Abstract

Recently, the global economy has experienced recurrent episodes of safe asset shortages. In this paper we present a model that shows how such shortages can generate macroeconomic phenomena similar to those found in liquidity trap scenarios. Despite the similarities, there are also subtle but important differences which carry significant impacts on the relative effectiveness of economic policy and potential market solutions to the underlying problem. For example, while forward guidance policies are typically more effective than quantitative easing ones in the standard liquidity trap environment, the opposite holds in safety trap contexts. Also, while asset bubbles (market solutions) and public debt are both effective in liquidity traps, only the latter are in safety traps. Essentially, a safe asset shortage is a deficit of a particular form of wealth (safe wealth), which the government has comparative advantage in supplying. Forward guidance and financial bubbles, which increase risky wealth and stimulate the economy in liquidity traps, fail to do so in safety traps as they are dissipated through higher spreads.
1 Introduction

One of the main structural features of the global economy in recent years is the apparent shortage of safe assets. This deficit provided one of the key macroeconomic forces for the financial engineering behind the subprime crisis, it was a paramount factor in determining the spike in funding costs when European economies switched from the core to the periphery, and has put new constraints on the effectiveness of monetary and fiscal policy.

In this paper we provide a model of the most acute manifestation of a safe asset shortage and its economic policy implications. In this form, safe asset shortages can generate macroeconomic phenomena similar to those found in liquidity trap scenarios, such as severe recessions and (safe) interest rates so low that bonds and money become perfect substitutes. We call this scenario a safety trap.

Despite these similarities, there are subtle but important differences between liquidity and safety traps that carry significant impacts on the relative effectiveness of economic policy and potential market solutions to the underlying problem. For example, while forward guidance policies are typically more effective than quantitative easing ones in the standard liquidity trap argument, the opposite holds in safety trap contexts. Also, while asset bubbles (market solutions) and public debt are both effective in liquidity traps, only the latter are effective in safety traps. Essentially, a safe asset shortage is a deficit of a particular form of wealth (safe wealth), which the government has comparative advantage in supplying. Forward guidance and financial bubbles, which increase risky wealth and stimulate the economy in liquidity traps, fail to do so in safety traps as they are dissipated by higher spreads.\(^1\)

The emergence of an “excess” demand for safe assets triggers a variety of symptoms, the most direct of which is a strong downward pressure on the Wicksellian “natural” real safe interest rates (the real safe interest rate consistent with full capacity utilization). With strong nominal rigidities, nominal and real interest rates essentially coincide. Moreover, because money is a safe asset, nominal interest rates cannot drop below zero. As a result, when safe natural real interest rates are negative, full capacity utilization cannot be sustained and output falls below potential. In a nutshell, away from the zero bound, an excess demand for safe assets is resolved through a reduction in safe interest rates. Instead, at the zero lower bound, equilibrium is restored through a reduction in output.

\(^1\)In reality safety and liquidity traps features are likely to coexist (perhaps with the safety trap dominating in the most acute phase of a recession and gradually mutating into a more conventional liquidity trap as the recovery progresses), but our purpose in this paper is to isolate the implications of the former.
Public debt (and “helicopter” money) plays a central role in a safe asset shortage episode as typically the government owns a disproportionate share of the capacity to create safe assets while the private sector owns too many risky assets. The key concept then is that of fiscal capacity: How much public debt can the government credibly pledge to honor should a major macroeconomic shock take place in the future?\footnote{At the zero lower bound, short-term public debt and money are perfect substitutes. Issuing money and issuing public debt are therefore equivalent. In both cases, fiscal capacity limits how much issuance can be undertaken by the government. With public debt, it is because the government must be able to pay down its debt. With money, it is because the government must be able to retire the extra money when the economy exits the safety trap and money demand subsides, or otherwise face the costly consequence of an overstimulated economy.} As long as the government has spare fiscal capacity to back safe asset production, it can increase the supply of safe assets by issuing public debt. This reduces the root imbalance in the economy.

