A Bird in the Hand is Worth Two in the Grave

Risk Aversion and Life-Cycle Savings*

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Abstract

We explore the role of risk aversion on life-cycle savings and portfolio choices. We consider a setup where agents are endowed with recursive preferences, enabling us to disentangle risk aversion and intertemporal elasticity of substitution. Agents face mortality, income, and investment risks, and assign a positive value to being alive. In this framework, the overall impact of risk aversion is theoretically ambiguous, as shown by Bommier, Chassagnon and LeGrand (2012). We carefully calibrate this life-cycle model, with particular attention to the value of a statistical life, and find clear-cut results: greater risk aversion implies smaller (not larger!) savings and safer investment strategies. The impact of mortality risk therefore dominates the other ones.

Keywords: recursive utility, life-cycle model, risk aversion, saving choices, portfolio choices.

JEL codes: .

1 Introduction

There are many sources of risk in life, including mortality, health, income, and investment risks, which impact both future resources and the ability to derive satisfaction out of them. As it was already emphasized by Fisher (1930), a theory of savings has therefore to embed at theory of

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behavior under risk. The economic literature provides however a rather blurred view on how risk aversion may influence life-cycle consumption-saving strategies. The first reason is that the literature has long focused on the additively separable model that makes it impossible to study the role of risk aversion in isolation. The second reason is that the few studies that use frameworks that allow to disentangle risk aversion from intertemporal elasticity of substitution end up providing mix messages. Gomes and Michaelides (2005, 2008) –who consider mortality, income and investment risks– explain that risk aversion should increase savings. Hugonnier, Pelgrin and Saint-Amour (2012) (henceforth, HPSA) include health risks in addition to the aforementioned risks, with source dependent risk aversion. They explain that mortality risk aversion increases or decreases savings depending on whether the intertemporal elasticity of substitution is larger or smaller than one. This contrasts with the papers of Bommier (2006, 2013) and Drouhin (2015), which focus on mortality risk exclusively and in which risk aversion is found to always increase time discounting and thus to decrease savings. Bommier, Chassagnon and LeGrand (2012) (henceforth, BCL) discuss how risk aversion impacts savings in simple two-period models where different sources of risk are separately considered. Simple dominance arguments lead to conclude that risk aversion has a positive impact in some cases (e.g., with income risk) and a negative impact in other cases (e.g., with mortality risks). The least that we can say is that these several articles fail to provide a clear and unified picture. In Table 1, we gather these contradictory findings about the role of risk aversion on savings.

The objective of this paper is to help clarifying this question: what does economic theory predict as to the impact of risk aversion on life-cycle financial strategies? This is a question of importance as it may hep understanding individual behaviors and, in particular, the low level of retirement savings that are empirically observed. In the main part of this paper, we address the question with models that make it possible to disentangle risk aversion, in particular with Epstein and Zin (1989) and risk-sensitive preferences. In line with Gomes and Michaelides, we consider mortality, income, and investment risks but leave health risk aside.[1]

When these different sources of risk are considered simultaneously, we know from the theoretical contribution of BCL that the overall impact of risk aversion will be the result of several opposing effects. The intuition underlying these opposing effects can be stated as follows. Risk aversion basically involves greater concern for bad state realizations. Thus, the impact of an increase in risk aversion on savings depends on whether an agent would save more or less if anticipating a bad state realization for sure. The effects are of different signs and magnitudes, depending on whether

[1]Despite of being of natural relevance, we choose not to model health risk. One of the reasons is that health being much more difficult to quantify (compared to monetary variables or to survival), it is difficult to find precise estimates on the willingness-to-pay to improve health.

a bad state realization is death, a low income or a low financial rate of return. When the three risks about income, mortality and investment are simultaneously at play, there is therefore no hope to derive a general simple conclusion that would hold for all risk and preference parameters. The conclusion has to rely on a quantitative analysis, based on carefully calibrated life-cycle models. In particular, as mortality risk is taken into account, it is necessary to use models which predict a reasonable value of life.

Our findings are simple and interestingly contradict some "conventional wisdom": we indeed find that greater risk aversion leads to save less and to opt for safer investment strategies (with lower stock market participation and a larger share of savings invested in bonds). In fact, when using models that provide a value of life consistent with empirical estimates, it is found that mortality is the main source of uncertainty on lifetime utility. This simply reflects that an early death has a much more dramatic impact on welfare than a (permanent) shock on income risk, or a low return on savings. As a consequence, the effects discussed in Bommier (2006, 2013) and Drouhin (2015), which solely focus on mortality risk, dominate the others. Labor income risk does generate a prudence effect (with more savings) which is magnified by risk aversion, but this contributes quite little compared to the effect of mortality risk. The impact of asset return risks are also visible, but are also of lower magnitude compared to those of mortality risk.

In a second part of the paper we explain why our results differ from those of Gomes and Michaelides (2005, 2008) and HPSA. The explanation relates to two channels. First, Gomes and Michaelides (2005, 2008), as well as many others that use Epstein-Zin preferences with mortality risk, consider specifications with which the value of life is most often negative. This leads to inverse the impact of risk aversion. Indeed, if long life is considered as an adverse realization, greater risk aversion leads to put a larger weight on the utility derived in case of long life, and therefore leads to save more. The results of Gomes and Michaelides (2005, 2008) are then opposed to ours, but this is imputable to the negative value of life of the agents in their economy. HPSA assume a positive value of life, but in the case where the intertemporal elasticity of substitution is smaller than one, their model assumes that consumption and survival are substitute rather than complementary. In other words, the marginal utility of consumption at a given age decreases with the probability of being alive at that age, so that mortality makes agents more patient, instead of making them more impatient. When survival at a given age becomes almost impossible, marginal utility of consumption tends to infinity, which leads the agent to keep a lot of resources to increase consumption at this old age. This increase in patience due to mortality is magnified by risk aversion. The result is contrary to ours, but again related to an unintuitive aspect of preferences that are used.

Last, we also present empirical evidence that supports the results of our model. The German

	Risk aversion increases savings	Risk aversion decreases savings
Income risk only	BCL.	
Investment risk only	Kihlstrom and Mirman (1974) and BCL if IES<1.	Kihlstrom and Mirman (1974) and BCL if IES >1.
Mortality risk only	*HPSA if IES* <1.	Bommier (2006, 2013), BCL, Drouhin (2015), HPSA if IES >1.
Income, investment, and mortality risk	*Gomes and Michaelides (2005, 2008).*	This paper.

Note: Models in *italics* have results that are inconsistent with ours for reasons that are explained in the introduction and with further details in Section 6. All others provide a consistent view. BCL stands for Bommier, Chassagnon LeGrand (2012), HPSA for Hugonnier, Pelgrin and Saint-Amour (2012) and IES for "intertemporal elasticty of substitution". HPSA also consider cases where simultaneous risks are at play, using several source dependent ceofficients of risk aversion. The corresponding results cannot be reported in the above table.

Table 1: Impact of risk aversion on savings

Socio-economic panel has a measure of general risk aversion that has been shown to be a good predictor for a variety of risky behavior (Dohmen, et al. (2011)). We find that risk aversion decreases total savings, which confirms the main finding of the model. As a further evidence supporting the model, more risk averse households save more when income risk increases, but save less as mortality risk increases.

The rest of the paper is organized as follows. We present our setup in Section 2. We specify utility functions in Section 3. We describe our calibration in Section 4 and present the related results in Section 5. In Section 6, we discuss how our results compare to the literature. Section 7 presents the empirical evidence. We conclude in Section 8.

2 The setup

We consider a partial equilibrium economy populated by an agent endowed with recursive preferences and facing several risks: a mortality risk, an income risk and an investment risk through risky financial returns. The agent may save through bonds and a risky asset (similar to a stock). We are interested in studying her lifecycle portfolio allocation and in particular the impact on saving choices of some preference feature, such as risk aversion and the value of life. Time is discrete and indexed by $t = 0, 1, \ldots$ The period between two dates is one year. As it is standard, t refers to the agent's adult age and is equal to her age minus 20. The initial date $t = 0$ corresponds to age 20. There is a single consumption good, whose price serves as a numeraire. We now describe the setup, starting with risks faced by the agent.

Mortality risk. The agent faces a mortality risk, which is assumed to be independent of any other risk in the economy. While alive at date $t-1$, the agent faces the probability $p_{t|t-1}$ to be still alive at date t. Thus, $1 - p_{t+1|t}$ denotes the probability of dying at the beginning of period t. The agent is alive at date 0, so that we have $p_{0|-1} = 1$. We denote by $m_{t|0}$ (resp. $p_{t|0}$) the probability of living exactly (resp. at least) until date t. These probabilities relate to each other as follows:

$$m_{t|0} = (1 - p_{t+1|t}) \prod_{k=1}^{t} p_{k|k-1} \quad \text{and} \quad m_{0|0} = 1 - p_{1|0},$$

$$p_{t|0} = \prod_{k=1}^{t} p_{k|k-1} \quad \text{and} \quad p_{0|-1} = 1.$$

The agent will die for sure at some age and there exists a maximal date T_m, such that the probability to live after T_m is 0: $p_{T_m+1|T_m} = 0$.

