Financial Innovation and Asset Prices*

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Abstract

We study the effects of financial innovation on the asset allocation decisions of experienced and inexperienced investors and the dynamics asset prices. We show that when some investors, such as households, are less well informed about the new asset but learn about it, many “intuitive” results are reversed: financial innovation increases the volatility of investors’ portfolios along with the return volatility and risk premium for the new asset, all of which decline to their pre-innovation levels only slowly. If the new asset is illiquid, shocks to the new asset spill over to the traditional asset, increasing their return correlation and giving rise to a liquidity premium for the new asset. Despite the substantial increase in volatility, financial innovation improves the welfare of both inexperienced and experienced investors.

Keywords: household finance, household portfolio choice, differences in beliefs, parameter uncertainty, Bayesian learning, spillover effects, recursive utility.

JEL: G11, G12

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1 Introduction

A fundamental force in modern economies is financial innovation, which makes available new asset classes to investors who previously did not have access to them. In recent years, markets have been transformed by the rapid pace of financial innovation.\(^1\) Our objective in this paper is to understand the impact of financial innovation on the asset allocation decisions of experienced and inexperienced investors, such as households, and the dynamics of asset prices.\(^2\)

Should inexperienced investors include these new assets in their portfolios at all? How should they revise their portfolio over time as they gain experience? What are the dynamics of the optimal risk-sharing arrangement between experienced and inexperienced investors? What is the impact of investors’ trading decisions on the return volatilities and risk premia of new and traditional assets upon the introduction of the new asset, and how do these to change in the long run? Should financial regulators be concerned about the impact of new assets on capital markets? What are the welfare consequence of investing in new assets for inexperienced and experienced investors?

The traditional view about financial innovation (see, for example, Kihlstrom, Romer, and Williams (1981), Allen and Gale (1988), Weil (1992), and Elul (1997)) is that it facilitates risk sharing across investors while also improving diversification of each investor’s portfolio. Thus, financial innovation should smooth consumption, leading to a decrease in the return volatility and risk premium of the new asset and a substantial increase in its price.

The main contribution of our work is to show in a theoretical setting that the traditional view depends crucially on the assumption of common beliefs; that is, the new (inexperienced) investors need to have the same knowledge about the new asset class as the (experienced) investors who have been invested in it for some years already. In contrast, if inexperienced investors, who represent households in our model, are less well informed about the new asset

\(^1\)These new asset classes include hedge funds, private equity (buyout, venture, distressed), private placement (in public equity and catastrophe bonds), emerging-market equity and debt, mezzanine and distressed debt, real estate (commercial and timberland), infrastructure, natural resources, art and other collectibles, commodities, precious metals, and cryptocurrencies.

\(^2\)For comprehensive surveys of household finance, see Guiso, Haliassos, and Jappelli (2002), Haliassos (2002), Campbell (2006), and Guiso and Sodini (2013). Campbell (2016) provides an excellent discussion of the importance of regulating financial innovation when all households are not equally sophisticated. The report by Ramadorai (2017) provides a detailed empirical analysis of how financial education affects financial decision making in India.
class but learn about it over time, many of these “intuitive” results are reversed, as we explain below.

In order to understand the economic mechanisms through which financial innovation influences asset prices, we study a dynamic general-equilibrium economy that explicitly accounts for financial innovation and in which the asset-allocation decisions of investors incorporate the feedback effects from asset prices. In our model, there are three asset classes: a risk-free bond and two risky assets—a traditional asset class representing publicly traded equities and a new asset class. There exist two groups of investors with identical recursive preferences and a preference for early resolution of uncertainty. Initially only one group of investors, called “experienced investors,” has access to the new asset class. Financial innovation is modeled as the process through which the other group—“inexperienced investors”—gains access to the new asset class.

Our model is “standard” in all respects except one: to capture the salient feature of a new asset class, we allow experienced and inexperienced investors to disagree about its future prospects, though both classes of investors are fully Bayesian. Experienced investors, who have been invested in the new asset for some years already, are more confident about their views regarding the new asset than inexperienced investors, who are new to this asset class.

The model yields many surprising predictions about the impact of financial innovation on asset prices. To understand the changes induced by financial innovation, we start by noting that before financial innovation gives inexperienced investors access to the new asset, the portfolios of both groups of investors are highly concentrated: Inexperienced investors, i.e. households, overweight the traditional asset, whereas experienced investors hold all of the new asset. Moreover, because inexperienced investors have no access to the new asset, they bear more risk than experienced investors and thus, invest in the risk-free bond.

Upon financial innovation, in the traditional setting with common beliefs, investors would instantly switch to holding a well-diversified portfolio. This would lead to a decline in the average volatility of the investors’ stochastic discount factors because of a better sharing of the risks associated with the new asset. Consequently, the new asset’s risk premium and return

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3The difficulties in assessing new asset classes are discussed by: Phalippou (2009), Phalippou and Gottschalg (2009), and Ang and Sorensen (2013) for private equity; Dhar and Goetzmann (2005) for real estate; and Ang, Ayala, and Goetzmann (2014) for hedge funds and private equity. For evidence on how experience with new asset classes affects investment behavior, see Blackstone (2016, p. 11).
volatility would decline. In contrast, in the presence of differences in beliefs, inexperienced investors’ portfolios remain highly concentrated in traditional assets. In particular, they underweight the new asset class because of a negative intertemporal hedging demand to protect against downward revisions in the perceived dividend-growth rate of the new asset.\(^4\) On the other hand, experienced investors have levered positions that overweight the new asset.

As a result of these asset-allocation decisions, the impact of financial innovation on the volatility of the stochastic discount factor is also very different under belief disagreement. That is, because the portfolios of experienced investors are still tilted toward the new asset after financial innovation, the volatility of their stochastic discount factor declines only marginally. At the same time, the combination of parameter uncertainty and Bayesian learning leads to a substantial increase in the volatility of the inexperienced investors’ stochastic discount factor.

The effect of financial innovation on the new asset’s return volatility and its risk premium is also strikingly different under belief disagreement: The new asset’s return volatility increases substantially when it becomes available to the inexperienced investors because learning amplifies the fluctuations in the new asset’s dividends. For example, positive cash-flow news implies an upward revision in the perceived dividend-growth rate. If investors have a preference for early resolution of uncertainty, this improvement in their investment opportunity set implies that they save more.\(^5\) Moreover, because of the higher expected dividend-growth rate for the new asset, inexperienced investors dynamically allocate a larger fraction of their savings to the new asset. Both effects increase the demand for the new asset, which leads to a higher price-dividend ratio, thus increasing the return volatility. Thus, the new asset, which seemed particularly promising prior to financial innovation, performs relatively poorly after financial innovation so that late entrants into new asset classes do not earn the same rewards as experienced investors. Differences in beliefs also induce a strong countercyclical variation in the new asset’s return moments, in contrast to the cyclical behavior under common beliefs.

\(^4\)Inexperienced investors wish to hold a portfolio that performs well when marginal utility is high or, equivalently, the new asset’s perceived dividend-growth rate is low. This is achieved through a negative intertemporal hedging position in the new asset because its return is positively correlated with the perceived dividend-growth rate.

\(^5\)When investment opportunities improve, the substitution effect induces investors to consume less and save more. In contrast, the income effect induces them to do the opposite. If investors have a preference for early resolution of uncertainty, the substitution effect dominates.
The effects described above—the concentration of investors’ portfolios and the resulting increase in the return volatility and risk premium of the new asset—last for several decades. These quantities decline only slowly over time, as the inexperienced investors’ estimate of the dividend-growth rate becomes more precise.

In spite of the increase in volatility, financial innovation increases the welfare of inexperienced investors. Intuitively, financial innovation allows them to improve portfolio diversification and enhance risk sharing. Moreover, the magnitude of the shocks transmitted from the new asset to existing assets is quite small because of the relatively small size of the new asset relative to the magnitude of existing public equities, indicating that regulatory concerns on this account should be small as well.

In our baseline model, discussed above, financial innovation can occur only at a predetermined point in time and the decision about financial innovation is exogenous. In a first extension of the model, we endogenize the decision to introduce the asset; in particular, we consider the case in which financial innovation is more likely if an asset class has performed well and demand for it is high. Next, we also endogenize the timing of financial innovation. In both cases, the results from our baseline model continue to hold. We also show that the results are robust to changes in the parameter values used to model the beliefs of inexperienced investors.

Our paper is related to four strands of the literature. First, we consider the theoretical literature on financial innovation. Simsek (2013a,b) focuses on its impact on portfolio risks and shows that, in the presence of belief disagreement, financial innovation can lead to an increase in portfolio risks for two reasons: one, investors take on new bets, and two, they increase the magnitude of their existing bets. We highlight a third channel that causes an amplification of portfolio risks: an endogenous increase in the new asset’s return volatility. Iachan, Nenov, and Simsek (2016) show that access to new risky assets, together with heterogeneous beliefs, induces investors to save more, which, in turn, can explain the decline in returns of various asset classes over the last few decades. We demonstrate that in a dynamic setting, the implications of financial innovation are even more complex and surprising. That is, return volatilities and risk premia increase before they slowly converge to their (lower) long-term levels.\footnote{This dynamic process of financial innovation, in combination with Bayesian learning, shares similarities with Pástor and Veronesi (2012), who study the impact of new government policies on stock prices when a representative investor faces uncertainty about government policies.} None of these studies analyzes learning, which plays an important role here. Gennaioli, Shleifer, and Vishny
(2012) focus on the security-issuance aspect of financial innovation; in their model, financial intermediaries cater to investors’ preferences for safe cash-flow patterns and their biased beliefs (due to neglected risk), leading to excessive issuance and, in the long-run, fragile markets. Basak and Pavlova (2016) show that the entry of new, institutional investors in the commodity markets leads to higher prices and more volatile returns. In their model, these effects arise because the new investors care about their performance relative to a commodity index, whereas in our model they are a consequence of new investors being less well informed than experienced investors.

Second, our work is related to the literature on learning in general equilibrium. In a comprehensive analysis, Collin-Dufresne, Johannes, and Lochstoer (2016a) show that parameter learning strongly amplifies the impact of macro shocks when a representative investor has a preference for early resolution of uncertainty. In our paper, the focus is on the interaction between learning and financial innovation. Thus, in contrast to their model, we consider an economy with multiple risky assets—a traditional asset and a new asset class—with parameter uncertainty about only the new asset’s expected dividend-growth rate. Moreover, in our model there are two groups of investors with differences in beliefs. Interestingly, even though learning in our model is only about a fraction of aggregate consumption—that represented by the new asset class—and for a fraction of investors—those who are inexperienced—it still has substantial effects on asset prices, which strengthens their findings.

