Dynamic Adverse Selection with a Patient Seller

Juan Beccuti*   Marc Möller†

Abstract

This paper considers dynamic bilateral trade with short-term commitment. We show that, when the seller is more patient than the buyer, there exist systematic differences between the optimal selling and renting mechanisms. While the former consists of simple price-posting, the latter induces the buyer to choose between a secure- and a random-delivery contract. Allowing for mechanisms more general than price-posting reduces the seller's cost of learning the buyer's valuation in the renting case. Renting leads to more learning than selling but only when general mechanisms are available. Our results contrast with the common view that the restriction to price-posting is innocuous and that informational asymmetries are more persistent under renting than under selling.

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*Department of Economics, University of Bern. Email: juan.beccuti@vwi.unibe.ch

†Department of Economics, University of Bern. Email: marc.moeller@vwi.unibe.ch

1 Introduction

In one of the most fundamental economic transactions, a monopolistic seller offers an indivisible durable product to a buyer whose valuation constitutes his private information. An important insight of the literature on dynamic adverse selection holds that the seller's inability to commit to the future terms of trade represents an obstacle for learning. Fudenberg and Tirole (1983) and Sobel and Takahashi (1983) show that, without commitment, the seller will offer a decreasing sequence of prices, inducing him to learn the buyer's type only *gradually*. When the product is rented rather than sold, informational asymmetries turn out to be even more persistent. Hart and Tirole (1988) argue that, due to the so-called ratchet effect, the seller cannot do better than by offering a rental price so low as to prevent himself from learning *anything* about the buyer's type for almost all of the time horizon.[1]

More recent results seem to suggest that allowing for more sophisticated mechanisms than simple price posting does not alter these conclusions. Indeed, using a selling model with a continuum of types, Skreta (2006) finds that price-posting constitutes the seller's optimal selling mechanism. In this paper, we argue that in the renting case, learning can be improved by use of a mechanism whose outcome is not implementable via simple price-posting. In particular, we show that if (and only if) general mechanisms are available, the seller's optimal renting mechanism induces *more* learning than the seller's optimal selling mechanism. Our results come at a surprise as they contrast with the view, suggested by the existing literature, that the restriction to price-posting is innocuous and that informational asymmetries are more persistent under renting than under selling.

Our setting resembles a two-period version of Hart and Tirole's (1988) seminal model of dynamic bilateral trade with short-term commitment. In each period, the buyer has a

[1] A similar conclusion is obtained by the dynamic incentive models of Laffont and Tirole (1987,1988) which bear a certain resemblance to the renting model of Hart and Tirole (1988). They show that full separation is not feasible with a continuum of types or might be dominated by pooling when there are only two types.

unit demand for the seller's product. The buyer's per-period valuation can be either high or low and remains constant across periods. Our analysis encompasses both the renting and the selling case. While in the former, the seller offers his product to the buyer in every period, in the latter the interaction ends once a delivery has taken place. We differ from Hart and Tirole (1988) in that we choose a general mechanism design approach.[2] At the start of each period, the seller commits to a mechanism which, due to his lack of long-term commitment, must be optimal given his (potentially updated) belief about the buyer's type. A mechanism specifies a probability with which the product is delivered to the buyer and a transfer from the buyer to the seller, both conditional on the buyer's message. Our analysis focuses on the (interesting) case where the seller is more patient than the buyer.[3]

Applying the revelation principle of Bester and Strausz (2001), we find that with a patient seller there exists a range of (moderate) prior beliefs for which the seller induces the buyer to separate (in the first period) although pooling would be optimal in a static context. The reason for this result is straight forward and the same as in the price-posting models of Fudenberg and Tirole (1983) and Sobel and Takahashi (1983). When the seller is more patient, information about the buyer's type is more valuable to the seller than to the buyer, giving the seller an incentive to separate types even at the cost of a sub-optimal (first-period) allocation.

Perhaps surprisingly, heterogeneous discounting generates systematic differences between the optimal (first-period) renting and selling mechanisms that are absent when

[2] Their model allows for general mechanisms in the analysis of long-term contracting with renegotiation, but in their treatment of the non-commitment case they "restrict attention to deterministic offers" (Hart and Tirole (1988), p.512).

[3] Although a common assumption in bargaining models (e.g. Fudenberg and Tirole (1983), Sobel and Takahashi (1983)), the literature on dynamic mechanism design has abstracted from heterogeneous discounting. Heterogeneous discounting renders dynamic contracting problems ill-defined when parties are able to commit to inter-temporal transfers (Krähmer and Strausz, 2015). In order to facilitate comparisons with the commitment case (as in Hart and Tirole, 1988), the literature has maintained the assumption of homogeneous discounting even when, as in this paper, parties are unable to commit to future payments.

discount factors are the same. In particular, we find that the optimal renting mechanism consists of a menu containing a secure-delivery and a random-delivery contract whereas the optimal selling mechanism can be implemented through simple price-posting. To understand this result, an important insight is that in the renting case, the presence of the ratchet effect requires the low type's trade to fall short of the high type's trade by a strictly positive amount for separation to be feasible.[4] Hence, when the seller chooses to separate, he will set the low type's likelihood of trade equal to its upper bound. By doing so, the seller minimizes his loss from implementing a sub-optimal first-period trade, resulting in an allocation that is unattainable through simple price-posting. Random-delivery contracts are useful in the renting case because they allow the seller to elicit the buyer's type without excluding low valuation buyers from trade all together.

We conclude our analysis with a comparison between renting and selling with respect to the persistence of informational asymmetries. We find that renting induces the (immediate) separation of types for a wider range of parameters. Notably, this conclusion hinges on the availability of random-delivery contracts. When the seller is restricted to simple price-posting, the result is reversed, i.e. selling leads to more separation than renting. This result is driven by the fact that, under renting, learning is not only more beneficial but also more costly than under selling. When the seller is restricted to price-posting, the cost-disadvantage turns out to be dominant. Allowing for general mechanisms lowers the costs of learning in the renting case, making learning become more desirable than under selling. This finding highlights the importance of allowing for general mechanisms in models of dynamic bilateral trade with short-term commitment.

The plan of the paper is as follows. Following a discussion of the related literature and the description of the model (Section 2) we provide a motivating example (Section 3) that serves to highlight our subsequent results. After a brief review of the static benchmark

[4] It will become clear that this requirement, reminiscent of the monotonicity constraint in the static context, is necessary to induce *both* types of buyers to reveal their type truthfully (with positive probability).

(Section 4), our general analysis commences in Section 5. There we first derive properties of the seller's optimal mechanism that hold in both the renting and the selling case before completing our characterization of the optimal mechanism for each of the two cases in separation. In order to simplify the exposition, our analysis in Section 5 restricts attention to the case of a *soft* seller whose prior beliefs about the buyer's type are rather low. The case of a *tough* seller is the subject of Section 6. For a tough seller, price-posting turns out to be optimal both in the selling and the renting case. Section 7 concludes.

Related literature

This paper contributes to the literature on dynamic adverse selection, initiated by the regulation and procurement models of Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1990). A common theme in this literature is that, in the absence of long-term commitment, the ratchet effect obstructs information from being revealed. This view has been challenged recently by Gerardi and Maestri (2015) who find immediate and full information revelation in an infinite horizon model where in each period the informed party has the choice between staying in the relationship or leaving forever to claim a strictly positive outside option. Our finding that, conditional on the availability of general mechanisms, renting induces more learning than selling adds to Gerardi and Maestri's argument that the ratchet effect might be less of an obstacle than commonly expected.

More generally, this paper contributes to our understanding of how to sell indivisible products to buyers with unit demands. In settings with multiple buyers, auctions have been shown to be optimal by the pioneering work of Harris and Raviv (1981), Myerson (1981) and Riley and Samuelson (1981). For a multi-product monopolist, Thanassoulis (2004), Manelli and Vincent (2007), and Pavlov (2011) have proven the optimality of product-lotteries, stipulating the purchase of a product with uncertain identity. The random-delivery contracts that emerge in our renting framework share with these mecha-

nisms the buyer's choice between a (more) certain consumption at a high price versus an uncertain consumption at a low price.

Finally, our model allows for the interpretation of random-delivery contracts as "damaged products" in the spirit of Deneckere and McAfee (1996). This connects our model to the literature on monopolistic screening with a quality dimension initiated by Mussa and Rosen (1978). The special feature of our model is that the "costs" of supplying different levels of quality are the same (zero) for all qualities below a certain maximum feasible level. It is important to note that with this cost structure, intermediate levels of quality cannot be part of the seller's optimal menu. In our setting, the randomness of the optimal contract is an intimate consequence of the dynamic nature of contracting.

2 Model

We consider a buyer and a seller who interact for two periods $n \in \{1, 2\}$. The seller offers an indivisible durable product and the buyer has a unitary demand. For simplicity, we abstract from production costs.[5] Our model distinguishes between two cases. In the *renting case*, the product is allocated to the buyer for only one period at a time. In the *selling case*, the product is allocated to the buyer for all remaining periods.

