# Asymmetric Information and Security Design under Knightian Uncertainty

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- E.g., it is common knowledge that cash flows are lognormal and the mean is drawn from some distribution.
- Equilibrium features pooling on risky debt under certain conditions (Nachman and Noe, 1994).
- Foundation for the "pecking order" theory of capital structure (Myers and Majluf, 1984; Myers, 1984).

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**Mixed empirical evidence:**

- Works fine for large mature firms (Shyam-Sunder and Myers, 1998);
- But poorly for small high-growth firms (Frank and Goyal, 2003; Leary and Roberts, 2010).

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What if investor has only a vague idea about possible distributions of cash flows?

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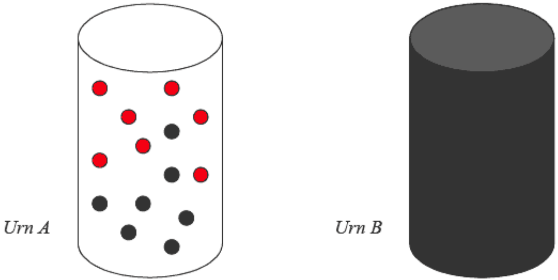
**The problem we study:**

- The investor thinks that distribution is in some uncertainty set but lacks confidence to assign prior.
  - Modeled via multiple priors ("models of the world").
- The issuer signals some information with security offer.
- The investor "demands robustness": evaluates the security according to the worst-case rationalizable model.

# Ellsberg Paradox




- Observed preference:
  (A,Black) $\simeq$ (A,Red) $\succ$ (B,Black) $\simeq$ (B,Red).
- No expected utility representation of these preferences.

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# Overview of Results

1. Two most common financial contracts – risky debt and standard outside equity – arise in eqm
   - Both are special contracts but for different reasons.
2. Optimal security depends on the degree of investor's uncertainty
   - Small uncertainty $\Longrightarrow$ "usually" risky debt
   - Large uncertainty $\Longrightarrow$ outside equity
3. Type of uncertainty matters: new project v.s. assets in place
   - Latter case: "usually" risky debt (if financing occurs) regardless of uncertainty, but never equity.

# Literature

**Security Design under Asy Info:** Myers and Majluf, 1984; Nachman and Noe, 1994; DeMarzo and Duffie, 1999; Fulghieri and Lukin, 2001; DeMarzo, 2005; Fulghieri, Garcia, and Hackbarth, 2015; Yang, 2015; Ortner and Schmalz, 2016; Szydlowski, 2017

**Robust Contracting:** Carroll, 2015; Antic 2015; Chassang, 2013; Bergemann and Schlag, 2011; Zhu, 2015; Lee and Rajan, 2016

**Ambiguity in Corporate Finance:** Dicks and Fulghieri, 2015; 2016; Garlappi, Giammarino, and Lazrak, 2016

# Model

New project requires investment $K$

Issuer has $W$, needs to raise $I = K - W$ through security sale

Distribution of project's cash flows $f \in \Delta(Z)$, $Z = \{0, z_1, \ldots, z_N\}$

- Privately known by the issuer
- Three states $0 < z_1 < z_2$
- General $N$ in the paper

# Uncertainty Set

Investor does not know $f$, but knows $f$ in the uncertainty set $B$

- $B$ is a neighborhood around some reference distribution $g$.

$B$ is a set of all distributions within Prokhorov neighborhood of radius $\nu$ around base distribuion $g$.

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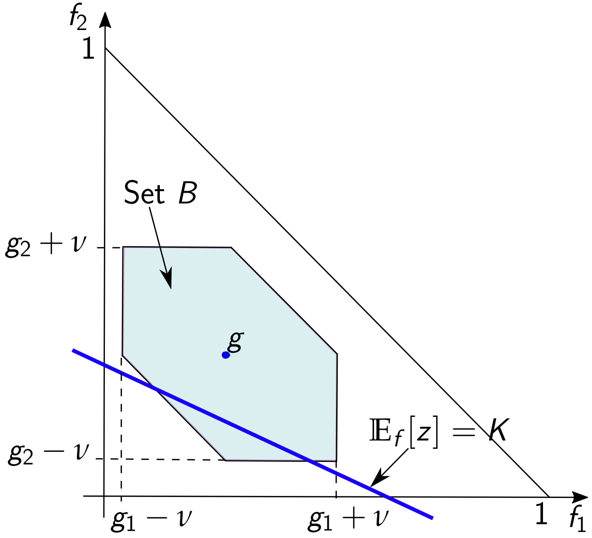
If $(0, z_1, z_2)$ are sufficiently far apart, then $B$ is a set of distributions $f = (f_0, f_1, f_2)$ that satisfy

$$g_n - \nu \leq f_n \leq g_n + \nu,\ n \in \{0, 1, 2\}.$$

# Illustration of Uncertainty Set $B$



# Actions

Issuer offers security $s$ that pays $s_n$ in state $z_n$

$s$ satisfies:

