that the effects of a change in \( v \) or \( a \) on the value of housing demand and saving is critically determined by \( \gamma \). Namely,
\[ \frac{\partial \log q_{h}^{d}}{\partial \log v} = \hat{q}_{v}^{h} + \hat{h}_{v}^{d} = \hat{q}_{v}^{h}(1 - \gamma)\phi^{h}, \quad \frac{\partial \log q_{h}^{d}}{\partial \log a} = \hat{q}_{a}^{h} + \hat{h}_{a}^{d} = \hat{q}_{a}^{h}(1 - \gamma)\phi^{h}, \]
\[ \hat{s}_{v} = \hat{q}_{v}^{h}(1 - \gamma)\phi^{h} c^{v}/s, \quad \hat{s}_{a} = \hat{q}_{a}^{h}(1 - \gamma)\phi^{h} c^{v}/s. \]
In other words, ignoring \( \tau \), the value of housing demand and saving are unaffected by \( v \) and \( a \) when \( \gamma = 1 \).

### 4.3 Reduced Form Characterization and Regular Intersections

Now we are in a position to derive a reduced form characterization of a PBSS in terms of the two variables, \( b \) and \( v \), and the two equations defining, respectively, market clearing the market for housing and a capital market equilibrium. First, we notice that, by the full employment condition, (17), and the supply of housing, (25), we can derive the average capital-labor ratio and the value of land as
\[ k = ak^{m} - \Delta v, \quad q^{z}\xi = (\delta + n)S_{z}^{h}v/r^{*}, \]
where
\[ \Delta = \left( \frac{\delta + n}{1 + r^{*}} \right) S_{k}^{h} \left( \frac{ak^{m} - k^{h}}{k^{h}} \right). \]
Observe that the sign of the term \( \Delta \) depends on the difference between the capital intensities in manufacturing and construction, which is, in turn, a function of factor
shares. In particular,
\[
\frac{ak^m - k^h}{k^h} = \frac{1}{1 - S_k^h} \left( S_k^h (S_k^m - S_k^h) - S_k^h (1 - S_k^m) \right) .
\] (40)

The above will be called the *capital intensity differential* across the two sectors and it will be shown to be a key variable for the comparative statics of the model. It is worth mentioning that \( S_k^m > S_k^h \) if and only if the housing price, \( q^h \), is decreasing in the interest rate, \( r \). Evidently, \( \Delta \) is, in general, a function of the pair \((v, a)\), through the land policy and the dependence of \( \lambda \) on \((v, a)\). However, when \( \sigma^h = 1 \) (\( F_h \) Cobb-Douglas), \( \Delta \) is parametric, as, in this case, \( S_k^h = \alpha \) and \( S_k^z = \eta \).

Finally, from (38), (39) and the market clearing conditions (19), (20) (for housing and capital markets) at steady state, we derive the excess demand for housing wealth and the excess supply of saving (over investment) as

\[
G_d(b, v, a) \equiv q^h(v, a) h^d(b, v, a) - v,
\]
\[
G_s(b, v, a) \equiv \frac{1}{1 + n} s(b, v, a) - ak^m - (1 - \Delta + (\delta + n) S_k^h / r^*) v.
\]

Then, the steady state specification of the equilibrium conditions (19) and (20) define the the reduced form equilibrium steady state conditions for any given \( a \) as

\[
G_d(b, v, a) = 0, \quad (41)
\]
\[
G_s(b, v, a) = 0, \quad (42)
\]

and a PBSS is a positive pair, \((b^*(a), v^*(a))\), such that

\[
0 = G_d(b^*(a), v^*(a), a) = G_s(b^*(a), v^*(a), a). \quad (43)
\]

We establish now a set of conditions for which the above characterization provides
a unique equilibrium as an intersection between a demand and a supply of housing wealth with some appropriate crossing conditions. Namely, let $v^d(b,a)$ and $v^s(b,a)$ be the solutions for $v$ to (41) and (42), respectively, for a given pair $(b,a)$. Assuming that these exist and are well defined, we say that $v^d$ is the demand and $v^s$ the supply of housing wealth. Intuitively, $v^d$ is the households' real expenditure for the stock of available housing and $v^s$ defines the amount of housing wealth that is consistent with a capital market equilibrium, i.e., with the available amount of savings, business capital and land value.

General conditions for a unique equilibrium and well defined demand and supply of housing wealth are not easy to derive. Hence, we follow the strategy of concentrating on a subset of economies for which these conditions are satisfied, which we consider particularly relevant based on theoretical and empirical motivations. Namely, these economies are characterized by the following restriction on capital intensities

$$1 + \delta S^h_z/r^* > \Delta > \delta S^h_z/r^*.$$ 

(44)

and parameter configurations sufficiently close to a benchmark model that we denominate as unit-elastic model, i.e., a model characterized by (i) unit own price elasticity of housing demand ($\gamma = 1$), (ii) Cobb-Douglas technology in construction ($\sigma^h = 1$) and (iii) zero population growth, i.e., $n = 0$.

In particular, for the unit-elastic model, we show that, if a PBSS exists, the demand and supply functions, $v^s(b,a)$, $v^d(b,a)$, are (locally) well defined and have a unique intersection such that $v^s$ is steeper than $v^d$, a property that we label as regular intersection, and in the next section we will use this property to provide comparative statics results for small perturbations of the unit-elastic economy and show that a regular intersection is instrumental for generating a housing cost disease under some additional conditions.
To see why these properties imply regular intersections, we observe that, by inspection of the aggregate demand and saving functions, \( h(b, v, a), s(b, v, a) \), the impact of \( v \) on \( G^d \) and \( G^s \) are measured as

\[
G^d_v = q^h h^d (1 + \hat{h}_n^d) + h^d \tau_v - 1,
\]
\[
G^s_v = \frac{s}{1 + n} \left( \frac{q^h}{q^n} \right) \hat{s}_\pi + \frac{1}{1 + n} s \tau_v - 1 + \Delta + \Delta_v v - \hat{q}_v (\delta + n) S^h_z / r^*,
\]

where \( \tau_v \) is the partial derivative of \( \tau \) with respect to \( v \). Now assume \( \sigma^h = \gamma = 1 \) and \( n = 0 \). Recalling that \( \sigma^h = 1 \) implies \( \Delta_v = 0 \), \( \hat{q}_v = 1 \), \( \gamma = 1 \) implies \( \hat{h}_n^d = -1 \), \( \hat{s}_\pi = 0 \), and that \( \tau = 0 \) for \( n = 0 \), the unit-elastic economy is such that

\[
G^d_v = -1, \quad G^s_v = -1 + \Delta - (\delta + n) S^h_z / r^*.
\]

Then, for the unit-elastic economy, we have \( 1 = -G^d_v > -G^s_v > 0 \) if and only if (44) is verified. Now remember that, by proposition 4, the effect of a rising \( b \) on saving is higher than the effect on housing demand. Then, we have \( G^s_b > G^d_b > 0 \) and, then, the above inequalities imply

\[
v^s_b - \frac{G^s_b}{-G^s_v} > \frac{G^d_b}{-G^d_v} = v^d_b.
\]

These findings are summarized in the following proposition.