Third, our work is related to the literature on heterogeneous investors and portfolio constraints. Chabakauri (2015) studies, among others, the effect of a limited-participation constraint. He demonstrates that this leads to a higher market price of risk in order to compensate unconstrained investors for holding more stocks to clear the market, excess volatility, and a lower price-dividend ratio. While our results are similar to his results for the period prior to financial innovation, our focus is on the impact of asset prices at and after financial innovation. Hong, Scheinkman, and Xiong (2006) study the impact of a change in asset float (tradeable shares)

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7For an extensive discussion of the broader set of issues related to financial innovation, see Merton (1992), Allen and Gale (1994), and the World Economic Forum Report (2012).

on asset prices, documenting that volatility and prices increase when fewer shares are available. Panageas (2005) studies the implications of heterogeneous beliefs and portfolio constraints for physical investment. Other work includes Basak and Cuoco (1998), Basak and Croitoru (2000), Gallmeyer and Hollifield (2008), Prieto (2013), and Chabakauri (2013). In contrast to this literature, we focus on financial innovation, which can be interpreted as the relaxation of the constraint on the portfolio of inexperienced investors. In particular, we show that in the presence of belief disagreement, many of the results about asset prices and returns from models that assume common beliefs can be reversed. Our paper also contributes to this literature by allowing for parameter uncertainty and Bayesian learning.

Fourth, there are several papers that study the effect of heterogeneous beliefs in the absence of portfolio constraints; for instance, Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Xiong and Yan (2010). In contrast to these papers, in which investors have risk-neutral or time-additive preferences, we incorporate recursive preferences, in the presence of which parameter learning leads to an additional risk premium and excess volatility. Moreover, our focus is on the changes in the dynamics of the assets’ returns over time, and the interaction between heterogeneous beliefs and financial innovation.

The rest of the paper is organized as follows. In Section 2, we describe the baseline model of the economy we study, equilibrium in this economy, and our solution approach. In Section 3, we analyze the effect of financial innovation on portfolio positions, consumption policies, and asset prices. Section 4 summarizes the insights from our robustness analysis, including the results from various extensions of the baseline model. Section 5 concludes. Technical details are relegated to the Appendix.

2 A Model of Financial Innovation

In this section, we describe the general-equilibrium model that we use to study the impact of financial innovation on asset prices, with our modeling assumptions being motivated by the desire to have the simplest possible setting. In Section 4, we discuss various generalizations of this baseline model.
The model is set in discrete time with time interval $\Delta t$ and a finite horizon $T$; that is, $t \in \{0, 1, \ldots, T\}$. The key feature of the economy is that it is populated by two groups of investors who differ in their access to financial assets and disagree how to value the new asset. The first group, experienced investors, can invest in all three financial assets that are available. The second group, inexperienced investors, can initially trade only in the risk-free bond and the traditional risky asset; they gain access to the new asset class only after financial innovation. Moreover, inexperienced investors are uncertain about the new asset’s expected dividend-growth rate and learn about it over time. We describe the details of our model below.

2.1 Economic Framework

**Investors:** The two groups of investors, indexed by $k \in \{1, 2\}$, are assumed to have identical Epstein and Zin (1989) and Weil (1990) preferences over consumption $C_{k,t}$ of the single consumption good.\footnote{It would be straightforward to allow for heterogeneity in investors’ preferences; we assume identical preferences so that we can focus on the effects of financial innovation.} Specifically, lifetime utility $V_{k,t}$ is defined recursively as

$$V_{k,t} = \left(1 - \beta\right) C_{k,t}^{1 - \psi} + \beta E_t^k \left[V_{k,t+1}^{\frac{1}{\phi}}\right]^{1 - \psi},$$  \hspace{1cm} (1)

where $E_t^k$ denotes the time-$t$ conditional expectation under an investor’s subjective probability measure, $\beta$ is the rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, $\psi > 0$ is the elasticity of intertemporal substitution (EIS), and $\phi = \frac{1 - \gamma}{1 - 1/\psi}$.

**Financial Assets:** There exist three financial assets. The first asset is a risk-free single-period discount bond in zero net supply, indexed by $n = 0$. In addition, there are two risky assets, indexed by $n \in \{1, 2\}$—each in unit supply and modeled as a claim to the dividend stream, $D_{n,t}$, of a Lucas (1978) tree. Specifically, we assume that, for each tree, log dividend growth $\Delta d_{n,t+1} \equiv \ln[D_{n,t+1}/D_{n,t}]$ can be described by an IID Normal model with expected dividend-growth rate $\mu_n$ and dividend-growth volatility $\sigma_n$

$$\Delta d_{n,t+1} = \mu_n + \sigma_n \varepsilon_{n,t+1},$$ \hspace{1cm} (2)

where $\varepsilon_{n,t+1} \sim \mathcal{N}(0, 1)$, and $\varepsilon_{1,t+1}$ and $\varepsilon_{2,t+1}$ are assumed to be uncorrelated.\footnote{Even though dividends are uncorrelated, asset returns, which depend on equilibrium prices, will be correlated. A detailed description of the economic mechanism that drives this correlation is given in footnote 36.} We interpret the first risky asset as a traditional asset that has been available to all investors for some time...
already, for example, public equity, whereas the second risky asset represents a new asset class. For our main analysis, we assume that all financial assets are perfectly liquid; that is, they can be traded without incurring costs. In Section 4, we also consider the case in which the new asset is illiquid.

**Financial Innovation:** Experienced investors \((k = 1)\) can always trade all three financial assets. In contrast, we assume that inexperienced investors \((k = 2)\) can trade only the risk-free bond and the traditional risky asset until date \(t = \tau - 1\). The new asset class becomes available to inexperienced investors only if *financial innovation* occurs. Specifically, we assume that at time \(t = \tau\), the new asset is made available to inexperienced investors with probability \(p\), who therefore cannot fully anticipate whether financial innovation will occur. The variable that indicates whether or not financial innovation has occurred is \(I_\tau \in \{0, 1\}\). One can think of our framework as a setting in which a subset of sophisticated experienced investors has access to a new asset class before it becomes available to the rest of the investors; for example, for many recently introduced asset classes, such as private equity, venture capital, hedge funds, and commodities, initially only a fraction of investors were able to participate.

Note that in our baseline model financial innovation can only occur at a predetermined point in time. In practice, the introduction of a new asset class would be undertaken by financial intermediaries whose decision would depend on the demand for the new asset. Therefore, in Section 4, we consider extensions of the model in which the timing and the decision to introduce the new asset are endogenous, and we find that the insights from our baseline model remain unchanged in these more general cases.

### 2.2 Learning

For ease of exposition, we assume that there is no uncertainty regarding the dynamics of the dividends of the traditional asset and also that experienced investors have perfect knowledge of the new asset’s dividend dynamics.\(^{11}\) Both groups of investors also know that the two assets’ dividends are uncorrelated. In contrast, when inexperienced investors gain access to the new

\(^{11}\)The model can be extended in a straightforward way to incorporate generalizations, such as parameter uncertainty for both investors and/or parameter uncertainty about both risky assets. Also, one could allow inexperienced investors to learn about the new asset class starting from date \(t = 0\) instead of \(t = \tau\), even though the new asset becomes available for trading only at date \(\tau\). The results in these cases are similar to the ones for the baseline model.
asset class, these investors are less well informed about the new asset than experienced investors, who had been holding the asset until then, but learn about it by observing realized dividend growth.\textsuperscript{12} Moreover, inexperienced investors do not learn from the other investors’ behavior; rather, they “agree to disagree.”\textsuperscript{13}

Specifically, we assume that inexperienced investors are uncertain about the new asset’s expected dividend-growth rate, $\mu_2$, but know its dividend-growth volatility, $\sigma_2$.\textsuperscript{14} The investors start, at date $\tau$, with a conjugate prior $\mu_2 \sim N(\mu_{2,\tau}, A_\tau \sigma_2^2)$ and update their beliefs based on realized dividend growth using Bayes’ rule.\textsuperscript{15} The prior, combined with the dividend dynamics in (2), implies a time-$t$ posterior density function $p(\mu_2|d_2^t) = N(\mu_{2,t}, A_t \sigma_2^2)$, where $d_2^t$ denotes the history of all observed dividend-growth realizations up to time $t$: $d_2^t = \{\Delta d_2,s\}, s \in \{\tau, \ldots, t\}$, $t \geq \tau$ and the dynamics of $\mu_{2,t}$ and $A_t$ are described by

$$\mu_{2,t+1} = \mu_{2,t} + \left(\Delta d_{2,t+1} - \mu_{2,t}\right) \frac{A_t}{1 + A_t},$$  \hfill (3)

$$A_{t+1} = \frac{1}{1/A_t + 1}. \hfill (4)$$

Consequently, even though the dividend dynamics of the new asset are driven by an IID model with constant parameters, from the perspective of inexperienced investors the expected dividend-growth rate $\mu_{2,t}$ is time-varying; with the inexperienced investors sometimes being more optimistic and at other times being more pessimistic than the experienced investors.

Panel A of Figure 1 plots, for a simulated path of the economy, the expected dividend-growth rate of the new asset as perceived by the inexperienced investors. When realized dividend growth is higher than expected, highlighted in shaded gray, the investors revise their beliefs about $\mu_2$ upward and vice versa for lower than expected shocks. In particular, Bayesian learning implies that posterior estimates of the parameters are martingales. Thus, revisions in beliefs constitute permanent shocks to the inexperienced investors’ perceived dividend-growth rate. However, the

\textsuperscript{12}There is substantial empirical evidence about the difficulties in assessing a new asset class because of limited data and investor inexperience; see footnote 3.

\textsuperscript{13}Morris (1995) explains why it is reasonable to assume that investors have different priors and that this is fully consistent with rationality. There is a large literature that uses this formulation; see, for example, Basak (2005), Dumas, Kurshev, and Uppal (2009), and the papers cited therein.

\textsuperscript{14}Merton (1980) shows that the mean is much more difficult to learn than volatility. For instance, in a continuous-time setting with Brownian motions, Bayesian learning would immediately reveal the volatility of the underlying process.

\textsuperscript{15}$A_\tau \sigma_2^2$ represents the reciprocal of the prior precision, so that the prior density converges to a uniform distribution as $A_\tau$ approaches infinity and converges to a single value as $A_\tau$ approaches zero. Intuitively, $A_\tau$ denotes the number of quarters of data used to update an initially flat prior.
posterior variance $A_t \sigma^2_t$ converges deterministically to zero (Panel B of Figure 1), because $A_t$ in (4) converges to zero. This implies ever smaller updates to the perceived growth rate over time, because the impact of the realized dividend growth $\Delta d_{2,t+1}$ on $\mu_{2,t+1}$ declines as $A_t$ declines. Hence, the investors’ perceived growth rate converges in the long run—though not necessarily to the true value $\mu_2$. Panel C shows the probability density function of the new asset’s expected dividend-growth rate as perceived by the inexperienced investors, $\mu_{2,t}$, for different dates. Even though, the distribution converges quickly (always being nicely centered around the true mean), one can observe substantial variation in the beliefs for all horizons.