Labor income risk. The agent is supposed to exogenously retire at the age of 65, which corresponds to $T_R = 45$. In retirement, the agent gets a constant income y_R, which is therefore riskless from any date perspective. When active, the agent earns at any date $t < T_R$ a risky income y_t, which is affected by the combination of two shocks, a persistent one π_t and a transitory one u_t:

$$y_t = y_0 \exp\left(\mu_t + \pi_t + \varepsilon_t^y\right), \tag{1}$$

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t^\pi, \tag{2}$$

where the two independent processes $(\varepsilon_t^\pi{}_t)_{t\geq 0}$ and $(\varepsilon_t^y{}_t)_{\geq 0}$ are IID and normally distributed with mean 0 and respective variance σ_π^2 and σ_y^2. The quantity y_0 in (1) is the constant riskless component of income, while $(\mu_t)_{t\geq 0}$ is a deterministic process that contributes to fit the wage process to the data and in particular the humped-shape pattern of income during active age. We will discuss this

later on in Section 4. The parameter ρ in (2) drives the persistence of the process π and will be assumed to be very close to 1.

Financial risk and security markets. The agent has the opportunity to save through either a riskless one period asset (similar to a T-Bill) and a risky asset (similar to a stock). The bond is a security of price 1 which pays R^f as a riskless gross return in the subsequent period. The rate of interest R^f is constant and exogenous. The risky asset is similar to the bond except that the gross return R_t^s is risky and time dependent. More precisely, we make the following assumption regarding the evolution of the risky return:

$$\ln R_t^s = \ln R^f + \nu + \varepsilon_t^R,$$

where ν interprets as the average risk premium of stocks over bonds, while the financial risk $(\varepsilon_t^R)_{t\geq 0}$ is an IID normally distributed process with mean 0 and variance σ_R^2. The financial risk is assumed to be possibly correlated to both income shocks (ε_t^π) and (ε_t^y). The correlation with each income process is assumed to be constant and is denoted respectively $\kappa_{R,\pi}$ and $\kappa_{R,y}$.

The participation to the stock market is not free, as for example in Gomes and Michaelides (2008). The agent must pay a fixed cost $F \geq 0$ to participate to the stock market, which may interpret as transactions costs or as an opportunity cost to discover how the stock market works. This is a once-in-a-life cost: if the cost is paid at a given date t, the agent can freely trade stocks at date t and at any date afterwards.

Timing and notations. At the beginning of every period, the agent first learns the realizations of financial and labor shocks and whether she is alive or not. She thus knows the amount of her current savings and, if she is alive, what will be her income over the period to come. More precisely, at any date t, we assume that the agent knows the entire history of all shocks up to date t, which is formalized by the natural filtration $(\mathcal{F}_t)$ generated by the processes (u_t), (v_t) and (ε_t). The alive agent then decides her consumption level c_t, her savings in bonds b_t and in stocks s_t and her stock market participation status η_t (if she has never paid the participation cost before and never participated). The bequest of a dead agent is denoted w_t.

Constraints.

1. If the agent is dead at date t, she bequeaths all her wealth:

$$w_t = R^f b_{t-1} + R_t^s s_{t-1}. \tag{3}$$

The stock holding s_{t-1} may be null if the agent has never participated to the stock market.

Moreover, the total wealth is null if the agent was already dead at date $t-1$, since it has already been bequeathed.

2. If the agent is alive, her resources at the beginning of the period is made of stock and bond payoffs plus the labor income y_t of the period. Resources cover consumption, bond and stock savings. The agent can only invest in stocks if the participation cost has been paid at some date prior to t, i.e., if $\eta_t = 1$. Moreover, the agent may also have to pay the participation cost F at date t if she participates at date t in the stock market for the first time, i.e., if $\eta_t = 1$ and $\eta_{t-1} = 0$. The budget constraint at date t of the alive agent can then be expressed as follows:

$$c_t + b_t + s_t 1_{\eta_t=1} + F 1_{\eta_t=1} 1_{\eta_{t-1}=0} = y_t + R^f b_{t-1} + R_t^s s_{t-1}, \tag{4}$$

where $1_{\eta_s=1}$ is an indicator function equal to 1 if $\eta_s = 1$ and 0 otherwise. The agent is prevented from short-selling bonds and stocks and her consumption must also be strictly positive. Formally, at date t, we have the following borrowing and consumption positivity constraints:

$$b_t \geq 0 \text{ and } s_t \geq 0, \tag{5}$$

$$c_t > 0. \tag{6}$$

A feasible allocation is a sequence of choices $(c_t, b_t, s_t, \eta_t)_{t\geq 0}$ satisfying the constraints (3)–(6). The set of feasible allocations is denoted $\mathcal{A}$.

Preferences. The agent enjoys instantaneous felicity from current spending, in either consumption or bequest, depending on her survival status. We denote by $u(c_t) : \mathbb{R}^+ \to I$ the instantaneous felicity she gets when being alive and consuming c_t and by $v(w_t)$ the utility she derives when being dead and bequeathing the amount w_t. Preferences are separable over time and future instantaneous utilities are discounted by a factor $\beta \in (0,1)$ representing the agent's exogenous time preference.

Regarding risk preferences, we consider recursive utilities *à la* Kreps and Porteus (1978). Agents value the certain equivalent of a concave transformation of the future utility stream. More precisely, for an increasing concave function $\Phi : \mathbb{R} \to \mathbb{R}$, the utility U_t at date t expresses as follows:[2]

$$U_t = (1-\beta)u_t + \beta\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi(U_{t+1})\right]\right), \text{ with } u_t = \begin{cases} u(c_t), & \text{if the agent is alive at } t, \\ v(w_t), & \text{if the agent is dead at } t. \end{cases} \tag{7}$$

[2]Formally speaking, preferences are defined over the set of temporal lotteries, allowing for preferences for late or early uncertainty resolution. See Epstein and Zin (1989) or Wakai (2007) for a formal treatment.

In the above equation, $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ the conditional expectation operator with respect to the information available at date t. Formally, the information is the filtration $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t\geq 0}$, where $(\mathcal{G}_t)_{t\geq 0}$ is the filtration generated by the independent mortality process.

In such models, if there were no uncertainty, the utility U_t would be independent of the function Φ and the recursion (7) would reduce to $U_t = (1-\beta)u_t + \beta U_{t+1}$. We thus have a possible separation between preferences over certain consumption streams (determined by the functions u, v and the scalar β) and risk preferences driven by the function Φ. A more concave Φ implies lower certainty equivalents $\Phi^{-1}\left(E_t^{\mathcal{F}\times\mathcal{G}}\left[\Phi(U_{t+1})\right]\right)$ and therefore greater risk aversion.

Our specification of recursive preferences nests some of the most standard cases, including the additive specification, or the Epstein and Zin (1989) isoelastic specification or the risk-sensitive specification introduced by Hansen and Sargent (1995) in their work on robustness. We will introduce below (see in Section 3) some additional properties that we will be made on the functions u and v and make precise the functions Φ which correspond to these different popular specifications. Our results will make it possible to discuss the impact of these specifications on saving decisions.

Agent's program. We can now write the agent's program that can be expressed recursively by taking advantage of the structure of preferences. We denote as U_t^D the intertemporal utility at date t of a dead agent and U_t^A the one of an alive agent. Regarding the dead agent, note that the instantaneous utility of an agent dead for more than two periods is constant (all bequests take place in the first period after death) and simply equals to $v(0)$. From the recursive formulation (7), we deduce that there is no actual optimization and that the program of a dead agent can then be expressed as:

$$U_t^D(w_t) = (1-\beta)v(w_t) + \beta v(0). \tag{8}$$

For an alive agent, the agent maximizes her intertemporal utility by picking up the proper feasible allocation $(c_t, b_t, s_t, \eta_t)_{t\geq 0}$ in the set $\mathcal{A}$. The utility U^A of the alive agent depends on four state variables: the beginning-of-period holdings in stocks s_{t-1} and bonds b_{t-1}, the permanent shock π_{t-1} of labor income and the stock market participation $\eta_{t-1} \in \{0, 1\}$. The latter is discrete, while the three former ones are continuous. From the recursive formulation (7) and feasibility constraints (3)–(6) and using the fact that the mortality risk is assumed to be independent of other risks, the program of an alive agent at date t can be expressed as follows:

$$\begin{aligned} U_t^A(s_{t-1}, b_{t-1}, \eta_{t-1}, \pi_{t-1}) = &\max_{(c_t, s_t, b_t, \eta_t)\in\mathcal{A}} (1-\beta)u(c_t) \\ &+ \beta\Phi^{-1}\left(p_{t+1|t}E_t[\Phi(U_{t+1}^A(s_t, b_t, \eta_t, \pi_t))] + (1-p_{t+1|t})E_t[\Phi(U_{t+1}^D(w_{t+1}))]\right), \end{aligned} \tag{9}$$

where $E_t[\cdot]$ is the expectation for an alive agent with respect to the filtration $\mathcal{F}$ (i.e., made of all

past shock realizations but the death). Note that we distinguish it from the expectation $E_t^{\mathcal{F}\times\mathcal{G}}[\cdot]$ with respect to the whole information $(\mathcal{F}_t \otimes \mathcal{G}_t)_{t\geq 0}$ (including death information). It should be noted that the program (9) has a finite-horizon since there exists a maximal age for the agent, who cannot live beyond date T_m, such that $p_{T_m+1|T_m} = 0$.

