Selection versus Talent Effects on Firm Value*

Briana Chang† Harrison Hong‡

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Abstract

Measuring the value of labor market hires on stock prices, be it the choice of underwriters when firms go public (IPOs) or chief executive officers (CEOs), is difficult due to selection. For instance, firms facing a higher cost of capital might pay more for talent to raise their stock valuations. Using an assignment model, we show that selection affects both the cross-sectional correlation of firm value and talent and the convexity of the wage distribution. The net of these two quantities is invariant to selection and can be used to understand changes over time in IPO underpricing and CEO wages.

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†University of Wisconsin, Madison

‡Columbia University and NBER

1. Introduction

Measuring the value of labor market hires for stock prices is a fundamental question in financial economics. Two labor markets are particularly important and widely studied. The first is the market for underwriters when firms issue equity. Firms compete and spend significant resources to hire reputable underwriters (typically investment banks with track records of successful placements) for their initial public offering (IPO) so as to alleviate adverse selection (see, e.g., Allen and Faulhaber (1989), Carter and Manaster (1990), Welch (1989)). Measuring the value of prestigious underwriters is a long-standing goal in the IPO literature (for reviews, see, e.g., Ritter and Welch (2002)). The second is the market for Chief Executive Officers (CEOs). There is a large literature on the value-added of a CEO (see, e.g., Bertrand and Schoar (2003)). In particular, valuing different attributes of a CEO, be it intelligence or other personality attributes, remains a widely researched topic (see, e.g., Kaplan et al. (2012), Graham et al. (2013)).[1]

Regardless of the situation, quantifying these hires' impact on stock prices is difficult due to selection or sorting in competitive labor markets (Becker (1973) and Rosen (1974)). Firms that hire better underwriters might have more uncertainty or information asymmetry and higher costs of capital to begin with. So correlating a firm's stock valuation with the status of the underwriter it hired is potentially problematic due to this selection. Indeed, recent empirical work suggests that such competitive selection effects between firms and underwriters (Fernando et al. (2005), Akkus et al. (2013)) might inform the long-running debate on IPO underpricing, the first-day return thought to compensate investors for adverse selection. Prestigious underwriters were associated (correlated) with less underpricing before the Internet period but became associated with more underpricing during and after the Internet period. This change in the sign of this underpricing-prestige correlation remains puzzling.

[1] Due to CEO wage inequality, this question continues to be important in the popular press and across various disciplines such as strategy and management (see, e.g., "Do CEOs Matter?" by *The Atlantic* in the June 2009 issue which surveyed a variety of contrasting views of whether CEOs add any value.).

Selection effects arising from assortative matching in a competitive labor market for CEO talent are even more problematic. On the one hand, Terviö (2008) and Gabaix and Landier (2008) point to a positive assortative matching between managerial talent and firm market capitalization due to complementarities in firm production functions. On the other hand, firms with greater uncertainty and a higher cost of capital might want to hire a talented CEO as well (see, e.g., Lev (2011)). An example is Marissa Mayer of Yahoo, whose goal arguably was not to increase the fundamental value of Yahoo but to increase Yahoo's stock valuation for a sale.[2] This type of selection would then lead to a negative correlation between stock valuation and managerial talent.

To understand the role of selection from the direct effect of agent talent, we develop an assignment model where firms compete to hire a talented agent, be it an underwriter or a CEO, to raise their stock valuations. Specifically, the value of the match depends on the role of the agent in the asset market. In the labor market for underwriters when firms go IPO, our asset market follows the classical IPO underpricing set-up (Rock (1986), Benveniste and Wilhelm (1990), Habib and Ljungqvist (2001)), in which adverse selection generates underpricing. More prestigious underwriters are assumed to be able to bring in more uninformed investors and hence alleviate the need for underpricing.

In the labor market for CEOs, a talented CEO increases firm valuation in two ways. First, talent raises long-term fundamental value as in Terviö (2008) and Gabaix and Landier (2008). Second, talent increases investor confidence by offering more precise public signals regarding a firm's fundamental value. The price of the stock is determined in a traditional noisy rational expectations framework (Grossman and Stiglitz (1980); Diamond and Verrecchia (1981); Hellwig et al. (2006)).

We characterize the assignment equilibrium that maps (assigns) these multiple dimensions of firms into agent talent by building on Chiappori et al. (2015) and solve for the wage

[2]The importance of CEOs for firm cost of capital is also reflected in CEOs of even large companies invariably leading disclosures to reduce investors' uncertainty. Quarterly conference calls with investors are one of the few managerial tasks that never gets delegated and is a skill that firms value.

function and stock prices. For the labor market for underwriters and IPO underpricing, we create an index of firm opacity (depending on firm investment, issuance size and volatility) that characterizes the positive assortative matching between firms with differing opacity and agents (i.e. underwriters of different prestige or CEOs of different talent).

There are two effects that shape the relationship between the cost of capital (measured with underpricing) and agent talent (measured with the prestige of the underwriter). The first is the "direct effect" of hiring a more talented agent: all else equal, a firm with a better agent will have a lower cost of capital since the agent will bring more uninformed investors to the IPO. The second is the "selection effect", whereby firms with greater opacity and hence greater adverse selection and higher cost of capital all else equal, benefit more and pay more for a better agent. Which of these two effects dominates then empirically determines the sign of the correlation between the cost of capital and agent talent.

We establish that the strength of the selection effect increases with heterogeneity in opacity across firms relative to the talent distribution of agents. If firm opacity is tightly distributed in the population, the highly opaque firms still hire the most talented agent and so the direct effect dominates and firms with a better agent have a lower cost of capital. As dispersion of firm types increases this association flips signs from negative to positive; i.e. firms with a better agent have a higher cost of capital. This selection effect can potentially explain the change in the underpricing-prestige relationship during Internet period to the extent there is greater dispersion in firm opacity as young Internet firms with little cashflows went IPO.

To derive a test of whether such selection effects are indeed driving these stylized patterns, we show that a stronger selection effect leads also to a relatively more convex wage distribution. Since the wage distribution becomes more convex and the association of the cost of capital is more positively associated with worker talent due to selection, we can use these two quantities to purge or net out the effect of selection. We prove that the difference in appropriately scaled versions of these two quantities picks up just the convexity of the

contribution of talent to firm-worker matching surplus.

Given that wage distributions are often observable, empirical researchers can use this metric to measure the direct effect of underwriter prestige on firm value.[3] This time variation in IPO underpricing patterns has spawned competing explanations of selection effects versus structural changes in underwriter technology or incentives (Loughran and Ritter (2002)). To the extent that such selection effects are driving the association (correlation) of underpricing and underwriter prestige over time, our model predicts that wage convexity should move proportionately with this association. That is, the residual between these two quantities should remain constant. Thus, any change in this residual suggests changes in the impact of prestige (either through the talent distribution or the impact of prestige). The residual is higher when the direct effect is more convex; i.e. when prestige is relatively important.

In the labor market for CEOs, the selection effect driven by the desire of firms with high fundamental value to hire talented CEOs gives rise to a positive correlation between firm size and CEO talent. But the selection effect driven by the desire to reduce the cost of capital leads to a negative correlation. These off-setting effects can potentially explain empirical findings which show only a modest correlation in the cross-section between firm size and CEO pay, to the extent pay is naturally a proxy for CEO talent (Frydman and Saks (2010)).

Selection effects due to firm productivity and CEO talent have been argued to explain the rise of CEO wages since the 1980s (Gabaix and Landier (2008)). The main test is to show that the increase in the average size of the firm in the stock market can explain the rise in CEO wages. But there is debate on whether this coincident trend is causal (Frydman and Jenter (2010), Frydman and Saks (2010)). Our model offers an alternative test. We show that the decoupling of selection and from the direct effect of agent talent by taking into account wage convexity applies also to our CEO model. That is, we can calculate wage convexity using the CEO wage distribution and also calculate the correlation of firm size

[3] Adjusting for selection (see, e.g., Heckman (1977), Roberts and Whited (2012)) is empirically challenging since it generally requires instrumental variables for the selection equation. Moreover, important firm characteristics that might drive the selection, such as investors' information asymmetry or uncertainty, are also unobservable to econometricians and at a minimum difficult to measure.

and CEO pay or talent. Our model predicts that the residual between wage convexity and association of firm size and CEO talent should remain constant over time if the rise in CEO wages is driven by such selection. Otherwise, changes in the talent distribution of CEOs or in the impact of that talent to matching surplus might also be behind the rise in CEO wages.

The literatures on underwriters and CEOs have evolved largely separately but they are united, as we have hoped to show, in fundamental ways through the role of selection in labor markets. There are other labor markets we have not covered, such as the labor market for venture capitalists or other types of financial intermediaries (see, e.g., Sørensen (2007)). The matching surplus functions are similar to those considered here. As such, our model likely applies to these settings as well.

