Information and Optimal Trading Strategies with Dark Pools

Anna Bayona\textsuperscript{1} Ariadna Dumitrescu\textsuperscript{1} Carolina Manzano\textsuperscript{2}

\textsuperscript{1}ESADE Business School
\textsuperscript{2}Universitat Rovira i Virgili

CEPR-Imperial-Plato
Inaugural Market Innovator (MI3) Conference
Motivation

"Today’s markets enjoy historically narrow spreads, low transaction costs, and increased displayed liquidity. This suggests that the proliferation of lit exchanges and dark trading venues in recent years has not harmed investors, at least not to any measurable degree.”
Commissioner Luis A. Aguilar, SEC, 2015.
European stock markets are getting darker

Dark pools' share of European equity trading

Source: Rosenblatt Securities Inc.
* Includes 19 pools and estimates on some banks' broker-crossing networks
The Model in a Nutshell

- Sequential trading model with asymmetric information
- Rational traders have access to different types of trading venues (lit market and dark pools) and order types (market orders, limit orders, dark orders)

Adding a Dark Pool alongside an exchange

- Increases the welfare at $t = 1$ of rational traders and does not affect the welfare of liquidity traders
- Decreases expected price informativeness at $t = 1$
- Cross-sectional differences in stocks characteristics have important implications for market liquidity, volume of trading and segmentation of informed and uninformed traders in the two trading venues
Theoretical analysis which analyses the effects of adding a dark pool alongside an exchange:

- Hendershott and Mendelson (2000)
- Degryse, Van Achter, and Wuyts (2009)
- Ye (2011)
- Buti, Rindi and Werner (2017)
Theoretical analysis which analyses the effects of adding a dark pool alongside an exchange:

- Hendershott and Mendelson (2000)
- Degryse, Van Achter, and Wuyts (2009)
- Ye (2011)
- Buti, Rindi and Werner (2017)

We study the dynamic order strategies and interaction of the LOB with a dark pool in the presence of asymmetric information.
The model

- Single risky asset with a liquidation value $\tilde{\nu}$ that can take two values $\nu \in \{\nu^H, \nu^L\}$ with equal probabilities, $E(\tilde{\nu}) = \mu$.

- The asset can be traded either in an exchange or in a dark pool.
The model

- Single risky asset with a liquidation value $\tilde{\nu}$ that can take two values $\nu \in \{\nu^H, \nu^L\}$ with equal probabilities, $E(\tilde{\nu}) = \mu$.

- The asset can be traded either in an exchange or in a dark pool.
Trading in the exchange

- Exchange (lit market) is organised as a limit order book (LOB). We assume that:
  - At any point in time, all the information of the LOB is available to all market participants (full pre-trade transparency).
  - Initial LOB has only two prices on each side of the book. Ask and bid prices are on a grid; tick size is equal to $\tau$.
    \[ v^H = \mu + k_3\tau; \quad v^L = \mu - k_3\tau \]
  - Initial depth of the LOB at any price is 1.
  - The LOB has price and time priority rules.
Trading in the dark pool

- Dark pools - typically agency/broker or exchange-owned pools.
  - Completely opaque in the sense that an order submitted to the dark pool is not observable to anyone but the trader who submitted it.
  - Anonymous matches orders - brings together a variety of sources of non-displayed liquidity - has an execution probability $\theta$ that is exogenous and does not change over time.
  - If the order is attended then it is executed at the price equal to the corresponding midpoint of the bid and the ask prevailing in the lit exchange at time $t$.
  - If the order is *not* attended in the dark then it returns to the lit market at $t + 2$.
  - Dark pool orders are similar to a common type of order in the dark: Immediate-or-Cancel (IOC) orders.
Type of traders and tree of events

Bayona, Dumitrescu, & Manzano
Information and Optimal Trading Strategies with Dark Pools
We focus on a symmetric Perfect Bayesian Equilibrium (PBE) in pure strategies.
Equilibrium in the benchmark model without dark pool: $t = 2$

The optimal submission strategy at $t = 2$ is as follows:

**Informed traders** always choose $MO$.

**Uninformed traders** choose $MO$ or $NT$ depending on the LOB.

<table>
<thead>
<tr>
<th>State of the book</th>
<th>UB</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(A_1, B_1)$</td>
<td>$NT$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_2, B_1)$</td>
<td>$\begin{cases} MO &amp; \text{if } X &gt; \frac{k_2}{k_3} \ NT &amp; \text{if } X \leq \frac{k_2}{k_3} \end{cases}$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_1, B_1 + \tau)$</td>
<td>$\begin{cases} MO &amp; \text{if } Y &gt; \frac{k_1}{k_3} \ NT &amp; \text{if } Y \leq \frac{k_1}{k_3} \end{cases}$</td>
<td>$NT$</td>
</tr>
<tr>
<td>$(A_1, B_2)$</td>
<td>$NT$</td>
<td>$\begin{cases} MO &amp; \text{if } X &gt; \frac{k_2}{k_3} \ NT &amp; \text{if } X \leq \frac{k_2}{k_3} \end{cases}$</td>
</tr>
<tr>
<td>$(A_1 - \tau, B_1)$</td>
<td>$NT$</td>
<td>$\begin{cases} MO &amp; \text{if } Y &gt; \frac{k_1}{k_3} \ NT &amp; \text{if } Y \leq \frac{k_1}{k_3} \end{cases}$</td>
</tr>
</tbody>
</table>

