Exclusion Bias in the Estimation of Peer Effects

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Motivation

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- Counter-intuitive but common finding in the literature: $\hat{\beta}^{2SLS} > \hat{\beta}^{OLS}$
- OLS maybe affected by yet another bias?

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  - x_{ikl} is pre-determined characteristic of individual i in peer group k in cluster l
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- We refer to this bias as the 'exclusion bias'

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  - No formal proof
  - Limited to test of random peer assignment; Do not consider exclusion bias in the estimation of endogenous peer effects

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   - We derive an exact formula for the OLS inconsistency caused by exclusion bias and discuss underlying parameters (peer group size and pool size)

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2. We generalize results to case $\beta_1 \geq 0$ & non-overlapping peer groups of size $K = 2$
   $\Rightarrow$ **Estimation of endogenous peer effects** (basic model)

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   - We derive exact formulas for both exclusion bias and reflection bias
   - We determine conditions under which exclusion bias dominates reflection bias, changing the sign of peer effect estimates
   - We show that exclusion bias is significantly stronger when cluster FEs are added at the level of the selection pool (e.g. classroom dummies)

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6. Simulation results confirm all theoretical predictions of the paper
7. We (will) review the literature and discuss the type of peer effects studies that are likely to be/not to be affected by exclusion bias, and how

Exclusion bias in the test of random peer assignment ($\beta_1 = 0$)

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where $K=$ size of peer group and N_P = size of pool from which peers are drawn

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1. $\frac{\triangle |bias|}{\triangle N_P} < 0$: *Ceteris paribus*, exclusion bias is less severe in datasets with a larger pool of potential peers.
2. $\frac{\triangle |bias|}{\triangle K} > 0$: *Ceteris paribus*, exclusion bias is more severe in datasets with larger peer groups.

Cluster FEs versus Pooled OLS

- Another important result:
  1. When peers are selected at the level of the entire population Ω (e.g. school) and clusters are formed independently from peer group formation:
  $$E(\hat{\beta}_1^{FE}) = E(\hat{\beta}_1^{POLS})$$
  2. When peers are selected within clusters indexed by $l \subset \Omega$ (e.g. classroom within a school):
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- **Intuitive?**

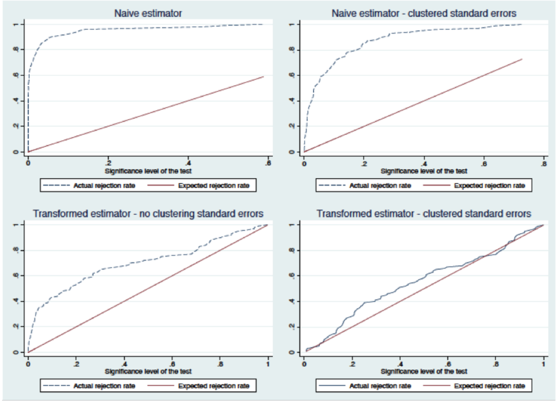
Illustration of the magnitude of the exclusion bias

Table: Exclusion bias when true $\beta_1 = 0$

		$N_P = 20$	$N_P = 50$	$N_P = 100$
K = 2	plim($\hat{\beta}_1$)	-0.053	-0.020	-0.010
K = 5	plim($\hat{\beta}_1$)	-0.250	-0.087	-0.042
K = 10	plim($\hat{\beta}_1$)	-0.818	-0.220	-0.099

Note: N= 1000 and cluster fixed effects added to all models

Figure: Expected versus actual rejection rate of $H_0 : \beta_1 = 0$; $N = 1000$; $N_P = 20$; K = 5; Cluster FE model



Note: Monte Carlo simulation results based only 100 repetitions

Exclusion bias in the estimation of endogenous peer effects ($\beta_1 \geq 0$)

Model setup

- The peer effects model we seek to estimate has the following form:

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- To make this illustration as clear as possible, to start:
  - We assume peer groups of size $K = 2$
  - We abstract from exogenous peer effects and contextual effects
  - We abstract from unobserved common shocks and other correlated effects

Simple model (K = 2) - Reflection bias ($\beta_1 \geq 0$)

- We consider a system of equations similar to that of Moffit (2001):

$$\begin{aligned} y_1 &= \alpha + \beta_1 y_2 + \epsilon_1 \\ y_2 &= \alpha + \beta_1 y_1 + \epsilon_2 \end{aligned} \tag{2}$$

$0 < \beta < 1$, $E[\epsilon_1] = E[\epsilon_2] = 0$ and $E[\epsilon^2] = \sigma_\epsilon^2$