The buyer's (per-period) valuation of the good, θ, is constant across periods. It can take a high value normalized to $\theta_H = 1$ or a low value $\theta_L \in (0, 1)$. For brevity we define $\Delta\theta \equiv 1 - \theta_L$. If in period n, the buyer makes a transfer t_n to the seller then the buyer's (instantaneous) payoff is given by $\theta - t_n$ if the product is allocated to him. If the product is not allocated to the buyer during that period, his payoff is given by $-t_n$. The seller's payoff is equal to his revenue t_n.

Buyer and seller discount future payoffs with discount factors $\delta_B \in (0, 1)$ and $\delta_S \in (0, 1)$, respectively.[6] We focus on the case where the seller is more patient than the buyer

[5] All results remain valid when the seller has a constant unit cost of production sufficiently small for trade to be efficient independently of the buyer's valuation.

[6] As we will see, in the renting case separation cannot be achieved when the second period counts more

by assuming that $\delta_S > \delta_B$. It will become clear below that, in our setting, $\delta_S > \delta_B$ represents the "interesting case", whereas for $\delta_S \leq \delta_B$ the model's predictions remain the same as under homogeneous discounting.

The buyer's valuation θ constitutes his private information and will therefore be denoted as his type. We let $\beta_1 \in (0, 1)$ be the probability with which the buyer has a high type. At the start of the relationship, the seller's prior belief about the buyer's type is given by β_1. After observing the buyer's behavior in period 1 the seller may update his belief about the buyer's type. We denote by $\beta_2 \in [0, 1]$ the seller's (updated) belief about the buyer's type at the start of period 2. In order to simplify the exposition, most of our analysis focuses on the case where, in the language of Fudenberg and Tirole (1983), the seller is *soft*. A soft seller, being characterized by a sufficiently low prior, $\beta_1 < \theta_L$, would sell to both types of buyer at a low price if the setting was static rather than dynamic. The analysis of the case of a *tough* seller, $\beta_1 \geq \theta_L$, is postponed until Section 6.

While earlier articles on dynamic adverse selection have analyzed settings with long-term commitment, the focus of the literature has shifted to the (more realistic but also more complicated) case in which no such commitment is available. We follow this tendency by assuming that in period 1 the seller can commit to the "terms of trade" for period 1 but cannot commit to the "terms of trade" for period 2. Using Perfect Bayesian Equilibium as our solution concept, this assumption implies that the seller's offer in period 2 must be (sequentially) optimal given the trading history in period 1.

Besides our assumption of short-term commitment, we put no restrictions on the way in which the seller and the buyer interact. In particular, we do not restrict attention to the possibility that the seller may simply post a price for his product. Instead, we use a mechanism design approach to determine the seller's revenue maximizing strategy. Following Bester and Strausz (2001), we can assume without loss of generality, that in each

than the first. We assume discounting in order to study the optimality of separation in a setting where it is feasible.

period n the seller offers a *direct mechanism* in which the buyer is asked to report his type and conditional on his message $m_n \in \{l, h\}$, the mechanism specifies an *outcome* consisting of a transfer $t_n(m_n) \in \Re$ from the buyer to the seller and a likelihood $x_n(m_n) \in [0,1]$ with which the product is allocated to the buyer. A pair of probabilities $(x_n(l), x_n(h))$ will be denoted as an *allocation*. We say that a mechanism's outcome can be implemented by *price-posting* when $x_n(h), x_n(l) \in \{0,1\}$ and there exists a price p_n such that $t_n(m_n) = p_n x_n(m_n)$.

3 Motivating example

In this section we motivate our subsequent analysis by way of an example. The example focuses on the renting case in order to demonstrate the usefulness of random-delivery contracts. It assumes that the buyer's valuation for the seller's product is $\theta_H = 1$ with probability $\beta_1 = 30\%$, or $\theta_L = 1/2$ with probability $1 - \beta_1 = 70\%$. The seller's discount factor is assumed to be given by $\delta_S = \frac{3}{4}$, whereas the buyer's discount factor is $\delta_B = \frac{1}{4}$.

Suppose first that the seller's strategy consists of posting a price in each period. If he posts a low price $p_1 = 1/2$ in period 1, it will be accepted by both types. In this case, the seller learns nothing from his observation of the buyer's first period behavior and given that the seller is soft he optimally chooses $p_2 = 1/2$ again in period 2. Following this strategy, the seller's payoff is $1/2 + \frac{3}{4} \cdot 1/2 = 7/8$.

Alternatively, the seller may post a high price in period 1 that is accepted only by the high type. As this allows the seller to learn the buyer's type, the seller will set the period 2 price equal to the buyer's true valuation. In order to make the high type accept in period 1 the seller has to compensate him for facing a high price $p_2 = 1$ rather than a low price $p_2 = 1/2$ in period 2 by charging $p_1 = 1 - \frac{1}{4}(1 - 1/2) = 7/8$. Following this strategy, the seller's payoff is $30\% \cdot (7/8 + \frac{3}{4} \cdot 1) + 70\% \cdot \frac{3}{4} \cdot 1/2 = 6/8 < 7/8$. Hence when the seller's strategy is restricted to simple price-posting, he will refrain from inducing the

buyer to reveal his type.

What if we allow the seller to use more sophisticated renting mechanisms? Consider the possibility that in period 1 the seller offers the buyer the following choice: Make a transfer of $t_1 = 1/2$ and obtain the product for sure or make a smaller transfer of $t_1 = 3/8$ and obtain the product only with probability 75%. This menu makes it optimal for the high and the low type to choose secure- and random-delivery respectively. To see that the high type buyer has no incentive to pool with the low type note that his payoff from random-delivery $75\% \cdot 1 - 3/8 + \frac{1}{4}(1 - 0.5) = 1/2$ is just the same as his payoff from secure-delivery $1 - 1/2 = 1/2$. The seller's payoff is $30\% \cdot (1/2 + \frac{3}{4} \cdot 1) + 70\% \cdot (3/8 + \frac{3}{4} \cdot 1/2) = 72/80 > 7/8$. Hence, when we allow for general mechanisms, the seller will no longer refrain from inducing the buyer to reveal his type. Random renting contracts allow the seller to induce the separation of types at a lower cost.

4 Static benchmark

We start our analysis with a brief review of the standard adverse selection setting where the seller interacts with the buyer only once. The static framework will serve as a benchmark for the comparison with the dynamic case. Although most readers should be familiar with the analysis, it will help us to understand why price-posting is optimal in the static setting but may not be optimal in a dynamic context. The static analysis also constitutes the first step of the backwards induction necessary to determine the Perfect Bayesian equilibrium in our dynamic setting.

Applying the (standard) revelation principle, a seller with belief $\beta \in [0, 1]$ chooses the

menu $\{(x(l), t(l)), (x(h), t(h))\}$ that solves the following static program:

$$\max_{x(.),t(.)} \beta t(h) + (1-\beta)t(l) \tag{1}$$

$$\text{subject to}$$

$$x(h) - t(h) \geq x(l) - t(l) \tag{2}$$

$$x(l)\theta_L - t(l) \geq x(h)\theta_L - t(h) \tag{3}$$

$$x(h) - t(h) \geq 0 \tag{4}$$

$$x(l)\theta_L - t(l) \geq 0. \tag{5}$$

It is well known that in this program the incentive constraint of the high type (2) and the participation constraint of the low type (5) must be binding whereas the participation constraint of the high type (4) is redundant. Moreover, adding the two incentive constraints they can be substituted by the so called monotonicity constraint (7) which requires that the probabilities of delivery must be increasing in the buyer's reported type. In summary, the seller's program therefore simplifies to

$$\max_{x(.)} \beta[x(h) - x(l)\Delta\theta] + (1-\beta)x(l)\theta_L \tag{6}$$

$$\text{subject to}$$

$$x(h) \geq x(l). \tag{7}$$

Note from this program that, due to the information rent of the high type, $x(l)\Delta\theta$, the seller's payoff is decreasing in the low type's likelihood of trade, $x(l)$, unless the seller's belief β is smaller than the threshold θ_L. As a consequence, it is optimal for the seller to implement either full trade or no trade with the low type. More specifically, the optimal mechanism in the static setting is given by:

$$(x^s(h), t^s(h)) = \begin{cases} (1,1) & \text{if } \beta \geq \theta_L \\ (1,\theta_L) & \text{if } \beta \leq \theta_L \end{cases} \tag{8}$$

$$(x^s(l), t^s(l)) = \begin{cases} (0,0) & \text{if } \beta \geq \theta_L \\ (1,\theta_L) & \text{if } \beta \leq \theta_L. \end{cases} \tag{9}$$

Note for future reference that the optimal mechanism is separating if $\beta \geq \theta_L$ whereas it is pooling for $\beta \leq \theta_L$. In the static setting, the optimal mechanism can be implemented via price-posting by simply setting $p = 1$ for $\beta \geq \theta_L$ or $p = \theta_L$ for $\beta \leq \theta_L$.

