School Competition and Product Differentiation

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Abstract

Policies that encourage competition are increasingly popular as a means to improve school quality around the world, especially in poor countries, where a large share of students enroll in private schools. Yet policy-makers and researchers have suggested that private school competition can harm poorer students. This paper provides the first theoretical and empirical analysis of the impact of private school entry on private schools' strategic choices of quality, with an emphasis on distributional effects. First, I develop a model in which schools select their quality when (1) the match between a school and a student affects test scores, and (2) wealthier students' enrollment decisions are more responsive to quality. Then, I study the effect of a new private school entering a village in rural Pakistan on students' outcomes. As the model predicts, entry leads to a 0.1 SD increase in within-school inequality in test scores between rich and poor students in the average private school. Structural estimates of the key strategic variables in my model – private schools' equilibrium choices of match-specific quality – show that private schools respond to entry by competing more intensively for wealthier students, consistent with the reduced-form findings. More broadly, my structural estimates indicate that match-specific quality is quantitatively important, suggesting that improving the match between schools and students may be an effective way to increase learning.

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1 Introduction

Policies that boost school competition are increasingly popular as tools to increase educational quality in both high and low income countries. For example, Chile has instituted a national voucher program, and with the passage of the Right to Education Act, India will require that private schools set aside 25 percent of their seats for poor students with vouchers. Moreover, the private schooling market in low income countries is large and fast-growing with private enrollment at 28 percent in rural India (Pratham, 2012) and 65 percent in urban India (Desai et al., 2008).[1] In the United States, the increasing importance of charter schools means that public schools also experience competitive pressures. Economists have made a great deal of progress in understanding the effects of competition when school quality benefits all students in the same way (that is, when quality is vertical), but there has been less research on how schools compete when they can choose characteristics that affect different students differently.

In this paper, I depart from the previous literature on competition by showing, both theoretically and empirically, how competition – in the form of an additional school in the market place – affects the distribution of test scores within profit-maximizing private schools. My model of how schools choose their quality in the presence competition incorporates two important, novel facts: (1) the match between a school and a student affects student learning, as measured by test scores, and (2) rich and poor students are not equally responsive to their predicted test score gains when they make enrollment decisions. There is growing evidence that both these elements of the model are important aspects of school competition.

In an experiment in Kenya, Duflo et al. (2011) provide evidence that match-specific quality (hereafter referred to as horizontal quality) is an important determinant of learning. In particular, Duflo et al. (2011) show that the match between instructional level and a student's ability has a large effect on students' test scores. This result suggests that schools can affect their match-specific quality by choosing an advanced instructional level to benefit high ability students or a remedial instructional level to benefit low ability students.[2] More generally, if school or teacher characteristics

[1] The statistics for sub-saharan Africa are similar with 10-54 percent of primary school children enrolled in private schools in sub-Saharan African countries in 2012 (World Bank Development Indicators, 2014).

[2] Instructional level is a particularly important type of school quality for schools in low income countries since schools typically have one teacher (or less) per grade. Therefore, it is difficult for a

have heterogeneous effects, schools can affect their match-specific quality by choosing costly characteristics that disproportionately benefit some students over others.[3]

Intuitively, the second aspect of the model – that rich and poor students are not equally responsive to school quality when they make enrollment decisions – is also likely to be important empirically. Poorer students may have limited information about their match with all the available schools or their parents may not choose schools to maximize their learning.[4] Since students have little opportunity to experiment with different schools and directly observe educational gains in their new schools, imperfect information about the match between students and schools is likely to persist over time.

To model how private schools choose their equilibrium quality in the presence of these two important aspects of school competition, I develop a new heterogeneous Hotelling model in which a student's location on the Hotelling line (her optimal instructional level) is correlated with how responsive a student is to her match with a school when she makes enrollment decisions and schools can choose both horizontal quality and vertical quality (a dimension of quality that affects all students equally). I derive predictions for a simplified version of this model, where students choose schools solely based on horizontal quality and schools can only choose their location on the Hotelling line. This restricted model suggests that competition can have adverse consequences since schools will be incentivized to cater to students who are more responsive to their match to a school when they make enrollment decisions. A key implication is that when wealthier students are better able to identify their best-match schools, this imposes a negative externality on poorer students because schools shift their offerings to cater more to the rich students. Therefore, the entry of an additional private school into the market can lead learning to increase for wealthy

school to simultaneously teach at multiple instructional levels.

[3]For example, Glewwe et al. (2009) find that providing textbooks to students in rural Kenya did not help the median student but did benefit more advanced students. Providing textbooks to students does not harm poor students directly, but it comes at a cost since the same money could be used to provide remedial tutoring (Banerjee et al., 2007) or some other school characteristic that benefits disadvantaged students.

[4]For example, research by Mullainathan and Shafir (2013) shows that disadvantaged students and their families may have fewer cognitive resources to devote to enrollment decisions, Hastings and Weinstein (2008) show that students and their parents have limited information about school quality, Bayer et al. (2007) show that higher income households have a higher willingness to pay to live in neighborhoods where schools have higher average test scores, and Ajayi (2011) and Hoxby and Avery (2012) show that poor students make less sophisticated decisions when they choose schools. Relatedly, Dizon-Ross (2015) shows that poor parents have less information about their children's academic performance than rich parents do.

students and decrease for poor students *in the same school*.

I test this prediction using data from the Learning and Educational Attainment in Punjab Schools data (LEAPS), a panel data set from 2003-2007, which includes geocoded data on Pakistani students and schools. The private schooling market in Pakistan is a natural place to study competition and horizontal differentiation. First, there is a large, low cost private schooling market, with 35 percent of students enrolled in private schools as of 2005 (Andrabi et al., 2006). Second, horizontal differentiation is likely to be particularly important for learning outcomes in low income countries. In a review of the literature, Kremer et al. (2013) find that it is generally difficult to shift student test scores by increasing the quality of classroom inputs but that interventions that more closely tailor the instructional level to a student's own level are highly effective in low income countries.

Exploiting variation in the number of private schools in a village caused by the exit and entry of private schools over time, I find that an increase in the number of private schools in a village increases yearly test score gains for wealthier students, but reduces yearly test scores gains for poorer students in the average private school. The addition of one more private school in the market increases within-school inequality in test score gains by 0.1 standard deviations. Consistent with the idea that schools change their instructional level to target higher ability, wealthier students when there are more schools in the market place, the entry of an additional private school leads poorer students to perform worse on easier questions and richer students to perform better on harder questions. Importantly, the effect of increased competition on the average private school student is not statistically significantly different from zero since these positive and negative effects cancel one another out. Therefore, ignoring heterogeneity in the effects of competition would lead a researcher to erroneously conclude that competition had little effect on school quality or student test scores.

I next structurally estimate the parameters of the theoretical model. I first estimate the determinants of students' demand for different schools. I allow a student's perceived utility in a school to depend differentially on distance to the school, school fees, and predicted test score gains in a school for wealthier and poorer students. I then estimate the parameters of the perceived utility function by matching students' predicted and observed school choices. Consistent with the key assumption of the model, the coefficient for predicted test score gains for wealthier students is about five times as large as the coefficient for predicted test score gains in a school for

poorer students. Altogether, given the current set of school characteristics, the estimates suggest that on average a school that increases its predicted test score gains for high types will increase its share of students by six times as much as a school that increases its predicted test score gains for low types.

Finally, using the parameters from the demand estimation, I directly test the model by computing each private school's profit-maximizing match-specific quality given the characteristics chosen by the other schools in the market. Reassuringly, the equilibrium choices of horizontal quality implied by the structural model, which allows students' school choice decisions to depend on a variety of school characteristics, including distance and fees, confirm the implications of the simplified model. On average, private schools increase their horizontal quality for wealthy students and decrease it for poor students when there are more private schools in the market. According to the estimates, a 1 percent increase in the number of private schools in the market increases the difference in test scores between the wealthier and poorer halves of the private school population due to horizontal quality by 6 percent.

Overall, this paper makes two key contributions. First, it establishes the importance of the match between schools and students for student learning. My structural estimates suggest that on average schools strongly cater to the needs of richer students and that there is substantial scope for schools to improve horizontal quality for poorer students. According to my estimates, a school can increase test scores for poorer students by as much as 0.5 standard deviations by moving from wealthier students' optimal instructional level to poorer students' optimal instructional level. Given my findings, in some cases, economic stratification across schools may be beneficial if poorer students have different needs than richer students. A great deal of recent research has documented the low level of learning in low income countries (for a comprehensive discussion, see Pritchett (2013)). Much of the past research has focused on finding interventions that increase "vertical quality." Increasing the match between students and schools and sorting students into their best-match school may ultimately offer a more cost-effective way to improve student learning.

Second, this paper demonstrates that the greater ability of rich students to sort into their best match schools imposes a negative externality on poorer students by incentivizing schools to skew their quality toward wealthier students. I show that this mechanism has real consequences for the learning of poor students. Moreover, while this paper focuses on the private schooling market, the mechanisms that my

model highlights have broader implications. They may be relevant for understanding competition and its effects on inequality in any environment where consumers have little opportunity to experiment with products and information is limited, such as health care and voting.

Understanding the mechanisms underlying school competition in low income countries opens the door to policies that will harness the benefits of competition for poorer students as well. For example, supplying students with information on their type-specific predicted test score gains within a school may allow poorer students to make better decisions and incentivize schools to cater to them more. Thus, small interventions could have big effects, pushing school competition closer to the ideal described by Milton Friedman where "The injection of competition would do much to promote a healthy variety of schools" (1962).

This paper relates to several literatures. First, it contributes to a growing literature on private school quality (Neal (1997) in the United States, Andrabi et al. (2010) in Pakistan, Muralidharan and Sundararaman (2015) in India, and Neilson (2013) in Chile). Researchers working in the same (Andrabi et al., 2010) or similar (Muralidharan and Sundararaman, 2015) contexts have demonstrated that private schools are more productive than public schools, delivering as great or greater levels of learning at lower costs. In contrast, this paper builds on this important result by showing how market structure can affect the distribution of learning *within* private schools.

Second, this paper contributes to a growing literature that structurally estimates models of demand (Carneiro et al. (2016) in Pakistan, Neilson (2013) in Chile, Walters (2014) among charter schools in Boston, Hastings et al. (2005) in North Carolina, and Dinerstein and Smith (2014) in New York) and supply (Dinerstein and Smith, 2014) in education. This paper contributes to this literature by allowing learning in the same school to differ across students and by allowing schools to choose both match-specific and vertical quality.

This paper also informs the literature on the effects of increased competition on student outcomes (Hoxby, 2000; Hoxby, 2003; Sass, 2006; Booker et al., 2008; Bayer and McMillan, 2010; Card et al., 2010; Neilson, 2013; Figlio and Hart, 2014; Jinnai, 2014; Hsieh and Urquiola, 2006; Bettinger, 2005; Bifulco and Ladd, 2006; Rothstein, 2007; Mehta, 2015; Imberman, 2011). Much of this literature finds small positive or zero effects of competition on the average student's learning. My findings suggest

that these small average effects could mask important heterogeneity when schools compete on horizontal quality and help explain why researchers have found small and sometimes negative effects of competition on student learning in the past.

Finally, this paper relates to a mainly theoretical literature on the effects of voucher systems (McMillan, 2005; Epple and Romano, 1998; Hoyt and Lee, 1998; Nechyba, 1996; Nechyba, 1999; Nechyba, 2000).[5] However, this paper differs from past work in this literature in important ways. First, I study the effects of competition due to the entry of additional schools rather than the introduction of vouchers. Increased competition due to school entry has very different implications than the creation of and expansion of voucher systems, and models of school entry are arguably more relevant for understanding the effects of charter schools, which are increasingly more popular among policymakers than vouchers. Second, most of this literature does not allow for any kind of match-specific school quality.[6] Third, and most importantly, this paper empirically estimates students' and schools' choices, whereas the previous literature has been mainly theoretical.

The paper is organized as follows. Section 2 presents a theoretical framework for understanding school competition. Section 3 discusses the context and the data, and section 4 reports the results of reduced-form tests of the theoretical framework's implications. Section 5 structurally estimates the model of demand for schools from section 2, section 6 structurally estimates schools' equilibrium choices of horizontal quality, and section 7 concludes.

2 Theoretical Framework

In this section, I develop a model of how students sort into schools and how schools choose their characteristics. I assume that there are two types of students, high and low types, with different optimal instructional levels and different responsiveness to school characteristics when they make enrollment decisions. In the first subsection, I specify student demand for schools as a function of students' and schools' char-

[5] In one of the few empirical papers in this literature, Hsieh and Urquiola (2006) estimate the effects of the Chilean voucher system and suggest that vouchers harmed students who remained in public schools with worse peers.

[6] McMillan (2005) is an exception. McMillan (2005) allows schools to select different effort levels for different students and models public schools' responses to voucher systems. However, this model implies that public schools will respond to competition due to increased vouchers by improving learning for both rich and poor students.

acteristics. In the next subsection, I specify the school's problem, and in the final subsection, I derive predictions for a simplified version of the model.

