

Sophisticated Sincerity under Incomplete Information

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Abstract

We aim to offer guidance for the practical design of sequential voting schemes that are widely used by legislatures and committees. Our main results describe a broad class of procedures that yield robust and desirable outcomes even in relatively complex strategic situations involving incomplete information about preferences. We uncover a strong rationale for content-based agendas (in contrast to agendas formed by procedural rules): if the outcome should not be sensitive to beliefs about others, nor to the deployment of strategic skills, the agenda needs to be built "from the extremes to the middle" so that more extreme alternatives are both more difficult to adopt, and are put to vote before other, more moderate options. We illustrate our findings with several case studies.

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1 Introduction

Sequential, binary voting procedures are widely used in democratic legislatures and in committees in order to select one among several alternatives. At each stage, one among two subsets of alternatives is adopted via a simple Yes/No vote by (possibly qualified) majority. This process is repeated until a unique alternative is singled out, and formally elected. There is significant diversity in the details of observed schemes: For example, *amendment* procedures are used in English-speaking and Scandinavian countries, while most continental European parliaments use the rather different *successive* procedure. Which sequential voting procedures have desirable properties in terms of the induced equilibria and their outcomes? Can we explain various voting patterns observed in reality? Can we offer some theoretical guidance for the practical design of sequential voting procedures? These are the issues discussed in this paper.

We analyze sequential, binary voting schemes in settings where several privately informed agents have single-peaked preferences on a finite set of alternatives, and we focus on robust equilibria that do not depend on assumptions about the players' beliefs about each other. Our main results identify two intuitive conditions on binary voting trees ensuring that sincere voting at each stage forms an ex-post perfect equilibrium in the associated extensive form game with incomplete information.¹ In other words, in the identified game trees voters cannot gain by manipulating their vote, regardless of their beliefs about others' preferences and regardless of the information disclosure policy along the voting sequence. An important corollary is that, as long as our conditions are satisfied and simple majority is required at each stage, the equilibrium outcome of the incomplete information game is always the Condorcet winner. We illustrate that this need not be the case if our conditions are violated, giving a strong rationale for using procedures that satisfy our conditions.

The two main conditions conducive to desirable outcomes in sequential voting procedures are:

1) *Convexity of divisions (CONV)*. Recall that each vote is taken by (possibly qualified) majority among two, not necessarily disjoint, subsets of alternatives. Convexity says that if two alternatives a and c belong to the left (right) subset at a given node, then any alternative b such that $a < b < c$ (in the order governing single-peakedness) also belongs to the left (right) subset. Intuitively, each of the Yes/No votes in the sequence must be among two options that cover a well-defined, coherent segment of positions in the respective ideological spectrum.

2) *Monotonicity of qualified majorities (MON)*. This condition says that, after a vote that resulted in the adoption of a left (right) subset of alternatives, a subsequent movement left (right) requires a qualified majority that is at least as large as the one that governed the previous move in the same direction. Intuitively, adopting consecutive and

¹We also show that the sincere equilibrium is *strong Nash*, which means that no coalition of voters can profitably deviate from sincere voting. This is particularly important for legislatures where voting is mostly according to party lines, so that coordinated deviations are likely.

more "extreme" positions should become more and more difficult. The standard case of keeping a constant majority requirement at each vote - such as a simple majority - satisfies monotonicity.

In order to understand the role played by the above conditions, note that, under incomplete information, the main determinants of optimality are the decisions at pivotality events: only such instances offer the opportunity to directly influence the outcome, and hence the optimal strategy must recommend a "correct" action whenever an agent is pivotal. Due to presence of incomplete information agents do not a priori know when and whether they are pivotal, and hence they need to make accurate inferences about future outcomes conditional on being pivotal. Roughly speaking, the combined effect of CONV and MON is to finely tune this inference: a pivotal agent is able to infer that a more preferred alternative will ultimately be elected, either because there are anyway enough other supporters for this alternative, or because the agent will continue to remain pivotal (and hence in control of the decision) at future stages. Thus, sincere voting - according to the preference relation restricted to the remaining set of alternatives - is optimal at each and every stage of the voting process.

Fortunately, many ubiquitous voting procedures do satisfy our conditions if simple and intuitive rules of agenda formation are respected. Let us consider two prominent examples:

1) In an *amendment* procedure, alternatives are paired, and the winner competes against the next alternative on the agenda, until all alternatives are exhausted. The amendment procedure will satisfy convexity if the pairing is such that the most "extreme" alternatives compete against each other at each round of voting. If the single-peakedness order is $1 < 2 < \dots < A$, the first vote should be among alternatives 1 and A . If 1 wins this contest then the next vote is among 1 and $A - 1$, whereas if A won at the first round, the second vote should be between A and 2, and so on. Monotonicity holds, for example, if the qualified majority needed to keep alternative 1 in the process is not decreasing along the voting tree.

2) In a *successive* procedure voters consider alternatives one after the other, and the process stops as soon as one alternative garners the required majority.² It will satisfy convexity if, at each stage, the considered alternative is one of the two most extreme ones. If the single-peakedness order is $1 < 2 < \dots < A$, convex agendas are, for example, to vote on the alternatives in the order $1, 2, 3, \dots, A$,³ in the order $A, A - 1, \dots, 1$ or even in a left-right alternating order such as $1, A, 2, A - 1, \dots, A/2$. If the successive procedure uses the order $1, 2, 3, \dots, A$, monotonicity says, for example, that the qualified majority required

²A famous case has been the vote to establish the capital of the united Germany: the successive method was used in order to decide among 4 alternatives, some including split locations of government and parliament. The likely Condorcet winner, Bonn, was ultimately not elected.

³For this particular procedure and agenda, monotonicity of qualified majorities has been shown to be necessary for a robust dynamic implementation by Gershkov, Moldovanu and Shi (2015). These authors were mainly concerned with identification of welfare maximizing mechanisms in settings where monetary transfers are not possible.

	Sequential Voting	Amendment Procedure
Always Vote on Most Extreme Alternative	Austria, Denmark, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Netherlands, Norway, Poland, Slovenia, Spain, European Parliament	Finland
Other procedural rule	Belgium, Czech Republic, Luxembourg, Portugal, Slovakia	Sweden, Switzerland, UK, (US)

Table 1: Parliamentary Floor Voting Procedures. Source: Rasch (2000)

to continue the voting process cannot decrease (i.e., the majority needed to accept the current alternative does not increase).

As should be clear from the above, for our conditions to be satisfied, the content of proposals (rather than purely procedural considerations) should determine the agenda: extremes should be voted on first. This is the main prescriptive design recommendation coming from our study. Of course, the interpretation of content may be ambiguous - leading to possible manipulations that could be prevented by rigid, procedural rules. Nevertheless, the basic idea of a content-based agenda is anchored, formally or informally, in the rules governing many legislatures. For example, the German parliament uses an informal rule, rooted in custom and practice, to vote on extreme alternatives first. The Standing Orders of its Second Chamber (the Bundesrat) prescribe explicitly that

“if several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo” (Article 30 (2), *Geschäftsordnung des Bundesrates* (1993)).

The above Table 1 (taken from Rasch 2000) summarizes the practices in European parliaments.

To illustrate the empirical and theoretical content of qualified majorities and monotonicity, recall first that many legislatures and committees use staggered supermajorities for the passing of various, special laws. Prominent among these are the supermajorities required for constitutional amendments in all democracies, and the Taxes and Expenditure laws (TEs) in most states in the U.S. (see also Knight 2000). For example, the legislature of Nebraska can vote to increase property taxes reflecting changes in the Consumer Price Index by simple majority, while larger increases up to 5% require a three-quarters majority. Further increases above 5% require a full referendum in the population. Another example, discussed in more detail below, is the U.S. Supreme court where accepting a case for review (*certiorari*) requires 4 out of 9 votes, but decisions on the accepted cases need to garner a simple majority (5 votes out of 9). From a theoretical point of view,

we show that the use of any particular voting tree satisfying convexity is not restrictive: varying the qualified majority at each node in the voting tree allows us to replicate, via any binary voting procedure that satisfies convexity, the entire set of anonymous, unanimous and dominant strategy implementable social choice functions for the domain of single-peaked preferences. In other words, we show that the use of any particular voting tree satisfying convexity is not restrictive. The dynamic implementation in an ex-post perfect and sincere equilibrium necessarily satisfies our monotonicity condition. The proof of these results builds upon the seminal contribution of Moulin (1980)