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- We obtain the following expression for reflection bias:

$$E[\widehat{\beta_1}] = \frac{2\beta_1}{1 + \beta_1^2} \neq \beta_1$$

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- We obtain:

$$E[\hat{\beta}_1] = \frac{2\beta_1 + (1 + \beta_1^2)\rho}{1 + \beta_1^2 + 2\beta_1 \rho} < \frac{2\beta_1}{1 + \beta_1^2} \neq \beta_1 \tag{5}$$

Illustration: Reflection bias vs. exclusion bias ($\beta_1 \geq 0$)

Table: Simulation results - Exclusion bias versus reflection bias in the estimation of endogenous peer effects

$K = 2$; $N_P = 10$; $N = 500$					
True β_1	Predicted reflection bias	Prediction exclusion bias	Total predicted bias	Predicted E($\hat{\beta}_1$)	Simulated E($\hat{\beta}_1$)
0.00	0.000	-0.111	-0.111	-0.111	-0.117
0.02	0.020	-0.111	-0.091	-0.071	-0.077
0.04	0.040	-0.111	-0.072	-0.032	-0.038
0.06	0.060	-0.111	-0.051	0.009	0.002
0.08	0.079	-0.110	-0.031	0.049	0.042
0.10	0.098	-0.109	-0.011	0.098	0.082
0.12	0.117	-0.108	0.009	0.129	0.122
0.14	0.135	-0.106	0.029	0.169	0.162
0.16	0.152	-0.104	0.048	0.208	0.201
0.18	0.169	-0.102	0.067	0.247	0.240
0.20	0.185	-0.099	0.086	0.286	0.279

Correction methods - Guryan et al (2009)

- Control for differences in mean outcome across selection pools by adding to the estimation equation the mean outcome $\bar{y}_{-i,l}$ of individuals other than i in selection cluster l:

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- Limitations:
  - Requires knowledge of the selection pool
  - Parameters β_1 and θ are separately identified only if there is variation in pool sizes N_P
  - Variation in N_P may be insufficient, resulting in multicollinearity and quasi-underidentification
  - Does not correct for reflection bias (mainly useful for test of random peer assignment)

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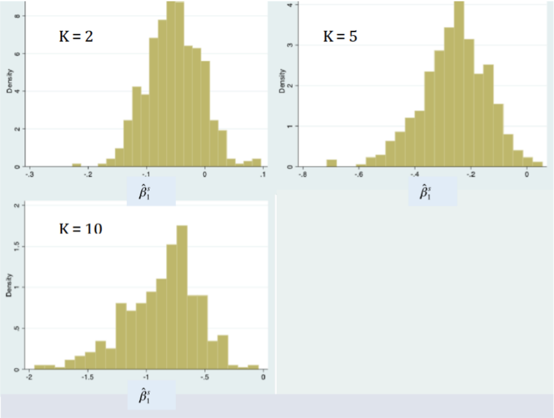
    - Repeat this process many times and trace the distribution of $\widehat{\beta}_1^{s}$ = distribution under the null
    - Obtain correct p-value by taking proportion of $\widehat{\beta}_1^{s}$ that are above $\widehat{\beta}_1^{naive}$ (similar to bootstrapping procedures)

Randomization inference - Example

Figure: Histogram $\hat{\beta}_1^s$ under null hypothesis ($N = 1000$; $N_P = 20$)



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- If interested in correcting point estimates as well as inference: No closed-form formulas available
- We can characterize the DGP under exclusion bias and use nonlinear method of moments estimation techniques to provide consistent estimates for β_1:

$$E[YY'] = (I-\beta G)^{-1}E[(\gamma X+\delta GX)(\gamma X+\delta GX)'](I-\beta G')^{-1}+(I-\beta G)^{-1}E[\epsilon\ \epsilon'](I-\beta G')^{-1}$$

Correction - generalized model (groups ≥ 2 or network data)

$$Y = \beta GY + \gamma X + \delta GX + \epsilon \tag{7}$$

- If simply interested in correcting inference for $H_0 : \beta_1 = 0$ then randomization inference method described before will do the trick
- If interested in correcting point estimates as well as inference: No closed-form formulas available
- We can characterize the DGP under exclusion bias and use nonlinear method of moments estimation techniques to provide consistent estimates for β_1:

$$E[YY'] = (I - \beta G)^{-1} E[(\gamma X + \delta GX)(\gamma X + \delta GX)'](I - \beta G')^{-1} + (I - \beta G)^{-1} E[\epsilon\ \epsilon'](I - \beta G')^{-1}$$

