Sequential Credit Markets

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ABSTRACT

Entrepreneurs who seek financing for projects typically do so in decentralized markets where they need to approach investors sequentially. We study how well such sequential markets allocate resources when investors have expertise in evaluating investment opportunities, and how surplus is split between entrepreneurs and financiers. The sequential nature of the market introduces endogenous adverse selection which leads to substantial investment inefficiencies and rents to investors that are not competed away even as the pool of potential investors grows large. Contrary to common belief, we show that the introduction of a credit bureau that tracks the application history of a borrower leads to more adverse selection, quicker market break downs, and higher rents for investors. Nevertheless, a sequential search market can be more efficient than a centralized exchange where excessive competition may impede information aggregation. We also show that investors who rely purely on hard information in their lending decisions can out-compete better informed investors with soft information, and that an introduction of interest rate caps can increase the efficiency of the market.

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The main role of primary financial markets is to channel resources to firms with worthwhile projects, a process that requires information about investment opportunities. Investors with expertise in evaluating projects, such as venture capitalists, business angels, or commercial banks, can therefore serve an important role for the productivity and growth of the real economy. Since no single investor usually has all the information for deciding whether a project should be pursued or not, there is a need for financial markets to aggregate information efficiently.

The extent to which markets can aggregate information and allocate resources efficiently depends on how they are organized. At least until very recently, the overwhelming majority of primary capital markets for small- and medium sized firms operate as decentralized search markets in which firms approach potential investors sequentially (one-by-one). This is true whether firms are seeking capital from banks or from equity investors such as business angels and venture capitalists. Historically, transparency of these markets has been limited. Advances in technology over the last decades has made these markets more transparent by allowing easier access to credit bureau information that track the application history of borrowers. Recently, innovations in financial technology has brought some of the market activity to centralized market places such as peer-to-peer lending and crowdfunding platforms.

How does the transparency and structure of primary capital markets affect the efficiency of resource allocation in the economy? How do improvements in information technology benefit entrepreneurs relative to financiers, and what are the implications for regulation? In this paper, we develop a general but tractable decentralized search model of credit markets to study these questions, and contrast the results to the ones we have developed in a companion paper on centralized markets (Axelson and Makarov (2016)). We show that perhaps contrary to common intuition, decentralized search markets can be more efficient at aggregating information than centralized markets. Even more surprisingly, we show that increased transparency can lead to more adverse selection in credit markets and higher rents for investors, with ambiguous affects on the efficiency of resource allocation. Finally, policies such as interest rate caps can lead to more efficient decentralized markets but higher rent for investors.

We consider a setting in which an entrepreneur with a project idea searches for credit by approaching potential financiers sequentially, until she either finds an investor who is willing to accept her terms for financing the project or runs out of options and abandons the project. Unlike standard search models which focus on the friction introduced by the cost of finding a counterparty, we are interested in the consequences of sequential interactions. We therefore assume that the entrepreneur is infinitely patient and has no search cost, so that all our results are driven by informational frictions. We assume that

there is uncertainty about whether the project is worthwhile or not. Each investor, if approached, can do due diligence which results in a private signal about the prospects of the project. Although the aggregate information held by all investors is valuable for distinguishing good projects from bad, a number of problems impede information aggregation.

First, whenever the entrepreneur comes to an agreement with an investor, information aggregation stops although there is potentially valuable information held by investors who have not yet been approached. Second, when the entrepreneur does not come to an agreement with an investor and continues her search, not all of the information held by the investor is passed on to the next investor she meets. In particular, each new investor faces an adverse selection problem created by the fact that the entrepreneur failed to receive financing in previous interactions with informed investors, which could potentially lead to a market break down.

We first show that even in the case where the entrepreneur commits to maximizing social surplus rather than her own expected profits, information aggregation typically fails and substantial investment mistakes occur relative to the case where all signals are used for the investment decision. Nevertheless, the market outcome will be as efficient as the one achieved in a centralized first-price auction where an optimally chosen set of market participants can be invited to compete for the financing of the project, and where the investment decision can be based on all bids received. The way this is accomplished is by asking for financing at an interest rate which is increasing in the number of rounds the entrepreneur has been on the market, where the interest rate is set such that each investor accepts if his signal is above the same optimally chosen cut-off.

We then turn to the more realistic case where the entrepreneur maximizes her own expected profits. As a benchmark, we first study the most transparent case where investors can observe the offers the entrepreneur has made to previous investors. We show that surplus is always smaller than in a centralized first-price auction with an optimal number of bidders. However, for the case where large markets are socially optimal, we show that as the number of potential investors grows large revenues and surplus approach those generated in an auction. The sequential market can also do better than large free-entry auctions when it is socially optimal to have few investors, because the sequential market will endogenously break down after a limited number of financing attempts.

We then turn to the more realistic case where terms of the offers to previous investors is not observable. We show that the efficiency of the market and the way surplus is split depends critically on whether the sequence in which investors is approached is

observable or not. Both cases are empirically relevant. In most developed countries, any bank that is approached for financing will submit a credit check to a credit bureau or credit registry, and will also learn from the credit bureau who else has performed a credit check on the borrower in the past. Hence, the investor will learn how many times the borrower has applied for financing previously. We refer to the case where the sequence is observable as the "credit bureau" case. In the second case, which we refer to as the "no credit bureau" case, a lender does not know how many other lenders an applicant has visited before. This is commonly the case in less developed countries, in informal lending markets, and in non-bank markets such as when an entrepreneur seeks angel- or venture capital financing.

We show that a credit bureau can have either positive or negative effect on social surplus and the profit of the entrepreneur relative to the case with no credit bureau. To understand the trade-off, consider the case with a credit bureau in place. Each time an entrepreneur is rejected, the rejection is recorded in the credit bureau so that remaining investors revise their beliefs about the quality of the project downwards. The impact of a rejection on the beliefs of remaining investors depends on the terms at which they believe the entrepreneur was rejected—if they believe the entrepreneur asked for financing at very favorable terms (a low interest rate), a rejection is not such bad news. Because these terms are not directly observable, the entrepreneur cannot affect the beliefs of investors and improve her prospects in future rounds by asking for more favorable terms in the current round. In equilibrium, this biases her towards asking for less favorable terms.

When there is no credit bureau, an investor cannot verify how many times an applicant has been rejected previously. This is potentially bad for an entrepreneur who has not been rejected, since she might be pooled with rejected entrepreneurs with worse credit quality. A first-time applicant therefore has an incentive to signal her type, and we show that she will always be able to do so by asking for more favorable financing terms (a lower interest-rate loan). This is a credible signal, because a request for more favorable terms has a higher probability of rejection, and rejection is less costly for a first-time applicant who has many investors left to visit. This logic extends to all rounds, leading to a fully separating equilibrium where the entrepreneur asks for slightly less favorable terms with each rejection. Thus, the need for signalling creates a credible way for the entrepreneur to ask for favorable terms early on.

Asking for favorable financing terms early on has two consequences. First, it reduces the rents to investors. We show that as the number of potential investors grows large, investors' rent is competed away in the case of no credit bureau. In contrast, in the case of a credit bureau, investors continue to earn significant rents even though the

entrepreneur has zero search costs and all the bargaining power.

Second, asking for favorable financing terms leads to more financing rounds relative to the case with a credit bureau because credit quality deteriorates slower with each rejection. In the case of no credit bureau, the entrepreneur can visit all the available investors. In contrast, in the case of a credit bureau, the entrepreneur might get locked out of the market after a single rejection even when there is a large set of potential investors.

The benefits of having extended search depend on the informational content of the signal distribution. The way many financing rounds are sustained is by asking for offers that only the most optimistic investor would accept, while less optimistic information is never incorporated in the financing decision. As a result, extended search is desirable in situations where the informational content of the signal distribution is concentrated towards the top. We show that for these situations, as the number of potential investors grows large, the social surplus without a credit bureau approaches that attained in a large first-price auction, which is also the maximal possible one.

However, extended search can lead to less informative financing decisions in situations where the informational content of the signal distribution is not concentrated towards the top. For these situations, the market with a credit bureau and few financing rounds turns out to be more efficient and can dominate even a centralized auction market. Although we have established that a central auction market with an optimally chosen number of investors is always better than a sequential market, it may not always be easy to commit to limit the number of participants in an auction. In the credit bureau market, there is no need for such a commitment—the market breaks down endogenously after a limited set of rounds. Hence, the market with a credit bureau can lead to higher social surplus than a large auction market because it restricts the competition among investors, allowing them to utilize their information more efficiently. Surprisingly, the increased surplus can more than compensate for the higher rent left to investors, so that the entrepreneur can also be better off than in an auction market.

We also show that the sequential market with a credit bureau can have multiple equilibria, due to the feedback effect of equilibrium beliefs. When investors believe that rejected borrowers have low credit quality, rejection is more costly for entrepreneurs. Therefore, entrepreneurs will be more likely to ask for unfavorable financing terms in early rounds to avoid rejection, which means that rejection is a signal of worse quality—a self-fulfilling prophesy. Hence, equilibria with few financing rounds and equilibria with more financing rounds can coexist. The equilibria with few financing rounds are often worse for entrepreneurs because of the unfavorable financing terms,

but can be good for social surplus. This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. An interest rate cap will eliminate "sub-prime" markets for rejected borrowers, and hence will eliminate the socially inefficient equilibria with many financing rounds.