- Limited Liability: $0 \leq s_n \leq z_n$
- Monotonicity: $s_n$ and $z_n - s_n$ are weakly monotone

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Investor accepts $(\sigma = 1)$ or rejects $(\sigma = 0)$ security $s$ in exchange for $I$

- If $s$ accepted, investor gets $s - I$ and issuer gets $z - s$
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Strategies $s^*(f)$ and $\sigma^*(s)$.

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Investor demands robustness: evaluates each security $s$ by the "worst-case" rationalizable model, i.e.,

$$P(s) = \min_{f \in B(s)} \mathbb{E}_f[s].$$

Multi-prior maximin expected utility of Gilboa and Schmeidler (1989).

- But $B(s)$ is affected by the issuer's signaling.

# Valuation under Knightian uncertainty

*In uncertain environments when it is impossible to define a complete list of scenarios and related probabilities, it is impossible to calculate the expected value of different strategies. However, establishing the range of scenarios should allow managers to determine how robust their strategy is. (Courtney et al. HBR'97).*

# Equilibrium

**Definition:** $(\sigma^*, s^*, B(\cdot))$ constitute an equilibrium if

1. Issuer's rationality:

$$s^*(f) \in \arg\max_{s \in S} \mathbb{E}_f[z - s - W]\sigma^*(s),$$

   and $s^*(f) = \mathbf{0}$ if $\max_{s \in S^*} \mathbb{E}_f[z - s - W] < 0$, where $S^* \equiv \{s : \sigma^*(s) = 1\}$.

2. Investor's rationality:

$$\sigma^*(s) = 1 \iff P(s) \geq I.$$

3. For any $s \in S$, $B(s)$ is a set of rationalizable models.

# Rationalizable Models

Test that is similar to Intuitive Criterion: any securitiy $s$, even unexpected, is interpreted as a signal.

> *For each model $f$, investor contemplates: "If I were to accept offer $s$, would issuer $f$ be weakly better off than if he instead issued an equilibrium security $s^*(f)$ or chose not to invest in the project entirely?"*

**Set of Rationalizable Models:**

$$B(s) = \left\{ f \in B : \mathbb{E}_f[z - s] \geq 1_{\{s^*(f) \in S^*\}} \mathbb{E}_f\left[z - s^*(f)\right] + 1_{\{s^*(f) = \mathbf{0}\}} W \right\}$$

whenever the set is non-empty. Otherwise, $B(s) = B$.

For $s \in S^*$ it is similar to the model of learning under ambiguity of Epstein and Schneider (2003).

# Uniqueness

Equilibrium is generically unique and takes a semi-pooling form.

# Effect of Uncertainty

**Result 1:** More uncertainty ($B$ expands),
then equity ($s = \frac{I}{K} z$) becomes a dominant security.

# Effect of Project Quality

**Result 2:** Higher investment cost ($\uparrow K$),
then equity ($s = \frac{I}{K} z$) becomes a dominant security.

# Assets in Place

**Result 3:** In assets-in-place model, equity is never optimal.

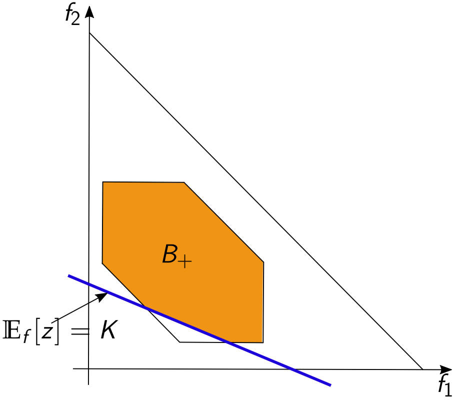
# Preliminary Analysis

Recall $P(s) = \min_{f \in B(s)} \mathbb{E}_f[s]$.

**Lemma 1:** Wlog, focus on $s$ such that $P(s) = I$.

- No point to raise more money than you need.