- Where $E[\epsilon\epsilon'] = \sigma_\epsilon^2 \begin{bmatrix} B & 0 & ... \\ 0 & B & ... \\ ... & ... & ... \end{bmatrix}$ and where B is a $K \times K$ of the form:

$$B = \begin{bmatrix} 1 & \rho & ... \\ \rho & 1 & ... \\ ... & ... & ... \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{N_P - 1} & ... \\ -\frac{1}{N_P - 1} & 1 & ... \\ ... & ... & ... \end{bmatrix}$$

Simulation results correction method - General groups

	K = 2 ; $L = 20$; $N = 100$			K = 5 ; $L = 20$; $N = 100$		
	(1)	(2)	(3)	(4)	(5)	(6)
True β_1	0.0	0.1	0.2	0.0	0.1	0.2
Naive E($\hat{\beta}_1$)	-0.05	0.14	0.33	-0.26	-0.04	0.17
E($\hat{\beta}_1$) correction reflection bias only	-0.03	0.07	0.17	-0.12	-0.01	0.10
E($\hat{\beta}_1$) correction reflection and exclusion bias	0.00	0.10	0.20	0.00	0.10	0.20

Note: Cluster fixed effects added in all regressions; Simulations $\hat{\beta}_1$ over 100 Monte Carlo repetitions.

Example randomization inference on one fictional dataset - General groups

	K $= 2$; $L = 10$; $N = 500$			K $= 5$; $L = 10$; $N = 500$		
	(1)	(2)	(3)	(4)	(5)	(6)
True β_1	0.0	0.1	0.2	0.0	0.1	0.2
Naive $\hat{\beta}_1$	-0.15	0.05	0.25	-0.60	-0.34	-0.08
Naive p-value	0.00	0.27	0.00	0.00	0.00	0.43
Corrected p-value	0.63	0.01	0.00	0.72	0.11	0.03

Note: Cluster fixed effects added in all regressions; Randomization over 100 Monte Carlo replications.

Simulation correction method - Network data

p = probability of link between i and j within a cluster	$p = 0.1$; $L = 20$; $N = 100$			$p = 0.3$; $L = 20$; $N = 100$		
	(1)	(2)	(3)	(4)	(5)	(6)
True β_1	0.0	0.1	0.2	0.0	0.1	0.2
Naive E($\hat{\beta}_1$)	-0.09	0.08	0.24	-0.31	-0.13	0.04
E($\hat{\beta}_1$) correction reflection bias only	-0.04	0.03	0.11	-0.11	-0.05	0.01
E($\hat{\beta}_1$) correction reflection and exclusion bias	0.01	0.10	0.19	0.02	0.11	0.19

Note: Cluster fixed effects added in all regressions; Simulations $\hat{\beta}_1$ over 100 Monte Carlo repetitions.

Example randomization inference on one fictional dataset - Network data

p = probability of link between i and j within a cluster	$p = 0.1$; $L = 10$; $N = 500$			$p = 0.3$; $L = 10$; $N = 500$		
	(1)	(2)	(3)	(4)	(5)	(6)
True β_1	0.0	0.1	0.2	0.0	0.1	0.2
Naive $\hat{\beta}_1$	-0.14	0.03	0.19	-0.40	-0.24	-0.06
Naive p-value	0.02	0.60	0.00	0.00	0.01	0.52
Corrected p-value	0.65	0.09	0.00	0.40	0.01	0.00

Note: Cluster fixed effects added in all regressions; Randomization over 100 Monte Carlo replications.

Example - Sacerdote (2001)

Parameters	**Values**	**Example Sacerdote (2001)**
Sample size N	1589	Number of graduate students enrolled in 1998-1999
Number of peer selection pools	42	Number of housing preference blocks within which students are randomly allocated to dorms and dorm rooms
Pool size N_P	38	Average number of students in each housing preference block
Peer group size K	53% in double room ($K = 2$) 44% in triple rooms ($K = 3$) 3% in quad rooms ($K = 4$)	Number of students in each dorm room
Exclusion bias: $-\frac{K-1}{N_P - K + 1}$	-0.04	Weighted average of biases for different K: $0.53 * (bias \mid K = 2)$ $+0.44 * (bias \mid K = 3)$ $+0.03 * (bias \mid K = 4)$