In our main analysis, all investors have access to privately observed "soft information." We also consider an extension where some investors do not have such information, or can commit not to use it and instead only rely on publicly available "hard information" in their lending decisions. Surprisingly, we show that such lenders are sometimes able to out compete soft information lenders when there is a credit bureau, even though they have strictly less information. The reason is that a hard information lender never makes any rents, which for high credit quality entrepreneurs can make them more attractive despite the lower surplus created.

Related literature: [INCOMPLETE] Our paper is related to three bodies of work. The first one studies the ability of financial markets to aggregate information. The vast majority of papers studying information aggregation in financial markets focuses on endowment economies in which information has no real value, or settings in which trading takes place on organized exchanges.[1] We contribute to this literature by studying information aggregation in a production economy with a decentralized sequential search market. Notable exceptions are Lauermann and Wolinsky (2016) and Zhu (2012), who consider a decentralized search setup, but in endowment economies with a seller searching for buyers. Lauermann and Wolinsky (2016) assumes that there is an infinite number of buyers and that a search history is not observable. As a result, the focus of Lauermann and Wolinsky (2016) is more narrow. Its main conclusion is that search markets are always worse at aggregating information than the centralized markets, which is not necessarily true in our more general setup. Zhu (2012) focuses on the case, in which buyers have private values and assumes that a search history is not observable.

Our paper is also related to the vast literature on search markets. Search models have been used extensively in labor economics, but also in models of financial over-the-counter markets (see, e.g., Duffie, Garleanu and Pedersen (2005)). These models focus on the friction introduced by the cost of finding a counterparty in private value environments. We differ from this literature by focussing on the consequences of sequential interactions in a common-value environment, where the entrepreneur is infinitely patient and has no search cost. Hence, all our results are driven by informational frictions that are absent in the standard search literature.

[1] An incomplete list of papers includes Grossman (1976), Milgrom and Weber (1982), Kremer (2002).

Our paper is also related to the literature on relationship lending, e.g. Rajan (1992). These papers have in common with ours the adverse selection created for other borrowers when an informed lender refuses credit, but in the context of an existing borrower rather than a first-time borrower.

1. Setup

We consider a penniless entrepreneur seeking outside financing for a new project from a set of $N < \infty$ investors indexed by $i \in \{1, ..., N\}$.[2] All agents are risk neutral. The project requires one unit of investment, and can be of two types: good (G) and bad (B), where the unconditional probability of the project being good is π. If the project is good it pays $1 + X$. Otherwise, it pays 0. We denote the net present value, or NPV, of the project by V, a random variable that takes value X if the project is good and value -1 if the project is bad.

No one knows the type of the project but investors have access to a screening technology. When an investor makes an investigation, he gets a privately observed informative signal $s \in [0, 1]$ drawn from a distribution $F_G(s)$ with density $f_G(s)$ in case the project is good and from a distribution $F_B(s)$ with density $f_B(s)$ in case the project is bad. We make the following assumption about the signal distribution:

ASSUMPTION 1: *Signals satisfy the monotone likelihood ratio property (MLRP):*

$$\forall s > s', \quad \frac{f_G(s)}{f_B(s)} \geq \frac{f_G(s')}{f_B(s')}.$$

Both $f_G(s)$ and $f_B(s)$ are continuously differentiable at $s = 1$, $f_B(1) > 0$, and $\lambda \equiv f_G(1)/f_B(1) > 1$.

Without loss of generality, we will also assume that $f_G(s)$ and $f_B(s)$ are left-continuous and have right limits everywhere. Assumption 1 ensures that higher signals are at least weakly better news than lower signals. Assuming that densities are continuously differentiable at the top of the signal distribution simplifies our proofs, but is not essential for our results.

We denote the likelihood ratio at the top of the distribution by λ, a quantity that will be important in our asymptotic analysis. Assuming $\lambda > 1$ ensures that MLRP is strict over a set of non-zero measure, which in turn implies that as $N \to \infty$, an

[2] Although we assume the entrepreneur has zero wealth to invest in the project, this is not essential for our results. Our results generalize to situations where the entrepreneur has either wealth or other assets to pledge against the project.

observer of all signals would learn the true type with probability one. Therefore, for large enough N, the aggregate market information is valuable for making the right investment decision.

To exclude trivial cases, we assume that the signal of a single investor can be sufficiently optimistic for the expected value of the project to be positive:

ASSUMPTION 2: $E(V|S_i = 1) > 0$.

Although the signal space is continuous with no probability mass points, it can be used to represent discrete signals by letting the likelihood ratio $f_G(s)/f_B(s)$ follow a step-function which jumps up at a finite set of points. All signals within an interval over which the likelihood ratio is constant are informationally equivalent and represent the same underlying discrete signal. Following Pesendorfer and Swinkels (1997), we call such intervals "equivalence intervals." Representing discrete signals as equivalence intervals is a convenient way of making strategies pure when they are mixed in the discrete space: one can think of a continuous signal s as a combination of a discrete signal and a random draw from the equivalence interval, where a different draw can result in a different strategy even when the underlying discrete signal is the same.

The entrepreneur contacts investors sequentially in a random order. When contacting investor i the entrepreneur makes a take-it-or-leave-it offer, in which she asks for the loan size of one in an exchange for the repayment of $1 + r_i$ in case the project is successful. Based on the signal, the investor decides whether to approve the application or not. If the offer is accepted the project is financed and production commences. If the offer is rejected the entrepreneur goes to the next investor in line. We assume that the entrepreneur commits not to visit the same investor twice, and that the approved offer cannot be taken to other investors.[3]

2. Maximal social surplus and profit

We start by analyzing the most transparent setting where investors know both the order in which the entrepreneur approaches them and can observe all previous offers r_i. We first show that picking a vector of offers $\{r_i\}_{i=1}^N$ is equivalent to picking a set of screening thresholds $\{s_i^*\}_{i=1}^N$ such that the project gets started if any investor i has a signal S_i above the threshold s_i^*. To see this, consider an investor i who is approached with an offer of financing the project at interest rate r_i. The investor conditions on

[3] It is clearly in the interest of the entrepreneur to commit not to re-visit the same investor when there is only one investor available. It is an open question whether this result holds for any number of investors.

the history Ω_i, which contains the information that each previous investors $j < i$ has rejected the project at interest rate r_j. His expected profit from accepting to finance the project given his own signal $S_i = s$ is then given by

$$\Pr(G|\Omega_i, S_i = s)r_i - \Pr(B|\Omega_i, S_i = s).$$

The investor accepts the offer if and only if

$$r_i \geq \frac{\Pr(B|\Omega_i, S_i = s)}{\Pr(G|\Omega_i, S_i = s)} = \frac{\Pr(B|\Omega_i)}{\Pr(G|\Omega_i)} \frac{f_B(s)}{f_G(s)}, \tag{1}$$

where the last equality follows from Bayes' rule and the independence of signal S_i and history Ω_i conditional on the true state of the project. MLRP implies that the right-hand side decreases in s. Therefore, the project is either rejected for any signal, or there is a unique screening level s_i^* such that the offer is accepted if and only if $S_i \geq s_i^*$.

2.1. The social planner's problem

As a benchmark, we now study a social planner's problem where offers are set to maximize social surplus. The surplus the social planner can achieve will give us an upper bound on equilibrium surplus, and will also be useful for establishing some of our results.

Section 2 shows that we can write the social planner's problem as a choice of screening thresholds $\{s_i^*\}_{i=1}^N$, which amounts to trading off rejection of good projects versus acceptance of bad projects:

$$\max_{\{s_i^*\}_{i=1}^N} \pi X \left(1 - \prod_{i=1}^N F_G(s_i^*)\right) - (1-\pi)\left(1 - \prod_{i=1}^N F_B(s_i^*)\right). \tag{2}$$

Note that not every choice of screening thresholds $\{s_i^*\}$ is implementable with feasible interest rates $r_i \leq X$, but we show below that the optimal solution to (2) is always implementable:

PROPOSITION 1: *The socially optimal screening policy is to use the same screening threshold $s_n^* < 1$ for $n \leq N$ rounds and set the screening level at 1 for remaining rounds. The optimal screening threshold is an increasing function of n and is the lowest signal at which investor n breaks even at the maximal interest rate X:*

$$\Pr(G|S_n = s_n^*, S_1, ..., S_{n-1} \leq s_n^*)X - \Pr(G|S_n = s_n^*, S_1, ..., S_{n-1} \leq s_n^*) \geq 0. \tag{3}$$

The social surplus is the same as that generated in a first-price auction where n in-

vestors bid with interest rates for the right to finance the entrepreneur.

If $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ *is a strictly decreasing function of* s *then* $n = N$ *and the expected surplus strictly increases with the number of screenings. If* $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ *is a strictly increasing function for* $s \in [s_n^*, 1]$ *then the maximal expected surplus is achieved with no more than* n *screenings.*

Proof: See the Appendix.