The key assumption of the model is that students from similar socioeconomic backgrounds have similar optimal instructional levels, and a student's ability to sort into her best-match school is correlated with her optimal instructional level (her location on the unit line). This is consistent with the idea that it is less costly for wealthier students or parents to attain information about school quality. For example, if wealthier parents are more educated, they will be better able to assess their children's learning in subjects like English and math.

Importantly, the model allows school quality to have both a horizontal and vertical component. In the model, low types are assumed to have an optimal instructional level of 0 on a unit line, while high types have an optimal instructional level of 1. This set-up is similar to a standard Hotelling model. A school j can choose both its location on the line (h_j) and its vertical quality $\overline{VA_j}$. Then, if a school chooses h_j, a low type's learning in the school will be

$$VA_{j,low} = \overline{VA_j} - \beta h_j^2 \tag{1}$$

and a high type's learning in a school will be

$$VA_{j,low} = \overline{VA_j} - \beta(1 - h_j)^2. \tag{2}$$

Here, β weights the importance of horizontal quality relative to vertical quality for students' outcomes.

2.1 Students' Decision Problem

A student i of type $z \in \{low, high\}$ chooses the school j that maximizes her perceived utility

$$u_{ijz} = \delta_z VA_{jz} + \mathbf{\Gamma_z} X_{ij} + \xi_j + \epsilon_{ij}, \tag{3}$$

where X_{ij} is the set of other student-school characteristics, such as fees and distance, ξ_j is unobserved school-specific quality, and ϵ_{ij} is an idiosyncratic shock drawn from the type 1 extreme value distribution that may either capture parents' misperception

of school quality or idiosyncratic taste. A student's choice set includes both private and public schools, and a student may also choose the outside option (not enrolling in school), which is normalized to have a utility of zero. To capture the idea that low types are less responsive to school quality, I assume $\delta_{low} < \delta_{high}$. In section 5, I verify that this assumption is strongly supported by the data.

Since ϵ_{ij} is drawn from the type 1 extreme value distribution, the probability that a student i of type z attends a school j can be written as

$$p_{ijz} = \frac{e^{\delta_z VA_{jz} + \mathbf{\Gamma_z} X_{ij} + \xi_j}}{\sum_k e^{\delta_z VA_{kz} + \mathbf{\Gamma_z} X_{ik} + \xi_k}},$$

where k indexes the other schools in a student's choice set. In practice, a student's choice set consists of the schools in her village.

2.2 Schools' Decision Problem

A private school j chooses its characteristics (including its fee, fee_j, and its vertical quality, $\overline{VA_j}$) simultaneously with other private schools to maximize its profits. The profits of a private school j are given by

$$\pi_j = fee_j \times s_j(\overline{VA_j}, h_j, fee_j, X_{ij}, \xi_j) - c(\overline{VA_j}, h_j, fee_j, X_j, \xi_j, s_j(\overline{VA_j}, h_j, fee_j, X_{ij}, \xi_j)),$$

where c is the school's cost function and $s_j = \sum_i p_{ijz}$ is the share of students attending j. Differentiating π_j with respect to h_j yields

$$\frac{\partial \pi_j}{\partial h_j^*} = \left(fee_j - \frac{\partial c}{\partial s_j} \right) \frac{\partial s_j}{\partial h_j^*} - \frac{\partial c}{\partial h_j^*} = 0.$$

If we assume that changing horizontal quality is costless and that c is weakly concave as a function of s_j, for h_j^* to be profit-maximizing, it must be the case that $\frac{\partial s_j}{\partial h_j^*} = 0$.[7]

Unlike private schools, public schools do not have any incentive to compete for students and take their characteristics as given. In section 4, I verify this is the case by showing that public school quality does not change for either wealthy or poor students when there are more private schools in the marketplace.

[7] The cost,c, will be weakly concave in the share of students if there are constant or increasing economies of scale in providing education.

2.3 Equilibrium

A simplified version of the model is useful to develop intuition for how new entry can incentivize the average private school to increase h_j, increasing learning for wealthy students and decreasing it for poorer students. For simplicity, let $\mathbf{\Gamma_z}$ and ξ_j be 0, so that students only respond to horizontal quality, and fix fee_j. Let h_j be the only characteristic that a school can choose. Furthermore, assume there is no outside option besides the public school. Thus, a private school j maximizes s_j, and a student i of type z's utility in a school is

$$u_{ijz} = -\delta_z(o_z - h_j)^2 + \epsilon_{ij},$$

where type z's preferred location, o_z, is 1 for high types and 0 for low types. Furthermore, assume that δ_{high} is infinite so that high types always select their best-match option (as in the traditional Hotelling model), while $0 < \delta_{low} < \infty$. This model, in which students choose their best-match school subject to an idiosyncratic shock, relates to work on the Hotelling model by De Palma et al. (1987) and De Palma et al. (1985), but here, the relative importance of the shock is correlated with a student's location on the Hotelling line.

To see how competition can affect a private school's incentives, consider the cases with one or two private schools in the market, and a public school that gives utility $u_0 \leq 0$.

Proposition 2.1. *For N=1, there is a unique equilibrium where the single private school chooses* $h^* = \max(1 - (-u_o)^{1/2}, 0)$.

Proof. When there is one private school, it maximizes its share when it minimizes the share lost to the public school. The school will discontinuously receive all the high types as long as $-(1-h)^2 \geq u_o$. It can never receive all the low types, so it will always choose its location to receive all the high types at $h \geq 1 - (-u_o)^{1/2}$. Then, the school minimizes the loss of the low types subject to this constraint at $h^* = \max(1 - (-u_o)^{1/2}, 0)$. □

Proposition 2.2. *For N=2, if* δ_{low} *is sufficiently small, the unique equilibrium is* $(h_1, h_2) = (1, 1)$.

Proof. If 1 chooses $h_1 < 1 - (-u_o)^{1/2}$, then 2 will place at $h_2 = 1 - (-u_o)^{1/2}$ and take all the high types. If $1 > h_1 \geq 1 - (-u_o)^{1/2}$, h_2 will choose $h_1 + \epsilon$, $\epsilon > 0$, and

again take all the high types. Therefore, x_1 has only two possible choices: $\frac{1}{2}$ and 1. If $h_1 = 1$, h_2 will choose 1 as well if

$$\frac{1}{2} + \frac{e^{-\delta_{low}}}{e^{-\delta_{low} u_o} + 2e^{-\delta_{low}}} > \frac{1}{1 + e^{-\delta_{low} u_o} + e^{-\delta_{low}}}.$$

We can see that when δ_{low} is sufficiently low, this will be the case since the derivative of the left-hand side (LHS) with respect to δ_{low} is always negative:

$$\begin{aligned} \frac{\partial LHS}{\partial \delta_{low}} &= \frac{-e^{-\delta_{high}}(e^{-\delta_{high} u_o} + 2e^{-\delta_{high}}) - e^{-\delta_{high}}(-u_o e^{-\delta_{high} u_o} - 2e^{-\delta_{high}})}{(e^{-\delta_{high} u_o} + 2e^{-\delta_{high}})^2} \\ &= \frac{(u_o - 1)e^{-\delta_{low} u_o}}{(e^{-\delta_{high} u_o} + 2e^{-\delta_{high}})^2} < 0 \end{aligned} \tag{4}$$

and the derivative of the right-hand side (RHS) with respect to δ_{low} is always positive:

$$\frac{\partial RHS}{\partial \delta_{low}} = \frac{-(-u_o e^{-\delta_{low} u_o} - e^{-\delta_{high}})}{(e^{-\delta_{high} u_o} + e^{-\delta_{high}} + 1)^2} > 0.$$

This shows that there is a single crossing in the profit functions from placing at 1 and $\frac{1}{2}$, implying that there exists a δ^*_{low} such that, for all $\delta_{low} < \delta^*_{low}$, the equilibrium is $(1, 1)$. □

Comparing the equilibrium for $N = 1$ and $N = 2$ shows that the implications of this model are very different from a standard Hotelling model, such as the one developed by Eaton and Lipsey (1975). In a standard Hotelling models, increasing the number of schools in the market typically leads to symmetric product differentiation. However, this is no longer the case when there is heterogeneous sorting.

When there is only one school in the market, it caters more to the low types because it is more likely to lose them. As long as the high types prefer the single school to the outside option, the school gains nothing by becoming more attractive to them. On the other hand, it gains low types continuously as it moves closer to zero. However, when a second school enters, it changes the first school's incentives. Now, it must compete aggressively to retain the high types, and since they are more responsive to horizontal quality, competing for them has a higher payoff. As a result, high types benefit disproportionately from increasing school competition.

Importantly, the implications of this model are different from those of a model where schools can influence a single quality characteristic (vertical quality) that im-

pacts all students equally. In that model, schools may vertically differentiate in response to competition such that some schools compete as “high price/high quality” schools, while others compete as “low price/low quality” schools. In this case, if increased competition results in greater vertical differentiation, it may increase inequality in students’ outcomes *across* schools, but it should not increase inequality *within* schools. Rich and poor students alike should benefit from being in a “good school” and suffer from being in a “bad school.” In contrast, in my model, inequality increases between rich and poor students in *the same school*.

For simplicity, in this section, I have focused on cases where the pure strategy Nash equilibrium strategy exists. When δ_{low} is not sufficiently low, in this special case, the pure strategy Nash equilibrium does not exist when $N = 2$. However, in sections 5 and 6, where I estimate δ_{high} and δ_{low} and then compute the equilibrium for the full model, I do not impose any restrictions on δ_{high} and δ_{low}. For a proof that the equilibrium does not exist when $N = 2$ and δ_{low} is sufficiently high, see the mathematical appendix.

In the following sections, I test the model’s predictions. In section 4, I report reduced-form estimates showing that the entry of an additional private school into the market reduces test score gains for poor students and increases them for wealthy students. In sections 5 and 6, I estimate the model structurally, and show that the average private school will still respond to competition by increasing h_j in more realistic settings where schools also differ in terms of fees and distance, and there are more than two private schools in a market.

3 Context and Data

3.1 Context

Pakistan is a natural setting to study the private schooling market in low income countries. Like many low income countries, Pakistan has experienced a rapid increase in low-cost, secular private schooling over the past two decades. Pakistani private schools are virtually unregulated and are, for the most part, unsupported by the government. As a result, they offer us a glimpse into what mechanisms may be important in a purely private market for education. In my rural sample, the private school system is quite competitive. In 2007, the median village in my sample had 2

private schools (the average had 2.8). This suggests that the equilibrium of the model for $N = 1$ and $N = 2$ described earlier may be particularly relevant in this empirical context.

Private schools do not merely cater to the wealthy. Andrabi et al. (2008) show that grassroots, rural private schools are even affordable for day laborers, noting that the average school charges the equivalent of "a dime a day." Indeed, private schools typically spend less per student than government schools, largely because government school teachers earn about five times as much as private school teachers (Andrabi et al., 2010). Government and private schools do not compete to hire teachers from the same labor market since government schools are typically constrained to hire teachers who have completed post-secondary degrees.[8] Thus, changes in the private school market are unlikely to affect public schools through the market for teachers.

Unlike private schools, government schools face relatively little competitive pressure. Historically, most teachers were hired on permanent contracts and are difficult to fire. School budgets are not determined by the number of students enrolled, and schools face little threat of closure if enrollments drop.

An additional advantage of studying private schooling in rural Pakistan is that, at the primary level, villages act as closed educational markets. Villages are typically far apart or separated by natural barriers, and students are very sensitive to distance when they make enrollment decisions (Andrabi et al. (2010), Carneiro et al. (2016)). Therefore, I can consider how competition effects equilibrium school and student outcomes at the market level.

The match between schools and students is also likely to be an important determinant of students' learning in rural Pakistan because schools' ability to cater to students' instructional needs is limited. Both government schools and private schools have less than 1 teacher per grade on average, suggesting that within-grade tracking is extremely rare and cross-grade mixing within the same class is quite common. In the median private school, the average teacher teaches 1.11 grades, while in the median government school, the average teacher teaches 1.25 grades. In this context, students requiring many different instructional levels are likely to be present in the same classroom.

[8] Government school teachers typically have more education and more teaching training than private school teachers (Andrabi et al., 2008). However, in on-going work with Jishnu Das, I show that these characteristics do not correlate with teacher value-added estimates (Bau and Das, 2016). These results suggest that while government school teachers are more qualified, they are not necessarily more effective.

Finally, neither public nor private schools in rural Pakistan typically exclude students. The majority of schools in 2007 had some form of admissions procedure (97 percent of private schools and 95 percent of government schools), which frequently consisted of an oral exam (81 percent of private schools, 54 percent of government schools) or the perusal of previous school reports (14 percent of private schools and 33 percent of government schools). However, even if a student was deemed "weak" based on this assessment, only 11 percent of private schools and 3 percent of government schools said they would refuse admission. Therefore, a student typically attends her first choice school. Furthermore, a student's primary school is unlikely to impact whether she is admitted to a given secondary school apart from its impact on her learning.