Swapping risky private assets for safe public debt, which with some abuse of terminology we refer to as Quantitative Easing (QE) type policies, and which encompass QE1 in the U.S. and LTRO in Europe, as well as many other lender of last resort central bank interventions, have positive effects on output.\footnote{In a previous version of this paper, we argued that the benefit of QE1 type policies are unlikely to extend to the swapping of short-run public debt for long-run public debt (which we refer to as Operation Twist (OT), and which encompass the recent QE2 and QE3 in the U.S). In fact, OT can be counterproductive since long term public debt, by being a “bearish” asset that can be used to hedge risky private assets, has a safe asset multiplier effect that short term public debt lacks. That is, long term public debt is not only a safe asset in itself, but also makes risky private assets safer through portfolio effects. Of course, part of the benefit of OT policies is to support the bearish nature of long term public debt, and in this sense it is the commitment to future support of these assets, should conditions deteriorate, that generates the benefit, for reasons similar to those we highlight in QE type policies. We refer the reader to a previous version of this paper (Caballero and Farhi 2013) for a detailed exposition.} In contrast, commitments to low interest rates in future good states—the way forward guidance type policies are usually discussed—are ineffective precisely because they attempt risky assets revaluation rather than the safe asset expansion that a safe asset shortage requires.

By the same token, while in liquidity traps private asset production and bubbles are good substitutes for public debt, they are not in a safe asset shortage since they mostly increase the supply of risky rather than of safe assets.

Related literature. Our paper is related to several strands of literature. First and most closely related is the literature that identifies the shortage of safe assets as key macroeconomic fact (see e.g. Caballero 2006 and 2010, Caballero and Krishnamurthy 2009, Bernanke et al 2011, Barclay’s 2012). Our paper provides a model that captures many of the key insights in that literature and that allows us to study the main macroeconomic policy implications of this environment more precisely.
Second, the literature on aggregate liquidity (see e.g. Woodford 1990 and Holmström and Tirole 1998) analyzes the shortage of liquidity (stores of value). It has emphasized the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Our paper shares the idea that liquidity shortages are important macroeconomic phenomena, and that the government has a special role in alleviating them. However, it shifts the focus to a very specific form of liquidity—safe assets—and works out its distinct and unique consequences.

Third, there is a literature that documents significant deviations from the predictions of standard asset-pricing models—patterns which can be thought of as reflecting money-like convenience services—in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically (Krishnamurthy and Vissing-Jorgensen 2011, 2012, Greenwood and Vayanos 2010, Duffee 1996, Gurkaynak, Sack and Wright 2006, Bansal and Coleman 1996). Our model offers a distinct interpretation of these stylized facts, where the “specialness” of public debt is its safety during bad states of the economy.

Fourth, there is an emerging literature which emphasizes how the aforementioned premium creates incentives for private agents to rely heavily on short-term debt, even when this creates systemic instabilities (Gorton and Metrick 2010, 2012, Gorton 2010, Stein 2012, Woodford 2012, Gennaioli, Shleifer and Vishny 2012). Greenwood, Hanson and Stein (2012) consider the role of the government in increasing the supply of short-term debt and affecting the premium. Gorton and Ordonez (2013) also consider this question but in the context of a model with (asymmetric) information acquisition about collateral where the key characteristic of public debt that drives its premium is its information insensitivity. Our model also illustrates how this premium affects private agent’s balance sheets, but it offers distinct mechanisms for its source, and on how it affects the economy and macroeconomic policy.

Fifth, there is the literature on liquidity traps (see e.g. Keynes 1936, Krugman 1998, Eggertsson and Woodford 2003, Christiano, Eichenbaum and Rebelo 2011, Correia, Farhi, Nicolini and Teles 2012, Werning 2012, Kocherlakota 2013). This literature emphasizes that the binding zero lower bound on nominal interest rates presents a challenge for macroeconomic stabilization. In most models of the liquidity trap, the corresponding asset shortage arises from an exogenous increase in the propensity to save (a discount factor shock). Some recent models (see e.g. Guerrieri and Lorenzoni 2011, and Eggertsson and Krugman 2012) provide deeper microfoundations and emphasize the role of tightened borrowing constraints in economies with heterogeneous agents (borrowers and savers). Our model of a safety trap shares elements of the Keynesian liquidity trap story. However, in our model the key interest
rate is the safe interest rate, and the root cause of safety traps is an acute safe asset shortage. This distinction has important policy implications.

