Habits and Leverage

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Abstract

Many stylized facts of leverage, trading, and asset prices can be explained by a frictionless general equilibrium model in which agents have heterogeneous endowments and external habit preferences. Our model predicts that aggregate leverage increases in good times when stock prices are high and volatility is low, it should predict low future returns and it is positively correlated with a "consumption boom" of levered agents. In addition, negative aggregate shocks induce levered agents to deleverage by "fire selling" their risky positions as their wealth drops. While such agents' total leverage decreases, their debt/wealth level increases as wealth value is especially sensitive to changes in aggregate risk aversion.

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1. Introduction

The financial crisis has elicited much research into the understanding of the dynamics of aggregate leverage and its impact on asset prices and economic growth. Recent empirical and theoretical research has produced a variety of results that many claim should inform a reconsideration of existing frictionless models. Amongst these we have (i) the evidence that excessive credit supply may lead to financial crises;[1] (ii) the growth in household debt and the causal relation between the deleveraging of levered households and their low future consumption growth;[2] (iii) the idea that active deleveraging of financial institutions generates "fire sales" of risky financial assets which further crashes asset prices;[3] (iv) the evidence that the aggregate leverage ratio of financial institutions is a risk factor in asset pricing;[4] (v) the view that balance sheet recessions are critical components of business cycle fluctuations;[5] and many others. Most of these explanations rely on some types of market frictions and behavioral biases, and point at a causal effect of leverage onto aggregate economic and financial phenomena. In this paper, we put forward a simple frictionless general equilibrium model with endogenous leverage that offers a coherent explanation of most of these relations between agents' leverage, their consumption, and asset prices.

The mechanism emphasized in this paper is standard in the asset pricing literature. We posit an economy populated with agents whose preferences feature external habits. Specifically agents' utilities are determined by the distance between their own level of consumption and the level of aggregate endowment, appropriately scaled; roughly agents care about consumption inequality. How much agents care about this distance varies across agents and over the business cycle. In particular, agents care more about their relative standing in bad times than in good times and there are some agents who care more than others about this comparison between their own level of consumption and habits. This cross sectional heterogeneity introduces motives for risk sharing and asset trading in general. Agents also differ in their level of wealth, which is also an important determinant of their risk bearing capacity. The model aggregates nicely to standard external habit models such as Campbell and Cochrane (1999) and Menzly, Santos and Veronesi (2004) and thus inherits the asset pricing properties of these models and in particular the dynamics of risk and return that were the original motivation for these models.

[1] See for instance Jordà, Schularick and Taylor (2011).
[2] See Justiniano, Primiceri and Tambalotti (2013) and Mian and Sufi (2015).
[3] See e.g. Shleifer and Vishny (2011).
[4] See He and Krishnamurthy (2013) and Adrian, Etula and Muir (2014).
[5] See Huo and Ríos-Rull (2013) and Mian, Rao and Sufi (2013).

External habit models feature strong discount effects, which, as shown by Hansen and Jagannathan (1991), are required to explain the Sharpe ratios observed in financial markets. We argue that these strong discount effects are also important to understand the dynamics of risk sharing. Standard risk sharing arguments require that agents with large risk bearing capacity insure those with low risk bearing capacity. In models where, for instance agents have CRRA preferences, such as Longstaff and Wang (2013), this means the agents who provide the insurance consume a large share of aggregate consumption when this is large and a low share when instead aggregate consumption is low. This is obviously also the case in our framework, but in addition the share of consumption will also depend on whatever state variable drives discount effects, which introduces additional sources of non-linearities in the efficient risk sharing arrangement. The reason is that in our model risk aversion changes depending on the actual realization of the aggregate endowment and thus so do the efficiency gains associated with risk sharing.

We decentralize the efficient allocation by allowing agents to trade in the aggregate endowment process and debt that is in zero net supply and provide a full characterization of the corresponding competitive equilibrium. We show that agents with large risk bearing capacity provide insurance by issuing the debt that the more risk averse agents want to hold to insure against fluctuations in their marginal utility of consumption. A striking property of the competitive equilibrium is that aggregate leverage is procyclical, an intuitive result but one that does not obtain in standard models. The reason hinges on the decrease in aggregate risk aversion in good times, which makes agents with high risk bearing capacity willing to take on a larger fraction of the aggregate risk by issuing more risk-free debt to agents with lower risk bearing capacity. Thus, procyclical leverage emerges naturally as the result of the optimal trading of utility maximizing agents in an equilibrium that in fact implements an optimal risk sharing allocation.

Besides procyclical aggregate leverage, our model has several additional predictions that are consistent with numerous stylized facts. First, higher aggregate leverage should be correlated with (*i*) high valuation ratios, (*ii*) low volatility, (*iii*) lower future excess returns, and (*iv*) a "consumption boom" of those agents who lever up, who then should experience a consumption slump relative to others, on average. The reason is that as explained above, in good times leverage increases as aggregate risk aversion declines. Lower risk aversion implies high valuation ratios and lower stock return volatility, as well as lower future excess returns, explaining (*i*) through (*iii*). In addition, levered agents who took up levered positions do especially well when stock market increases, implying higher consumption in good times. Mean reversion, however, implies that these same agents should also expected a relatively lower expected future consumption growth after their consumption binge, explaining (*iv*).

Second, our model also implies active trading. For instance, a series of negative aggregate shocks induces deleveraging of levered agents through the active sales of their positions in risky stocks. It follows that stock price declines occur exactly at the time when levered agents actively sell their risky positions to reduce leverage. This commonality of asset sales and stock price declines give the impression of a "selling pressure" affecting asset prices, when in fact they are the equilibrium outcome of business cycle variation and its differential impact on risk aversion. Indeed, our model make transparent the fact that equilibrium prices and quantities comove due to aggregate state variables, but there is no causal relation between trading and price movements. In the special case of identical agents, in fact, our model reverts back to a standard representative agent model such as in Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004), which feature no trading. Yet, all of the asset pricing implications are identical.

Third, while our model implies that during bad times aggregate leverage declines, levered agents' debt-to-wealth ratios increase, as wealth declines faster than debt due to severe discount-rate effects. This implies that while the aggregate level of debt is pro-cyclical (*i.e.* lower aggregate debt in bad times), the debt-to-wealth ratio of levered agents is countercyclical (*i.e.* higher debt-to-wealth in bad times). Broadly interpreting levered agents as intermediaries (they receive funding from unlevered agents to increase their risky positions), the model is thus also consistent with the recent literature about the ambiguous impact of negative aggregate shocks on the leverage of intermediaries. Specifically, we find that the level of leverage decreases but debt-to-equity ratios increase, as equity drop faster than leverage due to discount rate effects. Moreover, because such leverage is endogenously negatively related to aggregate risk aversion, it would naturally become "a pricing factor" for asset prices, as emphasized in recent literature.

Finally, our model has predictions about the source of the variation in wealth inequality. We show that heterogeneity in preferences and in endowments work together to exacerbate wealth inequality during good times, but they work in opposite directions in bad times. That is, lower discount rates (due to the lower aggregate risk aversion) and heterogeneity in stock positions (due to heterogeneous preferences) both increase wealth dispersion when times are good. However, when times are bad, higher discount rates tend to decrease the wealth dispersion due to initial different endowments, but heterogeneity in habits still tend to exacerbate them. These effects imply a complex dynamics of wealth dispersion that depends both on endowments, but also on optimal actions of agents that are affected by differential habits. Once again, the model emphasizes that while asset prices affect wealth inequality, the converse does not hold, as asset prices are identical with homogeneous agents, and hence in the same model without wealth dispersion.

Clearly many explanations have been put forth to explain the growth of leverage and of household debt in particular during the run up to the crisis. For instance, among others, Bernanke (2005) argues that the global savings glut, the excess savings of East Asian nations in particular, is to blame for the ample liquidity in the years leading up to the Great Recession, which reduced rates and facilitated the remarkable rise in household leverage; Shin (2012) shows how regulatory changes, the adoption of Basel II, led European banks to increase lending in the US; Pinto (2010), Wallison (2011) and Calomiris and Haber (2014) argue that the Community Reinvestment Act played a pivotal role in the expansion of mortgage lending to risky households (but see Bhutta and Ringo (2015)); Mendoza and Quadrini (2009) show how world financial integration leads to an increase in net credit. The list goes on. When the crisis came, the crash in prices and the rapid deleveraging of households and financial intermediaries was interpreted appealing to classic inefficient runs arguments a la Diamond and Dybvig (1983) as in Gorton and Metrick (2010) or contagion. He and Krishnamurthy (2008) connect the fall in asset prices to the shortage of capital in the intermediation sector. Finally, much research has focused on the impact that the crisis had on the consumption patterns of households. For instance Mian and Sufi (2014) argue that debt overhang is to blame for the drop in consumption in counties where households were greatly levered.

Our point here is not to claim that these frictions are not important but simply to offer an alternative explanation that is consistent with complete markets and that matches what we know from the asset pricing literature. We argue for instance that when debt overhang is put forth as an explanation for low consumption patterns amongst levered households the alternative hypothesis of efficient risk sharing cannot be dismissed outright. Both explanations operate in the same direction and thus assessing the quantitative plausibility of one requires controlling for the other.

Our model has the considerable advantage of simplicity: All formulas for asset prices, portfolio allocation, and leverage are in closed form, no numerical solutions are required, and their intuition follows from basic economic principles. Moreover, because our model aggregates to the representative agent of Menzly, Santos, and Veronesi (2004), except that we allow for time varying aggregate uncertainty, we can calibrate its parameters to match the properties of aggregate return dynamics. As such, our model has not only qualitative implications – as most of the existing literature – but quantitative implications as well.

This paper is related to the literature on optimal risk sharing, starting with Borch (1962). Our paper is closely related to Dumas (1989), Wang (1996), Bolton and Harris (2013), and Longstaff and Wang (2013). These papers consider two groups of agents with constant

risk aversion, and trading and asset prices are generated by aggregate shocks through the variation in the wealth distribution. While similar in spirit, our model considers a continuum of agents whose risk preferences are time varying due to their agent-specific external habit preferences. Our main source of variation is aggregate economic uncertainty – which is absent in these earlier papers – as it correlates with agents' risk aversion. Our model is also related to Chan and Kogan (2002), who consider a continuum of agents with habit preferences and heterogeneous risk aversion. In their setting, however, the risk aversions of individual agents are constant, while in our setting risk aversions of individual agents are time varying in response to variation in aggregate uncertainty, a crucial ingredient in our model. Finally, our paper also connects to the recent literature that tries to shed light on the determinants of the supply of safe assets; see for instance Barro and Mollerus (2014) and Caballero and Fahri (2014), though this literature is more interested in the implications of the shortage of safe assets for macroeconomic activity.

The paper is structured as follows. The next section presents the model. Section 3 characterizes the optimal risk sharing arrangement and Section 4 decentralizes the efficient risk sharing allocation and characterizes the competitive equilibrium. Section 5 evaluates the model quantitatively and Section 6 concludes. All proofs are in the Appendix.

2. The model

Preferences. There is a continuum of agents endowed with log utility preferences defined over consumption C_{it} in excess of agent-specific external habit indices X_{it}:

$$u\left(C_{i,t}, X_{i,t}, t\right) = e^{-\rho t} \log\left(C_{it} - X_{it}\right)$$

Agents are heterogeneous in the habit indices X_{it}, which are given by

$$X_{it} = g_{it}\left(D_t - \int X_{jt}dj\right) \tag{1}$$

That is, the habit level X_{it} of agent i is proportional to the excess aggregate output D_t over average habit $\int X_{jt}dj$, which we call *excess output* henceforth. A higher excess output decreases agent i's utility, an effect that captures a notion of "Envy the Joneses." The excess output $\left(D_t - \int X_{jt}dj\right)$ is in fact an index of the "happiness" of the Joneses – their utility is higher the higher the distance of D_t from average habit $\int X_{jt}dj$ – a fact that makes agent i less happy as it pushes up his habit level X_{it} and thus reduces his utility. Our model is thus an external habit model defined on utility – as opposed to consumption – in that other people happiness is negatively perceived by agent i.

The sensitivity of agent i's habit X_{it} to aggregate excess output $(D_t - \int X_{jt} dj)$ depends on the agent-specific proportionality factor g_{it}, which is heterogeneous across agents and depends linearly on a state variable, to be described shortly, Y_t:

$$g_{it} = a_i Y_t + b_i \tag{2}$$

where $a_i > 0$ and b_i are heterogeneous across agents and such that

$$\int a_i di = 1 \ .$$

Endowment. Aggregate endowment – which we also refer to as dividends or output – follows the process

$$\frac{dD_t}{D_t} = \mu_D \ dt \ + \ \sigma_D(Y_t) \ dZ_t \tag{3}$$

where the drift rate μ_D is constant.[6] The volatility $\sigma_D(Y_t)$ of aggregate endowment – which we refer to as *economic uncertainty* – depends on the state variable Y_t, which follows

$$dY_t = k \ (\overline{Y} - Y_t) \ dt - v \ Y_t \ \left[\frac{dD_t}{D_t} - E_t \left(\frac{dD_t}{D_t} \right) \right] \tag{4}$$

That is, Y_t increases after bad aggregate shocks, $\frac{dD_t}{D_t} < E_t \left(\frac{dD_t}{D_t} \right)$, and it hovers around its central tendency $\overline{Y}$. It is useful to interpret Y_t as a *recession indicator*: During good times Y_t is low and during bad times Y_t is high. We assume throughout that Y_t is bounded below by a constant $\lambda \geq 1$. This technical restriction is motivated by our preference specification above and it can be achieved by assuming that $\sigma_D(Y_t) \to 0$ as $Y_t \to \lambda$ (under some technical conditions). We otherwise leave the diffusion terms $\sigma_D(Y_t)$ in (3) unspecified for now, although, to fix ideas, we normally assume that economic uncertainty is higher in bad times, i.e. $\sigma'_D(Y_t) > 0$.