Proposition 1 shows that it is optimal to use the same screening threshold for the first $n \leq N$ investors, and completely ignore the rest of the signals. The screening thresholds correspond to a set of interest rate offers that increase in each round until they reach the maximal feasible rate X in the n^{th} round.

The screening threshold s_n^* is set such that the project just breaks even when $\max\{s_1, s_2, \ldots, s_n\} = s_n^*$. The project is financed if and only if the maximal of n signals is higher than s_n^*. In Axelson and Makarov (2016) we show that this is also the investment outcome realized in a first-price auction with n bidders. Thus, no sequential credit market can generate higher surplus than a first-price auction if the number of investors in the auction is chosen optimally.

Note that the investment outcome is equivalent to the decision of a social planner who observes only the first-order statistic of n signals when making his investment decision. Hence, there is a potentially substantial loss of efficiency relative to the first-best setting where all signals are used in the decision making. In Axelson and Makarov (2016) we show that unless the likelihood ratio $f_G(s)/f_B(s)$ goes to infinity at the top of the signal distribution information aggregation fails. As a result, investment mistakes are not eliminated even when the market becomes infinitely large. The next lemma provides an upper bound on the maximal expected surplus that can be achieved with a screening technology that satisfies $f_G(1)/f_B(1) = \lambda$.

LEMMA 1: *The maximal expected social surplus with a screening technology that satisfies* $f_G(1)/f_B(1) = \lambda$ *is no larger than* $\max(\pi X - (1-\pi)/\lambda), 0)$.

Proof: See the Appendix.

If all signals were perfectly observable to the social planner, he would be able to eliminate all investment mistakes in the limit as the number of signals goes to infinity, and create expected surplus of πX. Lemma 1 shows that a sequential market will do strictly worse. We will use the upper bound on surplus derived in Lemma 1 to study the behavior of screening thresholds in Section 3.

Proposition 1 shows that the social planner may find it optimal to restrict the number of screening rounds. This surprising result is due to the fact that the investment

decision is based only on the information contained in the first-order statistic of signals. For some signal distributions, as outlined in the conditions of the proposition, it is more informative to rely on the highest signal in a small sample rather than a large sample.

2.2. Maximal profits

We now turn to the problem of an entrepreneur who can commit to a set of interest rates to maximize her profits, where we still assume that investors can observe the interest rates offered in each round. Again, we can express the problem as a choice of screening thresholds $\{s_i^*\}_{i=1}^N$ such that an investor will accept the offer in round i if his signal is above s_i^*. Investor i knows that the entrepreneur has been rejected in previous rounds with screening thresholds $\{s_j^*\}_{j=1}^{i-1}$. Using Bayes' rule and Equation (1) then implies that the interest rate in round i is

$$r_i = \frac{1-\pi}{\pi} \frac{f_B(s_i^*)}{f_G(s_i^*)} \Pi_{j=1}^{i-1} \frac{F_B(s_j^*)}{F_G(s_j^*)}. \tag{4}$$

The maximization problem of the entrepreneur is then

$$\max_{s_1^*,\ldots,s_N^*} \pi \sum_{i=1}^{N} \Pi_{j=1}^{i-1} F_G(s_j^*)(1-F_G(s_i^*))(X-r_i), \tag{5}$$

where we have used the fact that the entrepreneur makes profits only when the project is good, and where $\Pi_{j=1}^{i-1} F_G(s_j^*)(1-F_G(s_i^*))$ is the probability of being rejected in rounds $j < i$ and being accepted in round i when the project is good.

The next proposition shows how the surplus generated when the entrepreneur maximizes profits compares with the socially optimal solution.

PROPOSITION 2: *Suppose the likelihood ratio $f_G(s)/f_B(s)$ is continuous. For any finite N, the surplus generated when the entrepreneur maximizes profits is strictly lower than the surplus generated by the social planner. If $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s, both the social planner and the entrepreneur use all rounds, and the profits of the entrepreneur approach the maximal social surplus as N goes to infinity.*

Proof: The first part of the Proposition follows from the fact that the social planner uses a policy such that when the last round is reached, every positive NPV project is financed. This is achieved by setting the interest rate at X so that all rents accrue to the investor. The entrepreneur is always better off deviating from this policy by setting a lower interest rate at the expense of social surplus.

To prove the second part of the proposition, note that when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s, the social planner sets the same screening threshold s_N^* in all N rounds. From Equation 3, it follows that this screening threshold must go to one with N. Hence, the interest rate offer in any given round is such that only the most optimistic investors accept it. The rent captured by an investor with signal $s \in [s_N^*, 1]$ who accepts the offer is given by

$$\Pr(G|\Omega_i, S_i = s)r_i - \Pr(B|\Omega_i, S_i = s),$$

where r_i is set such that an investor with signal s_N^* just breaks even. Hence, the rent to the investor equal

$$\Pr(G|\Omega_i, S_i = s)\left(\frac{\Pr(B|\Omega_i, S_i = s_N^*)}{\Pr(G|\Omega_i, S_i = s_N^*)} - \frac{\Pr(B|\Omega_i, S_i = s)}{\Pr(G|\Omega_i, S_i = s)}\right).$$

Rents are proportional to the difference in break-even interest rates for an investor with a signal at the threshold and an investor with a signal above the threshold. From the continuity of the likelihood ratio, the rent goes to zero as the screening threshold goes to one. This implies that an entrepreneur who replicates the strategy of the social planner is able to capture the maximal surplus as N goes to infinity. Because this maximal surplus is an upper bound on profits, profits under the entrepreneur's maximizing strategy (which typically does not involve a fixed screening threshold) must also approach this limit. *Q.E.D.*

Proposition 2 implies that as the market grows large, and when using many rounds is socially optimal, the entrepreneur leaves no rent to investors and is able to generate the same surplus and profits as a centralized first price auction in the limit. We next show that the sequential market can do strictly better than a centralized market with free entry, both with respect to surplus and revenues.

To see this, note that if an entrepreneur runs a first-price auction for financing the project but cannot commit to restrict the number of participating investors, all investors will show up and the entrepreneur is better off ex post taking all bids (even unsolicited) into account. This can be suboptimal ex ante, as is explained in Proposition 1—for some signal distributions, the entrepreneur would like to restrict the number of participating bidders. For example, in Axelson and Makarov (2016) we show that in the case of $f_B(s) = 1$ for all $s \in [0, 1]$, and $f_G(s) = 0$ for $s \in [0, 1/2]$ and $f_G(s) = 2$ for $s > 1/2$, the surplus in the first-price auction is maximized with a single investor, with s_1^* being set to $1/2$. With free entry, the surplus will be strictly lower.

In the sequential market with observed interest rate, suppose the entrepreneur sets

$s_1^* = 1/2$. This generates the maximal surplus, and all surplus is captured by the entrepreneur. There will be no second round, because if the project is rejected by the first investor, the updated credit quality is so low that no investor would be willing to finance the project at any interest rate. Hence, the market closes down endogenously. The sequential nature of the market gives the entrepreneur endogenous commitment power to limit the number of investors, as opposed to a centralized setting where all investors show up at the same time.

The following proposition shows that this result is more general:

PROPOSITION 3: *As the number of potential investors N goes to infinity, surplus and profits in the sequential market with observable offers either equal surplus and profits in a first-price auction with free entry, or are strictly higher.*

Proof: The proof of Proposition 2 shows that the entrepreneur in the sequential market can always replicate surplus and profits of a free entry auction in the limit as N goes to infinity, and that all surplus goes to the entrepreneur using this strategy. When using all N rounds does not maximize surplus, the entrepreneur in the sequential market may be able to earn strictly more profits with a smaller set of investors as shown in the example above. These profits are higher than the surplus in the free entry auction, and hence surplus in the sequential market (which is always weakly greater than entrepreneurial profits) must also be strictly higher in the sequential markets. *Q.E.D.*

Note that an auction in which the entrepreneur can commit to restrict entry and can set an optimal reserve price to maximize profits may well create more profits and surplus than the sequential market.

3. Equilibrium with a credit bureau

In this section, we assume that the offer an entrepreneur makes to an investor is not observable to other investors. However, an investor has access to information collected by a credit bureau. In practice, credit bureaus keep track of the history of loans and repayments, as well as any credit checks made in the past on the applicant. Because there is only one project in our setting, there is no history of previous loans or repayments for an entrepreneur seeking financing. However, the information contained in the number of credit checks made on the entrepreneur allows an investor to infer how many times the entrepreneur has been rejected previously. Importantly, as in practice, the terms at which an entrepreneur was rejected are not observable.