Finally, our paper relates to an extensive literature, both policy and academic, on fiscal sustainability and the consequences of current and future fiscal adjustments (see, e.g., Giavazzi and Pagano 1990, 1996, Alesina and Ardagna 1998, IMF 1996, Guihard et al 2007). Our paper revisits some of the policy questions in this literature but highlights the government’s capacity to create safe assets at the margin, as the key concept to determine the potential effectiveness of further fiscal expansions as well as the benefits of future fiscal consolidations.4

The paper is organized as follows. Section 2 describes our basic model and introduces the key mechanism of a safety trap. Section 3 introduces public debt and considers the effects of QE policies. Section 4 analyzes the role of forward guidance (monetary policy commitments). Section 6 develops a version of a liquidity trap in the context of our model and explains the similarities and differences with a safety trap. Section 7 concludes.

2 A Model

In this section we describe the safe asset shortage equilibrium without government intervention. When a shortage of safe assets arises, interest rates need to drop to reduce the return on these assets and hence their demand. However, if there is a lower bound for interest rates, then a safety trap emerges and asset markets are cleared through a recession instead.

2.1 No Lower Bound on Safe Interest Rates

Setup. The basic model has output exogenous at its maximum level. The goal is to characterize demand and supply of safe assets, and their impact on equilibrium returns.

Output is constant, $X$, unless a Poisson shock takes place. There are two Poisson processes. First there is a good Poisson process with intensity $\lambda^+$. Second there is bad Poisson

4Our paper is also related to a strand of literature on global imbalances. Caballero, Farhi, and Gourinchas (2008a,b) developed the idea that global imbalances originated in the superior development of financial markets in developed economies, and in particular the U.S. Global imbalances resulted from an asset imbalance. Although we do not develop the open economy version of our model here, our model could capture a specific channel that lies behind global imbalances: The latter were caused by the funding countries’ demand for financial assets in excess of their ability to produce them, but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to produce them.
process with intensity $\lambda^-$. If the good Poisson shock takes places, output jumps to $\mu^+X > X$ forever. If the bad Poisson shock takes place, output drops to $\mu^-X < X$ forever. Whichever Poisson shock takes place first (which we refer to as the Poisson event) removes the possibility of any other Poisson shock, and therefore uncertainty disappears. We focus on the period before the Poisson event, when uncertainty has not yet been resolved. We simplify the notation by studying the limit as $\lambda^+ \to 0$ and $\lambda^- \to 0$.

Population has a perpetual-youth overlapping generations structure with death and birth rates $\theta$. Agents consume only when they die, which yields a simple aggregate consumption function $C_t = \theta W_t$, where $C_t$ and $W_t$ represent aggregate consumption and wealth, respectively. Note that equilibrium in goods markets pins down the equilibrium value of aggregate wealth $W_t$ at

$$W = \frac{X}{\theta}.$$ 

Next, we introduce two key ingredients of the model, one about asset demand, and one about asset supply. On the asset demand side, there are two types of agents in the economy: Neutral and (locally) Knightian. Neutral agents are risk-neutral. Knightian agents are infinitely risk averse (over short time intervals): between $t$ and $t + dt$, when forming their portfolio, they act as if the Poisson shock happens in the next infinitesimal time interval with probability one. The fraction of Neutrals in the population is $1 - \alpha$. The fraction of Knightians is $\alpha$. We denote the wealth of Neutral and Knightian agents by $W^N_t$ and $W^K_t$ with