### 2.3 Investors’ Optimization Problem and Equilibrium

The objective of investors in group $k$ is to maximize their expected lifetime utility (1), by choosing their consumption, $C_{k,t}$, and their holdings in the available financial assets, $\theta_{k,n,t}, n \in \{0, \ldots, N_{k,t}\}$, subject to the budget equation

$$C_{k,t} + \theta_{k,0,t} S_{0,t} + \sum_{n=1}^{N_{k,t}} \Delta \theta_{k,n,t} S_{n,t} \leq \theta_{k,0,t-1} + \sum_{n=1}^{N_{k,t}} \theta_{k,n,t-1} D_{n,t}, \quad (5)$$

where $\Delta \theta_{k,n,t}$ denotes the change in the shares held of asset $n$ and $S_{n,t}$ denotes the price of asset $n$. In particular, while experienced investors always have access to all financial assets ($N_{1,t} = 2, \forall t$), inexperienced investors initially have access to only the risk-free bond and the traditional risky asset ($N_{2,t} = 1, t < \tau$) and gain access to the second risky asset only if financial innovation occurs ($N_{2,t} = 2$ for $t \geq \tau$ if $I_\tau = 1$).

The left-hand side of budget equation (5) describes the amount allocated to consumption, the purchase or sale of the (newly issued) short-term bond, and changes in the portfolio positions in the risky assets. The right-hand side reflects the available funds, stemming from the unit payoff of the (old) short-term bond as well as the dividends from the holdings of the available risky assets. In Appendix A, we derive the corresponding first-order conditions.

Equilibrium in the economy is defined as a set of consumption and asset-allocation policies, along with the resulting price processes for the financial assets such that the consumption policies of all investors maximize their lifetime utility, that these consumption policies are financed by the asset-allocation policies that investors choose, and that markets for the financial assets and the consumption good clear.
Figure 1: Learning

This figure illustrates rational learning in our model. Panel A depicts the dividend-growth rate for the new asset, as perceived by inexperienced investors, for a simulated path of the economy. The shaded gray areas indicate periods of positive dividend realizations for the new asset. Panel B shows the (deterministic) decline in the variance of the inexperienced investors’ posterior, $A_t \sigma^2_t$, over time. Panel C shows the distribution of the expected dividend growth rate of the new asset as perceived by the inexperienced investors, $\mu_{2,t}$ after the elapse of 30, 70, and 100 periods. This figure is based on the following parameter values: $\Delta_t = 1/4 \text{ year}$, $\mu_{2,\tau} = 0.0045$, $A_\tau = 20$, and $\sigma^2 = 0.0275$.

There are four state variables in the economy: the dividend share of the first risky security $\delta_{1,t} \in (0, 1)$, the dynamics of which follow from the joint dividend dynamics in equation (2); the expected dividend-growth rate of the new asset class as perceived by the inexperienced investors, $\mu_{2,t}$, with the dynamics specified in equation (3); the (deterministic) posterior variance of the inexperienced investors’ beliefs, $A_t \sigma^2_t$, with the dynamics specified in equation (4); and the consumption share of the experienced investors, $\omega_{1,t} \in (0, 1)$. 
2.4 Solving for the Equilibrium

If financial markets are complete, one can separate the task of identifying the equilibrium into two distinct steps by exploiting the condition that investors can achieve perfect risk sharing. First, one identifies the optimal allocation of aggregate consumption across investors, and then, in the second step, determines asset prices and also the portfolio policy of each investor that supports this consumption allocation.

However, in economies in which financial markets are incomplete, such as the one considered here, one must solve simultaneously for the consumption and investment policies of the two groups of investors along with asset prices, leading to an equation system that requires backward and forward iteration in time, instead of a purely recursive system. Dumas and Lyasoff (2012) show how this backward-forward system of equations can be reformulated to obtain a purely recursive system, using a “time shift” for a subset of the equations that characterize the equilibrium. We solve for the equilibrium numerically, extending the algorithm proposed by Dumas and Lyasoff (2012) to the case of parameter uncertainty with Bayesian learning, multiple risky assets, and Epstein-Zin-Weil utility functions. Details of the numerical algorithm are provided in Appendix B.

3 Effects of Financial Innovation

We now illustrate the impact of financial innovation on the dynamics of the investors’ asset allocation decisions, their consumption policies, and the resulting effect on the stochastic discount factors as well as the dynamics of asset prices and returns. We conclude with a brief discussion of the welfare effects of financial innovation.

3.1 Parameter Values

The parameter values used in our numerical illustrations are summarized in Table 1. We solve the model at a quarterly frequency (Δ_t = 1/4 year) for T = 800 periods; that is, for a total of 200 years. We assume that the inexperienced investors get access to the new asset at time τ = 40 (that is, after ten years) with probability p = 0.50, and we report the dynamics for the quantities of interest for the following 100 periods (that is, 25 years). The distant terminal
Table 1: Model Parameters
This table reports the parameter values used for our numerical illustrations. The choice of these parameter values is explained in Section 3.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Base Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_t$</td>
<td>Trading (and observation) interval</td>
<td>1/4 year</td>
</tr>
<tr>
<td>$T$</td>
<td>Total number of trading dates (quarters)</td>
<td>800</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Date of introduction of the new asset (quarters)</td>
<td>40</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of introduction of the new asset</td>
<td>0.50</td>
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<td>$\beta$</td>
<td>Rate of time-preference (per quarter)</td>
<td>0.993</td>
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<td>$\gamma$</td>
<td>Relative risk-aversion</td>
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<tr>
<td>$\psi$</td>
<td>Elasticity of intertemporal substitution</td>
<td>1.50</td>
</tr>
<tr>
<td>$w_{2,0}$</td>
<td>Initial wealth share of the inexperienced investors</td>
<td>2/3</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>Expected dividend growth (per quarter)</td>
<td>0.45%</td>
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<tr>
<td>$\sigma_n$</td>
<td>Dividend growth volatility (per quarter)</td>
<td>2.75%</td>
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<td>$\rho$</td>
<td>Correlation between dividend growth rates</td>
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<td>$\delta_{1,0}$</td>
<td>First asset’s share of total initial dividends</td>
<td>0.80</td>
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<td>$\lambda$</td>
<td>Leverage factor</td>
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<td>$\mu_{2,\tau}$</td>
<td>Initial mean of inexperienced investor’s prior distribution</td>
<td>0.45%</td>
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<tr>
<td>$A_\tau$</td>
<td>Initial precision of inexperienced investor’s prior distribution</td>
<td>20</td>
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<tr>
<td>$\mu_2, \mu_2$</td>
<td>Truncation boundaries for beliefs of inexperienced investors</td>
<td>$[-0.65%, 1.55%]$</td>
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</tbody>
</table>

date of $T = 800$ is chosen to minimize any effects resulting from the finite horizon. The results reported below are based on 500,000 simulations of the economy.

For the preference parameters, we use a (quarterly) rate of time-preference $\beta = 0.993$, a coefficient of relative risk-aversion, $\gamma = 10$, and an elasticity of intertemporal substitution $\psi = 1.50$—common choices in the literature (see, for instance, Bansal and Yaron (2004) and Collin-Dufresne, Johannes, and Lochstoer (2016a)).

We assume that at time $t = 0$, inexperienced investors are endowed with $2/3$ of the total wealth, as it seems reasonable that initially a majority of investors does not participate in the new asset class. The initial wealth of the inexperienced investors is fully concentrated in the market portfolio; that is, they do not have any debt and are not endowed with any shares of the new asset class. On the other hand, experienced investors hold all the shares of the new asset class with the rest of their wealth in the traditional risky asset; they, too, start out with zero debt.
For both trees, \( n \in \{1, 2\} \), we set the expected dividend-growth rate \( \mu_n \) to 0.45% per quarter and the corresponding volatility \( \sigma_n \) to 2.75%.\(^{16}\) Together with an initial dividend share of the first tree, \( \delta_{1,0} \), of 0.80,\(^{17}\) this implies an expected growth rate of aggregate consumption of 1.80% per year and a time-average annual aggregate consumption growth volatility of 3.73%. These numbers are close to the annual time-averaged mean and volatility of U.S. per capita consumption growth of 1.72% and 3.28% from 1889 to 1994; the higher consumption-growth volatility is required to get a reasonable equity risk premium. When computing the returns of the claims to the dividend trees we apply a leverage factor of \( \lambda = 2.5 \) to accommodate the fact that most assets are implicitly levered (a common assumption in the literature—Collin-Dufresne, Johannes, and Lochstoer (2016a, p. 21) use a value of 2.5 and Bansal and Yaron (2004, Table II) use a value of 3).

We set the mean of the inexperienced investors’ prior distribution, \( \mu_{2,\tau} \), equal to the true expected dividend-growth rate \( \mu_2 \), leading to, on average, unbiased beliefs. The parameter governing the initial precision, \( A_\tau \), is set to 20 and we truncate the inexperienced investors’ beliefs at \( \mu_2 = -0.65\% \) and \( \bar{\mu}_2 = 1.55\% \).\(^{18}\) In the figures below that are used to illustrate our results, in addition to this case of differences in beliefs, we also always report the results for the case of common beliefs; that is, for the case in which both groups of investors know the true growth rate of the new asset.

The choice of equal means and volatilities for the two dividend trees guarantees a stable mean dividend share over time. Thus, we abstract from any effects arising from a drift in the mean dividend share. Note, however, that in the long-run one approaches a bimodal distribution with dividend shares of zero and one—a standard result for such models; see, for example, Cochrane, Longstaff, and Santa-Clara (2008). While this might pose some problems for a long-term analysis, it is less important for our analysis, because our focus is on the transitional dynamics for the initial years following financial innovation. In fact, even after 100 years, the distribution is well behaved. In one of the extensions that we consider of the baseline model, we consider a stationary distribution for the dividend share and find that our results remain unchanged.

The market capitalizations implied by a long-term mean dividend share for the new asset class of 20% are realistic for a variety of asset classes. For example, assets under management for private equity and hedge funds are about $3.5 and $1.7 trillion, respectively—relative to $18 trillion for U.S. stock-market capitalization.

Accordingly, we use a truncated Normal prior for \( \mu_2 \), which results in a posterior that is truncated Normal with the same truncation bounds as the prior (see the online appendix of Collin-Dufresne, Johannes, and Lochstoer (2016a)). Conveniently, the updating equations for the hyperparameters, \( \mu_{2,t+1} \) and \( A_{t+1} \) remain the same—though \( \mu_{2,t+1} \), in general, no longer corresponds to the subjective conditional mean of the new asset’s dividend growth. Truncation is needed for an elasticity of intertemporal substitution that differs from one, to guarantee the existence of equilibrium (see the discussion in the online appendix of Collin-Dufresne, Johannes, and Lochstoer (2016a)). In Section 4, where we examine the robustness of our modeling assumptions, we document that the effect of the truncation bounds is negligible and that the choice of initial precision, \( A_\tau \), has no effect on the qualitative results.