Besides Moulin’s work, our study builds upon a large literature (in Economics, Law and Political Science) that has studied both theoretical and institutional/empirical aspects of sequential voting procedures.⁴ Farquharson (1969) is the first to study voting trees from a game-theoretic perspective, and Miller (1977), McKelvey and Niemi (1978) and Moulin (1979) provide important theoretical contributions. Ordeshook and Schwartz (1987) and Schwartz (2008) describe institutional aspects of sequential procedures used in parliaments, and Bjurulf and Niemi (1978), Enelow (1981), and Ladha (1994) empirically analyze voting behavior in sequential voting procedures.⁵ Almost the entire previous literature assumed that agents are completely informed about the preferences of others. Under complete information, the associated extensive form games are amenable to analysis by backward induction: if a simple majority is used at each stage, a Condorcet winner - which always exists with single-peaked preferences - must be selected by sophisticated voters independently of the particular structure of the binary voting tree and independently of its agenda (Miller 1977). A pioneering analysis of strategic, sequential voting under incomplete information (and of the subtle effect of pivotality in this case) is Ordeshook and Palfrey (1988) who were able to construct Bayesian equilibria for an *amendment* procedure with three alternatives and with three possible preference profiles that potentially lead to a Condorcet paradox.⁶

We conclude the paper with an analysis of case studies from the Legal and Political Science literature, where a lively debate has revolved around the question whether strategic behavior is a common phenomenon in real life voting situations. Our discussion complements the existing, often strained, explanations of strategic manipulations and of failures to implement the Condorcet outcome, which assume complete information: if preferences are known, why do some manipulations succeed while others backfire? Why can’t the “victims” of an attempted manipulation anticipate it, and respond by changing their strategy?

We first illustrate several environments where strategic manipulations seem rare, and argue that, in these examples, our conditions were satisfied. We next review several case studies that identified strategic manipulations and observe that, in each case, conditions

⁴Our general treatment of varying qualified majorities at each stage generalizes the constant, simple majority rule assumed in almost the entire previous literature.

⁵See also Volden (1998), Wilkerson (1999), and Caldeira, Wright and Zorn (1999).

⁶See also Krehbiel and Rivers (1990), who study the voting on the school construction bill of 1956 as an incomplete information game.

CONV and MON were not satisfied. Under incomplete information, such procedures offer ample space for strategic manipulations that may or may not succeed, depending on the particular beliefs that voters entertain, and on the actually realized profile of preferences (which is not common knowledge). As a consequence, the Condorcet winner need not be elected in equilibrium. Note that this result is in stark contrast to the complete information case, where, independently of the procedure used, the Condorcet winner always emerges in a sophisticated equilibrium (Miller 1977).

The rest of the paper is organized as follows. In Section 2 we present several simple examples that illustrate our results. In Section 3 we present the voting model, and the convexity and monotonicity conditions. In Section 4 we present our main results for voting procedures where sincere voting is a robust equilibrium. In Section 5 we characterize the class of social choice functions that can be robustly and dynamically implemented in our setting. In Section 6 we discuss several case studies in light of our findings. Section 7 concludes.

2 Illustrative Examples

The purpose of this section is to illustrate the main ideas and results in the simplest non-trivial setting with three privately informed agents and three alternatives, labeled $\{1, 2, 3\}$. We assume that preferences are strict and single-peaked with respect to the order on natural numbers.⁷ This assumption yields four possible individual preferences: $1 > 2 > 3$, $2 > 1 > 3$, $2 > 3 > 1$, and $3 > 2 > 1$.

2.1 The Role of Convexity

Assume that voters use an amendment procedure where the first vote is by simple majority between alternative 1 and 2, and the second vote is by simple majority between the winner of the first vote and alternative 3. This can be represented by the voting tree illustrated in Figure 1.

The second (and last vote) is either between 1 and 3, or between 2 and 3. Voting sincerely at the last vote (which is always a simple binary choice between two deterministic outcomes) is clearly optimal for all types of all agents.

Consider now the first vote between alternatives 1 and 2. Sincere voting calls for agents with peaks on 1 to vote for alternative 1, and for agents with peaks on 2 and 3 to vote for alternative 2. But sincere voting does not form of an ex-post perfect equilibrium: consider for example the case where our three agents have the following profile of preferences: $1 > 2 > 3$, $2 > 3 > 1$, $3 > 2 > 1$. Then alternative 2 is ultimately chosen under sincere voting, but a deviation to the left of the agent with peak on alternative 3 at the first vote (vote for 1) would result in a second vote among alternatives 1 and 3, where 3 would win.

⁷By relabeling alternatives, this is without loss of generality.

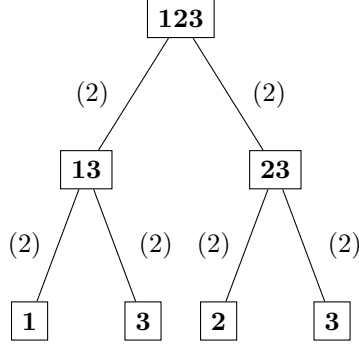


Figure 1: Illustration of an amendment procedure

Hence such a deviation may be profitable, and our agent may regret his first sincere vote. Thus, sincere voting is not an equilibrium, and the reader may easily work out that only trivial ex-post perfect equilibria exist in this game.

In the following example, we analyze a Bayesian equilibrium for this amendment procedure, and illustrate how strategic deviations from sincerity can prevent the Condorcet winner from being selected.

Example 1. *There are 3 voters that use the voting procedure illustrated in Figure 1. Each of the four possible single-peaked ordinal preferences is associated with a "type": preference $1 > 2 > 3$ with type t_1 , $2 > 1 > 3$ with type t_2 , $2 > 3 > 1$ with type t_3 , and $3 > 2 > 1$ with type t_4 . Types are I.I.D. and, for each agent, type t_i is realized with probability q_i . For an analysis of Bayesian equilibria, we need a cardinal description of utilities: we assume here that each voter values his most preferred alternative by 1, his second-most preferred alternative by v , and his least preferred alternative by 0. We show in the Appendix that, if $v \leq \frac{\min\{q_2, q_3\}}{q_2 + q_3}$, then the following strategies form a Bayes-Nash equilibrium: all types except t_4 vote sincerely, and voters of type t_4 vote left at the first stage and vote sincerely at the second.*

Even though they are aware of the deviation by type t_4 voters, it is optimal for type t_1 voters to vote sincerely because a deviation would significantly reduce their chances of getting their most preferred alternative. Type t_4 voters vote strategically (for their worst alternative!) at the first stage in order to increase the chances of their most preferred alternative at the second stage. This is optimal for type t_4 voters as long as v is small enough.

The strategic manipulations imply that a Condorcet winner, which always exists, will not always be selected in equilibrium: If each alternative is the peak of one of the voters, alternative 2 is the Condorcet winner. However, because the voter with peak at alternative 3 votes left in the first stage, the Condorcet winner is eliminated at the first stage. Note that by voting insincerely, a voter with peak at alternative 3 runs the risk that his least preferred alternative is finally selected in equilibrium.

Consider now the same amendment procedure, but with a different agenda: the first vote is between alternatives 1 and 3, and the second vote between the winner of the first

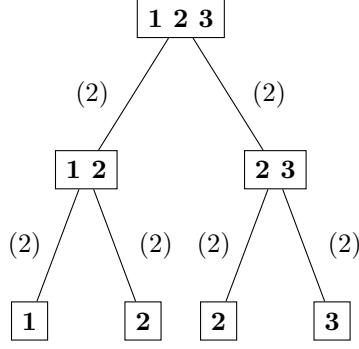


Figure 2: Illustration of an amendment procedure satisfying CONV

vote and alternative 2. This procedure can be represented by the voting tree illustrated in Figure 2.

The second vote is either between 1 and 2, or between 2 and 3, and voting sincerely at the last vote is again optimal for all types of all agents. Consider now the first vote between alternatives 1 and 3. Sincere voting calls for agents with peaks on 1 to vote for alternative 1 and this is indeed optimal since every outcome that can be reached following the left branch at the origin is (weakly) preferred by such a player to any outcome that can be reached following the right branch. An analogous reasoning shows that sincere voting is optimal for agents with peaks on alternative 3.

Consider next an agent i with preference profile $2 > 1 > 3$, for whom sincere voting recommends voting for alternative 1 at the first vote. This vote matters only if such an agent is pivotal, thus only in case there is exactly one other agent that votes for alternative 1, and exactly one agent that votes for alternative 3. But then our agent i can be sure that voting sincerely at the first vote will ultimately lead to a second vote between 1 and 2 where his most preferred alternative is elected. An analogous reasoning for an agent i with preference profile $2 > 3 > 1$ completes the proof that sincere voting is an ex-post perfect equilibrium in this amendment procedure. This equilibrium has the desirable property that the Condorcet winner will always be selected.