At time 0 each agent is endowed with a fraction w_i of the aggregate endowment process D_t. The fractions w_i satisfy $\int w_i di = 1$, and the technical condition

$$w_i > \frac{a_i(\overline{Y} - \lambda) + \lambda - 1}{\overline{Y}} \tag{A1}$$

which ensures that each agent has sufficient wealth to ensure positive consumption over habit in equilibrium, and hence well defined preferences. A1 is assumed throughout.

Discussion. Our preference specification differs from the standard external habit model of Campbell and Cochrane (1999) and Menzly, Santos and Veronesi (2004, MSV henceforth).

[6] As will be shown below the drift μ_D does not play any role into any of relevant formulas, except for the risk-free rate. The main results of the paper are thus consistent with a richer specification of the drift μ_D.

In particular, notice that our model is one *without* consumption externalities as habit levels depend only on exogenous processes and not on consumption choices. This, as shown below, will allow the application of standard aggregation results which will considerably simplify the characterization of optimal sharing rules.

Second our model features two relevant sources of variation across agents: Wealth, as summarized by the distribution of ω_i, and the sensitivity of individual habits X_{it} to excess output, as summarized by g_{it}, which results in differences in attitudes towards risk. These two dimensions seem a natural starting point to investigate optimal risk sharing as well as portfolio decisions.[7] Clearly, one could contemplate other sources of variation in the cross section of households such as differences in beliefs or in investment opportunity sets to which the agents have access.

Notice though that our model features no idiosyncratic shocks to individual endowment as agents simply receive a constant fraction w_i of the aggregate endowment process. Individual endowment processes are thus perfectly correlated and thus they are not the driver of risk sharing motives. Instead in our model risk sharing motives arise exclusively because agents are exposed differently to business cycle fluctuations through their sensitivity to habits. Indeed how sensitive agents are to shocks in excess output depend on the state variable Y_t. Economically, assumption (2) implies that in bad times (after negative output shocks) the habit loadings g_{it} increase, making habit preferences become more important on average. However, different sensitivities a_i imply that changes in Y_t differentially impact the external habit index as g_{it} increase more for agents with high a_i than for those with low a_i. We set $b_i = \lambda(1 - a_i) - 1$, which ensures $g_{it} > 0$ for every i and for every t (as $Y_t > \lambda$), and allows for a simple aggregation below. This assumption does not affect the results.

Finally, we note that the case of homogeneous preferences ($a_i = 1$ for all i) and/or homogeneous endowments ($w_i = 1$ for all i) are special cases, as is the case in which habits are constant ($v = 0$ in (4)). We investigate these special cases as well below.

[7] For instance, two recent theoretical contributions that consider these two sources of cross sectional variation are Longstaff and Wang (2012) and Bolton and Harris (2013). Empirically these sources of variation have been investigated by, for example, Chiappori and Paeilla (2011) and Calvet and Sodini (2014), though the results in these two papers are rather different.

3. Optimal risk sharing

As already mentioned, markets are complete and therefore standard aggregation results imply that a representative agent exists, a planner, that solves the program

$$U\left(D_t, \{X_{it}\}, t\right) = \max_{C_{it}} \int \phi_i u\left(C_{it}, X_{it}, t\right) di \qquad \text{subject to} \qquad \int C_{it} di = D_t \tag{5}$$

where all Pareto weights $\phi_i > 0$ are set at time zero, renormalized such that $\int \phi_i di = 1$ and are consistent with the initial distribution of wealth in a way to be described shortly. The first order condition implies that

$$u_C(C_{it}, X_{it}, t) = \frac{\phi_i e^{-\rho t}}{C_{it} - X_{it}} = M_t \qquad \text{for all } i \tag{6}$$

where M_t is the Lagrange multiplier associated with the resource constraint in (5).[8] Straightforward calculations[9] show that

$$M_t = \frac{e^{-\rho t}}{\left(D_t - \int X_{jt} dj\right)} \qquad \text{and} \qquad C_{it} = \left(g_{it} + \phi_i\right)\left(D_t - \int X_{jt} dj\right). \tag{7}$$

The optimal consumption of agent i increases if the aggregate excess output $\left(D_t - \int X_{jt} dj\right)$ increases or if the habit loading g_{it} increases. This is intuitive, as such agents place relatively more weight on excess output and thus want to consume relatively more. In addition, agents with a higher Pareto weight ϕ_i also consume more, as such agents have a larger relative endowment.

The optimal consumption plans (7) seems to indicate that as g_{it} increases, the consumption of all agents will increase (recall that g_{it} are perfectly correlated), and thus exceed total output D_t. This does not happen because of the effect of g_{it} on the habit levels X_{it}, which decreases the excess output $D_t - \int X_{it} di$. In fact, we can aggregate total optimal consumption and impose market clearing to obtain

$$D_t = \int C_{it} di = \left[\int \left(g_{it} + \phi_i\right) di\right]\left(D_t - \int X_{it} di\right) \tag{8}$$

Using $\int \phi_i di = 1$, we can solve for the equilibrium excess output as

$$D_t - \int X_{it} di = \frac{D_t}{\int g_{it} di + 1} > 0 \tag{9}$$

[8] This result was first derived by Borch (1962, equation (1) p. 427).

[9] It is enough to solve for C_{it} in (6), integrate across agents (recall $\int \phi_i di = 1$), and use the resource constraint to yield M_t. Plugging this expression in (6) yields C_{it}.

This intermediate result also shows that individual excess consumption $C_{it}-X_{it}$ is positive for all i, which ensures all agents' utility functions are well defined.[10] Notice also an important implication of (9) and it is that preferences can be expressed as

$$u\left(C_{i,t}, X_{i,t}, t\right) = e^{-\rho t} \log\left(C_{it} - \psi_{it} D_t\right) \qquad \text{with} \qquad \psi_{it} \equiv \frac{g_{it}}{\int g_{it} di + 1}.$$

Individual agents compare their own consumption to aggregate endowment properly scaled by ψ_{it}, which is agent specific and dependent on Y_t. Roughly agents care about their relative standing in society, which is subject to fluctuations. It is these fluctuations what introduces motives for risk sharing. The next proposition solves for the Pareto weights and the share of the aggregate endowment that each agent commands and illustrates the basic properties of the optimal risk sharing rules in our model.

Proposition 1 *(Efficient allocation). Let the economy be at its stochastic steady state at time 0, $Y_0 = \overline{Y}$, and normalize $D_0 = \rho$. Then (a) the Pareto weights are*

$$\phi_i = a_i \lambda + (w_i - a_i)\,\overline{Y} + 1 - \lambda \tag{10}$$

(b) The share of the aggregate endowment accruing to agent i is given by

$$C_{it} = \left[a_i + (w_i - a_i)\frac{\overline{Y}}{Y_t}\right] D_t \qquad \text{or} \qquad s_{it} \equiv \frac{C_{it}}{D_t} = a_i + (w_i - a_i)\frac{\overline{Y}}{Y_t} \tag{11}$$

Pareto weights (10) are increasing in the fraction of the initial aggregate endowment w_i and decreasing in habit sensitivity a_i. The first result is standard. To understand the second, given optimal consumption (7), agents with higher sensitivity a_i have a higher habit loading $g_{it} = a_i(Y_t - \lambda) + \lambda - 1$ and thus would like to consume more. Given (7), for given initial endowment w_i, the Pareto weight ϕ_i must then decline to ensure that such consumption can be financed by the optimal trading strategy.

Equation (11) captures the essential properties of the optimal risk sharing rule, that is, agents with high w_i or low a_i enjoy a high consumption share $s_{it} = C_{it}/D_t$ during good times, that is, when the recession indicator Y_t is low, and vice versa. To grasp the intuition consider first the curvature of the utility function of an individual agent, which we refer to as "risk aversion" for simplicity:

$$Curv_{it} = -\frac{C_{it} u_{cc}(C_{it}, X_{it}, t)}{u_c(C_{it}, X_{it}, t)} = 1 + \frac{a_i(Y_t - \lambda) + \lambda - 1}{w_i\overline{Y} - a_i(\overline{Y} - \lambda) - \lambda + 1} \tag{12}$$

[10] To see this, substitute the excess output into (7) and use (1). Given g_{it} in (2), we have $\int g_{it} di + 1 = Y_t$.

It is the combination of wealth and sensitivity to excess output what determines the agent's attitude towards risk: Agents with higher wealth w_i or lower habit loading a_i have lower risk aversion. Moreover, an increase in recession indicator Y_t increases the curvature of every agent, but more so for agents with a high habit loading a_i.

These two effects combine to determine the planner's transfer scheme needed to support the optimal allocation. Let $\tau_{it} > 0$ be the transfer received by agent i at time t above her endowment $w_i D_t$; if instead the agent consumes below her endowment then $\tau_{it} < 0$. Trivial computations prove the next corollary.[11]

Corollary 2 *The transfers that implement the efficient allocation are given by*

$$\tau_{it} = -(w_i - a_i)\left(1 - \frac{\overline{Y}}{Y_t}\right) D_t. \tag{13}$$

Notice that agents for whom $w_i - a_i > 0$ receive transfers, $\tau_{it} > 0$, when $Y_t < \overline{Y}$, that is in good times and pay $\tau_{it} < 0$ in bad times, when $Y_t > \overline{Y}$. The opposite is the case for the agents for whom $w_i - a_i < 0$. In effect, optimal risk sharing requires agents with $w_i - a_i > 0$ to insure agents with $w_i - a_i < 0$.

The intuition of expression (11) is now clear. The consumption of agent i depends on both the aggregate consumption, D_t, and the state variable Y_t, our recession indicator. There are two effects in equation (11). First for a given Y_t agents with high wealth or low sensitivity to excess output, for whom $w_i - a_i > 0$, have more risk bearing capacity than agents for whom $w_i - a_i < 0$ and thus insure the poorer or more risk averse agents. As a result the shares of consumption of the agents with large risk bearing capacity fluctuate more with aggregate consumption. This is the standard result as found for instance in Longstaff and Wang (2012) (see their Proposition 1 as well as equation (16) in that paper). The second effect is due to the Joneses feature of our preferences: As Y_t drops the risk bearing capacity of the agents with larger risk bearing capacity ($w_i - a_i > 0$) increases further and thus the fluctuations of the shares is even stronger than in the case where agents have, say, simply power utility functions. This result is reminiscent of Chan and Kogan (2002, Lemma 1). An important property of the share of consumption of agent i, s_{it}, is that it inherits the stationarity properties of Y_t. Thus, unlike models in which agents differ in their degree of constant relative risk aversion the distribution of wealth does not degenerate, as for example in Dumas (1989) and Wang (1996), where the proportion of wealth held by the least risk averse agents converges to one.[12]

[11] Simply subtract from the optimal consumption allocation (11) the consumption under autarchy, $w_i D_t$.

[12] A standard modeling device to obtain stationary distributions and avoid degeneracy in the long run is to

We emphasize an important attribute of our model and that is that habits are key to deliver all the results in our paper. Indeed, assume that $Y_t = \overline{Y}$ for all t (i.e. $v = 0$ in (4)). In this case our model collapses to an economy populated with agents with log preferences, the share of consumption of each agent is simply $s_{it} = w_i$ and, as it will be shown below, no trading occurs amongst agents. Thus, our model does not deliver risk sharing motives beyond what is induced by the habit features of our preference specification.

4. Competitive equilibrium

4.1. Decentralization

Financial markets. Having characterized the optimal allocation of risk across agents in different states of nature we turn next to the competitive equilibrium that supports it. Clearly we can introduce a complete set of Arrow-Debreu markets at the initial date, let agents trade and after that simply accept delivery and make payments. It was Arrow's (1964) original insight that decentralization can be achieved with a sparser financial market structure. There are obviously many ways of introducing this sparser financial market structure but here we follow many others and simply introduce a stock market and a market for borrowing and lending. Specifically we assume that each of the agents i is endowed with an initial fraction w_i of a claim to the aggregate endowment D_t. We normalize the aggregate number of shares to one and denote by P_t the price of the share to the aggregate endowment process, which is competitively traded. Second, we introduce a market for borrowing and lending between agents. Specifically we assume that there is an asset in zero net supply, a bond, with a price B_t, yielding an instantaneous rate of return of r_t. Both P_t and r_t are determined in equilibrium. Because all quantities depend on one Brownian motion (dZ_t), markets are dynamically complete.

The portfolio problem. Armed with this we can introduce the agents' problem. Indeed, given prices $\{P_t, r_t\}$ agents choose consumption C_{it} and portfolio allocations in stocks N_{it} and bonds N_{it}^0 to maximize their expected utilities

$$\max_{\{C_{it}, N_{it}, N_{it}^0\}} E_0 \left[\int_0^\infty e^{-\rho t} \log \left(C_{it} - X_{it} \right) dt \right]$$

subject to the budget constraint equation

$$dW_{it} = N_{it}(dP_t + D_t dt) + N_{it}^0 B_t r_t dt - C_{it} dt$$

have agents die and be replaced by "children" with randomly assigned coefficients of relative risk aversion. For a recent application of this idea see Barro and Mallerus (2014).

with initial condition $W_{i,0} = w_i P_0$.

Definition of a competitive equilibrium. A competitive equilibrium is a series of stochastic processes for prices $\{P_t, r_t\}$ and allocations $\{C_{it}, N_{it}, N^0_{it}\}_{i \in \mathcal{I}}$ such that agents maximize their intertemporal utilities and markets clear $\int C_{it} di = D_t$, $\int N_{it} di = 1$, and $\int N^0_{it} di = 0$. The economy starts at time 0 in its stochastic steady state $Y_0 = \overline{Y}$. Without loss of generality, we normalize the initial output $D_0 = \rho$ for notational convenience.

The competitive and the decentralization of the efficient allocation. We are now ready to describe the competitive equilibrium and show that it indeed supports the efficient allocation. We leave the characterization of the equilibrium for the next section.