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3.2 Asset Allocation

We start by studying the asset allocation decisions of the two groups of investors. Figure 2 shows the dynamics of the investors’ portfolio shares, that is, the proportion of the investors’ wealth allocated to each of the three financial assets.

First, note that before the introduction of the new asset at time $\tau = 40$, the portfolios of both groups of investors are basically the same across the cases of common beliefs and differences in beliefs; that is, in all panels of Figure 2, the two lines essentially coincide for dates $t < 40$. However, there are large differences in the portfolios of experienced and inexperienced investors. The portfolios of inexperienced investors consist of a constant fraction of their wealth invested in the risk-free bond (Panel A of Figure 2), with their remaining wealth invested in the traditional asset (Panel E). On the other hand, the portfolio of experienced investors consists of a short position in the risk-free bond (Panel B) and long positions in the two risky assets (Panels D and F). *Neither group of investors holds the market portfolio*:\(^{19}\) Inexperienced investors are overinvested in the traditional asset (Panel E); in contrast, experienced investors are overin invested in the new asset (Panel D)—in both cases because the inexperienced investors have no access to the new asset. As a consequence of this under-diversification, the risky parts of the portfolios of both groups of investors are *more volatile* than if all investors had access to the new asset, resulting in a precautionary-savings motive from both investors. However, because inexperienced investors have no access to the new asset and therefore bear more risk than experienced investors, they have a stronger precautionary-savings motive; thus, in equilibrium, it is the inexperienced investors who invest in the bond.

*After* the introduction of the new asset at date $\tau = 40$, the portfolios of the two groups of investors differ substantially across the cases of common beliefs and differences in beliefs. In the case of common beliefs, that is, when inexperienced investors know the true dividend-growth rate of the new asset, both groups of investors hold the market portfolio and have zero position in the risk-free bond. In contrast, if inexperienced investors are uncertain about the dynamics of the new asset class, then the portfolios of the two groups of investors are very different—even though the new asset is now available to all

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\(^{19}\) The weights in the market portfolio are simply equal to the relative size, measured by market capitalization, of the two risky assets. Specifically, in our setting, they are equal to about 73% and 27% for the traditional and the new asset, respectively.
Figure 2: Asset Allocation

This figure shows the dynamics of the portfolio shares of inexperienced (left column) and experienced investors (right column) over time, based on the parameter values described in Section 3.1. The results are conditional on financial innovation taking place ($I_r = 1$) and averaged across simulation paths. The dotted line shows the average “myopic” portfolio component of the inexperienced investors, which is the portfolio allocation ignoring changes in the investment opportunity set arising from fluctuations in beliefs. Panels A and B show the average proportion of wealth invested in the risk-free bond for the inexperienced and experienced investors, Panels C and D the average proportion of wealth invested in the new asset, and Panels E and F the average fraction of wealth invested in the traditional risky asset.
investors—and still substantially different from the market portfolio. Inexperienced investors have a positive but declining position in the risk-free bond. This is because, with risk-aversion $\gamma > 1$, parameter uncertainty leads to precautionary savings. To illustrate this, the figure also shows the average “myopic” portfolio component of the inexperienced investors for the case of differences in beliefs (dotted line); that is, the portfolio allocation ignoring fluctuations in beliefs, or equivalently, ignoring fluctuations in the investment opportunity set.\textsuperscript{20} In that case, the portfolio allocations for the risk-free bond are, on average, essentially zero, so that the bond holdings in the case with differences in beliefs (solid line) can be fully attributed to the precautionary savings motive stemming from fluctuations in beliefs. Over time, as the posterior variance of the inexperienced investors declines (see Panel B of Figure 1), so does their precautionary-savings motive, and thus, the portfolio share allocated to the bond declines over time (solid line in Panel A of Figure 2).

As soon as financial innovation gives inexperienced investors access to the new asset class, they invest into it for diversification reasons (Panel C of Figure 2). However, compared to the case in which they know the true growth rate (dashed line), the investment is rather small in the presence of parameter uncertainty and Bayesian learning (solid line). In particular, they wish to hold a portfolio that performs well when marginal utility is high or, equivalently, the new asset’s perceived dividend-growth rate is low. This is achieved through a negative intertemporal hedging position in the new asset because its return is positively correlated with the perceived growth rate. In particular, the average myopic portfolio component of the inexperienced investors’ portfolios (dotted line) is essentially equal to the portfolio allocation in the case of common beliefs.\textsuperscript{21} Consequently, the entire difference between the new asset’s portfolio allocations for the cases of common beliefs and differences in beliefs can be attributed to the desire to hedge against fluctuations in beliefs. Eventually, the perceived dividend-growth rate settles down, as the posterior variance declines deterministically to zero. Accordingly, the magnitude of the negative hedging demand declines and inexperienced investors gradually increase their holdings in the new asset. But, even twenty-five years after financial innovation

\textsuperscript{20}In the case of common beliefs, the myopic component and the final portfolio coincide because there are no fluctuations in beliefs. For ease of exposition, we therefore do not show this case in the figure.

\textsuperscript{21}The inexperienced investors’ myopic portfolio component conditional on their current beliefs will differ substantially from the portfolio in case of common beliefs. In particular, if inexperienced investors are currently optimistic (pessimistic) regarding the new asset, they will over-invest (under-invest) in it. However, with the investors’ beliefs being centered around the true expected dividend-growth rate, the average myopic portfolio allocations are very similar to the portfolio allocation for common beliefs.
(that is, at $t = 140$), inexperienced investors still assign a substantially smaller weight to the new asset relative to experienced investors.

The investment into the new asset class by the inexperienced investors following financial innovation leads to a reduction in the portfolio share of the traditional asset relative to the pre-innovation period (Panel E of Figure 2). However, because in the presence of learning the investor is underinvested in the new asset, the portfolio share of the traditional asset with belief disagreement (solid line) still exceeds that for the case of common beliefs (dashed line).

The portfolio shares of the experienced investors in the presence of differences in beliefs follow naturally from the positions of the inexperienced investors. That is, for reasons of market clearing, experienced investors take a short position in the bond, which is (in absolute terms) declining over time (Panel B of Figure 2). Moreover, to clear the markets for the two risky assets, experienced investors, even after the new asset becomes available to all investors, are overinvested in the new asset (solid line in Panel D) and underinvested in the traditional asset (solid line in Panel E) relative to the case of common beliefs, in which they would be holding the market portfolio.

3.3 Stochastic Discount Factor

The stochastic discount factor (SDF) of investors in group $k$, $M_{k,t+1}$, is given by

$$M_{k,t+1} = \beta \delta_t \exp \left( -\frac{1}{\psi} \Delta c_{k,t+1} - (\gamma - 1/\psi) v_{k,t+1} \right),$$

(6)

where $\delta_t = E^k_t \left[ \exp \left( (1 - \gamma) v_{k,t+1} \right) \right]^{(\gamma - 1/\psi)/(1 - \gamma)}$, $c_{k,t+1}$ denotes log consumption, and $v_{k,t+1} = \log(V_{k,t+1})$ denotes the log of lifetime continuation utility. Hence, shocks to the log SDF, $m_{k,t+1} = \log(M_{k,t+1})$, can be written as

$$m_{k,t+1} - E^k_t[m_{k,t+1}] = -\left( \frac{1}{\psi} \right) \left( \Delta c_{k,t+1} - E^k_t[\Delta c_{k,t+1}] \right) - (\gamma - 1/\psi) \left( v_{k,t+1} - E^k_t[v_{k,t+1}] \right).$$

(7)

Expressing the SDF in terms of these two components makes it clear that, if relative risk aversion is not equal to the reciprocal of EIS ($\gamma \neq 1/\psi$), then shocks to future continuation utility, $v_{k,t+1}$, are a source of priced risk—in addition to shocks to log consumption growth, $\Delta c_{k,t+1}$. In particular, the volatility of the SDF is driven by: (i) consumption-growth volatility,
Figure 3: Stochastic Discount Factors

This figure shows the dynamics of the volatility of the stochastic discount factor and consumption growth over time, based on the parameter values described in Section 3.1. Panels A and B show the average conditional volatility of the stochastic discount factor for the inexperienced and experienced investors, respectively. Panels C and D show the average conditional consumption growth volatility for the inexperienced and experienced investors, respectively. The results are conditional on financial innovation taking place ($I_r = 1$), averaged across simulation paths and computed under the subjective beliefs.

Panels A and B of Figure 3 illustrate the volatility of the SDF for the inexperienced and experienced investors, respectively. With common beliefs, the volatility of the SDF of inexperienced investors increases (slightly) at the introduction of the new asset; in contrast, the volatility of the SDF of experienced investors falls at introduction. In contrast, if inexperienced investors are uncertain about the true dividend-growth rate but learn about it over time, the (ii) variation in future continuation utility, which is almost exclusively due to variation in posterior beliefs, and (iii) their covariance.

Panels A and B of Figure 3 illustrate the volatility of the SDF for the inexperienced and experienced investors, respectively. With common beliefs, the volatility of the SDF of inexperienced investors increases (slightly) at the introduction of the new asset; in contrast, the volatility of the SDF of experienced investors falls at introduction. In contrast, if inexperienced investors are uncertain about the true dividend-growth rate but learn about it over time, the
volatility of the SDF of both groups of investors is higher than for the case with common beliefs. In particular, the volatility of the SDF of the inexperienced investors increases substantially at the introduction of the new asset.

To better understand the changes in the volatility of the stochastic discount factor, we now study the two components of the SDF in (7) separately. First, Panels C and D of Figure 3 show the volatility of the investors’ consumption policies. Similar to the results for portfolio shares, before financial innovation occurs, consumption-growth volatilities are essentially the same across the cases of common beliefs and belief disagreement. In particular, the consumption of inexperienced investors is less volatile than that of experienced investors. Intuitively, the desire for precautionary savings leads inexperienced investors to hold a portfolio that is less risky (a large proportion of wealth invested in the risk-free asset), which allows them to better smooth their consumption across states; in contrast, experienced investors, whose portfolio is levered, have consumption that is more volatile.

After financial innovation makes the new asset available to inexperienced investors, the consumption-growth patterns differ substantially across the cases of common beliefs and belief disagreement. If the two groups of investors have common beliefs, investors are identical and, hence, consume a constant fraction (equal to their wealth share) of aggregate consumption. Therefore, their consumption policies inherit the dynamics of aggregate consumption growth, which leads to a reduction (increase) in the volatility for the experienced (inexperienced) investors.