**Proposition 5.** Consider a unit-elastic economy (\( \gamma = \sigma^h = 1, n = 0 \)) and satisfying condition (44)). Then, a PBSS is characterized by a regular intersection, i.e.,

\[
v^s_b(b, a) > v^d_b(b, a) > 0.
\]  

The motivations for concentrating on economies characterized by the capital intensity restriction (44) and close enough to the unit-elastic model are both theoretical and empirical. One reason to concentrate on this case is that, as it will be shown in the
next sections, the unit-elastic model represents a benchmark case such that changes in the relative labor productivity in manufacturing is neutral with respect to wealth-to-income ratios, average labor productivity and labor allocations across sectors. Hence, it is useful and instructive to look at the consequences of departing slightly from this parameter specification. More generally, assuming \( \sigma^h = 1 \) and \( n = 0 \) has clear advantages for the comparative statics of the model studied in the next section. The first restriction allows to ignore effects of \( v \) and \( a \) on \( \Delta \) and the second shuts down any effect on housing demand and saving from the government land policy.

Regarding the restrictions implied by (44), notice that this condition states that the manufacturing technology is more capital intensive than the construction technology and that the (relative) capital intensity differential is bounded by a term that depends crucially on the land share of income. In particular, (44) is equivalent to

\[
\left( \frac{1 + r^*}{r^*} \right) \left( \frac{r^* + \delta S^h_z}{\delta S^h_k} \right) > \frac{ak^m - k^h}{k^h} > \left( \frac{1 + r^*}{r^*} \right) \frac{S^h_z}{S^h_k},
\]

and, then, \( ak^m > k^h \) is an "almost" sufficient characterization of (44) for \( \delta \) and \( S^h_z \) close to zero. We should stress that the non-negativity of \( \Delta \) is more than a simplifying assumption. In the next section we will show that, together with the assumption \( \gamma < 1 \) (i.e., inelastic housing demand), this is a key restriction for generating the rising wealth-to-income ratios that are part of the housing cost disease defined in this paper. Furthermore, we observe that the upper bound on \( \Delta \) in (44) is a requirement for a meaningful steady state equilibrium when bequests are not too large. In appendix C (proposition 8) we show that \( \Delta < 1 + \delta S^h_z/r^* \) is, in fact, necessary for generating \( \lambda \leq 1 \) at equilibrium, i.e., non-negative shares of labor in the two sectors, under the assumption that aggregate saving falls short of the wage bill. This is a natural restriction, which is clearly satisfied when total bequests are not too large (and, a fortiori, in the canonical overlapping generations model).
From an empirical standpoint, there is consistent support for the assumptions that the construction sector is less capital intensive than manufacturing and little consensus on the most plausible values for the elasticity of substitution between capital and labor. In particular, Valentinyi and Herrendorf (2008) sets the capital share in manufacturing and construction at 0.4 and 0.2, respectively, confirming that $\Delta > 0$. Moreover, although the lower bound on $\Delta$ defined in (44) is higher than zero, this is defined in terms of two variables, $\delta$ and $S^h$, that are likely to be small. In particular, Neels (1982) provide an estimate of the output elasticity of land (i.e., our measure of $S^h$) between 0.03 and 0.06, and Davis and Heathcote (2005) set this value at 0.106 for their own calibration\(^8\). Regarding the elasticity of substitution between capital and labor, Piketty (2014) assumes this to be greater than one (an assumption that justifies his belief that a rising capital-output ratio generates a rising capital share), Chirinko (2008) provides a comprehensive survey of the available empirical evidence about the elasticity of substitution between capital and labor, as well as his own estimates, and puts the most likely range for $\sigma^j$ between 0.5 and 0.6. Finally, we observe that there exists strong evidence that housing demand responds by less than a percentage point to a one per cent rise in price is strong. In particular, Hanushek and Quigley (1980), Mayo (1981) and Ermisch et al. (1996) provide estimates of the housing demand elasticity in the range $(-0.8, -0.5)$.

\section{The Housing Cost Disease}

\subsection{Definition and Preliminary Observations}

In this section we study the comparative statics of the model at a PBSS, generated by exogenous productivity improvements, i.e., an increase in the value of the parameter

\footnote{This value is based on an unpublished 2000 memo from Dennis Duke to Paul L. Hsen entitled "Summary of the One-Family Construction Cost Study"}
a. In particular, we are going to test the validity of a housing cost disease, i.e., the possibility that this model may replicate some of the features of a two-sector economy studied by Baumol (1967), with construction of housing playing the role of the stagnant sector and manufacturing the role of the progressive sector.

We recall that the Baumol’s cost disease holds if, following a rise in productivity in the dynamic sector, (a) the relative price of the stagnant sector output increases (price increase), (b) the stagnant industry takes a rising share of nominal output (unbalanced growth) and (c) the changing composition of output across stagnant and dynamic industries reduces the effect of the productivity improvement on the average productivity (adverse effect on productivity). More formally, we restate the Baumol’s cost disease result in the present framework as follows. Define the average labor productivity

\[ y = (1 - \lambda)ay^m + \lambda q^h y^h = aw + (1 + r^*)k + r^*q^z \xi, \]  

(46)

and the wealth to income ratio,

\[ \beta = \frac{k + v + q^z \xi}{y} = \beta^k + \beta^h + \beta^z, \]

where \( \beta^k = k/y \) is the business capital, \( \beta^h = v/y \) the housing and \( \beta^z = q^z \xi/y \) the land components, and let \((q^{h*}(a), \beta^{h*}(a), \beta^*(a), b^*(a))\) be the steady state equilibrium values of the housing price, housing and total wealth to output ratio and bequests. Then, we say that there exists a housing cost disease if

\[(HA) \quad \partial q^{h*}/\partial a > 0 \quad \text{(housing appreciation)},\]

\[(IW) \quad \partial \beta^{h*}/\partial a > 0, \quad \partial \beta^*/\partial a > 0 \quad \text{(increasing wealth-to-income ratios)},\]

\[(IN) \quad \partial b^*/\partial a > 0 \quad \text{(increasing wealth inequality)},\]

\[(SP) \quad \partial y^*/\partial a < y^a/a \quad \text{(stagnant average productivity)}.\]

Observe that (IW) may be defined, alternatively, as an increase in the share of labor
employed in construction (i.e., as \( \lambda_a > 0 \)). However, this phenomenon and the one defined in (IW) are strictly related, since, by equation (25), we have derived \( \lambda = \lambda(v,a) \).

We impose, now, some key restrictions on technology and preferences that may be responsible for the housing cost disease. In particular, we consider small enough perturbations of the benchmark unit-elastic economy defined in the previous section. Such perturbations are sufficiently small to preserve the property of a regular intersection at steady state (expressed by (45)) and to keep \( \Delta \) in the range defined by (44), but allow for an inelastic demand for housing. These specifications are summarized in the following assumption.

**Assumption 3.** The economy is characterized by \( n = 0, \sigma^h = 1, \gamma \leq 1 \) and, at a PBSS, conditions (44) and (45) are verified.

Some motivations for the above assumptions have been discussed in the previous section. Here we stress that these are far from being necessary, as they only provide the most favorable environment for the housing cost disease phenomenon. In particular, a low price elasticity in the demand for the good produced in the stagnant sector is a key assumption in Baumol (1967)’s model. Similarly, by studying structural change in a multi sector growth model with identical production functions and a CES specification of individuals’ preferences, Ngai and Pissarides (2007) assumes a low (below one) elasticity of substitution across final goods in order to show that employment is gradually shifting to sectors with low TFP growth. In the present model, a rise in the efficiency of labor in manufacturing, \( a \), by rising the relative price of housing, generates a relatively small effect on the demand of this good. Then, the demand for housing wealth \( (v = qh) \) rises with \( a \) and generates a reallocation of production and labor to the less productive sector. The assumption \( \Delta > 0 \) has no analogous counterpart in the literature following the Baumol’s cost disease proposition. The role of this assumption is explained above. In the next sections we will provide some analytical about the total elasticities of bequests, housing wealth, asset prices, labor productivity and wealth-to
income ratios with respect to $a$, and we will show that assumption 3 is a sufficient condition for a housing cost disease.