5 Optimal mechanism

In the dynamic setting, the seller will update his belief about the buyer's type from the observation of the buyer's past behavior. Since the buyer adjusts his behavior accordingly, the design of the optimal mechanism in period 1 requires the seller to take account of the mechanism's effect on information revelation. This distinguishes the dynamic setting from the static benchmark discussed in the previous section.

Bester and Strausz (2001) have shown that, in a dynamic setting the revelation principle does not restrain the buyer to reveal his true type in the first period but only requires that he does so with a strictly positive probability. We denote by $q_L \in [0, 1)$ and $q_H \in (0, 1]$ the probabilities with which the high type and the low type report the message $m_1 = h$ respectively. In a *separating mechanism* both types of buyer report their type truthfully ($q_L = 0$, $q_H = 1$), allowing the seller to determine with certainty whether the buyer has a high or a low valuation, i.e. $\beta_2 \in \{0, 1\}$. In contrast, in a *pooling mechanism*, $q_L = q_H$, and the seller's updated belief remains equal to his prior, $\beta_2 = \beta_1$. More generally, the seller's updated beliefs conditional on the buyer having reported type h or l are given by

$$\beta_2(h) \equiv \frac{\beta_1 q_H}{Q} \quad \text{and} \quad \beta_2(l) \equiv \frac{\beta_1(1 - q_H)}{1 - Q}, \tag{10}$$

where for brevity we have defined $Q \equiv \beta_1 q_H + (1 - \beta_1) q_L$. Note that since messages can be renamed we can assume without loss of generality that $\beta_2(h) \geq \beta_1 \geq \beta_2(l)$ or equivalently $q_H \geq q_L$.

In period 1 the seller induces the buyer to use the reporting strategy (q_L, q_H) by offering the menu $\{(x_1(l), t_1(l)), (x_1(h), t_1(h))\}$. Letting $V(m_1)$, $U_H(m_1)$, and $U_L(m_1)$ denote the seller's and the buyer's second-period continuation payoffs, the optimal first-

period mechanism must solve the following program:

$$\max_{x_1(.),t_1(.),q_L,q_H} Q[t_1(h) + \delta_S V(h)] + (1-Q)[t_1(l) + \delta_S V(l)] \tag{11}$$

subject to

$$x_1(h) - t_1(h) + \delta_B U_H(h) \geq x_1(l) - t_1(l) + \delta_B U_H(l) \tag{12}$$

$$\theta_L x_1(l) - t_1(l) + \delta_B U_L(l) \geq \theta_L x_1(h) - t_1(h) + \delta_B U_L(h) \tag{13}$$

$$x_1(h) - t_1(h) + \delta_B U_H(h) \geq 0 \tag{14}$$

$$\theta_L x_1(l) - t_1(l) + \delta_B U_L(l) \geq 0. \tag{15}$$

The constraints (12)-(15) are the dynamic analog of the incentive and participation constraints (2)-(5) in the static benchmark. The incentive constraints have to hold with equality whenever the seller wishes to induce the buyer to misreport his type with positive probability. That is, (12) must hold with equality when $q_H < 1$ and (13) must hold with equality when $q_L > 0$. Letting $\Delta U_H \equiv U_H(l) - U_H(h)$ and $\Delta U_L \equiv U_L(l) - U_L(h)$ denote the buyer's (potential) gain in continuation value from reporting to be a low rather than a high type, the following lemma is obtained via the same steps as in the static analysis.

Lemma 1. *In the optimal mechanism, the first period allocation* $(x_1(l), x_1(h))$ *and reporting strategy* (q_L, q_H) *solve the following program:*

$$\max_{x_1(.),q_L,q_H} Q\{x_1(h) - x_1(l)\Delta\theta - \delta_B \Delta U_H + \delta_S V(h)\} \tag{16}$$
$$+(1-Q)[x_1(l)\theta_L + \delta_S V(l)] + \delta_B U_L(l)$$

subject to

$$x_1(h) - x_1(l) \geq \max\left\{0, \frac{\delta_B(\Delta U_H - \Delta U_L)}{\Delta\theta}\right\} \quad \textit{with equality for } q_L > 0. \tag{17}$$

Proof: See the Appendix.

Note two important differences with respect to the corresponding static program. First, the dynamic monotonicity constraint (17) can be more demanding than its static

counterpart (7). This happens when the gain in continuation value from reporting a low valuation is higher for the high type than for the low type. The existence of future information rents drives a wedge between the incentive compatible first period allocations requiring them not only to be monotonically increasing in the reported type but to differ from each other by a strictly positive amount. To understand the origin of this constraint, note from (12) and (13) that the transfer difference $t_1(h) - t_1(l)$ has to be smaller than $x_1(h) - x_1(l) - \delta_B \Delta U_H$ for the high type to prefer $(x_1(h), t_1(h))$ but larger than $[x_1(h) - x_1(l)]\theta_L - \delta_B \Delta U_L$ for the low type to prefer $(x_1(l), t_1(l))$. For $\Delta U_H > \Delta U_L$ this is possible only when the allocations $x_1(h)$ and $x_1(l)$ are sufficiently different.

The second difference with respect to the static program is that the maximization includes the reporting strategy (q_L, q_H) induced by the mechanism. This is because in the dynamic setting the seller is not required to induce truth-telling but may choose to allow the buyer a certain degree of type-misrepresentation. This is the sense in which the seller accounts for information-revelation in the design of the optimal mechanism.

Our next lemma is an important step towards the characterization of the optimal mechanism:

Lemma 2. *Let $\beta_1 < \theta_L$. The optimal mechanism creates no distortion at the top, i.e. $x_1^*(h) = 1$. Moreover, no mechanism can give the seller a higher revenue than what he can achieve by either pooling ($q_L = q_H$) or separating ($q_L = 0, q_H = 1$).*

Proof: See the Appendix.

Lemma 2's first statement is familiar from the static setting and follows from the fact that an increase in $x_1(h)$ raises the seller's objective in (16) while simultaneously relaxing the dynamic monotonicity constraint (17). Trading with the high type is beneficial because it creates surplus while simultaneously improving the seller's ability to elicit the buyer's type.

To understand the intuition for the second statement, note that, in our setting, the

only reason for allowing the buyer to misrepresent his type is to substitute for the seller's lack of commitment. Indeed, if the seller is tough then by offering a mechanism that restricts his learning he can effectively "commit" to offer a high price θ_H in period 2, thereby increasing the high type's willingness to accept his offer $(x_1(h), t_1(h))$ in period 1. A soft seller cannot benefit from such a mechanism because by preventing himself from learning the buyer's type he "commits" to a low price θ_L rather than a high price θ_H in period 2. As a consequence, the seller either pools types or separates them completely by inducing them to tell the truth.

Lemmas 1 and 2 state properties of the optimal mechanism that must hold independently of whether the good is rented or sold. In the remainder of this section we will complete our characterization of the optimal mechanism for these two cases separately before discussing potential differences. In each case, the seller's problem in period 2 (after renting or not selling in period 1) is equivalent to the static benchmark analyzed in Section 4 with the seller's belief given by his posterior $\beta_2(m_1)$. Using the results of the static benchmark, the continuation values $V(m_1)$, $U_H(m_1)$, and $U_L(m_1)$ are therefore readily determined, and with the help of Lemmas 1 and 2 the characterization of the optimal mechanism becomes straight forward.

5.1 Renting

In the renting case, the seller offers his product to the buyer in period 2 independently of his trade in period 1. The seller will post a low rental price unless his posterior belief

$\beta_2(m_1)$ exceeds the threshold θ_L.[7] Hence second period continuation values are given by

$$V(m_1) = \max\{\theta_L, \beta_2(m_1)\}. \tag{18}$$

$$U_H(m_1) = \begin{cases} 0 & \text{if } \beta_2(m_1) > \theta_L \\ 0 \text{ or } \Delta\theta & \text{if } \beta_2(m_1) = \theta_L \\ \Delta\theta & \text{if } \beta_2(m_1) < \theta_L \end{cases} \tag{19}$$

$$U_L(m_1) = 0. \tag{20}$$

It follows from Lemma 2 that in order to determine the seller's revenue maximizing mechanism, it is sufficient to compare the maximum payoff he can obtain from pooling with the maximum payoff from separating the buyer's types. It should be clear (and we show formally in the proof of Lemma 2) that no pooling mechanism can give the seller a higher payoff than $(1 + \delta_S)\theta_L$. This is due to the fact that for any pooling mechanism, the second-period continuation payoffs are given by $V = \theta_L$ and $U_H = \Delta\theta$, independently of the buyer's first-period message, making the mechanism design problem resemble its static equivalent. The seller can obtain the payoff $(1 + \delta_S)\theta_L$ by simply posting a price equal to the low type's valuation in both periods.