Our paper proceeds as follows. We describe our model in Section 2. We provide the solution in Section 3. We consider a number of implications of our model in Section 4. We conclude in Section 5.

2. Model

The model lasts for three dates. There is a unit measure of heterogeneous firms that issue equity through stock markets. Firms can hire agents (e.g., underwriters or executives) through a competitive labor market at date 0. These agents, who differ in their ability, can increase the share price for the firm. Finally, at date 2, the cash flow is realized, and all players in the economy consume their realized gains.

To proceed, we first introduce a general framework for the labor market. Firms, which differ in multiple dimensions, choose the optimal agent to maximize the firm's expected payoff. They rationally anticipate how different agents affect their stock price at date 1. We then specify the relationship between agents' talents and share prices by considering two classical models.

Agents: There is a distribution of heterogeneous agents whose ability is indexed by $h \in$

$H \equiv [h^L, h^U]$. Let $G^A(h)$ denote the talent distribution. This ability in the context of underwriters is prestige bringing in more uninformed investors to an IPO. In the context of CEOs, this talent increases fundamental value and investor confidence.

Firms: Each firm owns a risky project with capital stock k. The payoff of the project for the firm with capital k, is given by $k\theta$, where θ is a firm-specific payoff with mean $\hat{\theta}(h)$. The mean $\hat{\theta}(h)$ aims to capture firms' long-term fundamental, which can potentially affected by the type of agent a firm hires. The riskiness of the project is indexed by σ. A firm originally owns $(1+\psi)$ measure of shares and wants to raise capital by issuing one measure of its equity to investors. The characteristic of a firm is then summarized by $y = (k, \psi, \sigma) \in Y \subseteq \mathbb{R}^3_+$. The types of firms are distributed according to a probability measure ν^F on Y, which is assumed to be absolutely continuous with respect to the Lebesgue measure.

Labor Market ($t = 0$): At date 0, each firm can hire at most one agent assuming it hires any at all. The fee paid to the agent is denoted by $\omega(h)$, which will be determined competitively in equilibrium. The payoff of agent h is then given by $\omega(h)$. The end-of-period cash flows for firm y are then the profit from its project minus the fee that it commits to pay: $\pi_y = k\theta - \omega(h)$.

At date 0, given the fee required to hire agent $\omega(h)$, a firm of type y chooses the optimal agent to maximize its expected payoff. Let $\tilde{p}_{hy}$ denote the realized share price at date 1 for firm y if it hires agent h. A firm's optimization problem yields:

$$U^*(y) \quad = \quad \max_{h \in H \cup \{\emptyset\}} E\left[\left(\frac{\psi}{1+\psi}\right)(k\theta - \omega(h)) + \tilde{p}_{hy}\right]. \tag{1}$$

The equilibrium in the labor market consists of an assignment $\mu(y)$: $Y \rightarrow H \cup \{\emptyset\}$ and competitive fee for agents $\omega(h) : H \rightarrow \mathbb{R}^+$ such that (1) the optimality conditions for both firms and agents are satisfied, i.e. given wage $\omega(h)$, $\mu(y)$ is the type of agent that firm y optimally chooses to hire. So that, $\mu(y)$ maximizes (1). And (2) the market-clearing condition holds for the labor market.

Financial Market ($t = 1$): We now consider two classical settings that explicitly gives the relationship between agents' talents and share prices.

Labor Market for Underwriters: In the setting of Rock (1986), Benveniste and Wilhelm (1990), and Habib and Ljungqvist (2001), the key friction is the asymmetric information among investors, where informed investors know the quality of the firm, while uninformed investors do not. As the winner's curse increases in proportion to the fraction of informed investors, so does the necessary amount of underpricing.

The main value of the underwriters is to reduce the cost of capital by attracting uninformed investors. One can thus interpret agents as the underwriters in this setting, who differ in terms of their ability to attract uninformed investors. Specifically, let $\beta(h)$ denote the fraction of uninformed investors that are attracted by underwriter h. We assume that the higher is h, the more prestigious the underwriter, the higher fraction of investors will be uninformed participating in the IPO: $\beta'(h) = b_0 > 0$.

We further set $\hat{\theta}(h) = \bar{\theta}$, which captures the fact that underwriters do not affect the fundamental, but can help reduce the cost of captial. Specifically, the payoff of the project is given by $\theta \in \{\bar{\theta} - \sigma, \bar{\theta} + \sigma\}$ with equal probability. Informed investors know the realized value of θ, but uninformed investors do not.

For any given fraction of uninformed investors, the price at which shares are sold to investors must be such that uninformed investors expect to break even on average. Thus $\tilde{p}_{hy}$ must solve

$$0 = \frac{1}{2}\beta(h)\left(\frac{k(\bar{\theta} + \sigma) - \omega(h)}{1 + \psi} - \tilde{p}_{hy}\right) + \frac{1}{2}\left(\frac{k(\bar{\theta} - \sigma) - \omega(h)}{1 + \psi} - \tilde{p}_{hy}\right).$$

To interpret this break-even condition, if $\beta(h) = 1$, and all investors are uninformed, there is no winner's curse since the uninformed investor will with 50% chance get the asset when the valuation is low and 50% chance get the asset when the valuation is high. But if $\beta(h) < 1$ and hence some fraction of the investors are informed, the informed investors

will only buy when the valuation is good. That is, from the viewpoint of an uninformed investors, when the project has a high valuation, the probability that an order is filled is $\beta(h) < 1$. Hence, the share price for firm y that hires an underwriter with ability h is then given by

$$\tilde{p}_{hy} = \frac{k\left(\beta(h)(\bar{\theta}+\sigma) + (\bar{\theta}-\sigma)\right)}{(1+\beta(h))(1+\psi)} - \frac{\omega(h)}{1+\psi}.$$

As is standard in the IPO literature, underpricing (which is also the expected return) is defined as

$$R(y,h) \equiv UP(y,h) \equiv \mathbb{E}_\theta\left[\left(\frac{k\theta - \omega(h)}{1+\psi}\right) - \tilde{p}_{hy}\right] = \left(\frac{k\sigma}{1+\psi}\right)\frac{(1-\beta(h))}{(1+\beta(h))}. \tag{2}$$

Note that $\frac{\partial UP(h,y)}{\partial h} = -\left(\frac{k\sigma}{1+\psi}\right)\frac{2\beta'(h)}{(1+\beta(h))^2} < 0$. Since a better (more prestigious) underwriter can attract more uninformed investors, a better agent helps reduce the amount of underpricing by more than a worse agent. Firms' optimization problem in Equation (1) can then be rewritten as

$$U^*(y) = \max_h \left\{k\bar{\theta} - R(y,h) - \omega(h)\right\}. \tag{3}$$

Labor Market for CEOs: CEOs may affect firms' value through varied channels. One standard channel (Terviö (2008), Gabaix and Landier (2008)) is to increase the firms' fundamental. To capture this, we assume that the firm-specific payoff θ is drawn from a Normal distribution with mean $\hat{\theta}(h) = \bar{\theta}h$ and variance σ^2. That is, a better CEO increases the average payoff of the project.

Another aspect of CEO talent is their ability to communicate with investors and thus reduce the cost of capital. Specifically, we assume the agent can produce a report about the firm, which is a noisy, unbiased signal regarding the payoff: $z = \theta + \sigma_h\eta$, where $\eta \sim N(0,1)$, and the variance of the report is given by $\sigma_h^2 = \frac{1}{h}$. Thus, higher values of h denote more precise agents. All investors observe the public signal z produced by the agent and know its precision.

The stock price is determined in a noisy REE in the spirit of Grossman and Stiglitz (1980) with risk-averse investors who are imperfectly and heterogeneously informed. For simplicity, we assume that each investor only has access to one market, and in each market, there is a unit measure of a continuum of risk-averse investors. Each investor receives a private signal $x_i = \theta + \sigma_x \epsilon_i$, and can submit his demand based on his information set.[4] There are also noise traders in each market. To solve the model in closed form, we assume that noise traders purchase a random quantity $\Phi(u)$ of stock, where $u \sim N(0, \sigma_u)$ and Φ is the standard normal CDF. The specific functional form assumed here is close to that in Hellwig et al. (2006).