Because offers are not observable, investors have to form beliefs about the terms at which the entrepreneur has been rejected previously. Beliefs are formed about the thresholds used in previous rounds. Denote by $\mathbf{s_{i-1}} = \{s_j\}_{j=1}^{i-1}$ the vector of thresholds used prior to round i. Denote by $\mathbf{\hat{s}_{i-1}}$ investor i's beliefs about previous thresholds. Given beliefs $\mathbf{\hat{s}_{i-1}}$, the interest rate offer in round i that corresponds to a screening threshold s_i is then given by

$$r_i(s_i, \mathbf{\hat{s}_{i-1}}) = \frac{1-\pi}{\pi} \frac{f_B(s_i)}{f_G(s_i)} \Pi_{j=1}^{i-1} \frac{F_B(\hat{s}_j)}{F_G(\hat{s}_j)}. \tag{6}$$

We define a pure-strategy equilibrium as a vector of thresholds $\mathbf{s_N^*}$ that solves

$$\max_{\mathbf{s_N}} \pi \sum_{i=1}^{N} \Pi_{j=1}^{i-1} F_G(s_j)(1 - F_G(s_i))(X - r_i(s_i, \mathbf{s^*_{i-1}})). \tag{7}$$

Note that our equilibrium definition assumes that out-of-equilibrium beliefs coincide with equilibrium beliefs. In other words, an investor who expects to receive an interest rate offer r_i does not change his beliefs about previous thresholds if he is offered an out-of-equilibrium interest rate r_i'. This is a natural requirement in our setting, because the optimal action of an entrepreneur entering round i is independent of his previous history. Hence, there is no room for the entrepreneur to affect investor beliefs through any type of signalling.

The difference between problem (7) and problem (5) where interest offers are observed is that the choice of threshold in the credit bureau case does not affect interest rates in future rounds. It is instructive to rewrite problem (7) in a recursive way. In the last round, the entrepreneur solves the following maximization problem:

$$V_N = \max_{s_N} (1 - F_G(s_N)) \left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right),$$

where we condition on the project being good, since the entrepreneur gets zero pay-off regardless of his strategy when the project is bad. $1-F_G(s_N)$ is the probability that the interest rate offer $r_N(s_N, \mathbf{s^*_{N-1}})$ gets accepted, conditional on the project being good. In the second to last round, the entrepreneur solves

$$V_{N-1} = \max_{s_{N-1}} (1 - F_G(s_{N-1})) \left(X - r_{N-1}(s_{N-1}, \mathbf{s^*_{N-2}})\right) + F_G(s_{N-1}) V_N.$$

Note that the continuation value V_N after rejection in round $N-1$ does not depend on the choice of the threshold s_{N-1} since investor N does not observe the interest rate

offer r_N. Iterating backwards, the maximization problem in round i is

$$V_i = \max_{s_i}(1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s}^*_{\mathbf{i-1}})\right) + F_G(s_i)V_{i+1}. \tag{8}$$

An equilibrium can then also be defined as a vector of screening thresholds $\mathbf{s}^*_{\mathbf{N}}$ such that s^*_i solves problem (8) for all rounds i (where we define $V_{N+1} \equiv 0$).

The recursive formulation helps in our proof of existence, and for illustrating the economics of the problem. Below we provide sufficient conditions for the existence of a pure-strategy equilibrium.

PROPOSITION 4: *Suppose that $f_B(s)/f_G(s)$ is a continuous function and for any y*

$$(1 - F_G(s))\left(y - \frac{f_B(s)}{f_G(s)}\right) \tag{9}$$

is a quasi-concave function of s. Then there exists a pure-strategy equilibrium in the game with any number of investors N and publicly recorded rejections.

Proof: Because the entrepreneur cannot affect investors' beliefs one can view an optimization problem of setting the optimal screening threshold level at each round i, $i = 1, 2, \ldots, N$ as if it is done by a fictitious agent i. Each fictitious agent i takes decisions of other agents as given and solves (8), which is equivalent to maximizing (9) with an appropriately chosen y. By assumption the payoff of each agent i is quasi-concave in his own action and continuously depends on the actions of other agents. Therefore, by the theorem of 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium. *Q.E.D.*

The main difference in the credit bureau case relative to the case where offers are observed is that whenever the entrepreneur decides on the interest rate offer, he does not internalize the effect this has on the continuation value. Because of this, the entrepreneur will tend to offer higher interest rates (lower screening thresholds) to early investors relative to the case where offers are observed. This has several consequences: First, profits will be lower. Second, the entrepreneur will tend to leave more rents to investors. Third, the entrepreneur will not be able to replicate the strategy of capturing maximal surplus in large markets where using all investors is socially optimal:

PROPOSITION 5: *Suppose MLRP holds strictly, and the likelihood ratio is continuous. The entrepreneur in the credit bureau case uses a strictly lower screening threshold in the first round than in the case where offers are observed, and there is an $\varepsilon > 0$ such that for any number of investors N the total expected profit of all investors in equilibrium with N investors and a credit bureau is greater than ε. For the case where*

$\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s, the profits and surplus are strictly lower than in a first-price auction with free entry as N goes to infinity.

Although the entrepreneur in the credit bureau case does worse than a centralized large market when having many rounds is optimal for a social planner, she can do better when it is optimal to restrict the number of rounds. Proposition 6 shows how the equilibrium number of screenings depends on the behavior of the likelihood ratio. When $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function, and therefore having as many investors as possible is socially optimal, the entrepreneur visits all available investors in equilibrium with a credit bureau. If $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at some neighborhood of $s = 1$, and therefore, smaller markets are preferred, the entrepreneur is able to apply to only a finite number of investors.

PROPOSITION 6: *If $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s then the entrepreneur applies to all available investors. If $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at some neighborhood of $s = 1$ then the entrepreneur applies only to a finite number of investors.*

Proof: See the Appendix.

Just as was the case when interest rates are observed, Proposition 6 suggests that the sequential credit market with a credit bureau can do a better job than an auction if the number of potential investors is outside the entrepreneur's control. In fact, for the example above where one round is optimal when offers are observed, the credit bureau case becomes equivalent to the case with observed offers and dominates a free-entry auction. Again, the endogenous market depth gives the entrepreneur commitment power to restrict the number of investors.

Proposition 6 shows that when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s the interest rates are set in such a away that the entrepreneur is able to apply to all available investors. However, as we showed above, the entrepreneur is still not able to extract all rents from investors. We now turn to the least transparent case where there is no credit bureau, and show the surprising result that the entrepreneur will then be able to extract all rents in large markets.

4. Equilibrium without a credit bureau

In this section we study the least transparent case where neither previous offers nor rejections are observed by an investor that is approached for financing. In contrast to the credit bureau case, the entrepreneur now has private information about how many

times she has been rejected. An investor observes only the interest rate he is being offered, from which he has to make a conjecture about how many times the entrepreneur has been rejected previously. The investor is more likely to accept a given interest rate offer if he believes that the entrepreneur has been rejected fewer times. This opens up the possibility that the entrepreneur can use the interest rate offer to signal her type.

In general, as is typical in dynamic signaling games, there could exist many equilibria unless one imposes some restrictions on out-of-equilibrium beliefs. A pooling equilibrium would be one in which the entrepreneur asks for the same interest rate in different rounds, while a separating equilibrium would be one in which the interest rate offer is different in each round so that investors can infer how many times the entrepreneur has been rejected. We show below that only separating equilibria in which the entrepreneur increases her interest rate offer every time she is rejected survive the Cho and Kreps (1987) intuitive criterion:

PROPOSITION 7: *Suppose there are N investors and a rejection by any investor is not observed by other investors. Then any equilibrium that survives the Cho and Kreps intuitive criterion must be separating.*

Proof: See the Appendix.

To prove that there can be no pooling equilibrium in which an entrepreneur asks for the same interest rate in early and late rounds we show that an "early" entrepreneur can always profitably deviate to a lower interest rate which a "late" entrepreneur would find too costly to mimic. When such a deviation exists, the equilibrium does not survive the Cho-Kreps intuitive criterion.

To understand this result, note that an entrepreneur in a late round finds rejection costlier than an entrepreneur in an early round, because she has fewer opportunities left to seek financing. By making a low interest rate offer that is sufficiently unlikely to be accepted, an "early" entrepreneur can therefore credibly separate from "late" entrepreneurs.

Suppose that investors believe that any deviation to a lower interest rate comes from the "early" type. If the deviation is to an interest rate only slightly below the pooling rate, investors would be more likely to provide financing than at the pooling rate because they perceive the project to have higher quality. Hence, such a deviation is not credible, because all entrepreneurs would prefer it. However, by lowering the interest rate sufficiently, the probability of acceptance can be made arbitrarily small even under optimistic beliefs. Therefore there exists an interest rate low enough such that a "late" entrepreneur is indifferent between this offer and the pooling rate, and such that she is strictly worse off with even lower interest rate offers. Because rejections

are costlier for the "late" entrepreneur than for an "early" entrepreneur the "early" entrepreneur is strictly better off by deviating and asking for the interest rate at which the "late" entrepreneur is indifferent. Hence, the "early" entrepreneur always separates herself from the "late" entrepreneur, which makes a pooling equilibrium impossible.

The next proposition characterizes separating equilibria:

PROPOSITION 8: *In any pure-strategy equilibrium satisfying the Cho-Kreps intuitive criterion, interest rates strictly increase and screening thresholds strictly decrease with the number of rejections. Furthermore, a screening threshold s_N^* solves*

$$V_N \equiv \max_{s_N} \pi(1 - F_G(s_N)) \left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right),$$

and for each $i < N$, s_i^* solves

$$V_i \equiv \max_{s_i} \pi(1 - F_G(s_i)) \left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+1}, \tag{10}$$

$$s.t. \quad (1 - F_G(s_i)) \left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+2} \leq V_{i+1}. \tag{11}$$

Under the conditions of Proposition 4 there exists a pure-strategy equilibrium.