Proposition 3 *(Competitive equilibrium). Consider the following prices and portfolio allocations*

1. *Stock prices and interest rates*

$$\widehat{P}_t = \left(\frac{\rho + k\overline{Y} Y_t^{-1}}{\rho (\rho + k)} \right) D_t \tag{14}$$

$$\widehat{r}_t = \rho + \mu_D - (1 - v)\sigma_D^2(Y_t) + k \left(1 - \frac{\overline{Y}}{Y_t} \right) \tag{15}$$

2. *The position in bonds $\widehat{N}^0_{it} \widehat{B}_t$ and stocks $\widehat{N}_{it}$ of agent i at time t are, respectively,*

$$\widehat{N}^0_{it} \widehat{B}_t = -v \left(w_i - a_i \right) H_0 \left(Y_t \right) D_t \tag{16}$$

$$\widehat{N}_{it} = a_i + (\rho + k)(1 + v) \left(w_i - a_i \right) H_0 \left(Y_t \right) \tag{17}$$

where

$$H_0 \left(Y_t \right) = \frac{\overline{Y} Y_t^{-1}}{\rho + k(1 + v)\overline{Y} Y_t^{-1}} > 0 \tag{18}$$

Then the processes $\left(\widehat{P}_t, \widehat{r}_t, \widehat{N}_{it}, \widehat{N}^0_{it} \right)$ constitute a competitive equilibrium which supports the efficient allocation, (11).

We comment on these results in the next few subsections.

4.2. Asset prices

The stock price in Proposition 3 is identical to the one found in MSV, which was obtained in the context of a representative consumer model. The reason is that our model does indeed aggregate to yield a representative consumer which is similar to the one posited in that paper. Indeed, having solved for the Pareto weights (10) and the individual consumption allocations we can substitute back in the objective function in (5) and obtain the equilibrium state price density associated with the representative agent, which we characterize in the next Proposition.

Proposition 4 *(The stochastic discount factor). The equilibrium state price density is*

$$M_t = e^{-\rho t} D_t^{-1} Y_t, \tag{19}$$

which follows

$$\frac{dM_t}{M_t} = -r_t dt - \sigma_{M,t} dZ_t \qquad \textit{with} \qquad \sigma_{M,t} = (1+v)\sigma_D(Y_t), \tag{20}$$

and where r_t is given by (15).[13]

This state price density is similar to one obtained in the representative agent, external habit models of Campbell and Cochrane (1999) and MSV. Indeed, we can define the surplus consumption ratio as in Campbell and Cochrane (1999)

$$S_t = \frac{D_t - \int X_{it} di}{D_t} = \frac{1}{Y_t} \tag{21}$$

where the last equality stems from (9). The recession indicator Y_t is then the inverse surplus consumption ratio of MSV. Indeed, as in this earlier work, Y_t can be shown to be linearly related to the aggregate risk aversion of the representative agent (see footnote 4 in MSV). As in MSV, we sometimes refer to Y_t as the *aggregate risk aversion* of the economy.

In what follows, we express the results as functions of the surplus consumption ratio $S_t = 1/Y_t$ for notational convenience. The surplus consumption ratio increases after positive aggregate shocks, that is, S_t is high in good times. With a small abuse of notation, we denote functions of Y_t, such as output volatility $\sigma_D(Y_t)$, simply as functions of S_t, $\sigma_D(S_t)$. So, for instance, the function $H_0(Y_t)$ in (18) becomes

$$H_0(S_t) = \frac{\overline{Y} S_t}{\rho + k(1+v)\overline{Y} S_t} > 0 \tag{22}$$

[13] In what follows and to lighten up the notation we drop the $\widehat{\cdot}$ from the competitive equilibrium.

We are now ready to discuss the asset prices in Proposition 3. Start, briefly, with the risk free rate r_t. The terms $\rho + \mu_D - \sigma_D^2(S_t)$ in (15) are the standard log-utility terms, namely, time discount, expected aggregate consumption growth, and precautionary savings. The additional two terms, $k(1 - \overline{Y} S_t)$ and $v\ \sigma_D(S_t)$, are additional intertemporal substitution and precautionary savings terms, respectively, associated with the external habit features of the model (see MSV for details).

As for the stock price, given that $S_t = Y_t^{-1}$, we can write

$$P_t = E_t \left[\int_t^\infty \frac{M_\tau}{M_t}\ D_\tau d\tau \right] = \left(\frac{\rho + k\overline{Y} S_t}{\rho \left(\rho + k \right)} \right) D_t.$$

The intuition for this is by now standard (Campbell and Cochrane (1999) and MSV). A negative aggregate shock $dZ_t < 0$ decreases the price directly through its impact on D_t, but it also increases the risk aversion Y_t, and thus reduces S_t, which pushes down the stock price P_t further. Notice that in the steady state, when $S_t = \overline{Y}^{-1}$, the price of the stock is $P_t = \rho^{-1} D_t$, which is the price that obtains in the standard log economy. External habit persistence models thus generate variation in prices that are driven not only by cash-flow shocks but also discount effects. Indeed, we show in the Appendix the volatility of stock returns is

$$\sigma_P(S_t) = \sigma_D(S_t) \left(1 + \frac{v k \overline{Y} S_t}{\rho + k \overline{Y} S_t} \right) \tag{23}$$

In addition, as shown in (20), the market price of risk also is time varying, not only because of the variation in consumption volatility ($\sigma_D(S_t)$) but also because of the variation in the volatility of aggregate risk aversion, given by $v\sigma_D(S_t)$. In MSV, a lower surplus consumption ratio S_t increases the average market price of risk and makes it time varying. This generates the predictability of stock returns. Indeed

$$E_t \left[dR_P - r(S_t) dt \right] = \sigma_M(S_t) \sigma_P(S_t) dt \tag{24}$$

where $dR_P = (dP_t + D_t dt)/dt$. The risk premium $E_t[dR_P] - r(S_t)dt$ increases compared to the case with log utility both because the aggregate amount of risk $\sigma_P(S_t)$ increases and because the market price of risk $\sigma_M(S_t)$ increases.

An important property of asset prices (P_t and r_t) in our model is summarized in the next Corollary.

Corollary 5 *Asset prices are independent of the endowment distribution across agents as well as the distribution of preferences. In particular the model has identical asset pricing implications even if all agents are identical, i.e.* $a_i = 1$ *and* $w_i = 1$ *for all* i.

The asset pricing implications of our model are thus "orthogonal" to its cross sectional implications: P_t in equation (14) and r_t in (15) are independent of the distribution of either current consumption or wealth in the population. This distinguishes our model from, for instance, Longstaff and Wang (2012, Proposition 2 equation (18)) or Chan and Kogan (2002, Lemma 2). For instance Chan and Kogan (2002) consider "catching up with the Joneses" preferences as in Abel (1990), $(1-\gamma)^{-1}(C/X_t)^{1-\gamma}$, where agents differ in the degree of curvature γ. Pricing in that paper depends on the cross sectional distribution of γ. Instead, the present paper uses preferences that are, roughly, a logarithmic version of the external habit model of Campbell and Cochrane (1999) and they aggregate so as to eliminate from the Lagrange multiplier associated with the resource constraint, expression (6), all dependence from the distribution of Pareto weights ϕ_i.[14]

In both Chan and Kogan (2002) and in Longstaff and Wang (2012) variation in risk premia is driven by endogenous changes in the cross-sectional distribution of wealth. Roughly more risk-tolerant agents hold a higher proportion of their wealth in stocks. A drop in stock prices reduces the fraction of aggregate wealth controlled by such agents and hence their contribution to the aggregate risk aversion. The conditional properties of returns rely thus on strong fluctuations in the cross sectional distribution of wealth. Instead in the present paper agents' risk aversions change inducing additional variation in premia and putting less pressure on the changes in the distribution of wealth to produce quantitatively plausible conditional properties for returns. Indeed, Corollary 5 asserts exactly that asset pricing implications are identical even when agents are homogeneous and thus there is no variation in cross-sectional distribution of wealth. This of course does not mean that secular changes in the distribution of wealth cannot affect long run trends in asset prices. For instance, Hall (2016) has recently proposed that changes in the proportion of wealth of the more risk averse agents in society explain the secular decline in interest rates in the USA.[15]

Corollary 5 allows us to separate cleanly the asset pricing implications of our model from its implications for trading, leverage and risk sharing, which we further discuss below. In particular, the corollary clarifies that equilibrium prices and quantities do not need to be causally related to each other, but rather comove with each other because of fundamental state variables, such as S_t in our model.

[14] To see this notice that the equilibrium price-dividend ratio in that paper depends on the shadow price of the resource constraint which in turn depends on the weight the planner attaches to the agent whose attitudes towards risk are given by γ, which is $f(\gamma)$ (see expression (9) and (13) in Chan and Kogan (2002)).

[15] Hall (2016) also emphasizes differences in beliefs as a second source of heterogeneity across agents.

4.3. Leverage and risk sharing

We turn next to the characterization of the portfolio strategies in Proposition 2. The next Corollary follows immediately from that Proposition.

Corollary 6 *(Individual leverage).*

(a) The position in bonds is $N_{it}^0 B_t < 0$ *if and only if* $w_i - a_i > 0$. *That is, agents with* $w_i > a_i$ *take on leverage.*

(b) The investment in stock of agent i *in proportion to wealth is*

$$\frac{N_{it}P_t}{W_{it}} = \frac{1 + v\left(1 - \frac{\rho}{\rho + \overline{Y}[k + (\rho + k)(w_i - a_i)/a_i]S_t}\right)}{1 + v\left(1 - \frac{\rho}{\rho + \overline{Y}kS_t}\right)} \tag{25}$$

Therefore

$$\frac{N_{it}P_t}{W_{it}} > 1 \qquad \text{if and only if} \qquad w_i - a_i > 0$$

Recall that, as shown in equation (13), optimal risk sharing requires transfers from agents with $w_i - a_i > 0$ to those with $w_i - a_i < 0$ when Y_t is high (or S_t is low) and the opposite when Y_t is low (or S_t is high). Equations (16) and (17) show the portfolios of stocks and bonds needed to implement the efficient allocation. This is achieved by having the agents with large risk bearing capacity, agents with $w_i - a_i > 0$, issue debt in order to insure those agents with lower risk bearing capacity, $w_i - a_i < 0$. Part (b) of Corollary 1 shows that indeed agents with $w_i - a_i > 0$ lever up to achieve a position in stocks that is higher than 100% of their wealth.

Expression (25) shows that for given level of habit sensitivity a_i, agents with higher wealth w_i invest comparatively more in stocks, a result that finds empirical support in Wachter and Yogo (2010). Indeed, as in their paper, our habit preferences imply that utility is not homothetic in wealth (due to habit), thereby implying that agents with a higher endowment invest comparatively more in the risky asset.

Expressions (16) and (17) show that the amount of leverage and asset allocation depend on the function $H_0(Y_t)$, which is time varying as the recession indicator Y_t moves over time. We discuss the dynamics of leverage in the next section.

4.4. The supply of safe assets: Leverage dynamics

A particular feature of our model is that that the risk attitudes of the agents in the economy fluctuate with the recession indicator Y_t (see equation (12)). As Y_t increases, for instance, the risk bearing capacity of the agents for whom $w_i - a_i > 0$ decreases precisely when the demand for insurance by the agents with $w_i - a_i < 0$ increases. The supply of safe assets, to use the term that has become standard in the recent literature, may decrease precisely when it is most needed.[16] This question has been at the heart of much discussion regarding the determinants of the supply of safe assets.[17] In this section we focus on the dynamics of the aggregate leverage ratio, which we define as

$$L(S_t) = \frac{-\int_{i:N_{i,t}^0<0} N_{it}^0 B_t di}{D_t}$$

where the negative sign is to make this number positive and recall $S_t = 1/Y_t$. It is immediate to see that aggregate leverage/output ratio is

$$L(S_t) = vK_1 H_0(S_t) \qquad \text{where} \qquad K_1 = \int_{i:(w_i-a_i)>0} (w_i - a_i)\, di > 0 \tag{26}$$

and the function $H_0(S_t)$ is in (22).

The following corollary characterizes the dynamics of aggregate leverage:

Corollary 7 *(Aggregate leverage). $H_0(S_t)$ is strictly increasing in S_t. Hence, aggregate leverage $L(S_t)$ is procyclical, increasing in good times (high S_t) and decreasing in bad times (low S_t).*

To gain intuition on Corollary 7, we proceed in steps. First, if habit $S_t = 1/\overline{Y}$ is constant, i.e. $v = 0$, then $\widehat{N}_{it}^0 B_t = 0$ and $\widehat{N}_{it} = w_i$ in (16) and (17). That is, there is no leverage and each agent i simply holds the fraction w_i of shares with which they are initially endowed and the model reverts to the standard log-utility case.[18]

[16] In our framework the debt issued by the agents with the largest risk bearing capacity is safe because they delever as negative shocks accumulate in order to maintain their marginal utility bounded away from infinity.

[17] See for instance Barro and Mollerus (2014), who propose a model based on Epstein-Zin preferences to offer predictions about the ratio of safe assets to output in the economy. Gorton, Lewellen and Mettrick (2012) and Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence regarding the demand for safe assets. In all these papers the presence of "outside debt" in the form of government debt plays a critical role in driving the variation of the supply of safe assets by the private sector, a mechanism that is absent in this paper.

[18] In this case the model is similar to a log-utility model with subsistence, $\ln(C_{it} - X_{it})$, with $X_{it} = \psi_i D_t$. Given that X_{it} is just proportional to aggregate output D_t, preferences are homothetic in wealth and standard log-utility results obtain.

When S_t is time varying and responds to aggregate output shocks, leverage is procyclical. Intuitively, during good times (S_t high) agents with high endowment w_i and low habit a_i have lower risk aversion compared to other agents (see expression (12)). As a result, they become even more willing to take on aggregate risk in such times compared to bad times, which they do by issuing even more risk-free debt to agents with lower risk bearing capacity, i.e. those with low endowment or a higher habit loading. Procyclical leverage then emerges as a natural outcome in equilibrium.