Value of life. A very important concept in our paper is the value of a statistical life, or value of life, which is crucial for determining saving behaviors in presence of death risk. The value of life denoted VSL_t at the age of date t can be expressed as the opposite of the marginal rate of substitution between the mortality rate and consumption at that age. Noting $q_{t+1|t} = 1 - p_{t+1|t}$ the mortality rate at the age of date t, we formally define the value of life as follows:

$$VSL_t = -\frac{\frac{\partial U_t^A}{\partial q_{t+1|t}}}{\frac{\partial U_t^A}{\partial c_t}} = \frac{\frac{\partial U_t^A}{\partial p_{t+1|t}}}{\frac{\partial U_t^A}{\partial c_t}}.$$

The value of life VSL_t is equal to the quantity of consumption an agent would be willing to give up for a marginal decrease in the hazard rate. Our definition of the value of life is standard and consistent with that of Johansson (2002) for example. Using equation (9), we derive the following expression for the value of a statistical life:

$$VSL_t = \frac{\beta}{1-\beta} \frac{E_t\left[\Phi\left(U_{t+1}^A\right) - \Phi\left((1-\beta)v(w_{t+1}) + \beta v(0)\right)\right]}{u'(c_t)\Phi'\left(\frac{1}{\beta}\left(U_t^A - (1-\beta)u(c_t)\right)\right)}. \tag{10}$$

There exists a whole line of applied research that estimates the value of a statistical life from empirical data. See for example Viscusi and Aldy (2004) for a survey. We will make use of these estimates for the calibration of the model.

3 Specifications of utility functions

We now specify the functional forms for felicity functions u and v and for the aggregator Φ.

3.1 Felicity function specification

We begin with specifying u and v. We assume that the agent has a constant intertemporal elasticity of substitution, which means that $-\frac{u'(c)}{c\,u''(c)}$ is constant. This implies that u is equal, up to an affine transformation, to:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma} & \text{if } \sigma \neq 1, \\ \ln(c) & \text{if } \sigma = 1. \end{cases} \tag{11}$$

where the parameter $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. The above specification is such that $u(1) = 0$. It embeds therefore a normalization assumption, which is without generality loss. The case $\sigma = 1$ is obtained by continuity from the general case.

The felicity derived from bequeathing the wealth w for a dead agent is assumed to have the following functional form:

$$v(w) = \begin{cases} -v_0 + \frac{\theta}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right] & \text{if } \sigma \neq 1, \\ -v_0 + \theta \ln\left(\frac{\hat{w}+w}{\hat{w}}\right) & \text{if } \sigma = 1. \end{cases} \tag{12}$$

where $v_0 \in \mathbb{R}$, $\theta \geq 0$ and σ is the inverse of the intertemporal elasticity of substitution used in the expression (11) of the felicity of u. As for u, the case $\sigma = 1$ in (12) is obtained by continuity from the general case.

We can distinguish two components in the specification of v in equation (12). The first one is the constant v_0, which corresponds to the difference in utility between being alive and consuming 1 unit or being dead and bequeathing nothing. A higher (resp. lower) value of v_0 will be associated with a higher (resp. lower) valuation of being alive, compared to being dead. The value of v_0 thus strongly connects to the value of life. The second part, $\frac{1}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]$ measures the contribution of bequest to post-mortem felicity. This extra felicity derived from bequest is assumed to be continuous in zero, increasing in the amount of bequest and exhibiting bounded and decreasing marginal felicity. The rationale for this functional expression is the following one. Heirs may dispose of individual resources summarized by the quantity $\hat{w}$ and they enjoy bequest in addition to these resources $\hat{w}$. The felicity derived by heirs from bequest is proxied by the quantity $\frac{1}{1-\sigma}\left[(\hat{w}+w)^{1-\sigma} - \hat{w}^{1-\sigma}\right]$.[3] The agent values the felicity of her heirs with the weight θ that can therefore be interpreted as a parameter for the altruism intensity. With $\hat{w} > 0$, bequests are a luxury good, as reported in the data (e.g., in Hurd and Smith, 2002). Moreover, the derivative $v'(0)$ is finite, so that agents bequeath only when their wealth is large enough. This functional form has been chosen for example in De Nardi (2004), De Nardi et al. (2010), Ameriks et al. (2011), and Lockwood (2012, 2014). It is sometimes assumed in the literature that $v_0 = \frac{1-\theta\hat{w}^{1-\sigma}}{1-\sigma}$ so that $v(w) = \frac{1}{1-\sigma}\left[\theta(\hat{w}+w)^{1-\sigma} - 1\right]$, which has some advantage in terms of tractability.[4] However, this constraint on v_0 implies to impose a nontrivial relationship between the utility of bequest and the value of life. In particular, if θ is set to zero (no altruism) and $\sigma > 1$, then the utility of being dead is always higher to that of being alive, implying a negative value of life. We will not make assumptions of these kinds as we want our model to match standard empirical estimates for the value of life.

[3]The proxy is exact if (i) heirs have the same intertemporal elasticity of substitution as the donator and (ii) heirs fully annuitize their wealth.

[4]Additional tractability can then be obtained by setting $\hat{w} = 0$, in order to have homogeneous specifications, as in Inkmann, Lopes and Michaelides (2011) for example.

3.2 Risk-sensitive preferences

Regarding Φ that determines risk preferences, we consider several functional forms. First, we consider risk-sensitive preferences:

$$\Phi(u) = \begin{cases} -\frac{1}{k}\left(\exp(-ku) - 1\right) & \text{if } k \neq 0, \\ u & \text{if } k = 0, \end{cases} \tag{13}$$

where k is a constant driving risk aversion. The case $k = 0$ corresponds to the usual additive model and is obtained by continuity of the general case. For an agent endowed with risk-sensitive preferences to be more risk averse than in the usual additive model, we need to assume that $k > 0$. Risk-sensitive preferences have been introduced in Hansen and Sargent (1995) and axiomatized in Strzalecki (2011). As shown in Bommier, Kochov and LeGrand (2016), this is the only functional form Φ for which preferences represented by the utility function in recursion (7) are monotone. The monotonicity of preferences has to be understood as the monotonicity with respect to first-order stochastic dominance. Preference monotonicity means that if two uncertain consumption streams are available and that the first one is preferred to the other one in any possible state of the world, the former will always be preferred to the latter. This is distinct from and not implied by the fact that more certain consumption is preferred to less, which is in our setup equivalent to an increasing felicity function u. Moreover, as proved in Bommier and LeGrand (2014), risk-sensitive preferences are well-ordered with respect to risk aversion, both "in the large" (i.e., in terms of willingness-to-pay to eliminate all risks) but also "in the small" (i.e. in terms of willingness-to-pay for marginal risk reductions). This last aspect is important when addressing problems where complete risk elimination is not possible, or simply not optimal, as it is the case in the portfolio choice that we study.

3.3 Epstein-Zin preferences

We now present Epstein-Zin isoelastic preferences, which correspond to the following functional form for Φ:

$$\Phi(u) = \begin{cases} \frac{1}{1-\gamma}(1 + (1-\sigma)u)^{\frac{1-\gamma}{1-\sigma}} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\sigma}\ln(1 + (1-\sigma)u), & \text{if } \gamma = 1 \text{ and } \sigma \neq 1, \\ \frac{1}{1-\gamma}e^{(1-\gamma)u} - \frac{1}{1-\gamma}, & \text{if } \gamma \neq 1 \text{ and } \sigma = 1, \\ u, & \text{if } \gamma = 1 \text{ and } \sigma = 1, \end{cases} \tag{14}$$

where $\gamma \in \mathbb{R}$ and $1 + (1-\sigma)u \geq 0$.[5] Remark that whenever $\gamma = \sigma$ (but possibly different from 1), we get $\Phi(u) = u$ and Epstein-Zin preferences are additive. It is also well-known from Tallarini (2000), and directly visible from the last two lines of (14), that when $\sigma = 1$ Epstein-Zin preferences coincide with risk-sensitive preferences. Thus, the cases where $\sigma = 1$ are already addressed with risk-sensitive preferences and do not need further consideration. We will therefore exclude them whenever we refer to Epstein-Zin preferences below. For $\sigma \neq 1$, the constraint $1 + (1-\sigma)u \geq 0$ is not trivial. It holds whenever the agent is alive, since we have $1 + (1-\sigma)u(c) = c^{1-\sigma}$, but imposes constraints on the felicity of bequest defined in equation (12). The constraint $1 + (1-\sigma)u \geq 0$ is equivalent to

$$\begin{cases} v_0 \leq \frac{1}{1-\sigma} & \text{if } \sigma < 1, \\ v_0 \geq \frac{1-\theta\hat{w}^{1-\sigma}}{1-\sigma} & \text{if } \sigma > 1. \end{cases}$$

Isoelastic Epstein-Zin preferences are very popular in macroeconomics and finance, one of their main advantage being that they usually provide homogeneous specification, which is key for tractability. Note that this is not the case in our setup, where Epstein-Zin preferences are not homothetic. The reason is not the normalization that we made in equation (14), but stems from our choice of imposing a plausible value of statistical life.