3.2 Data

For this paper, I use the Learning and Educational Achievement in Pakistan Schools study (LEAPS). The LEAPS data consists of four rounds of data collected between 2004 and 2007 in a stratified random sample of 112 rural villages in Attock, Rahim Yar Khan, and Faisalabad districts of Punjab. To be included in the sample, villages were required to have at least one private school in 2003. Therefore, the sampled villages are somewhat more populous and wealthier on average than the average village in the state. The LEAPS data allow for the construction of two partially overlapping samples containing data on household wealth: a sample of tested students surveyed in schools and a sample of students whose parents were surveyed in their geocoded homes.

Surveys were administered to schools, children attending the schools, and to a sub-sample of households. Surveyors collected geocoded data from the universe of schools within a 15 minute walk of the village. The initial sample included 823 schools in the first round (2004), and additional schools were added to the sample as they entered the market. In the first round of data collection, all third graders in a school were tested using low-stakes tests administered by the surveyors in math, Urdu, and English. These students were followed and tested in subsequent years, and an additional sample of third graders was added in 2005 and tested subsequently in 2006. In all, the panel includes 71,167 student-year test score observations (31,382 unique students). Test scores on the exams were calculated using item response theory (see

Das and Zajonc (2010) for details), so that the mean test score in the population is 0 and the standard deviation is 1. A random sub-sample of students were also given an additional survey on their household assets leading to a sample of 28,449 student-year observations for which both test score and survey data is available. I refer to this sub-sample as the *tested sample.*

A smaller sub-sample of the tested sample (1,269 households) can be matched to a household survey administered to a panel of 1,740 households in each year. Drawing from a baseline census of the villages, 16 households were sampled in each village. 12 households were randomly chosen among those who had a child attending grade three in the first survey round and 4 were chosen among households where a child eligible to be enrolled in grade 3 was not enrolled. Importantly, in round 1, the location of households was geocoded. As a result, I can compute the geographic distance between a household and the schools in the village for the 7,216 children aged 5-15 who appear in the household survey. Households were also asked about their enrollment decisions for each of the children in each year and about their asset ownership, although the asset questionnaire differed slightly from the asset questionnaire administered to the children in the tested sample. I refer to the sample of 7,216 children from the household survey as the *household sample.*

Appendix table A1 presents summary statistics for the four rounds of LEAPs data at the school level. Because of the panel structure of the data, each observation is at the school-year level. As Appendix table A1 shows, there is substantial variation across schools in terms of facilities, and private schools generally have better facilities. The average private school is mechanically located in villages with more private schools. Appendix table A2 summarizes the characteristics of the children in the tested sample; each observation is at the child-year level. While students in private schools tend to have higher test scores and more assets, there is again substantial variation in student performance and wealth. Finally, appendix table A3 reports summary statistics for the household sample, which unlike the tested sample, includes students who are not enrolled. One statistic from this table is particularly noteworthy: students do not travel far to school. Both the average public and private school student attend schools that are within 1 km of their home.

Before moving on to the analysis, it is important to describe how I estimate two key components of the analysis: children's types and schools' type-specific quality. In the next subsection, I describe how I assign types in both the tested children data

set and the household survey data set. In the subsection after that, I combine this type information with the test scores in the tested data to calculate my measure of type-specific school quality, type-specific school value-added.

3.3 Assignment of Low and High Types

Notably, neither the LEAPS questionnaire administered to the tested children nor the one administered to the household sample directly measures income. Even if the questionnaires did, measures of income are unlikely to measure socioeconomic status well in rural Pakistani villages where the majority of the population is engaged in agriculture. However, the LEAPS survey administered to tested children does ask "yes" or "no" questions about asset ownership for beds, radios, TVs, refrigerators, bicycles, ploughs, small agricultural tools, tables, chairs, fans, tractors, cattle, goats, chickens, watches, motor rickshaws, motorcycles, cars, telephones, and tubewells. To synthesize this data into a single measure that captures the relationship between wealth and learning outcomes, I adapt a simple machine learning technique.

I first regress an indicator variable for ownership of each asset on year and village fixed effects and predict the residual from this regression. This helps ensure that my measure picks up differences in wealth *within villages*. From the perspective of the schools in my model, which compete for students within a village, across-village or across-year differences in wealth are not relevant. I then interact these residual terms to generate double and triple interactions of all the asset ownership measures. Finally, I regress average student test scores across math, Urdu, and English, which have also been residualized, on these 211 covariates using a lasso regression to choose the final specification. Lasso regression is a technique for selecting the most predictive covariates when there are a large number of possible covariates. Put simply, lasso selects the covariates that maximize predictive power subject to a penalty for each additional covariate, and the penalty is chosen to maximize the out of sample predictive power of the covariates. For more details, see Tibshirani (1996).

I then use a simple rule to determine which students are "high" or "low" types based on their predicted average test scores. I code students as high types if their predicted average test scores are above the median for the private school-going population and as low types otherwise. Therefore, by construction, 50 percent of private school students in the tested data are low types. Since private school students are

wealthier than the general population, in the full student tested data, 73 percent of students overall are low types.

Now, I also need to assign types in the household data, where test scores are not available for most children. The household survey asks about a slightly different set of assets than the tested child survey. For example, the household survey asks about VCR, gun, and thresher ownership, while the tested child survey does not. To generate types for this data set, I again regress the indicator variables for each of these assets on village and year fixed effects and create double and triple-interactions of these residualized asset ownership variables, generating 299 variables. Then, for the 1,269 children who appear in both data sets, I use a logistic version of the lasso regression to select the covariates that are most predictive of their assigned types, resulting in the selection of 14 covariates. I then use the estimated coefficients to predict the probability of being a high type for all the children in the household sample. For the children who are assigned both a type from the first procedure and a probability of being a high type from the second procedure, the two measures are strongly related with a correlation of 0.58. To summarize, in the tested sample, children are assigned to their type with certainty, while in the household sample, children are assigned a probability of being each type.

3.4 Estimation of Match-Specific School Quality

To proxy for the match between each type and a particular school, I draw on the value-added literature in education economics (for example, see Chetty et al. (2014a), Chetty et al. (2014b), Rivkin et al. (2005), Kane and Staiger (2008), and Rockoff (2004)), to estimate the predicted test score gains of a low or a high type attending a given school conditional on the number of competing schools (hereafter referred to as a school's type-specific value-added). In other words, I allow a school's value-added to vary over time with how many private schools are in the market, consistent with the theoretical framework that suggests that schools will be incentivized to change their horizontal quality when the number of private schools changes. To calculate a school's type-specific value-added, I estimate the following regression for math, English, and Urdu:

$$y_{it} = \tau_{gz,1}y_{i,t-1} + \tau_{gz,2}y_{i,t-1}^2 + \omega_{gz} + \alpha_{zt} + \eta_{zjn} + \epsilon_{it} \tag{5}$$

where y_{it} is the outcome variable consisting of normalized test scores in math, English, or Urdu, i indexes an individual, g indexes a grade, z indexes a type, j indexes a school, n indexes the number of private primary schools in the market place, and t indexes a year. $\tau_{gz,1}$ is a grade-by-type specific coefficient on the lagged test score, $\tau_{gz,2}$ is a grade-by-type specific coefficient on the lagged test score squared, α_{zt} is a year-by-type fixed effect, ω_{gz} is a grade-by-type fixed effect, and η_{zjn} is a fixed effect at the school-type-number of schools in the market place level. Then, the type-specific value-added for a type z in a school j under competitive regime n is given by η_{zjn}. To construct a single measure of quality, I average across a school's estimated type-specific value-addeds in math, Urdu, and English to create a type-specific value-added in mean test scores.

The goal of this method is to estimate the causal effect of attending a school on test scores separately for low and high types. The regressions account for variation in test scores that is explained by year of test-taking, grade of test-taking, and a student's past performance. The remaining unaccounted for variation in test scores is then attributed to the school that students attended. Then, the fixed effect η_{zsn} is the average of the unaccounted for variation in test score gains for different types of students in different schools. Therefore, for these measures to be unbiased, the underlying assumption is that controlling for the lagged test scores and fixed effects accounts for most of the selection of students into schools.

In Appendix Table A4, I test whether these measures are in fact strong predictors of student test scores when students change schools.[9] I show that the type-specific value-added measures are indeed highly predictive of a student's out-of-sample gains from attending a given school. In fact, the coefficient is approximately 1, as we would expect if equation (5) is the correctly specified equation for student test score outcomes. Appendix Table A4 also indicates that the type-specific value-addeds are similarly predictive of test score gains for both high and low types and that, while own type-specific value-added is a strong predictor of a child's test scores gains, the type-specific value-added for the other type has no additional predictive power.

With these value-added estimates, I can also directly compare value-addeds for high and low types in the same schools to see if horizontal quality appears to be important for students' outcomes. Figure 1 plots each private school's value-added

[9]To avoid any spurious correlation between the student's own test scores and the school's estimated value-added, I recalculate each school's type-specific value-added excluding the student whose test scores are being predicted.

estimate for high types against its value-added estimate for low types. There is a strong correlation between the two of 0.61, indicating that vertical quality plays an important role in determining students' outcomes. However, there is still evidence of substantial variation in the figure, with some schools performing well for low types but poorly for high types and vice versa. A correlation of 1 would indicate horizontal quality is unimportant, but the correlation of 0.61 leaves substantial scope for match-specific school quality to affect student test scores. Moreover, this low correlation is not merely due to attenuation bias: an F-test of the estimated interactions between school fixed effects and an indicator variable for being a high type in equation (5) rejects the possibility that these effects are jointly equal to 0 with a F-statistic of 1,310 when the outcome variable is mean test scores.

4 Reduced-Form Evidence

In this section, I provide several pieces of reduced-form evidence in support of the model's assumptions and predictions. I first report descriptive evidence that wealthy students are more informed about their educational options and more sensitive to school quality when they make enrollment decisions using the LEAPS household survey, consistent with the model's assumption that $\delta_{high} > \delta_{low}$. Then, using the sample of tested children, I exploit the exit and entry of private schools into the education market over time to test how increased school competition affects private school students' test scores and provide evidence that the estimates from the exit-entry regressions are not driven by selection or omitted variable bias. I then present reduced-form evidence that the results in the previous subsections are not driven by private schools increasing their match to wealthier students so that they can charge higher prices. Finally, in the last subsection, I show that changes in vertical school quality alone cannot account for the entire effect of school competition on inequality in test scores.

4.1 Descriptive Evidence

To test whether the assumption that high types are more responsive to type-specific quality when they make enrollment decisions is reasonable, I estimate five associations in Table 1 using the household survey. First, I regress an indicator variable that

is equal to 1 if a child ever changes schools over the course of the survey on the probability that the child is a high type. Here, I restrict the sample to children who are always enrolled in school between 2004 and 2007. Column 1 shows that there is a strong positive and statistically significant relationship: moving from having a 0 probability of being a high type to a probability of 1 increases the likelihood that a child changes schools between 2004 and 2007 by 31 percentage points. In Column 2, I regress an indicator variable for whether a child's parents' know her teacher's name on the probability that a child is a high type. Again, there is a strong and statistically significant relationship: moving from a 0 probability of being a high type to a probability of 1 increases the likelihood that parents know the teacher's name by 26 percentage points. In Column 3, I regress an indicator variable equal to 1 if parents report choosing a child's school based on distance on the probability the child is a high type. Here, the sample size drops substantially since parents were only asked why they chose a given school in 2004. I find that high types are less likely to choose a school based on distance, though this effect is not significant. In Column 4, I run a similar regression with an indicator variable for choosing a school based on quality as the outcome. Column 4 indicates that high types are significantly more likely to report choosing a school based on quality. Finally, in Column 5, I regress a school's estimated type-specific value-added for a household on parents' ranking of that school's quality (from 1-5), the household's probability of being a high type, and the interaction of these two variables. The interaction term, though marginally significant, indicates that moving from having a 0 probability of being a high type to a probability of 1 nearly doubles the association between a parent's rankings of school quality and the school's type-specific value-added.

Taken together, the associations reported in Table 1 show that low types both report caring less about quality when they make enrollment decisions and have less information with which to make these decisions. This suggests that the model's assumption that $\delta_{high} > \delta_{low}$ is reasonable. Nonetheless, in Section 5, I will test this assumption more directly by estimating δ_{low} and δ_{high}.

4.2 Evidence From School Entry and Exit

If wealthier students (high types) are hetter able to sort into their best match schools, the model predicts that, on average, the match between high types and private schools

may increase at the expense of poorer students (low types) in response to competition. To test whether competition makes high types better off relative to low types, I exploit variation in the number of private schools due to private school entry and exit. I examine whether this variation in the number of private schools has heterogeneous effects on low and high types using a difference-in-differences framework.