In contrast, if the two groups of investors disagree on how to value the new asset, then the consumption-growth volatility of both groups of investors increases upon the introduction of the new asset (Panels C and D of Figure 3). For experienced investors, the increase is due to a higher portfolio volatility, driven by a sizable increase in the return volatility of the new asset (discussed in detail below), which is overweighted in their portfolios. This effect dominates the dampening effect from a slightly lower leverage. For inexperienced investors, the increase in consumption-growth volatility stems from smaller precautionary savings in the bond and the investment in the new asset, which now is more volatile. Over time, the decline in leverage and return volatility of the new asset lead to a decline in consumption-growth volatility for both groups of investors. However, the reduction in consumption volatility over time is much less pronounced for inexperienced investors, because the reduction in the new asset’s return
volatility is offset by smaller precautionary savings that lead to a reduction in the investment in the risk-free bond.

In the case in which both groups of investors know the true growth rate of the new asset’s dividends, and thus there is no variation in posterior beliefs, the volatility of the SDF matches exactly the pattern for consumption growth volatility (that is, the dashed lines in Panels A and B as well as Panels C and D of Figures 3). In contrast, the economic mechanism driving the increase in the volatility of the SDF of inexperienced investors for the case of differences in beliefs is the second component in equation (7), in which one can further decompose the continuation-utility component using the total derivative

\[ v_{k,t+1} - E_t^k[v_{k,t+1}] = \frac{\partial v_{k,t}}{\partial \mu_{2,t}} \left( \mu_{2,t+1} - E_t^k[\mu_{2,t+1}] \right) + \frac{\partial v_{k,t}}{\partial \omega_{1,t}} \left( \omega_{2,t+1} - E_t^k[\omega_{2,t+1}] \right) + \frac{\partial v_{k,t}}{\partial \delta_{1,t}} \left( \delta_{1,t+1} - E_t^k[\delta_{1,t+1}] \right). \]

Intuitively, shocks to investor’s log continuation utility depend on its sensitivity to the state variables and (unexpected) variations in the state variables. The three stochastic state variables are: (i) the new asset’s dividend-growth rate as perceived by the inexperienced investors, \( \mu_{2,t} \) (only relevant for \( t \geq \tau \)); (ii) the experienced investors’ share of aggregate consumption, \( \omega_{1,t} \); and (iii) the traditional asset’s share of total dividends, \( \delta_{1,t} \).

Figure 4 shows the sensitivities of the inexperienced investors’ continuation-utility component of the SDF (Panel A) as well as the volatility of changes in the state variables (Panel B) over time for the case of differences in beliefs.\(^{22}\) Notably, the sensitivity with respect to perceived dividend-growth rate is (at least) an order of magnitude bigger than the sensitivities with respect to the experienced investors’ consumption share and the traditional asset’s dividend share; that is, investors with recursive utility who have a preference for early resolution of uncertainty are highly averse to the permanent shocks to the perceived dividend-growth rate of

\(^{22}\)In the case of common beliefs, that is, in the absence of learning, the sensitivity with respect to the perceived dividend-growth rate is irrelevant because there are no fluctuations in beliefs. The sensitivities with respect to the other two state variables are similar to the case with belief disagreement.
Figure 4: Continuation Utility Component of Inexp. Investors’ SDF

This figure shows the average sensitivity of the inexperienced investors’ continuation utility component of their stochastic discount factor with respect to the state variables (Panel A) as well as the volatility of changes in the state variables (Panel B) over time for the case of differences in beliefs; based on the parameter values described in Section 3.1. The results are conditional on financial innovation taking place ($I_r = 1$), averaged across simulation paths and expectations are computed under the objective beliefs.

Moreover, the variations in the second and third state variables are basically unaffected by financial innovation.\textsuperscript{24}

Hence, the substantial increase in the volatility of the inexperienced investors’ SDF can be fully attributed to uncertainty regarding the dividend-growth rate of the new asset and Bayesian learning. Note that the sensitivity with respect to the perceived dividend-growth rate is actually increasing over time. However, this increase is offset by an even stronger decrease in the volatility of the changes in beliefs (Panel B),\textsuperscript{25} explaining the decline in the volatility of the inexperienced investors’ SDF over time.

\textsuperscript{23}Intuitively, the sensitivity of the inexperienced investors continuation utility with respect to the consumption share of the experienced investors is negative because lower consumption implies lower utility. Moreover, because the inexperienced investors over-weight the traditional asset in their portfolio, the sensitivity with respect to the dividend share of the traditional asset is positive.

\textsuperscript{24}The volatility of changes in the consumption share increases slightly at the introduction of the new asset and then declines over time, mimicking the behavior of investors’ consumption-growth volatility. And, as expected, the volatility of changes in the dividend share is constant over time because its dynamics are exogenous and, thus, not affected by financial innovation.

\textsuperscript{25}This figure is reminiscent of Panel C of Figure 1 that shows the decline in the inexperienced investors’ posterior variance.
3.4 Moments of Asset Returns

We now focus on a key objective of the paper: to study the implications of financial innovation for asset returns. Figure 5 shows the return moments for the two risky assets in our model. Similar to the results regarding optimal consumption and portfolio policies discussed in the sections above, before financial innovation makes the new asset available to inexperienced investors, the return moments are very similar across the cases of common beliefs and belief disagreement. Also, the return moments are very stable until \( t = \tau - 1 \); this is because the averages of the investors’ consumption shares and the dividend share (which are the only state variables during this period) are essentially constant over time.

Turning to the levels of the return volatilities and the risk premia on the two risky assets, we start by highlighting the finding in Cochrane, Longstaff, and Santa-Clara (2008) that in a standard economy in which all investors have access to all assets, assets are symmetric and there is no parameter uncertainty, the risk premium of the asset with the larger dividend share would be higher than that of the other asset. Specifically, because of the larger dividend share, the asset’s dividends constitute a larger fraction of aggregate consumption, and thus, co-vary more with it. The return volatility of the larger asset would also typically be slightly higher than that for the asset with the smaller share of aggregate dividends.

In contrast, in our model prior to financial innovation \((t < \tau)\) the risk premium of the new asset, despite having a smaller dividend share, is of the same magnitude as the risk premium of the traditional asset (Panels C and D of Figure 5). Intuitively, prior to financial innovation, experienced investors need to be compensated for bearing all the risk of holding the new stock to clear the market. The pattern for the return volatilities (Panels A and B) is comparable to the results in the standard economy; that is, the volatility is slightly lower for the new asset. Thus, before the new asset becomes available to all investors, the Sharpe ratio of the new asset is endogenously higher than that of the traditional asset (Panels E and F).

After financial innovation occurs, there is a dramatic change in the return moments of the new asset, with major differences depending on inexperienced investors’ beliefs. If both groups of investors know the true growth rate of the new asset’s dividends, the new asset’s risk premium drops substantially (dashed line in Panel C of Figure 5) when it becomes available to inexperienced investors because the risk of holding the new asset is now shared between
Figure 5: Moments of Asset Returns

This figure shows the dynamics of the return moments of the financial assets over time, based on the parameter values described in Section 3.1. The results are conditional on financial innovation taking place (I_t = 1), averaged across simulation paths and computed under the objective beliefs. Panels A and B show the average conditional return volatilities, Panels C and D the average conditional risk premia, and Panels E and F the average Sharpe ratio of the new asset and the traditional asset, respectively.
the two groups of investors. Because the return volatility is largely unchanged (dashed line in Panel A), the Sharpe ratio of the new asset drops substantially (dashed line in Panel E) at the introduction of the new asset. On the other hand, the return moments of the traditional asset are only marginally affected by the introduction of the new asset (Panels B, D, and F).

In contrast, if inexperienced investors have to learn about the new asset’s dynamics, the implications of financial innovation are substantially different and in stark contrast to “conventional wisdom.” For example, at the introduction of the new asset, the return volatility of the new asset increases considerably (solid line in Panel A of Figure 5) and is much higher than the volatility of the traditional asset, which does not change much at introduction (solid line in Panel B). To understand the reason for the higher return volatility of the new asset, recall that positive (negative) cash-flow news for the new asset class leads to an upward (downward) revision in its perceived dividend-growth rate by inexperienced investors. If the substitution effect dominates the income effect, that is, with \( \text{EIS} > 1 \), an upward revision in the inexperienced investors’ perceived dividend-growth rate, that is, an improvement in the investors’ investment opportunity set, implies that they save more. Moreover, in reaction to the higher expected dividend-growth rate, inexperienced investors dynamically allocate a larger fraction of their savings to the new asset. Both effects increase the demand for the new asset and vice versa for a downward revision in the perceived dividend-growth rate. This is illustrated in Panel A of Figure 6, which depicts the inexperienced investors’ optimal holdings in the new asset, measured in number of shares, for a particular simulated path of the economy. Whenever realized dividend growth is higher than expected (highlighted in shaded gray), the inexperienced investors increase their holdings of the new asset and vice versa.\(^{26}\)

As a consequence, positive (negative) cash-flow news lead to a higher (lower) price-dividend ratio for the new asset class.\(^{27}\) Accordingly, positive dividend news coincides with a higher price-dividend ratio, and vice versa for negative news, thereby amplifying the variations in its dividend and creating “excess volatility.” Panel B of Figure 6 shows the realized return of the new asset for the same simulated path. In particular, the amplification mechanism is apparent in that for periods of higher than expected dividend growth (highlighted in shaded gray), the inexperienced investors increase their holdings of the new asset and vice versa.\(^{26}\)

\(^{26}\)The figure also illustrates the (slow) upward trend in the holdings of the new asset discussed in Section 3.2. In the case of common beliefs, the two groups of investors are homogeneous, so that no trading takes place and both groups of investors hold the number of shares equal to their wealth share at time \( \tau \).

\(^{27}\)The relation between the two assets’ price-dividend ratios and the dividend-growth rate as perceived by the inexperienced investors is discussed in further detail in Appendix C.
Figure 6: New Asset: Holdings and Realized Return over a Simulated Path
This figure illustrates the inexperienced investors’ optimal holdings in the new asset, measured in number of shares (Panel A) and the new asset’s realized return (Panel B), for a particular simulated path of the economy. The results are conditional on financial innovation taking place ($\tau = 1$) and based on the parameter values described in Section 3.1. The results are plotted in event time, with time $\tau$ marking the date of financial innovation and the shaded gray areas indicate periods of positive dividend realizations for the new asset.

Realized return is always higher in the case of differences in beliefs than in the case of common beliefs and vice versa for lower than expected dividend growth. Only over time, the decline in the posterior variance dampens this amplification mechanism, explaining the slow decline in volatility. However, even after 25 years of being introduced (that is, at $t = 140$), the return volatility of the new asset (Panel A of Figure 5) is higher than that of the traditional asset (Panel B of Figure 5).