As we shall see below, the crucial difference between the two agendas is a (discrete) convexity idea: given the order under which preferences are single-peaked, the votes in the second example are always among sets of alternatives "without holes": the generated divisions are $\{12|23\}$, $\{1|2\}$ and $\{2|3\}$. In contrast, in the first example, the divisions are $\{13|23\}$, $\{1|3\}$ and $\{2|3\}$ and the division $\{13|23\}$ contains the non-convex set $\{13\}$ with a "hole" in place of alternative 2. This creates an uncertainty about the actions of others (and hence about the outcome) that cannot be satisfactorily resolved - i.e., without ever experiencing regret - under incomplete information.

2.2 The Role of Monotonicity

In order to explain the role of our second condition, let us look at the successive voting procedure illustrated in Figure 3.

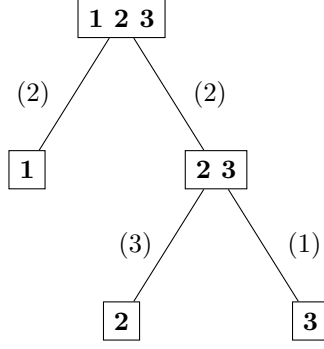


Figure 3: Illustration of a successive voting procedure

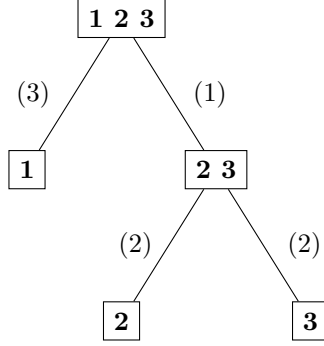


Figure 4: Successive voting procedure satisfying MON

At the first vote the agents decide whether to accept or reject alternative 1. If 1 is accepted voting ends, and otherwise a vote is taken whether to accept or reject alternative 2. If this alternative is accepted voting ends, and otherwise alternative 3 is elected.

Assume first that alternative 1 is adopted by simple majority (so that two votes are sufficient to reject it) while alternative 2 is adopted by unanimity (so that one vote is sufficient to reject it, and hence elect alternative 3). Consider an agent i with preferences $2 > 1 > 3$. Then sincere voting calls for such an agent to vote against alternative 1 at the first vote. However, in case there is one agent with a peak on 1 and one agent with a peak on 3, sincere voting would lead to alternative 3 being elected. Agent i would then be better off by deviating and voting for alternative 1 at the first vote, which would lead to the implementation of alternative 1. Therefore, sincere voting is not an ex post perfect equilibrium for the above voting procedure.

Note that the partitions generated by this procedure are convex, without "holes": these are $\{1|23\}$ at the first vote and $\{2|3\}$ at the second. So the difficulty lies here elsewhere, namely in the specific thresholds for rejection of consecutive alternatives.

To correct the problem consider the same voting tree, but where alternative 1 is adopted by unanimity (so that one vote is sufficient to reject it) while alternative 2 is adopted by simple majority (so that two votes are required to reject it, and hence to elect alternative 3) (see Figure 4).

Then sincere voting is an ex-post perfect equilibrium: this is obvious for the agents

with peaks on alternative 1 and 3 and for an agent with preferences $2 > 3 > 1$, so consider again an agent i with preferences $2 > 1 > 3$. If such an agent is pivotal at the first vote (to accept or reject 1) then the two other agents have peaks on alternative 1, so that at the next vote it must be the case that alternative 2 will be chosen. Hence it is optimal for our agent to also vote sincerely.⁸

This example shows that the thresholds at consecutive votes must be such that pivotal agents can infer (from the instance of them being pivotal) that, roughly speaking, they can still control the outcome at future stages.

3 The Sequential, Binary Voting Model

There is a finite set of alternatives, $A = \{1, 2, \dots, |A|\}$, and a finite set of voters, N . Each voter $i \in N$ has a strict preference ordering $>_i$ over the elements of A . A preference ordering $>_i$ is single-peaked (with respect to the natural order \leq on A) if there is an alternative $a \in A$ such that $c < b \leq a$ or $a \leq b < c$ imply $b >_i c$. We denote by \mathbf{P}^{SP} the set of preference profiles that are single-peaked, and assume below that each voter has single-peaked preferences.

A binary tree is a pair $\mathbf{V} = (V, Q)$ where V is a finite set whose elements are called *nodes*, and $Q \subseteq V \times V$ is a partial ordering over V whose elements are called *branches*. It is assumed that V has a unique least element (the *origin*), and that for any $v, v' \in V$ there is at most one Q -chain between v and v' . Any maximal element of V is called a *terminal node*, and we denote by V' the set of non-terminal nodes. If $v Q v'$ for $v \neq v'$, we say v' is a *successor* of v . Every non-terminal node has exactly two successors. If two distinct nodes are successors of the same node, we call them *neighbors*. If there are nodes $v_0 = v, v_1, \dots, v_l = v'$ such that $v_{k-1} Q v_k$ for $k = 1, \dots, l$, we say that v' *follows* v .

Given a binary tree, a *division correspondence* $L : V \Rightarrow A$ associates with each terminal node a unique alternative, and with each node $v \in V'$ all the alternatives that are associated with terminal nodes following v . We assume that each alternative is associated with at least one terminal node and that for any v and any v' following v , $L(v) \neq L(v')$. The set $L(v)$ denotes the alternatives that are under consideration by the voters at node v .

A *voting tree* (\mathbf{V}, L) is a binary tree \mathbf{V} together with a division correspondence L .

Definition 1. A voting tree (\mathbf{V}, L) satisfies convexity of divisions (CONV) if,

$$\text{for all } v \in V, a \leq b \leq c \text{ and } a, c \in L(v) \text{ imply } b \in L(v).$$

⁸Claims in the literature indicate that sincere voting should be a dominant strategy for this procedure. This is not true as the following example illustrates: Suppose agent 1 has preferences $2 > 1 > 3$, agent 2 votes left in the first vote and right in the second, and agent 3 always votes right. Then sincere voting by agent 1 yields alternative 3 being selected. However, if agent 1 deviates and votes left in the first vote instead, then alternative 1, which he prefers to alternative 3, will be selected.

We denote by $\min_{SP} L(v)$ and $\max_{SP} L(v)$ the smallest and largest alternatives of the set $L(v)$ in the order underlying the single-peaked preferences. Given a voting tree (\mathbf{V}, L) and two neighbors $u, v \in V$, we label one of the branches leading to u by ℓ and the other by r . If (\mathbf{V}, L) satisfies CONV, we label the branches leading to u and v as follows: If $\min_{SP} L(u) < \min_{SP} L(v)$, we label the branch leading to node u by ℓ , the *left* branch, and the branch leading to v by r , the *right* branch (and vice versa).⁹

We can identify a node in terms of the branches that lead to this node starting from the origin. Given a path $v \in \{r, \ell\}^k$, we denote by $v \oplus r$ the path of length $k + 1$ whose first k entries coincide with the entries of v and whose last entry is r (and similarly for ℓ).

A *system of thresholds* for a voting tree (\mathbf{V}, L) is a tuple of functions $\tau^\ell : V' \rightarrow \{1, 2, \dots, n\}$ and $\tau^r : V' \rightarrow \{1, 2, \dots, n\}$ such that for any $v \in V'$,

$$\tau^\ell(v) + \tau^r(v) = n + 1.$$

For any non-terminal node, the thresholds determine the number of votes needed in order to continue to the successor node of the left and right branch, respectively. The sum of the two thresholds is $n + 1$, so that no ties can occur. A *voting procedure* (\mathbf{V}, L, τ) is a voting tree (\mathbf{V}, L) together with a system of thresholds τ .

Definition 2. A voting procedure (\mathbf{V}, L, τ) satisfies *monotonicity of thresholds (MON)* if

- (i) for every $u, v \in V'$ such that $\max_{SP} L(u \oplus \ell) = \max_{SP} L(v \oplus \ell) = k$ and $\min_{SP} L(u \oplus r) = \min_{SP} L(v \oplus r) = k + 1$, it holds that $\tau^\ell(u) = \tau^\ell(v)$, and
- (ii) for every $u, v \in V'$ and $s \in \{\ell, r\}$ such that v follows $u \oplus s$, it holds that $\tau^s(u) \leq \tau^s(v)$.

For voting procedures with convex divisions, part (i) is a mild consistency condition requiring that, whenever the vote is among the same set of disjoint alternatives, the same threshold is used, irrespective of the previous voting history. Condition (ii) states that after a vote that resulted in the adoption of the left (right) division, a subsequent movement left (right) requires a qualified majority that is at least as large as the one that governed the previous move in the same direction.

Figure 5 shows an example of a general voting procedure satisfying CONV and MON.

Together with a given set of agents, their preferences and beliefs, a voting procedure describes a game of incomplete information: At each non-terminal node, players simultaneously vote either for the left or the right branch. If there are at least $\tau^\ell(v)$ voters voting for the left branch at node v , then the game advances to node $v \oplus \ell$; otherwise the game advances to node $v \oplus r$. If a terminal node v' is reached, then alternative $L(v')$ is implemented.