Each investor can purchase at most one share for each stock or none at all based on his information set. That is, investors submit a price-contingent demand schedule for stock y, which specifies their demand $d_i(p) \in \{0, 1\}$ conditional on price p to solve:[5]

$$\max_{d \in \{0,1\}} \left\{ d\mathbb{E}[(\frac{k\theta - \omega(h)}{1+\psi} - p)|x_i, z, p] - \frac{\gamma_I}{2} d^2 Var(\frac{k\theta - \omega(h)}{1+\psi} - p|x_i, z, p) \right\}. \tag{4}$$

These bid functions determine the aggregate demand of informed investors. Together with the demand of the noisy traders, the auctioneer selects a price to clear the market.

In the Appendix, we show that investors' demand function can be characterized by a cutoff investors $\hat{x}$ such that an investor buys the asset if and only if his private signal is larger than $\hat{x}$. As a result, the market clearing price $P(\theta, z, u|h, y)$, which depends on the realized signals and noise traders, is pinned down so that the cut-off investor is indeed indifferent, given his information set. Thus,

$$P(\theta, z, u|h, y) = \mathbb{E}\left[\left(\frac{k\theta - \omega(h)}{1+\psi} \right) |x_i = \hat{x}, z, \hat{x} \right] - \frac{\gamma_I}{2} \frac{k^2}{(1+\psi)^2} Var(\theta|x, z, p). \tag{5}$$

[4] For simplicity, we allow each investor to trade only one stock. We can allow investors to trade multiple stocks as long as markets are incomplete and idiosyncratic risk is priced.

[5] We maintain the assumption about demand for tractability. Alternatively, one can allow investors to submit a bidding schedule $d_i(p) \in \mathbb{R}$, which will not change the key economic results. In this alternative setup, one would need to use a different assumption on the noise traders' demand. Specifically, the noise traders' demand is given by $\tilde{u}$ instead of $\Phi(\tilde{u})$. As standard, there exists a price which is linear in $(\theta, z, \tilde{u})$.

The second term represents the risk premium for firm y that hires analyst h. The (unconditional) expected asset return in this case is then the risk premium, which yields the following expression:

$$R(y,h) = \mathbb{E}\left[\left(\frac{k\theta - \omega(h)}{1+\psi}\right) - P(\theta, z, u|h, y)\right] \tag{6}$$

$$= \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x,z,p) = \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}\left(\frac{1}{\tau(y)+h}\right), \tag{7}$$

where $\tau(y)$ is a one-dimensional transparency index that summarizes the information characteristic of firm: $\tau(y) \equiv \frac{1}{\sigma_x^2} + \frac{1}{\sigma_x^2\sigma_u^2} + \frac{1}{\sigma^2}$. That is, a higher level of precision h decreases the risk premium charged by investors, since it improves an investor's estimation of the fundamental pay-off. Firms with higher volatility have a lower transparency index. All things being equal, the risk premium is then higher for less transparent firms, since investors have to bear more risk for those firms. Similarly, the risk premium is higher for firms with a larger scale.

Firms' optimization problem in Equation (1) can then be rewritten as

$$U(y,h) = k\hat{\theta}(h) - R(y,h) - \omega(h). \tag{8}$$

The only difference from Equation (3) is that the CEO can also affect the fundamental, captured by the first term $\hat{\theta}(h)$. Similar to Equation (3), the second term is the cost of capital. While these two labor markets have different functional forms of $R(y,h)$, they are otherwise similar in the following senses. First, an agent with higher ability can reduce the cost of capital for a firm more effectively. That is, $R_h(y,h) < 0$ in both cases. Second, all things being equal, the cost of capital is higher for firms with riskier projects (σ), a larger size (k), and a higher proportion of issued shares ($\frac{1}{1+\psi}$).

3. Labor Market Hiring

Taking into account how the agent affects the price in the asset market, we now characterize the assignment function and wage function. As is well-known in a matching model, the allocation depends on the property of the matching surplus of the firm and agent. Specifically, as discussed above, the firm's expected utility when hiring agent h can be conveniently rewritten as the expected payoff k of the project minus the underpricing/risk-premium $R(y, h)$ and the agent's fee.

Note that, although the firm only pays some proportion of the fees at the end of the period (i.e., $\frac{\psi}{1+\psi}\omega(h)$), the reduction in the asset price due to the hiring is $\frac{\omega(h)}{1+\psi}$. Hence, from a firm's view point, the total cost is simply the agent's fee. As a result, the surplus between firm y and agent h, which is the sum of their payoff minus their outside option, yields

$$\begin{aligned}\Omega(y,h) &\equiv U(y,h) - U(y,\emptyset) + \omega(h) \\ &= k\left(\hat{\theta}(h) - \hat{\theta}(\emptyset)\right) + R(y,\emptyset) - R(y,h),\end{aligned} \tag{9}$$

where $\emptyset$ denotes the case in which a firm hires no agent (i.e., the firm's autarky value) and the workers' unemployed value is normalized to zero. The first two terms thus represents the gain of firm y when it hires agent h relative to no hiring. The third term represents the payoff of a worker, which is the fee. Thus, the surplus is simply the change in fundamental plus the reduction in the cost of capital (relative to no hiring).

Technically, given the multiple characteristics of a firm, our environment is a multidimensional-to-one matching problem. As established in Chiappori et al. (2016), given our surplus function in (9) and that the measure of firms ν^F is absolutely continuous with respect to the Lebesgue measure, stable matching exists and the assignment function $\mu(y)$ is unique and pure. That is, each firm hires a unique agent instead of using mixed strategies.

3.1. Sorting

Proposition 1 first establishes the property of the assignment function in terms of firms' characteristics.

Proposition 1. *All else equal, a firm with a riskier project* (σ)*, a larger size* (k)*, and a higher proportion of issued shares* ($\frac{1}{1+\psi}$) *hires a more talented agent. That is,* $\mu_\sigma(k,\psi,\sigma) > 0$*,* $\mu_k(k,\psi,\sigma) > 0$ *and* $\mu_\psi(k,\psi,\sigma) < 0$*.*

These results can be seen from the firms' optimization problem. Specifically, given that all firms face the same cost function $\omega(h)$, a firm that has a higher marginal benefit of increasing precision must hire a better agent in equilibrium. In the IPO setting, according to Equation (2),

$$\frac{\partial U(y,h)}{\partial h} = \left(\frac{k\sigma}{1+\psi}\right)\frac{2\beta'(h)}{(1+\beta(h))^2} - \omega_h(h). \tag{10}$$

In other words, there is a *complementarity* between the prestige of an agent and firms' scale and riskiness. Thus, firms with a higher scale or more volatility benefit more from hiring a prestigious underwriter.

A similar intuition holds for the CEO setting. This again can be seen formally by looking at the marginal value of precision, derived from Equation (8)

$$\frac{\partial U(y,h)}{\partial h} = k\bar{\theta} + \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}\frac{1}{(\tau(y)+h)^{\,2}} - \omega_h(h). \tag{11}$$

In either case, $\Omega(y,h)$ increases with h and thus, firms essentially compete for more talented agents. Thus, a more talented agent must earn a higher fee: $\omega_h(h) > 0$.

3.2. Full Characterization

To proceed, we first establish a full characterization for a general surplus function and then apply the developed solution to both IPO underpricing and REE settings. As discussed in Chiappori et al. (2016), when type spaces are multidimensional, it is generally not possible to

derive a closed-form solution for the assignment function. Nevertheless, one can see that the characteristics of firms can be further reduced to aggregated indices in our setting, thereby simplifying our characterization.

Facing equilibrium fee $\omega(h)$, $\omega_h(h)$ represents the marginal cost of a particular precision from the view point of firms. From the first-order condition, if firm y chooses to match with agent $\mu(y)$ in equilibrium, then his marginal benefit of precision must equal the marginal cost. That is, $\Omega_h(y, \mu(y)) = \omega_h(\mu(y))$.

In other words, once we have figured out the value of $\omega_h(h)$, one can then find the set of firms matched to agent h. Note that when firms differ in multiple dimensions, two different types of firms may have the same marginal value of h. To facilitate the analysis, define the set of firms y whose marginal benefit of h is given by a value of m :

$$\Upsilon(h, m) \equiv \{y \in Y \mid \Omega_h(y, h) = m\}.$$

That is, if the marginal cost of hiring agent h is given by $\omega_h(h)$, then $\Upsilon(h, \omega_h(h))$ is the set of firms matched to agent h.

Clearly, $\omega_h(h)$ is an equilibrium object that depends on the underlying distribution. We now consider the following algorithm that allows us to construct an explicit solution for this multi-dimensional environment. The basic idea of the equilibrium construction is the following.

First, for each h, we will need to choose some level $m \in R$ that satisfies the following condition:

$$\underline{Y}(h, m) \equiv \nu^F \left(\{y \in Y \mid \Omega_h(y, h) \leq m\}\right) = G^A(h). \tag{12}$$

By choosing m properly for each agent h, the measure of firms whose marginal benefit of h is lower than m exactly coincides with the measure of agents below h. Intuitively, if m were the price for ability h, all firms within (outside of) the set $y \in \underline{Y}(h, m)$ find this type of agent to be too expensive (cheap).