**Lemma 2:** For $\forall s$ s.t. $P(s) = I$, it holds $P(s) = \min_{f \in B_+} \mathbb{E}_f[s]$.

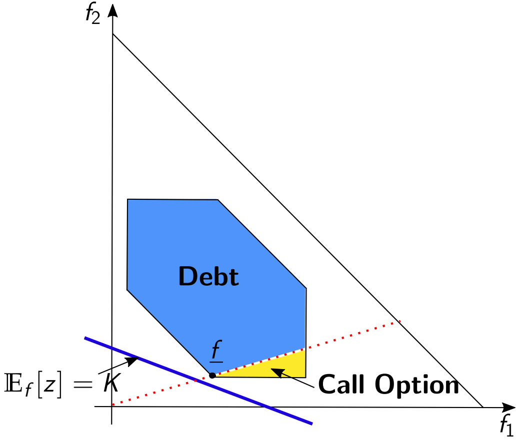


# Small Uncertainty

**Small** $B$**:** all $f \in B$ have positive NPV ($\mathbb{E}_f[z] > K$).

- For any $s \in S^*$, worst-case rationalizable model is given by $\underline{f}$.

**Result:** Debt optimal under MLRP ordering of $\underline{f}$ and $f$ (weaker than MLRP ordering of types)



# Why is Debt Special?

**Intuition:**

- Because the investor fears adverse selection, he is cautious at evaluating any risky security.
- Any risky security will be (weakly) underpriced.
- But debt is less underpriced than any other security, because it gives highest downside protection to investor.

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- This is exactly the informal intuition for "folklore proposition for debt" (e.g., the way I explain it to MBA students).
- But this is not the formal reason for optimality of debt in Nachman and Noe (1994):
  - In equilibrium, any issued security is, on average, priced fairly, not underpriced.
  - Non-debt is not an equilibrium, because if the investor unexpectedly observed debt, he will believe that the project is great.

# Extreme Uncertainty

**Extreme $B$:** the investor is worried that <u>any</u> distribution is possible

Why does risky debt become a bad security?

- Suppose the issuer issues debt with face value $F$, such that $P(s) = I$.
- Investor's valuation problem:

$$\min_{f_1, f_2} f_1 \min\{z_1, F\} + f_2 F$$

$$\text{s.t. } f_1 \max\{z_1 - F, 0\} + f_2 (z_2 - F) \geq W$$

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- To convince the investor to put $I$, the face value must be $F = \frac{I}{I+W} z_2$.

# Extreme Uncertainty

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- Suppose the issuer issues fraction $\alpha$ of equity such that $P(s) = I$.
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$$\min_{f_1, f_2} \alpha \left( f_1 z_1 + f_2 z_2 \right)$$

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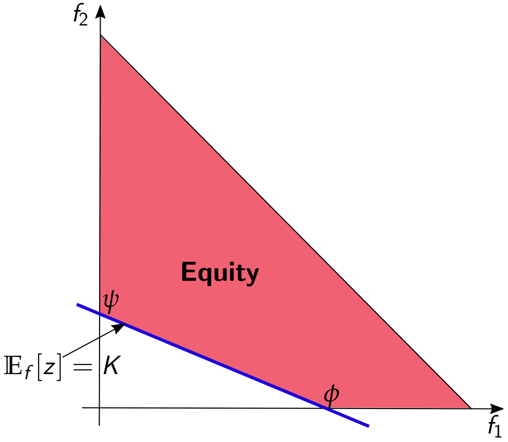
- To convince the investor to put $I$, the issuer must issue $\alpha = \frac{I}{I+W}$.
- But any issuer prefers $s(z) = \frac{I}{I+W} z$ over $s(z) = \min\left\{z, \frac{I}{I+W} z_2\right\}$.

# Extreme Uncertainty



# Why is Equity Special?

**Intuition:**

- Outside equity serves as a very credible signal that the project is "good enough", because the investor and the issuer both hold the security with the same shape:

  *Issuer: "I know you are worried about cash flow distribution you are getting. However, the fact that I want to do the project while keeping $\frac{W}{I+W}$ of equity is a proof that I think the project is positive NPV. Since you also hold equity, you will break even."*

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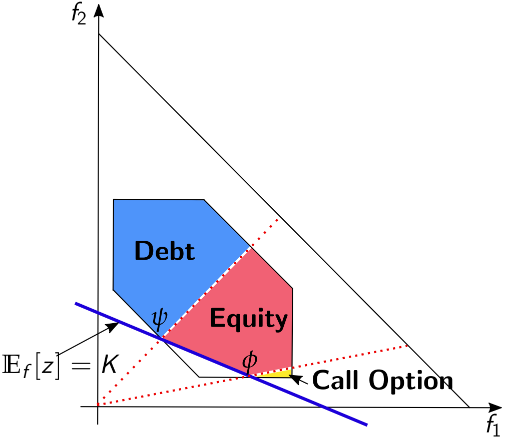
- Any non-linear security only sends the message that the security the issuer keeps is good enough.
  - If the issuer offers debt, the investor is worried that he will not profit from the upside.
  - If the issuer offers a convex security, the investor is worried that the project has little upside.