Similarly, in the case in which inexperienced investors have to learn about the dynamics of the new asset, the new asset’s risk premium is increased by financial innovation (Panel C of
Figure 5)—even though innovation allows the investors to share the risk of holding the new asset. To understand the economic mechanism driving this result, note that an asset’s risk premium is given by the covariance between its return and the SDF:

\[ E_t[r_{n,t}] - r_{0,t} = -\text{Cov}_t(M_{k,t+1}, r_{n,t+1}) \]
\[ = -\text{Corr}_t(M_{k,t+1}, r_{n,t+1}) \text{Vol}_t(M_{k,t+1}) \text{Vol}_t(r_{n,t+1}), \tag{8} \]

where \( r_{0,t} \) denotes the risk-free rate and \( r_{n,t} \) the return on asset \( n \). With differences in beliefs, both—the volatility of the new asset’s return and the volatility of the investors’ SDFs—increase at financial innovation. At the same time, the correlation of the SDF with the new asset’s return increases only marginally (that is, it is smaller in absolute terms). Consequently, the risk premium on the new asset increases. Only over time, as the volatility of the SDF and the volatility of the new asset’s return decline, so does its risk premium.

Taken together, the stronger increase in the return volatility relative to the increase in the risk premium of the new asset leads to a substantial decline in its Sharpe ratio at the time of financial innovation. The reduction in the risk premium over time is stronger than the decline in the return volatility, which causes the Sharpe ratio to decline further. An alternative way to think about the change in Sharpe ratio is that the availability of the new asset to all investors increases aggregate demand for the asset. Thus, because the asset is in fixed supply, its attractiveness for investors must decrease, which is achieved by a drop in its Sharpe ratio. As the posterior variance declines over time, and consequently, the demand strengthens further, this “equilibrium incentive” must also strengthen, implying a further decline in the Sharpe ratio over time.

Learning also has direct implications for the cyclicality of the new asset’s return. With common beliefs, a positive dividend shock for the new asset implies a higher share of aggregate dividends such that the new asset’s dividends co-vary more with aggregate consumption (and, thus, the SDF), leading to a higher risk premium and return volatility. Consequently, the new asset’s risk premium, return volatility and Sharpe ratio co-vary positively with changes in its dividends and, hence, are high in good times—inconsistent with what is observed empirically. In contrast, differences in beliefs induce strong counter-cyclical variation in the new asset’s return volatility, risk premium, and Sharpe ratio; that is, they are all high in bad times, which are defined as periods following negative dividend shocks. Intuitively, negative dividend
shocks for the new asset lead to a downward revision in the inexperienced investors’ perceived dividend-growth rate, which leads to a lower price and, accordingly, higher subsequent returns.\textsuperscript{28}

3.5 Asset Prices

Recall that dividend-growth rates in the model are exogenous and IID, so that price-dividend ratios are driven entirely by discount rates. Hence, the price-dividend ratios are roughly the inverse of the expected returns and can be understood as such.

Upon financial innovation, the price-dividend ratio of the new asset increases (Panel A of Figure 7). With common beliefs, the reason for this is that the risk premium on the new asset drops considerably when it becomes available to all investors (see Panel C of Figure 5 and the discussion in Section 3.4), leading to a reduction in the asset’s expected return, and consequently, a substantial increase in its price-dividend ratio. Instead, if inexperienced investors need to learn about the dividend-growth rate of the new asset, the increase in the price-dividend ratio is considerably smaller. In particular, the initial increase in the asset’s risk premium (which dominates a small reduction in the risk-free rate) dampens the effect of lower long-term expected returns.\textsuperscript{29}

The effect of the introduction of the new asset on the price-dividend ratio of the traditional asset is substantially smaller in relative terms. In particular, if investors have common beliefs, the increase in the interest rate (due to the absence of a precautionary savings motive) leads to an increase of the asset’s expected return and, thus, a small drop in the price-dividend ratio (dashed line in Panel B). In contrast, in the case of belief disagreement, the price-dividend ratio is basically unaffected because the effects on the interest rate and the asset’s risk premium offset each other (solid line).

3.6 Welfare

Finally, we briefly discuss the implications of financial innovation for investors’ welfare, measured by their certainty-equivalent consumption (normalized by total output). Note that in our model, the beliefs of experienced investors coincide with the objective beliefs. Also, while

\textsuperscript{28}The correlations between changes in the dividend of the new asset and changes in its return moments are discussed in more detail in Appendix D.

\textsuperscript{29}With differences in beliefs, the asset’s expected return is lower than that in the pre-innovation period after about eighty periods (twenty years), which has a positive impact on the date-\texttau price-dividend ratio.
Figure 7: Asset Prices

This figure shows the dynamics of the prices of the financial assets over time, based on the parameter values described in Section 3.1. The results are conditional on financial innovation taking place ($\mathcal{I}_\tau = 1$) and averaged across simulation paths. Panels A and B show the average price-dividend ratio for the new asset and the traditional asset, respectively.

inexperienced investors do not know the true dividend-growth rate of the new asset, their beliefs are, on average, unbiased and their learning is fully Bayesian. So, in this sense, neither group of investors has distorted beliefs, and hence, we compute welfare using the subjective beliefs of each group of investors; that is, according to (1).\(^{30}\)

Recall that inexperienced investors are endowed with a majority of the wealth. Thus, their level of certainty-equivalent consumption (Panel A of Figure 8) is higher in general than that of experienced investors (Panel B). In the pre-innovation period ($t < 40$), the certainty-equivalent consumption of the inexperienced investors is declining over time (Panel A), whereas it is increasing for the experienced investors (Panel B). This is because of the higher portfolio returns, and in turn, higher consumption growth.

The introduction of the new asset at $\tau = 40$ is beneficial for both groups of investors, and leads to a jump in welfare levels. In particular, inexperienced investors see their welfare increase

\(^{30}\)We computed welfare also under the objective beliefs for both investors. For experienced investors this has no impact because their beliefs coincide with the objective measure, so Panel B of Figure 8 is unchanged. But, also for inexperienced investors (Panel A), the pattern is very much the same, with the introduction of the new asset at $\tau = 40$ leading to an improvement in welfare. Intuitively, at introduction, inexperienced investors actually have the correct beliefs (that is, their prior is unbiased); later on, their beliefs will differ from true ones, but on average their beliefs remain unbiased. For a more detailed discussion of computing welfare under distorted beliefs, see Brunnermeier, Simsek, and Xiong (2014). For a more general discussion of the social welfare consequences of financial innovation, see Lerner and Tufano (2009).
Figure 8: Welfare

This figure shows the dynamics over time of the investors’ certainty-equivalent consumption, normalized by total output. The results are conditional on financial innovation taking place ($I_\tau = 1$), averaged across simulation paths and computed under the investors’ subjective beliefs. Panels A and B show the average certainty-equivalent consumption for the inexperienced and the experienced investor, respectively. The figure is based on the parameter values described in Section 3.1.

with financial innovation because having access to a new asset improves their investment opportunity set, which allows them to achieve higher expected long-term portfolio returns, and consequently, higher expected consumption growth. This effect is stronger if inexperienced investors know the true dividend-growth rate for the new asset, and hence, are willing to allocate substantial wealth to the new asset more quickly.

Experienced investors also benefit from financial innovation because it allows them to achieve better risk sharing; that is, they can share the risk of holding the new asset. Again, the welfare gains at introduction ($\tau = 40$) are stronger in the case of common beliefs because the improvements in risk sharing take place faster. However, in the long-run, experienced investors are better off if inexperienced investors need to learn the dividend-growth rate because then they have an information advantage.

That said, we do not attempt to make any policy recommendations because financial innovation might have implications for welfare through other channels not modeled here.
4 Extensions and Alternative Specifications

To study the robustness of our results, we now evaluate the impact of financial innovation for several variations of the baseline economy. For each of these variations, we reproduce all the quantities studied for the baseline model, but to save on space, we report only our main object of interest, the moments of asset returns for the new asset.

4.1 Extension: Financial Innovation

So far, the decision to introduce the new asset was exogenous. In practice, new asset classes are introduced endogenously by financial intermediaries with profit incentives. Thus, a new asset class may become available only if there is sufficient demand for it and thus intermediaries can extract some profits by charging fees. Instead of modeling these institutional details explicitly, we adopt a reduced-form approach to extend the baseline model so that the decision to introduce the new asset is endogenous.

In the first extension, we consider the case in which financial innovation is more likely if an asset class has performed well and demand for it is high or, in the parlance of the investment industry, when the new asset class is “hot.” Technically, we assume that at time $\tau = 40$, the new asset is introduced only if its share of aggregate dividends is higher than some threshold value, which we set to 20%.\(^{31}\) Again, we see from Figure 9 that qualitatively, our predictions for the impact of financial innovation remain unchanged. Even quantitatively, the predictions for the new asset’s return volatility and its risk premium after introduction are basically unaffected.

In the second extension, we study the case in which both the introduction and the timing of financial innovation are endogenous. Recall that so far we had assume that if the new asset was not introduced at a predetermined date $\tau$, it would never be introduced. In this extension, we endogenize the timing of financial innovation; that is, we assume that the new asset can be introduced at any date, if its share of total dividends is above a threshold of 22.5\%.\(^{32}\) As a

\(^{31}\)The threshold is set so that the unconditional probability (at date $t = 0$) of the new asset being introduced at $\tau = 40$ is still about 50%.

\(^{32}\)This threshold level is chosen so that the unconditional probability (at $t = 0$) of the new asset being available for trading for the inexperienced investors at time $\tau = 40$ (that is, the asset has been introduced at some point $t \leq \tau$) is about 50%. Using a threshold of 20% as in the previous case, has no implications for the results, but increases the unconditional probability of the new asset being introduced.
This figure shows the dynamics of the new asset’s return moments over time. The figure is based on the extension of the basic model described in Section 4.1. The results are conditional on financial innovation taking place ($I_\tau = 1$), averaged across simulation paths and computed under the objective beliefs. Panels A and B show the average conditional return volatility and the average conditional risk premium, respectively. The results are plotted in event time, with time $\tau$ marking the (endogenous) date of financial innovation.

result, the timing of financial innovation is now stochastic. Once again, we see from Figure 9 that, qualitatively and quantitatively, the predictions remain unchanged.

4.2 Extension: Illiquidity

Next, we consider an economy in which the new asset is illiquid after being introduced. In practice, trading in new asset classes often entails sizeable trading costs, which we model as a proportional transaction cost; that is, each investor has to pay a constant fraction $\kappa = 2.5\%$ of the (dollar) value of the trade. Qualitatively, our main insights are not affected by illiquidity, and, as we see from Panels A and B of Figure 10, even quantitatively the effect of transaction costs is small.