Choosing m for each h is thus as if we are choosing the price for any given ability. Equation (12) requires that, in equilibrium, the price for any particular ability must be chosen in a way so that the measure of firms that find this type of agent to be too expensive exactly coincides the measure of agents below this agent.

As established in Chiappori et al. (2015), this algorithm works only in the environment where the constructed $\omega_h(h)$ in the above procedure satisfies the following *nested* condition:

$$\underline{Y}(h, \omega_h(h)) \subset \underline{Y}(h', \omega_h(h')) \, \forall h' > h. \tag{13}$$

The construction of the fee schedule is such that if a firm finds that hiring agent h is too expensive, then it must find a more better agent $h' > h$ to be too expensive as well.

Observe that Condition (13) together with Condition (12) guarantee that (1) the set of firms that found h to be too expensive are always matched to firms below agent h and (2) the market clears in the sense that the measure that firms that hire agents below h coincides with the measure of agents below h. As a result, the optimality condition of firms and market-clearing condition are satisfied. Proposition 2 summarizes the characterization.

Proposition 2. *Let $\omega_h(h)$ be the value that solves $\underline{Y}(h, \omega_h(h)) = G^A(h)$. Under nested matching (i.e., if condition (13) holds), the optimal assignment is characterized by $\mu^{-1}(h) = \Upsilon(h, \omega_h(h))$.*

3.2.1. Underwriters

We now apply to the developed solution to the setting of IPO underpricing. Observe from Equation (10), firms' marginal value for hiring can be simply summarized by the one-dimensional index

$$q(y) = \left(\frac{k\sigma}{1+\psi} \right). \tag{14}$$

Thus, two different firms will choose the same agent in equilibrium if they have same index $q(y)$. In this instance, firm's marginal value of precision can be ranked by index $q(y)$ and

such ranking is the same for any precision h. This feature, however, may not hold for general cases, as shown in our next example for the CEO setting.

With this particular feature, the model can be solved similarly as in the standard model with one-dimensional heterogeneity. For any agent h, his marginal gain in equilibrium (represented by $\omega_h(h)$) in Proposition 2 is such that the measure of firms that that found h to be too expensive coincides with the measure of firms with lower q index than his optimal hiring firm.

Formally, let $G^F(\tilde{q})$ denote the measure of firms with an index lower than $\tilde{q}$: $G^F(\tilde{q}) \equiv \nu^F(\{y \in Y : q(y) \leq \tilde{q}\})$. Given that a firm y hires an agent $\mu(y)$, the set of firms that finds this agent too expensive is then given by $G^F(q(y))$. Thus, Equation (12) can then be rewritten as $G^F(q(y)) = G^A(\mu(y))$. That is, the assignment function $\mu(y)$ must satisfy the familiar market clearing condition, as standard in the model with one-dimensional heterogeneity.

Observe that Condition (13) is always satisfied in this case. To see this, if a firm y finds an agent h too expensive (i.e., $y \in \underline{Y}(h, \omega_h(h))$, it must be that the index for this firm $q(y)$ is smaller than the firm that actually hires this agent in equilibrium: $q(y) \leq q(\mu_h^{-1}(h))$. Given that the more prestigious agent $h' > h$ is matched to a firm with higher q, this thus implies that such a firm will also find it too expensive to hire $h' > h$. That is, his index $q(y)$ must also be lower than $q(\mu_h^{-1}(h'))$. Thus, if $y \in \underline{Y}(h, \omega_h(h))$, then $y \in \underline{Y}(h', \omega_h(h'))$, which establishes that Condition (13) holds.[6]

The wage profile is then characterized by

$$\omega_h(h) = \Omega_h(\mu^{-1}(h), h) = q(\mu^{-1}(h)) \frac{2\beta'(h)}{(1+\beta(h))^2}. \tag{15}$$

That is, the marginal increase in the fee of agent h is his contribution to the surplus $\Omega_h(\mu^{-1}(h), h)$ within the match, given his optimal assignment firm with index $q(\mu^{-1}(h))$.

[6]Formally, for any $h' > h$, $\underline{Y}(h, \omega_h(h)) = \{y \in Y \mid q(y) \leq q(\mu^{-1}(h))\} \subset \{y \in Y \mid q(y) \leq q(\mu^{-1}(h'))\} = \underline{Y}(h', \omega_h(h'))$.

3.2.2. CEOs

To illustrate the solution for the CEO labor market, we consider the case where firms y only differ in their size k and their transparency index $\tau(y)$. For each value of h, the set of firms whose marginal value of h is m is given by:

$$\Upsilon(h, m) = \left\{(k, \tau) \in Y \mid k\bar{\theta} + \frac{\gamma_I}{2} \frac{k^2}{(1+\psi)^2} \frac{1}{(\tau + h)^2} = m\right\}.$$

This is illustrated in Figure 1.

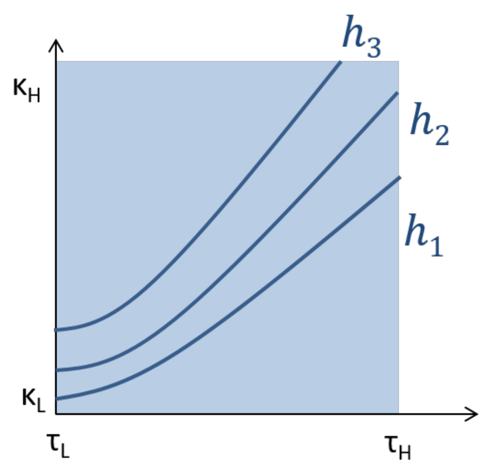

Figure 1: Figure 1 illustrates the set of firms that hire an agent with ability h, where $h_3 > h_2 > h_1$. Each line is given by $k\bar{\theta} + \frac{\gamma_I}{2} \frac{k^2}{(1+\psi)^2} \frac{1}{(\tau+h)^2} = \omega_h(h)$ and $\omega_h(h)$ is chosen so that the measure of firms below the line coincides with $G^A(h)$.

Firms below this curve thus constitute the set of firms whose marginal benefit of precision is lower than m. By setting $m = \omega_h(h)$, it is only optimal for firms in set $\Upsilon(h, \omega_h(h))$ to match with agent h. The set of firms matched to h is illustrated in Figure 1, where each line is given by $k\bar{\theta} + \frac{\gamma_I}{2} \frac{k^2}{(1+\psi)^2} \frac{1}{(\tau+h)^2} = \omega_h(h)$ and $\omega_h(h)$ is chosen so that Equation (12) is satisfied.

In contrast to the one-dimensional index case, Condition (13) may not always hold. Specifically, it holds if and only if the constructed indifference set in Figure 1 never intersects. As discussed in Chiappori et al. (2015), whether Condition (13) actually holds generally depends on the underlying measure of agents and firms. Chiappori et al. (2015) further provide criteria for this condition to hold and establish that it is always possible to find such

an underlying distribution that satisfies this condition.

4. Implications

4.1. IPO Underpricing and Underwriter Prestige

We now derive implications of our model for the level of IPO underpricing and the underpricing-prestige relationship. The stylized facts are well summarized in various review papers (see, e.g. Ritter and Welch (2002), Loughran and Ritter (2002)). First, underpricing before the Internet era of the late nineties averaged a few percent and firms that hired prestigious underwriters had lower underpricing. Second, the underpricing became much larger after the late nineties, averaging nearly 20% and coverage by a prestigious underwriter is associated more underpricing. Explanations have typically centered on structural changes in firm objective functions, such as firms underpricing to benefit friends or family in the late nineties, and changes in underwriter strategies, such as underpricing to pay buy-side clients.

We use our model to examine the extent to which selection effects induced by competitive sorting in the labor market for underwriters can account for these stylized facts. The main driver will be the distribution of firm opacity, summarized by our q index. Our model provides a direct link between underpricing and the underlying distributions of firms (as characterized by q the one-dimensional index of opaqueness) and underwriter ability or prestige (h). We first order the agents or firms by their type. Let $q[i]$ denote the ith quantile firm sorted based on the one-dimensional index of firm opaquenss given by Equation (14). Let $h[i]$ denote the value of an ith quantile agent sorted based on underwriter ability or prestige. Due to sorting and labor market clearing, an ith percentile underwriter based on prestige then must match with an ith percentile firm based on opaqueness.