Formally, to estimate the heterogeneous effects of the number of private schools on the learning of high and low types, I estimate the regression

$$\begin{aligned} y_{it} = \rho_0 + \rho_1 num\_pri_{vt} + \rho_2 num\_pri_{vt} \times \mathbb{1}_{high} + \eta_{zs} + \alpha_{zt} + \omega_{gz} \\ + \lambda_{gz} y_{i,t-1} + \phi_{gz} y_{i,t-1}^2 + \epsilon_{it}, \end{aligned} \tag{6}$$

where i indexes students, g indexes grades, z indexes types, v indexes villages, j indexes schools, and t indexes years. y_{it} is then a student-year level test score in math, Urdu, or English, α_{zt} are year-by-type fixed effects, ω_{gz} are grade-by-type fixed effects, and η_{zj} are school-by-type fixed effects. λ_{gz} and ϕ_{gz} are grade-by-type specific coefficients on test scores and lagged test scores. Standard errors are clustered at the village-level.

This regression allows me to estimate the change in the test scores of low and high type individuals induced by the entry of a new private school. The coefficients of interest are ρ_1 and ρ_2. Intuitively, controlling for the school-by-type fixed effects and the rich function of lagged test scores means that ρ_1 and ρ_2 are identified by changes to the value-added for a type within a school when another school exits or enters the market.

Table 2 reports the results from this exit-entry identification strategy for math, Urdu, English, and mean test scores. Columns 1, 4, 7, and 10 restrict ρ_2 to be zero and estimate the effect of adding an additional private school on the average private school student. With the exception of math (Column 1), where there is a marginally significant and negative effect of adding a private school, increased competition does not have any strong or significant effect on the *average* student.

In Columns 2, 5, 8, and 11, ρ_1 is no longer restricted to be zero. The new results reveal that estimates of the effect of competition on the average student conceal important heterogeneity in the effects of competition. Adding an additional private school has a negative effect on average test scores for low types and a positive effect for high types (Column 11). An additional private school in the market place increases

inequality in yearly test score gains by 0.08 standard deviations. In Columns 3, 6, 9, and 12, I add additional controls that might capture alternative explanations for the heterogeneous effects of competition. I control for average student-teacher ratios, allowing them to affect low and high types differently, and for mean lagged test scores within a school, again allowing them to affect high and low types differently. The second control is intended to capture any peer effects on outcomes due to increased sorting of students into schools by ability or wealth when the number schools grows. The inclusion of these controls only strengthens the evidence of heterogeneous effects.

These heterogeneous effects are consistent with the model. In section 2, I suggested that schools largely cater to students by choosing their instructional level to better match students' needs. I can use data on how students perform on specific questions to test this hypothesis more directly. For the set of questions asked in all four years, I code questions that less than one-third of students answered correctly in the first year as hard and questions that more than two-thirds answered correctly as easy. Now, I re-estimate equation (6) with the share of easy questions answered correctly and the share of hard questions answered correctly as the outcome variables. Since I am combining questions across subjects, I control for lagged mean test scores.

If schools respond to competition by moving their instructional level closer to the needs of more advanced, wealthier students, we expect high types to perform better on hard questions when there are more private schools in the market and low types to perform worse on easy questions. Table 3 confirms that this is the case. An additional private school in the market place reduces the share of easy questions low types answer correctly by 1 percentage point (significant at the 5 percent level) and increases the share that high types answer correctly by 3 percentage points (significant at the 1 percent level).

Using the same identification strategy, I can also test the assumption in the model that public schools do not respond to competitive incentives. In table 4, I re-estimate equation (6) for public school students. Consistent with the idea that public schools do not respond to increased competition from private schools, num_pri_{vt} and $num\_pri_{vt} \times \mathbb{1}_{high}$ have small and insignificant effects on the test scores of public school students.

4.3 Robustness of School Exit and Entry Results

In this section, I discuss several possible sources of bias in the estimation of ρ_1 and ρ_2 and test whether my estimates are robust to them. If the number of private schools in the market is correlated with increasing inequality in test scores for low types and high types or other factors that lead to greater inequality in student outcomes, the estimates of ρ_1 and ρ_2 may be biased. To test whether this is the case, I first test whether the number of private schools is associated with changes over time in different village-level measures of wealth. I create a measure of household wealth, following Filmer and Pritchett (2001), by predicting the first principal component from a principal components analysis over indicator variables for asset ownership for the different assets in the household survey (see appendix table A2). I then take the village-year average of this measure and regress it on the number of private schools in the market, controlling for village and year fixed effects. The coefficient for number of private schools (0.017) is small and statistically insignificant. Using the gini coefficient for this asset index as the outcome variable or the percent of households that own land in a village-year yields similar results; the coefficients are again small (0.001 and -0.001 respectively) and insignificant. Thus, there is no strongly relationship between the number of private schools in a village and village-level trends in wealth or inequality.

In table 5, I test whether the results are driven by pre-trends in test score outcomes more explicitly. I include the forward lag of the number of private schools and its interaction with being a high type in equation (6). These forward lags are placebo tests; they test whether the number of private schools in year $t+1$ had an effect on outcomes in year t before the entry or exit event took place. If the main effects are indeed driven by pre-trends, one would expect the estimates of the forward lags to be similar to the estimates of ρ_1 and ρ_2. There is no evidence that this is the case. Across all subjects and for both high and low types, the forward lags are small and statistically insignificant.[10]

The estimates of ρ_1 and ρ_2 may also be biased if the creation of new private schools leads new students to enroll in private schools who would otherwise attend public schools. These new students may be unobservably worse than the low types already attending private schools, leading to a negative correlation between the number of

[10] The smaller sample size in table 5 relative to table 2 is due to the fact that the forward lag for the number of private schools is missing in 2007, causing these observations to be dropped.

private schools in the market place and the performance of low types in private schools. To ensure that my results are robust to this possibility, I re-estimate equation (6) restricting my sample to students who always attend private schools. Appendix table A5 reports the results of this regression. The results are similar: the addition of a private school in the village increases inequality in test scores between low and high types by a statistically significant 0.11 standard deviations.

4.4 Role of School Fees

In this paper, I have emphasized the role low and high types' differential responsiveness to school quality when they make enrollment decisions can play in incentivizing schools to respond to competition by increasing their horizontal quality for high types (and reducing it for low types). However, low types and high types are likely to differ in their responsiveness to prices as well. If high types are less price sensitive than low types, it is possible that schools are increasing their horizontal quality for high types so that they can charge higher prices, maximizing their profits. In this subsection, I provide some reduced-form evidence that this is not the case.

First, if schools are increasing their horizontal quality predominantly to increase their prices, we would expect private schools to be socioeconomically segregated with high types attending more expensive schools. In fact, this is not the case. Private schools are quite mixed; the median school is 43 percent high types. Figure 2, which plots the distribution of the share of high types in a private school-year illustrates this fact. Since the share of high types will mechanically appear balanced in villages with only one private school, the figure only plots the share of high types for private schools in villages with two or more private schools. As the figure indicates, few schools are extremely segregated.

Second, if schools are in fact increasing their horizontal quantity for high types so that they can charge higher fees and this effect is driving the results in table 2, then the schools where inequality in test scores is increasing should be more expensive. In appendix table A6, I directly test if this is the case. I re-estimate equation 6 for private school students but now include $num\_pri_{vt} \times \mathbb{1}_{high} \times fee_{jt}$ and $num\_pri_{vt} \times fee_{jt}$, where fee_{jt} is the fee for school j in year t measured in 1000s of Rupees. Appendix table A6 reports the results of these regressions. Appendix table A6 indicates that the results in table 2 are not driven by high fee schools. The triple interaction terms

$num\_pri_{vt} \times \mathbb{1}_{high} \times fee_{jt}$ and $num\_pri_{vt} \times fee_{jt}$ are typically small and insignificant.

4.5 Role of Vertical Quality

The reduced-form results in table 2 suggest that increasing the number of private schools in the market increases inequality in test scores between high and low types. However, these effects may be driven by increases in inequality in test scores across schools in addition to increases in inequality within schools. Changes in vertical quality may contribute to increased inequality in test scores if high types are more likely to attend schools that respond to competition by increasing their vertical quality or low types are more likely to attend schools that decrease their vertical quality. Understanding the total effect of competition on inequality in learning is important in its own right, but it is also useful to directly test whether there is reduced-form evidence that changes in horizontal quality alone lead to greater inequality in learning.

To do so, I take advantage of the fact that growth in test score inequality between high and low types due to changes in vertical quality must be driven by the composition of schools. Since vertical quality benefits both high and low types in the same school equally, growth in inequality in test scores can only arise if there are more high types in schools where vertical quality is increasing (or low types in schools where vertical quality is decreasing). Intuitively, if I re-weight my observations so that there are equally many high and low types in each private school, then inequality in the outcomes of high and low types (the coefficient ρ_2) cannot be driven by vertical quality. In the weighting appendix, I discuss a weighting process that removes the effect of vertical quality on inequality in test scores in more detail.

Appendix table A7 re-estimates regression 6 with the weighted observations. Consistent with the previous results, as Column 4 indicates, an additional private school increases inequality in mean test scores between high and low types by a statistically significant 0.1 standard deviations. The fact that the estimates are so similar to those in Table 2 may suggest that compositional differences do not drive most of the results, consistent with the fact that schools are not highly segregated in this context.

5 Empirical Analysis of School Choice

The reduced-form estimates are consistent with private schools responding to competition by increasing their horizontal quality for wealthier students and reducing it for poor students. In the next two sections, I explicitly test the model's assumptions and predictions by estimating the model outlined in section 2. In this section, I estimate the parameters of equation (4) using a discrete choice model with unobserved school quality in the spirit of Berry et al. (2004). Since my data has repeated observations of schools' characteristics and students' enrollment decisions over time, I now add the t subscript, for the year the data was collected, to equation (4).

Then, the new equation for the utility of a student i of type z in school j and year t is

$$u_{ijzt} = \delta_z VA_{jzn} + \mathbf{\Gamma_z^{indiv}} X_{ijt}^{indiv} + \zeta_{jn} + \epsilon_{ijt}, \tag{7}$$

where X_{ij}^{indiv} is the set of characteristics affecting school choice that vary at the individual-level, consisting of the interaction of school fees and an indicator variable for being a high type, the effect of distance on high and low types, a control for a child having been in school j in the previous period interacted with type, and controls for a boy attending a school marked as an all boys school or a girl attending a school marked as an all girls school.[11] ζ_{jn} is the school fixed effect, which is allowed to vary non-parametrically with the number of private schools in the market, n, and is equal to

$$\zeta_{jn} = \xi_{jn} + \mathbf{\Gamma_z^{school}} X_{jn}^{school}. \tag{8}$$

Here, ξ_{jn} is the school's unobserved quality and X_{jn}^{school} is a school j's average, inflation-adjusted fees under competitive regime n. Therefore, my parameters of interest are the coefficients in a student's utility function: $\{\mathbf{\Gamma_z^{school}}, \mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}\}$.

The distinction between equation (7) and equation (8) is important. The variables $VA_{j,low,n}$, $VA_{j,high,n}$ and X_{ijt}^{indiv} vary at the individual-level while X_{jn}^{school} varies at the school-level. Therefore, the coefficients $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}\}$ can be estimated

[11] $\mathbf{\Gamma_z^{indiv}}$ does not include the interaction between school fees and being a low type since including both this control and the interaction of school fees with being a high type would be collinear with the school fixed effect. Instead, I estimate the baseline effect of school fees on school choice, $\mathbf{\Gamma_z^{school}}$, separately.

together with the school fixed effects, ζ_{jn}, using a maximum likelihood procedure with individual-year level choice data. Intuitively, including the school fixed effects accounts for any unobserved characteristics of the school that may affect school choice and be correlated with a school's type-specific value-added, which would otherwise bias the estimates of δ_z. For example, if schools with higher value-addeds also have other attractive features like toilets, the estimates of δ_{low} and δ_{high} would be positively biased in the absence of school fixed effects.

$\mathbf{\Gamma_z^{school}}$ is not identified in this procedure since X_{jn}^{school} is collinear with the school effects. For this reason, I separate my estimation into two stages. In the first stage, I estimate $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}\}$ using maximum likelihood, and in the second stage, I estimate $\mathbf{\Gamma_z^{school}}$ using the general method of the moments. As I will show section 6, it is not necessary to estimate $\mathbf{\Gamma_z^{school}}$ to estimate private schools' equilibrium choices of horizontal quality since the baseline effect of school fees is subsumed by the estimates of ζ_{jn}. Nonetheless, estimates of $\mathbf{\Gamma_z^{school}}$ are useful since they allow us to measure the importance of value-added relative to fees for both high and low types and since the baseline effect of prices should be strongly negative, estimating this coefficient provides an additional check of the structural model. Therefore, the estimation procedure for $\mathbf{\Gamma_z^{school}}$ is documented in estimation appendix A.