5.2 Effects on Bequests and Housing Values

Here we show that the set of restrictions in assumption 3 are instrumental in generating more wealth inequality (i.e., higher bequests) as a result of a rising value of $a$. To see this, define the equilibrium elasticities of bequests and housing wealth with respect to $a$

$$\hat{b}_a^* = \frac{\partial \log b^*(a)}{\partial a}, \quad \hat{v}_a^* = \frac{\partial \log v^*(a)}{\partial a}.$$  

Then, by direct computation, we derive that, if a PBSS is characterized by a regular intersection, and $n = 0$, $\sigma^h = 1$, we have

$$\text{sign}\{\hat{b}_a^* - 1\} = \text{sign}\{\hat{v}_a^* - 1\} =$$

$$\text{sign} \left\{ (1 + \hat{h}_\pi)(S_l^h + S_z^h) v(1 - \Delta + \delta S_z^h / r^*) + s\hat{s}_\pi (S_l^h - S_z^h - 2(1 + \hat{h}_\pi) S_l^h S_z^h) > 0 \right\}. \quad (47)$$

Then, recalling (37) and (44), we can state the following proposition.

**Proposition 6.** Under assumption 3, we have $\hat{b}_a^* = \hat{v}_a^* = 1$ for $\gamma = 1$, and $\hat{b}_a^* > 1$ and $\hat{v}_a^* > 1$ under the sufficient condition

$$1 > \gamma > 1 - \frac{1}{1 - \phi^h} \left( \frac{S_l^h - S_z^h}{2S_l^h S_z^h} \right). \quad (47)$$

We may get some intuition about the above findings through a demand-supply type of analysis. What makes the effect of $a$ on bequests ambiguous is that a rise in $a$ produces an upward shift in both demand and supply of housing wealth. In fact, a higher $a$ generates higher wages and higher housing prices. In turn, since $\gamma < 1$, a housing price appreciation increases the expenditure share on housing and lowers the one on young age consumption, so that households save more. Now recall that
\( \hat{v}_b^s > \hat{v}_b^d \), i.e., the demand for housing wealth crosses the supply from above at \( b^* \). Then, a higher \( a \) has a positive effect on bequests only if demand shifts up by more than supply. The above discussion shows that, when (47) holds, the upward shift in the demand for housing wealth (generated by higher wages and housing prices) is greater than the upward shift in supply, and this imbalance can only vanish through higher steady state bequests and housing wealth.

### 5.3 Effects on Asset Prices and Wealth-to-Income Ratios

Now observe that, since the equilibrium value of housing wealth, \( v^* \), depends on \( a \), the total elasticities of \( q^h \) and \( q^z \) with respect to \( a \) are

\[
\hat{q}_a^h \big|_{tot} = \hat{q}_v^h \hat{v}_a^* + \hat{q}_a^h, \\
\hat{q}_a^z \big|_{tot} = \hat{q}_v^z \hat{v}_a^* + \hat{q}_a^z,
\]

where \( \hat{v}_a^* \) denotes the elasticity of the equilibrium value of housing wealth with respect to \( a \). Then, using (28) and (29), we get

\[
\hat{q}_a^h \big|_{tot} = (1 - S_h^h) + \hat{q}_v^h (\hat{v}_a^* - 1) + (1 - \sigma^h) S_k^h, \\
\hat{q}_a^z \big|_{tot} = \frac{1}{\sigma^h} [(\hat{v}_a^* - 1) (1 - \hat{q}_v^h + \sigma^h \hat{q}_a^h) + (S_k^h + \sigma^h (1 - S_k^h)) (1 - \hat{q}_v^h)].
\]

For the Cobb-Douglas case, (30) implies that the above reduce to

\[
\hat{q}_a^h \big|_{tot} = (1 - S_k^h) + S_h^h (\hat{v}_a^* - 1), \\
\hat{q}_a^z \big|_{tot} = (1 - S_z^h) + (\hat{v}_a^* - 1).
\]

Then, by proposition 6, both the housing and the land prices increase with \( a \) when housing demand is sufficiently inelastic. In particular, under assumption 3, price increases occur irrespectively of the own price housing demand elasticity. Additionally, the strength of the housing appreciations in enhanced by a low capital share in the
housing sector and a low housing demand elasticity (which, in turn, implies $\hat{v}_a^* > 1$).

Now we turn to wealth-to-income ratios, $\beta$ and $\beta^h$. Using (25) in (46) we obtain $y$ as a function of $v$ and $a$ by defining

$$y = y(v, a) \equiv ay^m + r^* q^z(v, a)\xi - (1 + r^*) \Delta(v, a)v,$$

(52)

Then, if $\Delta > 0$, both the average capital and output per worker fall short of their respective values in the manufacturing sector by a margin that depends on the size of housing wealth. This has a strong implication for the housing wealth-to-output ratio. In particular, using again (25) and (52), we derive

$$\beta^h = \beta^h(v, a) \equiv \frac{v}{ay^m + r^* q^z(v, a)\xi - (1 + r^*) \Delta(v, a)v},$$

(53)

$$\beta = \beta(v, a) \equiv \frac{ak^m + q^z(v, a)\xi + v(1 - \Delta)}{ay^m + r^* q^z(v, a)\xi - (1 + r^*) \Delta(v, a)v}.$$  

(54)

Notice that, when $\Delta \in (0, 1)$ and land prices are sufficiently small, the higher is the housing wealth ($v$), the lower is the average labor productivity ($y$), relative to its level in the manufacturing sector ($y^m$), and the higher is the housing wealth to income ratio, when all else is unchanged.

Using the results derived in this section, we can compute the total elasticities of $y$, $\beta^h$ and $\beta$ from (46), (53), (54). Consistently with assumption 3, we provide the analytical expressions for the Cobb-Douglas case ($\sigma^h = 1$) only, i.e.,

$$\hat{y}_a|_{tot} = 1 - \beta^h((1 + r^*)\Delta - \delta S^h_z)(\hat{v}_a^* - 1),$$  

(55)

$$\hat{\beta}^h_a|_{tot} = (1 + \beta^h((1 + r^*)\Delta - \delta S^h_z)(\hat{v}_a^* - 1),$$  

(56)

$$\hat{\beta}_a|_{tot} = \left[\beta^h((1 + r^*)\Delta - \delta S^h_z) + \frac{\beta^h}{\beta}(1 - \Delta + \delta S^h_z / r^*)\right](\hat{v}_a^* - 1).$$  

(57)
Now recall that, by (44),

\[(1 + r^*)\Delta - \delta S^h > \Delta > \delta S^h \geq 0, \quad 1 - \Delta + \delta S^h / r^* > 0,\]

Then, we can make the following claim.

**Proposition 7.** Under assumption 3,

- \(\hat{y}_a|_{\text{tot}} = \hat{\beta}_a|_{\text{tot}} = \hat{\beta}^h_a|_{\text{tot}} = 0\) for \(\gamma = 1\),
- \(\hat{y}_a|_{\text{tot}} < 1, \hat{\beta}_a|_{\text{tot}} > 0, \hat{\beta}^h_a|_{\text{tot}} > 0\) for \(\gamma\) satisfying (47).