Hence, combining Equation (2), Equation (14) and Proposition 1, underpricing for an

ith quantile underwriter is given by

$$UP[i] \equiv R(q[i], h[i]) = q[i] \left(\frac{1 - \beta(h[i])}{1 + \beta(h[i])} \right) \tag{16}$$

This formula highlights that underpricing is a combination of the characteristics of firms and quality of the underwriter, derived from the equilibrium assignment function.

The relationship between underpricing and underwriters' prestige can then be captured by the derivative of underpricing with respect to the quantile i. Specifically, we look at the percentage change of underpricing for the ith quantile underwriter. The expression yields:

$$\frac{UP'[i]}{UP[i]} = \frac{q'[i]}{q[i]} - \frac{2\beta'(h[i])h'[i]}{((1 - \beta(h[i])^2)} \tag{17}$$

This expression shows two offsetting effects. The first term is the selection effect and is positive: more prestigious underwriters match with firms that are more opaque (i.e., $q'[i] \geq 0$). The second term is the direct effect of prestige (which is also often referred to as the certification effect) and is negative: all else being equal, more prestigious underwriters lead to lower underpricing, since they can attract more uninformed investors (which is captured by the fact that $\beta'(h) > 0$). This relationship can be captured in data by the correlation of firm underpricing and the prestige of the underwriter. When the selection effect is large, the correlation is positive in our model. When the selection effect is small, the correlation is negative.

Comparison to the No-Sorting or Random Matching Benchmark To highlight the selection effect, we further consider a counterfactual environment where agents and firms are matched randomly. That is, the type of agent hired by each firm is randomly drawn from distribution $G^A(h)$. Underpricing is always negatively correlated with prestige in this benchmark. To see why, suppose that underwriters need to pay a fixed cost C to

attract uninformed investors. In this case, firm y will hire underwriter h as long as surplus $\Omega(y,h)$ is positive (i.e., $\Omega(y,h) = R(y,\emptyset) - R(y,h) - C > 0$). Hence, a underwriter with lower quality will be hired only if firms are relatively more opaque since an underwriter with higher quality will match with all types of firms. This suggests then that the average value of q for firms that hire a higher quality underwriter is in fact lower (i.e., $q'[i] < 0$). As a result, underpricing is always negatively correlated with prestige (i.e., $UP'[i] < 0$ under random matching).

Proposition 3. *(Prestige vs. Selection Effect) Under a no-sorting or random matching benchmark, underpricing is always negatively correlated with underwriter prestige. Under a competitive sorting benchmark, underpricing can either be positively or negatively correlated with underwriter prestige. It is positively correlated with prestige when the selection effect dominates.*

4.1.1. Comparative Statics

Whether the selection effect dominates then crucially depends on the underlying distribution of firms and talents. We now proceed to establish the comparative statics results on how the underlying distributions affect underpricing.

Given that the Internet period produced a sizeable Internet (new technology) sector that had little cashflows before going IPO, a plausible hypothesis is that the q distribution most likely has a higher mean, i.e. the average firm becomes more opaque, and also more dispersed or heterogeneous. While opacity is itself difficult to measure, we can evaluate the importance of each of these changes to the firm distributions by examining the level of underpricing, the correlation between underpricing and underwriter prestige.

An Uniform Increase in Firm Opaqueness We first consider a comparative static for the scaling effect, where all firms become larger or less transparent: $\tilde{q}[i] = \lambda q[i]$. Intuitively, higher q represents higher demand for prestige. However, since the increase is uniform across

firms, it will not change the matching pattern. That is, all firms match with exactly the same underwriter, but the matching surplus within each pair is simply scaled up by the same constant λ.

As shown in Proposition 4, underpricing is scaled up by the same constant in this case. This also implies that such a uniform change will not affect the strength of the selection effect. That is, $\frac{UP'[i]}{UP[i]}$ remains the same.

Proposition 4. *When all firms are scaled by some constant $\lambda > 1$, that is, $\tilde{q}[i] = \lambda q[i]$. Then, underpricing also changes by the same multiple: $\tilde{\omega}[i] = \lambda\omega[i]$ and $\tilde{UP}[i] = \lambda UP[i]$. But, the strength of the selection effect remains the same.*

From Proposition 4, a uniform increase in opaqueness leads to a higher level of underpricing and higher overall wages in the underwriting industry. However, it cannot explain why the sign of the underpricing-prestige correlation changed signs from negative pre-Internet to positive during and after the Internet period.

An Increase in Firms' Heterogeneity To explain this change in signs, one needs an increase in firm heterogeneity in opacity. In particular, the strength of the selection effect depends on $q'[i]$. Intuitively, a steep $q'[i]$ means that an i-th quantile firm has a higher q index relative to the competitor right below it. Similarly, the strength of the direct effect depends on $h'[i]$. As a result, which effect dominates crucially depends on the ratio of these two, which can be mapped onto the ratio of the density function:

$$\frac{q'[i]}{h'[i]} = \frac{dG^A(h[i])}{dG^F(q[i])}. \tag{18}$$

A higher (lower) ratio thus represents that firms are relatively dispersed (homogeneous) relative to the talent distribution, resulting in a stronger selection effect. For the sake of illustration, consider a simple case where both firms and agents follow a uniform distribution. This term is then a constant ratio of the density function (i.e., $\frac{dG^A(h)}{dG^F(\phi(h))} = \frac{(q^U - q^L)}{(h^U - h^L)}$). Hence, this shows that the selection effect is stronger whenever firms are dispersed relative to the

talent. The same intuition holds for more general distributions: for any given distribution of talent, the selection effect is stronger when firms are more heterogeneous in the sense that there is a smaller mass for a given q. Propositions below formalize this effect.

To see this dispersion effect in q clearly, consider a change in distribution such that $\tilde{q}'[i] \geq q'[i]$ for i and $\tilde{q}[1] = q[1]$. That is, by fixing the maximum value of the distribution, we increase the heterogeneity of firms.

Proposition 5. *Consider two distributions, where the heterogeneity of firms is higher under $\tilde{q}[i]$ in the sense that $\tilde{q}'[i] \geq q'[i]$ for i and $\tilde{q}[1] = q[1]$. The selection effect is stronger (i.e. a higher $\frac{UP'[i]}{UP[i]}$) under $\tilde{q}[i]$.*

In other words, our model points out that both a greater mean and heterogeneity in opaqueness of IPOs during the Internet period are needed to rationalize both sets of stylized facts. Indeed, as shown in Ritter and Welch (2002), the composition of firms that go IPO changed during the Internet period. In particular, during this period, firms are much younger and many of them have negative earnings, which thus leads to a higher level of heterogeneity in opacity.

An Increase in the Impact of Talent To better understand this dispersion effect, we now study the direct effect of talent on the underpricing-prestige relationship. Assume that $\beta(h) = b_0 + b_1 h$, where $b_1 > 0$. The parameter b_1 captures the impact of prestige. From Equation (17), one can see that it leads to a stronger direct effect. Proposition 4 summarizes the prediction when the impact of prestige becomes larger.

Proposition 6. *A increase in the impact of underwriters' prestige (b_1) leads to lower $\frac{UP'[i]}{UP[i]}$.*

Proposition 5 and 6 together highlight that an increase in the dispersion of firms (talents) leads to a stronger selection (direct) effect, resulting in different effects on the level of underpricing and the sign of the underpricing-prestige relationship.

4.2. Firm Size and CEO Talent

The theoretical literature on CEO wages has mostly focused on the sorting between skill and firm size. It predicts a positive correlation between firm valuation or size and CEO talent (as in Gabaix and Landier (2008)). Such a force is captured by $\hat{\theta}(h) = h\bar{\theta}$ in our model, where a better agent increases a firm's long-run fundamental. Our model highlights a neglected aspect of CEO talent, which is their ability to reduce the cost of capital. In this case, fixing firm size, a more talented CEO may work for a more volatile firm with a lower valuation. Hence, while sorting plays an important role in both channels, the selection effect under the cost of capital channel generates a negative correlation between firm valuation and CEO talent. This negative correlation can have an offset the positive correlation predicted under the traditional theory.

To see this clearly, the risk premium for firm y who hires an agent with ability $\mu(y)$ is given by:

$$R(y, \mu(y)) = \frac{\gamma_I}{2} \frac{k^2}{(1+\psi)^2} \left(\frac{1}{\tau + \mu(y)} \right).$$

Conditional on scale, a more opaque firm will hire a more talented agent, who gets a higher wage. This thus immediately shows that the correlation between wage and firm value can go either way. That is, the selection effect leads to a higher risk premium or lower firm valuation to be associated with agent talent. Without taking into account the selection effect on this dimension, one may mistakenly conclude that these hiring have little impact (or even a negative impact) on the firm value.