In order to determine the maximum payoff from separating we set $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = 1$ in the seller's problem in Lemma 1 which leaves us with the following simple program:

$$\max_{x_l \in [0,1-\delta_B]} \beta_1[1 - x_1(l)\Delta\theta] + (1 - \beta_1)x_1(l)\theta_L + \delta_S\theta_L + \beta_1(\delta_S - \delta_B)\Delta\theta. \tag{21}$$

Note from its similarity to the static program (6) and the fact that $\beta_1 < \theta_L$, that the seller should choose the likelihood of trade with the low type, $x_1(l)$, as high as possible. The difference to the static benchmark is that in the dynamic setting, separation of types requires that $x_1(l)$ does not exceed the upper threshold $1 - \delta_B$.

Comparing the seller's payoff from pooling, $(1+\delta_S)\theta_L$, and his optimized payoff from separating, with $x_1^*(l) = 1 - \delta_B$ substituted in (21), we find that separating is optimal if

[7] As for $\beta_2 = \theta_L$ the seller is indifferent between pooling or separating in period 2, in period 1 he can credibly promise to pursue any of these strategies. In order to reduce the buyer's reluctance to reveal his type, the seller will promise separation when $\beta_2(l) = \theta_L$ and pooling when $\beta_2(h) = \theta_L$, making the buyer's second period payoff independent of his message.

and only if

$$\beta_1 \delta_S \Delta\theta > \beta_1 \delta_B \Delta\theta + \{\theta_L - \beta_1[1 - x_1^*(l)\Delta\theta] - (1-\beta_1)x_1^*(l)\theta_L\}. \tag{22}$$

This inequality compares the benefits (LHS) and costs (RHS) from inducing the buyer to reveal his type in period 1. The benefits originate from the fact that without separation, the seller would charge a low price to a high type in period 2 whereas he can charge a high price if he knows the buyer's type to be high. The costs consists of the compensation that the high type must receive to give up his future information rent, plus the period 1 misallocation that is necessary to separate types. The term in parentheses is positive since separation requires the seller to restrict his trade with the low type although his prior favors pooling in the static benchmark. For $\delta_S > \delta_B$, separation can be optimal in the dynamic context, because what the seller gets ($\beta_1 \delta_S \Delta\theta$) is more than what he has to pay ($\beta_1 \delta_B \Delta\theta$). This happens when the seller's prior is sufficiently close to θ_L because in this case the cost of a suboptimal period 1 allocation becomes negligible. From (22) we find that separation is optimal if and only if $\beta_1 > \underline{\beta}_1^R$ with

$$\underline{\beta}_1^R \equiv \frac{\delta_B \theta_L}{\delta_S - (\delta_S - \delta_B)\theta_L} \in (0, \theta_L). \tag{23}$$

Hence we have shown the following:

Proposition 1. *Let $\beta_1 < \theta_L$. In the renting case, the seller's revenue is maximized by choosing a (first-period) mechanism with the following characteristics:*

- *Pooling: For $\beta_1 \in (0, \underline{\beta}_1^R]$ the seller posts a price $p_1 = \theta_L$ which is accepted by both types of buyer.*
- *Separation: For $\beta_1 \in (\underline{\beta}_1^R, \theta_L)$ the seller offers a menu consisting of a random-delivery contract $(x_1^*(l), t_1^*(l)) = (1 - \delta_B, (1 - \delta_B)\theta_L)$ and a secure-delivery contract $(x_1^*(h), t_1^*(h)) = (1, \theta_L)$ inducing the high and the low type to choose secure- and random-delivery respectively.*

Note that for $\beta_1 \in (\underline{\beta}_1^R, \theta_L)$ the seller's optimal mechanism requires a random allocation $x_1^*(l) \in (0,1)$. In particular, the outcome of the mechanism that induces separation optimally cannot be replicated by price-posting. When the seller is restricted to price-posting, he will separate in a smaller interval of priors, by posting $p_1 = 1 - \delta_B \Delta\theta$ if and only if $\beta_1 > \underline{\beta}_1^{RPP}$ where

$$\underline{\beta}_1^{RPP} \equiv \frac{\theta_L}{1 + (\delta_S - \delta_B)\Delta\theta} \in (\underline{\beta}_1^R, \theta_L). \tag{24}$$

It is in this sense that allowing the seller to choose more general mechanisms increases his ability to elicit the buyer's type.

Proposition 1 contrasts with the view, suggested by the existing literature on dynamic bilateral trade with short-term commitment, that the restriction to price-posting is innocuous. If the seller is more patient than the buyer then there exists a non-empty range of priors for which price-posting ceases to be optimal in the renting case. It is intuitive and follows immediately from (23) that this range increases when discounting becomes more heterogeneous.

5.2 Selling

In the case of selling, the buyer and the seller interact in period 2 only if the product has failed to be delivered to the buyer in period 1. Moreover, if the product has been allocated in period 1, the buyer obtains a benefit from its consumption in both periods. Taking account of these differences, the seller's and the buyer's continuation payoffs in period 2 are as follows:

$$V(m_1) = [1 - x_1(m_1)] \max\{\theta_L, \beta_2(m_1)\} \tag{25}$$

$$U_H(m_1) = \begin{cases} x_1(m_1) & \text{if } \beta_2(m_1) > \theta_L \\ x_1(m_1) + [1 - x_1(m_1)]\Delta\theta & \text{if } \beta_2(m_1) \leq \theta_L \end{cases} \tag{26}$$

$$U_L(m_1) = x_1(m_1)\theta_L. \tag{27}$$

The seller's maximum payoff from any pooling mechanism is given by the payoff he can achieve by posting the price $(1+\delta_B)\theta_L$ in period 1 (see proof of Lemma 2). As before, we

determine the seller's maximum payoff from separating by setting $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = 1$ in the seller's problem in Lemma 1 which leaves us with the following program:

$$\max_{x_1(l)\in[0,1]} [1 - x_1(l)][\beta_1(1 + \delta_B\theta_L) + (1 - \beta_1)\delta_S\theta_L] + (1 + \delta_B)x_1(l)\theta_L. \tag{28}$$

Note that in this linear program, the objective is decreasing in $x_1(l)$ unless $\beta_1 < \underline{\beta}_1^S$ where

$$\underline{\beta}_1^S \equiv \frac{1 - (\delta_S - \delta_B)}{1 - (\delta_S - \delta_B)\theta_L}\theta_L \in (0, \theta_L). \tag{29}$$

Moreover, for $x_1(l) = 1$ the seller's payoff from separating becomes equal to the payoff $(1 + \delta_B)\theta_L$ from pooling. Thus we have shown the following:

Proposition 2. *Let $\beta_1 < \theta_L$. In the selling case, the seller's revenue is maximized by choosing a (first-period) mechanism with the following characteristics:*

- *Pooling: For $\beta_1 \in (0, \underline{\beta}_1^S]$ the seller posts a price $p_1 = (1 + \delta_B)\theta_L$ which is accepted by both types of buyer.*
- *Separation: For $\beta_1 \in (\underline{\beta}_1^S, \theta_L)$ the seller posts a price $p_1 = 1 + \delta_B\theta_L$ which is accepted by the high type but rejected by the low type.*

The seller's choice between pooling and separation is based on the following comparison. Separation is optimal if and only if

$$\beta_1\delta_S\Delta\theta - \beta_1(\delta_S - \delta_B)\Delta\theta > \beta_1\delta_B\Delta\theta + \{\theta_L - \beta_1 - (1 - \beta_1)(\delta_S - \delta_B)\theta_L\}. \tag{30}$$

Similar to the renting case, this inequality compares the benefits (LHS) and costs (RHS) of separation. Again, the costs of separation originate from the compensation necessary to induce the high type to reveal his information and from the loss of not serving the low type in period 1 (in parentheses). The benefits of separation are lower than in the renting case, because of the absence of the ratchet effect. Only in the renting case, the seller is able to extract the additional rent $\beta_1(\delta_S - \delta_B)$ due to his higher level of patience. However, in the selling case, separation also comes at a different cost. The costs of separation

are reduced by the fact that the exclusion of the low type from first period trade gives rise to the opportunity of a second period sale. Selling the second-period access to the seller's product to the low valuation buyer in period 2 rather than in period 1 creates the additional payoff $(\delta_S - \delta_B)\theta_L$ when the seller is more patient than the buyer.

Using a selling model with an arbitrary number of periods and a continuum of types, Skreta (2006) shows that price-posting is optimal when the seller and the buyer have equal discount factors. Proposition 2 shows that, in a model with two periods and two types, this insight remains valid when discount factors differ. Heterogeneous discounting only affects the prices charged. In particular, when the seller is more patient than the buyer, then for moderate priors, the seller will separate types by charging a high price in period 1 whereas under homogeneous discounting both types would be served at a low price.