### 5.4 Effects on the Capital Share

A final note in this section is that the housing cost disease may be responsible for a higher capital share of income, when the latter is affected by imputed rents. In a series of influential papers, Piketty and Zucman (2014), Piketty (2015a) and Piketty (2015b) have argued that the rising capital share of income is a consequence of the joint hypothesis of a steadily rising capital-output ratio and low diminishing returns to capital. Most of the evidence on the rising capital-output ratios and capital shares provided in this literature reflects the role of housing. In particular, housing values are included in the definition of capital (and represent a big part of it) and capital shares are evaluated based on a definition of income that includes imputed rents. For instance, Rognlie (2014), using Picketty’s data, claims that housing “accounts for nearly 100% of the long-term increase in the capital/income ratio, and more than 100% of the long-term increase in the net capital share of income.” (Rognlie (2014), p. 3). On a similar ground, according to Bonnet et al. (2014), the capital income ratio has dropped or remained roughly constant, when we take housing capital aside. The question we raise here is whether the phenomena highlighted by Piketty and Zucman (2014), Piketty (2015a) and Piketty (2015b) may be a consequence of a rising efficiency of labor in manufacturing, *i.e.*, as a byproduct of a housing cost disease.
The most natural definition of the capital share of income is

\[ \zeta(a, v) = 1 - aw/y. \]

Then, quite trivially,

\[ \hat{\zeta}_a \big|_{tot} = \frac{aw^*}{y - aw^*(\hat{y}_a - 1)}, \]

i.e., the average capital share of income always falls in the presence of a housing cost disease, as wages grow proportionally with \( a \) and average labor productivity grows by less than proportionally with \( a \). However, suppose that, following the prevailing practice, we include imputed rents in the definition of GDP. This would provide the following alternative definition of capital share

\[ \zeta^h = 1 - \frac{aw}{y + \pi h} = 1 - \frac{1 - \zeta}{1 + \left(\frac{\delta + r^*}{1 + r^*}\right) \beta_h}. \]

Hence, \( \zeta^h \) is typically larger than \( \zeta^k \) and it is increasing in \( \beta^h \) for any given \( y \). By straightforward computations we derive that, when \( \hat{v}_a^* > 1 \),

\[ \hat{\zeta}_a \big|_{tot} \geq 0 \quad \Leftrightarrow \quad \Delta (1 + r^*) \leq \left(\frac{\delta + r^*}{1 + r^*}\right) + \delta \xi^h. \]

By (44), the above requires \( \Delta < (\delta + r^*)/(1 + r^*) \). Hence, when the definition of income includes imputed rents, a rising relative productivity may generate a rising capital share if the growth in housing wealth is sufficiently strong.

### 5.5 Numerical Simulation

In this section we study numerically the consequences of a rising relative productivity on stationary equilibrium variables and show the impact of the housing cost disease effect on housing prices, total and housing wealth, average productivity and inequality.
We assume that production functions are Cobb-Douglas in both manufacturing and housing construction and that households’ preferences are CES. We further assume that labor efficiency in construction is time invariant and normalized to one so that $A^m = a$ and $A^h = 1$. Table 3 reports all the parameters used in the numerical simulation. We set preferences parameters such that the weight on consumption when young and old is the same, and equal to 0.4, while the weight on housing services is equal to 0.2. The elasticity of substitution for the households’ preferences is equal to 0.5. We focus on the two-type of households case (i.e., rich and poor), described in section 3.3, and set the preference for altruism of the rich household to $\theta_r = 0.75$. Recall that $\theta_r$ pins down the gross interest rate, which is equal to $1/\theta_r$. As a result, we implicitly set a mean annual real interest rate of about 1% over a generation. The depreciation of the housing stock is set equal to $\delta = 20\%$, implying complete depletion over five generations as in Deaton and Laroque (2001). Since we assume Cobb-Douglas production functions in both sectors, the elasticities of substitution between inputs are all unity. We also assume that the manufacturing sector is more capital intensive than the housing sector, and set accordingly $\alpha_m = 2/3$ and $\alpha_h = 1/5$. The land share in housing is small and equal to $\eta_h = 0.10$. If $\eta_h = 0$, the model has just labor and capital, and no land. To check the robustness of the results to different values of $\eta_h$, we also run simulations for $\eta_h = 0$ and $\eta_h = 0.25$, changing the share of labor in housing accordingly. Finally, we set the fraction of rich households to $m_r = 10\%$ and the parameter $\xi$ in the land policy equation 13, to 1, so that the amount of available land for construction adjusts proportionally to any change in the population.

We summarize the main results, under the proposed parametrization, in figures 3 to 5. Several of the variables of the model are directly pinned down by our parametrization (i.e., $R$, $w$). Therefore, in this section we focus on the variables that are instead determined in general equilibrium. In particular, in figure 3, we plot the cumulated

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Table 3: Model parameters

<table>
<thead>
<tr>
<th>Preferences</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Weight consumption young:</td>
<td>$\chi^y$</td>
<td>0.40</td>
</tr>
<tr>
<td>Weight consumption old:</td>
<td>$\chi^o$</td>
<td>0.40</td>
</tr>
<tr>
<td>Weight housing services:</td>
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<tr>
<td>Elasticity of substitution preferences:</td>
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<td>Inter-generational discount factor rich:</td>
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</table>

<table>
<thead>
<tr>
<th>Technology</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Housing depreciation (%):</td>
<td>$\delta$</td>
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</tr>
<tr>
<td>Capital share in housing:</td>
<td>$\alpha_h$</td>
<td>0.20</td>
</tr>
<tr>
<td>Land share in housing:</td>
<td>$\eta_h$</td>
<td>0.10</td>
</tr>
<tr>
<td>Capital share in manufacturing:</td>
<td>$\alpha_m$</td>
<td>0.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Economy structure</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population growth rate (%):</td>
<td>$n$</td>
<td>0.00</td>
</tr>
<tr>
<td>Fraction of rich households (%)</td>
<td>$mr$</td>
<td>0.10</td>
</tr>
<tr>
<td>Land policy rate:</td>
<td>$\xi$</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: In this table we report the model’s parameters used to simulate different steady-states for different values of the exogenous parameter $a$. For robustness, we also run simulations for values of $\eta^h = 0, 0.25$.

percentage changes of steady-state values of four key variables: the equilibrium supply of bequests ($b$); the total wealth to income ratio ($\beta$) and the housing wealth to income ratio ($\beta^h$), with respect to their initial values when $a = 1$. The black solid lines correspond to the baseline specification of table 3. To check the robustness of the results, we include also results for the case of zero (red dot-dashed line) and higher (blue dashed line) share of land in the production of houses ($\eta^h$). Bequests (right panel) respond strongly, and proportionally, to an increase in relative labor productivity in manufacturing. The response of bequest is very robust to different values of $\eta^h$. Recall that, in the model, bequests are a proxy for inequality that, therefore, increases with $a$. Both the total, and the housing, wealth-to-income ratios increase with $a$. However, the response of housing wealth is almost twice as large as that of total wealth, and the effect is stronger the higher the share of land in the production of houses. In figure 4, we plot
the steady-state changes of the labor share in construction \((\lambda)\), the average productivity \((y)\), the housing stock \((h)\) and the housing price \((q^h)\). Note how the housing price increases almost proportionally with \(a\), while the housing stock increases by about 50\% for a 100\% increase in \(a\). The share of labor in construction increases by about 30\%, going from about 4.5\% to approximately 6\%. Average productivity increases, but less than proportionally. The latter result depends on the shift in composition of output, from manufacturing to construction, resulting from the increase in relative productivity.