Formally, given the solution $\mu(k, \tau)$, we can establish the prediction regarding the impact of an executive. To show this, we now condition on the market for firms with scale k and their executives. Specifically, among firms with scale k, by abuse of notation, let $q(y) = -\tau(y)$ represent the opacity of a firm. Similar to before, among these firms, a firm with higher q has a higher demand for information and thus hires a better agent. Conditional on k, let $q[i|k]$ denote the opacity for ith quantile firm within the subsample with scale k.

We then rank all agents who work for firms with scale k by their wages (denoted by $\omega[i|k]$), which maps to their ability (denoted by $h[i|k]$). The performance of the ith quantile agent is measured as the risk premium of his firm $RP[i|k] = \left(\frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}\right)\frac{1}{-q[i|k]+h[i|k]}$. Thus, the correlation between the agent's ability and his performance yields

$$RP'[i|k] = \left(\frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}\right)\frac{q'[i|k]-h'[i|k]}{\left(-q[i|k]+h[i|k]\right)^2}.$$

As before, the strength of the selection effect depends on the density ratio, captured by the term $\frac{q'[i|k]}{h'[i|k]}$. The key difference here is that such a ratio depends on the aggregate distribution for all firms, since other firms with different combinations of (k,τ) are also competing for the agents. Thus, while we are looking at the subsample of firms with scale k, the density ratio for each value h, depends on the measure of talents $G^A(h)$ and the measure of all firms that have the same marginal value $v^F\{\Upsilon(h,\omega_h(h))\}$, which is given by

$$\frac{q'[i|k]}{h'[i|k]} = \frac{dG^A(h[i|k])}{v^F\{\Upsilon(h[i|k],\omega_h(h[i|k]))\}}. \tag{19}$$

This thus shows that the intuition for Proposition 5 remains the same. That is, the strength of selection effect is stronger when firms are more heterogeneous relative to talents. Despite the density ratio in Equation (19) now depending on the whole distribution and we are unable to express the result in a closed form, any underlying change in the aggregate distribution that leads to a smaller measure of $v^F\{\Upsilon(h,\omega_h(h))\}$ (i.e., when there are less firms that are similar) leads to a stronger selection effect. When such an effect dominates, an executive with higher pay would in fact work for a firm with a higher risk premium, suggesting a negative correlation between the ability of executive and firm value, as established in Proposition 7.

Proposition 7. *An increase in the heterogeneity of firms relative to talent in the sense that* $\frac{\tilde{q}'[i|k]}{\tilde{h}'[i|k]} \geq \frac{q'[i|k]}{h'[i|k]}$ *for* i *leads to a stronger selection effect: if* $RP'[i|k] > 0$, *then* $\tilde{RP}'[i|k] > 0$.

This proposition can potentially rationalize empirical findings which show only a modest

correlation in the cross-section between firm size and CEO pay, to the extent pay is naturally a proxy for CEO talent (Frydman and Saks (2010)).

4.3. Decomposition of Selection and Talent Effects

One advantage of our model is we can explicitly derive the relationship between underlying distributions, underpricing, and wages. We now proceed to establish the connection between the wage schedule and the selection effect.

Consider a multiplicatively separable output function $V(a,b) = ab$ in an environment under which a firm's type can be summarized by one aggregate index $a[i]$ and an agent's type is given by $b[i]$ (where $a'[i] > 0$ and $b'[i] > 0$).[7] Given positive sorting, the output created by the ith percentile agent is thus $V[i] = a[i]b[i]$. In our underpricing environment, this value is measured by underpricing $V[i] = -UP[i]$, where $a[i] = q[i]$ represents the opacity of firm i, and $b[i] = -\left(\frac{1-\beta(h[i])}{1+\beta(h[i])}\right)$ represents the value generated by underwriter $h[i]$. In the standard CEO literature (e.g., Gabaix and Landier (2008)), $a[i]$ represents the firm size and $b[i]$ represents the ability of the ith quantile CEO. Specifically, this can be nested in our framework by shutting down the risk premium channel (i.e., setting $\gamma = 0$). Thus, $a[i] = k[i]$ represents the firm size and $b[i] = \hat{\theta}(h[i])$ represents the payoff of the project generated by ith quantile agent.

Let $\omega[i]$ denote the wage for the ith percentile agent. Since the marginal wage increase for the ith percentile agent is given by his contribution to the surplus within the match, given his optimal assignment with the ith firm, the wage profile as a function of the ith percentile agent can then be rewritten as $\omega'[i] = V_b(a[i], b[i])b'[i] = a[i]b'[i]$. Given that the output function and the wages depend on the characteristics of firms and agent, it is thus difficult to tell whether the change in output/income is driven by either talent or selection

[7] As explained in Terviö (2008), a central feature of the assignment is that the characteristics a and b are essentially ordinal. It is thus without loss of generality to consider a simple multiplicative function $V(a,b) = ab$. Any separable function, for example, $Aa^{\gamma}b^{1-\gamma}$, can be nested in this expression.

effects. Formally, the difference in the output yields

$$\frac{V'[i]}{V[i]} = \frac{a'[i]}{a[i]} + \frac{b'[i]}{b[i]}. \tag{20}$$

Thus, the first (second) term is the selection (talent) effect, which captures the characteristics of firms (talent).

With Lemma 1, we establish a new metric to address this well-known issue.

Lemma 1. *For any separable output function, the relative convexity of the wage schedule and the association of firm value and agent talent is given by the following:*

$$\frac{\omega''[i]}{\omega'[i]} = \frac{V'[i]}{V[i]} + \left(\frac{b''[i]}{b'[i]} - \frac{b'[i]}{b[i]}\right).$$

Lemma 1 has two important messages. First, fixing any talent distribution, the relative convexity of wage and the percentage change in firms' valuation will move one to one in response to a change in firms' distribution. Formally, we say that wage distribution $\tilde{\omega}[i]$ is convex relative to another wage distribution $\omega[i]$ if and only if

$$\frac{\tilde{\omega}''[i]}{\tilde{\omega}'[i]} \geq \frac{\omega''[i]}{\omega'[i]} \; \forall i. \tag{21}$$

Another way to measure this convexity is to look at the ratio of slope of the wage distribution at different points i_a and i_b on the distribution:

$$\Gamma(i_a, i_b) \equiv \frac{\omega'[i_a]}{\omega'[i_b]}, \text{ where } i_a > i_b. \tag{22}$$

The latter ratio of slopes measure is likely to be more empirically implementable. A higher heterogeneity in firms leads to a higher ratio.[8]

In the market for CEOs, the implications are direct. In the market for underwriters, given that $V[i] = -UP[i]$, this value then represents the difference in the underpricing

[8]See detailed derivation in Appendix A.2.5 and A.2.6.

$\frac{V'[i]}{V[i]} = \frac{UP'[i]}{UP]i]}$ across agents of different ability. Recall that, according to Proposition 5, an increase in firm heterogeneity increases the selection effect and thus a higher $\frac{UP'[i]}{UP]i]}$. Hence, combined with Lemma 1, our model predicts that the wage schedule must become more convex for underwriters.

Second, notice that $\left(\frac{b''[i]}{b'[i]} - \frac{b'[i]}{b[i]}\right)$ only depends on the talent distribution and increases with the relatively convexity of talent. Given that both the wage schedule $\omega[i]$ and firm valuation $V[i]$ are observable, the difference between the relative convexity of wages and the correlation of firm value and talent (i.e. $V'[i]/V[i]$) is a metric to directly measure the relative convexity of talent without worrying about selection.

More generally, the relationship between wage convexity and the correlation of firm value and agent talent depends on the surplus function. Nevertheless, one may use a similar method to isolate the talent effect. Consider the other extreme case of our model where CEO only affects the cost of capital (but not the fundamental). Thus the surplus function is simply the risk premium. Analogous to Lemma 1, Lemma 2 establishes the relationship between the relative convexity and the selection effect, within any subsample of firms with scale k. As a result, our main insight remains the same: the relative convexity of talent can be directly measured by looking at the wage convexity and change in the firm value.

Lemma 2. *The relationship between relative convexity of wage schedule and change in firm valuation with agent talent is given by*

$$\frac{\omega''[i|k]}{\omega'[i|k]} = 2\frac{RP'[i|k]}{RP[i|k]} + \frac{h''[i|k]}{h'[i|k]}.$$

Lemmas 1 and 2 form the core of our new empirical predictions. Selection leads to greater convexity of the wage distribution and also to a stronger association or correlation between firm value and agent talent. Both of these quantities are observable. Notice that the difference or residual of these two quantities is independent of selection. Hence, based on two observable outcomes, one can use this new metric to directly measure the relative

convexity of talents, without worrying about the selection effect.