Rather than assuming that the transaction cost is a deadweight loss to society, we assume that it is added back to the consumption of investors after they have made their consumption and portfolio decisions, thereby eliminating any wealth effects. The transaction cost paid by the group of investors buying shares is equal to that paid by the group selling shares, so the total transaction cost is redistributed equally between the two groups. However, if one were to assume that the transaction cost is a deadweight cost, it would not affect any of our results because investors optimize their trading decisions, and therefore, the transaction costs incurred in equilibrium are small. Details of how the model with transaction costs is solved (by extending the dual formulation of Buss and Dumas (2017)) are given in the Internet Appendix.
Figure 10: Extension: Illiquidity

This figure shows the dynamics of the new asset’s return moments over time. The figure is based on the extension of the basic model to allow for transaction costs, as described in Section 4.2. The results are conditional on financial innovation taking place ($I_t = 1$), averaged across simulation paths and computed under the objective beliefs. Panels A to C show the average conditional return volatility, the average conditional risk premium and the average conditional return correlation between the traditional asset and the new asset, respectively. The results are plotted in event time, with time $\tau$ marking the date of financial innovation.

The main effect of transaction costs is on the trading decisions of investors. In the presence of transaction costs, inexperienced investors rebalance their holdings in the new asset less aggressively following updates in their beliefs, which slightly limits the increase in the new asset’s return volatility (Panel A). Moreover, instead of trading the new asset at a cost, inexperienced investors often use the correlated traditional asset as a substitute. Consequently, shocks to the cash flows of the new asset spill over to the traditional asset, substantially increasing the return correlation between the new and traditional risky assets in the post-innovation period relative to the baseline model with zero transaction costs; thus, decreasing the diversification.
benefits from the new asset (compare the solid line to the dashed line in Panel C of Figure 10).\(^{34}\) Transaction costs also give rise to a *liquidity premium*, which increases the total risk premium of the new asset (Panel B).

### 4.3 Alternative Specifications

Finally, Figure 11 highlights the implications of alternative choices for the beliefs of the inexperienced investors. A more precise prior \(A_\tau = 40\) instead of \(A_\tau = 20\) for the baseline model implies that the inexperienced investors’ demand for the new asset is less sensitive to cash-flow news due to smaller revisions in beliefs. Consequently, when the new asset becomes available, the increase in the new asset’s return volatility and, in turn, its risk premium is smaller compared to the baseline model. Similarly, if inexperienced investors start learning at time $1$ instead of at time $\tau$ \((A_1 = 1\) instead of \(A_\tau = 20\) for the baseline model), the effects at introduction are smaller because the inexperienced investors’ posterior beliefs are more precise when the asset becomes available (implicitly, \(A_\tau\) is equal to 40), but the results in the pre-innovation period are unchanged. Conversely, a less precise prior would strengthen the effects of financial innovation on the moments of asset returns. If financial innovation occurs later (at \(\tau = 80\) instead of \(\tau = 40\) for the baseline model) or is more likely \((p = 0.75\) instead of \(p = 0.50\) for the baseline model), the predictions remain—even quantitatively—unchanged.

Unreported results\(^{35}\) show that varying the truncation bounds has no impact, which implies that our results are not driven by extreme beliefs. Moreover, reducing the dividend-growth volatilities to match aggregate consumption volatility or using a mean-reverting dividend share (so that it has a stationary distribution), has qualitatively no effect. Similarly, changes in risk-aversion \((\gamma)\), intertemporal elasticity of substitution \((\psi)\), initial wealth shares \((w_k,0)\), the initial dividend shares \((\delta_i,0)\), the dividend-growth volatilities of the two assets, the horizon \((T)\), and the frequency \((\Delta_t)\) lead to only small quantitative changes.

\(^{34}\)The economic mechanism that drives the correlation between the returns of the two assets is the following: Before the new asset is introduced, the correlation between returns of the traditional and new asset is high because positive cash-flow news for the new asset implies a wealth transfer in favor of experienced investors who hold all of this asset. For diversification reasons, this wealth transfer leads to disproportionately strong demand for the traditional asset, increasing its price, and hence, higher correlation. Once the new asset becomes available to inexperienced investors, who have to learn about its expected dividend-growth rate, the correlation drops. That is, now investors’ portfolios are less concentrated, which reduces the wealth transfer and limits its positive effect on correlation.

\(^{35}\)These results are reported and discussed briefly in the Internet Appendix.
Figure 11: Robustness: Alternative Specifications
This figure shows the dynamics of the new asset’s return moments over time. The figure is based on the alternative model specifications described in Section 4.3: a more precise prior ($A_\tau = 40$ instead of $A_\tau = 20$ for the base case), inexperienced investors learning from time 1 onwards ($A_1 = 1$ instead of $A_\tau = 20$), financial innovation occurring later (at $\tau = 80$ instead of $\tau = 40$ in the base case), and financial innovation being more likely ($p = 0.75$ instead of $p = 0.50$ in the base case). The results are conditional on financial innovation taking place ($I_\tau = 1$), averaged across simulation paths and computed under the objective beliefs. Panels A and B show the average conditional return volatility and the average conditional risk premium, respectively. The results are plotted in event time, with time $\tau$ marking the date of financial innovation.

5 Conclusion
In this paper, we study how asset prices are affected by financial innovation. The traditional view is that new asset classes, such as private equity, hedge funds, emerging market equity and debt, natural commodities, real assets, and cryptocurrencies should play a crucial role in the portfolios of all investors because of the substantial diversification benefits and potentially higher returns they offer. Moreover, the resulting consumption smoothing should lead to a decrease in the return volatility and risk premium of the new asset class.

Our main contribution is to show that when the new (inexperienced) investors are less well informed about the new asset class than the experienced investors who have been holding this asset, then many of these “intuitive” results are reversed. For instance, even after financial innovation occurs, there are large differences in the portfolios of experienced and inexperienced investors, resulting from a large negative intertemporal hedging demand of inexperienced investors, which is a consequence of their uncertainty regarding the expected dividend-growth rate of the new asset. Financial innovation leads to an increase in the volatility of the portfolios of
investors, and even after several decades have elapsed, inexperienced investors allocate only a small fraction of their capital to the new asset class. Moreover, the learning of the inexperienced investors amplifies the volatility of the stochastic discount factor. As a result, the new asset’s risk premium goes up and its returns become more volatile. Differences in beliefs also induce a strong countercyclical variation in the new asset’s return volatility, risk premium, and Sharpe ratio; that is, they are all high after negative shocks to the dividends of the new asset.

Our analysis shows that it is important to account for differences in beliefs when making predictions about the effects of financial innovation. When investors differ in their beliefs, the consequences of financial innovation can be counter-intuitive and very different from those in a setting with common beliefs, even if learning is fully Bayesian.

Finally, in spite of the increase in volatility, financial innovation increases the welfare of inexperienced and experienced investors. Moreover, the magnitude of the shocks transmitted from the new asset to existing assets is quite small because of the relatively small size of the new asset relative to the magnitude of existing public equities, indicating that regulatory concerns about financial innovation should be small as well.
## A Optimality Conditions and Equilibrium

### A.1 Investors’ Optimality Conditions

The objective of each investor \( k \) is to maximize her expected lifetime utility given in equation (1), by choosing consumption, \( C_{k,t} \), and the holdings in the available financial assets, \( \theta_{n,k,t}, n \in \{0, \ldots, N_{k,t}\} \):\(^{36}\)

\[
V_{k,t}(\{\theta_{k,n,t-1}\}) = \max_{C_{k,t},(\theta_{k,n,t})} \left[ (1 - \beta) C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[ V_{k,t+1}(\{\theta_{k,n,t}\}) \right]^{1-\gamma} \right]^\frac{1}{1-\gamma},
\]

subject to the budget equation (5).

Denoting the Lagrange multiplier associated with the budget equation by \( \eta_{k,t} \), the Lagrangian can be written as

\[
L_{k,t} = \sup_{C_{k,t},(\theta_{k,n,t})} \inf_{\eta_{k,t}} \left[ (1 - \beta) C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{\psi}} \right] (1 - \beta) \left( 1 - \frac{1}{\psi} \right) C_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} + \eta_{k,t} \left( \theta_{k,0,t-1} + \sum_{n=1}^{N_{k,t}} \theta_{k,n,t-1} D_{n,t} - C_{k,t} - \theta_{k,0,t} S_{0,t} - \sum_{n=1}^{N_{k,t}} \Delta \theta_{k,n,t} S_{n,t} \right),
\]

and the corresponding first-order conditions are given by

\[
\frac{\partial L_{k,t}}{\partial C_{k,t}} = \frac{1}{1 - \frac{1}{\psi}} \left\{ (1 - \beta) C_{k,t}^{1-\frac{1}{\psi}} + \beta E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{\psi}} \right\} \left( 1 - \beta \right) \left( 1 - \frac{1}{\psi} \right) C_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} = 0,
\]

\[
(1 - \beta) C_{k,t}^{\frac{1}{\psi}} V_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} \equiv 0, \quad (A1)
\]

\[
\frac{\partial L_{k,t}}{\partial \eta_{k,t}} = \theta_{k,0,t-1} + \sum_{n=1}^{N_{k,t}} \theta_{k,n,t-1} D_{n,t} - C_{k,t} - \theta_{k,0,t} S_{0,t} - \sum_{n=1}^{N_{k,t}} \Delta \theta_{k,n,t} S_{n,t} \equiv 0, \quad (A2)
\]

\[
\frac{\partial L_{k,t}}{\partial \theta_{k,n,t}} = \frac{1}{1 - \frac{1}{\psi}} V_{k,t}^{\frac{1}{\psi}} \beta \left( 1 - \frac{1}{\psi} \right) E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} (1 - \gamma) E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \left( 1 - \beta \right) \left( 1 - \frac{1}{\psi} \right) C_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} S_{n,t} = 0.
\]

\[
(1 - \beta) V_{k,t}^{\frac{1}{\psi}} E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} E_t^k \left[ V_{k,t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \left( 1 - \beta \right) \left( 1 - \frac{1}{\psi} \right) C_{k,t}^{\frac{1}{\psi}} - \eta_{k,t} S_{n,t} \equiv 0. \quad (A3)
\]

---

\(^{36}\)For ease of exposition, in the following derivations we do not explicitly write the dependence of \( V_{k,t} \) on the incoming (i.e., date \( t - 1 \)) asset holdings, \( \{\theta_{k,n,t-1}\} \).
Using the Envelope Theorem we can compute the derivatives of the value function $V_{k,t}$ with respect to $\theta_{k,n,t-1}$:

$$
\frac{\partial V_{k,t}}{\partial \theta_{k,0,t-1}} = \frac{\partial L_{k,t}}{\partial \theta_{k,0,t-1}} = \eta_{k,t},
$$

(A4)

$$
\frac{\partial V_{k,t}}{\partial \theta_{k,n,t-1}} = \frac{\partial L_{k,t}}{\partial \theta_{k,n,t-1}} = \eta_{k,t} (D_{n,t} + S_{n,t}), \quad n \in \{1, 2\}.
$$

(A5)

In summary, the optimality conditions for each investor $k$ are given by the following set of equations. First, the budget equation arising from (A2):

$$
C_{k,t} + \theta_{k,0,t} S_{0,t} + \sum_{n=1}^{N_{k,t}} \Delta \theta_{k,n,t} S_{n,t} = \theta_{k,0,t-1} + \sum_{n=1}^{N_{k,t}} \theta_{k,n,t-1} D_{n,t},
$$

(A6)

which equates the uses and sources of funds. Second, the pricing equations, arising from equations (A3) to (A5), which equate the price of an asset to the expected payoff from holding it:

$$
S_{0,t} = E_t^k [M_{k,t+1}],
$$

$$
S_{n,t} = E_t^k [M_{k,t+1} (S_{n,t+1} + D_{n,t+1})], \quad n \in \{1, 2\},
$$

where the stochastic discount factor $M_{k,t+1}$, given in equation (6) on page 19, subsumes the Lagrange multiplier $\eta_{k,t}$ from equation (A1).