5.3 Comparison

It is well known that, in a model with two periods, renting and selling lead to the same outcome when the seller's and the buyer's discount factors are identical (see Chapter 9 in Bolton and Dewatripont (2005)). In this section we argue that heterogeneous discounting introduces non-trivial differences between these two modes of trade.

The first difference that arises from a simple comparison of Propositions 1 and 2 is that renting may require a more sophisticated, random contract than the simple price-posting characterizing the optimal selling mechanism. The reason for this difference is that in the renting case the existence of future information rents drives a wedge between the incentive compatible first period trades, whereas in the selling case the ratchet effect has no bite. This difference matters under heterogeneous but not under homogeneous discounting because a patient seller may prefer separation even when implementing the same trade with both types is optimal from a static viewpoint.

More importantly, Propositions 1 and 2 also allow us to compare renting and sell-

ing with respect to the amount of information that becomes revealed. In particular, a comparison of the intervals of priors for which separation occurs gives us the following:

Corollary 1. *Let $\beta_1 < \theta_L$. Under renting the seller's optimal mechanism induces more separation than under selling, i.e. $\underline{\beta}_1^R < \underline{\beta}_1^S$. The opposite holds when the seller is restricted to price-posting, i.e. $\underline{\beta}_1^{RPP} > \underline{\beta}_1^S$.*

Proof: See the Appendix.

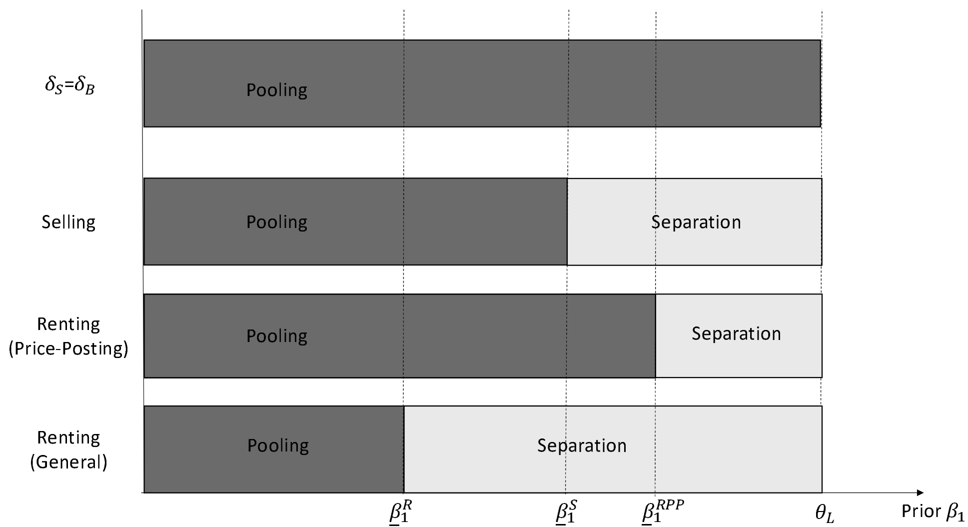

Figure 1: **Learning in Dependence of the Soft Seller's Prior.** The case of homogeneous discounting, $\delta_S = \delta_B$ serves as a benchmark. When the seller is more patient than the buyer, selling and renting differ with respect to learning, and the comparison depends on whether or not general mechanisms are available.

The result is pictured in Figure 1. There we depict the range of prior beliefs for which the optimal mechanism induces the buyer to reveal (completely) his type. As a benchmark, the first bar shows the case of homogeneous discounting. Under homogeneous discounting, renting and selling are equivalent and the optimal mechanism pools the buyer's types for all priors $\beta_1 < \theta_L$. Allowing for heterogeneous discounting shows that, generically, renting and selling are different. Renting leads to more learning than selling but only when general mechanisms are available.

The intuition for Corollary 1 can be obtained from a comparison of the benefits and the costs of separation under the two modes of trade. Consider first the case where the seller is restricted to simple price-posting. Setting $x_1^*(l) = 0$ in (22) and comparing with (30) we see that selling reduces the costs of separation by $(\delta_S - \delta_B)(\theta_L - \beta_1\theta_L)$ while lowering the benefits of separation by only $(\delta_S - \delta_B)(\beta_1 - \beta_1\theta_L)$. In other words, when the seller is restricted to price posting, the elimination of the ratchet effect through selling rather than renting improves the seller's ability to learn the buyer's type. Allowing the seller to use general mechanisms reverses the cost-benefit comparison in the favor of renting, by lowering the costs of separation in the renting case. With the help of a random-delivery, the seller can achieve separation under renting without excluding the low type buyer from first period trade altogether. When this effect is taken into account, the seller's incentive to separate is stronger under renting than under selling.

Corollary 1 asserts that, when the seller discounts the future less strongly than the buyer, more information will be revealed in a renting framework than in a selling framework. Although we have obtained this result in a two period model, it comes at a surprise in light of Hart and Tirole's (1988) finding that (under homogeneous discounting) renting leads to *less* separation than selling in a model with more than two periods.[8] Interestingly, Corollary 1's conclusion hinges on the seller's ability to use mechanisms that are more sophisticated than the deterministic mechanisms considered in Hart and Tirole (1988). When the seller is restricted to use price-posting, then his cost of separating types in the renting framework increases and we obtain the same conclusion as in Hart and Tirole (1988).

[8] Their result is due to the interplay between the seller's desire to lower the rental for the low type and his incentive to increase the rental for the high type which is absent in models with two periods.

6 Tough seller

Our previous analysis has focused on the case where the seller is soft. In this section we complement our analysis by considering a tough seller, characterized by a prior $\beta_1 \geq \theta_L$.

Hart and Tirole (1988) have shown that (under the assumptions of price-posting and homogeneous discounting), a tough seller will choose either separation or semi-separation. Under semi-separation, the seller posts a very high price in period 1 inducing the high type to mix between accepting and rejecting. The probability of acceptance is chosen sufficiently low to ensure that, when observing a rejection, the seller's updated belief $\beta_2(l)$ remains above θ_L. Semi-separation enables the seller to maintain a high price in period 2 and turns out to be optimal when the seller's prior is sufficiently high.

In this section we show that for a tough seller, the restriction to simple price-posting is indeed innocuous, thereby confirming the results of the earlier literature. We also show that, when the seller is more patient than the buyer, renting induces less semi-separation than selling. This is in line with our finding for the case of a soft seller that (in the presence of general mechanisms) informational asymmetries are more persistent under selling than under renting.

The formal analysis of the tough seller case follows roughly the same steps as our analysis in Section 5. We therefore relegate it to an Online-Appendix. There we proof the following result:

Proposition 3. *Let $\beta_1 \geq \theta_L$. Both in the renting and the selling case, the seller's revenue is maximized by simple (first-period) price-posting:*

- *Separation: For $\beta_1 \in [\theta_L, \overline{\beta}_1)$ the seller posts a price $\tilde{p}_1$ which is accepted by the high type and rejected by the low type.*

- *Semi-separation: For $\beta_1 \in [\overline{\beta}_1, 1]$ the seller posts a price $\hat{p}_1 > \tilde{p}_1$ which is accepted by the high type with probability $\frac{\beta_1 - \theta_L}{\beta_1 \Delta\theta} < 1$ and rejected by the low type.*

Renting and selling only differ in the prices posted ($\tilde{p}_1^R = 1-\delta_B\Delta\theta$, $\hat{p}_1^R = 1$, $\tilde{p}_1^S = 1+\delta_B\theta_L$, $\hat{p}_1^S = 1+\delta_B$) and in the threshold $\overline{\beta}_1$. Renting induces more separation than selling, i.e. $\overline{\beta}_1^R > \overline{\beta}_1^S$.

Proof: See the Online-Appendix.

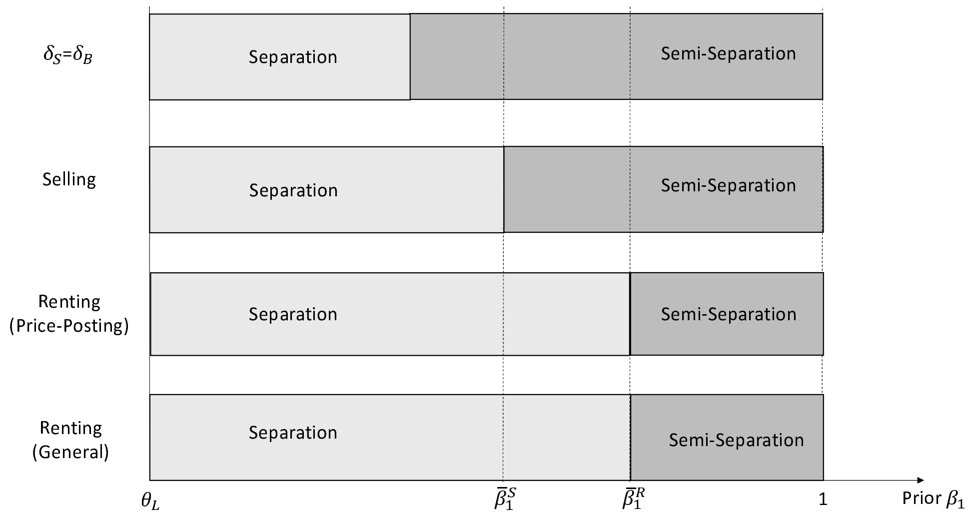

Figure 2: **Learning in Dependence of the Tough Seller's Prior.** The case of homogeneous discounting, $\delta_S = \delta_B$ serves as a benchmark. When the seller is more patient than the buyer, renting leads to more learning than selling, independently of whether or not general mechanisms are available.