To allow students' choices of schools to depend on distance, I use the household data rather than the tested data (which does not include distances) to estimate the demand for schooling. Additionally, to ensure that there is no correlation between the estimated value-addeds and the errors in the discrete choice model, I drop children in the household survey data who also appear in the tested sample. In other words, I estimate the school characteristics $VA_{j,low,n}$, $VA_{j,high,n}$ from the tested sample, but use the child characteristics and observed choices in the household sample to estimate the model.

5.1 Identification of $\{\mathbf{\Gamma_z^{school}}, \mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}\}$

Since I have previously assumed that ϵ_{ijt} is a type 1 extreme value error, the probability that a student i of type z attends school j in year t can be written as

$$p_{ijtz} = \frac{e^{\delta_z VA_{j,z,n} + \mathbf{\Gamma_z^{indiv}} X_{ijt}^{indiv} + \zeta_{jn}}}{\sum_k e^{\delta_z VA_{k,z,n} + \mathbf{\Gamma_{high}^{indiv}} X_{ikt}^{indiv} + \zeta_{kn}}}.$$

However, recall that for each child in the household survey, I have estimated the probability of being a high type rather than assigning each child a binary type. Therefore, I write the expression for the probability that a child i attends a school j in year t as follows:

$$p_{ijt} = P(type_i = high)\frac{e^{\delta_{high}VA_{j,high,n}+\mathbf{\Gamma^{indiv}_{high}}X^{indiv}_{ijt}+\zeta_{jn}}}{\sum_k e^{\delta_{high}VA_{k,high,n}+\mathbf{\Gamma^{indiv}_{high}}X^{indiv}_{ikt}+\zeta_{kn}}} \\ + (1 - P(type_i = high))\frac{e^{\delta_{low}VA_{j,low,n}+\mathbf{\Gamma^{indiv}_{low}}X^{indiv}_{ijt}+\zeta_{jn}}}{\sum_k e^{\delta_{low}VA_{k,low,n}+\mathbf{\Gamma^{indiv}_{low}}X^{indiv}_{ikt}+\zeta_{kn}}}, \tag{9}$$

where $P(type_i = high)$ is the probability that i is a high type that was previously estimated with the logistic lasso regression and $VA_{j,z,n}$ is type z's value-added in school j during competitive regime n.

Using equation (9), I choose the parameters $\{\mathbf{\Gamma^{indiv}_z}, \delta_{low}, \delta_{high}, \zeta_{zn}\}$ that maximize the log likelihood function

$$\sum_{ijt} \mathbb{1}_{ijt} log(p_{ijt}),$$

where $\mathbb{1}_{ijt}$ is an indicator variable equal to 1 if i attends school j in year t. Intuitively, this estimation procedure chooses the parameters which make students' observed enrollment decisions most likely. More details of the estimation procedure are described in estimation appendix A.

5.2 Estimates

Table 6 reports the estimates for the key parameters of the utility function. Reassuringly, the directions and relative magnitudes of the coefficients are reasonable. Both types respond negatively to distance, but low types are much more sensitive to distance, consistent with the descriptive evidence in table 1. Both types also respond negatively to fees (measured in 1000s of Rupees), but the wealthier high types are somewhat less sensitive to school fees.

Importantly, the estimates confirm the model's assumption that low types are less responsive to match-specific quality relative to high types ($\delta_{high} > \delta_{low}$). While low types do respond positively to their predicted test score gains in a school, high types are much more responsive. An increase in predicted test score gains of 1 standard

deviation increases the utility of attending a school for high types by five times as much as it increases the utility for low types.

Since coefficients in the utility function are difficult to interpret, I also compare the effects of type-specific value-added for high and low types in two other ways. First, I can compare what change in fees is equivalent to a 1 student-level standard deviation increase in value-added. For high types, an increase in value-added of 1 student-level s.d. is equivalent to a reduction in fees of 574 Rupees. For low types, it is only 93 Ruppees. Second, I can calculate the average derivative of each school's enrollment with respect to type-specific value-added and compare these derivatives for the value-addeds of high and low types. I find that, on average, a school will increase its student population by six times as much by increasing value-added for high types relative to low types.

6 Equilibrium Choice of Horizontal Quality

While the simplified model in Section 2.3 illustrates how poor students' lower responsiveness to school quality can lead schools to respond to competition by selecting a higher horizontal quality for wealthier students (h_{jt}), the restricted model ignores many important dimensions of school competition and school choice. It does not take into account price competition, and it does not allow students to make enrollment decisions based on other school characteristics besides school quality. In this section, I combine the estimates from section 5 with the fact that a school's equilibrium choice of h^*_{jt} must satisfy the condition $\frac{\partial s_j}{\partial h^*_{jt}} = 0$ (see section 2.2) to estimate schools' implied equilibrium choices of h^*_{jt} given their other characteristics and the other characteristics of schools in the market. This allows me to check if a richer model that allows for vertical quality, distance, and fees would also predict that private schools respond to competition from new entrants by increasing their horizontal quality for wealthy students at the expense of poorer students.

6.1 Estimation Strategy

Using the condition that $\frac{\partial s_j}{\partial h^*_{jt}} = 0$, I can identify each school's h^*_{jt} given their other characteristics, including vertical quality, and the characteristics of the other schools

in the market. To see this, rewrite $\frac{\partial s_{jt}}{\partial h_{jt}}$ as

$$\frac{\partial s_{jt}}{\partial h_{jt}} = \sum_{it} P(type_i = high)\frac{\partial p_{ij,high,t}}{\partial h_j} + (1 - P(type_i = high))\frac{\partial p_{ij,low,t}}{\partial h_j}, \tag{10}$$

where

$$\begin{aligned}\frac{\partial p_{ij,low,t}}{\partial h_{jt}} &= 2\delta_{low}\beta h_j(p^2_{ij,low,t} - p_{ij,low,t}) \\ \frac{\partial p_{ij,high,t}}{\partial h_{jt}} &= \delta_{high}p_{ij,high,t}(2\beta - 2\beta h_j)(1 - p_{ij,high,t}).\end{aligned} \tag{11}$$

Then $p_{ij,high,t}$, the probability a student i attends school j in year t if she is a high type, and $p_{ij,low,t}$ can be estimated using the demand system from the previous section. As before, $P(type_i = high)$ is given by the lasso logistic regression.

I assume that the schools in the data choose their equilibrium characteristics. If I observed vertical quality for a school, I would be able to directly estimate that school's choice of horizontal quality using equation (10). However, in practice, I only observe my estimates of a school's type-specific value-addeds for low and high types. As equations (1) and (2) show, these are combinations of a school's horizontal and vertical value-added. Observing the estimates of the type-specific value-added, $\widehat{VA_{j,high,n}}$ and $\widehat{VA_{j,low,n}}$ is not sufficient to identify a school's choice of vertical quality, $\overline{VA_{jn}}$. However, if β is known, manipulating equations (1) and (2) shows that $\overline{VA_{jn}} = \frac{VA_{j,high,n} - VA_{j,low,n}}{2\beta} + \frac{1}{2}$. Therefore, to identify schools' equilibrium choices of horizontal quality, I solve for both β and h_{jt}.

I estimate

$$\min_{\beta,h_{jt}} \sum_j \sum_z \left(\widehat{VA_{j,z,n}} - (\overline{VA_{jn}} + \beta(o_z - h_{jt})^2)\right)^2$$

subject to

$$\frac{\partial s_{jt}}{\partial h_{jt}} = 0 \quad \text{for each } j.$$

This is equivalent to choosing the β that minimizes the distance between the type-specific value-addeds of schools predicted by the model and the type-specific value-addeds observed in the data subject to the constraint that schools choose their profit maximizing horizontal quality. Estimation appendix B describes how this procedure

is implemented in greater detail.

Finally, my estimation strategy assumes that a pure strategy equilibrium exists and that schools are playing a pure strategy equilibrium. While the existence of a pure strategy equilibrium is not guaranteed in this model, the fact that I can successfully solve for a h_{jt} for each school j in each year t that satisfies the constraint $\frac{\partial s_{jt}}{\partial h_{jt}} = 0$ imposed by profit-maximization proves that a pure strategy equilibrium exists in the data.

6.2 Estimates

I estimate that β is 0.482. This implies that horizontal quality can play a large role in students' outcomes; choosing an instructional level that is optimal for high types will reduce low types' test scores by 0.482 standard deviations relative to choosing the instructional level that is optimal for low types and by 0.362 standard deviations relative to choosing an instructional level in the center of the Hotelling line. The average value of h^*_{jt} is 0.838, implying that private schools typically choose instructional levels that strongly advantage high types over low types.

The estimates of h^*_{jt} are consistent with the prediction that horizontal quality for wealthy students increases when there are more private schools in the market place. A regression of the estimated h^*_{jt} on the number of private schools in the market shows that an additional private school in the market place increases h^*_{jt} by 0.004 (with a standard error of 0.001). Since the number of private schools is right skewed, I also regress the log difference in test scores between high and low types due to a school's choice of horizontal quality on the log number of private schools. In this regression, a 1 percent increase in the number of private schools increases the difference in test scores between high and low types due to horizontal quality by a statistically significant 6 percent (se=0.004).

Figure 3 plots the effect of the number of private schools on the equilibrium choice of h^*_{jt} and the implied effects on the mean value-addeds for high and low types. An interesting implication of the estimates (due to the high average value of h^*_{jt}) is that increasing the number of private schools in a village should increase the test scores for high types less than it decreases the test scores of low types. This prediction matches the reduced-form findings in table 2, which suggest that the negative effect of school entry on the low types' test scores is greater than the positive effect on the

high types' test scores. Thus, the structural estimation of the equilibrium values of h^*_{jt} using the data confirms the intuition from the simplified model and is consistent with the reduced-form findings. Private schools respond to increased competition by increasing their horizontal quality for high types and reducing it for low types. This leads to lower test scores for low types and higher test scores for high types in the same schools.

7 Conclusion

In this paper, I develop a Hotelling-style model of competition where consumers' responsiveness to their match with a product is correlated with their type. While I tested the implications of this model in educational markets in Pakistan, the theoretical contribution is more general. When individuals are differentially informed (or differentially responsive to match-specific quality) and how informed they are is related to their type, increased competition can have adverse impacts on less informed consumers. These results may apply to other situations where information about horizontal quality is limited and opportunities to experiment with products are rare. For example, political markets and other product markets, such as health care, often have these features.

I showed that a model where schools compete on horizontal quality is consistent with school choice behavior and educational outcomes in rural Pakistan, where there is a thriving low cost private schooling market. Consistent with the assumptions of the model, high types are quite responsive to their predicted test score gains when they choose schools, while low types are not very responsive. I found that while wealthier "high" types benefit from increased competition, "low" types suffer. My estimates imply that an additional private school in the market will lead to a nearly 0.1 standard deviation gap in the gains of low and high types. Over the course of five years of primary schooling, the magnitude of this gap will be half the size of the black-white test score gap in the United States (Fryer and Levitt, 2004).

I further confirm that the implications of my simplified theoretical model are relevant for Pakistan by estimating schools' equilibrium choices of horizontal quality, taking into account the population characteristics and market structure of different villages and allowing distance and prices to affect students' enrollment decisions. This direct test of the model shows that private schools do respond to increased competition

by increasing their match with wealthy students and reducing their match with poor students.

The recent growth of low cost private schools in Pakistan mirrors that of much of the rest of the developing world. Educational markets and enrollment rates in private schooling are similar across South Asia and Sub-Saharan Africa. Given the growth of private schooling markets in low income countries, it is important to understand how the interaction of market mechanisms and information can impact the educational outcomes of different groups of students. This model may also have implications for schooling in the United States, where school districts are increasingly adopting some form of school choice and policymakers are promoting a high accountability approach to improving educational quality. As schools compete for students in districts with school choice like Boston and New York, some of the same market mechanisms may play out. This is an area that warrants further research.

My findings suggest that improving the ability of poor students to sort into their best match schools will have long-run supply-side effects on schools' horizontal quality. Interventions which help disadvantaged types sort will even aid those who don't respond to the intervention by changing schools' choices of horizontal quality. In general, these results suggest that as some policymakers move toward offering vouchers on a large scale, as they have already done in Chile, they may also want to adopt informational interventions.

More broadly, this paper illustrates the importance of horizontal quality for students' educational outcomes. My estimate of β suggests that a school's choice of horizontal quality can decrease test scores for a type by as much as 0.482 standard deviations. This suggests that interventions that affect the match between schools and students, rather than vertical school quality, could play an important role in increasing students' outcomes. In light of low levels of learning in many low income countries, increasing the match between schools and students may be an important and relatively low cost policy tool for improving student outcomes.

Finally, this paper raises an important methodological point. As my results show, estimates of the effect of school competition that do not take into account heterogeneity in the effects of competition on poorer and wealthier students may find moderate or null effects. However, these moderate effect sizes may mask large positive effects on wealthy students and negative effects on poorer students.