# Large Uncertainty

**Large** $B$**:** some $f \in B$ have negative NPV

For any $s \in S^*$, worst-case rationalizable model

- has zero NPV;
- $\psi$ for concave $s$ and $\phi$ for convex $s$.



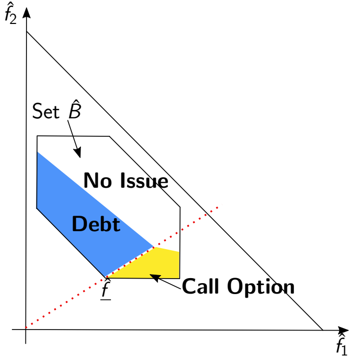
# Assets in Place

- Issuer does not have $W$, **but** pledges assets in place with cash flow $z \sim f \in B$
- New project's quality is common knowledge
  - shifts the probability mass $\delta$ from 0 to $z_2$ so that $\delta z_2 > K$ for any $f \in B$
- New type $f \to \hat{f}$ is distribution after investment in $\hat{B}$

# Assets in Place

**Result:** Equity never optimal irrespective of uncertainty

- Intuition: the issuer with worse assets is always more willing to pledge them $\Longrightarrow$ similar to small uncertainty case



# Implications

1. In the literature, mixed empirical evidence of pecking-order theory
    - works best for large mature firms (Shyam-Sunder and Myers (1999))
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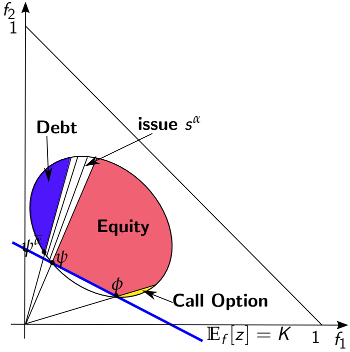
2. Suggest evolution of optimal financing:
    - young firm with little assets in place and large uncertainty: equity
    - mature firm with lots of assets in place and small uncertainty: debt

# Alt. Uncertainty Set and $N > 3$

General Insight:

$$\text{Equity} = B_+ \cap \text{Cone generated by zero-NPV segment}$$

More uncertainty/adv. selection $\implies$ Cone expands $\implies$ Equity expands

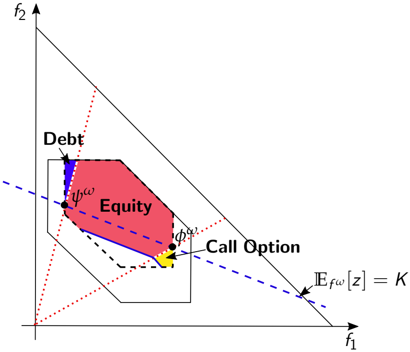


# Robustness: Valuation of Securities

Both worst- and best-case scenario (Hurwitz's criterion)

$$P^{\omega}(s) = \omega \min_{f \in B(s)} \mathbb{E}_f[s] + (1-\omega) \max_{f \in B(s)} \mathbb{E}_f[s].$$

"Equivalent" to shrinking set $B$



# Robustness: Learning from Offers

Alternative specification:

$$B(s) = \{f \in B : \mathbb{E}_f[z - s] \geq W\}.$$

- Does not require knowledge of equilibrium strategy $s^*(\cdot)$ by investors, ONLY knowledge of rationality.
- Results unchanged

# Conclusion

Classic problem of designing a security to an investor facing Knightian uncertainty

Two most popular securities, risky debt and standard outside equity, arise in equilibrium with only friciton one friction – asymmetric info

- Risky debt is special, because it gives the highest payoff in low states, which is valued by a cautious investor.
- Outside equity is special, because it gives a cautious investor certainty that the project is "good enough".

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Result 1: Small uncertainty + MLRP of $f$ and $\underline{f} \Longrightarrow$ risky debt.

Result 2: Large uncertainty $\Longrightarrow$ outside equity

Result 3: Assets in place: risky debt