4 Calibration and computation

In this section, we first give an overview of our calibration strategy. We then discuss the resulting parametrization in detail. The last section discusses the major difficulties in solving the model computationally.

4.1 Calibration strategy

Our calibration shares many common aspects with the related literature but differs mainly in that we target the value of a statistical life explicitly.[6] Given a realistic value of life, the objective of the calibration exercise is to highlight the impact of risk aversion. To this aim, we consider three agents: one with standard additively separable preferences, one with Epstein-Zin preferences corresponding to the aggregator (14), and one with risk-sensitive preferences corresponding to the

[5]Epstein Zin preferences are often introduced with a different but equivalent normalization for the function u (e.g. using $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ instead $u(c) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma}$), and therefore different functions Φ (the constant 1 being no longer needed). Our (equivalent) approach has however the advantage to have the cases $\sigma = 1$ or $\gamma = 1$ directly obtained as limit cases of the others, while keeping v_0 or $\hat{w}$ independent of σ and γ. This normalization choice has no impact on our results.

[6]Note however that some aspects of the current calibration are still preliminary.

aggregator (13).[7] We will henceforth refer to the three agents as the additive, the Epstein-Zin and the risk-sensitive agent, respectively. Importantly, we calibrate only the additive agent to the data. The other two only differ in that they have a marginally larger risk aversion.

We now describe our strategy for calibrating the additive agent; the resulting parameter values are discussed in the following sections. We set the intertemporal elasticity of substitution, $\frac{1}{\sigma}$, to a standard value. We then jointly calibrate the discount factor, β, the bequest motive, θ, and the life-death utility gap, v_0, to match the following three targets. The first target is an estimate of the value of a statistical life at age 45, VSL_{45}, as defined in equation (10). Targeting this is central to our exercise. The second and third targets are average assets at age 45 and average bequests at age 85, respectively. All other parameters are set to values that are taken directly from available data or related studies. Table 2 summarizes the parameter values, displaying endogenously calibrated values in italics.

After having calibrated the additive agent, we set the risk aversion of the Epstein-Zin agent to a slightly higher value $\gamma^{EZ} > \sigma$, while keeping all other parameters the same. This isolates how in a given economy risk aversion impacts the lifecycle savings choice and the value of a statistical life, among others. Similarly, we increase risk aversion k of the risk-sensitive agent by setting $k > 0$. More precisely, we calibrate k to produce the same average savings at age 45 as the Epstein-Zin agent, i.e., such that $E_0[s_{45}^{RS} + b_{45}^{RS}] = E_0[s_{45}^{EZ} + b_{45}^{EZ}]$. We do this, because we want the increases in risk-aversion to have a similar meaning when compared to the additive agent. Thus, we can compare on one hand the Epstein-Zin agent with the additive one, and on the other hand the risk-sensitive agent with the additive one. Note that the Epstein-Zin and risk-sensitive agents are not comparable with each other in terms of risk aversion.

4.2 Demographics

Agents start being economically active in the model at the working age of 21. They exogenously retire at the fixed age of 65, which corresponds to the statutory retirement age in the U.S. Mortality rates are taken from the Human Mortality Database for the U.S. for 2007. The maximum biological age is capped at 100, since mortality estimates become inaccurate after that.

4.3 Preferences

The intertemporal elasticity of substitution is set to 0.5, a common value in the literature, so that its inverse is $\sigma = 2$. For the Epstein-Zin agent we increase the risk aversion parameter moderately

[7] Recall from Section 3 that the additively separable case is nested in Epstein-Zin preferences when $\gamma = \sigma$, and in risk-sensitive preferences when $k = 0$.

Table 2: Parameterization in baseline economy

Parameter	Value	Source, empirical counterpart, or target
Demographics		
Biological age at $t = 1$	21	age at labor force entry (college)
Model age at retirement, T_R	45	S.S.A. statutory retirement age of 65
Model age maximum, T_M	80	Biological maximum age of 100
Cond. mortality rates, $\{m_{t+1\|t}\}$		Human Mortality Database, U.S. 2007
Preferences		
Inverse IES, σ	2.0	
Risk aversion, Epstein-Zin, γ^{EZ}	3.0	
Risk aversion, risk-sensitive, k	*0.08*	Assets of EZ at age 45
Life-death utility gap, v_0	*30.0*	VSL_{45} = US$ 6.5m (add. pref.)
Discount factor, β	*0.96*	$Assets_{45}$ = US$ 100000 (add. pref.)
Strength of bequest motive, θ	20.0	Bequests at age 85 (add. pref.)
Exogenous offspring endowment, $\hat{w}$	1.5	
Endowments		
Average wage, y_0	0.3	
Pension, y_R	0.1	
Age productivity, $\{\mu_t\}$	cf. app.	Earnings profiles (PSID)
Labor income autocorrelation, ρ	0.95	Storesletten, et al. (2004)
Var. of persistent innovation, σ_π	0.3	Storesletten, et al. (2004)
Correlation with stock return, $\kappa_{R,\pi}$	0.15	Gomes and Michaelides (2005)
Var. of transitory innovation, σ_y	0.0	Preliminary
Asset Markets		
Gross risk-free return, R^f	1.01	Bond return (Shiller)
Equity premium, ν	0.02	Preliminary
Stock volatility, σ_R	0.18	Shiller data
Participation cost, F	0.2	Preliminary

Notes: Parts of this calibration are still preliminary. Values in italics have been calibrated to their respective targets.

to $\gamma^{EZ} = 3$, since we do want not deviate too much from the additive agent. Last, for the risk-sensitive agent, we calibrate the risk aversion parameter to match the same average savings as the Epstein-Zin agent at the age of 45, which yields $k = 0.08$.

We then calibrate v_0 and β jointly so that the additive agent has a value of a statistical life at age 45 and assets at age 45 that match their empirical counterparts. For VSL we target US$ 6.5 million, which is in the middle of available estimates.[8] For average individual assets we target US$ 100 000, which is consistent with Census data. This yields $v_0 = 30.0$ and $\beta = 0.96$, which we keep constant for the three agents.

The strength of the bequest motive will be calibrated jointly with v_0 and β in order to match average bequests at age 85 but at the moment is set to a preliminary value of $\theta = 20$. The exogenous lifetime endowment of the offspring, $\hat{w}$, shifts bequest utility and is mostly needed for making bequests a luxury good. We set it to $\hat{w} = 1.5$.

4.4 Endowments

The average wage in this economy is just a scaling factor and we set it to $y_0 = 0.3$ to keep the bounds of the state space small for computational purposes.[9] Pensions are set to 30 percent of the average wage, in line with the U.S. social security replacement rate. The deterministic age-productivity profile is taken from Harenberg and Ludwig (2015), who compute it from PSID data using the method of Huggett, Ventura and Yaron (2011). The values are displayed in the computational appendix.[10]

The values for the persistent income process are taken from Storesletten, Telmer and Yaron (2004). Using PSID data, they find an autocorrelation $\rho = 0.95$ and a variance of shocks of $\sigma_\pi = 0.3$. The correlation of the persistent income shocks with stock returns is set to the same value as in the baseline of Gomes and Michaelides (2005), $\kappa_{R,\pi} = 0.15$. In our preliminary calibration, the variance of the temporary income shocks, σ_y, is set to zero.[11]

4.5 Asset markets

The parameter values for asset markets are mostly preliminary. The gross risk-free rurn is set to the average bond return of the last 50 years in the data of Robert Shiller, $R^f = 1.01$ percent.[12] The equity premium takes a preliminary value of $\nu = 0.02$ percent (to be increased to 0.06 in the

[8] Viscusi and Aldy (2003) provide a discussion of available estimates of the value of life.

[9] Assets (or cash at hand) can become very large because we treat income shocks as unbounded. That is why a small y_0 is helpful for computational reasons.

[10] The computational appendix is available on request.

[11] This will be changed in further versions.

[12] Robert Shiller's data are freely available at `http://www.econ.yale.edu/~shiller/data.htm`.

next version). Stock volatility is $\sigma_R = 0.18$, again as measured from Robert Shiller's data over the last 50 years. Participation cost is set to a preliminary value of $F = 0.2$ to get a reasonable stock market participation rate for the additive agent.

4.6 Computational solution

From a computational perspective, there are two difficulties when solving the model that are worth discussing. The first difficulty is that we want to numerically approximate the risks as precisely as possible, since the impact of the various risks is at the core of this paper. This is important for the autoregressive process driving the persistent income shock and for the correlation between income shocks and stock returns. The well-known discretization method of Tauchen (1986) and Tauchen and Hussey (1991) has been shown to be sensitive in both statistical and economic outcomes (e.g., Flodén 2008) and cannot handle cross-correlated processes (see Galindev and Lkhagvasuren 2010, for example). While several improvements have been proposed by Kopecky and Suen (2010) or Galindev and Lkhagvasuren (2010) for instance, it is not fully understood how sensitive to the discretization utility and choices are in a given model. Instead of relying on a finite-state approximation, we keep the continuous representation in equations (1) and (2) and treat π_t as an additional, continuous state variable. We use 24 gridpoints to approximate this continuous state and evaluate the expectations with Gauss-Hermite quadrature, for which convergence is well-known. To evaluate continuation utility at points off the grid, we use cubic two-dimensional B-splines. Details are provided in the computational appendix.