Time Variation in IPO Underpricing-Prestige Relationship Our model makes a clear prediction: to the extent that selection effects are driving the association of underpricing and underwriter prestige over time, we expect wage convexity to move one for one with this association over time. That is the difference between these two quantities should remain constant over time. If it is not, there are structural changes in the talent distribution, which are perhaps consistent with alternative explanations.

For each year t, we can calculate the correlation of underpricing of IPOs and underwriter prestige. An important component of underwriting services is security analysts who issue research on companies going IPO. We can look to the wage distribution of such analysts to gauge the extent to which selection effects are important. We can gather financial analysts wages data for each year t using the American Community Survey produced by the Census Bureau.

While our test needs to be implemented formally, there is anecdotal evidence at least to suggest that wages of analysts did become more convex during the Internet period. Recall that the Internet period is also the time when selection effects become stronger in the sense that the underpricing-prestige relationship changes from negative to positive, consistent with our prediction. The compensation of All-American analysts (Stickel (1992)), the most talented ones who work for prestigious investment banks, became highly skewed during the Internet Bubble Period of 1997-2000. Top analysts such as Henry Blodgett and Mary Meeker during this period earned payoffs similar to the most well-paid investment bankers. These high wages led to worries among regulators of conflicts of interest, and subsequent reforms such as Regulation-FD that were supposed to minimize such conflicts. The skewness of wages diminished after Regulation-FD in the early 2000s. However, these reforms also coincided with the end of the Internet IPO wave in 2002. As such, the reduced skewness in wages could have simply reflected less dispersion in q as a result of the end of IPO activities.

The magnitudes of underpricing have returned to Internet period levels again with the advent of the second Internet Bubble in social media stocks.[9] Importantly, media recently reported the return of skewed wages for star analysts. A number of commentators expressed surprise given the regulations on conflicts of interest that were established after the Internet Period.[10] Our model shows that such a selection effect can occur even independent of regulatory concerns about conflicts of interest.

Time Variation in CEO Wages There is debate on whether selection effects due to firm productivity and CEO talent can explain the rise of CEO wages since the 1980s in the CEO literature. The main test in the literature thus far is to show that the increase in the average size of the firm in the stock market can explain the rise in CEO wages (Gabaix and Landier (2008)). But Frydman and Saks (2010)) find that in the pre-1980s period, wages of CEOs did not rise eventhough the average size of the firm in the stock market rose. They point out concerns regarding coincident trends. Alternative explanations such as managerial entrenchment might also be part of the story (see Frydman and Jenter (2010) for a review).

Our model offers an alternative test. We show that the decoupling of selection from the direct effect of agent talent by taking into account wage convexity applies also to our CEO model. That is, we can calculate the difference between wage convexity using the CEO wage distribution and the correlation of firm size and CEO pay or talent. Specifically, if one believes that the rise in CEO compensation in time-series is driven by a change in the distribution of firms, one would expect this residual wage convexity metric to stay the same. On the other hand, any change in this value must be driven by the change in the talent distribution.

[9] For instance, the mean underpricing of IPOs which hit a low of 9.1% in 2002 have returned to average 15% in the 2011-2016 period (according to Jay Ritter's calculations on his website).

[10] Susan Craig, "Star Analysts Are Back (No Autographs, Please), August 20, 2011, NYTIMES DealBook

5. Conclusion

Firms hire in labor markets, be it underwriters when going public or executives on an ongoing basis, to improve firm value. But assessing the value of these hires for stock prices is challenging because of selection since more opaque firms with higher costs of capital and lower valuations to begin with might hire better agents. By developing an assignment model, where matching surpluses, firm valuation, and wages emerge from a stock-market equilibrium, we derive a decomposition of selection from the direct effect of agent talent. We use this decomposition to understand changes over time in IPO underpricing and the surge in CEO wages since the 1980s.

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A. Appendix

A.1. Detailed Derivation for REE Setting

We now provide detailed derivation for the market clearing price. When firm y hires agent h, investors thus obtain a public signal with precision $h = \frac{1}{\sigma_h^2}$. Aggregating the demand decisions of all investors in market (h, y), market clearing then implies

$$\int D(x_i, z, p|h, y) dF(x_i|\theta) + \Phi(\tilde{u}) = 1. \tag{23}$$

From the investor's optimization problem,

$$\begin{aligned} D(x_i, z, p|h, y) &\in \arg\max_{d \in \{0,1\}} \left\{ d\mathbb{E}[\left(\frac{k\theta - \omega(h)}{1+\psi} \right) - p|x_i, z, p] - \frac{\gamma_I}{2} d^2 Var(\frac{(k\theta - \omega(h))}{1+\psi} - p|x_i, z, p) \right\} \\ &= \arg\max_{d \in \{0,1\}} \left\{ d\mathbb{E}[\left(\frac{k\theta - \omega(h)}{1+\psi} \right) - p|x_i, z, p] - \frac{\gamma_I}{2} \frac{k^2 d^2}{(1+\psi)^2} Var(\theta|x_i, z, p) \right\}, \end{aligned}$$

Since $Var(\theta|x_i, z, p)$ is constant over the realization of (x_i, z, p), the demand $D(x_i, z, p|h, y) \in \{0, 1\}$ can be characterized by a cutoff $\hat{x}(z, p)$, such that $D(x_i, z, p|h, y) = 1$ if and only if $x_i > \hat{x}(z, p)$.

Recall that each investor receives a private signal $x_i = \theta + \sigma_x \epsilon_i$, where $\epsilon_i \sim N(0, 1)$. This cut-off equilibrium then implies that only investors with good signals will buy, i.e. those investors with $\epsilon_i > \frac{\hat{x}(z,p)-\theta}{\sigma_x}$. With our specifications, the market-clearing condition can then be conveniently rewritten as

$$1 - \Phi(\frac{\hat{x}(z, p) - \theta}{\sigma_x}) + \Phi(u) = 1. \tag{24}$$

For the market to clear,

$$\hat{x} = \theta + \sigma_x u.$$

Hence, observing price in our model is informationally equivalent to a public signal (i.e. this

cut-off value $\hat{x}$) with the precision $\frac{1}{\sigma_x^2\sigma_u^2}$.[11] An investor's information set can be summarized by $I_i = (x_i, z, \hat{x})$. Thus, the conditional expectation of the fundamental is then given by

$$\mathbb{E}[\theta|I_i] = \frac{\sigma_\theta^{-2}\bar{\theta} + \sigma_x^{-2}x_i + (\sigma_x\sigma_u)^{-2}\hat{x} + hz}{\sigma_\theta^{-2} + \sigma_x^{-2} + (\sigma_x\sigma_u)^{-2} + h}. \tag{25}$$

For the cut-off investor $\hat{x}$, the price must be equalized to the payoff of holding one share. Hence,

$$\begin{aligned} P(\theta, z, u|h, y) &= \mathbb{E}\left[\left(\frac{k\theta - \omega(h)}{1+\psi}\right)|x_i = \hat{x}, z, \hat{x}\right] - \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x, z, p) \\ &= -\frac{\omega(h)}{1+\psi} + \frac{k}{1+\psi}\mathbb{E}[\theta|x_i = \hat{x}, z, \hat{x}] - \frac{\gamma_I}{2}\frac{k^2}{(1+\psi)^2}Var(\theta|x, z, p), \end{aligned} \tag{26}$$

which gives Equation (5). The expression for the risk premium yields:

$$Var(\theta|x, z, p) = \sigma_\theta^2 - \begin{bmatrix} \sigma_\theta^2 \\ \sigma_\theta^2 \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2}{(h+\tau_m+\tau_\theta)} \end{bmatrix}^T \Sigma_{dd}^{-1} \begin{bmatrix} \sigma_\theta^2 \\ \sigma_\theta^2 \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2}{(h+\tau_m+\tau_\theta)} \end{bmatrix} = \frac{1}{(\tau_m + h + \frac{1}{\sigma_\theta^2})},$$

where $\Sigma_{dd} \equiv \begin{bmatrix} \sigma_\theta^2 + \sigma_x^2 & \sigma_\theta^2 & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2}{(h+\tau_m+\tau_\theta)} \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_h^2 & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2 + h\sigma_h^2}{(h+\tau_m+\tau_\theta)} \\ \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2}{(h+\tau_m+\tau_\theta)} & \frac{k}{1+\psi}\frac{(h+\tau_m)\sigma_\theta^2 + h\sigma_h^2}{(h+\tau_m+\tau_\theta)} & \left(\frac{k}{1+\psi}\right)^2\frac{(\tau_\theta+\tau_m)\tau_\theta^{-1} + h + \tau_m^2(\sigma_x^2\sigma_u^2)}{(\tau_\theta+h+\tau_m)^2} \end{bmatrix}.$

Hence, the expression for the risk premium is given by

$$R(y, h) = \gamma_I k^2 \frac{Var(\theta|x, z, p)}{2(1+\psi)^2} = \frac{\gamma_I k^2}{2(1+\psi)^2(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_x^2\sigma_u^2} + \frac{1}{\sigma_\theta^2} + h)}.$$

[11] In general, as shown in Albagli et al. (2011), there exists a random variable that is only a function of θ and $\tilde{u}$, and contains the same information as the price.