Proposition 3 is depicted in Figure 2 which compares the extent of learning induced by the two modes of trade. In comparison to the benchmark of homogeneous discounting, a patient seller induces more separation, independently of whether he rents or sells. This is similar to the case of a soft seller and due to the mere fact that information is more valuable to the seller than to the buyer. A second similarity is that renting induces more separation than selling. However, in contrast to the case of a soft seller, this conclusion no longer hinges on the availability of mechanisms more general than price-posting as, for a tough seller, price-posting turns out to be optimal under both modes of trade.

To build intuition for this result, it is again helpful to consider the seller's benefits and costs of inducing the buyer to reveal his type. In the renting case we find that separation

is optimal if and only if

$$(1-\beta_1)\delta_S\theta_L + \beta_1\left[1 - \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta}\right] > \beta_1\delta_B\Delta\theta. \tag{31}$$

The benefits of separation (LHS) originate from two sources. First, by learning the buyer's type, the seller avoids the failure of implementing a zero trade with the low type in period 2. Second, when preventing himself from learning is not an issue, the seller is able to implement full trade with the high type in period 1. The costs of separation (RHS) are given by the compensation the high type needs to receive to give up his future information rent.

In the selling case, the cost-benefit comparison looks very similar. In particular, under selling separation is optimal if and only if

$$(1-\beta_1)\delta_S\theta_L + \beta_1\left[1 - \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta}\right] - \beta_1(\delta_S - \delta_B)\left[1 - \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta}\right] > \beta_1\delta_B\Delta\theta.$$

Selling differs from renting only in that the benefits of separation are reduced by the term $\beta_1(\delta_S - \delta_B)\left[1 - \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta}\right]$. By allowing for the possibility of not selling to the high type in period 1, the seller enables a trade with the high type in period 2. Postponing the sale of (the second-period access to) the seller' product is beneficial when the seller is more patient than the buyer. For this reason, selling leads to less separation than renting in the case of a tough seller.

7 Conclusion

In this paper we have applied a mechanism design approach to a model of dynamic bilateral trade with short-term commitment. Our analysis encompasses both the renting and the selling case. Allowing the seller to be more patient than the buyer reveals two differences between these alternative modes of trade which become overlooked under the simplifying assumption of homogeneous discounting. First, the optimal renting mechanism requires a random allocation of the seller's product whereas the outcome of the

optimal selling mechanism can be implemented via simple price-posting. Second, renting induces the buyer to reveal his type for a wider range or parameters than selling. Both results contrast with the view suggested by the existing literature which holds that the restriction to price-posting is innocuous and that informational asymmetries are more persistent under renting than under selling. The fact that our second result relies on the availability of a random allocation underlines the importance of a general mechanism design approach for models of dynamic adverse selection.

Appendix - Proofs

Proof of Lemma 1: This proof has three steps. (i) The participation constraint for the high type is redundant: As $\theta_L < \theta_H = 1$ and $U_H(l) \geq U_L(l)$, (12) and (15) together imply (14). (ii) The participation constraint of the low type and the incentive constraint of the high type are binding at the optimum: If $x_1(l)\theta_L - t_1(l) + \delta_B U_L(l) > 0$ then there exists an $\epsilon > 0$ such that raising both prices to $t_1(l) + \epsilon$ and $t_1(h) + \epsilon$ increases the seller's revenue while keeping all constraints satisfied. Hence $t_1^*(l) = x_1(l)\theta_L + \delta_B U_L(l)$, i.e. (15) must be binding in the optimal mechanism. If $x_1(h) - t_1(h) + \delta_B U_H(h) > x_1(l) - t_1^*(l) + \delta_B U_H(l)$ then there exists an $\epsilon > 0$ such that raising the price for the buyer reporting a high type to $t_1(h) + \epsilon$ increases the seller's revenue while keeping all constraints satisfied. Hence $t_1^*(h) = t_1^*(l) + x_1(h) - x_1(l) + \delta_B \Delta U_H$, i.e. (12) must be binding in the optimal mechanism. Substitution of $t_1^*(l)$ and $t_1^*(h)$ into (11) gives the objective in (16). (iii) Finally, adding both incentive constraints (12) and (13) gives (17). Since the incentive constraint of the high type is binding at the optimum it follows that (17) must be binding whenever $q_L > 0$.

■

Proof of Lemma 2: This proof considers separately the cases of renting and selling, using the corresponding continuation values derived in Sections 5.1 and 5.2.

Renting: (i) In the renting case, the continuation values (18), (19), and (20) are

independent of $x_1(h)$. Hence an increase in $x_1(h)$ raises the objective (16) and relaxes the constraint (17). The optimal mechanism must therefore set $x_1^*(h) = 1$. (ii) Consider a mechanism for which $\beta_2(h) > \theta_L$ so that $\Delta U_H = \Delta\theta$, $\Delta U_L = 0$, $V(h) = \beta_2(h)$, $V(l) = \theta_L$, and $U_L(l) = 0$. If $q_L > 0$ then (17) must be binding and we can substitute $x_1(h)^* = 1$ and $x_1(l) = 1 - \delta_B$ into (16) which leaves us with the simplified program

$$\max_{q_L, q_H} \theta_L + \delta_S \beta_1 q_H + [1 - \beta_1 q_H - (1 - \beta_1) q_L](\delta_S - \delta_B)\theta_L. \tag{32}$$

Since (32) is decreasing in q_L, the mechanism must optimally set $q_L^* = 0$. Finally, setting $x_1^*(h) = 1$ and $q_L^* = 0$ in (16) makes the objective become equal to

$$[x_1(l) + \delta_S]\theta_L + \beta_1 q_H [1 - x_1(l) + (\delta_S - \delta_B)(1 - \theta_L)] \tag{33}$$

which is increasing in q_H for all $x_1(l)$ satisfying the constraint $x_1(l) \leq 1 - \delta_B$. Hence the mechanism must optimally set $q_H^* = 1$. Now consider a mechanism with $\beta_2(h) \leq \theta_L$ so that $\Delta U_H = \Delta U_L = 0$, $V(h) = V(l) = \theta_L$, and $U_L(l) = 0$. If $q_L > 0$ then from the binding constraint (17) it must hold that $x_1(l) = x_1(h) = 1$ making the seller's revenue equal to $(1 + \delta_S)\theta_L$ independently of q_H, i.e. setting $q_H = q_L$ is optimal. If $q_L = 0$ then the objective in (16) becomes

$$\beta_1 q_H [1 - x_1(l)\Delta\theta] + (1 - \beta_1 q_H) x_1(l)\theta_L + \delta_S \theta_L. \tag{34}$$

As $\beta_1 q_H \leq \beta_1 < \theta_L$ the objective is increasing in $x_1(l)$, i.e. the seller's revenue cannot be higher than what he gets for $x_1(l) = 1$ which is again $(1 + \delta_S)\theta_L$. We have therefore shown that no renting mechanism can give the seller a higher payoff than what he gets by either separating or pooling.