where $X, Y$ are the beliefs that a $MO, LO$ order, respectively, was submitted by an informed trader. These values differ for each optimal strategy profile at $t = 1$. 
In equilibrium the following results hold at \( t = 1 \):

**Informed traders** never choose \( NT \).

**Uninformed traders** never choose \( MO \).
Optimal strategy profiles at $t = 1$

Proposition 1

The optimal strategy profiles at $t = 1$ are:

$$(BMO, SMO, BLO, SLO), \quad (BMO, SMO, NT, NT),$$

$$(BLO, SLO, BLO, BLO), \quad (BLO, SLO, NT, NT),$$

These are found by maximising informed and uninformed traders’ expected profits given the beliefs about the strategies of other traders.

Each optimal strategy profile at $t = 1$ implies a set of consistent beliefs (according to Bayes rule) for the traders at $t = 2$.  

Bayona, Dumitrescu, & Manzano
Information and Optimal Trading Strategies with Dark Pools
Optimal Trading Strategies at $t = 1$ without a dark pool: Graphical Representation

Bayona, Dumitrescu, & Manzano
Information and Optimal Trading Strategies with Dark Pools
Proposition 2

The optimal strategy profiles at $t = 1$ are:

- $(BMO, SMO, BLO, SLO)$, $(BMO, SMO, NT, NT)$,
- $(BLO, SLO, BLO, BLO)$, $(BLO, SLO, NT, NT)$,
- $(BDO, SDO, BLO, SLO)$, $(BDO, SDO, NT, NT)$,

These are found by maximising informed and uninformed traders’ expected profits given the beliefs about the strategies of other traders.

Each optimal strategy profile at $t = 1$ implies a set of consistent beliefs (according to Bayes rule) for the traders at $t = 2$. 
Equilibrium in the model with a dark pool

Bayona, Dumitrescu, & Manzano
Information and Optimal Trading Strategies with Dark Pools
Optimal Trading Strategies at $t = 1$ with a dark pool: Graphical Representations

Bayona, Dumitrescu, & Manzano

Information and Optimal Trading Strategies with Dark Pools
Optimal Trading Strategies at $t=1$ with a dark pool: Graphical Representations

Bayona, Dumitrescu, & Manzano

Information and Optimal Trading Strategies with Dark Pools
Equilibrium in the model with a dark pool

Informed traders at \( t = 1 \)

- Informed traders at \( t = 1 \) never choose \( NT \).
- Effect of adding a dark pool alongside an exchange on the optimal strategy profiles of informed traders at \( t = 1 \):
  - They replace \( MO \) by \( DO \) when large price improvements in the dark are expected (i.e. when the probability that a liquidity trader arrives is low) and execution risk in the dark is not too high.
  - When the execution risk reduces further, informed traders eventually also replace \( LO \) by \( DO \) since they lead to higher expected profits.
  - When execution risk is very small, all informed traders trade in the dark.
Equilibrium in the model with a dark pool

Uninformed traders at $t = 1$

- Uninformed traders at $t = 1$ never choose MO or DO.
- Effect of adding a dark pool alongside an exchange on the optimal strategy profiles of uninformed traders at $t = 1$:
  - They never choose DO when the book is uninformative ($t = 1$).
  - When execution risk in the dark is medium-high: uninformed choose the same strategy as if there was not a dark pool (i.e. either LO or NT).
  - When the execution risk in the dark is low: uninformed traders choose LO (may switch from NT to LO).
Equilibrium in the model with a dark pool

Informed traders at $t = 2$

- When the state of the LOB has not changed $(A_1^1, B_1^1)$ then informed traders choose between MO and DO depending on how high the execution risk in the dark is.
- As the dark execution risk decreases, informed traders gradually replace MO by DO for all informative states of the book since the expected price advantage outweighs the execution risk.

Bayona, Dumitrescu, & Manzano
Equilibrium in the model with a dark pool

Uninformed traders at $t = 2$

Learn from the state of the LOB and choose among $MO$, $DO$, $NT$.