$$W^N_t + W^K_t = W.$$ 

On the asset supply side, we assume that a fraction $\delta$ of output $X$ is pledgeable and accrues as the total dividend of Lucas trees (each tree capitalizes a stream of $\delta$ units of goods per period). The rest, $(1 - \delta)X$, is distributed to newborns. The total value of assets before the Poisson event is $V$, and from financial market equilibrium we have:

$$V = W = \frac{X}{\theta}.$$ 

We assume that only a fraction $\rho$ of these assets can be tranched to split the risky and riskless component of returns. We denote by $V^S$ and $V^R$ the supply of safe and risky assets with

$$V = V^S + V^R.$$ 

$^5$We relax this assumption only when needed in Sections 4 and 6.
A safe asset is one whose value does not change when the Poisson event takes place. Thus, we can find $V^S$ by solving backwards and noting that by construction a fraction $\rho$ of the total value of assets after a bad Poisson shock is safe:

$$V^S = \rho \mu \frac{X}{\theta}.$$  

Risky assets (before the Poisson event) are worth the residual $V - V^S$:

$$V^R = (1 - \rho \mu^-) \frac{X}{\theta}.$$  

Since Knightian agents only hold safe assets, their wealth holdings, $W^K_t$, must satisfy:

$$W^K_t \leq V^S.$$  

Let $r$, $r^K$, and $\delta^S$ denote the (ex-ante) rate of return on risky assets, the rate of return on safe assets, and the dividend paid by safe assets, respectively. Then equilibrium is characterized by the following equations:

$$r^K V^S = \delta^S X,$$
$$r V^R = (\delta - \delta^S) X,$$
$$\dot{W}^K_t = -\theta W^K_t + \alpha (1 - \delta) X + r^K W^K_t,$$
$$\dot{W}^N_t = -\theta W^N_t + (1 - \alpha) (1 - \delta) X + r W^N_t,$$
$$W^K_t + W^N_t = V^S + V^R.$$  

Note that in the limit that we consider ($\lambda^+ \to 0$ and $\lambda^- \to 0$), $\mu^+$ does not appear in the equilibrium equations before the Poisson event. Only $\mu^-$ does, because it determines the supply of safe assets.

**Two regimes.** There are two regimes, depending on whether the constraint $W^K_t \leq V^S$ is slack (unconstrained regime) or binding (constrained regime).

In the unconstrained regime, since Neutrals are the marginal holders of safe assets, safe and risky rates must be equal. A few steps of algebra show that in this case:

$$\delta^S = \delta \rho \mu^-.$$
\[ r = r^K = \delta \theta. \]

The interesting case for us is the \textit{constrained} regime, which captures the safe asset shortage environment. In it, Knightians gobble up all safe assets and wish they had more, so that:

\[ W^K = V^S = \rho \mu - \frac{X}{\theta}. \]

It is easy to verify that this regime holds (after possibly a transitional period) as long as \( \alpha > \rho \mu^- \). \(^6\) The latter is the \textit{safe asset shortage condition}, which we shall assume holds henceforth. In this case we have:

\[ \delta^S = \delta \rho \mu^- - (\alpha - \rho \mu^-)(1 - \delta) < \delta \rho \mu^-, \]

\[ r^K = \delta \theta - (1 - \delta) \frac{\theta \alpha - \rho \mu^-}{\rho \mu^-} < \delta \theta, \]

\[ r = \delta \theta + (1 - \delta) \frac{\theta \alpha - \rho \mu^-}{1 - \rho \mu^-} > \delta \theta. \]

It follows that in this region there is a safety premium

\[ r - r^K = (1 - \delta) \frac{\theta \alpha - \rho \mu^-}{\rho \mu^- (1 - \rho \mu^-)} > 0. \]

The supply of safe assets is determined by the severity of the potential bad shock (\( \mu^- \)) and the ability of the economy to create safe assets (\( \rho \)). In fact \( \rho \) and \( \mu^- \) enter the equilibrium equations only through the sufficient statistic \( \rho \mu^- \). Similarly, the demand for safe assets is summarized by the fraction of Knightians (\( \alpha \)). Together, these sufficient statistics determine whether we are in the unconstrained regime (\( \alpha \leq \rho \mu^- \)) or in the constrained regime (\( \alpha > \rho \mu^- \)).