**Figure 3**: Simulation results (I/III)

*Notes: This figure plots the percentage changes in the steady-state values of total \((\beta)\) and housing \((\beta^h)\) wealth to income ratios and the level of bequests \((b)\), for different values of the relative productivity sector \(a = 1, \ldots, 2\), with respect to their value for \(a = 1\). The back solid lines correspond to the baseline parametrization. The blue dashed line is plotted for a high value of the share of land in construction equal to \(\eta^h = 0.25\). The red dot-dashed line is plotted for \(\eta^h = 0\), which corresponds to the case of a model with only capital and labor (and no land).*
Recall that the capital share is defined as \( \zeta^k = 1 - aw/y \). Since \( aw \) respond proportionally to an increase in \( a \), while \( y \) increases less than proportionally, we expect the capital share to decline. This is depicted in figure 5. The left panel plots \( \zeta^k \) and shows that it decreases by about 12 percent for a 100 percent increase in relative labor efficiency. In section 5.4 we showed that an alternative measure of capital share, that includes imputed rents, can theoretically increase if the increase in housing wealth is large enough. However, under our parametrization, also this latter alternative definition of capital share declines, but by less than \( \zeta^k \).

Figure 4: Simulation results (II/III)

Notes: This figure plots the percentage changes in the steady-state values of the labor share in construction (\( \lambda \)), the average productivity (\( y \)), the per capita housing stock (\( h \)) and the price of houses (\( q^h \)) for different values of the relative productivity sector \( a = 1, \ldots, 2 \), with respect to their value for \( a = 1 \). The back solid lines correspond to the baseline parametrization. The blue dashed line is plotted for a high value of the share of land in construction equal to \( \eta^h = 0.25 \). The red dot-dashed line is plotted for \( \eta^h = 0 \), which corresponds to the case of a model with only capital and labor (and no land).
Figure 5: Simulation results (III/III)

Notes: This figure plots the percentage changes in the steady-state values of the capital share of income ($\zeta$) for different values of the relative productivity sector $a = 1, \ldots, 2$, with respect to their value for $a = 1$. We consider two alternative definitions: the left panel corresponds to the standard definition of capital share $\zeta^k = 1 - aw/y$; the right panel corresponds instead to a definition that includes imputed rents as in $\zeta^{k,h} = (y + \pi h - aw)/(y + \pi h)$. The back solid lines correspond to the baseline parametrization. The blue dashed line is plotted for a high value of the share of land in construction equal to $\eta^h = 0.25$. The red dot-dashed line is plotted for $\eta^h = 0$, which corresponds to the case of a model with only capital and labor (and no land).

In this section, we presented results of numerical simulations of the model showing that, under our parametrization, it generates a large housing cost disease. In the next section, we evaluate the implications of the housing cost disease in terms of welfare.

6 Welfare

In this section we address the housing cost disease problem from a welfare point of view. Is the fact that housing takes a large share of private wealth undesirable?
Within a similar overlapping generations model, Deaton and Laroque (2001) find that the presence of a demand for housing generates a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the Golden Rule level. They take this result as a possible reason for confiscating property and giving it to consumers at no charge. An other reason to argue in favor of a reduction in housing wealth may be based on Piketty’s argument that a rising wealth to income ratio, especially when caused by a rising housing stock appreciation, may lead to increasing inequality, especially when housing takes a sizable share of intergenerational bequests. For example, Auerbach and Hassett (2014) note that reducing the tax benefits for owner-occupied housing in a progressive manner (or a deregulation in land use) may be more effective than a wealth tax in addressing the inequality problem (as proposed by Piketty).

However, this conclusions must be taken with some caution. Regarding Deaton and Laroque (2001)’s analysis, it should be noted that allocations departing from the Golden Rule are inconsistent with a social optimum only if we endorse a specific social welfare criterion, such as a weighted sum of all generations’ utilities with rate of time preference equal to the population growth rate. In fact, any market allocation at which the rate of interest is larger than the population growth rate is Pareto optimal and, in these cases, reducing the value of the housing stock may have adverse effects on some generation’s welfare10. Conversely, when the real interest rate falls short of the population growth rate, a case that can only occur when both type of households leave zero bequests, Pareto improvements can be obtained by decreasing investment in housing as well as in the capital stock. In other words, crowding out of capital induced by housing demand and exchange across generations may, in fact, be desirable to avoid

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10Using standard arguments, and recalling that population is stationary, one can show that the equilibrium allocation derived in the above sections is Pareto efficient if and only there exists no bounded sequence, \( \{\xi_t\}_{t=0}^{\infty} \), such that \( \xi_{t+1} \geq R_{t+1} \xi_t, \xi_0 > 0 \) for all \( t \geq 0 \) and \( \xi_0 > 0 \). Evidently, this requires that the sequence of gross interest rates is below one for an infinite number of periods. In particular, at the steady state, we need \( R < 1 \), which can only verified in a ZBSS.
an over-accumulation of capital.

In this section, we examine how a change in the reallocation of resources and prices induced by a rise in relative productivity affects social welfare in a market economy, under an egalitarian criterion. There are a number of reasons why this question may be interesting. First of all, since we have shown that rising housing prices generate more bequests, \textit{i.e.}, more wealth inequality, the housing cost disease may be social welfare diminishing from an egalitarian perspective. However, since a non paternalistic Planner takes into account the households’ subjective discount rates, \textit{i.e.}, their degree of paternal altruism, some inequality is compatible with an egalitarian planning optimum. A second observation is that, in a competitive equilibrium, poor households are unable to leave any bequests to their children because of one-sided altruism. Hence, by raising the poor old individuals’ wealth, a rise in housing prices may relax the non-negativity constraint on bequest values and generate the increase in old age consumption that the market is preventing under one-sided altruism. Finally, we notice that housing is a consumption good as well as an asset. Then, a housing appreciation may decrease welfare as it makes housing less affordable.

Recall that, by forward iteration, the initial old type-\textit{i} household’s utility can be written as

\[
V^{-1,i} = u(c_{-1}^{-1,i}, c_{0}^{-1,i}, h_{0}^{-1,i}) + \sum_{t=0}^{\infty} (\theta_t (1 + n))^{t+1} u(c_{t}^{t,i}, c_{t+1}^{t,i}, h_{t+1}^{i}).
\]

Then, the objective of the Egalitarian Planner is

\[
U = \sum_{i} m_i V^{-1,i}.
\]

One can easily verify that the only possible differences between a planning optimum and a competitive equilibrium are in the allocation of bequests and the fact that a market allocation does not explicitly provide a transversality condition. However, the
latter condition is always verified at any equilibrium such that \( R_{t+1} > (1+n) \). In turn, this inequality is always verified when at least one type of households leaves positive bequests at all periods. Regarding bequests, we have

\[
\begin{align*}
u^i_{2,t} &= \theta_i u^i_{1,t+1} \\
\end{align*}
\]

for all \( i \) at a planning optimum and \( u^i_{2,t} \geq \theta_i u^i_{1,t+1} \) at a competitive equilibrium. In other words, the Planner can implement negative bequests. Notice, also, that, although the Planner is egalitarian, she takes into account the subjective discount factors, \( \theta_i \), representing their degree of altruism with respect to the offsprings, in allocating resources. In fact, since, at a planning optimum, the first order conditions for individual optimality must be verified with equality and because \( \theta_r > \theta_p \), we get

\[
\begin{align*}u^r_{1,t}/u^r_{1,t+1} > u^p_{1,t}/u^p_{1,t+1},\end{align*}
\]

i.e., the Planner’s allocation is such that the poor (less altruistic) type young households end up with a lower consumption than the rich (more altruistic) type.