Proof: The system of equations (10) is similar to the system of equations (8), which describes an equilibrium with a credit bureau. The main difference is the presence of the incentive compatibility constraint (11). In any fully separating equilibrium, it must be that no type wants to mimic another type. In our case, the proof of Proposition 7 shows that early entrepreneurs never want to mimic late entrepreneurs. Therefore, the only incentive compatibility constraints that need to be imposed are the ones that ensure that late entrepreneurs do not mimic early entrepreneurs, which is the constraint (11). We also show in the proof of Proposition 8 below that because the cost of rejection increases with the number of rounds, the only potentially binding incentive compatibility constraints are the ones that ensure that an entrepreneur in round $i + 1$ does not mimic an entrepreneur in round i. In equilibrium $s_i^* > s_{i+1}^*$ because the constraint (11) is always violated at $s_i^* = s_{i+1}^*$. Hence both interest rate offers and the probability of financing increase strictly in each round, and the entrepreneur can always visit all available investors. Finally, as in Proposition 4, because the entrepreneur cannot affect investors' beliefs one can view an optimization problem of setting the optimal screening threshold level at each round i, $i = 1, 2, \ldots, N$ as if it is done by a fictitious agent i. Each fictitious agent i takes decisions of other agents as given and solves (10). By assumption the payoff of each agent i is quasi-concave in his own action and continuously depends on the actions of other agents. Also, quasi-concavity of the payoff ensures that the action space of every agent that satisfies the constraint is a

concave set. Therefore, by the theorem of 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium. *Q.E.D.*

The next proposition shows that if the MLRP holds strictly then as the number of investors increases the entrepreneur's expected profit approaches that generated in the first-price auction.

PROPOSITION 9: *Suppose MLRP holds strictly and there exists a separating equilibrium in the case of N investors and no credit bureau. Then as N goes to infinity the entrepreneur's surplus converges to that generated in the first-price auction with free entry.*

Proof: See the Appendix.

In the proof we show that because of the incentive compatibility constraints (11) all screening thresholds converge to one and the interest rate in the last screening round converges to X. Using the expression for the interest rate (6) we can write the latter as

$$\frac{1-\pi}{\pi}\frac{f_B(s_N^*)}{f_G(s_N^*)}\frac{\prod_{i=1}^{N-1}F_B(s_i^*)}{\prod_{i=1}^{N-1}F_G(s_i^*)} \to X.$$

Comparing the above expression to (3) we can see that the probabilities of being rejected in the sequential search and the first-price auction must converge, which leads to the statement of the proposition.

In Section 2.1 we show that if $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function then the first-price auction generates the maximal possible surplus, and the surplus increases with the number of investors. Proposition 9 thus implies that under the same conditions, in the sequential credit market without a credit bureau the entrepreneur benefits from the large market and manages to receive the maximum possible surplus in the limit.

The result that all surplus can be extracted contrast with the result when there is a credit bureau. The need for signalling when there is no credit bureau creates commitment power for the entrepreneur to ask for low interest rate offers early on. On the other hand, the need for signalling makes it impossible to limit market depth when few rounds is socially optimal.

5. Hard vs. Soft Information

So far we have assumed that all investors have access to privately observed "soft information." Next we consider an extension where some investors do not have such information, or can commit not to use it and instead only rely on publicly available

"hard information" in their lending decisions. In this section we use the following interpretation of our main setup. We assume that "hard information" about the project quality is public and therefore, is summarized by the likelihood ratio $z \equiv \pi/(1-\pi)$. We assume that z is sufficiently high so that the project can be financed without any additional due diligence process.

As before, there are N investors who get "soft information" about the project quality. Unlike hard information, soft information is private and is modelled as an informative signal $s \in [0,1]$, the realization of which is privately observed by an investor and is an independent draw from a distributions $F_G(s)$ and $F_B(s)$.

In addition, we assume that there are investors who either do not have access to soft information, or can commit not to use it. We assume that these investors are competitive. The break-even interest rate is $1/z$ so that the entrepreneur gets expected profit of $\pi(X-1/z)$ if she applies to this set of investors. We show next that such lenders are sometimes able to out compete soft information lenders when there is a credit bureau, even though they have strictly less information.

Figure 1 shows the entrepreneur's profit in two cases: if she obtains financing from investors that use only hard information (blue line) and if she obtains financing from investors that use both hard and soft information (red line). It is assumed that $X = 1$, $N = 100$, $f_B(s) \equiv 1$, and $f_G(s) = 2s$. We can see that if the ex ante quality of the project is good then the entrepreneur is better off if she does not apply to investors that use soft information. The intuition for the above result is that a hard information lender never makes any rents. With a credit bureau in place Proposition 5 shows that investors earn some rent in equilibrium. The example demonstrates that the rent can be so large so that it outweighs the benefits of using information, even though the market as an aggregate possesses perfect information.

Our results in Axelson and Makarov (2016) imply that if the entrepreneur gets financing via auctions then she is always better off obtaining financing from investors who use both hard and soft information. Since as we show in Proposition 9 a large sequential market without a credit bureau delivers the same surplus to the entrepreneur as the first-price auction, we see that hard information lenders cannot compete with soft information lenders if there is no credit bureau.

6. Multiple equilibria

So far we have focused on large markets. We now consider small markets. In this section we show that there can be multiple equilibria under quite natural assumptions. In particular, we provide an example in which two equilibria exist in the case of two

investors and a credit bureau. Suppose that $X = 1$, $f_B(s) \equiv 1$ and $f_G(s)$ is given by the following equation:

$$f_G(s) = 0.25 + \frac{1}{\exp\left(-100\left(s - \frac{1}{3}\right)\right) + 1} + \frac{0.25}{\exp\left(-100\left(s - \frac{2}{3}\right)\right) + 1}. \tag{12}$$

Panel A of Figure 2 draws densities $f_B(s)$ and $f_G(s)$. The so defined densities $f_B(s)$ and f_G represent a smoothed version of the case when investors' signals takes three values: low, medium and high as depicted in Figure 2 Panel B. If the project is bad then any of the values is equally likely. If the project is good then the respective probabilities of low, medium and high signals are 1/12, 5/12, 1/2.

Figure 3 plots the expected profit of the entrepreneur as a function of the screening threshold s if there is only one investor available. Panels A, B, and C correspond to the three initial values of the likelihood ratio: $z = 0.9$, $z = 0.95$, and $z = 1$. We can see that two flat areas of $f_G(s)$ lead to two humps in the expected surplus. Confirming the results of Lemma **??**, at high values of z the entrepreneur's profit is maximized at low screening thresholds while at low values of z the profit is maximized at high screening thresholds. There is a value of z (Panel B, $z = 0.95$) at which the same expected surplus is achieved at two different values of s. Even though if $z \neq 0.95$ there is a unique equilibrium in case of a single investor two equilibria can realize in the case of two investors.

In the first equilibrium, the second investor believes that the entrepreneur asks for a low screening threshold from the first investor. This makes it optimal for the entrepreneur to ask for a low screening threshold because the rejection then is very costly for the entrepreneur: If she is rejected she can no longer obtain financing from the second investor even with the most optimistic signal.

In the second equilibrium, the second investor believes that the entrepreneur asks for a high screening threshold. In this case, the cost of rejection is not so high because even if rejected the entrepreneur has still a chance to obtain financing from the second investor. As a result, it is optimal for the entrepreneur to try for a low interest rate and high screening threshold from the first investor.

For the two equilibria to exist it must be that the entrepreneur's choice of thresholds is consistent with investors' beliefs. This happens if the likelihood ratio z is such that $z > 0.95$ and $z(X - V_1) < 0.95$, where V_1 is the expected profit of the entrepreneur in the second equilibrium after she is rejected by the first investor.

If z is just below 0.95 then only the second equilibrium with two screenings exists because even with a single investor the entrepreneur is better off with a high screening threshold. Therefore, no matter what the second investor believes, the entrepreneur

will ask the first investor for a high screening threshold. If z is just above 1.03 then only the first equilibrium with one screening exists because even if the second investor believes that the screening threshold at the first investor is high the entrepreneur will find it profitable to deviate and ask for a low screening threshold. As a result, the entrepreneur can no longer take advantage of two investors and therefore can no longer attain a high expected surplus.

Panel A of Figure 4 plots the entrepreneur's expected profit in the two equilibria as a function of her initial likelihood ratio z. Panel B plots social surplus. The blue line corresponds to the first equilibrium with one screening; the red line to the second equilibrium with two screenings. We can see that the entrepreneur is better off in the second equilibrium, in which she can be screened twice. Social surplus, however, is higher in the first equilibrium, in which the entrepreneur is screened only once.

This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. Figure 5 shows interest rates in the two equilibria. The blue line shows an interest rate in the first equilibrium with one screening. The red and magenta lines show interest rates in the second equilibrium. Naturally, an interest rate increases if the entrepreneur is rejected by the first investor. If there is an interest rate cap so that the rejected entrepreneur can no longer obtain financing in the second round then the second equilibrium is no longer sustainable. Thus, an interest rate cap can eliminate sub-prime markets for rejected borrowers, and hence can eliminate the socially inefficient equilibria with many financing rounds.