While an aggregate procyclical leverage may seem intuitive, it is not normally implied by, for instance, standard CRRA models with differences in risk aversion. In such models, less risk averse agents borrow from more risk averse agents, who want to hold riskless bonds rather than risky assets. As aggregate wealth becomes more concentrated in the hands of less risk-averse agents, the need of borrowing and lending declines, which in turn decreases aggregate leverage. Moreover, a decline in aggregate uncertainty – which normally occur in good times – actually decreases leverage in such models, as it reduces the risk-sharing motives of trade. In our model, in contrast, the decrease in aggregate risk aversion in good times make agents with high-risk bearing capacity even more willing to take on risk and hence increase their supply of risk-free assets to those who have a lower risk bearing capacity.

Corollary 7 finally implies that aggregate leverage $L(S_t)$ is high when S_t is high. However, good times are times when expected excess returns are likely low, as the market price of risk $\sigma_M(S_t)$ is low, and aggregate uncertainty $\sigma_D(S_t)$ is likely low.[19] Therefore, under these assumptions, high aggregate leverage $L(S_t)$ should predict low future excess returns.

4.5. Individual leverage and consumption

The following corollary follows immediately from Proposition 1 and Corollary 6.

Corollary 8 *Agents with higher leverage enjoy higher consumption share during good times.*

After a sequence of good economic shocks aggregate risk aversion declines. Thus, agents with positive $(w_i - a_i)$ increase their leverage and experience a consumption "boom". The two effects are not directly related, however. The increase in consumption is due to the higher investment in stocks which payoffs in good times. Because good times also have lower aggregate risk aversion, moreover, these same agents also increase their leverage at these

[19]Note that we have not made any assumptions yet on $\sigma_D(S_t)$, except that it vanishes for $S_t \to \lambda^{-1}$.

times. Hence, our model predicts a positive comovement of leverage and consumption at the household level.

An implication of this result is that agents who took on higher leverage during good times are those that suffer a bigger drop in consumption growth as S_t mean reverts. In particular, we have the following corollary about agents' consumption growth:

Corollary 9 *Agent i's consumption growth satisfies*

$$E\left[\frac{dC_{it}}{C_{it}}\right]/dt = \mu_D + \frac{(w_i - a_i)\overline{Y}S_t}{a_i + (w_i - a_i)\overline{Y}S_t}F(S_t) \tag{27}$$

with

$$F(S_t) = k(1 - \overline{Y}S_t) + v\ \sigma_D^2(S_t) \tag{28}$$

If $\sigma_D(S_t)$ is decreasing in S_t with $\sigma_D(\lambda^{-1}) = 0$, then the function $F(S)$ has $F'(S) < 0$ and $F(0) > 0$ and $F(\lambda^{-1}) = k(1 - \lambda^{-1}\overline{Y}) < 0$. Thus, there exists a unique solution S^* to $F(S^*) = 0$ such that for all i and j with $w^i - a^i > 0$ and $w^j - a^j < 0$ we have

$$E\left[\frac{dC_{it}}{C_{it}}\right] < \mu_D < E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad \text{for} \qquad S_t > S^* \tag{29}$$

$$E\left[\frac{dC_{it}}{C_{it}}\right] > \mu_D > E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad \text{for} \qquad S_t < S^* \tag{30}$$

This corollary shows that cross-sectionally agents with high $w_i - a_i > 0$ have a lower expected growth rate of consumption when S_t is high. We know that these are also times when such agents are heavily leveraged. It follows then that agents who are heavily leveraged enjoy both a high consumption boom in good times, but a lower *future* expected consumption growth. These agents also expect a higher consumption growth when S_t is low. Therefore, Corollaries 7 and 9 imply the following:

Corollary 10 *Periods with high aggregate leverage $L(S_t)$ forecast lower consumption growth of agents who were highly leveraged compared to those with lower leverage.*

That is, according to Corollary 10, periods of very high aggregate leverage should follow on average by periods in which levered agents "retrench" and experience consumption growth that is comparatively lower than those agents who did not take on leverage.

This implication of our model speaks to some of the recent debate regarding the low consumption growth of levered households following the Great Recession. Some argue that

the observed drop in consumption growth was purely due to a wealth effect, as levered households tend to live in counties that experienced big drops in housing values, whereas others have emphasized the critical role of debt in explaining this drop.[20] Clearly these effects are important but our contribution is to show that high leverage followed by low consumption growth is precisely what arises from risk sharing arguments in models that can address the observed conditional properties of asset returns, as external habit models do. Indeed, suppose that such cross-sectional differences have a spatial nature, e.g. counties with richer agents or agents with lower risk aversion. Then, such counties should experience a credit boom during good times with high consumption growth, followed by a relative consumption slump during bad times. Corollary 10 then highlights the crucial role of proper identification strategies when asserting that higher leverage *casually* make levered counties (or countries) suffer comparatively slower growth in the future. If there is any residual correlation between the instrument employed in the identification strategy and S_t in previous equations, then the causality interpretation of standard instrumental variable or diff-in-diff estimators is undermined.

4.6. Active trading in stocks and bonds

The following corollary explores the implications of our model for stock trading.

Corollary 11 *(Active trading)*

(a) Agents with positive leverage (i.e. with $w_i - a_i > 0$) increase their stock position in good times (when S_t increases), and decrease their stock position in bad times, (when S_t declines.)

(b) Agents with higher absolute difference $|w_i - a_i|$ trade more in response to changes to the aggregate surplus consumption ratio S_t.

Corollary 11-a says that agents with positive leverage increase the number of units of stocks purchased in good times, and decrease them in bad times. Notice that there is an *active trading* from the part of these agents. In a model with passive investors, an agent who is long stocks may mechanically find himself with a higher allocation in stocks during good times because the stock yields good returns in good times. Even if the units of stocks purchased to do not change, N_{it} = constant, such an agent would still have a higher position

[20]See for instance Mian and Sufi (2014, in particular pages 39-45) for a nice exposition of this debate.

in stocks during good times than during bad times. Corollary 11-a instead says that an agent who is leveraged ($w_i - a_i > 0$) actively *increases leverage* in good times to buy more shares of stocks in such times.

Conversely, and importantly, Corollary 11-a also implies that such levered traders *actively deleverage* as times are getting worse (S_t declines) by actively *selling* the risky assets. The model thus implies active deleveraging from levered investors, which is a typical behavior of levered institutions at the onset of financial crisis. Because deleveraging occurs as both the stock price plunges and in fact, also the wealth of levered investors drops, it may appear that a "selling pressure" of levered agents and who are deleveraging is the cause of the drop in the stock price. While in reality such effects may occur, in our model the joint dynamics of deleveraging and price drop happens for the simple reason that during bad times aggregate risk aversion increases.

Corollary 11-b predicts that some agents trade more than others. From formulas (16) and (17) for bond and stock positions, we see that agents with $w_i = a_i$ do not change their position in stocks and bonds at all. They have zero leverage, and they just purchase a_i units of stock. Everyone else not only have higher or lower stock position than their preference-based position a_i, but they trade in response to aggregate shocks. Those with highest difference in $|w_i - a_i|$ trade the most. This model therefore has specific predictions on the cross-sectional difference in trading, which depend on heterogeneous endowments and preferences.

Corollary 12 *("panic deleveraging") The function $H_0(S_t)$ in (22) is concave in S_t. Therefore, both leverage and asset holdings of levered agents decrease by an increasing larger amount as time get worse, i.e. as S_t declines.*

Corollary 12 shows that $H_0(S_t)$ is not only increasing in S_t but it is concave in it. Such concavity has an important additional economic implication: during good times (S_t high) we should observe higher aggregate leverage and higher asset holdings of levered agents, but less variation of both compared to bad times (S_t low). This implies that as S_t declines, levered agents decrease their leverage by an increasingly larger amount, giving the impression of a "panic deleveraging" of levered agents during bad times.

One may be tempted to link asset sales by levered agents with the decline in asset prices but recall that, as shown in Corollary 5, the same asset pricing implications obtain without heterogeneity and hence no trade: The asset pricing implications of our model are orthogonal to its cross sectional implications. Our model then should caution against the excessive

reliance on the simple intuition of price declines due to the trading of some agents in the economy and the corresponding price pressure arguments. In our model, both trading and asset prices are jointly determined in equilibrium and influenced by time varying economic uncertainty and risk preferences.

4.7. The dynamics of wealth and wealth dispersion

We finally characterize agents' wealth and the ensuing dispersion of wealth. Recall that in our setting, all wealth is financial in nature, as it is composed by positions in stocks and bonds

$$W_{it} = N_{it} P_t + N_{it}^0 B_t$$

Because of dynamically complete markets, agents wealth equal the present value of future consumption

$$W_{it} = E_t \left[\int_t^\infty \frac{M_\tau}{M_t} C_{i\tau} d\tau \right]$$

We have the following proposition:

Proposition 13 *The wealth-consumption ratio of agent i is given by*

$$\frac{W_{it}}{C_{it}} = \frac{1}{\rho + k} \left\{ 1 + \frac{k w_i \overline{Y} S_t}{\rho \left(a_i + (w_i - a_i) \overline{Y} S_t \right)} \right\} \tag{31}$$

The wealth-output ratio of agent i is given by

$$\frac{W_{it}}{D_t} = \frac{1}{\rho} \left[\frac{\rho}{\rho + k} a_i \left(1 - \overline{Y} S_t \right) + w_i \overline{Y} S_t \right] \tag{32}$$

Expression (32) shows that the wealth to output ratio depends on agents' share of aggregate endowment w_i and their average habit sensitivity a_i. Higher w_i increases agents' wealth in good times because their higher endowment allow them to take on more leverage and thus reap the gains of an increase in stock market prices. For given w_i, however, agents with higher a_i have wealth that increases less or even decrease in good times compared to agents with lower a_i. As discussed earlier, the latter type of agents tends to take on more leverage to increase their stock holdings, which increase their wealth when stock market increases, and vice versa.

Finally, when the economy is at the aggregate steady state, i.e. $\overline{Y} S_t = 1$, then heterogeneity in preference does not matter. The reason is that we determined the Pareto

weights at time 0 under the assumption that $\overline{Y}S_0 = 1$. The Pareto weights thus "undo" the heterogeneity in preferences at the steady state.

We now turn to study the impact of the model on the cross-sectional dispersion in the wealth distribution at time t. Exploiting the characterization of wealth in equation (32), we obtain the following result:

Proposition 14 *Let $Var^{CS}(a_i)$, $Var^{CS}(w_i)$, and $Covar^{CS}(a_i, w_i)$ denote the cross-sectional variance of preference characteristics a_i and in share w_i of aggregate endowment, and their covariance, respectively. Then, the cross-sectional variance of wealth/output ratio is*

$$\begin{aligned} Var_t^{CS}\left(\frac{W_{it}}{D_t}\right) &= \frac{Var^{CS}(a_i)}{(\rho+k)^2}\left(1-\overline{Y}S_t\right)^2 + \frac{Var^{CS}(w_i)}{\rho^2}\left(\overline{Y}S_t\right)^2 \\ &\quad + 2\frac{Cov^{CS}(a_i,w_i)}{(\rho+k)\rho}\left(1-\overline{Y}S_t\right)\left(\overline{Y}S_t\right) \end{aligned} \tag{33}$$

To understand the intuition behind (33), first recall that when $\overline{Y}S_t = 1$, the economy is at its stochastic steady state, which is the initial condition at time 0 when agents' wealth is $W_{i0} = w_i$, their initial endowment. Thus, expression (33) shows that when the system is at its stochastic steady state, the wealth dispersion is given by the dispersion in endowments w_i.

Consider first the case in which the cross-sectional covariance between endowment and preferences is zero, $Cov^{CS}(a_i, w_i) = 0$. During good times the surplus consumption ratio S_t increases. Whether this variation brings about an increase or decrease in wealth distribution, however, depends on whether the relative importance of the heterogeneity in preferences $Var^{CS}(a_i)$ compared to the dispersion in shares of aggregate endowment across the population. For instance, if $Var^{CS}(a_i) = 0$, then during good times (high S_t) the dispersion in wealth increases, while it decreases during bad times. Intuitively, when $Var^{CS}(a_i) = 0$, all agents differ from each other only in shares of aggregate endowment, and the fair value of such shares of aggregate endowments increase during good times because of lower discount rates. Thus, the relative valuation of initial endowments increases, and thus so does the dispersion of wealth distribution, as it increases the stock market.

However, if $Var^{CS}(w_i) = 0$, then the dispersion in wealth is null at the aggregate stochastic steady state $\overline{Y}S_t = 1$, but it otherwise increases, both in good or in bad times, due to heterogeneous preferences. The intuition stems immediately from (32) and the discussion in previous section: Differential stock holdings across agents induced by differential preferences

generate an increase in dispersion both during good times, as agents heavily invested in stocks outperform, and symmetrically in bad times, when they underperform.

Once both dispersion in income shares and dispersion in preferences are taken into account and if $Cov^{CS}(a_i, w_i) > 0$, the impact is ambiguous in bad times, but it is un-ambiguous in good times: wealth dispersion will be higher in good times, as both forces go in the same direction.

We conclude this section with a characterization of agents' returns on investments:

Proposition 15 *The return on wealth portfolio of agent i's is*

$$dR_{W,i} = (r(S_t) + \mu_{W,i}(S_t))dt + \sigma_{W,i}(S_t)dZ_t \tag{34}$$

where the excess return and wealth volatility are, respectively

$$\mu_{W,i}(S_t) = E_t[dR_{W,i} - r(S_t)dt] = \beta_i(S_t)E[dR_P - r(S_t)dt] \tag{35}$$

$$\sigma_{W,i}(S_t) = \sigma_D(S_t)\left(1 + \frac{v\,(k + (\rho + k)(w_i - a_i)/a_i)\,\overline{Y}S_t}{\rho + (k + (\rho + k)(w_i - a_i)/a_i)\,\overline{Y}S_t}\right) \tag{36}$$

The market beta of agent i's portfolio is given by

$$\beta_i(S_t) = \frac{Cov_t(dR_{W,i}, dR_P)}{Var_t(dR_P)} = \frac{1 + v\ \left(1 - \frac{\rho}{\rho + [k + (\rho + k)(w_i - a_i)/a_i]\overline{Y}S_t}\right)}{1 + v\ \left(1 - \frac{\rho}{\rho + k\overline{Y}S_t}\right)}$$

In particular, $\beta_i(S_t) > 1$ *if and only if* $w_i > a_i$.