⁹This labeling procedure is well-defined given that (\mathbf{V}, L) satisfies CONV.

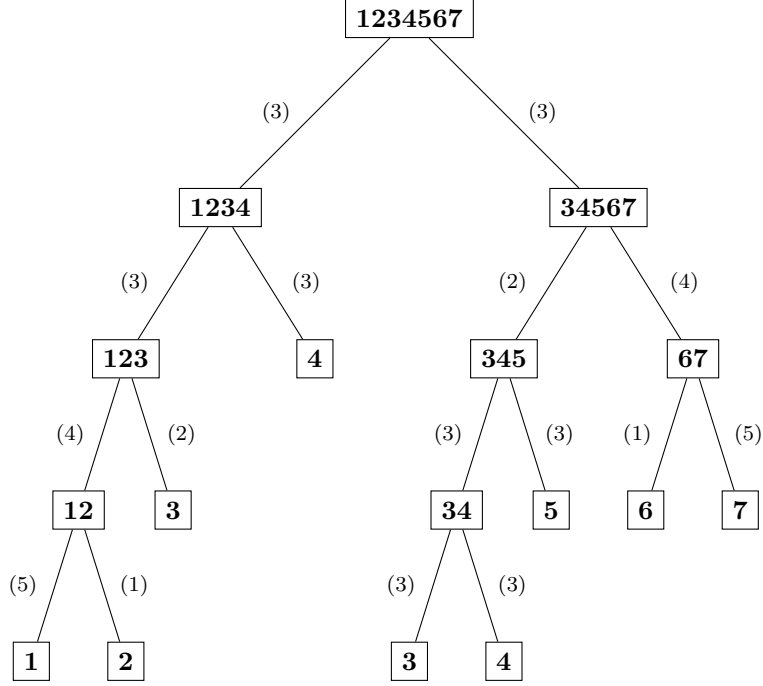


Figure 5: Example of a voting procedure satisfying CONV and MON.

Ex-post Perfect Equilibrium and Sincere Voting

Even with restrictions on the set of admissible preferences (such as single-peakedness), the analysis of the extensive form voting trees can be daunting if agents are privately informed. Optimal strategies generally depend on the specific, cardinal representation of utilities and on the beliefs about others. In turn, these beliefs are influenced by inferences that can be drawn from the ex-ante probabilities attached to the different possible profiles of preferences, and from new information generated by the employed strategies in the respective institutional setting.¹⁰ We focus instead on the much more robust, *ex-post perfect* equilibrium: in such an equilibrium, the resulting optimal strategies do not depend on ex-ante beliefs, and continue to be optimal irrespective of the information that is revealed during the sequence of votes.¹¹ This is the strongest form of equilibrium possible in our setting since, if there are more than two alternatives, desirable outcomes cannot be dynamically implemented in dominant strategies.

Let H_i^v denote the part of the history of play that is observable to player i at node v .¹² One sensible specification is that H_i^v consists of the aggregate number of left and right votes at each previous node, and i 's own voting behavior at all previous nodes. Another

¹⁰In particular, *manipulations* may occur also via voting behavior that attempts to influence the beliefs of other voters, and hence their future behavior (signaling effect), an effect that is absent under complete information.

¹¹Ex-post equilibria in settings with cardinal utility and with monetary transfers have been studied in the literature on *robust* mechanism design, e.g., by Bergemann and Morris (2005) and Jehiel, Meyer-ter Vehn, Moldovanu and Zame (2006).

¹²And $H^v \subset \times_i H_i^v$ the set of consistent profiles of histories.

possible specification is that H_i^v includes the individual voting behavior of every player at all previous nodes. A strategy of player i associates to each node and each history an action.¹³ None of our results below depend on the exact specification of H_i^v .

Definition 3. *A strategy profile constitutes an ex-post perfect equilibrium if for every non-terminal node, and following any history, the agents play best responses for every realization of preferences.*¹⁴

The ex-post equilibrium embodies a notion of no-regret: even if all private information is revealed, no voter regrets her equilibrium strategy, given the equilibrium strategies of the other voters. It is particularly robust because it does not depend on the beliefs voters entertain.

For any preference ordering \succ_i over A we denote, by the same symbol, its lexicographic extension over sets of alternatives.¹⁵ This allows us to define sincere voting as:

Definition 4. *A voting strategy is sincere given preference \succ_i if it prescribes at each node $v \in V'$ to vote ℓ if and only if $L(v \oplus \ell) \succ_i L(v \oplus r)$.*

Under sincere voting, each voter votes for the set that contains his most preferred alternative. If this alternative is contained in both sets, he votes for the set containing his second-most preferred alternative, and so on. This definition of sincere voting goes back to Farquharson (1969) (see also Miller (2010) for a recent discussion of this definition).

4 Sophisticated Sincerity

In order to make our arguments as transparent as possible, we first treat the class of partitional voting procedures, where the sets of alternatives associated with two neighbors (i.e., successors of the same node) are disjoint. A well known example is the successive voting procedure, mentioned in the Introduction: alternatives are considered one at a time, so that, for any non-terminal node, one successor node leads to a single alternative while its neighbor leads to all other remaining alternatives that were not yet eliminated. We shall afterwards analyze the somewhat more complex case where the voting tree need not be partitional.

¹³Formally, a strategy for voter i is a mapping $\sigma_i : \bigcup_{v \in V'} H_i^v \times P^{SP} \rightarrow \{\ell, r\}$.

¹⁴To formally define the equilibrium, let $g_{v,h^v}(\sigma, \succ)$ be the alternative that is selected if voters with preference profile \succ use the strategy profile σ in the subgame starting at node v after observed history $h^v \in H^v$. The strategy profile σ is an ex-post perfect equilibrium if, for all $i, v \in V', h^v \in H^v$, and for all $\succ \in \mathbf{P}^{SP}$, $g_{v,h^v}(\sigma, \succ) \succeq_i g_{v,h^v}(\sigma'_i, \sigma_{-i}, \succ)$ holds for all strategies σ'_i .

¹⁵That is, given two subsets B, C of A , we write $B \succ_i C$ if i 's most preferred alternative in B is preferred by him to his most preferred alternative in C or, if the most preferred alternative is the same in both sets, his second-most preferred alternative in B is preferred to the second-most preferred alternative in C and so on.

4.1 Partitional Voting Procedures

We call a voting procedure (\mathbf{V}, L, τ) *partitional* if $L(u) \cap L(v) = \emptyset$ for all neighbors $u, v \in V$.

Theorem 1. *Consider any partitional voting procedure satisfying CONV and MON. Then the profile of strategies where each player votes sincerely constitutes an ex-post perfect equilibrium. Moreover, if simple majorities are required at each stage then the Condorcet winner will be selected in this equilibrium.*

Proof

Assume, by contradiction, that sincere voting is not an equilibrium. Then there exist a preference profile \succ , a voter i , and a node v such that sincere voting prescribes a left vote for i at v , but this is not a best response to the sincere strategies used by the others (the arguments are analogous if sincere voting prescribes a right vote). Because sincere voting prescribes a left vote for i , his most preferred remaining alternative is contained in the left branch. Because preferences are single-peaked and the voting procedure is partitional and satisfies CONV, i prefers the largest alternative in the left branch, denoted by k , to any alternative in the right branch: $k \succ_i l$ for all $l \in L(v \oplus r)$. We now show that, if i always votes for the branch containing alternative k , then k will ultimately be selected. This contradicts our initial hypothesis that a left vote at v was not a best response.

Since, by assumption, a left vote is not a best response for voter i at v , it must be that i is pivotal at v . Hence, exactly $\tau^\ell(v) - 1$ other voters vote left at v , and therefore exactly $\tau^\ell(v) - 1$ of the other voters have a peak at k or to the left.

Since the voting procedure satisfies CONV, for every node v' following $v \oplus \ell$ it holds that, for some alternative $l < k$,

$$\max_{\text{SP}} L(v' \oplus \ell) = l \text{ and } \min_{\text{SP}} L(v' \oplus r) = l + 1.$$

Therefore, only agents with a peak (weakly) to the left of l will vote for the left branch at v' and hence at most $\tau^\ell(v) - 1$ of the other voters will vote left at v' . Since the voting procedure satisfies MON, $\tau^\ell(v) \leq \tau^\ell(v')$. Therefore, if i votes right at v' , the right branch is chosen. Since this holds for all nodes v' following $v \oplus \ell$, and since the right branch always contains alternative k (which is the largest remaining alternative), this alternative is selected at the final node. \square

Remark 1. *As is common in voting games, there are additional trivial equilibria where, for example, voters coordinate to always vote left, no matter what their preferences are. As long as no decision requires unanimity, no voter is then pivotal, and these strategies do form an ex-post perfect equilibrium. Call a strategy for voter i responsive if, for every node $v \in V$ and every history leading to this node, there is a preference realization such that i votes left at v and another preference realization such that i votes right at v . Then it follows that, for any partitional voting procedure satisfying CONV and MON, sincere voting is the unique ex-post perfect equilibrium in responsive strategies: given that all*

other voters use responsive strategies, there is a preference profile such that agent i is pivotal at a given node; by voting insincerely a strictly less preferred alternative will finally be elected. Hence, sincere voting is the unique best response to responsive equilibrium strategies. Thus, in our opinion, sincere voting constitutes the most compelling equilibrium for any partitional voting procedure satisfying CONV and MON.