Panel A also illustrates, perhaps surprisingly, that the entrepreneur's profit can be non-monotone in the ex-ante project's quality. This happens because of the entrepreneur's inability to commit to ask for a high screening threshold from the first investor, or in other words, for a low interest rate. As a result, investors get higher rent and the entrepreneur is worse off.

7. Conclusion

We have developed a sequential credit market model to analyze the efficiency of primary capital markets for new projects. We compare three regimes of differing level of transparency: A sequential market where lenders have no information about the search history of an entrepreneur, a sequential market where lenders can observe the search history via a credit bureau, and a centralized auction markets. None of these markets lead to first-best investment decisions, even when the number of potential investors grows so large that the aggregate information in the market allows for perfect investment decisions, and even when entrepreneurs are infinitely patient and there

are zero search costs. Moving to a more transparent market via the introduction of a credit bureau tends to increase rents to investors at the expense of entrepreneurs, leads to shorter search for financing by the entrepreneur, and has ambiguous effects on the efficiency or resource allocation. A centralized market is more efficient than decentralized markets if the number of investors who participate in the market can be chosen optimally, but may otherwise lead to excessive competition which impedes efficiency relative to decentralized markets.

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Appendix. Proofs

Proof of Proposition 1:

Consider the maximization problem (2). Let $n \leq N$ be the largest n such that the expected surplus generated with n screenings is strictly higher than that of generated with $n-1$. Then for all $i > n$, $s_i = 1$ and for all $i \leq n$, s_i satisfy the F.O.C.:

$$\pi X f_G(s_j) \prod_{i \leq n, i \neq j} F_G(s_i) = (1-\pi) f_B(s_j) \prod_{i \leq n, i \neq j} F_B(s_i) \quad j = 1, \ldots, n. \tag{A1}$$

Let $s_* = \min\left(\{s_i\}_{i=1}^n\right)$ and $s^* = \max\left(\{s_i\}_{i=1}^n\right)$. Suppose that $s_* \neq s^*$. Consider a change Δ in the expected surplus if one changes the threshold s^* (or any if there are multiple s^*) to s_*:

$$\Delta = \prod_{s_i \neq s^*} F_B(s_i) \left[(1-\pi) F_B(s^*) - \pi X F_G(s^*) \prod_{s_i \neq s^*} \frac{F_G(s_i)}{F_B(s_i)}\right] -$$

$$- \prod_{s_i \neq s^*} F_B(s_i) \left[(1-\pi) F_B(s_*) - \pi X F_G(s_*) \prod_{s_i \neq s^*} \frac{F_G(s_i)}{F_B(s_i)}\right] =$$

$$= \prod_{s_i \neq s^*} F_B(s_i) \left(\left[F_B(s^*) - F_G(s^*) \frac{f_B(s^*)}{f_G(s^*)}\right] - \left[F_B(s_*) - F_G(s_*) \frac{f_B(s^*)}{f_G(s^*)}\right]\right),$$

where we use the F.O.C. (A1). Because of the MLRP the function

$$F_B(s) - F_G(s) \frac{f_B(s^*)}{f_G(s^*)}$$

is nondecreasing between s_* and s^*. Thus, the maximum surplus is achieved when all s_i equal to s_*. The F.O.C. (A1) therefore becomes the F.O.C. (3). Equation (3) has a unique solution because of the MLRP.

To prove that the expected surplus strictly increases with the number of screenings if $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s we need to show that for any N the solution to the maximization problem (2) is interior. Suppose on the contrary that at some N it is optimal to set s_N to one. Let N be the lowest number of screenings when this happens. The optimal screening threshold level is the same in all $N-1$ screenings and solves the F.O.C.

$$\pi X f_G(s) F_G^{N-2}(s) = (1-\pi) f_B(s) F_B^{N-2}(s).$$

Taking the derivative of the surplus with respect to s_N at $s_N = 1$ we have

$$(1-\pi)f_B(1)F_B^{N-1}(s) - \pi X f_G(1)F_G^{N-1}(s) = f_B(1)F_B^{N-1}(s)\left(1 - \lambda\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}\right) < 0,$$

where we have used the F.O.C and where the last inequality follows from the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a decreasing function of s and therefore takes the lowest value λ^{-1} at $s = 1$. As a result, it is suboptimal to set s_N to 1 and the solution must be indeed interior.

Finally, we prove that the maximal expected surplus can be achieved with no more than n screenings if $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly increasing function for $s \in [s_n^*, 1]$. We prove earlier that Equation (3) has a unique solution, s_n^*, which is strictly increasing in n. We now show that the maximand in (2) is higher for $s_i = s_n^*$, $i = 1, 2, \ldots, n$ and $s_{n+1} = 1$ than for $s_i = s_{n+1}^*$, $i = 1, 2, \ldots, n+1$. To see this, we start at a point s_{n+1}^*, and move the first n screening thresholds s down while moving the $n+1$'s screening threshold s_{n+1} up to hold $F_B(s)^n F_B(s_{n+1})$ constant:

$$\frac{ds}{ds_{n+1}} = -\frac{F_B(s)f_B(s_{n+1})}{nf_B(s)F_B(s_{n+1})}.$$

This changes the maximand with an amount proportional to

$$-F_G(s)^n f_G(s_{n+1}) - nF_G(s)^{n-1}f_G(s)F_G(s_{n+1})\frac{ds_n}{ds},$$

which has the same sign as

$$\frac{F_G(s_{n+1})f_B(s_{n+1})}{F_B(s_{n+1})f_G(s_{n+1})} - \frac{F_G(s)f_B(s)}{F_B(s)f_G(s)}.$$

By the assumption of the Proposition, this change is positive for $s_{n+1} > s_n^*$, and hence the maximand is increased by setting $s_{n+1} = 1$. But at $s_{n+1} = 1$, it is optimal to set all first n screening thresholds to s_n^*.

Q.E.D.

Proof of Lemma 1: We first observe that the maximal expected surplus respects the order induced by MLPR on the space of signal distributions. Consider two cases of informative signals. Suppose that in both cases if the project is bad the signal is drawn from the same distribution $F_B(s)$. At the same time, if the project is good then in the first case, the signal is drawn from a distribution F_{G_1} with density f_{G_1}, and in the second case, from a distribution F_{G_2} with density f_{G_2}. Suppose that for all $s > s'$

$$\frac{f_{G_1}(s)}{f_{G_2}(s)} \geq \frac{f_{G_1}(s')}{f_{G_2}(s')},$$

then the maximal surplus in the first case is no less than that in the second case. This follows from the fact that MLRP implies the monotone probability ratio (Milgrom (1981)).

Suppose for now that $f_B(s) \equiv 1$. Then given λ, the maximal expected surplus is achieved with $f_G(s) = 0$ for $s \in [0, 1 - \lambda^{-1})$ and $f_G(s) = \lambda$ for $s \in [1 - \lambda^{-1}, 1]$. Setting a screening threshold level to $1 - \lambda^{-1}$ ensures that good projects are always financed and bad projects are financed with probability λ^{-1}. Thus, with a single screening the expected surplus is $\pi(X - (1-\pi)/(\pi\lambda)$. Direct computations show that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is an increasing function for $s \in [1-\lambda^{-1}, 1]$. Thus, by Proposition 1, $\pi(X - (1-\pi)/(\pi\lambda)$ is in fact the maximal expected surplus. Finally, notice that the assumption that $f_B(s) \equiv 1$ is innocuous. For an arbitrary $f_B(s)$ the maximal surplus is achieved with $f_G(s) = 0$ for $s \in [0, \bar{s})$ and $f_G(s) = \lambda f_B(s)$ for $s \in [\bar{s}, 1]$, where $\bar{s}$ is determined by the condition that $\int_{\bar{s}}^{1} \lambda f_B(s) ds = 1$. Hence, $\int_0^{\bar{s}} f_B(s) ds = 1 - \lambda^{-1}$.

Q.E.D.

Proof of Proposition 5: The maximization problem in the first round for the entrepreneur is

$$V_1 = \max_{s_1} \pi(1 - F_G(s_1))\,(X - r_1(s_1)) + F_G(s_1)V_2,$$

where

$$V_2 = \max_{s_2,\ldots,s_N} \pi \sum_{i=2}^{N} \Pi_{j=1}^{i-1} F_G(s_j)(1 - F_G(s_i))(X - r_i(s_i, \mathbf{s^*_{i-1}})).$$

The proof of Lemma 1 reveals that if MLRP holds strictly then there exists $\delta > 0$ such that for all N, $V_2 < \pi X - (1-\pi)/\lambda - \delta$. Note that s_1 increases in V_2. Therefore, there exists $s^* < 1$ such that for all N, $s_1^* < s^*$.

The first contacted investor is break even if he finances the project with the signal equal to s_1^* and makes positive expected profit if the signal is above s_1^*. Since s_1^* does not go to one with N, the expected profit does not vanish in the limit as N goes to infinity. For the same reason, the entrepreneur generates a stritly lower expected surplus in the limit compared to that in a large auction when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing funciton of s, and a first-price auction generates a maximum surplus.