Now we consider the effect on the Planner’s welfare function, \( \mathcal{U} \), of an unanticipated rise in the level of the relative labor efficiency, \( a = A^m/A^h \), at \( t = 0 \), for a constant labor efficiency in construction, \( A^h_t = 1 \) at a competitive equilibrium such that the poor-type households leave zero bequests and the rich-type leave positive bequests at all periods. By exploiting the equilibrium conditions (i.e., resource feasibility, first order conditions for individual optimality at equilibrium and budget constraints), and defining subjective prices, for \( i = p, r \), as

\[
\rho^i_t = (\theta_i(1+n))^{t+1} w^i_{1,t},
\]

50
we derive
\[
\frac{\partial U}{\partial a} = \sum_{t=0}^{\infty} \rho_t^r w_t (1 - \lambda_t) + m_p \Gamma, \tag{58}
\]
where
\[
\Gamma = \sum_{t=0}^{\infty} (\rho_t^p - \rho_t^r) \left( \frac{\partial c_{t,1}^p}{\partial a} + \frac{1}{1 + n} \frac{\partial c_{t,1-p}^l}{\partial a} + \pi_t \frac{\partial h_{t+1}^p}{\partial a} \right) + \sum_{t=0}^{\infty} \rho_t^p \left( \frac{u_{2,t-1}^p}{\theta_p u_{1,t}^p} - 1 \right) \frac{1}{1 + n} \frac{\partial c_{t,1-p}^l}{\partial a}.
\]
Since a First Best allocation is such that
\[
\rho_t^r = \rho_t^p, \quad \frac{u_{2,t-1}^p}{\theta_p u_{1,t}^p} = 1,
\]
for all \( t \geq 0 \), the first summation on the right hand side of (58) represents the undistorted component of the welfare effect of a rising \( a \), whereas \( \Gamma \) represents two possible distortions: the first one arising from the possibility that consumption is not allocated as dictated by the Planner across individuals of the same generation, and the second from the fact that the poor-type households are unable to leave positive bequests to their offsprings. Observe that, at equilibrium, \( u_{2,t-1}^p / \theta_p u_{1,t}^p > 1 \), whereas the sign of \( \rho_t^p - \rho_t^r \) is ambiguous, since it depends on the differences between consumptions across the two type of households (in the same age and time) and the discount factors. If the discount factors are very similar and the rich household has a much larger consumption compared with the poor household, we have \( \rho_t^p - \rho_t^r > 0 \). A further observation is that the second summation in \( \Gamma \) would vanish in the case of an equilibrium where both households are identical and leave zero bequests (the canonical overlapping generations model). In this case, equation (58) would imply that a rise in \( a \) generates a welfare benefit exceeding the undistorted (First Best) value if this parameter shift generates a higher consumption for the olds. In turn, the latter consumption is likely to rise if the increase in \( a \) has a positive effect on housing wealth (since old individuals are net suppliers of housing).
We provide now an equivalent more intuitive expression for $\partial U / \partial a$ based on individuals' budget constraints. For simplicity, we assume that these households own zero land. Then, by the lifetime budget constraints of the poor individuals,

$$\frac{\partial c_t^{p}}{\partial a} + \frac{1}{R_{t+1}} \frac{\partial c_t^{p}}{\partial a} + \pi_t \frac{\partial h_t^{p}}{\partial a} = w_t - \frac{\partial \pi_t}{\partial a} h_{t+1}^{p}, \quad \frac{\partial c_0^{-1,p}}{\partial a} = (1 - \delta) h_0^p \frac{\partial q_0}{\partial a}.$$

Then, by rearranging terms, we derive

$$\Gamma = (u_{-1}^p - u_{-1}^r) (1 - \delta) h_0^p \frac{\partial q_0}{\partial a} + \sum_{t=0}^{\infty} (\rho_t^p - \rho_t^r) \left( w_t - \frac{\partial \pi_t}{\partial a} h_{t+1}^{p} \right). \quad (59)$$

In other words, the distortionary component of the welfare effect of a rising $a$ depends on how consumption is allocated across households and on the impact of $a$ on wages and housing prices. The latter are important because they alter the old individuals' housing wealth and the affordability of housing (measured by $\pi$) for the young households.

Now consider a steady state, where $c_t^{i} = c^{y,i}$, $c_{t-1}^{i} = c^{o,i}$ for all $t \geq 0$ are the stationary young and old age consumptions of the two types, and define

$$z^i \equiv \sum_{t=0}^{\infty} \rho_t^i = \frac{\theta_i (1 + n)}{1 - \theta_i (1 + n)} u^i.$$

Using the above findings and evaluating the expression for $\Gamma$ in (59) at a steady state, we can decompose $\partial U / \partial a$ into two components: the effect arising from a change in $w$ and the effect arising from the housing appreciation, i.e., from a change in $q$. In particular, we have

$$\Gamma = (z^p - z^r) w + \frac{\partial q}{\partial a} h^p \left( \frac{(1 - \delta) \theta^r}{(1 + n) \theta^p} (z^p \theta^r - z^r \theta^p) - (z^p - z^r) \right).$$

Remember that $\theta^r > \theta^p$ and $\partial q / \partial a > 0$ at equilibrium. Then, the sign of $\Gamma$ at a steady state equilibrium is ambiguous. Observe that, at a steady state equilibrium with the
CES utility defined in the previous section and unit elasticity (i.e., $\gamma = 1$),

$$z^p > z^r \iff \frac{b}{W} > \frac{(1 + n)(\theta_r - \theta_p)}{(1 - (1 + n)\theta_r)(1 - (1 - \delta)\theta_r)}.$$

Suppose that the poor household is only marginally less altruistic than the rich one, i.e., $\theta_p \sim \theta_r$, but experiences a much smaller consumption. Then, $z^p > z^r$ and

$$\Gamma \sim (z^p - z^r) \left( \omega^p - h^p \frac{\partial q}{\partial a} \left( 1 - \frac{(1 - \delta)\theta_r}{1 + n} \right) \right).$$

In other words, the housing price appreciation reduces the social welfare benefit of the productivity improvement in manufacturing.

## 7 Conclusions

We have shown that a Baumol’s cost disease may explain the increase in total and housing wealth-to-income ratios and wealth inequality that took place in a large set of advanced economies in the last forty years. To show this, we have employed a simple life-cycle model with no financial frictions, two sectors (construction and manufacturing) and one-sided parental altruism. Key assumptions are that the construction sector less capital intensive than manufacturing and housing demand sufficiently inelastic. Under these assumptions, a rise in labor efficiency in manufacturing produces a strong upward pressure on housing prices, a rise in the total and housing wealth-to-income ratios and a rise in bequests. Finally, we have shown that housing appreciations may mitigate (relative to the First Best level) the beneficial effects of a rising productivity in manufacturing under an egalitarian welfare criterion when market allocations imply high enough consumption inequality and low enough heterogeneity in parental altruism.
References


A Data

In the paper we use two main source of data. The first is Piketty and Zucman (2014), which provide data on wealth to income, as well as saving and growth rates, for the top eight developed economies (the US, Germany, the UK, Canada, Japan, France, Italy and Australia). The second is the KLEMS database (O’Mahony and Timmer, 2009), from which we are able to build a measure of relative labor efficiency in manufacturing. For most countries, data from Piketty and Zucman (2014) are annual and covers the 1970-2010 sample; data from KLEMS are also annual, and covers the 1970-2007 sample. However, in both databases for some countries the available samples are shorter. As a general rule, for each country we use the longest available series. Table 4 specifies, for each country, the lengths of the samples for each of the main variables used in sections 2 and ?? of the paper. This section of the appendix presents additional details on the data, as well as robustness checks on the stylized facts and empirical relations discussed in the paper.