The second difficulty is that the model has a discrete choice—the stock market participation decision—which implies that the agent's problem is not (globally) differentiable in the continuous savings and portfolio choices.[13] As a consequence, we cannot rely on Euler equations and Newton-like nonlinear equation solvers. A brute-force maximization using discretization of the state space and the choices is also infeasible, because we have two continuous state variables, one binary state variable, 80 generations, along with two continuous and one discrete choice and want to calibrate the model to the data.[14] We solve this with a novel solution algorithm that is robust, fast, and generally applicable to finite-horizon problems. The main idea is to interpolate the expected continuation utility, $E_t U_{t+1}$, with a multi-dimensional cubic B-spline, because it can be proven

[13]Even recent, more general envelope theorems are of only very limited use in a computational application. E.g., the very powerful result in Clausen and Strub (2012) is not directly applicable, because in a numerical solution we search for an optimal choice and need to evaluate continuation utility also at points that are not optimal and may therefore not be differentiable. The computational appendix provides more details.

[14]The two continuous states are cash-at-hand, x_t, and the stochastic state, π_t, the discrete states are the 80 generations and the stock market participation indicator. The continuous choices are bond and stock investments and the discrete choice is stock market participation.

that $E_t U_{t+1}$ is twice differentiable. The divide and conquer algorithm of Gordon and Qiu (2015) is then used to quickly find a bracket for a global maximum on a fine grid. Then, the maximum is computed with a high precision using a Newton-like maximizing routine, which can be defended with the result of Clausen and Strub (2012) on the local differentiability around an optimum.

On top of that, we speed up the algorithm by making use of the fact that, after minor transformations, the optimal stock choice can be represented and computed as a function of the optimal savings choice. Programmed in Fortran 2008, the code is parallelized and runs on 24 cores. Further details are provided in the computational appendix.

5 Results

We now proceed to our results. We first describe the outcomes of the model, as calibrated in Section 4 and then provide some further explanation about the grounding of our findings.

5.1 Description

To present our results, we focus on the lifecycle profiles for agent choices. Each profile corresponds to the profile conditional on the agent surviving until the maximal age, averaged over all possible realizations for the income and investment risks. For agents that die before the maximal age, the savings-consumption profiles are simply truncated at the age of death. Lifecycle profiles –for savings for example– are computed as follows. For a given age, we compute the optimal saving response as a function of cash-at-hand as well as the distribution (conditional on surviving) of agents in terms of cash-at-hand.[15] From both optimal saving responses and conditional distribution, we directly get the average saving at each age.

Table 3: Selected Lifecycle Statistics

	Additive	Epstein-Zin	Risk-sensitive
Assets, age 45, in US \$ 1 000	*100.0*	58.5	*58.5*
VSL, age 45, in US \$ 1 000 000	*6.50*	12.8	15.3

Notes: Values in italics are calibrated as described in section 4.1.

Table 3 reports average life-cycle assets and value of life at the age of 45 for the additive, risk-sensitive and Epstein-Zin agents. Asset holdings of the additive agent match the empirical

[15]Since the shocks are continuous, we get a continuous distribution over cash-at-hand. We approximate this distribution with a piecewise linear function over 3600 points in the cash at hand grid. For details, see the computational appendix.

counterpart for individual savings at age 45 of US \$ 100 000, because that is a calibration target. Asset holdings for the other two agents are lower, amounting to US \$ 58 500. They are the same for both agents because we calibrated the parameters for the risk-sensitive agent to match the asset holdings of the Epstein-Zin agent at age 45, as is explained in Section 4.1. The value of a statistical life at age 45 is US \$ 6.5 millions for the additive agent, which was a calibration target. For the other two agents, the value of life at age 45 is higher, because we increase risk aversion while holding all other parameters—in particular v_0—constant. Thus, the agent is more averse to the risk of dying and values mortality risk reduction more. The values in Table 3 were part of the calibration strategy and represent a particular point of the full lifecycle profile, which we turn to next.

We now plot the lifecycle profile for savings in Figure 1. The three profiles, corresponding to the additive, risk-sensitive and Epstein-Zin agents exhibit a similar pattern. Agents build up savings during their working age, until they reach the exogenous retirement age of 65. During retirement, agents gradually decumulate their savings. The second element of this graph is that the savings of the additive agent are much larger that those of both the Epstein-Zin and risk-sensitive agents. Recall that the Epstein-Zin and risk-sensitive agents are both more risk averse than the additive one. The theoretical impact of risk aversion on savings is not clear cut when the three risks are simultaneously present,[16] but the main result of our quantitative exercise is unambiguous. An increase in risk aversion diminishes savings when investment, income and mortality risks are all present and properly calibrated. Our quantitative result means that the effect of the mortality risk dominates the effect of investment and income risks. This notably implies that it is crucial to properly take into account the value of life in the modeling and in the calibration. To conclude the discussion of saving profiles in Figure 1, note that the impact of a change in risk aversion (despite of a similar calibration) is not strictly identical for risk-sensitive and Epstein-Zin agents, except at age 45 (which is the age used for the value of life calibration). In particular, savings decrease more for the risk-sensitive agent at earlier age (before 45), but the pattern is reversed for later ages (after 45).

We relatedly plot the lifecycle consumption profiles in Figure 1. They are consistent with lifecycle saving profiles. Note that consumption profiles for the three agents are hump-shaped. Risk averse agents consume more at earlier ages (between ages 30 and 60) than the additive agent. The opposite holds at older ages, greater than 60. A greater risk aversion tends therefore to increase consumption at earlier age and to decrease it a later age. Again, this is consistent with the fact that the impact of mortality risk dominates the impact of other risks. A more risk averse

[16] See Table 1 and the related discussion in the introduction and in Section 5.2.1 for the impact of risk aversion on savings when only one the risk is present.

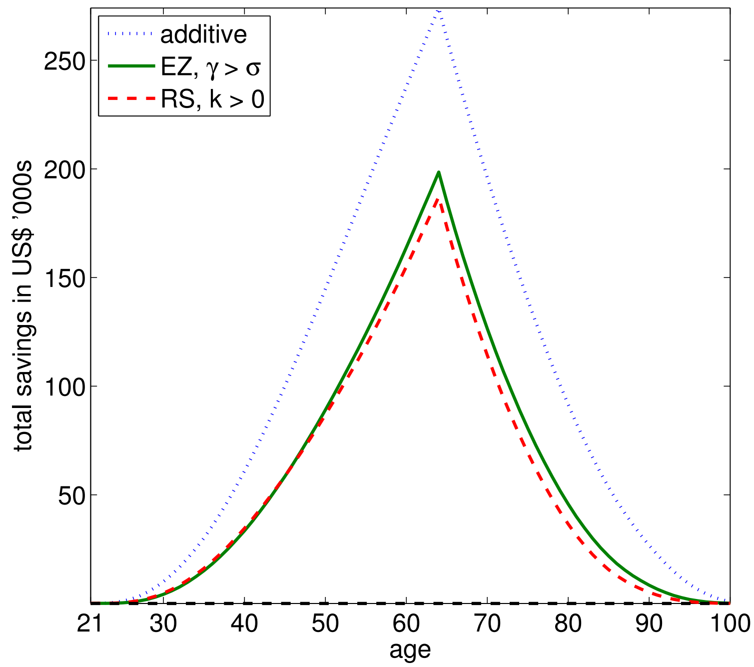

Figure 1: Lifecycle saving profiles

agent will be more prone to consume early and to save less in order to reduce the risk of dying while having large savings (even though they can be bequeathed).

We now plot the lifecycle participation rate as a function of age in Figure 3. First, since the participation cost is paid once in a life, the participation rate is an increasing function of age. Moreover, the other feature is that the stock market participation rate decreases with risk aversion for both the Epstein-Zin and the risk-sensitive agents. More risk averse agents choose to participate less in risky markets and therefore to have a smaller exposure to the investment risk. This is also consistent with the fact that more risk averse agents save less and are therefore less prone to pay a cost for participating to stock markets.

This pattern for participation rate is confirmed by the lifecycle profile of stock portfolio shares plotted in Figure 4. Conditional on the participation to stock markets, more risk averse agents hold a smaller share of their savings in stocks, except at later ages (above 85), where the wealth of agents becomes very low. More risk averse agent opt for less risky portfolios and a smaller exposure to the investment risk.

To summarize our findings in this quantitative exercise, we can state that in our environment in which agents simultaneously face investment, income, and mortality risks, more risk averse agents save less, choose to participate in the stock market less frequently and when participating, hold a smaller share of risky assets in their portfolios.