Proposition 15 shows that agents with a higher leverage enjoy higher average return on wealth than agents with lower leverage. Indeed, $\beta(S_t) > 1$ for all S_t if $w_i > a_i$. That is, independently on whether times are good or bad, agents with higher $w_i - a_i$ have a higher average return on their wealth. Clearly, this does not mean that on average, such agents will be infinitely wealthy in the infinite future – a standard result in models with agents with heterogeneous risk aversion – as we already know that the wealth distribution is stationary. The resolution of the puzzle is simply that such agents also take on more risk ($\sigma_{W,i}$ is larger) which implies larger losses than others during bad times. This argument shows that even if some agents enjoy higher average return on capital (wealth) all the time, this fact per se' does not lead the conclusion of a permanently more concentrated wealth distribution.

Expression (36) also allows to return to the intuition behind optimal leverage and the hedging strategy of agents' consumption plan. The wealth of agent i must satisfy

$$W_{it} = N_{it}P_t + N_{it}^0 B_t = E_t \left[\int_t^\infty \frac{M_\tau}{M_t} C_{i\tau} d\tau \right]$$

From the optimal risk sharing rule (11), agents with a high $w_i - a_i > 0$ have a high consumption share in good times, when S_t is high, and a low consumption share in bad times, when S_t is low. Such raising consumption profiles during good times makes their present values W_{it} especially sensitive to discount rate shocks in good times compared to bad times. As such, a rising leveraged position in the stock market is necessary to replicate the additional sensitivity of their wealth to discount rate shocks. Indeed, comparing (36) to (23), the wealth volatility $\sigma_{W,i}(S_t)$ of agents with $w_i - a_i > 0$ is higher than the stock volatility $\sigma_P(S_t)$ especially in good times, when S_t is high.

5. Quantitative Implications

We now provide a quantitative assessment of the effects discussed in previous sections. While the results in previous sections do not depend on the specific form of $\sigma_D(Y_t)$, we now make a specific reasonable assumption in order to make the model comparable with previous research. In particular, we assume

$$\sigma_D(Y_t) = \sigma^{max} \left(1 - \lambda Y_t^{-1}\right) \tag{37}$$

This assumption implies that dividend volatility increases when the recession index increases, but it is also bounded between $[0, \sigma^{max}]$.[21] This assumption about output volatility is consistent with existing evidence that aggregate uncertainty increases in bad times (see e.g. Jurado, Ludvigson, and Ng (2015)), it satisfies the technical condition $\sigma_D(Y) \to 0$ as $Y_t \to \lambda$, and it also allows us to compare our results with previous literature, as it generates a state price density similar to MSV, as we obtain

$$dY_t = k(\overline{Y} - Y_t)dt - (Y_t - \lambda)\overline{v} dZ_t$$

with $\overline{v} = v\sigma^{max}$ as in MSV.[22]

[21] The alternative of assuming e.g. $\sigma_D(Y)$ as linear in Y_t would result in $\sigma_D(Y)$ potentially diverging to infinity as Y_t increases.

[22] Technically, we also impose $\sigma_D(S_t)$ converges to zero for $S_t \leq \epsilon$ for some small but strictly positive $\epsilon > 0$ to ensure integrability of stochastic integrals. This faster convergence to zero for a strictly positive number can be achieved through a killing function, as in Cheriditto and Gabaix (2008). We do not specify such functions explicitly here, for notational convenience.

Table 1: **Parameters and Moments**

Panel A. Parameter Estimates

ρ	k	$\overline{Y}$	λ	$\overline{v}$	μ	$\overline{\sigma}$	σ^{max}
0.0416	0.1567	34	20	1.1194	0.0218	.0141	0.0641

Panel B. Moments (1952 – 2014)

	$E[R]$	$Std(R)$	$E[r_f]$	$Std(r_f)$	$E[P/D]$	$Std[P/D]$	SR	$E[\sigma_t]$	Std(σ_t)
Data	7.13%	16.55%	1.00%	1.00%	38	15	43%	1.41%	0.52%
Model	7.00%	24.58%	1.68%	5.84%	30.37	5.90	28.48%	1.42%	1.20%

Panel C. P/D Predictability R^2

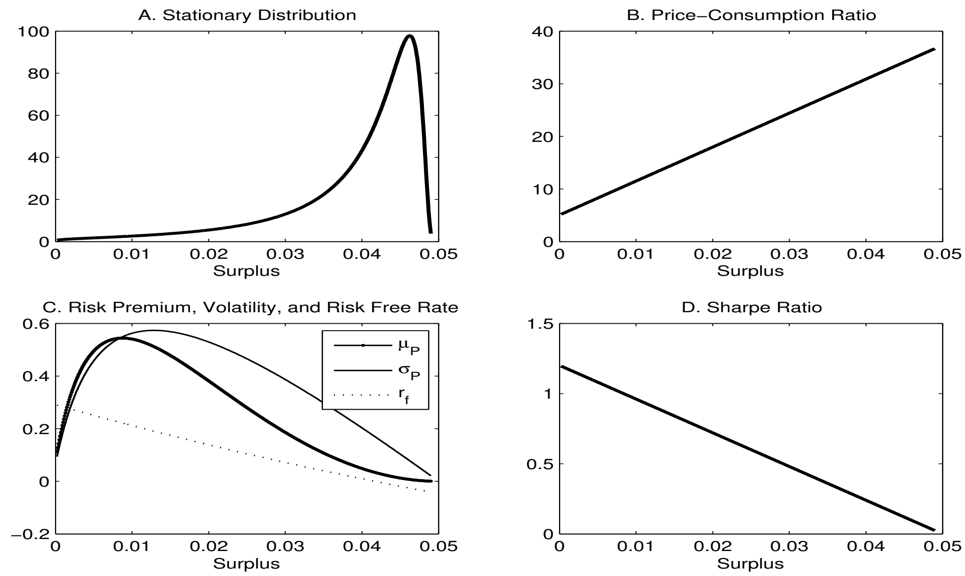
	1 year	2 year	3 year	4 year	5 year
Data	5.12%	8.25%	9.22%	9.59%	12.45%
Model	9.22%	14.47%	17.65%	20.13%	21.87%

For the calibration we use the same parameters as in MSV Table 1 to model the dynamics of Y_t. These are are reported in Panel A of Table 1. The only additional parameter is σ^{max}, which we choose to match the average consumption volatility $E[\sigma_D(S_t)] = std[\Delta \log(C^{data})]$, where the expectation can be easily computed from the stationary density of Y_t (see the Appendix in MSV).[23]

Figure 1 reports the conditional moments implied by the model as a function of the surplus-consumption ratio S_t. As in MSV Figure 1, Panel A reports the stationary distribution of the surplus-consumption ratio S_t and shows that most of the probability mass is around $\overline{S} = 0.0294$, although S_t drops considerably below occasionally. The price-dividend ratio is increasing in S_t (panel B), while volatility, risk premium and interest rates decline with S_t (panel C). Finally, the Sharpe ratio is also strongly time varying, and it is higher in bad times (low S_t) and lower in good times (high S_t). This figure is virtually identical to Figure 1 in MSV, which highlights that our mild calibration of consumption volatility (with

[23]In addition, note that in MSV, $\alpha = \overline{v}/\sigma$ and therefore we compute $\overline{v} = \alpha\sigma$.

Figure 1: Conditional Moments



a maximum of only 6.4%) is such to have a minor on impact on the level of asset prices.

Given the parameters in Panel A of Table 1, we simulate 10,000 years of quarterly data and report the aggregate moments in Panel B. As in MSV, Table 1, the model fits well the asset pricing data, though both the volatilities of stock returns and of the risk free rate are higher than the empirically observed one. Still, the model yields a respectable Sharpe ratio of 28.48%. Finally, the simulated model generates an average consumption volatility of 1.42% with a standard deviation of 1.20%. This latter variation is a bit higher than the variation of consumption volatility in the data (0.52%), where the latter is computed fitting a GARCH model to quarterly consumption data, and then taking the standard deviation of the annualized GARCH volatility. Our calibrated number is however lower than the standard deviation of dividend growth' volatility, which is instead around 1.50%.

The calibrated model also generates a strong predictability of stock returns (Panel C), with R^2 ranging between 9.22% at one year to 21.87% at 5 year. This predictability is stronger than the one generated in MSV and also the one in the data. This is due to the combined effect of time varying economic uncertainty (*i.e.* the quantity of risk) and time varying risk aversion (*i.e.* the market price of risk), which move in the same direction.

5.1. The Aggregate Behavior of Levered Agents

Inspection of the formulas derived throughout the paper reveals that the aggregate behavior of levered agents depends on two quantities:

$$K_0 = \int_{i:w_i - a_i > 0} a_i di; \qquad K_1 = \int_{i:w_i - a_i > 0} (w_i - a_i) di \tag{38}$$

It follows that the aggregate behavior of levered agents does not at this stage depend on any specific assumptions about the cross-sectional distribution of endowments w_i and preferences a_i but simply on K_0 and K_1. In what follows, and for illustrative purposes only, we set these two constants to $K_0 = 0.2483$ and $K_1 = 0.1526$, as these are the values that we obtain later in Section 5.2., when we discuss the cross-sectional implications of the model. Other values would result only on changes in levels but would leave the implications about the dynamics unaltered.

5.1.1. Leverage and Stock Holdings in Good and Bad Times

Consistently with the previous section, equation (26) shows that aggregate leverage is

$$L(S_t) = vH_0(S_t)K_1 \tag{39}$$

Similarly, from (17) the aggregate stock holding of levered agents can be written as

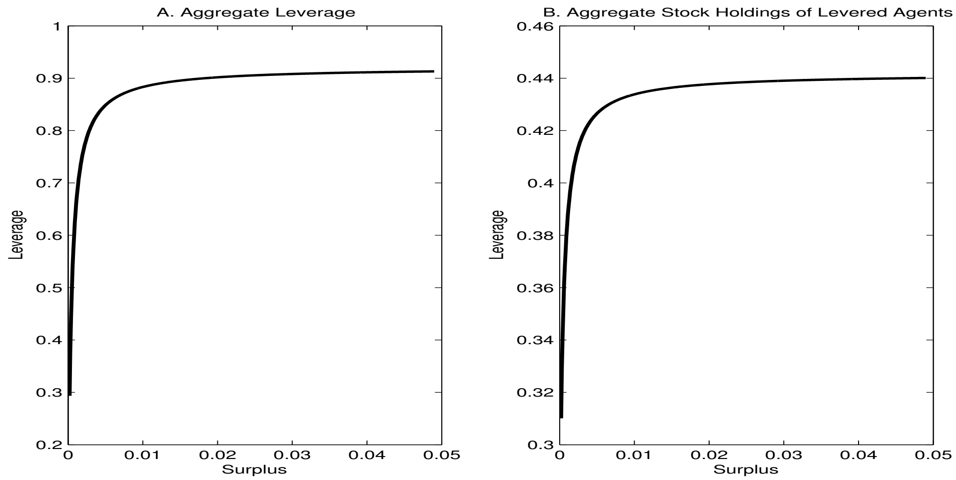
$$N^{Lev}(S_t) = K_0 + (\rho + k)(1 + v) K_1 H_0(S_t) \tag{40}$$

where recall $H_0(S_t)$ is given by (22).

As discussed in Corollaries 7 and 12, $H_0(S_t)$ is increasing and concave in S_t. That is, leverage and aggregate allocation to stocks are not only procyclical, but they also decline increasingly faster as times get worse, i.e. as S_t decline. Panels A and B of Figure 2 shows the patterns of $L(S_t)$ and the aggregate stock holdings of the levered agents, $N^{Lev}(S_t)$, under the parameter choices in Table 1. The concavity of $H_0(S_t)$ is especially strong for very low levels of S_t: Deleverage accelerates rapidly as bad times turn into severe distress. It is useful to recall that the function $H_0(S_t)$ is independent of the assumptions about the aggregate volatility $\sigma_D(S_t)$, and thus the strong concavity displayed in Figure 2 stems from the increase in aggregate risk aversion in bad times, with the implied decrease of differential sensitivity of stock prices and agents' wealth to discount rate shocks.