\(^6\) A delicate issue is the initial value of risky assets. It is possible for the value of risky assets (and hence total assets) to jump at date 0 and so we distinguish 0\(^-\) and 0\(^+\). We denote by \( \beta_0^S \) and \( \beta_0^R \) the fraction of safe assets and risky assets initially owned by Knightians. The initial budget constrained of Knightians implies

\[ \beta_0^R V_{0^-}^R \leq (1 - \beta_0^R) \rho \mu^- \frac{X}{\theta}, \]

and the absence of arbitrage for Neutrals requires that

\[ V_{0^-}^R \leq V_{0^+}^R = (1 - \rho \mu^-) \frac{X}{\theta}. \]

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Remark 1 Our model features two forms of market incompleteness. The first one is tied to our overlapping generations structure. As a result, our environment is non-Ricardian and asset supply ($\delta$) matters. The second market incompleteness is that only a fraction of trees ($\rho$) can be tranched. Tranching is desirable because it decomposes an asset into a safe tranche which can be sold to Knightian agents and a risky tranche that can only be sold to Neutral agents. Because agents cannot tranch assets at will, Modigliani-Miller fails in the sense that in the constrained regime the value $v^t$ of a unit of tranched tree is higher than that of an untranched tree $v^{nt}$. This gap widens as the safe asset shortage ($\alpha - \rho \mu$) worsens:

$$v^t - v^{nt} = \frac{1}{\theta} \frac{1}{\rho} \frac{1 - \delta}{1 - \rho \mu - (1 - \alpha)(1 - \delta)}.$$

2.2 The Safety Trap

As the potential shock becomes more extreme ($\mu^-$ drops), or the economy’s ability to create safe assets is more impaired ($\rho$ drops), the supply of safe assets shrinks ($\rho \mu^-$ drops). Alternatively or simultaneously, the demand for safe assets increases ($\alpha$ increases). Equilibrium is restored by a decline in $r^K$ which lowers demand for safe assets. But what if there is a lower bound $r^K \geq 0$ on the safe interest rate? In this section we address this issue and show how an excess demand for safe assets can trigger a recession similar to that which arises in a liquidity trap.

We develop our argument in two steps. In Section 2.2.1, we use a simple disequilibrium framework to isolate the mechanics of the interaction between an aggregate demand determined output and a zero lower bound on safe rates.\textsuperscript{7} In Section 2.2.2, we develop a New Keynesian model with a Cash-In-Advance constraint that provides an exact microfoundation for the mechanism we describe in the first step. The zero lower bound $r^K \geq 0$ arises from the interaction of two features. First, the possibility of arbitraging between money and other safe assets puts a zero lower bound on nominal interest rates $i \geq 0$. Second, prices are fixed (an extreme form of nominal rigidity), so that inflation is zero and nominal and safe real interest rates coincide $r^K = i$. The disequilibrium model developed in the first step is the cashless limit of the New Keynesian Cash-In-Advance model developed in the second step. Throughout the paper, we work with the disequilibrium model, seen as the cashless limit of the New Keynesian Cash-In-Advance model.

\textsuperscript{7}See e.g. Barro and Grossman 1971, Malinvaud 1977, Benassy 1986, as well as Hall 2011a,b, Kocherlakota 2012, and Korinek and Simsek 2013 for more recent applications.
2.2.1 The Mechanics of a Safety Trap

The extra restriction \( r^K \geq 0 \) forces us to consider disequilibrium in some other market, which we assume to be the goods market as this connects our discussion with standard Keynesian demand arguments. We introduce a distinction between potential output \( X \) and actual output \( \xi X \). When \( \xi < 1 \), output is below potential. We reinterpret endowments of goods \((1 - \delta) X\) and dividends \(\delta X\) as endowments of a non-traded input (say labor) that can be converted into output one-to-one. When \( \xi < 1 \), less of this input is converted into output. This modelling strategy essentially sidesteps the labor market. In every period, there is a goods market and an asset market. Dying old agents supply assets and demand goods. Survivors and newborns demand assets and supply goods.