A.2. Omitted Proofs

A.2.1. Proof for Proposition 1

Proof. From Equation (10) and (11), observe that $U_h(k, \psi, \sigma, h)$ increases with k, σ, and $\frac{1}{1+\psi}$. According to Milgrom and Segal (2002), $\mu(k, \psi, \sigma)$, the solution to Equation (1), must increase with k, σ, and $\frac{1}{1+\psi}$. Since $U_h(y, h) = \Omega_h(y, h)$, this is equivalent to looking at the complementarity of the surplus function, as is standard in matching models. □

A.2.2. Proof for Proposition 2

Proof. By construction, Equations (12) and (13) guarantee that the market clearing condition is satisfied: the measure of agents below h is the same as the measure of firms that hire agents whose precision is lower than h. We now examine firms' optimality condition.

Recall that $\Upsilon(h, \omega_h(h))$ is the set of firms that are matched to agent h,

$$\Upsilon(h, \omega_h(h)) \equiv \{y \in Y \mid \Omega_h(y, h) = \omega_h(h)\}.$$

Since $\Omega_h(y, h) = U_h(y, \mu(y))$, it thus shows that the FOC of firms is satisfied as

$$U_h(y, \mu(y)) = \omega_h(\mu(y)).$$

We now show that $\mu(y)$ is indeed the *maximum* of $U(y, h)$. Condition (13) suggests that, for any $h' > \mu(y)$, $y \in Y(\mu(y), \omega_h(\mu(y)) \subset Y(h', \omega_h(h'))$. That is, the marginal cost of a hiring a better agent h' is too high:

$$U_h(y, h') < \omega_h(h) \, \forall h' > \mu(y).$$

That is, $U(y, h)$ decreases with h for $h > \mu(y)$. Similarly, hiring an agent with lower precision is too cheap:

$$U_h(y, h') > \omega_h(h) \, \forall h' < \mu(y).$$

Thus, $U(y,h)$ increases with h for $h < \mu(y)$. Hence, the constructed $\mu(y)$ solves the firm's optimization problem.

$\square$

A.2.3. Proof for Proposition 3

Proof. Given that

$$UP'[i] = \frac{q'[i](1-\beta(h[i])^2) - 2q[i]\beta'(h[i])h'[i]}{(1+\beta(h[i]))^2}.$$

Under competitive sorting, we have $q'[i] > 0$. And thus $UP'[i] > 0$ if the selection effect dominates (i.e., $q'[i]$ is larger enough). Without sorting $q'[i] = 0$, or in a random matching with a fixed production cost, the average quality of firms that hire an underwriter i decreases in i, and thus $UP'[i] < 0$ for all distributions.

$\square$

A.2.4. Proof for Proposition 4

Proof. Given that $\tilde{q}[i] = \lambda q[i]$,

$$\tilde{UP}[i] = \tilde{q}[i]\left(\frac{1-\beta(h[i])}{1+\beta(h[i])}\right) = \lambda UP[i]$$

and

$$\tilde{\omega}[i] = \int \tilde{q}[j]\left(\frac{1-\beta(h[j])}{1+\beta(h[j])}\right) = \lambda\omega[i].$$

And, clearly, $\tilde{\Gamma}(i_a, i_b) = \Gamma(i_a, i_b)$.

$\square$

A.2.5. Proof for Proposition 5

Proof. From Equation (17), a change in the firm's distribution only changes the ratio of $\frac{q'[i]}{q[i]}$. Thus, for a change in the distribution that implies a higher heterogeneity (i.e., a higher $q'[i]$)

with the same maximum, it thus implies that $\tilde{q}[i] < q[i]\ \forall i < 1$. And hence, a larger $\frac{q'[i]}{q[i]}$, and a stronger selection effect.

Connection to wages: Since the distribution of talents remain the same, by Lemma (2), the wage then becomes more convex. Furthermore,

$$\Gamma(i_a, i_b) = \frac{q[i_a]}{q[i_b]}\frac{h'[i_a]}{h'[i_b]}\left(\frac{(1+\beta(h[i_b]))}{1+\beta(h[i_a])}\right)^2.$$

Given that $\tilde{q}[i_a] - \tilde{q}[i_b] = \int_{i_b}^{i_a} \tilde{q}'[j]dj > \int_{i_b}^{i_a} q'[j]dj = q[i_a] - q[i_b]$, we have

$$\frac{\tilde{q}[i_a]}{\tilde{q}[i_b]} - 1 > \frac{q[i_b]}{\tilde{q}[i_b]}\left(\frac{q[i_a]}{q[i_b]} - 1\right) \geq \left(\frac{q[i_a]}{q[i_b]} - 1\right)$$

and thus $\tilde{\Gamma}(i_a, i_b) > \Gamma(i_a, i_b)$.

□

A.2.6. Proof for Proposition 6

Proof. From (17), since $q[i]$ remains the same and the second term yields

$$\frac{b'[i]}{b[i]} = \frac{-2\beta'(h[i])h'[i]}{(1-\beta(h[i])^2)} = \frac{-b_1 h'[i]}{(1-(b_0+b_1 h[i_a])^2)}.$$

Thus, a higher b_1, a lower selection effect. Furthermore,

$$\frac{b''[i]}{b'[i]} = \frac{h''[i]}{h'[i]} - \frac{2b_1 h'[i]}{1+b_0+b_1 h[i]}$$

which decreases in b_1. Hence, by Lemma (2), the relative wage convexity decreases in b_1.

$$\Gamma(i_a, i_b) = \frac{q[i_a]}{q[i_b]}\frac{h'[i_a]}{h'[i_b]}\left(\frac{1+b_0+b_1 h[i_b]}{1+b_0+b_1 h[i_a]}\right),$$

we thus have $\frac{\partial\Gamma(i_a,i_b)}{\partial b_1} < 0$.

□

A.2.7. Proof for Proposition 7

Proof. Given any distribution, $RP'[i|k] \propto q'[i|k] - h'[i|k]$. a higher density ratio $\frac{q'[i|k]}{h'[i|k]}$ leads to a stronger selection effect: if $RP'[i|k] > 0$ under $\frac{q'[i|k]}{h'[i|k]}$, then it must be true for a larger ratio.

□

A.2.8. Proof for Lemma 1

Proof. Consider any multiplicatively separable production function, let $V[i] = a[i]b[i]$, where $b'[i] > 0$ and $y'[i] > 0$. Since $\omega'[i] = a[i]b'[i]$, $\omega''[i] = a'[i]b'[i] + a[i]b''[i]$. We thus have

$$\frac{\omega''[i]}{\omega'[i]} = \frac{a'[i]}{a[i]} + \frac{b''[i]}{b'[i]} = \frac{V'[i]}{V[i]} + \left(\frac{b''[i]}{b'[i]} - \frac{b'[i]}{b[i]}\right),$$

where we use the fact that $\frac{V'[i]}{V[i]} = \frac{a'[i]}{a[i]} + \frac{b'[i]}{b[i]}$.

Observe that the underpricing can be rewritten as $UP[i] = -a[i]b[i]$, where $b[i] \equiv -\left(\frac{1-\beta(h[i])}{1+\beta(h[i])}\right)$ and $b'[i] > 0$. Given that $UP[i] = -V[i]$, we have $\frac{UP'[i]}{UP[i]} = \frac{V'[i]}{V[i]}$.

□

A.2.9. Proof for Lemma 2

Proof. Conditioning on any given k, the risk premium yields:

$$\frac{RP'[i|k]}{RP[i|k]} = \frac{q'[i|k] - h'[i|k]}{(-q[i|k] + h[i|k])}.$$

And

$$\omega''[i|k] = k\frac{h''[i|k]\left(-q[i|k] + h[i|k]\right)^2 - 2h'[i]\left(-q[i|k] + h[i|k]\right)\left(-q'[i|k] + h'[i|k]\right)}{\left(-q[i|k] + h[i|k]\right)^4}.$$

We thus have

$$\frac{\omega''[i|k]}{\omega'[i|k]} = \frac{h''[i|k]}{h'[i|k]} + \frac{2(q'[i|k] - h'[i|k])}{(-q[i|k] + h[i|k])} = 2\frac{RP'[i|k]}{RP[i|k]} + \frac{h''[i|k]}{h'[i|k]}.$$

□