As already mentioned, this non-linear behavior of leverage and risky asset holdings of levered agents with respect to the surplus consumption ratio suggests that levered agents

Figure 2: Aggregate Leverage and Aggregate Stock Holdings of Levered Agents

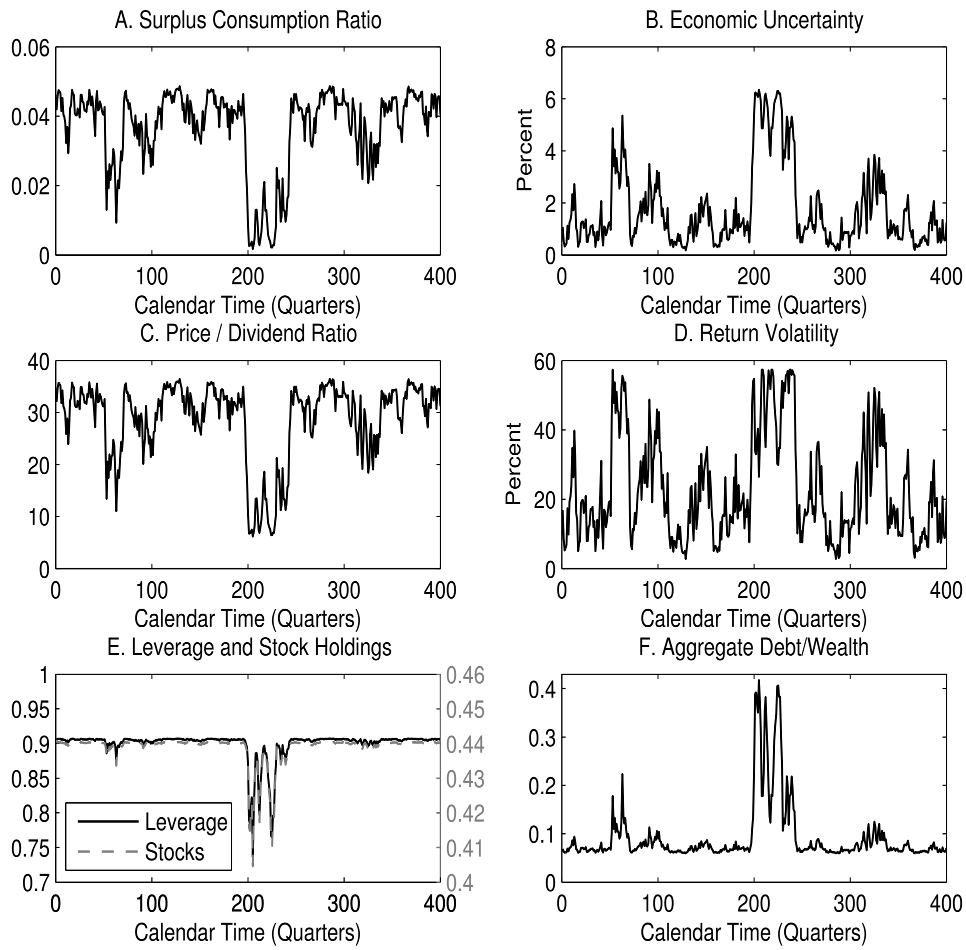


"fire sell" risky assets to decrease leverage in bad times. This is shown in the simulated path illustrated in Figure 3. Panel A shows 100 years of artificial quarterly data of the surplus consumption ratio S_t, while panel B report the corresponding economic uncertainty $\sigma_D(S_t)$. Panel C shows the variation in the price-dividend ratio due to variation in the surplus consumption ratio, with a visible drop of the stock price from 30 to less than 10 in the middle of the simulated sample. Panel D shows the corresponding stock return volatility, which increases dramatically during bad times, as it shoots up to almost 60% during the "crisis". Panel E demonstrates the impact of the variation of the surplus consumption ratio on the aggregate leverage-output ratio and the aggregate stock holdings of levered agents. As it is apparent, the variation of both quantities is rather limited most of the time, except during the extreme bad event visible in the middle of the sample. In this occasion, as the surplus consumption ratio drops and economic uncertainty increases, levered agents decrease their indebtness and liquidate their stock positions.

Finally, Panel F shows the debt-to-wealth ratio of the levered agents, and it highlights that the model is consistent with the observation that the efforts of all levered agents to delever simultaneously results in an increase in leverage ratios. Indeed, while Panel E shows that aggregate debt declines during bad times, Panel F shows that the aggregate debt-wealth ratio actually increases, as levered agents' wealth declines faster then the decline in debt leverage.

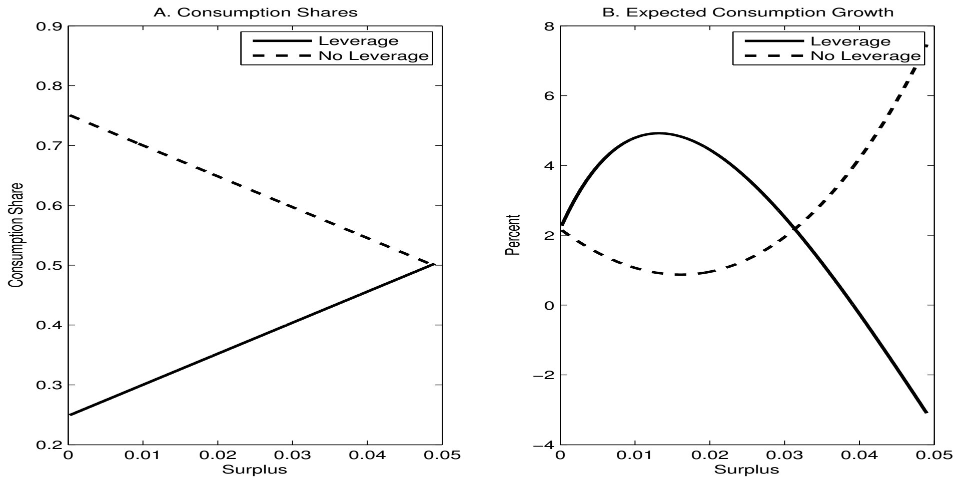
In sum thus, as economic conditions deteriorate (a drop in S_t) prices fall but agents only delever and liquidate stock positions slowly. As bad times turn into severely distressed conditions, deleveraging and stock liquidation accelerates, creating the impression of a panic

Figure 3: "Fire Sales" in a Simulation Run



selling episode. Leverage *ratios*, debt-to-wealth, increase sharply as prices drop faster than the deleveraging. In addition, as shown in Corollary 10 and discussed further below, the consumption of highly levered agent falls. These results obtain in the absence of any contagion effects, liquidity dry ups or debt overhang considerations. They are the result of the optimal trading of utility maximizing agents in an equilibrium that in fact implements an optimal risk sharing allocation. Our claim, again, is not that these particular frictions do not matter but rather to argue that the dynamics in quantities and prices observed in crises obtain naturally in risk sharing models that feature the strong discount effects needed to obtain reasonable asset pricing implications. Tests aimed at uncovering the aforementioned frictions have to control for the component of these dynamics that are the result of optimal risk sharing.

Figure 4: The Consumption of Levered Agents



5.1.2. The consumption of levered agents

Similarly to leverage and the stock-holdings of levered agents, the aggregate consumption of levered agents can be easily computed from the individual consumption shares as $C_t^{Lev}/D_t = K_0 + K_1\overline{Y}S_t$, where again K_0 and K_1 are in (38). The aggregate consumption share of unlevered agents is simply $C_t^{Unlev}/D_t = 1 - C_t^{Lev}/D_t$.

Panel A of Figure 4 shows the aggregate consumption share of levered agents (solid line) and unlevered agents (dashed line). As established in Proposition 1 and Corollary 6, the consumption share of levered agents increases in the surplus consumption ratio.

Panel B of Figure 4 shows the aggregate expected consumption growth of levered agents (solid line) and unlevered agents (dashed line), computed as

$$E\left[\frac{dC_t^{Lev}}{C_t^{Lev}}\right]/dt = \mu_D + \frac{K_1\overline{Y}S_t}{K_0 + K_1\overline{Y}S_t}F(S_t) \qquad (41)$$

and $F(S_t)$ is in (28). Consistently with Panel A, during good times levered agents are at the peak of their consumption share and therefore, they should expect lower consumption growth going forward. Conversely, during good times unlevered agents are at the bottom of their consumption share, and therefore should expect higher consumption growth going forward.

5.1.3. The Wealth Dynamics of Levered Agents

As in previous section, we can study the aggregate dynamics of levered agents without making specific assumption on the cross-sectional properties of preferences and endowments. Indeed, from (32) the wealth-to-output ratio of levered agents is[24]

$$\frac{W_t^{Lev}}{D_t} = \frac{1}{\rho}\left[\frac{\rho}{\rho+k}K_0\left(1-\overline{Y}S_t\right)+(K_0+K_1)\overline{Y}S_t\right] \tag{42}$$

The wealth-output ratio of unlevered agents is $W_t^{Unlev}/D_t = P_t/D_t - W^{Lev}/D_t$. From here, we can also compute the expected return and volatility of wealth, obtaining expressions similar to those in Proposition 15 .

Panel A of Figure 5 plots the wealth-output ratio of levered and unlevered agents. Not surprisingly, both the wealth-output ratios increase with the surplus consumption ratio, as the aggregate economy become wealthier. Panel B, however, shows that the share of aggregate wealth in the hands of levered agents increases with S_t, while unlevered agents see a reduction of their wealth share in such times. In addition, the increase in the wealth share of levered agents is concave, flattening out at high levels of S_t.

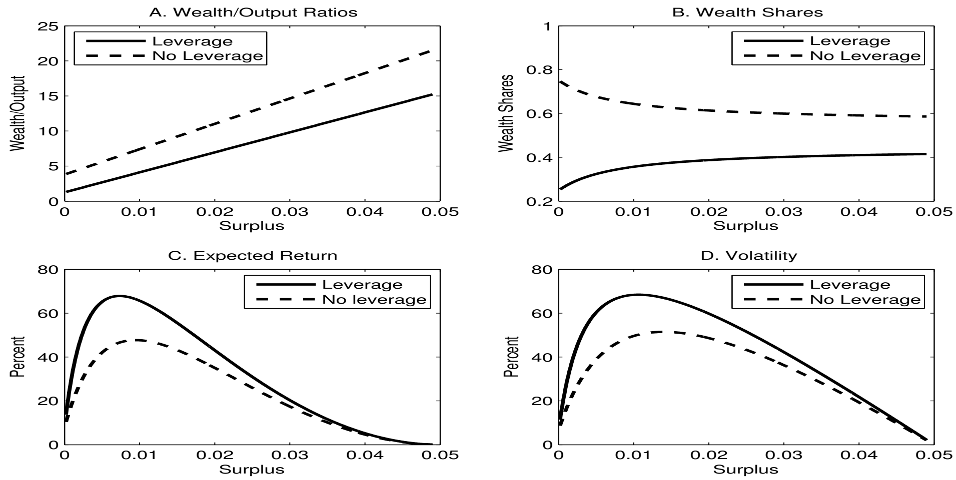
Panel C of Figure 5 shows that levered agents enjoy a uniformly higher expected return than unlevered agents. That is, as a group, those agents with $w_i - a_i > 0$, obtain higher average returns on wealth than unlevered agents. In many models with heterogeneous agents, this higher return would tend to generate an accelerated accumulation of capital of levered agents, who eventually would own the whole economy (see e.g. Dumas (1989)). This feature does not hold here because levered agents also take on more risk, as shown in Panel D of Figure 5. That is, even if agents with $w_i - a_i > 0$ have a higher average return on wealth, they also hold riskier portfolios, which leads to severe losses during downturns (S_t declining). As a consequence, the wealth share fluctuates as shown in Panel B.

5.2. The Cross Section of Agents' Behavior: Who levers?

The previous sections discuss the dynamics of aggregate quantities, such as aggregate leverage and aggregate stock holdings of levered agents. We now discuss the cross-sectional differences in agents' behavior. To this end, we finally make some assumptions about the dispersion of initial endowments w_i and of preferences a_i. For a_i we simply assume they are uniformly distributed between $\underline{a} = 0$ and $\overline{a} = 2$, so as $\int a_i di = 1$. Endowments w_i must

[24] Note that $\int_{i:w_i-a_i>0} w_i di = \int_{i:w_i-a_i>0} a_i di + \int_{i:w_i-a_i>0}(w_i-a_i)di = K_0 + K_1$.

Figure 5: The Wealth of Levered Agents



obey the technical assumption A1. While distributions can be found such that a_i and w_i are independent, restriction A1 severely restricts the dispersion of such distributions. We opt instead to rather assume that Pareto weights ϕ_i are distributed independently of preferences a_i and obtain the endowments by inverting (10):

$$w_i = \frac{\phi_i + a_i(\overline{Y} - \lambda) + \lambda - 1}{\overline{Y}} \tag{43}$$

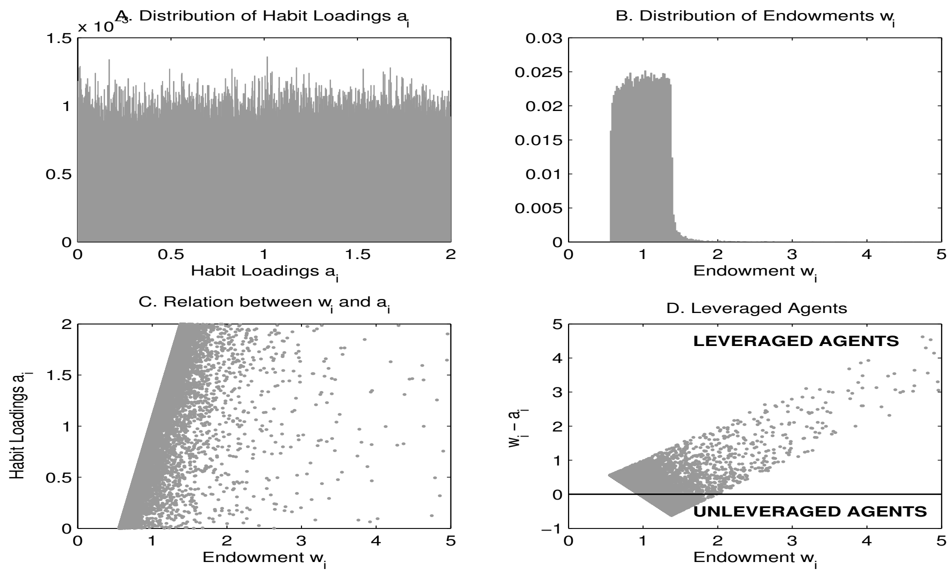
To ensure a skewed distribution of wealth, we assume

$$\phi_i = e^{-\sigma_w \varepsilon_i - \frac{1}{2}\sigma_w^2}$$

with $\varepsilon_i \sim N(0,1)$ and $\sigma_w = 2$. Thus, $\int_i \phi_i di = E^{CS}[\phi_i] = 1$. This procedure ensures that the Pareto weights are positive and hence all the constraints are satisfied. While all agents have random Pareto weights, and therefore contribute to the representative agent in a random manner, the procedure implies that agents with higher habit sensitivity a_i have also a higher endowment, a required condition to have well defined preferences in equilibrium.

Panel A and B of Figure 6 shows the resulting distribution of preferences and endowment in a simulation 100,000 agents. In particular, Panel B shows a markedly skewed distribution of endowments (we cut out the right tail of the distribution for better visual impression). Because of the restriction $\int w_i di = 1$, the distribution shows a large mass of agents with $w_i < 1$ to allow for some agents with a very large endowment. Panel C shows the relation between endowments on the x-axis and preference on the y-axis. The white area in the top-left corner is due to restriction A1: Agents with high habit loading a_i must have high initial endowment w_i to ensure a feasible consumption plan.