- $MO$ is chosen when the execution risk in the dark is high in relation to price improvement.
- $DO$ is chosen when the benefits of price improvement are large in relation to execution risk.
- $NT$ is chosen when expected profits of $MO$ or $DO$ are negative. This includes all the cases when the LOB is uninformative (i.e. $(A_1^1, B_1^1)$).
Expected Welfare & Market Performance

We study how the addition of the dark pool changes

- **Welfare**
  - Ex ante expected profits for each type of trader.

- **Market Performance**
  - Expected inside spread. The difference between the best bid and the best ask being quoted at the end of $t = 1$.
  - Expected volume of trade in the exchange, in the dark and overall.
  - Price informativeness

We calculate expected welfare/market performance indicators with respect to the *execution risk in the dark*. We assume that the probability of execution in the dark is a uniformly distributed random variable between 0 and 1.
## Expected Welfare and Market Performance at $t = 1$

<table>
<thead>
<tr>
<th>Low Liquidity Stocks</th>
<th>Medium Liquidity Stocks</th>
<th>High Liquidity Stocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(BLO, SLO, BLO, SLO)</td>
<td>(BLO, SLO, NT, NT)</td>
<td>(BMO, SMO, NT, NT)</td>
</tr>
<tr>
<td>• Price Informativeness (-)</td>
<td>• Price Informativeness (-)</td>
<td>• Price Informativeness (-)</td>
</tr>
<tr>
<td>• Total Welfare (+) - $E(\Pi)$ Rational Traders (+) - $E(\Pi)$ Liquidity Traders (=)</td>
<td>• Total Welfare (+) - $E(\Pi)$ Rational Traders (+) - $E(\Pi)$ Liquidity Traders (=)</td>
<td>• Total Welfare (+) - $E(\Pi)$ Rational Traders (+) - $E(\Pi)$ Liquidity Traders (=)</td>
</tr>
<tr>
<td>• Expected Inside Spread (+)</td>
<td>• Expected Inside Spread (+ if $\pi$ high, - if $\pi$ low)</td>
<td>• Expected Inside Spread (-)</td>
</tr>
<tr>
<td>• Expected Total Volume (+) - Expected Volume Lit (=)</td>
<td>• Expected Total Volume (+) - Expected Volume Lit (=)</td>
<td>• Expected Total Volume (-) - Expected Volume Lit (-)</td>
</tr>
</tbody>
</table>

Bayona, Dumitrescu, & Manzano
Information and Optimal Trading Strategies with Dark Pools
Empirical evidence/implications

The effect of adding a dark pool on liquidity

- Decreases
  - Buti, Rindi and Werner (2011)
  - Foley, Malinova, Park (2012)
  - Nimalendran and Ray (2014)
  - Kwan, Masulis and McInish (2014)
  - Comerton-Forde and Putninš (2015)

- Increases
  - Gresse (2006)
  - Aquilina et al. (2017)

- Does not affect
  - Foley and Putninš (2016)
Empirical evidence/ implications

The effect of adding a dark pool on liquidity

- Decreases
  - Buti, Rindi and Werner (2011)
  - Foley, Malinova, Park (2012)
  - Nimalendran and Ray (2014)
  - Kwan, Masulis and McInish (2014)
  - Comerton-Forde and Putninš (2015)

- Increases
  - Gresse (2006)
  - Aquilina et al. (2017)

- Does not affect
  - Foley and Putninš (2016)

- Depends on stock’s characteristics
  - liquidity
  - volatility
  - adverse selection
  - PIN
Empirical evidence

The effect of adding a dark pool

- harms price discovery
  - Hendershott and Jones (2005)
  - Hatheway, Kwan and Zheng (2014)
  - Weaver (2014)
  - Comerton-Forde and Putninš (2015) (when the proportion of non-block dark trades is high)
Næs and Odegaard (2006) show that there is substantial adverse selection costs when trading in a dark pool. This result is consistent with our result that uninformed traders do not trade in the dark pool in the first period, but as they infer more information about the liquidation value of the asset from the changes in the book, they decide to trade in the second period.

Dark pool trades do have informational content (Nimalendran and Ray, 2014)
Conclusions

- We are the first to study the interaction of the LOB with a dark pool in the presence of asymmetric information.
- Market participants’ strategic choice of trading venue and order type when traders have access to a dark pool and to a traditional exchange that is organised as a LOB.
  - Tradeoff between price improvement in the dark and execution risk in the dark.
  - Adverse selection.
- Adding a DP alongside an exchange may shift the optimal strategies of each type of rational traders.
- For each type of participant, expected welfare (at $t = 1$) is not lower with a dark pool than without it.
- Price informativeness is lower with a dark pool than without it.
- Market quality parameters vary according to stock characteristics.
- Both pooling and separating equilibria of trading in dark are possible (→ segmentation of the informed/uninformed order flow).