Table 4: Length of samples

<table>
<thead>
<tr>
<th>Countries</th>
<th>NWI</th>
<th>PWI</th>
<th>PWDI</th>
<th>HWI</th>
<th>s</th>
<th>g</th>
<th>a</th>
</tr>
</thead>
</table>

Notes: This table reports the available samples for the main variables used in sections 2 and ??: NWI is national wealth-to-income; PWI is private wealth-to-income; PWDI is private wealth-to-disposable-income; HWI is housing wealth-to-income; s is national saving as a fraction of income; g is real income growth; a is relative labor efficiency in manufacturing. The countries in the sample are: the US, Germany (DE), the UK, Canada (CA), Japan (JP), France (FR), Italy (IT) and Australia (AU). All variables are net of depreciation. Data are from Piketty and Zucman (2014) and O’Mahony and Timmer (2009).

A.1 Data on wealth ratios

Wealth ratios are from Piketty and Zucman (2014), who collect an incredibly rich dataset on wealth and income for the eight largest developed economies (the US, Germany, the UK, Canada, Japan, France, Italy and Australia) over the 1970-2010 period. Since the dataset is built according to the UN System of National Accounts (SNA), data are, to the best of our knowledge, homogeneous across countries. In the paper,
we use national wealth if not otherwise specified. Private wealth is the net wealth of households and non-profit institutions serving households. Assets include both non-financial assets (i.e., land, buildings, machines, etc.) and financial assets (i.e., stocks, bonds, shares in pension funds, etc.). Note that two types of assets are not included in the definition of wealth: pay-as-you-go public pensions and durable goods. Assets and liabilities are evaluated at market prices. National wealth is instead the sum of private and public wealth, where the latter is the net wealth of public administrations and government agencies. Note that national wealth can be also decomposed as the sum of domestic capital and net foreign assets so that in a closed economy it is exactly equal to the market value of domestic capital. The income definition used to build the wealth ratios is ”net-of-depreciation. Therefore, whenever we refer to saving or growth rates we implicitly consider their ”net” definitions. Figure 6 plots the evolution of the private wealth-to-income ratios over the 1970-2010 period. In section 2 of the paper we discussed stylized facts relative to national wealth. Ratios based on private, rather than national, wealth follow a similar patter. In fact, we observe, on average, a gradual increase (from 3.3 in 1970 to 4.7 in 2010, for a mean increase of 42%. Over the same period, the mean increase in the national wealth to income ratios was higher and equal to 79%. The larger increase in private wealth is in part explained by the fact that, over the period considered, public debt increased significantly. Holdings of domestic public debt are considered an asset for households in the computation of private wealth, but wash-out in the definition of national wealth as they are a liability for the government. In this respect, Italy and Japan are interesting examples as the public debt of these countries increased significantly in this period. For Italy, the percentage increase in the private wealth to income ratio was 180%, while the corresponding figure for national wealth was 95%. For Japan, the numbers are, respectively, 135% and 84%. Note that disposable income is a fraction of the total, and has been shrinking in the last decades as the size of the public sector expanded. Therefore, if we had computed wealth ratios with respect to disposable income, we would have observed steeper slopes.

In section 2 we show that wealth inequality is very concentrated in the top end of the wealth distribution and that wealth inequality has been gradually increasing since the ’70s. Since data on wealth are not as reliable as on income, we present in figure 7 the the evolution of the income share of the top 10 percent (top panel) and 1 percent (bottom panel) of the income distribution. As for wealth, the data show two simple facts: first, income is very unequally distributed; second, income inequality has been gradually increasing since the ’70s. In 2010, the average income share of the top 10 (top 1) percent of the income share was 37.6% (11.3%). On average, income inequality
increased dramatically in the 1970-2010 period. For example, in 1970 the income share of the top 10 (top 1) percent was 31.6 percent (7.6 percent) for an increase of 6 percentage points (3.7 percentage points). The United States is the country with the largest increase in income inequality, with the income share of the top 10 percent going up by about 15 percentage points. Over the same period, the same share increased by about 10 percentage points in Germany, the UK and Canada, while it increased by less then 5 percentage points in the rest of the countries in the sample.

A.2 Data on relative productivity

We use the KLEMS database, available at http://www.euklems.net/, and organized by O’Mahony and Timmer (2009), to construct a measure of relative labor efficiency in manufacturing. In particular, we use the ISIC Rev. 3 version of the KLEMS database (March, 2011), which reports data from 1970 up to 2007. For each of the countries in the sample, and for different sectors, we collect data on real gross valued added
Figure 7: Income distribution

Notes: The top (bottom) panel of this figure plots the evolution of the income share of the top 10 (top 1) percent of the income distribution. The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia.

\( Y^i \); real capital \( rK^i \) and labor compensation \( wL^i \); hours worked by employee; real capital \( K^i \) and labor inputs \( L^i \), where \( i \) is an index denoting the different sectors (we consider KLEMS sectors "total industries," as a proxy for manufacturing, and "construction"). Only for Canada, we consider the earlier March 2008 release which covers the sample 1970-2004; and use "wood and product of wood" as proxy for the manufacturing sector. In order to compute the relative labor efficiency, we compute labor productivities in manufacturing under the assumption of Cobb-Douglas technology, and in construction under the assumption of CES technology. For manufacturing, we first derive the capital share parameter in the production function \( (\alpha_m) \) by dividing real capital compensation, in this sector, by the corresponding real gross value added. Given the assumption of constant returns to scale, we assume that the labor share parameter is simply equal to \( 1 - \alpha_m \). We then derive the labor-augmented productivity
$A^m$ as:

$$A^m = \frac{1}{L^m} \left[ \frac{Y^m}{(K^m)^{\alpha_m}} \right]^{\frac{1}{1-\alpha_m}}. \quad (60)$$

For construction, we assume that the elasticity of substitution between capital and labor is equal to $\sigma = 2$, and that the parameter $\alpha_h$ in the CES production function is the same as for the case of a Cobb-Douglas specification (i.e., it is equal to the ratio between real capital compensation in construction and real gross value added in the same sector). Then, the labor-augmented productivity $A^h$ is derived as:

$$A^h = \left[ \frac{(Y^h)^{\frac{1}{\sigma}} - \alpha_h(K^h)^{\frac{1}{\sigma}}}{(1 - \alpha_h)(L^h)^{\frac{1}{\sigma}}} \right]^{\frac{\sigma}{\sigma-1}} \quad (61)$$

Relative labor efficiency in manufacturing in then simply equal to $a_t = A^m_t/A^h_t$.

**B Numerical simulations**

In this section we present additional results regarding the numerical simulations of the model. Figure 8 plots the steady state equilibrium level of bequests for two different values of the relative labor efficiency in manufacturing $a$. In particular, the figure plots the demand for housing wealth $v^d(b,a)$, blue line, and the supply of housing wealth $v^s(b,a)$ (red line) for different values of the level of bequests $b$. The demand for housing wealth is less steep that the supply so that it intersects the latter from below. At a steady state, demand and supply intersects. The figure shows that an increase in $a$ determines a large increase in the equilibrium level of bequests.