4.2 Non-Partitional Voting Procedures

There are important voting procedures that are non-partitional, the most common examples being the amendment procedures. At each stage of an amendment procedure, a vote is taken among two alternatives, with the winner advancing to the next stage. Thus, for any two neighbors, the intersection of the corresponding sets of alternatives is generally non-empty, and consists of all alternatives that were not yet eliminated, and that are not directly considered at the respective stage. An amendment procedure satisfies CONV if, at each stage, the two most extreme alternatives are voted on.

We now extend our results to a much broader class of voting procedures, which includes amendment procedures and partitional procedures as special cases. Specifically, we show that, for any voting procedure satisfying MON and CONV, sincere voting is an ex-post perfect equilibrium.¹⁶

Theorem 2. *Consider a voting procedure satisfying CONV and MON. Then sincere voting is a strong ex-post perfect equilibrium. . Moreover, if simple majorities are required at each stage, then the Condorcet winner will be selected in this equilibrium.*

The proof of Theorem 2 is found in the Appendix. Note that this theorem also implies that there is no coalition that can achieve a better outcome by deviating from sincere voting.

Remark 2. *By the nature of non-partitional procedures, where the same alternative may be obtained by following both branches of a given node, sincere voting need not be the unique equilibrium in responsive strategies. But, if the voting procedure satisfies MON and CONV, all responsive equilibria are outcome-equivalent.*

5 Dynamic Implementation

In this section we characterize the social choice functions (seen as mappings that associate to each profile of single-peaked preferences a social alternative) that can be implemented by the sincere, ex-post perfect equilibria of sequential, binary voting procedures. To do so, let us first focus on the social choice functions that are anonymous, unanimous and

¹⁶Ordeshook and Schwartz (1987) argue that the theoretical concept of amendment procedures as defined above is too narrow, and that many realistic agendas take a different form. Our characterization includes also *asymmetric* and *non-uniform* amendment agendas in Ordeshook and Schwartz's terminology. Note, however, that among amendment procedures, only their *continuous* agendas can satisfy CONV.

dominant-strategy implementable (DIC) on the domain of single-peaked preferences¹⁷. Unanimity is a weak form of Pareto-Optimality, and requires that an alternative is selected as the social choice when it is preferred by all agents to all other alternatives. Anonymity requires invariance of the social choice with respect to permutations of the voter's names. Note that we do not require neutrality since sequential, binary voting schemes and their agenda are per-se non-neutral.

The set of anonymous, unanimous and DIC social choice functions has been elegantly characterized by Moulin (1980) and further refined by Border and Jordan (1987) and Barbera, Gul and Stacchetti (1993). Moulin showed that any such mechanism can be described as choosing the median peak among n reported real peaks and $n - 1$ *phantom* peaks (these are constant for each mechanism, and do not depend on the reports of the real voters).¹⁸ In their study about optimal, utilitarian voting schemes Gershkov et al. (2015) showed that the successive voting procedure with a particular convex agenda can be used to replicate, in ex-post perfect equilibrium, every anonymous, unanimous and DIC mechanism. This is done by varying the adoption threshold associated with each alternative in the successive procedure.¹⁹ Our result below extends their observation to any convex, not necessarily partitional, voting procedure. It offers both a realistic implementation - via sequential, binary voting procedures - of a rich class of non-dictatorial social choice functions, and a much more transparent interpretation of the "phantom voters" that play the main role in Moulin's analysis. It also shows that the use of binary voting procedures that satisfy convexity and monotonicity is not restrictive.

Theorem 3. *Consider any unanimous and anonymous social choice function $f : \mathbf{P}^{SP} \rightarrow A$ that is implementable in ex-post equilibrium, and an arbitrary voting tree (\mathbf{V}, L) satisfying CONV. Then there exists a system of thresholds τ such that the voting procedure (\mathbf{V}, L, τ) satisfies MON and implements f in ex-post perfect equilibrium.*

This result implies that, as long as we are interested in robust implementation in ex-post equilibrium, we can, without loss, pick any binary voting tree satisfying CONV. In particular, there is no need to consider more complex voting procedures.

Proof. It follows from Moulin (1980) and Border and Jordan (1987) that f is a generalized median voting rule, with $\rho_k \geq 0$ phantom voters with peak on alternative k , such that $\sum_{k \in A} \rho_k = n - 1$. For each node $v \in V'$, set

$$\tau^\ell(v) = n - \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m \quad \text{and} \quad \tau^r(v) = \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m + 1.$$

By construction, $\tau^\ell(v) + \tau^r(v) = n + 1$ for all $v \in V'$, and this system satisfies the

¹⁷Given our private value environment, DIC is equivalent to implementability in ex-post equilibrium.

¹⁸Moulin assumed that the social choice functions depend only on the reported peaks, while Border and Jordan, among others, showed that the peak-only assumption is not needed.

¹⁹Note that in that procedure each alternative appears at a unique terminal node.

consistency condition (i) in the definition of MON. We now show that it also satisfies the monotonicity condition (ii). Suppose that node v follows $u \oplus \ell$. Then $L(v \oplus \ell) \subset L(u \oplus \ell)$ and hence $\tau^\ell(v) \geq \tau^\ell(u)$. Similarly, if v follows $u \oplus r$, then $L(v \oplus r) \subset L(u \oplus r)$. Hence, $\tau^\ell(v) \leq \tau^\ell(u)$, which is equivalent to $\tau^r(v) \geq \tau^r(u)$. Therefore, the system of thresholds τ satisfies MON, and Theorem 2 implies that sincere voting is an ex-post perfect equilibrium.

Fix now a preference profile $>$ in \mathbf{P}^{SP} and suppose that $f(>) = k$. We show that k is selected in the sincere voting equilibrium. Because $f(>) = k$, there are at least $n - \sum_{m=1}^k \rho_m$ agents with a peak weakly to the left of k and at most $n - \sum_{m=1}^{k-1} \rho_m - 1$ voters with a peak strictly to the left of k . By construction

$$\tau^\ell(v) = n - \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m \leq n - \sum_{m=1}^k \rho_m$$

for any node $v \in V'$ such that $k \in L(v \oplus \ell)$. Since there are at least $n - \sum_{m=1}^k \rho_m$ agents voting for left, the left branch is chosen in this case. If $k \notin L(v \oplus \ell)$, then, by construction,

$$\tau^r(v) = \sum_{m=k}^{\max_{SP} L(v \oplus \ell)} \rho_m + 1 \leq \sum_{m=1}^{k-1} \rho_m + 1.$$

By the above argument, there are at least $\sum_{m=1}^{k-1} \rho_m + 1$ voters with a peak weakly to the right of k , and therefore the right branch is chosen. Hence, at each node, a branch that contains alternative k is chosen, and consequently the final choice must be alternative k . \square

6 Case studies

Is strategic behavior common in real-life voting situations? Is the outcome of strategic voting nevertheless consistent with the Condorcet criterion? How are convexity and monotonicity implemented, and how do they perform in real life situations? In this Section we discuss several case studies in light of our findings.

Empirical research in Political Science has produced mixed results about the frequency of strategic voting. While individual case studies clearly identified strategic manipulations, systematic studies found strategic voting to be rare (see, for example, Groseclose and Milyo 2010, Wilkerson 1999, Poole and Rosenthal 1997, Volden 1998). Several explanations have been advanced for the relative rarity of clear, unambiguous examples where strategic behavior has been postulated: (1) Legislators may be bound either by party discipline or by the need to fulfill constituents' expectations, and hence cannot act opportunistically at each instance (Austen-Smith 1992, Denzau, Riker and Shepsle 1985); (2) Real life agendas are endogenous, and sincere voting is an equilibrium in the resulting game (of complete information) where the agenda is chosen in a first step (Austen-Smith 1987). These, and other existing explanations, require additional features beyond those captured by the

simple, isolated model of voting via a given, sequential binary procedure. In contrast, our above results offer a simple explanation for the relative rarity of observed strategic voting based on robustness to the potential presence of private information: sincere voting constitutes the most compelling equilibrium for all situations that can be described by convex and monotone voting procedures, and hence it cannot be empirically distinguished from sophisticated voting (with which it simply coincides) in those cases. Conversely, our results also delineate the conditions under which strategic behavior may be advantageous for some parties: we illustrate cases where a probable Condorcet winner has not been chosen under sophisticated voting, a phenomenon that cannot be satisfactorily explained under complete information.