Figure 6: Preference and Endowment Distribution



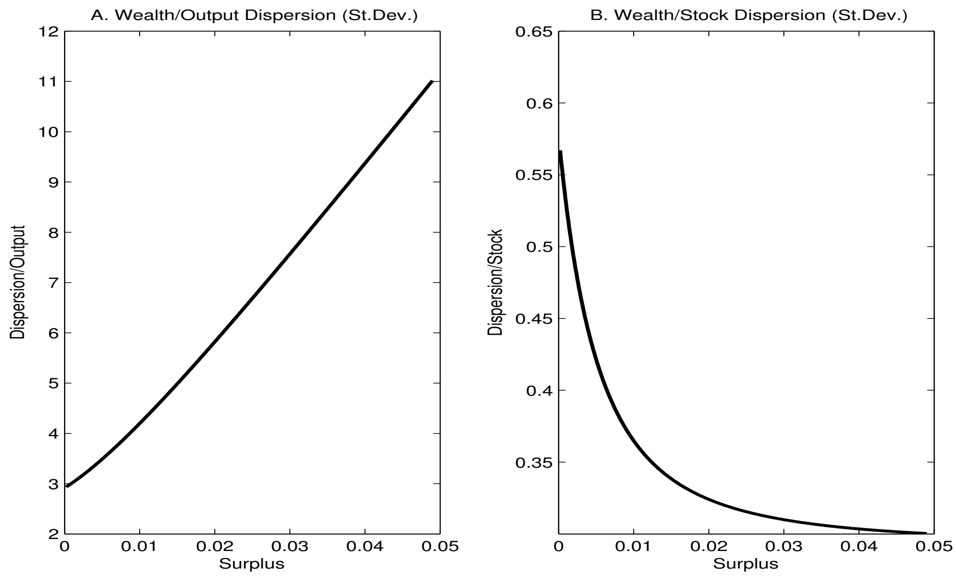
Finally, Panel D shows the relation between endowment w_i and leverage, namely, $w_i - a_i$. Indeed, recall that only agents with $w_i - a_i > 0$ lever up (see Corollary 6). Leverage has a sort of U-shape in the cross section in that two types of agents lever up, very poor agents but with very low sensitivity to surplus consumption shocks and rich agents. The group with intermediate endowment, in contrast, are those who purchase the risk-free asset.

5.2.1. Wealth Dispersion

We finally consider the wealth dispersion implied by the model in good and bad times. There are two measures of wealth dispersion. Panel A of Figure 7 plots the cross-sectional standard deviation of wealth-output ratios, as already introduced in equation (33). The plot shows a strongly increasing dispersion of wealth as times get better. This is a level effect: as the aggregate wealth increase, the *level* difference of wealth-output ratio increases. This pattern was in fact evident already in Panel A of Figure 5, as the wealth-output ratios of levered and unlevered agent diverges as times get better (S_t increases).

However, a second measure of wealth dispersion is the dispersion compared to aggregate wealth. Indeed, we know from Panel B of Figure 1 that the wealth increases seven-fold from very bad times to very good times, and so the question is how this increase in wealth is shared across agents in the model. Panel B of Figure 7 shows that the cross-sectional standard deviation of wealth normalized by aggregate wealth actually declines for the parameters of

Figure 7: Wealth Dispersion



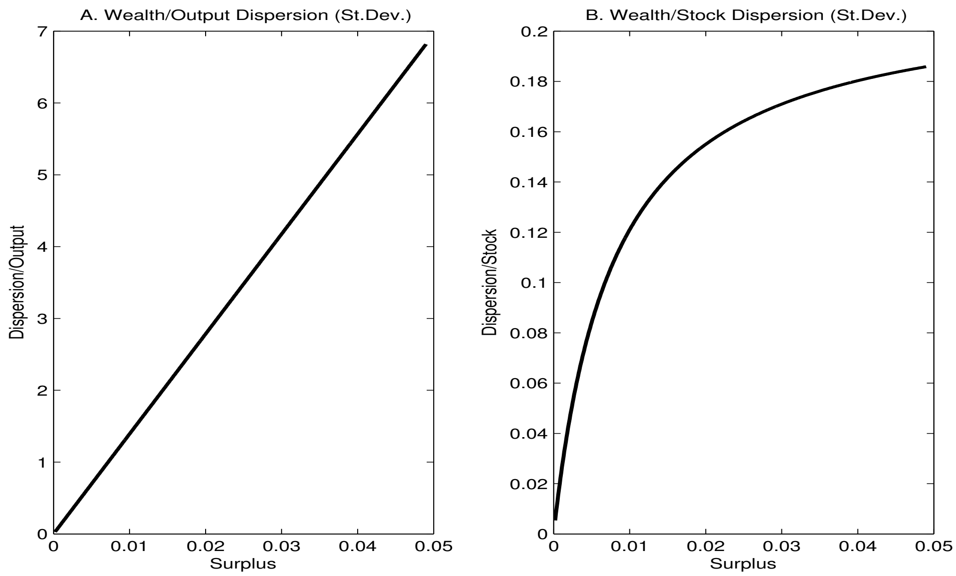
this section. This is consistent with the finding in Panel B of Figure 5 which shows the convergence of wealth shares between levered and unlevered agents.

5.2.2. The Role of the Dispersion in Endowments

We conclude this section with an illustration of the impact of initial endowments on the wealth inequality over the business cycle. Consider the case in which all agents have the same preferences, that is, $a_i = 1$ for all i but the distribution of Pareto weights is as above. This results in log-normally distributed initial endowments (this distributional assumption is irrelevant for the result).

Figure 8 is the analog of Figure 7 under the new assumptions. As is evident, Panel A is very similar to Panel A of the previous case due to the "level effect" on the wealth distribution. Panel B is instead markedly different: The model with homogeneous agents who have though differential endowment at time 0 generate a procyclical relative wealth inequality. That is, richer agents not only become richer during good times because of the level effect, but they become relatively richer compared to the rest of the economy. This is due to the fact that under this parametrization, rich agents borrow while poor agents lend. Thus, during good times, levered agents do better and become wealthier, even relative to the economy. The case discussed in Figure 7 had both very poor and very rich agents borrowing. Thus, the two extreme groups of the economy would do well in good times, reducing the

Figure 8: Wealth Dispersion with No Preference Heterogeneity



wealth inequality relative to the wealth of the economy.

6. Conclusions

Our general equilibrium model with heterogeneous agents, habits, and countercyclical uncertainty, is able to tie together several stylized facts related to leverage, consumption, and asset prices. For instance, our model predicts that aggregate leverage should be procyclical, it should correlate with high valuation ratios, low volatility, and with a "consumption boom" of levered agents. Agents actively trade in risky assets, moreover, and delever in bad times by "fire selling" their risky positions as their wealth decline and debt/wealth increase.

An important message of the paper is that leverage is an *endogenous* quantity and some caution must thus be taken when making casual statements about the impact of leverage on other economic quantities. For instance, in our model agents who increased leverage during good times will suffer low consumption growth in bad times. There is nothing inefficient of this allocation: Those agents who *decide* to take on higher leverage are implicitly providing insurance to the other agents who instead would like to buy safe assets. Similarly, the increase in leverage in good times is the result of an optimal, efficient risk-sharing allocation, and should predict low future asset pricing returns. Once again, it is not high leverage that implies that future return are low (because it increases the chance of a financial crisis, for

instance), but rather the fact that lower risk premia due to subsided discount rate shocks induce agents with higher risk bearing capacity to take higher leverage to achieve their optimal consumption profile.

Admittedly, our model is simple in that it only has one state variable and all quantities are driven by only one shock. Our simplifying assumptions thus imply that all quantities move in lock-step and there is a likely unrealistic perfect (positive or negative) correlation between leverage, prices, volatility, expected return, consumption, and so on. These simplifying assumptions allow us to obtain closed form solutions for all quantities in the model, and thus obtain a better understanding of the various economic forces that affect leverage and asset prices. Future research may attempt to generalize our simple setting to obtain more realistic dynamics.

Appendix: Proofs

Proof of Proposition 1. The Lagrangean

$$\mathcal{L}(C_i) = \int \phi_i u(C_{it}, X_{it}, t)\, di - M_t \left(\int C_{it} di - D_t \right)$$

implies that agents' marginal utilities satisfy

$$\phi_i u_c(C_{it}, X_{it}, t) = M_t. \tag{44}$$

Thus, consumption satisfies

$$C_{it} - X_{it} = \phi_i e^{-\rho t}\ M_t^{-1} \tag{45}$$

The individual excess consumption is inversely related to the Lagrange multiplier M_t. To obtain the equilibrium value of M_t, we integrate across agents

$$\int C_{it} di - \int X_{it} di = \left(\int \phi_i di \right)\ e^{-\rho t} M_t^{-1} = e^{-\rho t} M_t^{-1}$$

Using the market clearing condition $D_t = \int C_{it} di$ we find that the Lagrangean multiplier is

$$M_t = e^{-\rho t} \frac{1}{D_t - \int X_{it} di} \tag{46}$$

Finally, plugging this expression into (45) we obtain that agent i's consumption is given by

$$C_{it} - X_{it} = \phi_i \left(D_t - \int X_{jt} dj \right) \tag{47}$$

Each agent's excess consumption over habit is proportional to aggregate excess output. This condition also implies that in equilibrium, the ratio of any two agents' marginal utilities is constant (and equal to the ratio of Pareto weights), a standard result with complete markets. Substituting X_{it} from (1) and using (2) we obtain the optimal consumption of agent i in Proposition 1. □

Proof of Proposition 4. The Lagrange multiplier at time t in equation (46) provides the marginal utility of the representative agent. Using (9) we find:

$$M_t = e^{-\rho t} D_t^{-1} Y_t$$

The interest rate and SDF can be found by applying Ito's lemma to M_t. □

The pricing function for the consumption claim is

$$
\begin{aligned}
P_t &= E_t\left[\int_t^\infty \frac{M_\tau}{M_t} D_\tau d\tau\right] & (48)\\
&= D_t Y_t^{-1} E_t\left[\int_t^\infty e^{-\rho(\tau-t)} D_\tau^{-1} Y_\tau D_\tau d\tau\right] & (49)\\
&= D_t S_t E_t\left[\int_t^\infty e^{-\rho(\tau-t)} Y_\tau d\tau\right] & (50)\\
&= D_t S_t \int_t^\infty e^{-\rho(\tau-t)} E_t\left[Y_\tau\right] d\tau & (51)\\
&= D_t S_t \int_t^\infty e^{-\rho(\tau-t)} (\overline{Y} + (Y_t - \overline{Y}) e^{-k(\tau-t)}) d\tau & (52)\\
&= D_t S_t \left(\frac{\overline{Y}}{\rho} + \frac{(Y_t - \overline{Y})}{\rho + k}\right) & (53)\\
&= D_t S_t \left(\frac{\rho Y_t + k\overline{Y}}{\rho(\rho + k)}\right) & (54)
\end{aligned}
$$

Ito's lemma on P_t gives the further results about stock return volatility and expected return.

Finally, from market completeness, the wealth of agent i is always equal to the discounted value of his/her optimal consumption, which can be written as

$$
\begin{aligned}
C_{i,t} &= (g_{it} + \phi_i)\left(D_t - \int X_{jt} dj\right) & (55)\\
&= (a_i(Y_t - \lambda) + \lambda - 1 + \phi_i) S_t D_t & (56)
\end{aligned}
$$

We then have

$$
\begin{aligned}
W_{i,t} &= E_t\left[\int_t^\infty \frac{M_\tau}{M_t} C_{i,\tau} d\tau\right]\\
&= D_t S_t E_t\left[\int_t^\infty e^{-\rho(\tau-t)} D_\tau^{-1} S_\tau^{-1} C_{i,\tau} d\tau\right]\\
&= D_t S_t E_t\left[\int_t^\infty e^{-\rho(\tau-t)} (a_i(Y_\tau - \lambda) + \lambda - 1 + \phi_i) d\tau\right]\\
&= D_t S_t E_t\left[\int_t^\infty e^{-\rho(\tau-t)} (a_i Y_\tau - a_i\lambda + +\lambda - 1 + \phi_i) d\tau\right]\\
&= D_t S_t \left[a_i \frac{(Y_t - \overline{Y})}{\rho + k} + \frac{a_i(\overline{Y} - \lambda) + \lambda - 1 + \phi_i}{\rho}\right]
\end{aligned}
$$

where we used the fact that $E_t[Y_\tau] = \overline{Y} + (Y_t - \overline{Y}) e^{-k(\tau-t)}$. At time 0, the economy starts at its stochastic steady state, $\sigma_0 = \overline{\sigma}$, which implies $S_0 = \overline{S} = 1/\overline{Y} = 1/Y_0$. In addition, assume $D_0 = \rho$. Agent i's endowment is w_i. Therefore, we obtain that the budget constraint

implies

$$\begin{aligned}
w_i = W_{i,0} &= D_0 S_0 \left[a_i \frac{(Y_0 - \overline{Y})}{\rho + k} + \frac{a_i(\overline{Y} - \lambda) + \lambda - 1 + \phi_i}{\rho} \right] \\
&= D_0 S_0 \left[\frac{a_i(\overline{Y} - \lambda) + \lambda - 1 + \phi_i}{\rho} \right] \\
&= \overline{S} \left[a_i(\overline{Y} - \lambda) + \lambda - 1 + \phi_i \right]
\end{aligned}$$

$$w_i/\overline{S} = \left[a_i(\overline{Y} - \lambda) + \lambda - 1 + \phi_i \right]$$

or

$$\phi_i = w_i \overline{Y} - \left[a_i(\overline{Y} - \lambda) + \lambda - 1 \right] \ .$$

as in Proposition 1.

The curvature of the utility function can be obtained from the definition of curvature and by substituting C_{it} and ϕ_i in the resulting expression.