Figure 9 plots the two components that determine the increase in housing wealth as a result of an improvement in relative labor efficiency in manufacturing. The left panel plots the per capita housing stock, while the right panel the housing price. For the baseline parameterisation, the per capita housing stock responds almost linearly with $a$, while the housing price responds less than proportionally.

**C Propositions and Proofs**

**Proof of proposition 2**

Let

$$\phi(\lambda, a) = \frac{\lambda f_h(k^h(1/\theta_r, a), \xi/\lambda)}{f_{h,k}(k^h(1/\theta_r, a), \xi/\lambda)}$$
Figure 8: Steady-state with bequest

Notes: This figure plots the functions $v^d(b, a)$ (blue line) and $v^s(b, a)$ (red line) for different values of the level of bequests $b$. The solid line are for a value of $a = 1$, while the dashed lines are for $a = 2$.

and observe that, since $\lambda = \xi/z$,

$$\phi(\lambda, a) = \xi f_h(k^h, z)/zf_{h,k}(k^h, z).$$

Taking derivatives,

$$\frac{\partial \log(f_h/zf_{h,k})}{\partial z} = -(1 - S^h) - \frac{S^h}{\sigma^h} \left( \frac{f_h}{f_{h,k}} \right)^{\sigma^h - 1} < 0,$$

$$\frac{\partial \log(f_h/zf_{h,k})}{\partial a} = \sigma^h S^h + (1 - S^h) > 0.$$

Then,

$$\phi_\lambda = \left( \frac{f_h}{f_{h,k}} \right) \frac{\partial \log(f_h/zf_{h,k})}{\partial z} > 0, \quad \phi_a = \left( \frac{\lambda}{a} \right) \left( \frac{f_h}{f_{h,k}} \right) \frac{\partial \log(f_h/zf_{h,k})}{\partial a} > 0,$$

so that

$$\lambda_v(v, a) = \frac{(\delta + n)\theta_r}{\phi_\lambda} > 0, \quad \lambda_a(v, a) = -\frac{\phi_a}{\phi_\lambda} < 0.$$
Figure 9: Changes in wealth: volume and price effect

Notes: This figure plots the percentage change in the steady-state values of the housing stock and price for different values of the relative productivity sector $a = 1, \ldots, 3$, with respect to their value for $a = 1$. All variables are in intensive form. The back solid lines correspond to the baseline parametrization. The blue dashed line is plotted for a lower interest rate ($R = 1/\theta_r = 1.25$). The red dot-dashed line is plotted for a higher interest rate ($R = 1/\theta_r = 1.66$).

Proof of proposition 4

From the individuals’ budget constraints, and for any demand function, $(c^{y,i}, c^{o,i}, h^i)$, we have

$$s^i = W + b^i - c^{y,i}(.), \quad c^{y,i}(.) + c^{o,i}(.)/(1 + r^*) + \pi h^i(.) = 1,$$

where the subscript $I$ on each demand function denotes the partial derivative with respect to $I^i$. Then,

$$\frac{\partial s^r(.)}{\partial b} = 1 - \left(\frac{r^* - n}{1 + r^*}\right) c^{y,r}_I(.) = \left(\frac{r^* - n}{1 + r^*}\right) c^{y,r}_I(.) + \frac{c^{o,r}_I(.)}{1 + r^*} + \pi h^i(.)$$

and

$$\frac{\partial q^h h^r(.)}{\partial b} = \left(\frac{r^* - n}{1 + r^*}\right) q^h h^r_I.$$
Since $q^h = (1 + r^*)\pi/(r^* + \delta)$,

\[
\frac{\partial q^h r^*}{\partial b} = \left(\frac{r^* - n}{\delta + r^*}\right) \pi h^r < \pi h^r < \frac{\partial s^r(,)}{\partial b}.
\]

Since $\delta + n > 0$, we get the proposition.

**Proposition 8.** Assume that $b^r = 0$. Then, $\Delta < 1$ at equilibrium.

**Proof.** Since $\Delta = (\delta + n)(ak^m - k^h)/q^h y^h$, we have

\[
1 - \Delta = \frac{q^h y^h - (\delta + n)(ak^m - k^h)}{q^h y^h}.
\]

(62)

Since $v = \lambda q^h y^h/(\delta + n)$, $G^s(b, v, a) = 0$ implies

\[
\lambda(q^h y^h - (\delta + n)(ak^m - k^h)) = \frac{\delta + n}{1 + n}(s - (1 + n)(ak^m + q^z \xi)).
\]

Now let $1 - \Delta + q^z \xi/v < 0$. Then, $\lambda \in [0, 1]$ if and only if $s < (1 + n)(ak^m + q^z \xi)$ and

\[
s > \left(\frac{1 + n}{(\delta + n)}\right) q^h y^h + (1 + n)(k^h + q^z \xi).
\]

Since $\delta \leq 1$, the above requires

\[
s > q^h y^h \geq W.
\]

$$
\square
$$

**Proof of proposition 6**

Define the partial elasticities of housing demand as

\[
\hat{v}^j_a = v^j_a(b, a)a/v, \quad \hat{v}^j_b = v^j_b(b, a)b/v,
\]

for $j = d, s$, and we observe that, under assumption the maintained assumptions\(^{11}\),

\[
\hat{b}_a = \frac{\hat{v}_a^d - \hat{v}_a^s}{\hat{v}_b^s - \hat{v}_b^d}.
\]

\(^{11}\)Notice that the property (??) is preserved for $S^h_n > 0$ under the restriction $G^d_v < G^s_v < 0$, i.e.,

\[
\max\{1, 1 + q^z \xi/v - s\hat{s}_v S^h_2/v\} > \Delta > S^h_2(1 + \hat{h}_d^s) + q^z \xi/v - s\hat{s}_v S^h_2/v.
\]

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Then, by because of demand are homothetic, we get
\[ aG^j_a + bG^j_b = \Sigma^j, \quad \text{for } j = d, s, \]
where, by the assumption \( \sigma^h = 1, n = 0, \)
\[ \Sigma^d = v(S^h_d(1 + \hat{h}_d^d) + 1), \quad \Sigma^s = v(1 - \Delta) + q^z\xi - s\hat{s}_{\pi}S^h_l. \quad \text{(64)} \]

Using the above in (63), we get
\[ \hat{b}^*_a = 1 + M, \quad \hat{v}^*_a = 1 + \left(1 - S^h_k\right)(1 + \hat{h}_d^d) + \hat{v}_b^d\left(\hat{b}^*_a - 1\right), \quad \text{(66)} \]
where
\[ M = \frac{1}{\hat{v}_b^d - \hat{v}^*_b}\left(\frac{\Sigma^d}{-vG^d_v} - \frac{\Sigma^s}{-vG^s_v}\right). \]

Then, a sufficient condition for \( \hat{b}^*_a \geq 1 \) and \( \hat{v}^* > 1 \) is \( M \geq 0. \) By assumption (??) and equations (64), (65), we derive that the sign of \( M \) is equal to the sign of the following expression:
\[ (1 + \hat{h}_d^d)(S^h_l + S^h_z)(v(1 - \Delta) + q^z\xi) + s\hat{s}_{\pi}(S^h_l - S^h_z) - 2(1 + \hat{h}_d^d)S^h_lS^h_z. \]