Selling: (i) Using the continuation values in (25), (26), and (26), the dynamic monotonicity constraint (17) for a mechanism with $\beta_2(h) > \theta_L$ becomes

$$x_1(h) - x_1(l) \geq \delta_B[1 - x_1(h)] \tag{35}$$

whereas for a mechanism with $\beta_2(h) \leq \theta_L$, the constraint simplifies to the requirement that $x_1(h) \geq x_1(l)$. In both cases, an increase in $x_1(h)$ relaxes the dynamic monotonicity constraint. Moreover, the derivative of the seller's objective with respect to $x_1(h)$ is given by

$$Q[1 + \delta_B - \delta_S \max\{\theta_L, \beta_2(h)\}] > 0. \tag{36}$$

Hence the optimal mechanism must set $x_1^*(h) = 1$ which makes the dynamic monotonicity constraint become $x_1(h) \geq x_1(l)$ independently of whether $\beta_2(h) \leq \theta_L$ or $\beta_2(h) > \theta_L$. (ii) Consider a mechanism for which $\beta_2(h) > \theta_L$. Setting $x_1^*(h) = 1$ then gives $\Delta U_H = -[1 - x_1(l)]\theta_L = \Delta U_L$, $V(h) = 0$, $V(l) = [1 - x_1(l)]\theta_L$, and $U_L(l) = x_1(l)\theta_L$. If $q_L > 0$ then the constraint must be binding so that $x_1(l) = x_1^*(h) = 1$ and the seller's revenue becomes $(1 + \delta_B)\theta_L$ independently of q_H, i.e. setting $q_H = q_L$ is optimal. If $q_L = 0$ then the objective in (16) becomes

$$[1 - x_1(l)][\beta_1 q_H(1 + \delta_B\theta_L) + (1 - \beta_1 q_H)\delta_S\theta_L] + (1 + \delta_B)x_1(l)\theta_L. \tag{37}$$

which is increasing in q_H, making $q_H = 1$ optimal. Finally, consider a mechanism with $\beta_2(h) \leq \theta_L$ so that with $x_1^*(h) = 1$ we have $\Delta U_H = -[1 - x_1(l)]$, $\Delta U_L = -[1 - x_1(l)]\theta_L$, $V(h) = 0$, $V(l) = [1 - x_1(l)]\theta_L$, and $U_L(l) = x_1(l)\theta_L$. In this case, the dynamic monotonicity constraint is automatically satisfied and the seller's objective becomes equal to

$$x_1(l)(1 + \delta_B)\theta_L + [1 - x_1(l)][Q(1 + \delta_B) + (1 - Q)\delta_S\theta_L. \tag{38}$$

Note that this objective is increasing in Q. Moreover, it is increasing in $x_1(l)$ if Q is sufficiently close to 1. Since for $x_1(l) = 1$ the objective becomes identical to $(1 + \delta_B)\theta_L$, this shows that for $\beta_2(h) \leq \theta_L$ the seller cannot do better than by pooling through $x_1(l) = x_1(h) = 1$ and $q_L = q_H$. In summary, we have therefore shown that no selling mechanism can give the seller a higher payoff than what he gets by either separating or pooling. ■

Proofs of Proposition 1 and 2: The results are an immediate consequence of Lemma 2 and the derivations contained in the main text. ■

Proof of Corollary 1: First note that

$$\frac{\delta_B}{\delta_S - (\delta_S - \delta_B)\theta_L}\theta_L < \frac{1 - (\delta_S - \delta_B)}{1 - (\delta_S - \delta_B)\theta_L}\theta_L$$
$$\Leftrightarrow \delta_B[1 - (\delta_S - \delta_B)\theta_L] < [1 - (\delta_S - \delta_B)][\delta_S - (\delta_S - \delta_B)\theta_L]$$
$$\Leftrightarrow \delta_B - (\delta_S - \delta_B)\delta_B\theta_L < \delta_S - (\delta_S - \delta_B)\delta_S - (\delta_S - \delta_B)\theta_L + (\delta_S - \delta_B)^2\theta_L$$
$$\Leftrightarrow -\delta_B\theta_L < 1 - \delta_S - \theta_L + (\delta_S - \delta_B)\theta_L$$
$$\Leftrightarrow 0 < (1 - \delta_S)\Delta\theta.$$

As $\delta_S < 1$, this proves that $\underline{\beta}_1^R < \underline{\beta}_1^S$. Also note that

$$\frac{\theta_L}{1 + (\delta_S - \delta_B)\Delta\theta} > \frac{1 - (\delta_S - \delta_B)}{1 - (\delta_S - \delta_B)\theta_L}\theta_L$$
$$\Leftrightarrow 1 - (\delta_S - \delta_B)\theta_L > [1 - (\delta_S - \delta_B)][1 + (\delta_S - \delta_B)\Delta\theta]$$
$$\Leftrightarrow 0 > -(\delta_S - \delta_B)^2\Delta\theta.$$

As $\delta_S > \delta_B$, this proves that $\underline{\beta}_1^{RPP} > \underline{\beta}_1^S$. ■

Online-Appendix - The tough seller case

In this Appendix we derive the optimal renting and selling mechanisms for the case of a tough seller characterized by a prior $\beta_1 \geq \theta_L$. For this purpose, first note that Lemma 1 remains valid as it holds independently of the seller's prior. The following lemma is the equivalent of Lemma 2.

Lemma 3. *Let $\beta_1 \geq \theta_L$. The optimal mechanism creates no distortion at the top, i.e., $x_1^*(h) = 1$. Moreover, no mechanism can give the seller a higher revenue than what he can achieve by either pooling ($q_L = q_H$), separating ($q_L = 0, q_H = 1$), or semi-separating ($q_L = 0, q_H = \frac{\beta_1 - \theta_L}{\beta_1 \Delta\theta}$).*

Proof of Lemma 3: This proof follows the same structure as the one for Lemma 2. In particular, we consider separately the cases of renting and selling, using the corresponding continuation values derived in Sections 5.1 and 5.2.

Renting: (i) The continuation values (18), (19), and (20) are independent of $x_1(h)$. Thus, an increase in $x_1(h)$ raises the objective (16) and relaxes the constraint (17). The optimal mechanism must therefore set $x_1^*(h) = 1$. (ii) Consider a mechanism for which $\beta_2(l) \geq \theta_L$. Continuation values are $\Delta U_H = \Delta U_L = 0$, $V(h) = \beta_2(h)$, $V(l) = \beta_2(l)$, and $U_L(l) = 0$. If $q_L > 0$ then the dynamic monotonicity constraint (17) must be binding. Thus, we can substitute $x_1^*(h) = 1$ and $x_1(l) = 1$ into the objective (16) which makes the seller's revenue $\theta_L + \delta_S\beta_1$ independent of q_H. Hence, if $q_L > 0$, then setting $q_H = q_L$ maximizes the seller's revenue, i.e. pooling is optimal. If $q_L = 0$, then $x_1(l) \leq 1$ and (16) becomes

$$\beta_1 q_H[1 - x_1(l)\Delta\theta] + (1 - \beta_1 q_H)x_1(l)\theta_L + \delta_S\beta_1. \tag{39}$$

This objective is increasing in q_H. Hence the seller optimally chooses the largest q_H compatible with $\beta_2(l) \geq \theta_L$, i.e. $q_H^* = \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta}$. After substituting q_H^* into (39), the seller's objective is decreasing in $x_1(l)$ if $\beta_1 \geq \theta_L\Delta\theta + \theta_L$ and increasing otherwise. In the latter case, the seller achieves the same revenue as under pooling. Now consider a mechanism for which $\beta_2(l) < \theta_L$. In this case so that $\Delta U_H = \Delta\theta$, $\Delta U_L = 0$, $V(h) = \beta_2(h)$, $V(l) = \theta_L$, and $U_L(l) = 0$. If $q_L > 0$ then (17) is binding and $x_1(l) = 1 - \delta_B$. The objective in (16) becomes

$$\max_{q_L, q_H} \theta_L + \delta_S\beta_1 q_H + [1 - \beta_1 q_H - (1 - \beta_1)q_L](\delta_S - \delta_B)\theta_L. \tag{40}$$

Since (40) is decreasing in q_L, the mechanism must optimally set $q_L^* = 0$. Setting $x_1^*(h) = 1$ and $q_L^* = 0$ in (16) makes the objective become equal to

$$[x_1(l) + \delta_S]\theta_L + \beta_1 q_H[1 - x_1(l) + (\delta_S - \delta_B)(1 - \theta_L)] \tag{41}$$

which is increasing in q_H for all $x_1(l)$ satisfying the constraint $x_1(l) \leq 1 - \delta_B$. Hence the mechanism must optimally set $q_H^* = 1$. Therefore, we have shown that for a tough seller the optimal renting mechanism must be either pooling, separating or semi-separating.