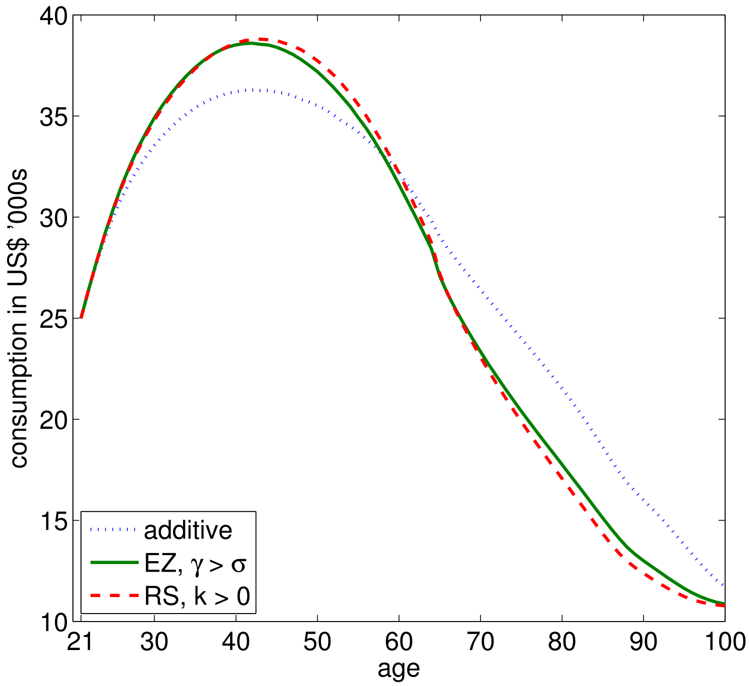

Figure 2: Lifecycle consumption profiles

5.2 Result discussion

Our discussion is threefold. First, we provide further intuitions about the fact that relationship between risk aversion and savings is ambiguous in presence of mortality, income, and investment risks. Second, we make a couple of back of the envelope computations to illustrate the orders of magnitude of mortality and income risks. Finally, we further investigate the importance of mortality risk and compute how much of the total variance in ex post utilities is explained by mortality. These two last computations show that mortality risk is the main risk from a lifecycle perspective. It is therefore not surprising that the effect of mortality risk dominates the ones of income and investment risks regarding the relationship between risk aversion and savings.

5.2.1 Why does economic theory predict an ambiguous relationship between risk aversion and savings?

To understand that the impact of risk aversion on savings is not clear-cut when the three risks (death, income and mortality) are present, let us start with focusing on situations where only one risk is present. To make the intuition more transparent, let us restrict to a two-state two-period economy. For each of the two states, there is an optimal state-specific saving in the first period, which corresponds to the optimal saving for a given state realization in absence of risk. The

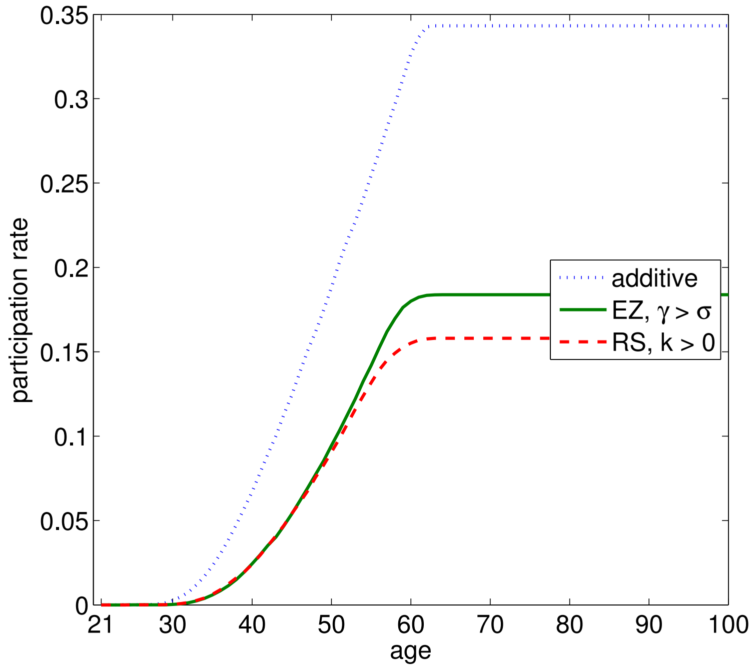

Figure 3: Lifecycle stock market participation

optimal saving in presence of risk can then be seen as a linear combination of these two state-specific optimal savings, where the weights depend on risk aversion. The more risk averse the agent, the greater the weight on the saving of the bad state. This reflects the intuitive fact that a risk averse agent cares a lot about bad state realizations.

We can now discuss the impact of risk aversion on savings in presence of one of the three aforementioned risks. In presence of income risk only, the bad state is a low income realization in the next period. The state-specific saving in the bad state is larger than the one in the good state. Therefore, in presence of income risk, a more risk averse agent will save more. When mortality is the only risk, the bad state is to die early and to leave savings, which are not consumed. In that case, a more risk averse agent will save less to avoid holding savings while dead. Finally, when the risk is an investment risk (through an uncertain financial rate of return), the state specific response mixes income and substitution effects and the best response depends on the intertemporal elasticity of substitution. When the intertemporal elasticity of substitution is greater (resp. lower) than one, the substitution (resp. income) effect dominates and bad-state-specific savings are lower (resp. higher) than the good state one. A more risk averse agent will thus save less (resp. more). These effects in presence of a single risk have been formally stated in BCL and summarized in Table 1.

In consequence, the direction of the impact of risk aversion on savings depends on the risk which is at stake, and when the three risks are present, this impact is possibly ambiguous and

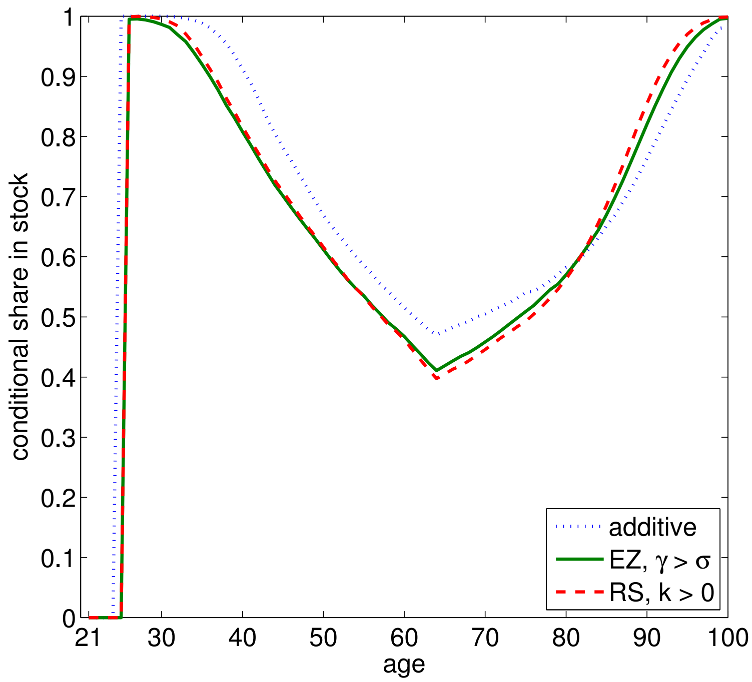

Figure 4: Share of wealth invested in stocks (conditional on participating)

will depend on the respective magnitude of individual effects. We explore magnitudes in the two following subsections.

5.2.2 How bad is the mortality risk compared to the income risk?

We make a couple of back of the envelope computations to assess a first order estimate for the magnitudes of mortality and income risks.

First regarding mortality risk, we obtain from our database that the life expectancy at age 20 amounts to 58.5 years (i.e., expected death at age 78.5) with a standard error of 14.5 years. This means that if we approximate the mortality risk by its standard error, the mortality risk amounts to 14.5 years. To convert this risk in monetary terms, note that we consider a value of life at age 45 equal to \$ 6.5 millions, while the life expectancy at that age is 35 years. The monetary value of one year of being alive is therefore approximately \$ 186000. We therefore deduce that as a first approximation, the mortality risk at age 20 amounts to \$ 2.7 millions in monetary terms.

Let us now consider the income risk from a lifecycle perspective. To measure this risk, we compute the distribution of future lifetime income seen from age 20. The lifetime income is simply the sum of all per period incomes discounted at the riskless interest rate to age 20. Since there is no income risk after age 65, we discard incomes after that age. Due to the discounting (and without

mentioning shock persistence), a bad income shock in earlier ages will have a much stronger impact on the lifetime income risk than the same bad shock at an older age. Using our calibration, we obtain an average lifetime income of \$ 1.1 million with a standard deviation of \$ 0.8 million.

Using these two proxies, we deduce that the mortality risk is approximately three times as "large" as the income risk. Moreover, as will be illustrated in the Section 5.2.3, the impact of the dispersion in lifetime incomes on lifetime utilities is dampened by the concavity of instantaneous utilities, while the impact of mortality risk on lifetime utilities is almost linear. In consequence, the mortality risk is not only greater than the income risk, but we also expect the mortality risk to have an even greater impact on the lifetime risk (measured as the dispersion of lifetime utilities).

5.2.3 How much of the lifetime risk is explained by mortality risk?

We finally assess the share of the total lifetime risk that can be explained by the mortality risk. We measure the lifetime risk as the risk associated to the distribution of (ex post) lifetime utilities. Any agent in our model enters the economy (alive) at age 20, experiences a sequence of income and investment shocks while alive, and finally dies at a given age. During her lifetime, the agent has made a sequence of consumption and saving decisions. It is therefore possible to compute the lifetime utility associated to this agent, once her life is over. Experiencing bad income shocks will negatively affect consumptions and thus lifetime utilities. Dying early, by shortening the number of consumption periods, will also have a negative effect on lifetime utilities.