Q.E.D.

Proof of Proposition 6:

Consider first the case when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing funciton of s. Suppose that there is a round $i < N$ such that $s_i^* < 1$ and the entrepreneur is unable to contact

another investor after being rejected in round i. In this round i, the entrepreneur solves

$$\max_{s_i^*} \pi(1 - F_G(s_i^*))\,(X - r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}}))\,,$$

where $r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}})$ is given by (6). Since $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s we have

$$\frac{F_B(s_i^*)}{F_G(s_i^*)}\frac{f_G(s_i^*)}{f_B(s_i^*)} < \frac{F_B(1)}{F_G(1)}\frac{f_G(1)}{f_B(1)} = \lambda.$$

Therefore, there exists $s_{i+1}^* < 1$ such that

$$r_{i+1} = r_i \frac{f_G(s_i^*)}{f_B(s_i^*)}\frac{F_B(s_j^*)}{F_G(s_j^*)} \times \frac{f_B(s_{i+1}^*)}{f_G(s_{i+1}^*)} < X.$$

Hence, the entrepreneur has a chance to get financing if she approaches another investor. Thus, the round i cannot be the last round and the entrepreneur profits increase in the number of rounds N.

Suppose now that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function of s in some neighbourhood of $s = 1$. We first show that this implies a bound on the derivative of the likelihood ratio at $s = 1$. For simplicity, we assume that $f_B(s) \equiv 1$. Note that

$$\left(\frac{F_G(s)}{F_B(s)}\right)'_{s=1} = \lambda - 1.$$

Since

$$\left(\frac{F_G(s)}{F_B(s)}\frac{1}{f_G(s)^2}\right)' = \left(\frac{F_G(s)}{F_B(s)}\right)'\frac{1}{f_G(s)^2} + \frac{F_G(s)}{F_B(s)}\left(\frac{1}{f_G(s)^2}\right)'$$

the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at $s = 1$ implies that

$$0 \le f_G'(1) < \frac{\lambda(\lambda - 1)}{2}. \tag{A2}$$

The idea of the proof is to show that relative flatness of the likelihood ratio leads to large screening thresholds. Note that the entrepreneur can contacts all available investors when N goes to infinity only if the number of screening thresholds bounded away from one is uniformly bounded. Suppose for a moment that round i is the last round. The entrepreneur then solves the following problem:

$$\max_{s_i^*} \pi(1 - F_G(s_i^*))\,(X - r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}}))\,,$$

where $r_i(s_i, \mathbf{\hat{s}_{i-1}})$ is given by (6). To simplify notation, let

$$z = \frac{\pi}{1-\pi} \Pi_{j=1}^{i-1} \frac{F_G(\hat{s}_j)}{F_B(\hat{s}_j)}.$$

Then

$$r_i(s_i, \mathbf{\hat{s}_{i-1}}) = \frac{1}{z f_G(s_i)}.$$

The F.O.C. to the above problem is

$$-(1 - F_G(s_i)) \left(\frac{1}{f_G(s_i)} \right)' = f_G(s_i) z X - 1. \tag{A3}$$

Let $\Delta s = 1 - s_i^*$, where s_i^* is a solution to (A3). Taking the Taylor's series of (A3) at $s_i = 1$ we have

$$\frac{f_G'(1)\Delta s}{\lambda} = \lambda z X - 1 - f_G'(1) z X \Delta s + o(\Delta s).$$

Hence,

$$\Delta s = \frac{\lambda(\lambda z X - 1)}{f_G'(1)(1 + \lambda z X)} + o(\lambda z X - 1). \tag{A4}$$

Therefore,

$$F_G(s_i^*)/F_B(s_i^*) = (1 - (\lambda - 1)\Delta s) + o(\lambda z X - 1) = \left(1 - \frac{\lambda(\lambda - 1)(\lambda z X - 1)}{f_G'(1)(1 + \lambda z X)} \right) + o(\lambda z X - 1).$$

Inequality (A2) implies that

$$\left(1 - \frac{\lambda(\lambda - 1)(\lambda z X - 1)}{f_G'(1)(1 + \lambda z X)} \right) < \frac{1}{\lambda z X}.$$

Therefore, if rejected the entrepreneur is unable to contact another investor.

Q.E.D.

Proof of Proposition 7:

We have to show that the entrepreneur who has been rejected i times would always like to separate herself from those who have been rejected more than i times. Denote the entrepreneur who has been rejected $i-1$ times by E_i, and her expected surplus by V_i, $i = 1, ..., N$. Suppose contrary to the statement of the proposition that there is some pooling in equilibrium. Let i be the first instance such that E_i pools with entrepreneurs rejected more than i times. Suppose that E_{i+1} pools with E_i (the proof easily extends to any type).

Let s^* be a screening threshold asked by E_i. Let π^* be an investor's belief that the project is good if the investor is asked for a screening threshold s^* and before the

investor observes his private signal. We have

$$\begin{aligned} V_i &= \pi(1 - F_G(s^*))\,(X - r(\pi^*, s^*)) + F_G(s^*)V_{i+1}, \\ V_{i+1} &= \pi(1 - F_G(s^*))\,(X - r(\pi^*, s^*)) + F_G(s^*)V_{i+2}, \end{aligned}$$

where

$$r(z, s^*) = \frac{1-\pi^*}{\pi^*}\frac{f_B(s^*)}{f_G(s^*)}.$$

Let $\hat{\pi}$ be an investor's belief that the project is good if the investor believe that the entrepreneur is of type E_i. Clearly, $\hat{\pi} > \pi^*$. Let $\hat{s}$ be the largest screening threshold such that

$$V_{i+1} \geq \pi(1 - F_G(\hat{s}))\,(X - r(\hat{\pi}, \hat{s})) + F_G(\hat{s})V_{i+2}. \tag{A5}$$

Suppose that investors believe that the entrepreneur is of type E_i if she asks for the screening threshold $\hat{s}$. Then the type E_{i+1} entrepreneur is indifferent between asking for s^* and $\hat{s}$. Note that $V_{i+1} > V_{i+2}$ because the type E_{i+1} entrepreneur can always follow the strategy of the type E_{i+2} entrepreneur. Therefore, Equation (A5) implies that

$$\pi(1 - F_G(\hat{s}))\,(X - r(\hat{z}, \hat{s})) + F_G(\hat{s})V_{i+1} > V_i.$$

Hence, E_i is better off by deviating and asking a screening threshold, which is slightly above $\hat{s}$. At the same time, E_{i+1} is worse off by deviating to this threshold. Thus, no pooling equilibrium survives the Cho-Kreps intuitive criterion.

Q.E.D.

Proof of Proposition 9:

The proof is done in two steps. First, we show that if $r_N(1, \mathbf{\hat{s}_{N-1}})$ goes to X as N goes to infinity then the entrepreneur's surplus converges to that generated in the first-price auction. Then, we show that $r_N(1, \mathbf{\hat{s}_{N-1}})$ must go to X in equilibrium as N goes to infinity.

Step 1. Suppose that $\lim_{N\to\infty} r_N(1, \mathbf{\hat{s}_{N-1}}) = X$. The expression for social surplus (2) implies that if $\prod_{i=1}^{N} F_G(s_i^*) \to F_G^N(s^*)$ and $\prod_{i=1}^{N} F_B(s_i) \to F_B^N(s^*)$, where s^* is a screening threshold in the first-price auction, then surpluses generated in a sequential credit market and in a first-price auction are asymptotically the same.

Using equation (6) for the interest rate $r_N(1, \mathbf{\hat{s}_{N-1}})$ we can see that

$$\lim_{N\to\infty} r_N(1, \mathbf{\hat{s}_{N-1}}) = X \Leftrightarrow \lim_{N\to\infty} \lambda X \frac{\pi}{1-\pi} \prod_{i=1}^{N-1} \frac{F_G(s_i)}{F_B(s_i)} = 1. \tag{A6}$$

If the entrepreneur is rejected $N-1$ times then in the last round she solves

$$V_N = \max_{s_N}(1 - F_G(s_N))\left(X - r_N(s_N, \hat{\mathbf{s}}_{\mathbf{N-1}})\right).$$

If $\lim_{N\to\infty} r_N(1, \hat{\mathbf{s}}_{\mathbf{N-1}}) = X$ and the strict MLRP holds then $\lim_{N\to\infty} s_N^* = 1$. Proposition 8 shows that $s_i^* > s_N^*$. Therefore for any i, $\lim_{N\to\infty} s_i^* = 1$. Let $\Delta s_i = 1 - s_i^*$. Taking the Taylor's series of (A6) we have

$$\sum_{i=1}^{N-1} \Delta s_i = a_1 + O(\Delta s_N), \quad a_1 = \frac{\ln(\lambda z X)}{\lambda - 1}. \tag{A7}$$

Therefore,

$$\begin{aligned}
\prod_{i=1}^{N} F_G(s_i) &= e^{-\lambda a_1} + O(\Delta s_N), \\
\prod_{i=1}^{N} F_B(s_i) &= e^{-a_1} + O(\Delta s_N).
\end{aligned}$$

We show in Axelson and Makarov (2016) that $F_G^N(s^*)$ and $F_B^N(s^*)$ converge to the same corresponding limits.