6.1 Stem Cell Research in Germany

In 2008 the German Parliament voted on several proposals to change the restrictive law regulating stem cell research. That law allowed the import of 40 specific stem cell lines developed abroad before 2002, while forbidding research using any newer lines.²⁰ According to a common procedure adopted in other cases involving ethical questions (such as laws on abortion, living-wills, assisted suicide, etc.), legislators were formally freed from any duty to vote along party lines, and thus were able to vote purely according to their "conscience". All proposed amendments were formulated by groups of legislators that crossed party lines (including those of the two large centrist ones), leading to considerable uncertainty about the outcome.²¹

There were 4 alternatives which we name and order here by their main characteristic, the number of allowed stem cell lines:

1. Proposal **0**, a ban on stem cell research.
2. Proposal **40**, retain the status quo.
3. Proposal **500**, a relative liberalization, implemented by moving the development deadline to 2007.
4. Proposal ∞ , a complete liberalization, freeing stem cell research from further regulation.

Due to the unambiguous, uni-dimensional issue, we assume single-peaked preferences according to the natural order. The German Parliament uses a successive voting procedure (see Table 1), and simple majority is required for each decision. It is important to note that the Parliament follows the "extremes first" prerogative; in this case there was consensus that the order of votes should be $\infty, 0, 500, 40$ (see Figure 6). Note how the agenda

²⁰Even researchers working abroad on newer stem cell lines were, theoretically, guilty of a felony. This particular issue has been reversed in a vote following the one discussed here.

²¹The 5 parties represented in the Parliament were CDU/CSU, FDP, SPD, Greens, and Leftists.

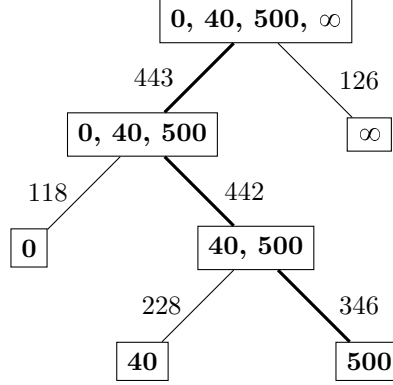


Figure 6: Voting tree and results of the votes on the regulation of stem cell research

oscillates from the extreme on one side to the extreme on the other side! As already mentioned in the Introduction, such an agenda is **convex**, and thus our theory predicts the election of the Condorcet winner via an equilibrium where all legislators vote sincerely.

In the actual vote, there were 3 openly conducted roll-calls. First, alternatives ∞ and $\mathbf{0}$ were defeated, and then alternative $\mathbf{500}$ won over alternative $\mathbf{40}$ with 346 to 228 votes and was formally elected. The analysis below is based on detailed results, available in disaggregated form at the level of the individual legislator.

For any convex voting procedure, we can compute the complete individual expected pattern of vote using only assumptions about the respective location of the individual peak (i.e. without knowing the precise ranking below the peak). Therefore, out of the $2^3 = 8$ possible voting profiles only 4 are consistent with sincere (and hence postulated equilibrium) behavior. Conversely, because in the successive procedure each alternative was actually put up for vote, we can estimate how many members of parliament had a peak on each alternative. Table 2 below summarizes the results.²²

	Peak on $\mathbf{0}$	Peak on $\mathbf{40}$	Peak on $\mathbf{500}$	Peak on ∞	Inconsistent
Equilibrium profile	ℓ, ℓ, ℓ	ℓ, r, ℓ	ℓ, r, r	r, r, r	$r, r, \ell; \ell, r, r$; etc.
Occurences of profile	115	97	211	116	7

Table 2: Predicted voting profiles and actual occurrences

Note how the vast majority of individual voting profiles, 539 out of 546, agrees with one of the 4 expected sincere voting patterns. Additional evidence for the hypothesis of sincere voting is obtained by juxtaposing: 1) the observed behavior (ℓ, ℓ, ℓ) of 33 out of 48 Green legislators with their party's traditional conservative view on this issue; 2) the observed behavior (r, r, r) of 47 out of 54 Liberal legislators with their party's liberal view.

²²We do not show the votes of 7 legislators who did not take part in all votes and of 28 legislators who abstained from a vote at least once. While not consistent with strict preferences, the behavior of all but one of these legislators was consistent with sincere voting according to weak single-peaked preferences.

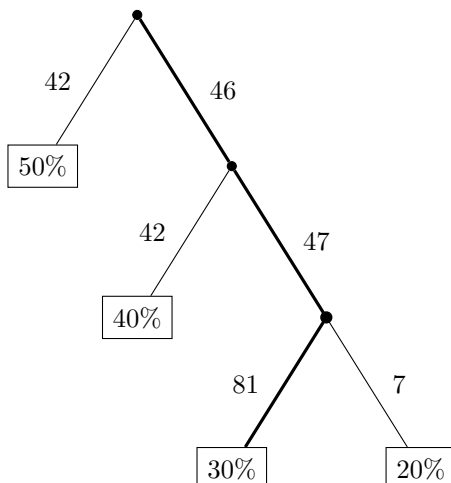


Figure 7: Successive voting procedure satisfying MON

6.2 Roll Calls in the U.S. Congress (Ladha, 1994)

This study is unusual because it looks at 200 votes selected from more than 8000 roll calls over a period of 8 years (Ladha 1994). The criterion for selection - the existence of several comparable votes on the same issue - was driven by the author's aim of carefully testing several direct empirical implications of a model proposed by Poole and Rosenthal (1997) in their monumental book on the U.S. Congress. It turns out that the series of votes that are presented, that cover quite diverse areas of legislation, describe situations where the order of votes on amendments closely follows their position on the Liberal-Conservative ideological spectrum. Let us illustrate these general findings with one of Ladha's examples, a series of 1977 Senate votes on the level of tip credit that enable hotels and restaurants to pay less than the minimum wage. The Human Resources Committee has proposed a 20% credit, i.e., allowing pay 20% under the minimum wage. Three amendments were considered, at levels of 50%, 40% and 30%, respectively, in this chronological order. Hence, each observed vote was indeed amongst the two most extreme remaining alternatives. The actual order of votes is depicted, together with the voting results, in Figure 7.

Ladha's empirical hypothesis, **assuming** sincere voting, was that the number of Yes votes (of legislators who prefer higher levels of credit) increases and the number of No votes (of legislators who prefer lower levels) decreases, as voting advances from the high to the low proposal. This is what the actual results suggest for this case, and basically the same picture emerges in all 200 examined cases: smooth increases of the number of Yes votes paralleled by smooth decreases in the number of No votes (or vice versa) as the amendment becomes more and more liberal or more and more conservative²³. Thus, there are never instances of large swings characteristic of strategic voting, as illustrated above. Since the analyzed roll calls were conducted according to an agenda where the order of vote followed the ideological one, we would indeed expect that incentives to manipulate

²³One senator voted only for the 40% level, which explains the small increase, from 46 to 47.

are significantly reduced. Therefore, the results are not at all surprising in light of our theoretical results: although we do not have precise information about the hypothetical complete agenda in all these cases, Ladha's many examples suggest indeed that trees with a convex agenda lead to sincere voting.

6.3 Pension and Women's Status in Switzerland (Senti, 1998)

As Table 1 shows, Swiss parliaments use an amendment procedure that does not always put the two most extreme alternatives up for vote. Therefore, the procedure will sometimes not be convex and we might expect strategic manipulations. The following example identifies a decision in which strategic manipulations led to the Condorcet winner being rejected.

The decision is about a pension reform pertaining to the status of married women, debated in the Swiss Chamber of Cantonal Representatives.²⁴ The alternatives were

(1) keep the status quo where women's benefits are mainly defined by their husbands' contribution,

(2) a moderate reform,²⁵ and

(3) a radical reform, that would make pension contributions and benefits individual rather than family based.

The likely preferences (see Senti 1998) were

- $1 > 2 > 3$ for a group of 18 legislators associated with the ruling, conservative party,
- $2 > 1 > 3$ or $2 > 3 > 1$ for a group of 21 legislators associated with moderate parties, and
- $3 > 2 > 1$ for a group of 6 legislators associated to left-leaning parties.

The first vote was among alternatives 2 and 3. Although 2 was the likely Condorcet winner, alternative 3, the radical proposal, won 24-19. The only possible explanation is that the conservatives voted strategically for their **worst** alternative in order to eliminate from further consideration the Condorcet winner and likely outcome of the overall vote.²⁶ In the second vote, alternative 1, the status quo preferred by the Conservatives easily won 30-13 against the radical proposal 3.