To measure how much of the lifetime risk is explained by mortality, we simply regress lifetime utilities on dummy variables capturing the different possible lifetimes. The coefficient of determination (i.e., the R^2) of this regression indicates how much of the variance in lifetime utilities is explained by the variance in lifetimes. In other words, this tells us the share of the lifetime risk, which is explained by the mortality risk.

Formally, for any agent i in our population sample I, we denote V^i her lifetime utility and T_i her lifetime. Consistently with our model notations, the lifetime is normalized to vary between 1 and T_m. The regression can be expressed as follows

$$\forall i \in I,\ V_i = \sum_{t=1}^{T_m} \alpha_t 1_{t=T_i} + \sigma_\nu \nu_i, \tag{15}$$

where $\{\nu_i\}_{i\in I} \sim \mathcal{N}(0,1)$ are IID standard normal variables.

[To be completed]

6 Relation to previous literature

Our calibrated life-cycle models predict that savings decrease with risk aversion. This contrasts with the predictions of Gomes and Michaelides (2005, 2008), and those of HPSA when the elasticity of substitution is smaller than one, where they find the opposite result. The divergence between these studies and ours comes from how mortality is taken into account. The simplest way to get the intuition is probably to refer to Section 5.2.1 of the current paper or to Section 4.3 of BCL which discusses the role of risk aversion in a simple two-period model, where mortality is the only risk at play. It is explained that if (i) living two periods is better than living one period and (ii) an agent with no second period labor income saves more when anticipating to live for two periods than when anticipating to live only for one period; then risk aversion decreases savings. Intuitively, greater risk aversion leads to take saving decisions which are closer to those that would be optimal in the bad state of nature (i.e., living for one period here), that is to save less. Assumption (i) means a positive value of life, while Assumption (ii) means that survival and consumption are complementary. Although BCL do not discuss it, their conclusion would be reversed if Assumption (i) or (ii) would be reversed, that is if assuming a negative value of life, or assuming that consumption and survival are substitute.

Several papers, including Gomes and Michaelides (2005, 2008) and Gomes, Michaelides and Polkovnichenko (2009) consider an homothetic version of Epstein-Zin preferences to explore the role of risk aversion under the presence of mortality risk. The utility conditional of being alive at time t, denoted V_t, can then be expressed as follows:

$$V_t = \left((1-\beta) c_t^{1-\sigma} + \beta E_t \left(p_t V_{t+1}^{1-\gamma} + (1-p_t) b w_{t+1}^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}} \tag{16}$$

where c_t is consumption in period t, p_t is the probability of being alive in period $t+1$ and w_{t+1} the amount bequeathed in case of death.[17] The parameter σ is the inverse of the elasticity of substitution, γ the coefficient of relative risk aversion and b the intensity of the bequest motives. This model assumes that survival and consumption are complementary, in line with Assumption (ii) mentioned above. As for the sign of the value of life, one may compute:

$$\frac{\partial V_t}{\partial p_t} = \frac{\beta}{1-\gamma} \left(E_t[V_{t+1}^{1-\gamma}] - b E_t[w_{t+1}^{1-\gamma}] \right) E_t \left(p_t V_{t+1}^{1-\gamma} + (1-p_t) b w_{t+1}^{1-\gamma} \right)^{\frac{\gamma-\sigma}{1-\gamma}} V_t^{\sigma}.$$

which may be positive or negative. The above Assumption (i) can therefore be contradicted. In particular if $\gamma > 1$, as is assumed in the papers mentioned above, a positive value of life is obtained only if $b > \frac{E_t[V_{t+1}^{1-\gamma}]}{E_t[w_{t+1}^{1-\gamma}]}$. The results of Gomes and Michaelides (2005, 2008) tend to indicate that such

[17] A formal derivation of equation (16) can be found in the appendix of Gomes, Michaelides and Polkovnichenko (2009).

an equality does not hold (at least not always) in their simulations. In particular, a negative value of life is systematically obtained when there is no bequest motives ($b = 0$), a case that Gomes and Michaelides consider in several instances, with no significant impact on their qualitative findings about the relation between risk aversion and savings. With a negative value of life, survival is an adverse realization, and risk aversion is then found to increase savings, providing a conclusion which is opposite to ours. One should moreover notice that if specification (16) were to be used with $\gamma > 1$ and a parameter b large enough to generate positive values of life, we would obtain a framework where the intensity of bequest motives would increase the willingness to pay for mortality risk reduction. This goes against intuition, since deriving utility from bequest reduces the welfare gap between life and death.

HPSA, who are interested in endogenous health investment, takes great care of using a model that assumes a positive value of life. When the elasticity of substitution is smaller than one, as in their fitted specification, their model assumes however that consumption and survival are substitute, rather than complementary. To clarify this, one may rewrite the model of HPSA while focusing on mortality risk and leaving health and financial risks aside. The HPSA utility function is then given by:

$$U = \left(\int_0^\infty e^{-\delta t} s(t)^{\frac{1-\sigma}{1-\gamma}} (c(t) - a)^{1-\sigma} dt \right)^{\frac{1}{1-\sigma}} \tag{17}$$

where a is a minimal subsistence level, δ the exogenous rate of time preference, $s(t)$ the probably of being alive age t, $c(t)$ consumption at age t, σ the inverse of the elasticity of substitution, and $\gamma \in [0, 1)$ mortality risk aversion (denoted by λ_{m_0} in their paper). When $\sigma > 1$, such specification may cause convergence problems if mortality rates goes above some level.[18] Let us assume that this technical point can be solved by truncating the integral after some finite time T and assuming that survival never reaches zero before T. The utility function (17) assumes that the marginal utility of consumption is positive and that survival increases the value function. The value of life is thus always positive, as is emphasized in HPSA. The model is well-behaved in that respect. Let us however look at the marginal rate of substitution between consumption in period t and consumption in period 0:

$$\frac{\frac{\partial U}{\partial c(t)}}{\frac{\partial U}{\partial c(0)}} = e^{-\delta t} s(t)^{\frac{1-\sigma}{1-\gamma}}$$

If $\sigma > 1$, this marginal rate of substitution is decreasing with $s(t)$. In other words, the less likely is survival in period t, the more valuable it is to make provision for consumption in that period. According to this model, the rate of time discounting at time t is in fact $\delta + \frac{1-\sigma}{1-\gamma}\mu(t)$ where $\mu(t) = -\frac{s'(t)}{s(t)} > 0$ is the mortality rate at age t. Thus if $\sigma > 1$, mortality reduces impatience. This

[18] This does not occur in HPSA which is a perpetual youth model à la Blanchard (1985). However convergence issues would systematically occur in realistic life-cycle models where mortality tends to be large at old age.

is reflected in HPSA analysis, where it is found that when $\sigma > 1$, the propensity to consume is decreasing with mortality (see their Theorem 1 and their discussion p. 678). The effect is limited in HPSA, since their calibrated model, which does not account for age effects, considers relatively low mortality rates. However, if applying the same model to realistic demographic data (as suggested in Appendix B of HPSA), the fact that $\mu(t)$ gets large at old age would imply (when $\sigma > 1$) that mortality would make people becoming extremely patient at the end of their life cycle. In fact, agents with utility (17) would keep most of their wealth for consumption at very old ages (e.g., ages greater than 110), precisely because these are ages they will most likely never reach. This is of course implausible, but a logical consequence of the assumption of substitutability between consumption and survival which is embedded in equation (17). This eventually explains the divergence between the results of HPSA and ours.

7 Supporting evidence

In this section we present empirical results that support the predictions of the model. Specifically, we use direct, validated survey questions on individuals' risk aversion and study their impact on household savings. Our results indicate that savings comove positively with risk aversion in the data and that the regression coefficient is highly significant. We start with presenting the data before explaining our regressions and the related results.

7.1 Data description

The data comes from the German Socio-economic Panel (SOEP), which is an annual panel of German households starting in 1984. It covers a wide range of information, including financial situation and personal attitudes. Importantly, it also has a measure of general risk aversion, which asks respondents to rate their willingness to take risks on a scale from 0 (not willing at all) to 10 (very willing). Dohmen, et al. (2011) translate the question from German as "How do you see yourself: are you generally a person who is fully prepared to take risks or do you try to avoid taking risks? Please tick a box on the scale, where the value 0 means: 'not at all willing to take risks' and the value 10 means: 'very willing to take risks'." The question is on purpose not focusing on a single risk and doesn't make use of a lottery. This is an advantage in our context, where we consider aversion towards very different risks. As shown by Dohmen, et al. (2011), this measure is a good predictor of risky behavior such as investing in stock or smoking. They also validate the measure by conducting a field experiment with an additional representative sample of 450 subjects. They conclude that there seems to be a general trait of risk aversion affecting behavior in different contexts and that the SOEP measure is a good approximation. We are therefore confident that we

can use it to study the impact of risk aversion on savings.