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Figures

Figure 1: Comparison of School Value-Addeds for High and Low Types

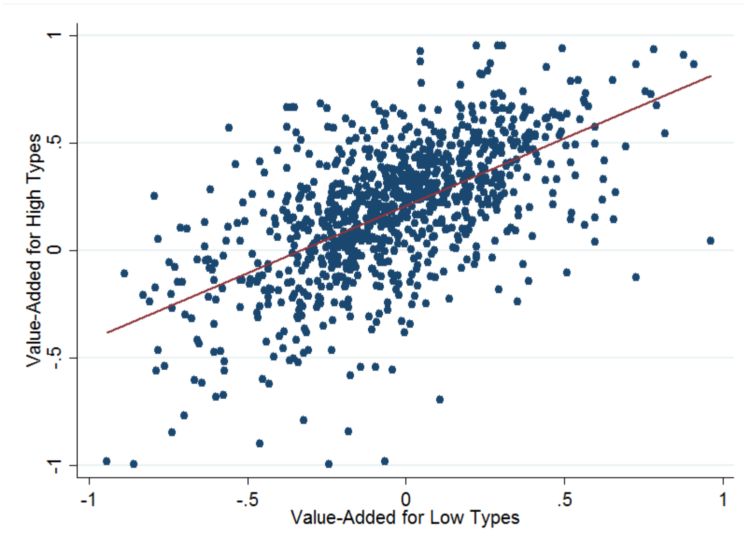


Figure 2: Distribution of the Percent High Types in Private Schools

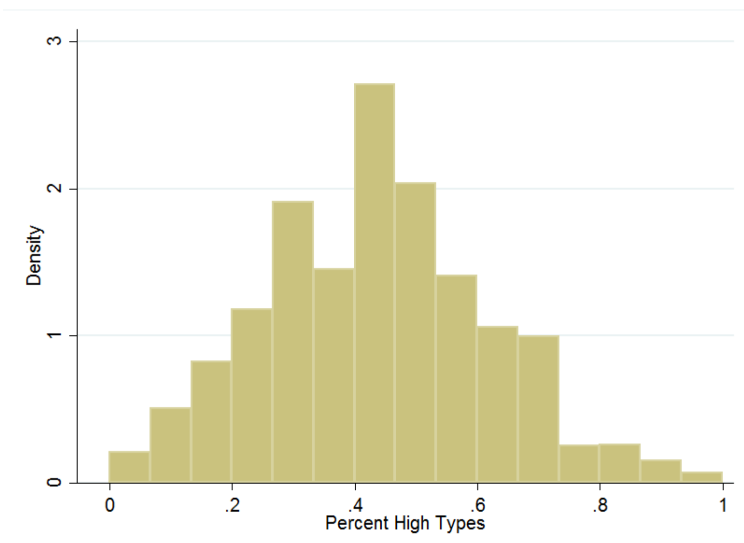
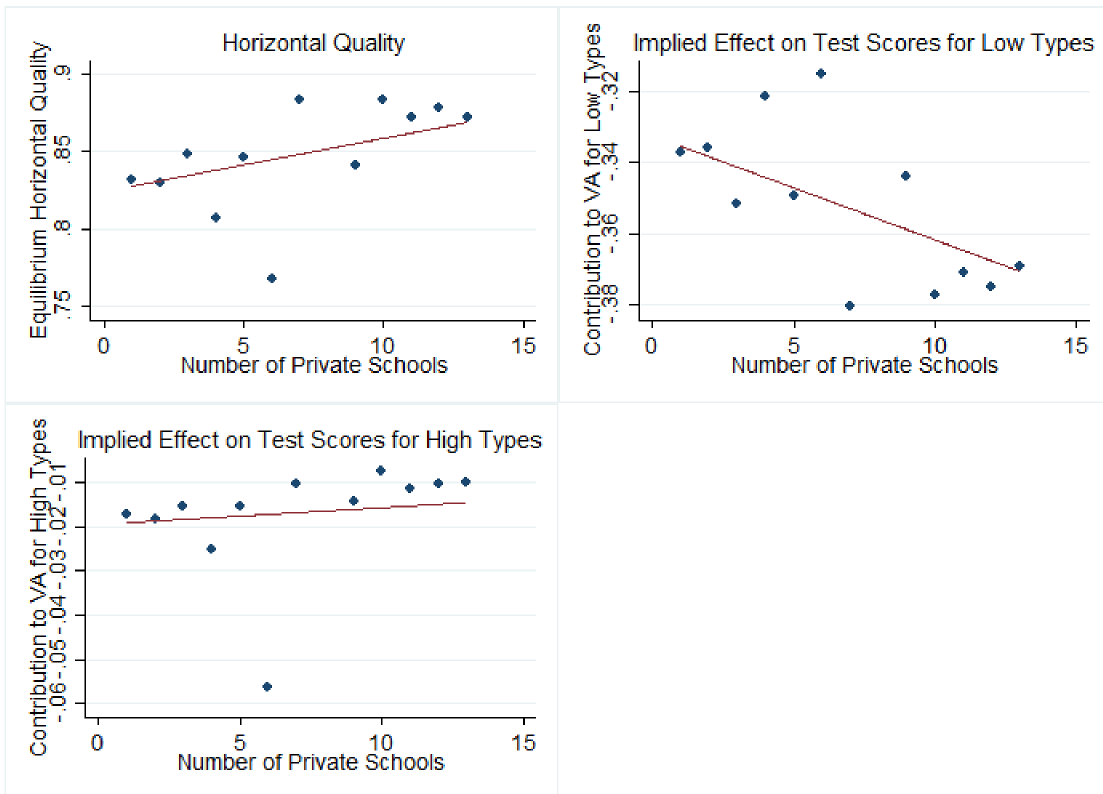


Figure 3: Relationship Between the Number of Private Schools and Equilibrium Choice of Horizontal Quality



Tables

Table 1: Knowledge of Educational Quality and Determinants of School Choice

	(1) Changed Schools	(2) Knows Teacher's Name	(3) Chose School for Distance	(4) Chose School for Quality	(5) Mean School VA
$P(type_i = high)$	0.310***	0.261***	-0.162	0.574***	0.153*
	(0.059)	(0.048)	(0.104)	(0.090)	(0.081)
$rank_{ij}$					0.050***
					(0.010)
$P(type_i = high) \times rank_{ij}$					0.043*
					(0.023)
Mean	0.347	0.532	0.427	0.210	0.022
Observation Level	Child	Child-Year	Child	Child	Parent-School-Year
Number of observations	5,621	13,645	2,873	2,873	22,826
Clusters	1,694	1,695	1,153	1,153	684
Adjusted R^2	0.008	0.005	0.002	0.038	0.031

This table reports descriptive statistics on high types' and low types' knowledge of educational markets and the determinants of their enrollment decisions in the household survey data. Column 1 regresses an indicator variable for changing schools at least once over the course of the study period on the probability of being a high type for children who were always enrolled in school; each observation is a child, and the standard errors are clustered at the household level. Column 2 regresses an indicator variable for whether a parent knows a child's teacher's name on the probability of being a high type; an observation is a child-year, and the standard errors are clustered at the household level. Column 3 regresses an indicator variable for if a parent reports distance is the main reason they choose their child's school on the probability of being a high type; an observation is a child, since the question was only asked round 1, and the standard errors are clustered at the household level. Column 4 regresses an indicator variable for if a parent reports quality is the main reason they chose their child's school on the probability of being a high type; an observation is a child, since the question was only asked in round 1, and the standard errors are clustered at the household level. Column 5 regresses a school's expected value-added for a household on parents' assessment of the school's quality, the household's probability of being a high type and their interaction. Each observation is at the parent-school-year level, and standard errors are clustered at the school level.

Table 2: Effect of Number of Private Schools on Test Scores

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		Math			English			Urdu			Mean	
$\mathbb{1}_{high} \times num\_pri_{vt}$		0.101*	0.102*		0.071*	0.078*		0.059	0.055		0.079**	0.083**
		(0.054)	(0.057)		(0.043)	(0.044)		(0.043)	(0.042)		(0.035)	(0.036)
num_pri_{vt}	-0.069*	-0.098**	-0.106***	-0.030	-0.049	-0.054	0.003	-0.024	-0.029	-0.028	-0.055*	-0.064**
	(0.039)	(0.038)	(0.036)	(0.048)	(0.042)	(0.044)	(0.040)	(0.037)	(0.035)	(0.039)	(0.031)	(0.027)
Peer Controls	N	N	Y	N	N	Y	N	N	Y	N	N	Y
Student-Teacher Ratio Controls	N	N	Y	N	N	Y	N	N	Y	N	N	Y
Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788
Clusters	106	106	106	106	106	106	106	106	106	106	106	106
Adjusted R^2	0.575	0.570	0.583	0.602	0.597	0.602	0.634	0.629	0.640	0.688	0.685	0.703

This table reports estimates of the effect of the number of private schools in the market place on test scores for high and low types attending private schools. The regressions use data from the LEAPS tested children data. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level.

Table 3: Effect of Number of Private Schools on Performance on Hard and Easy Questions

	(1) Easy Questions	(2) Hard Questions
$\mathbb{1}_{high} \times num\_pri_{vt}$	0.005	0.031***
	(0.010)	(0.011)
num_pri_{vt}	-0.010**	-0.014
	(0.005)	(0.009)
Peer Controls	Y	Y
Student-Teacher Ratio Controls	Y	Y
Lagged Test Score Controls	Y	Y
School by Type FE	Y	Y
Grade by Type FE	Y	Y
Year by Type FE	Y	Y
Mean	0.891	0.334
Number of observations	6,788	6,788
Clusters	106	106
Adjusted R^2	0.314	0.682

This table reports estimates of the effect of the number of private schools in the market place on performance on easy and hard questions for high and low types attending private schools. The regressions use data from the LEAPS tested children data set. Lagged test score controls consist of the mean lagged test score and its square interacted with grade by type fixed effects. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Observations are at the student-year level. Standard errors are clustered at the village level.

Table 4: Effect of Number of Private Schools on Test Scores in Government Schools

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{high} \times num\_pri_{vt}$	-0.043	-0.010	0.010	-0.015
	(0.037)	(0.040)	(0.039)	(0.035)
num_pri_{vt}	0.004	-0.012	-0.045	-0.016
	(0.050)	(0.041)	(0.034)	(0.039)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	13,829	13,829	13,829	13,829
Clusters	110	110	110	110
Adjusted R^2	0.599	0.617	0.631	0.709

This table reports estimates of the effect of the number of private schools in the market place on test scores for high and low types attending public schools. The regressions use data from the LEAPS tested children data. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level.

Table 5: Tests for Pre-Trends in the Effect of Number of Private Schools on Student Outcomes

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{high} \times num\_pri_{v,t+1}$	-0.019	-0.002	-0.005	-0.006
	(0.018)	(0.009)	(0.008)	(0.009)
$num\_pri_{v,t+1}$	0.009	-0.019	-0.0004	-0.008
	(0.015)	(0.020)	(0.016)	(0.009)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Number of Private Schools Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	2,644	2,644	2,644	2,644
Clusters	105	105	105	105
Adjusted R^2	0.568	0.624	0.627	0.712

This table reports estimates of the effect of the forward lagged number of private schools in the market place on performance on test scores. The regressions use data from the LEAPS tested children data. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Number of private schools controls consist of $num\_pri_{v,t}$ and its interaction with $\mathbb{1}_{high}$. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Observations are at the student-year level. Standard errors are clustered at the village level.

Table 6: Structural Estimates of Determinants of School Choice

	(1) Coefficient	(2) Se
$VA_{j,low,n} \times \mathbb{1}_{low}$	0.198	0.284
$VA_{j,high,n} \times \mathbb{1}_{high}$	1.093***	0.240
$distance_{ij} \times \mathbb{1}_{low}$	-1.590***	0.082
$distance_{ij} \times \mathbb{1}_{high}$	-0.285***	0.053
fee_{jn}	-2.131***	0.524
$fee_{jn} \times \mathbb{1}_{high}$	0.227*	0.118

This table reports estimates of the determinants of school choice using a discrete choice model where schools are allowed to have time-varying unobserved quality. $VA_{j,low,n}$ is the average of a school j's value-added for low types in math, Urdu, and English under competitive regime n, while $VA_{j,high,n}$ is the average value-added for high types. Distance is measured in kilometers, and fees are measured in 1000s of Rupees. The coefficients are estimated using the LEAPS household survey data.

Mathematical Appendix

In this appendix, I first prove that there is no pure strategy Nash equilibrium when $N = 2$ and δ_{low} is not sufficiently small in the simplified version of the model in subsection 2.3.