This is exactly the kind of manipulation we illustrated in our introductory example (see Figure 1) and is typical of "both extremes against the middle" examples of successful strategic voting discussed in the literature. But note that, under complete information such a manipulation should never be successful since a Condorcet winner must be elected irrespective of the agenda! In the present example, the 6 left leaning legislators could

²⁴This is the second Chamber of the Swiss Parliament, similar to the U.S. Senate.

²⁵In fact there were two moderate proposals, but this does not change the conclusions. We use the simplified version, following Senti.

²⁶This is analogous to the "killer amendments" discussed in the literature about the U.S. Congress.

have shifted their vote to alternative 2, the moderate reform, causing it to advance to the next stage where it would have most probably won against alternative 1. Thus, the above outcome can be explained by a complete information model only by attributing sophistication to some legislators and naivety to others, a not very satisfactory hypothesis. Instead, under incomplete information and the non-convex agenda, sincere voting is not an equilibrium, and hence various beliefs lead to maneuvers that succeed in some circumstances while failing in others, as illustrated in Example 1 above.

6.4 Gun Control in Sweden (Bjurulf and Niemi, 1978)

As shown in Table 1, the Swedish parliament also uses an amendment procedure with a procedural agenda setting that does not always put the most extreme alternatives up for vote. The Swedish parliament consists of two chambers, which potentially gives rise to additional possibilities for manipulating the outcome. The following case study, taken from Bjurulf and Niemi (1978), illustrates how a party votes strategically for its least preferred alternative, expecting its most preferred alternative finally to be elected. However, in the final vote its least preferred alternative got elected, which illustrates the risks associated with strategic voting under incomplete information (see also Example 1).

The Swedish parliament had to decide among three alternatives concerning the government's financial support for the national riflemen's association. These alternatives were

- (1) support the riflemen's association with 500.000 crowns,
- (2) support the riflemen's association with 470.000 crowns, and
- (3) do not support the association.

The main difference between alternatives 1 and 2, and the reason for the political controversy, were the extra 30.000 crowns, targeted at young riflemen aged 12-15. The Social Democrats had clear preferences $3 > 2 > 1$, while the Conservatives had opposed preferences $1 > 2 > 3$. The Farmers had preference $2 > 1 > 3$, while the preferences of the Liberals were uncertain, but most probably did not have a peak on alternative 1.

In each chamber, the procedure was an amendment agenda with 1 and 3 competing against each other at the first stage, with the winner competing against 2 at the second stage. In this case, the agenda used in each chamber **is** convex. However, this case is particularly interesting because of the entire structure of the legislative process: each Chamber decides independently, and if the elected alternatives are different, a joint vote is taken among the choices of the two Chambers. This voting architecture may lead to a violation of convexity - see Figure 8 for an illustration.

In one Chamber the result was unspectacular, with alternative 3 winning over alternative 2 with a 81-72 margin. But in the other Chamber, the binary supra-structure induced similar incentives for manipulation as a non-convex agenda: Indeed, the Social Democrats tried to achieve their preferred outcome by letting alternative 1 - their worst outcome - win in the other Chamber, so that the final contest would be between 1 and 3,



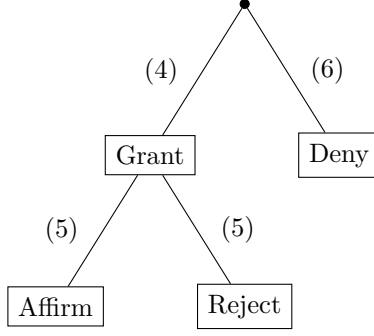


Figure 9: Structure of the decision process at the U.S. Supreme Court

where 1 was expected to loose. They were wrong: in the final vote alternative 1 defeated 3 with 197 to 168 votes, and the manipulation replaced the likely Condorcet winner, alternative 2, with the worst outcome for the manipulators (Bjurulf and Niemi 1978). It should be clear that, under an assumption of complete information, such a failure should never occur.

6.5 Certiorari at the U.S. Supreme Court (Caldeira et al., 1999)

We analyze here the voting procedure used in the Supreme Court in light of our findings. We argue that the procedure cannot be convex: therefore we should observe strategic manipulations.

Most cases decided by the U.S. Supreme Court arise from petitions to review decisions of lower courts.²⁷ The decision process is successive: first, a decision to grant or deny the cert is made, where granting requires at least 4 out of the 9 judges to be in favor (thus less than a simple majority).²⁸ This decision need not be explained to outsiders (the process is indeed rather secretive), and a denial does not constitute per-se an acknowledgment of the earlier decision. If a cert is granted, a decision "on merits", roughly speaking to affirm, or to reverse the opinion of the lower court, follows by simple majority (5 out of 9). A decision on merit is binding and therefore serves as precedent for all consecutive decisions in the lower courts. A decision to deny the cert keeps the status quo and allows more latitude to future decisions by lower courts (see Figure 9 for the structure of the decisions at the Supreme Court).

Caldeira et al. (1999) argue that these alternatives give rise to single-peaked preferences. Consider, for example, a cert to consider a case that has been decided in a perceived liberal way by a lower court (and analogously for a conservative decision). There are three possible outcomes that could be ordered on the liberal-conservative spectrum as follows:

- (1) Affirm the decision;
- (2) Deny the cert and keep the status quo;

²⁷Certiorari (or certs) is the Latin name for requests of more information, used because the Supreme Court requests the relevant documents from the lower court.

²⁸See Godefroy and Perez-Richet (2013) for an analysis of this rule.

(3) Reverse the decision.

Unless the earlier "status quo" decision is obviously wrong on pure judicial terms, and thus denying the cert is presumably the least preferred alternative of all judges, the assumption of single-peaked preferences on the liberal-conservative spectrum seems reasonable.

Note that the least extreme alternative is alternative 2, to deny the cert. However, the voting process is such that this alternative must be voted on first. Only if this alternative is eliminated and a cert is granted, will the Supreme Court obtain the record of a proceeding from a lower court. This leaves the second stage open for a contest among the more "extreme" alternatives 1 and 3. Thus, the procedure used by the Supreme Court cannot be convex: a convex successive procedure must start with the decision on one of the most extreme alternatives, which is simply infeasible here. In particular, the monotonicity of the adoption thresholds is only apparent! In other words, we should expect some strategic manipulation at the decision whether or not to grant the cert (while there is clearly a dominant strategy at the second stage)

Most certs are denied because they are deemed frivolous, but it has been argued, and frequently documented, that some cert decisions are "defensively" denied, for example, by liberal (conservative) judges that are afraid of a reversal (affirmation) of an earlier liberal decision.²⁹ In other words, judges do not vote sincerely at the first stage, but instead to deny the cert on defensive strategic considerations. Such a strategic vote is optimal for beliefs that attach a sufficiently high probability to a subsequent unfavorable outcome, and it is more likely to occur for courts where the number of moderates is high enough, so that pivotality at both stages matters. Indeed, the main source of uncertainty about the preferences of others is the presence of "moderate" judges who do not have strong ideological convictions that could be used to predict their opinions with high probability.

Strategic manipulations, as illustrated above, clearly affect the court's final decisions: Alone in 1982, Caldeira et al. (1999) empirically identify 18 potential cases where a cert was denied on defensive grounds even though a grant was to be expected under sincere voting.

7 Conclusion

Our study was motivated by observing actual voting procedures, and we aimed to offer guidance for the practical design of sequential voting schemes that are widely used by legislatures and committees. We described a broad class of procedures that yield robust and desirable results even in relatively complex strategic situations that involve incomplete information. We also uncovered a strong rationale for content-based agendas (in contrast to agendas formed by procedural rules): if the outcome should not be sensitive to beliefs

²⁹See, for example, the recommendation made by one of judge Marshall's clerks (cited in Caldeira et al. 1999): "In the normal case, this would be a pretty clear grant. Here, though, I would deny. [...] Because every abortion case on which cert is granted creates a new opportunity to overrule *Roe*, I would deny on defensive grounds. Tactical judgments aside, the case is a grant."

about others, nor to the deployment of strategic skills, the agenda needs to be built "from the extremes to the middle" so that more extreme alternatives are both more difficult to adopt, and are put to vote before other, more moderate options.

Our results also illuminate the empirical discussion about the relative frequency of strategic voting in real life situations. They imply that sincere voting is often a robust, ex-post perfect equilibrium, and hence that sincere and sophisticated voting are then observational equivalent. On the other hand, assuming incomplete information, we were also able to explain voting behavior in observed situations where the likely Condorcet winner was not elected (in contrast to the complete information analysis that does not allow this possibility, e.g. Miller 1977).