A.2 Characterization of Equilibrium

Equilibrium in the economy can then be characterized by the following set of equations: the budget equation (A6), the “kernel conditions” that equate the prices of the assets across investors:

$$
E_t^1 [M_{1,t+1}] = E_t^2 [M_{2,t+1}],
$$

(A7)

$$
E_t^1 [M_{1,t+1} (S_{n,t+1} + D_{n,t+1})] = E_t^2 [M_{2,t+1} (S_{n,t+1} + D_{n,t+1})], \quad n \in \{1, 2\},
$$

(A8)

and the market-clearing conditions:\textsuperscript{37}

$$
\sum_{k=1}^{2} \theta_{k,0,t} = 0, \quad \text{and} \quad \sum_{k=1}^{2} \theta_{k,n,t} = 1, \quad n \in \{1, 2\}.
$$

(A9)

\textsuperscript{37}By Walras’ law, market clearing in the asset markets guarantees market clearing for the consumption good.

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B Numerical Algorithm

We use the time-shift proposed by Dumas and Lyasoff (2012) to obtain a recursive system of equations characterizing equilibrium. That is, at date $t$, the “shifted” system of equations consists of the date-$t$ kernel conditions (A7) and (A8), the date-$t$ market-clearing conditions (A9), and the date-$t+1$ budget equations (A6):

$$C_{k,t+1,j} + \theta_{k,0,t+1,j}S_{0,t+1,j} + \sum_{n=1}^{N_{k,t}} (\theta_{k,1,t+1,j} - \theta_{k,1,t}) S_{1,t+1,j} \leq \theta_{k,0,t} + \sum_{n=1}^{N_{k,t}} \theta_{k,n,t} D_{n,t+1,j}, \forall k,j,$$

where the $J$ future states (nodes) are denoted by $j = 1, \ldots, J$. In total, we have a system of $2K + 2N_{k,t}$ equations with $2K + 2N_{k,t}$ unknowns: next period’s consumption, $C_{k,t+1,j}$, for both investors and $J$ states, and both investors’ holdings in the assets, $\theta_{k,n,t}$.

The system of equations is solved recursively, starting from $T - 1$. At each date $t$, we solve the equation system over the grid of the state variables. Next, when solving the system for date $t - 1$, we interpolate (over the grid) the optimal date-$t$ portfolio positions, $\theta_{k,n,t}$ and corresponding security prices, $S_{n,t}$, using the terminal conditions $\theta_{k,n,T} = 0$ and $S_{n,T} = 0$, $\forall n,k$. After solving the shifted system for all dates $t \in \{0, \ldots, T - 1\}$, one has solved all equations from the global system—except the date-0 budget equations, which have not been used because of the time shift. Thus, one only needs to solve the time-0 budget equations based on interpolating functions for the date-0 prices, $S_{n,0}$, and holdings, $\theta_{k,n,0}$. The endowed holdings $\theta_{k,n,-1}$ are exogenous to the system and reflect the incoming (endowed) wealth of the investors.

There are four state variables: (i) the dividend share of the first risky security $\delta_{1,t} \in (0,1)$, the dynamics of which follow from the joint dividend dynamics in (2); (ii) the expected dividend-growth rate of the new asset class as perceived by the inexperienced investors, $\mu_{2,t}$, with the dynamics specified in (3); (iii) the (deterministic) posterior variance of the inexperienced investors’ beliefs, $A_t \sigma^2_2$, with the dynamics specified in (4); and (iv) the consumption share of the experienced investors, $\omega_{1,t} \in (0,1)$. 

40
Figure A1: Price-Dividend Ratios

This figure shows the new asset’s and the traditional asset’s price-dividend ratio as a function of the new asset’s dividend-growth rate as perceived by the inexperienced investors ($\mu_{2,t}$) for various points in time, based on the parameter values described in Section 3.1. The results are for time-$t$ dividend shares of the traditional asset and time-$t$ consumption shares of the experienced investors equal to their averages across simulation paths.

C Price-Dividend Ratios

Figure A1 illustrates the relation between the new asset’s dividend-growth rate as perceived by the inexperienced investors, $\mu_{2,t}$, and the price-dividend ratio for the new and the traditional asset.

A change in the perceived dividend-growth rate has two distinct effects for the inexperienced investors’ portfolios. First, a higher perceived dividend-growth rate implies an improvement in the investors’ investment opportunity set. When investment opportunities improve, the substitution effect induces investors to consume less and save more. In contrast, the income effect induces them to do the opposite. If investors have a preference for early resolution of uncertainty, the substitution effect dominates; leading to an increase in the savings of the inexperienced investors and, hence, increasing the demand for all financial assets (a “savings effect”). Second, in reaction to a higher perceived dividend-growth rate, the inexperienced investors change their portfolio composition; that is, they allocate a larger fraction of their

We approximate the joint dynamics of the dividends in (2) using a four-node, equal probabilities tree with growth realizations $\{(u_1, u_2), (d_1, u_2), (u_1, d_2), (d_1, d_2)\}$, where $u_n \equiv \mu_n + \sigma_n$ and $d_n \equiv \mu_n - \sigma_n$ are chosen to match the expected dividend-growth rate and volatility of asset $n$. Under the inexperienced investors’ probability measure, the probabilities are set to $p_{2,t}/2$ and $(1 - p_{2,t})/2$ for the first and last two nodes, respectively. $p_{2,t}$ is chosen to match the inexperienced investors’ perceived dividend-growth rate, $\mu_{2,t}$.
savings to the new asset (a “portfolio allocation effect”). Accordingly, the demand for the new asset increases, whereas the demand for the traditional asset declines.

Both effects imply an increase in the demand for the new asset as the new asset’s dividend-growth rate as perceived by the inexperienced investors increases. As a consequence, its price-dividend ratio is monotonically increasing in the perceived dividend-growth rate (Panel A). In contrast, the two effects are of opposite sign for the traditional asset, so that the overall impact on the demand for the traditional asset and, in turn, on its price-dividend ratio, depends on the magnitudes of the two effects. For low perceived dividend-growth rates the portfolio allocation effect dominates, such that the traditional asset’s price-dividend ratio is declining in the new asset’s perceived dividend-growth rate. Instead, for high perceived dividend-growth rates, the savings effect dominates, so that the traditional asset’s price-dividend ratio is increasing in the new asset’s perceived dividend-growth rate. Thus, the relation for the traditional asset is U-shaped (Panel B)—though quantitatively weaker than for the new asset.

Note also that for both assets the effects are quantitatively stronger for later points in time. Intuitively, the higher precision of the inexperienced investors’ posterior at later dates implies that perceived dividend-growth rates are subject to smaller future changes and, thus, will be more persistent. Consequently, asset prices react stronger to changes in the new asset’s dividend-growth rate as perceived by the inexperienced investors.

**D Return Cyclicality**

The implications of financial innovation and differences in beliefs for the return cyclicality of the new asset are illustrated in Figure A2. In particular, the figure shows the correlation between changes in the new asset’s dividend and changes in the new asset’s risk premium, return volatility and Sharpe ratio. Thus, positive correlations indicate that return moments are pro-cyclical; that is, the risk premium, return volatility and Sharpe ratio are higher following positive dividend shocks. In contrast, negative correlations indicate that the new asset’s return moments are counter-cyclical and, therefore, the risk premium, return volatility and Sharpe ratio are higher in bad times.

In the pre-innovation period, the correlations are basically the same for the cases of common beliefs and differences in beliefs. There are two (opposing) effects taking place. First, a negative shock to the dividend of the new asset reduces its share of aggregate dividends. This, in turn, reduces its risk premium, return volatility and Sharpe ratio because the new asset’s dividends
**Figure A2: New Asset: Return Cyclicality**

This figure shows the dynamics of the correlations between changes in the new asset’s return moments and changes in its dividends over time, based on the parameter values described in Section 3.1. The results are conditional on financial innovation taking place ($I_{τ} = 1$), averaged across simulation paths and computed under the objective beliefs. Panels A to C show the average correlation between changes in the new asset’s dividend and the new asset’s risk premium, return volatility and Sharpe ratio, respectively.

constitute a smaller fraction of aggregate consumption, and thus, co-vary less with the SDF (see also Cochrane, Longstaff, and Santa-Clara (2008)). Second, a negative dividend shock leads to a reduction in the consumption share of the experienced investors because they are over-invested in the new asset. This increases the risk premium, return volatility and Sharpe ratio of the new asset because a smaller fraction of investors has to bear the risk of the new asset. In equilibrium, the first effect tends to dominate, leading to the positive correlations and, thus, pro-cyclicality in the new asset’s return moments.
With common beliefs, the new asset’s return moments are even more pro-cyclical following financial innovation. Intuitively, because both groups of investors hold the market portfolio, there are no fluctuations in the consumption share following financial innovation. Accordingly, the second effect, which had reduced the correlation between changes in the new asset’s dividends and its return moments, disappears; thereby increasing correlations.

In contrast, with differences in beliefs, the correlations between changes in the new asset’s dividends and its return moments are negative after financial innovation. Consequently, the return moments are counter-cyclical; that is, the new asset’s risk premium, return volatility and Sharpe ratio are higher in bad times—consistent with empirical stylized facts. Intuitively, a negative dividend shock for the new asset leads to a downward revision in the inexperienced investors’ perceived dividend-growth rate, which leads to a lower price (confer Figure A1) and, accordingly, a higher subsequent risk premium and Sharpe ratio. Over time, as the magnitude of the revisions in the inexperienced investors’ beliefs declines, this (third) effect weakens, so that correlations increase over time.
References