We use the SOEP waves 2004 and 2008-2014 because risk aversion is available only in those. We keep only observations where the household head answered the household questionnaire and associate the personal characteristics of the head to that household. After dropping observations with missing or inadmissible values for risk aversion, the sample contains 24.603 households.

Household monthly net income is available in the data. Since financial wealth is not directly available for all waves, we follow Fuchs-Schündeln and Schündeln (2005) (henceforth, FSS) and impute it from interest and dividend income. As an alternative, we also use the reported value of the house as an indicator for financial wealth.

7.2 Regressions

Our model predicts that the more risk averse an agent, the more she will save. To test this prediction, we will simply regress the logarithm of monthly savings on the risk aversion index, while controlling for several factors.[19] In particular, we need to control for the income process, the mortality risk, the wealth, as well as other saving motives. The first control we use is the logarithm of financial wealth, to account for the fact that savings are path-dependent and that our prediction in our model concerns agents endowed with the same initial wealth. For the other controls, we basically follow FSS. More precisely, for the income process, we control by the permanent income and a measure of the income risk. For the permanent income, we replicate the construction of FSS.[20] To limit the effect of measurement errors and of bias due to the small number of observations for every household, we run a two-stage regression, by instrumenting the logarithm of permanent income by education and age variables (including the square of these variables and the interaction term). For every household, we measure the income risk as the standard deviation of the difference between the actual income and the permanent one (at the household level). Second, we control the mortality risk as in FSS by using age and square of age variables. Other saving motives are proxied by the following control variables: education, square of education, gender, dummies for the marital status, size of the household, number of children and a dummy for the current residence (1 if in former West Germany and 0 otherwise). We also use dummies for the survey year. Finally, we cluster standard errors by household.

[19] Formally, the index in the data varies between 0 and 10 and the larger the index, the more the respondent is willing to take risks. We therefore then simply define our risk aversion index as 10−this index.

[20] See their Footnote 20, p. 1098. First, we compute the detrended income every year as the ratio of the household income divided by the average income for all households in

corresponding survey year. The permanent income is then equal to the average detrended household (computed for every household over all survey years) multiplied by

the average income of all households within each survey year.

In order to further test the model predictions, we include two other variables besides risk aversion. The first one is an interaction term between risk aversion and income risk. This term will help us check that a more risk averse agent save more because of income uncertainty. The model therefore predicts the regression coefficient to be positive. The second additional variable reflects the interaction between mortality and income risk. According to the model, a more risk averse facing a high mortality risk should diminish her savings. The regression coefficient is therefore expected to be negative.

On the whole, we run four regressions. The first regression is the one including risk aversion, as well as all control variables. It does not include any of the two interaction terms. The second and third regressions include one single additional interaction term, between risk aversion and income risk and between risk aversion and mortality risk respectively. Finally, the last regression includes both interaction terms together.

7.3 Results

The results of our four regressions are gathered in Table 4.

Dependent variable = log(savings)	Risk aversion only (1)	(1) + interaction with income risk (2)	(1) + interaction with mortality risk (3)	(1) + both interactions (4)
risk aversion	-0.013^{***}	-0.020^{***}	-0.010^{**}	-0.016^{***}
	(0.004)	(0.006)	(0.004)	(0.006)
risk aversion		0.036		0.033
× income risk		(0.026)		(0.026)
risk aversion			-0.200^{*}	-0.191^{*}
× mortality (. 10^4)			(0.118)	(0.118)
log(wealth)	0.088^{***}	0.088^{***}	0.088^{***}	0.088^{***}
	(0.004)	(0.004)	(0.004)	(0.004)
permanent income	−0.005	−0.010	−0.014	−0.018
	(0.208)	(0.208)	(0.208)	(0.208)
income risk	0.285^{***}	0.098	0.285^{***}	0.111
	(0.081)	(0.156)	(0.081)	(0.157)
age	0.010^{*}	0.011^{*}	0.007	0.007
	(0.006)	(0.006)	(0.006)	(0.006)
age squared	−0.009	−0.009	−0.004	-0.004^{***}
(. 10^2)	(0.006)	(0.006)	(0.004)	(0.004)

educ.	0.196***	0.196***	0.197***	0.197***
	(0.042)	(0.042)	(0.042)	(0.042)
educ. squared	−0.004***	−0.004***	−0.004***	−0.004***
	(0.001)	(0.001)	(0.001)	(0.001)
gender (1 if male)	0.008	0.008	0.008	0.008
	(0.011)	(0.011)	(0.011)	(0.011)
married and	0.195***	0.195***	0.196***	0.196***
not separated	(0.030)	(0.030)	(0.030)	(0.030)
married	−0.216***	−0.217***	−0.213***	−0.213***
and separated	(0.066)	(0.066)	(0.066)	(0.066)
divorced	−0.167***	−0.168***	−0.164***	−0.165***
	(0.039)	(0.039)	(0.039)	(0.039)
widowed	0.186***	0.186***	0.194***	0.194***
	(0.047)	(0.047)	(0.047)	(0.047)
not married and	0.514	0.508	0.521	0.514
living together	(0.442)	(0.447)	(0.445)	(0.449)
household size	0.202***	0.201***	0.202***	0.201***
	(0.020)	(0.020)	(0.020)	(0.020)
number of children	−0.235***	−0.234***	−0.233***	−0.232***
	(0.022)	(0.022)	(0.022)	(0.022)
current residence	0.322***	0.322***	0.323***	0.323***
(1 if West)	(0.025)	(0.025)	(0.025)	(0.025)
constant	1.918	1.991	2.027	2.090*
	(1.268)	(1.268)	(1.269)	(1.269)
year dummies	yes	yes	yes	yes
observations	22973	22973	22973	22973
R^2	21.56	21.57	21.57	21.58

*** indicates significance at the 1 percent level; ** significance at the 5 percent level; * significance at the 10 percent level.

Table 4: Regression results

The first conclusion that we can draw from our regression results is that risk aversion has a negative impact on savings. More risk averse agents, when controlling for wealth, income and mortality risks, tends to save less. The regression coefficient is consistently highly significant in all regressions. The second lesson is that a more risk averse agent facing a higher income risk increases

her savings. The regression coefficient of the interaction term between risk aversion and income risk is consistently positive, though not highly significant (p-values vary between 15% and 20%). This second finding, in spite of a weak significance, is also consistent with the predictions of our model. Finally, the interaction between risk aversion and mortality is reported to have a negative contribution on savings. The regression coefficient is significant at the 10% level. More risk averse agents facings a high mortality risk tend to save less. This last finding is also predicted by our model.[21] These empirical results tend to confirm the predictions of our model.

8 Conclusion

We have studied the role of risk aversion in a model that accounts for labor income, investment and mortality risks. The first message that one can take from our paper is that, under reasonable calibration, mortality is found to be the main source of risk in life. This resonates with basic intuition, death being generally considered to be more dramatic than bankruptcy or job loss. Discussing the role of risk aversion requires therefore to carefully account for mortality risk, with appropriate assumptions on the complementary between survival and consumption and reasonable (positive) levels for the value of life. Once this is done, economic analysis provides a simple message: the main impact of risk aversion is to decrease life-cycle savings. The basic intuition is the following: saving, which involves keeping resources for an uncertain future, is a risk taking behavior. As a risk taking behavior, it is found less appealing by more risk averse agents.

In addition to mortality risk, our analysis accounts for labor income and asset return uncertainty. Labor income risk typically leads the agents to save more, an effect that is amplified by risk aversion. This effect is however secondary compared that driven by mortality risk and only marginally reduces the impact of risk aversion on savings previously mentioned. Similarly, asset return uncertainty may reduce or amplify the effect of risk aversion on savings (depending on whether the elasticity of substitution is larger or smaller than one) but, again, this is of secondary importance. The main finding that risk aversion decreases savings is therefore quite robust.

Risk aversion does not only impact the level of saving, but also portfolio choice. Our results indicate that more risk averse agents participate less to the stock market, which reflects an indirect wealth effect: more risk averse accumulate less savings and are therefore less likely to pay the fixed participation costs that our model assumes. Last, for those who are active on the stock market, risk aversion leads them to choose less risky portfolio, in line with standard results in finance.

One of the implications of our study, is that relatively low level of savings could be explained by risk aversion. While the economic literature abounds of works arguing that observed saving

[21] We do not comment further other coefficients, but most of them have the same sign as in FSS.

behaviors have to reflect strongly myopic preferences or some form of irrationality, our analysis suggests on the contrary that saving little could just be a rational decision for risk averse agents who are well aware that life duration is uncertain. Of course, low saving levels typically result in having a majority of (surviving) elderly declaring that they failed to save enough. But this is not evidence of under-savings. If they could be questioned, those who died before retirement would surely answer that they actually saved too much.

Another message brought by our analysis is that mortality risk being the most significant risk in life, the impact of changes in mortality risks have to be considered with great attention. In particular, one should use well-behaved models that properly account for risk aversion. Preliminary exploration can be found in Bommier (2014), but this is the whole field of economics of aging that deserve to be revisited with non-additive models. That will unavoidably brings new technical challenges, as those we addressed for our numerical exercise, but this is a cost to pay to get a better modeling of the link between economics and demography.

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