Proposition A1. *When $N = 2$ and $\delta^* < \delta_{low} < \delta_{high}$, where δ^* is the value of δ_{low} that equalizes the profits from school 2 placing at 0 or 1 when school 1 places at 1, there is no pure strategy equilibrium.*

Proof. If $\delta_{low} > \delta^*_{low}$, this implies that the school 2's best response to school 1's choice of 1 will be to choose 0. However, if school 2 chooses 0, it is no longer school 1's best response to choose 1 since school 1 can choose $h_1 = 1 - (-u_o)^{1/2}$, retaining all the high types and gaining some of the low types. However, if 1 chooses any h_1 besides 1, it will no longer be in school 2's best interest to choose $h_2 = 0$. Instead 2 will choose a location $h_2 = \max(1 - (-u_o)^{1/2}, h_1 + \epsilon)$, $\epsilon > 0$, if $h_1 \neq 1$ and 0 otherwise. Since schools 1's and 2's best response functions are symmetric, we can see that there is no set of locations $\{h_1, h_2\}$ such that both schools are playing their best responses, and there is no pure strategy equilibrium. □

Weighting Appendix

In this appendix, I show how to weight the observations in the regression described by equation (6) so that ρ_2, the coefficient on $num\_pri_{vt} \times \mathbb{1}_{high}$, will not depend on vertical quality. According to the model, a student of type z's test scores in time t are given by

$$y_{it} = \rho_0 + \overline{VA}_{jzt} + H_{jzt} + \alpha_{zt} + \omega_{gz} + \lambda_{gz} y_{i,t-1} + \phi_{gz} y_{i,t-1}^2 + \epsilon_{it},$$

where $\overline{VA}_{jt}$ is the vertical quality of a school in year t and H_{jzt} is the school's horizontal quality for type z. In practice, I estimate

$$\begin{aligned} y_{it} = \rho_0 + \rho_1 num\_pri_{vt} + \rho_2 num\_pri_{vt} \times \mathbb{1}_{high} + \eta_{zs} + \alpha_{zt} + \omega_{gz} \\ + \lambda_{gz} y_{i,t-1} + \phi_{gz} y_{i,t-1}^2 + \epsilon_{it}. \end{aligned}$$

Then, ρ_1 is given by

$$\rho_1 = \frac{Cov(\widetilde{y_{it}}, \widetilde{num\_pri_{vt}})}{Var(\widetilde{num\_pri_{vt}})},$$

where the tilde denotes the residual of a regression of the variable on α_{zt}, ω_{gz}, $y_{i,t-1}$, and $y_{i,t-1}^2$. This can be rewritten as:

$$\rho_1 = \frac{Cov(\overline{VA}_{ijt}, \widetilde{num\_pri_{vt}})}{Var(\widetilde{num\_pri_{vt}})} + \frac{Cov(H_{ij,low,t}, \widetilde{num\_pri_{vt}})}{Var(\widetilde{num\_pri_{vt}})},$$

where $\overline{VA}_{ijt}$ is a school j's vertical quality in year t and $H_{ij,low,t}$ is the school's horizontal quality for low types in period t. Now note that $Cov(H_{ij,low,t}, \widetilde{num\_pri_{vt}})$ can be rewritten as

$$\begin{aligned} Cov(H_{ij,low,t}, \widetilde{num\_pri_{vt}}) &= E(\overline{VA}_{ijt}, \widetilde{num\_pri_{vt}}) - E(\widetilde{num\_pri_{vt}}) E(\overline{VA}_{ijt}) \\ &= \frac{1}{N_L} \sum_j \overline{VA}_{jt} \omega_{j,low,t} \widetilde{num\_pri_{vt}} \end{aligned}$$

where N_L is the total number of low types and $\omega_{j,low,t}$ is the number of low types in

school j in year t.[12] Similarly, ρ_2 is given by

$$\rho_2 = \frac{Cov(H_{ij,high,t}, \widetilde{num\_pri}_{vt})}{Var(\widetilde{num\_pri}_{vt})} + \frac{\frac{1}{N_H}\sum_j \overline{VA_{jt}}\omega_{j,high,t}\widetilde{num\_pri}_{vt}}{Var(num\_pri_{vt})} - \rho_1.$$

Importantly, we can see that if the terms that include vertical quality were equally weighted for high and low types, these terms would cancel out. Now, re-weight each low type observation with $w_{i,low,jt} = \frac{\omega_{j,high,t}/N_H}{\omega_{j,low,t}/N_L}$, while weighting high type observations with 1. Then, the new ρ_2^w from the weighted regression is

$$\rho_2^w = \frac{\frac{1}{N_H}\sum_j \overline{H_{j,high,t}}\omega_{j,high,t}\widetilde{num\_pri}_{vt}}{Var(num\_pri_{vt})} - \frac{\frac{1}{N_H}\sum_j \overline{H_{j,low,t}}\omega_{j,high,t}\widetilde{num\_pri}_{vt}}{Var(num\_pri_{vt})}.$$

Thus, ρ_2^w is not affected by changes in vertical quality.

[12] The removal of the constant means that $E(\widetilde{num\_pri}_{vt}) = 0$.

Estimation Appendix A: Empirical Analysis of School Choice

In the first subsection of this appendix, I discuss how I estimate the parameters $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}, \zeta_k\}$ from equation (4). In the second subsection, I discuss how I estimate the baseline effect of fees, $\mathbf{\Gamma_z^{school}}$, on utility.

Estimation of $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}, \zeta_k\}$

I estimate the parameters $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}, \zeta_k\}$ that maximize the log likelihood function

$$\mathcal{L} = \sum_{ijt} \mathbb{1}_{ijt} log(p_{ijt}), \tag{12}$$

where $\mathbb{1}_{ijt}$ is an indicator variable equal to 1 if a student i attends a school j in year t and p_{ijt} is the probability i attends j in year t given by equation (9). In practice, I do this using the Artelys Knitro package in matlab to minimize the negative log likelihood function. To reduce computational time, I provide the derivatives of equation (12) with respect to $\theta = \{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}, \zeta_k\}$. For notational simplicity, let X_{ijt} also include $VA_{j,high,n}$ and $VA_{j,low,n}$. Then, the derivative of equation (12) with respect to the vector θ is

$$\sum_{ijt} \mathbb{1}_{ijt} \frac{1}{p_{ijt}} \Big(P(type_i = high) \frac{\partial p_{ij,low,t}}{\partial \theta} + (1 - P(type_i = high)) \frac{\partial p_{ij,high,t}}{\partial \theta} \Big), \tag{13}$$

where p_{ijzt} is the probability that a student i chooses j in year t conditional on that student being type z, and

$$\frac{\partial p_{ijzt}}{\partial \theta} = p_{ijzt} \Big(X_{ijzt} - \frac{\sum_k X_{ikt} e^{\theta X_{ikt}}}{\sum_k e^{\theta X_{ikt}}} \Big)$$

for the elements of θ that are in the utility function for type z and 0 for the remaining elements of θ. To ensure that I find the global maximum of equation (12), I estimate $\{\mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}, \zeta_k\}$ with 20 randomly chosen start points and choose the parameter estimates that produce the largest value for the log likelihood function.

Finally, I estimate the standard errors using the fact that in general, for maximum likelihood estimation, $\sqrt{C}(\hat{\theta} - \theta^*) \to \mathcal{N}(0, I^{-1})$, where the information matrix $I(\theta)$

is given by the expectation of the outer-products of the first derivatives (given by equation (13)) of the log likelihood function and C is the number of observations (here, children). Therefore, the covariance matrix is:

$$\frac{1}{C}\left(\sum_i \frac{\partial \mathcal{L}}{\partial \theta}\frac{\partial \mathcal{L}'}{\partial \theta}\right)^{-1}.$$

Estimation of $\mathbf{\Gamma_z^{school}}$

Having estimated $\{\mathbf{\Gamma_z^{indiv}}, \delta_1, \delta_2, \zeta_{jn}\}$ in section 5.1, I return to equation (8) to estimate $\mathbf{\Gamma_z^{school}}$, which is the baseline effect of fees on a child's utility in a school. Equation (8) appears to be a linear regression. If school fees are unrelated to a school's unobserved quality, ξ_{jn}, then I can estimate $\mathbf{\Gamma_z^{school}}$ by regressing the estimated school fixed effects ζ_{jn} on X_{jn}^{school}. However, this assumption is unlikely to be satisfied. In theory, profit-maximizing schools with higher ξ_{jn} will charge higher prices. Therefore, to identify $\mathbf{\Gamma_z^{school}}$, I need an instrument for $\mathbf{\Gamma_z^{school}}$.

Following Nevo (2001) and Hausman (1996), I use a variable that shifts the cost of providing education as my instrument for price. In particular, I use geographic variation in teacher salaries as an instrument for schools' prices. To construct this instrument, I regress teacher salaries in private schools on teacher characteristics as follows:

$$salary_{ijt} = \mathbf{\Upsilon} Z_{it} + \eta_j + \alpha_t + \epsilon_{ijt},$$

where $salary_{ijt}$ is the salary of teacher i in school j in year t, η_j is a school fixed effect, α_t is a year fixed effect, and Z_{it} are teacher characteristics consisting of fixed effects for qualifications, experience, training, and age. I regress salaries on these characteristics to ensure that differences in the cost of teachers are not explained by differences in teacher quality, which could be related to ξ_{jn}. Then, I predict the residual:

$$\widehat{salary}_{ijt} = \widehat{\eta_j} + \widehat{\epsilon_{ijt}}.$$

For each village v, I create the leave-one out average measure $cost_v = \sum_{m \in T, m \neq v} \widehat{salary}_{ijt}$, where T is the set of villages in the same sub-district as v. I use a leave-one out es-

timate to ensure that differences in teacher salaries are not driven by competition over teachers in village v, which may also be related to ξ_{jn}. The key assumption here is then that any one village is too small to change prices in other villages in the sub-district, but villages in the same sub-district market are likely to have the same systematic differences in teacher labor supply. The final instrument is then the interaction of $cost_v$ with an indicator variable for whether a school j is private, controlling for $cost_v$ and whether the school is private. In equation form, the instrument is calculated from the regression

$$cost_v \times I(private)_j = \rho_0 + \rho_1 I(private)_j + \rho_2 cost_v + \mu_j, \tag{14}$$

where $I(private)_j$ is an indicator variable equal to 1 if a school is private and the final instrument is the estimate of the residual, $\hat{\mu}_j$. Therefore, variation in the instrument comes from being a private school in sub-district where private school teachers are more expensive. With this instrument, the parameters of equation (8) can be estimated with the general method of the moments under the assumption that $\mu_j \perp \xi_j$. The moment conditions are given by

$$\Phi = \begin{pmatrix} \xi'_{jn}\hat{\mu}_j \\ \xi'_{jn}\hat{\mu}_j^2 \\ \xi'_{jn}1 \end{pmatrix}$$

where $\xi_{jn} = \zeta_{jn} - \mathbf{\Gamma_z^{school}} X_{jn}^{school} - c$ and c is a constant. To estimate c and $\mathbf{\Gamma_z^{school}}$, I again use Knitro. I first solve for the parameters that minimize $\hat{\Phi}'\hat{\Phi}$. Given these parameters, I estimate the optimal weighting matrix, C (for details, see Cragg (1983) and Hansen (1982)). Having estimated C, I re-estimate the parameters, minimizing $\hat{\Phi}'C\hat{\Phi}$. I calculate the standard errors using the standard "sandwich formula" for GMM.

Estimation Appendix B: Equilibrium Choice of Horizontal Quality

In section 6.1, I jointly estimate β and the equilibrium value of horizontal quality for each school j in each year t, by finding the β and h_{jt} that solve

$$\min_{\beta, h_{jt}} \sum_j \sum_z \left(\widehat{VA_{j,z,n}} - (\overline{VA_{jn}} + \beta(o_z - h_{jt})^2) \right)^2$$

subject to

$$\frac{\partial s_{jt}}{\partial h_{jt}} = 0 \quad \text{for each } j.$$

In practice, the procedure to estimate h_{jt} and β proceeds as follows. For a given β, I solve for h_{jt} for each private school-year, jt, by solving for the value of h_{jt} that satisfies $\frac{\partial s_{jt}}{\partial h_{jt}} = 0$ given β, the data, the demand system estimates from section 5, the school's own non-horizontal quality characteristics, and the characteristics of the other schools in the market. I search over β's until I arrive at the β that minimizes the distance between the implied type-specific value-addeds and those observed in the data. I estimate β using the Nelder-Mead method (Nelder and Mead, 1965), which is implemented with the *fminsearch* routine in matlab.