Step 2. We now show $r_N(1, \hat{\mathbf{s}}_{\mathbf{N-1}})$ goes to X in equilibrium. Suppose on the contrary there exists $\varepsilon > 0$ such that for all N $r_N(1, \hat{\mathbf{s}}_{\mathbf{N-1}}) < X - \varepsilon$ for some . Note that only a bounded number of screening thresholds can stay away from one as N goes to infinity. Otherwise, the entrepreneur would not be able to obtain financing in the last round. Let M be the maximal index such that $\limsup_{N\to\infty} s_{N-M} = 1$ but $\limsup_{N\to\infty} s_{N-M+1} < 1$. Consider the problem of the entrepreneur who has been rejected $N-M-1$ times. She solves problem 10:

$$\begin{aligned}
V_{N-M} &\equiv \max_{s_{N-M}} \pi(1 - F_G(s_{N-M}))\left(X - r_i(s_{N-M}, \mathbf{s}^*_{\mathbf{N-M-1}})\right) + F_G(s_{N-M})V_{N-M+1}, \\
\text{s.t.} &\qquad (1 - F_G(s_{N-M}))\left(X - r_i(s_{N-M}, \mathbf{s}^*_{\mathbf{N-M-1}})\right) + F_G(s_{N-M})V_{i+2} \le V_{i+1}.
\end{aligned}$$

As in Proposition 5 one can show that the unconstrained solution to the above problem entails s_{N-M} to be bounded away from one. Since by assumption s_{N-M} goes to one it must be that the incentive compatibility constraint binds. However, with s_{N-M+1} being away from one, s_{N-M} converging to one, and $r_i(s_{N-M}, \mathbf{s}^*_{\mathbf{N-M-1}}) < X - \varepsilon$, the incentive compatibility constraint cannot bind.

Q.E.D.

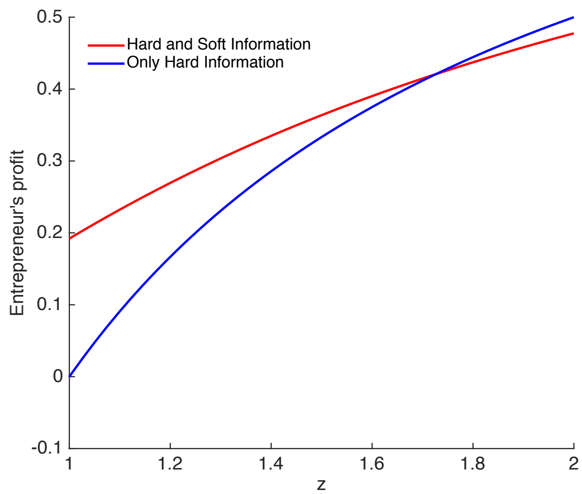


Figure 1. Hard vs. Soft information. Figure 1 blue line shows the entrepreneur's profit if she obtains financing from investors that use only hard information. The red line shows shows the entrepreneur's profit if she obtains financing from investors that use both hard and soft information.

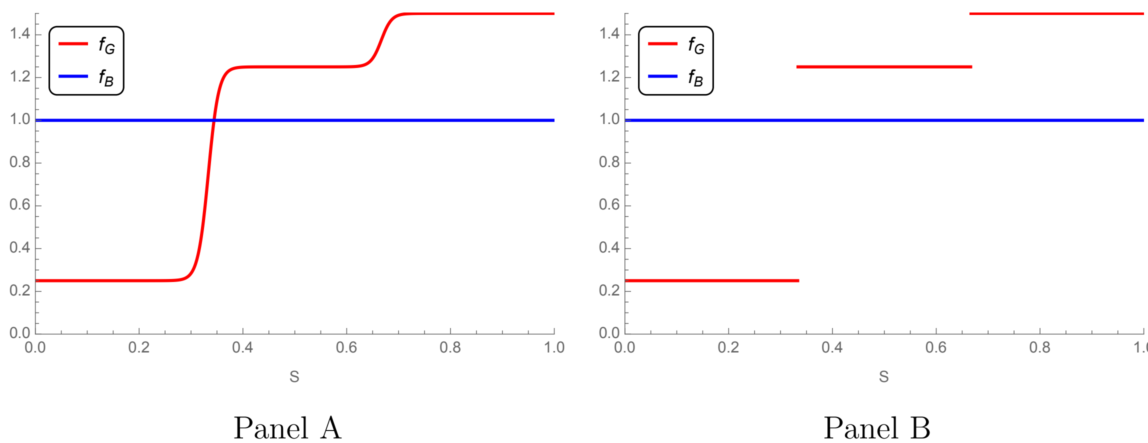
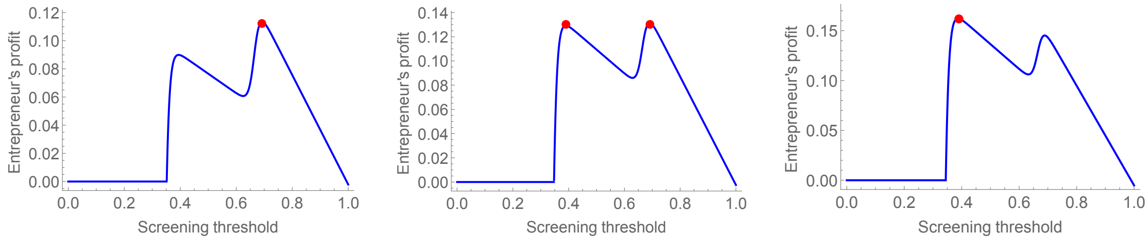
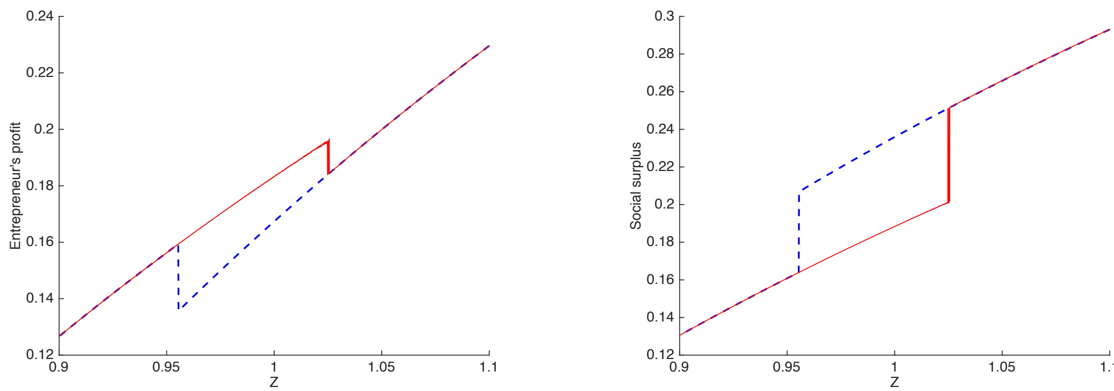


Figure 2. Multiple equilibria. Signal densities. Figure 2 Panel A plots densities $f_B(s)$ and $f_G(s)$. Function $f_G(s)$ is defined in equation (12), and is a smoothed version of the case shown in Panel B.



Panel A Panel B Panel C

Figure 3. Entrepreneur's profit. Panels A, B, and C display the entrepreneur's profit with one investor when $z = 0.9$, $z = 0.95$, $z = 1$ respectively. Other parameters are as follows: $X = 1$, densities f_B and f_G are displayed in figure 2 Panel A.



Panel A Panel B

Figure 4. Multiple equilibria. Entrepreneur's profit and social surplus. Figure 4 Panel A plots the entrepreneur's expected profit in the two equilibria described in Section 6 as a function of her initial likelihood ratio z. Panel B plots social surplus. The blue line corresponds to the first equilibrium with one screening; the red line - to the second equilibrium with two screenings.

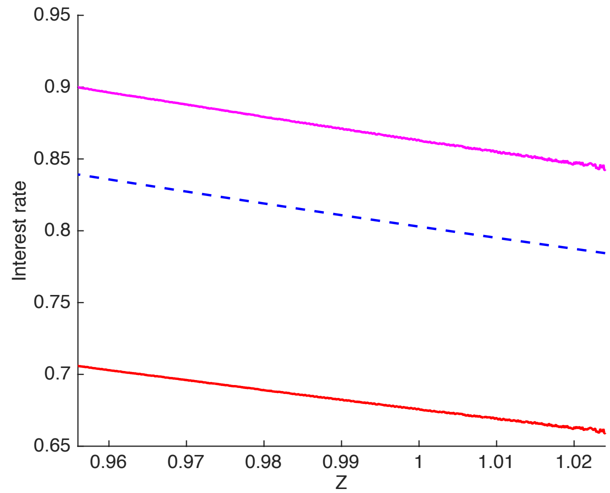


Figure 5. Multiple equilibria. Interest rate. Figure 5 shows interest rate the the entrepreneur asks from investors. The blue line shows the interest rate in the equilibrium with single screening. The red and magenta lines show the interest rates in the equilibrium with two screenings, with the highest rate being asked if rejected at the first investor.