Selling: (i) Using the continuation values in (25), (26), and (26), the dynamic monotonicity constraint (17) for a mechanism with $\beta_2(l) \geq \theta_L$ requires $x_1(h) \geq x_1(l)$ to satisfy $1 \geq \delta_B$ while for a mechanism with $\beta_2(l) < \theta_L$ the constraint becomes

$$x_1(h) - x_1(l) \geq \delta_B[1 - x_1(h)]. \tag{42}$$

In both cases, increasing $x_1(h)$ relaxes (17). Moreover, the derivative of the seller's objective with respect to $x_1(h)$ is given by

$$Q[1 + \delta_B - \delta_S \beta_2(h)] > 0. \tag{43}$$

Thus, the optimal mechanism must set $x_1^*(h) = 1$ which makes (17) become $x_1(h) \geq x_1(l)$ independently of whether $\beta_2(l) < \theta_L$ or $\beta_2(h) \geq \theta_L$. (ii) Consider a mechanism for which $\beta_2(l) \geq \theta_L$. Setting $x_1^*(h) = 1$ gives $\Delta U_H = -[1 - x_1(l)]$, $\Delta U_L = -[1 - x_1(l)]\theta_L$, $V(h) = 0$, $V(l) = [1 - x_1(l)]\beta_2(l)$, and $U_L(l) = x_1(l)\theta_L$. If $q_L > 0$ then (17) is binding, $x_1^*(l) = x_1^*(h) = 1$ and the seller achieves $(1 + \delta_B)\theta_L$ independently of q_H, i.e., setting $q_H = q_L$ is optimal. If $q_L = 0$ then the objective in (16) becomes equal to

$$x_1(l)(1 + \delta_B)\theta_L + [1 - x_1(l)][\beta_1 q_H(1 + \delta_B) + \delta_S \beta_1(1 - q_H)], \tag{44}$$

which is increasing in q_H. Hence, the seller optimally chooses the largest q_H compatible with $\beta_2(l) \geq \theta_L$, i.e. $q_H^* = \frac{\beta_1 - \theta_L}{\beta_1 \Delta\theta}$. Finally, consider a mechanism for which $\beta_2(l) < \theta_L$. In this case $\Delta U_H = -[1 - x_1(l)]\theta_L = \Delta U_L$, $V(h) = 0$, $V(l) = [1 - x_1(l)]\theta_L$, and $U_L(l) = x_1(l)\theta_L$. If $q_L > 0$ then (17) is binding, and again the seller achieves $(1 + \delta_B)\theta_L$ independently of q_H. If $q_L = 0$ the objective in (16) becomes

$$[1 - x_1(l)][\beta_1 q_H(1 + \delta_B \theta_L) + (1 - \beta_1 q_H)\delta_S \theta_L] + (1 + \delta_B)x_1(l)\theta_L \tag{45}$$

which is increasing in q_H, making $q_H = 1$ optimal. We have thus shown that, for a tough seller the optimal selling mechanism is either pooling, separating or semi-separating. ■

Proof of Proposition 3: To determine the seller's revenue maximizing mechanism, we compare the maximum payoffs he can obtain from pooling, separating, or semi-separating. As before, we perform the analysis separately for the renting and the selling case.

Renting: To determine the maximum payoff from separating, we set $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = 1$ in the seller's problem in Lemma 1. This leads the same reduced program as in the soft seller scenario:

$$\max_{x_l \in [0,1-\delta_B]} \beta_1[1 - x_1(l)\Delta\theta] + (1-\beta_1)x_1(l)\theta_L + \delta_S\theta_L + \beta_1(\delta_S - \delta_B)\Delta\theta. \tag{46}$$

However, since $\beta_1 \geq \theta_L$, the objective is now *decreasing* in $x_1(l)$, i.e. $x_1^*(l) = 0$ is optimal. The seller's maximized revenue from separating is thus given by $\beta_1 + (\delta_S - \delta_B)\beta_1\Delta\theta + \delta_S\theta_L$ which is strictly greater than the revenue $\theta_L + \delta_S\beta_1$ he achieves by pooling. Thus pooling can never be optimal for $\beta_1 \geq \theta_L$. To determine the maximum payoff from semi-separating, we set $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = \frac{\beta_1-\theta_L}{\beta_1\Delta\theta}$ in the seller's problem in Lemma 1. This gives us the following program:

$$\max_{x_l \in [0,1]} \frac{\beta_1 - \theta_L}{\Delta\theta}[1 - x_1(l)\Delta\theta + \delta_S] + (1 - \frac{\beta_1 - \theta_L}{\Delta\theta})[x_1(l)\theta_L + \delta_S\theta_L]. \tag{47}$$

In this program, the seller's objective is increasing in $x_1(l)$ if $\beta_1 < \theta_L(\Delta\theta + 1)$ and decreasing otherwise. For $\beta_1 < \theta_L(\Delta\theta + 1)$ the seller's maximized revenue from semi-separating coincides with his revenue from pooling, i.e. semi-separation cannot be optimal. For $\beta_1 \geq \theta_L(\Delta\theta + 1)$, the seller can obtain the revenue $\frac{\beta_1-\theta_L}{\Delta\theta} + \delta_S\beta_1$ by choosing $x_1^*(l) = 0$. Comparing this revenue with the seller's revenue from separating we find that separation is optimal if and only if $\beta_1 < \overline{\beta}_1^R$ with

$$\overline{\beta}_1^R \equiv \frac{\theta_L(1 + \delta_S\Delta\theta)}{(\theta_L + \delta_B\Delta\theta) + (\delta_S - \delta_B)\theta_L\Delta\theta}. \tag{48}$$

denoting the seller's threshold prior belief.

Selling: To determine the maximum payoff from separating, we set $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = 1$ in the seller's problem in Lemma 1. The seller's program becomes:

$$\max_{x_1(l)\in[0,1]} [1 - x_1(l)][\beta_1(1 + \delta_B\theta_L) + (1 - \beta_1)\delta_S\theta_L] + (1 + \delta_B)x_1(l)\theta_L. \tag{49}$$

This is the same program as in the soft seller case with the only difference that for a tough seller the objective is decreasing in $x_1(l)$ for *all* priors. Setting $x_1(l) = 0$ gives the seller's maximized revenue from separating: $\beta_1[1 + \delta_B\theta_L] + (1 - \beta_1)\delta_S\theta_L$. This revenue is larger than the revenue from pooling, $(1 + \delta_B)\theta_L$, i.e. pooling can never be optimal. To determine the maximum revenue from semi-separating, we set $x_1^*(h) = 1$, $q_L^* = 0$, and $q_H^* = \frac{\beta_1-\theta_L}{\beta_1\Delta\theta}$ in the seller's problem in Lemma 1. This yields the following program:

$$\max_{x_1(l)\in[0,1]} [1 - x_1(l)][\frac{\beta_1 - \theta_L}{\Delta\theta}(1 + \delta_B) + (1 - \frac{\beta_1 - \theta_L}{\beta_1\Delta\theta})\delta_S\theta_L] + (1 + \delta_B)x_1(l)\theta_L. \tag{50}$$

The objective is increasing in $x_1(l)$ if and only if $\beta_1 < \theta_L + \frac{[\theta_L-(\delta_S-\delta_B)\theta_L]\Delta\theta}{1+\delta_B-\delta_S\theta_L}$. When the objective is increasing, semi-separation cannot be optimal because it leads a revenue smaller or equal to the revenue under pooling. Only when $\beta_1 \geq \theta_L + \frac{[\theta_L-(\delta_S-\delta_B)\theta_L]\Delta\theta}{1+\delta_B-\delta_S\theta_L}$, semi-separation might be optimal and its maximized level of revenue is obtained by setting $x_1^*(l) = 0$, leading $\frac{\beta_1-\theta_L}{\Delta\theta}[1 + \delta_B\theta_L] + (1 - \frac{\beta_1-\theta_L}{\beta_1\Delta\theta})\delta_S\theta_L$. Comparing this revenue with the maximum revenue under separation we find that separation is optimal if and only if $\beta_1 < \overline{\beta}_1^S$ with

$$\overline{\beta}_1^S \equiv \frac{\theta_L(1 + \delta_B - \delta_S\theta_L)}{\theta_L + \delta_B\Delta\theta - (\delta_S - \delta_B)\theta_L^2}. \tag{51}$$

Finally, to see that $\overline{\beta}_1^S < \overline{\beta}_1^R$ note that

$$\begin{aligned}
\frac{1+\delta_B-\delta_S\theta_L}{\theta_L+\delta_B\Delta\theta-(\delta_S-\delta_B)\theta_L^2}\theta_L &< \frac{1+\delta_S\Delta\theta}{\theta_L+\delta_B\Delta\theta+(\delta_S-\delta_B)\theta_L\Delta\theta}\theta_L \\
\Leftrightarrow \frac{\theta_L+\delta_B\Delta\theta+(\delta_S-\delta_B)\theta_L\Delta\theta}{\theta_L+\delta_B\Delta\theta-(\delta_S-\delta_B)\theta_L^2} &< \frac{1+\delta_S\Delta\theta}{1+\delta_B-\delta_S\theta_L} \\
\Leftrightarrow \frac{(\delta_S-\delta_B)\theta_L}{\theta_L+\delta_B\Delta\theta-(\delta_S-\delta_B)\theta_L^2}+1 &< \frac{\delta_S-\delta_B}{1+\delta_B-\delta_S\theta_L}+1 \\
\Leftrightarrow \theta_L(1+\delta_B-\delta_S\theta_L) &< \theta_L+\delta_B\Delta\theta-(\delta_S-\delta_B)\theta_L^2 \\
\Leftrightarrow \theta_L &< 1-\theta_L+\theta_L^2.
\end{aligned}$$

This holds as $\theta_L < 1$. ■

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