Appendix

Arguments for Example 1

Sincere voting by all types in the second stage is a weakly dominant strategy. Moreover, sincere voting at the first stage is optimal for voters of types t_2 and t_3 : whenever they are pivotal, there is one other voter voting right at the first stage. Since only voters with a peak on alternative 2 vote right at the first stage, they can be sure to obtain their most preferred alternative by voting left at the second stage given that they are pivotal at the first stage.

It is optimal for voters of type t_1 to vote left if, conditional on being pivotal, they prefer the left branch:

$$\begin{aligned}\mathbb{E}U(\ell|t_1, \text{ pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_1(q_2 + q_3) + q_4q_2] \\ &= \frac{q_1}{q_1 + q_4} + \frac{q_2q_4}{(q_1 + q_4)(q_2 + q_3)} \\ \mathbb{E}U(r|t_1, \text{ pivotal}) &= v\end{aligned}$$

Similarly, it is optimal for type t_4 to vote left if, conditional on being pivotal, they prefer the left branch:

$$\begin{aligned}\mathbb{E}U(\ell|t_4, \text{ pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_4(q_2 + q_3) + q_1q_3] \\ &= \frac{q_4}{q_1 + q_4} + \frac{q_1q_3}{(q_1 + q_4)(q_2 + q_3)} \\ \mathbb{E}U(r|t_4, \text{ pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_4(q_2 + q_3) + q_1(q_2 + q_3)v] \\ &= \frac{q_4}{q_1 + q_4} + \frac{q_1}{q_1 + q_4}v.\end{aligned}$$

It can be easily verified that, given the strategies of the others, if $v \leq \frac{\min\{q_2, q_3\}}{q_2 + q_3}$, then voting left in the first stage is a best response for voters of type t_1 and t_4 . \square

Given a voting procedure satisfying CONV and MON, for any alternative k and given any $u \in V$ such that $\max_{SP} L(u \oplus \ell) = k$ and $\min_{SP} L(u \oplus r) = k + 1$, let $\tau(k) := \tau^\ell(u)$.³⁰

Lemma 1. *Consider an arbitrary voting procedure satisfying CONV and MON. Then the following statements hold:*

- (i) *If $k \in L(v \oplus \ell)$ but $k + 1 \notin L(v \oplus \ell)$, then $\tau^\ell(v) \geq \tau(k)$.*
- (ii) *If $k - 1 \notin L(v \oplus r)$ but $k \in L(v \oplus r)$, then $\tau^\ell(v) \leq \tau(k - 1)$.*
- (iii) *If $k \in L(v \oplus \ell)$ but $k \notin L(v \oplus r)$, then $\tau^\ell(v) \leq \tau(k)$.*
- (iv) *If $k \notin L(v \oplus \ell)$ but $k \in L(v \oplus r)$, then $\tau^\ell(v) \geq \tau(k)$.*

Proof. (i) If $L(v \oplus r) = \{k + 1, \dots\}$, then we are done. If $L(v \oplus r) = \{\dots, k + 1, \dots\}$, then there is a node v' following $v \oplus r$ such that $L(v' \oplus \ell) = \{\dots, k\}$ and $L(v' \oplus r) = \{k + 1, \dots\}$. Hence, $\tau^\ell(v') = \tau(k)$. MON implies $\tau^\ell(v) \geq \tau^\ell(v')$.

(ii) Analogously.

(iii) If $L(v \oplus \ell) = \{\dots, k\}$, then we are done. If $L(v \oplus \ell) = \{\dots, k, \dots\}$, then there is a node v' following $v \oplus \ell$ such that $L(v' \oplus \ell) = \{\dots, k\}$ and $L(v' \oplus r) = \{k + 1, \dots\}$. Hence, $\tau^\ell(v') = \tau(k)$. Because of MON, $\tau^\ell(v) \leq \tau^\ell(v') = \tau(k)$.

(iv) Analogously. □

Proof of Theorem 2.

Fix a preference profile \succ , a coalition $C \subseteq N$, and a node v . We show that there is no profitable deviation for C starting at v . This is done by proving that, for any possible deviation, the members of coalition C are weakly better off by voting sincerely at v and following specific strategies afterwards. Since this holds for any v , this implies that there is no coalition that has a profitable deviation, and hence that sincere voting is a strong ex-post perfect equilibrium.

To obtain a contradiction, suppose that C has a profitable deviation starting at v . If the decision at v was the same as under sincere voting, then the members of coalition C could, without incurring a loss, vote sincerely at v and then follow the actions prescribed by the deviation from the next node on. This holds because all other voters are assumed to vote sincerely, which is a Markovian strategy, i.e., a strategy that does not condition on the past history of play.

Therefore, there exists a subset $\tilde{C} \subseteq C$ such that sincere voting prescribes a left vote at v for $i \in \tilde{C}$, but the deviation prescribes a right vote (or vice-versa, which yields an analogous argument). Moreover, the left branch must be selected if all members of coalition \tilde{C} vote left, but the right branch is selected if all members follow the deviation

³⁰Since the voting procedure satisfies MON, this definition is independent of the exact choice of u .

and vote right. Let k be the alternative chosen if coalition C plays its deviation strategy profile, while all other voters vote sincerely. Hence, k must be contained in the right branch at v . We show first that k is also contained in the left branch.

Claim 1: $k \in L(v \oplus \ell)$

We show that $k \notin L(v \oplus \ell)$ implies that the outcome after the deviation is strictly worse for some members of C , contradicting the assumption that the deviation was profitable. Let l be chosen under sincere voting. By CONV, $l < k$. Because the deviation is profitable for C , $k \succ_i l$ for all $i \in C$. Since preferences are single-peaked, this implies $l + 1 \succ_i l$ for all $i \in C$.

We now show that $\max_{SP} L(v \oplus \ell) = l$. Suppose instead that $l + 1 \in L(v \oplus \ell)$, and consider any node v' following $v \oplus \ell$ such that $l + 1 \notin L(v' \oplus \ell)$. At this node, the right branch is chosen under sincere voting because, by MON, the threshold for a consecutive left decision is weakly higher than at v , and because coalition C votes for the right branch at v' . This implies that $l + 1$ is never rejected, which contradicts the assumption that l is selected. Thus, we can conclude that $l + 1 \notin L(v \oplus \ell)$.

Because l is the largest alternative in the left branch, and because a sincere vote at v for $i \in \tilde{C}$ is to vote left, single-peaked preferences imply that $l \succ_i k$ for all $i \in \tilde{C}$. This implies that the deviation is not profitable for $i \in \tilde{C}$, which contradicts our initial assumption. Thus, we can conclude that $k \in L(v \oplus \ell)$.

Claim 2: If coalition C votes sincerely at v , and afterwards always votes for the branch containing alternative k , then alternative k will be selected.

Because alternative k is contained in both branches at v , the right branch must contain a strictly larger alternative. Condition CONV implies then that $k + 1 \in L(v \oplus r)$. Since alternative k is selected if the right branch is chosen at v and since coalition C plays its profitable deviation, a node v' following $v \oplus r$ is reached such that the left branch is chosen at v' and $k + 1 \notin L(v' \oplus \ell)$. By Lemma 1 (i), $\tau^\ell(v') \geq \tau(k)$. Since the left branch is chosen at node v' , there are at least $\tau^\ell(k) - |C|$ voters in $N \setminus C$ having a peak weakly to the left of alternative k .

Since alternative k is selected if the right branch is chosen at v and since coalition C plays its profitable deviation, a node v'' following v is reached such that the right branch is chosen and $k - 1 \notin L(v'' \oplus r)$. By Lemma 1 (ii), $\tau^\ell(v'') \leq \tau(k - 1)$. Since the right branch is chosen, at most $\tau(k - 1) - 1$ of the voters in $N \setminus C$ have a peak weakly lower than alternative $k - 1$.

We show now that, if coalition C always votes for a division containing alternative k , then k is chosen following the left branch at v as well. Consider any node v' following $v \oplus \ell$ such that k is contained in the left branch, but not in the right branch: that is, $k \in L(v' \oplus \ell)$ and $k \notin L(v' \oplus r)$. By Lemma 1 (iii), $\tau^\ell(v') \leq \tau(k)$. Since there are at least $\tau^\ell(k) - |C|$ voters in $N \setminus C$ having a peak weakly lower than alternative k , the left division is chosen if coalition C votes left.

Analogous arguments imply that the right branch is chosen whenever alternative k is only in the right branch and when C votes for the right branch. As a consequence, the

branch containing alternative k is chosen at each node, and k is finally selected even if the left branch is chosen at v . To conclude, we have shown that, for any deviation of coalition C at v , the same outcome can be obtained by voting sincerely at v and following specific strategies thereafter. Hence, there is no profitable deviation starting at v , contradicting our initial hypothesis. \square

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