Example - Sacerdote (2001) - test of random peer assignment

$$x_{ikl} = \beta_0 + \beta_1 \bar{x}_{-i,k,l} + \delta_l + \epsilon_{ikl}$$

	SATH Math	SAT verbal	High school Academic class index	High school academic index
$\hat{\beta}_1^{Naive}$ - Sacerdote (2001)	-0.025 (0.028)	-0.009 (0.029)	0.010 (0.028)	-0.032 (0.028)
$\hat{\beta}_1^{Corrected}$ (Caveat! Not clustering s.e.)	0.015 (0.028)	0.031 (0.029)	0.05* (0.028)	0.008 (0.028)

Example - Sacerdote (2001) - estimation of peer effects

$$y_{ikl} = \beta_0 + \beta_1 \bar{y}_{-i,k,l} + \delta_l + \epsilon_{ikl}$$

	GPA test score
$\hat{\beta}_1^{Naive}$ - Sacerdote (2001)	0.07** (0.029)
$\hat{\beta}_1^{CorrectionReflectionOnly}$ Conservative correction (assuming $K = 2$ and no clustering s.e.)	0.03 (0.029)
$\hat{\beta}_1^{TotalCorrection}$ Conservative correction (assuming $K = 2$ and no clustering s.e.)	0.05* (0.029)

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- Certain studies that use an RCT as a means to estimate peer effects (e.g. Fafchamps and Vicente, 2013)

$$y_i = b_0 + b_1 \sum_{j \in N_i} T_j + b_2 T_i + u_i$$

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- Studies that use lagged outcome of peers and control for lagged outcome of individal i herself (e.g. Munshi, 2004)

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    - First stage: $\bar{y}_{-i,k} = \pi_0 + \pi_1 \bar{z}_{-i,k} + \pi_2 z_{ik} + v_{ik}$
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  - Example Bramouille et al (2009) or De Giorgi et al (2010):

$$y_i = b_0 + b_1 \bar{y}_j + b_2 \bar{x}_j + b_3 x_i + u_i$$

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  - Alternative explanation for the common but counter-intuitive tendency of peer effects studies to obtain $\hat{\beta}_1^{2SLS} > \hat{\beta}_1^{OLS}$

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- BUT, limitations to 2SLS:
  - Condition of controlling for z_{ik} not always satisfied
  - Requires suitable strong instruments (Bound et al, 1995)
  - Biased in finite samples (Bound et al, 1995)

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- BUT, limitations to 2SLS:
  - Condition of controlling for z_{ik} not always satisfied
  - Requires suitable strong instruments (Bound et al, 1995)
  - Biased in finite samples (Bound et al, 1995)
- We suggest a correction method that deals wit both reflection bias and exclusion bias and which does not require any IVs and that is valid even in small finite samples.

Concluding remarks

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- We suggest methods that can be used to correct point estimates and inference for both reflection bias and exclusion bias when no suitable instruments are available
- Next step: review the literature and demonstrate the impact of exclusion bias in practice

THANK YOU!

APPENDIX

APPENDIX SLIDES

Exclusion bias in test of random peer assignment ($\beta_1 = 0$)

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$$\bar{x}_{-i,k} = \bar{x}_{-i} + u_{ik} \quad (9)$$

where $\bar{x}_{-i}$ = average outcome of pool of $(N-1)$ potential peers and u_{ik} is random term

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- Through reduced form we derive formula for expected bias in $\hat{\beta}_1^{OLS}$:

$$E(\hat{\beta}_1^{OLS}) = -\frac{K(K-1)}{N + (N-K)(K-1)}$$

Formula - with clustered stratification

$$x_{ikl} = \beta_0 + \beta_1 \bar{x}_{-i,k,l} + \delta_l + \epsilon_{ikl} \tag{12}$$

where l is cluster of size $L < N$ at the level of which peers are randomised (e.g. classroom)

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- Cluster FE equation can be rewritten as follows:

$$x_{ikl} - \bar{x}_l = \beta_1 \left(\bar{x}_{-i,k,l} - \bar{\bar{x}}_{-i,l} \right) + \left(\epsilon_{ikl} - \bar{\epsilon}_l \right) \tag{13}$$

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- Proof proceeds in a similar way as in the non-stratified case but now based on a model in terms of deviations of outcomes from their respective cluster averages. We obtain:

$$E(\hat{\beta}_1^{FE}) = -\frac{K(K-1)}{L + (L-K)(K-1)} \tag{14}$$

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- The exclusion bias arises whenever:
  1. Individual i is excluded from her potential peer group (selection without replacement);
  2. But is a potential peer for other observed individuals j
  3. And one considers a peer effects model that regresses individual i's characteristic on an average characteristic of i's peer group without controlling for i's own characteristic

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- Studies that employ a particular type of 2SLS estimation that eliminates exclusion bias (e.g. Fletcher, 2012)