Note that, for a given β, the h_{jt} that solves the constraint $\frac{\partial s_{jt}}{\partial h_{jt}} = 0$ will be unique since increasing h_{jt} montonically decreases the share of low types who choose the school and monotonically increases the share of high types. To solve for the values of h_{jt} that satisfy the constraints, I loop through each school-year and using Knitro in matlab impose equation (10) as a non-linear constraint in a constrained optimization problem for a school-year jt. To minimize computational time, I provide Knitro with the derivative of the constraint, which is

$$\sum_{it} P(type_i = high) \frac{\partial^2 p_{ij,high,t}}{\partial h_{jt}^2} + (1 - P(type_i = high)) \frac{\partial^2 p_{ij,low,t}}{\partial h_{jt}^2},$$

where

$$\frac{\partial^2 p_{ij,high,t}}{\partial h_{jt}^2} = 2\delta_{low}(p_{ij,low,t}^2 - p_{ij,low,t}) + 2\delta_{low}\beta h_{jt}(2p_{ij,low,t}\frac{\partial p_{ij,low,t}}{\partial h_{jt}} - \frac{\partial p_{ij,low,t}}{\partial h_{jt}})$$

$$\frac{\partial^2 p_{ij,low,t}}{\partial h_{jt}^2} = -2\delta_{high}\beta(p_{ij,high,t} - p_{ij,high,t}^2) + \delta_2(2\beta - 2\beta h_{jt})(\frac{\delta p_{ij,high,t}}{\partial h_{jt}} - 2p_{ij,high,t}\frac{\delta p_{ij,high,t}}{\partial h_{jt}})$$

and $\frac{\delta p_{ij,high,t}}{\partial h_{jt}}$ and $\frac{\delta p_{ij,low,t}}{\partial h_{jt}}$ are given by equation (11). I further minimize the computational time by parallelizing the loop through jt.

Appendix Tables

Table A1: Summary Statistics for Public and Private Schools

	(1)	(2)	(3)	(4)	(5)	(6)
		Private			Government	
	Mean	SD	N	Mean	SD	N
Number of Private Schools	4.562	3.412	1,194	2.852	2.600	2,019
Number of Public Schools	4.812	3.251	1,194	6.045	3.818	2,019
Fee (Rupees)	1,360	963	1,166	11	155	1,928
Maximum Grade Offered	7.549	1.975	1,166	5.935	1.987	1,925
Student-Teacher Ratio	21.172	13.554	1,168	38.718	33.974	1,924
Has Library	0.392	0.488	1,168	0.224	0.417	1,930
Has Computer	0.266	0.442	1,168	0.010	0.101	1,930
Has Sports	0.349	0.477	1,168	0.110	0.313	1,930
Has Hall	0.195	0.397	1,168	0.069	0.253	1,930
Has Wall	0.962	0.190	1,168	0.658	0.474	1,930
Has Fans	0.942	0.233	1,164	0.476	0.500	1,926
Has Electricity	0.959	0.199	1,167	0.542	0.498	1,930
Number Permanent Classrooms	4.235	4.143	1,166	3.386	3.042	1,928
Number of Semi-Permanent Classrooms	1.854	2.990	1,168	0.664	1.526	1,929
Number of Staff Rooms	0.531	0.532	1,168	0.265	0.476	1,928
Number of Stores	0.428	0.571	1,168	0.269	0.623	1,929
Number of Toilets	0.668	0.851	1,168	0.315	0.744	1,929
Number of Blackboards	7.031	4.457	1,168	5.295	4.151	1,930

This table reports summary statistics at the school-year level for private and government schools in the LEAPS survey from 2004-2007.

Table A2: Summary Statistics for Tested Students in Public and Private Schools

	(1)	(2)	(3)	(4)	(5)	(6)
		Private			Government	
	Mean	SD	N	Mean	SD	N
Female	0.451	0.498	14,202	0.450	0.498	28,499
Age	10.338	1.820	14,200	10.582	1.819	28,496
Grade	4.120	1.027	14,202	4.133	0.988	28,499
Mother Some Primary	0.481	0.500	14,202	0.299	0.458	28,499
Father Some Primary	0.748	0.434	14,202	0.579	0.494	28,499
Beds	0.998	0.044	14,202	0.996	0.060	28,499
Radio	0.632	0.482	14,202	0.541	0.498	28,499
TV	0.761	0.427	14,202	0.589	0.492	28,499
Refrigerator	0.600	0.490	14,202	0.322	0.467	28,499
Bicycle	0.746	0.435	14,202	0.713	0.452	28,499
Plough	0.250	0.433	14,202	0.222	0.416	28,499
Small Ag. Tools	0.697	0.460	14,202	0.720	0.449	28,499
Tables	0.952	0.213	14,202	0.855	0.352	28,499
Chairs	0.952	0.215	14,202	0.846	0.361	28,499
Fans	0.974	0.161	14,202	0.924	0.265	28,499
Tractor	0.159	0.366	14,201	0.115	0.319	28,499
Cattle	0.529	0.499	14,201	0.601	0.490	28,499
Goats	0.527	0.499	14,201	0.663	0.473	28,499
Chicken	0.573	0.495	14,201	0.649	0.477	28,499
Watches	0.972	0.166	14,201	0.958	0.202	28,499
Motor Rickshaw	0.040	0.196	14,201	0.039	0.193	28,499
Motorcycle	0.295	0.456	14,200	0.169	0.375	28,499
Car	0.124	0.329	14,201	0.048	0.215	28,499
Telephone	0.577	0.494	14,201	0.364	0.481	28,499
Tubewell	0.233	0.423	14,201	0.158	0.364	28,499
Math	0.376	0.829	14,202	-0.044	0.972	28,499
Urdu	0.429	0.857	14,202	-0.099	0.972	28,499
English	0.537	0.753	14,202	-0.205	0.939	28,499
Yearly Gains in Math	0.395	0.641	6,828	0.390	0.717	14,402
Yearly Gains in Urdu	0.432	0.581	6,828	0.443	0.670	14,402
Yearly Gains in English	0.350	0.580	6,828	0.391	0.694	14,402

This table reports summary statistics at the student-year level for the sample of tested students in private and government schools in the LEAPS survey from 2004-2007.

Table A3: Summary Statistics for Household Sample of Children Aged 5-15

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Private			Government			Not Enrolled	
	Mean	SD	N	Mean	SD	N	Mean	SD	N
Female	0.434	0.496	2,892	0.479	0.500	6,851	0.593	0.491	4,169
Age	9.537	2.865	2,892	9.967	2.821	6,851	10.732	3.413	4,169
Distance to Current School (km)	0.484	0.687	2,892	0.724	0.927	6,851	–	–	–
Tables	0.717	0.451	2,892	0.623	0.485	6,851	0.475	0.499	4,169
Chairs	0.914	0.280	2,892	0.791	0.407	6,848	0.584	0.493	4,168
Fans	0.733	0.443	2,892	0.689	0.463	6,851	0.610	0.488	4,169
Sewing Machine	0.864	0.343	2,892	0.752	0.432	6,851	0.575	0.494	4,169
Air Cooler	0.153	0.360	2,892	0.074	0.262	6,851	0.042	0.201	4,169
Air Conditioner	0.268	0.443	2,892	0.247	0.431	6,851	0.220	0.414	4,169
Refrigerator	0.368	0.482	2,892	0.182	0.386	6,851	0.107	0.309	4,169
Radio	0.554	0.497	2,892	0.476	0.499	6,851	0.390	0.488	4,169
TV	0.524	0.500	2,892	0.419	0.493	6,851	0.318	0.466	4,169
VCR	0.112	0.315	2,892	0.051	0.220	6,851	0.036	0.186	4,169
Watches	0.908	0.290	2,892	0.907	0.290	6,851	0.849	0.358	4,169
Guns	0.129	0.336	2,892	0.071	0.256	6,851	0.052	0.222	4,169
Plough	0.173	0.378	2,892	0.134	0.340	6,851	0.105	0.306	4,169
Thresher	0.175	0.380	2,892	0.091	0.288	6,851	0.055	0.227	4,169
Tractor	0.092	0.289	2,892	0.059	0.236	6,851	0.042	0.201	4,169
Tubewell	0.221	0.415	2,892	0.166	0.372	6,851	0.133	0.339	4,169
Agricultural Machinery	0.263	0.440	2,892	0.239	0.427	6,851	0.206	0.404	4,169
Agricultural Hand Tools	0.660	0.474	2,892	0.657	0.475	6,851	0.618	0.486	4,169
Motorcycle	0.283	0.451	2,891	0.231	0.421	6,851	0.212	0.409	4,169
Car	0.084	0.278	2,889	0.032	0.176	6,851	0.030	0.169	4,169
Bicycle	0.615	0.487	2,891	0.635	0.482	6,851	0.590	0.492	4,169
Cows	0.654	0.476	2,891	0.715	0.451	6,851	0.710	0.454	4,169
Goats	0.485	0.500	2,891	0.570	0.495	6,851	0.602	0.490	4,169
Chickens	0.388	0.487	2,891	0.424	0.494	6,851	0.395	0.489	4,169

This table reports summary statistics at the child-year level for children aged 5-15 in the 1,740 surveyed households in the LEAPS survey from 2004-2007.

Table A4: Out of Sample Validation of Type-Specific School Value-Addeds

	(1) Math Score Low Type	(2) Math Score High Type	(3) English Score Low Type	(4) English Score High Type	(5) Urdu Score Low Type	(6) Urdu Score High Type	(7) Mean Score Low Type	(8) Mean Score High Type
$Math\,VA_{low}$	0.922***	0.101						
	(0.081)	(0.097)						
$Math\,VA_{high}$	-0.024	0.956***						
	(0.050)	(0.094)						
$English\,VA_{low}$			0.972***	0.067				
			(0.056)	(0.098)				
$English\,VA_{high}$			-0.006	0.940***				
			(0.041)	(0.110)				
$Urdu\,VA_{low}$					0.947***	0.130		
					(0.081)	(0.114)		
$Urdu\,VA_{high}$					0.041	1.028***		
					(0.059)	(0.107)		
$Mean\,VA_{low}$							1.012***	0.157
							(0.081)	(0.141)
$Mean\,VA_{high}$							0.026	1.032***
							(0.056)	(0.137)
Grade × Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	1,247	579	1,247	579	1,247	579	1,247	579
Clusters	100	93	100	93	100	93	100	93
Adjusted R^2	0.595	0.612	0.635	0.630	0.653	0.691	0.698	0.707

This table reports the coefficients for regressions of the test scores of high and low type students who change schools on their new school's value-addeds for high and low types, controlling for the grade-specific effects of lagged test scores and lagged test scores squared. The relevant type-specific value-added for a student was calculated leaving out that student's own outcomes. Standard errors are clustered at the school level.

Table A5: Effect of Number of Private Schools on Student Outcomes for Students Who Always Attend Private Schools

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{high} \times num\_pri_{vt}$	0.116*	0.114**	0.064	0.106**
	(0.070)	(0.048)	(0.046)	(0.043)
num_pri_{vt}	-0.107**	-0.059	-0.038	-0.070**
	(0.044)	(0.053)	(0.037)	(0.034)
Number of Private Schools Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	5,869	5,869	5,869	5,869
Clusters	106	106	106	106
Adjusted R^2	0.593	0.609	0.649	0.713

This table reports estimates of the effect of the number of private schools in the market place on test scores for students who always attend private schools during the sample period. The regressions use data from the LEAPS tested children data. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Number of private schools controls consist of $num\_pri_{v,t}$ and its interaction with $\mathbb{1}_{high}$. Observations are at the student-year level. Standard errors are clustered at the village level.

Table A6: Effect of Number of Private Schools on Student Outcomes by Private School Fees

	(1) Math	(2) English	(4) Urdu	(5) Mean
$\mathbb{1}_{high} \times num\_pri_{vt}$	0.105*	0.081*	0.055	0.086**
	(0.056)	(0.043)	(0.040)	(0.034)
num_pri_{vt}	-0.095**	-0.046	-0.019	-0.054**
	(0.038)	(0.040)	(0.032)	(0.027)
$\mathbb{1}_{high} \times num\_pri_{vt} \times fee_{jt}$	-0.001	-0.002	0.0002	-0.001
	(0.008)	(0.008)	(0.006)	(0.006)
$num\_pri_{vt} \times fee_{jt}$	-0.005	-0.003	-0.006*	-0.005
	(0.006)	(0.005)	(0.004)	(0.004)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	6,788	6,788	6,788	6,788
Clusters	106	106	106	106
Adjusted R^2	0.583	0.602	0.641	0.703

This table reports estimates of the effect of the number of private schools in the market place on on test scores for students, allowing those effects to vary by school fees. The regressions use data from the LEAPS tested children data. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Number of private schools controls consist of num_pri_{vt} and its interaction with $\mathbb{1}_{high}$. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Observations are at the student-year level. fee_{jt} is measured in 1000s of Rupees. Observations are at the student-year level. Standard errors are clustered at the village level.

Table A7: Effect of Number of Private Schools on Test Scores in Private Schools With Re-weighting

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{high} \times num\_pri_{vt}$	0.136**	0.052	0.098*	0.097**
	(0.056)	(0.050)	(0.052)	(0.042)
num_pri_{vt}	-0.154***	-0.061	-0.082*	-0.099***
	(0.042)	(0.049)	(0.045)	(0.038)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	5,451	5,451	5,451	5,451
Clusters	98	98	98	98
Adjusted R^2	0.579	0.587	0.640	0.698

This table reports estimates of the effect of the number of private schools in the market place on test scores for high and low types attending private schools, weighting observations as described in the weighting appendix. The regressions use data from the LEAPS tested children data. The peer controls consist of school average lagged test scores in year t, which are allowed to have different effects for high and low types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for high and low types. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level.