The consumption/output ratio (56) can then be written as

$$\begin{aligned}
\frac{C_{i,t}}{D_t} &= (a_i(Y_t - \lambda) + \lambda - 1 + \phi_i) S_t \\
&= (a_i(Y_t - \lambda) + w_i/\overline{S} - a_i(\overline{Y} - \lambda)) S_t \\
&= (a_i(Y_t - \overline{Y}) + w_i/\overline{S}) S_t \\
&= a_i(1 - S_t/\overline{S}) + w_i S_t/\overline{S}
\end{aligned}$$

Proof of Proposition 3. Given the results of Proposition 1 and the standard result that the efficient allocation maximize agents' utility, the only part left to show is the optimal allocation to stocks and bonds. From Cox and Huang (1989), the dynamic budget equation can be written as the present value of future consumption discounted using the stochastic discount factor. The optimal allocation can be found by finding the "replicating" portfolio, that is, the position in stocks and bonds that satisfies the static budget equation.

We denote for simplicity

$$\sigma_Y(Y) = v \sigma_D(Y) \tag{57}$$

First, note that the process for surplus consumption ratio is

$$\begin{aligned}
dS_t &= -Y_t^{-2} dY_t + Y_t^{-3} dY_t^2 \\
&= -Y_t^{-2} k(\overline{Y} - Y_t) dt + Y_t^{-1} \sigma_Y(Y) dZ_t + Y_t^{-1} \sigma_Y(Y)^2 dt \\
&= Y_t^{-1} k(1 - \overline{Y}/Y_t) dt + Y_t^{-1} \sigma_Y(Y) dZ_t + Y_t^{-1} \sigma_Y(Y)^2 dt \\
&= Y_t^{-1} \left(k(1 - \overline{Y}/Y_t) + \sigma_Y(Y)^2 dt \right) dt + Y_t^{-1} \sigma_Y(Y) dZ_t
\end{aligned}$$

Consider now the process for wealth dynamics in (62), which we write as

$$W_{i,t} = D_t \frac{1}{\rho}\left[a_i \frac{\rho}{\rho+k}(1-\overline{Y}S_t) + w_i \overline{Y} S_t\right] \tag{58}$$

$$= D_t \frac{1}{\rho(\rho+k)}\left[a_i\rho + (w_i(\rho+k) - a_i\rho)\,\overline{Y}S_t\right] \tag{59}$$

From Ito's lemma, the diffusion of wealth process $dW_{i,t}/W_{i,t}$ is

$$\sigma_{W,i}(S_t) = \sigma_D(S_t) + \frac{(w_i(\rho+k) - a_i\rho)\,\overline{Y}Y_t^{-1}\sigma_Y(Y_t)}{a_i\rho + (w_i(\rho+k) - a_i\rho)\,\overline{Y}Y_t^{-1}} \tag{60}$$

By market completeness (Cox and Huang (1989)), agent i's wealth is always equal to his/her allocation to stocks and bonds

$$W_{it} = N_{i,t}P_t + N_{i,t}^0 B_t$$

From this latter expression, N_{it} must be chosen to equate the diffusion of the portfolio to the diffusion of wealth. That is, such that

$$N_{it}P_t\sigma_P(S_t) = W_{i,t}\sigma_{W,i}(S_t)$$

Solving for N_{it} gives

$$\begin{aligned}
N_{it} &= \frac{W_{it}\sigma_{Wi}(Y)}{P_t\sigma_P(Y)} \\
&= \frac{\left(\rho a_i + (w_i(\rho+k) - \rho a_i)\,\overline{Y}/Y_t\right)}{\left(\rho + k\overline{Y}/Y_t\right)} \left(\frac{\sigma_D(Y) + \frac{(w_i(\rho+k)-\rho a_i)\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho a_i + (w_i(\rho+k)-\rho a_i)\overline{Y}/Y_t\right)}}{\sigma_D(Y) + \frac{k\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho+k\overline{Y}/Y_t\right)}}\right) \\
&= \frac{\left(\rho a_i + (w_i(\rho+k) - \rho a_i)\,\overline{Y}/Y_t\right)}{\left(\rho + k\overline{Y}/Y_t\right)} \left(\frac{\frac{\sigma_D(Y)\left(\rho a_i + (w_i(\rho+k)-\rho a_i)\overline{Y}/Y_t\right) + (w_i(\rho+k)-\rho a_i)\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho a_i + (w_i(\rho+k)-\rho a_i)\overline{Y}/Y_t\right)}}{\frac{\sigma_D(Y)\left(\rho+k\overline{Y}/Y_t\right) + k\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho+k\overline{Y}/Y_t\right)}}\right) \\
&= \left(\frac{\sigma_D(Y)\left(\rho a_i + (w_i(\rho+k) - \rho a_i)\,\overline{Y}/Y_t\right) + (w_i(\rho+k) - \rho a_i)\,\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\sigma_D(Y)\left(\rho + k\overline{Y}/Y_t\right) + k\overline{Y}Y_t^{-1}\sigma_Y(Y)}\right) \\
&= \frac{\sigma_D(Y)\,\rho a_i + \sigma_D(Y)\,\overline{Y}/Y_t\,(w_i(\rho+k) - \rho a_i) + w_i(\rho+k)\,\overline{Y}Y_t^{-1}\sigma_Y(Y) - \rho a_i\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\sigma_D(Y)\left(\rho + k\overline{Y}/Y_t\right) + k\overline{Y}Y_t^{-1}\sigma_Y(Y)} \\
&= a_i + (\rho+k)\frac{\sigma_D(Y)\,\overline{Y}/Y_t + \overline{Y}Y_t^{-1}\sigma_Y(Y)}{\sigma_D(Y)\left(\rho + k\overline{Y}/Y_t\right) + k\overline{Y}Y_t^{-1}\sigma_Y(Y)}(w_i - a_i) \\
&= a_i + (\rho+k)\frac{\sigma_D(Y)\,\overline{Y}/Y_t + \overline{Y}Y_t^{-1}\sigma_Y(Y)}{\sigma_D(Y)\,\rho + k\left(\sigma_D(Y)\,\overline{Y}/Y_t + \overline{Y}Y_t^{-1}\sigma_Y(Y)\right)}(w_i - a_i) \\
&= a_i + (\rho+k)\frac{\overline{Y}/Y_t\left[\sigma_D(Y) + \sigma_Y(Y)\right]}{\sigma_D(Y)\,\rho + k\overline{Y}/Y_t\left[\sigma_D(Y) + \sigma_Y(Y)\right]}(w_i - a_i) \\
&= a_i + (\rho+k)\frac{\overline{Y}/Y_t\sigma_M(Y)}{\sigma_D(Y)\,\rho + k\overline{Y}/Y_t\sigma_M(Y)}(w_i - a_i)
\end{aligned}$$

where

$$\sigma_M(Y) = \sigma_D(Y) + \sigma_Y(D)$$

Finally, substituting $\sigma_Y(Y) = v\sigma_D(Y)$ from definition (57) and deleting $\sigma_D(Y)$ throughout, the result follows.

Similarly, we have that the amount in bonds is

$$\begin{aligned}
N_{it}^0 B_t &= W_{it} - N_{it} P_t \\
&= D_t \frac{1}{\rho}\left(\frac{\rho}{\rho+k} a_i + \left(w_i - \frac{\rho}{\rho+k} a_i\right) \overline{Y}/Y_t\right) - N_{it} D_t \frac{\left(\rho + k\overline{Y}/Y_t\right)}{\rho\left(\rho+k\right)} \\
&= D_t \frac{1}{\rho\left(\rho+k\right)}\left[\left(\rho a_i + \left(w_i\left(\rho+k\right) - \rho a_i\right)\overline{Y}/Y_t\right) - N_{it}\left(\rho + k\overline{Y}/Y_t\right)\right] \\
&= D_t \frac{1}{\rho\left(\rho+k\right)}\left[a_i\left(\rho + k\overline{Y}/Y_t\right) + w_i\left(\rho+k\right)\overline{Y}/Y_t - a_i\left(\rho+k\right)\overline{Y}/Y_t - N_{it}\left(\rho + k\overline{Y}/Y_t\right)\right] \\
&= D_t \frac{1}{\rho\left(\rho+k\right)}\left[a_i\left(\rho + k\overline{Y}/Y_t\right) + \left(w_i - a_i\right)\left(\rho+k\right)\overline{Y}/Y_t - N_{it}\left(\rho + k\overline{Y}/Y_t\right)\right] \\
&= D_t \frac{1}{\rho}\left[\overline{Y}/Y_t - \frac{\overline{Y}/Y_t\sigma_M\left(Y\right)}{\sigma_D\left(Y\right)\rho + k\overline{Y}/Y_t\sigma_M\left(Y\right)}\left(\rho + k\overline{Y}/Y_t\right)\right]\left(w_i - a_i\right) \\
&= D_t \frac{1}{\rho}\left[\frac{\overline{Y}/Y_t\left[\sigma_D\left(Y\right)\rho + k\overline{Y}/Y_t\sigma_M\left(Y\right)\right] - \overline{Y}/Y_t\sigma_M\left(Y\right)\left(\rho + k\overline{Y}/Y_t\right)}{\sigma_D\left(Y\right)\rho + k\overline{Y}/Y_t\sigma_M\left(Y\right)}\right]\left(w_i - a_i\right) \\
&= -D_t\left[\frac{\overline{Y}/Y_t\left(\sigma_M\left(Y\right) - \sigma_D\left(Y\right)\right)}{\sigma_D\left(Y\right)\rho + k\overline{Y}/Y_t\sigma_M\left(Y\right)}\right]\left(w_i - a_i\right) \\
&= -D_t\left[\frac{\overline{Y}/Y_t\left(\sigma_M\left(Y\right)/\sigma_D\left(Y\right) - 1\right)}{\rho + k\overline{Y}/Y_t\sigma_M\left(Y\right)/\sigma_D\left(Y\right)}\right]\left(w_i - a_i\right)
\end{aligned}$$

Finally, substituting $\sigma_Y(Y) = v\sigma_D(Y)$ from definition (57) and deleting $\sigma_D(Y)$ throughout, the result follows. □

Proof of Corollary 1. Part (a) is immediate from the expression for N_{it}^0 in Proposition 2.

Part (b) can be shown as follows:

$$\begin{aligned}
\frac{N_{it}P_t}{W_{it}} &= \frac{\sigma_{Wi}\left(Y\right)}{\sigma_P\left(Y\right)} \\
&= \frac{\sigma_D\left(Y\right)+\frac{\left(w_i-\frac{\rho}{\rho+k}a_i\right)\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\frac{\rho}{\rho+k}a_i+\left(w_i-\frac{\rho}{\rho+k}a_i\right)\overline{Y}/Y_t\right)}}{\sigma_D\left(Y\right)+\frac{k\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho+k\overline{Y}/Y_t\right)}} \\
&= \frac{\sigma_D\left(Y\right)+\frac{(w_i(\rho+k)-\rho a_i)\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho a_i+(w_i(\rho+k)-\rho a_i)\overline{Y}/Y_t\right)}}{\sigma_D\left(Y\right)+\frac{k\overline{Y}Y_t^{-1}\sigma_Y(Y)}{\left(\rho+k\overline{Y}/Y_t\right)}} \\
&= \frac{\sigma_D\left(Y\right)+\sigma_Y\left(Y\right)\left(\frac{(w_i(\rho+k)-\rho a_i)\overline{Y}Y_t^{-1}}{\left(\rho a_i+(w_i(\rho+k)-\rho a_i)\overline{Y}/Y_t\right)}\right)}{\sigma_D\left(Y\right)+\sigma_Y\left(Y\right)\left(\frac{k\overline{Y}Y_t^{-1}}{\left(\rho+k\overline{Y}/Y_t\right)}\right)} \\
&= \frac{\sigma_D\left(Y\right)+\sigma_Y\left(Y\right)\left(1-\frac{\rho}{\rho+[k+(\rho+k)(w_i-a_i)/a_i]\overline{Y}/Y_t}\right)}{\sigma_D\left(Y\right)+\sigma_Y\left(Y\right)\left(1-\frac{\rho}{\left(\rho+k\overline{Y}/Y_t\right)}\right)}
\end{aligned}$$

Finally, substituting $\sigma_Y(Y) = v\sigma_D(Y)$ from definition (57) and deleting $\sigma_D(Y)$ throughout, the result follows. Q.E.D.

Proof of Corollary 2. Immediate from the expression of $H_0(S_t)$. Q.E.D.

Proof of Corollary 3. Immediate from the fact $L(S_t)$ is increasing and the fact that agents with $w_i - a_i > 0$ are leveraged and have C_{it}/D_t that is increasing in S_t. Q.E.D.

Proof of Corollary 4. The expression of $E[dC_{it}/C_{it}]$ stems from the application of Ito's lemma to the consumption $C_{it} = D_t[a_i + (w_i - a_i)\overline{Y}S_t]$. The remaing part is immediate from the statement in the corollary. Q.E.D.

Proof of Corollary 5. Immediate from Corollary 2 and 4. Q.E.D.

Proof of Corollary 6. Immediate from Corollary 2. Q.E.D.

Proof of Corollary 7. Immediate from Proposition 2 and 3. The state price density and the price of stocks are independent of cross-sectional quantities. Q.E.D.

Proof of Proposition 8 Substituting ϕ_i into the $W_{i,t}$

$$\frac{W_{i,t}}{D_t} = S_t\left[a_i\frac{(Y_t-\overline{Y})}{\rho+k}+\frac{a_i(\overline{Y}-\lambda)+\lambda-1+\phi_i}{\rho}\right] \tag{61}$$

$$= \frac{1}{\rho}\left[a_i\frac{\rho}{\rho+k}(1-S_t/\overline{S})+w_iS_t/\overline{S}\right] \tag{62}$$

which is the expression of wealth/output ratios in Proposition 8. The expression for consumption/wealth ratio follows from these last two results.

Proof of Proposition 9. Immediate from the definition of cross-sectional variance and the result in Proposition 8. Q.E.D.

Proof of Proposition 10. This proposition follows from an application of Ito's lemma to the wealth process in (62). Q.E.D.

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