We focus on the Keynesian regime of this disequilibrium model where output is demand determined, but the safe and risky asset markets are in equilibrium given output. This pins down a unique outcome.

Let us work backwards and recall that after the bad Poisson shock hits (which never literally happens in our model since \( \lambda^{-} \to 0 \)), uncertainty disappears and so does the safe asset shortage. This means that actual and potential output coincide after that shock and therefore the value of safe assets (before the shock) is still given by

\[
V^S = \rho \mu - \frac{X}{\theta}.
\]

Note that, mechanically, this disequilibrium model is identical to the basic model but with \( \rho \mu^\prime \) replaced by \( \frac{\rho \mu}{\xi} \) and \( X \) replaced by \( \xi X \). The requirement that \( r^K = 0 \) determines the severity of the recession \( \xi \):

\[
0 = \delta \theta - (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu}{\xi}}{1 - \frac{\rho \mu}{\xi}},
\]

yielding

\[
\xi = \frac{\rho \mu^-}{\rho \mu^\prime} < 1,
\]

where \( \rho \mu^- = \alpha (1 - \delta) \) corresponds to the value of these combined parameters for which zero is the Wicksellian equilibrium safe interest rate.\(^8\)

\(^8\)The risky interest rate \( r \) is increasing in \( \xi \), so that the deeper the recession, the lower is \( r \):

\[
r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu}{\xi}}{1 - \frac{\rho \mu}{\xi}}.
\]
Recession caused by a decrease in the supply of safe assets. The safe asset supply curve shifts left ($\rho \mu^{-} < \rho \mu^{-}$), the endogenous recession shifts the safe asset demand curve left ($\xi < 1$), and the safe interest rate remains unchanged at 0.

Because the safe interest rate $r^K$ cannot adjust downward, there is a recession. This mechanism is akin to that of a liquidity trap. We call it a “safety trap”. At full employment, there is an excess demand for safe assets. A recession lowers the absolute demand for safe assets while keeping the absolute supply of safe assets fixed and restores equilibrium. Figure 1 illustrates this mechanism, which we describe next.

The supply of safe assets is given by $V^S = \rho \mu^{-} \frac{X}{\theta}$ and the demand for safe assets is given by $W^K = \frac{\alpha (1 - \delta) \xi X}{\theta - r^K}$. Equilibrium in the safe asset market requires that $W^K = V^S$, i.e.

$$\frac{\alpha (1 - \delta) \xi X}{\theta - r^K} = \rho \mu^{-} \frac{X}{\theta}.$$ 

Consider an unexpected (zero ex-ante probability) shock that lowers the supply of safe assets (a reduction in $\rho \mu^{-}$). The mechanism by which equilibrium in the safe asset market is restored has two parts. The first part immediately reduces Knightian wealth $W^K$ to a lower level, consistent with the lower supply for safe assets $\rho \mu^{-} \frac{X}{\theta}$. The second part maintains Knightian wealth $W^K$ at this lower level.
The first part of the mechanism is as follows. The economy undergoes an immediate wealth adjustment (the wealth of Knightians drops) through a round of trading between Knightians and Neutrals born in previous periods. At impact, Knightians hold assets that now carry some risk. They react by selling the risky part of their portfolio to Neutrals. This shedding of risky assets catalyzes an instantaneous fire sale whereby the price of risky assets collapses before immediately recovering once risky assets have changed hands. Needless to say, in reality this phase takes time, which we have removed to focus on the phase following the initial turmoil.

The second part of the equilibrating mechanism differs depending on whether the safe interest rate $r_K$ is above or at the zero lower bound. If $r_K > 0$, then a reduction in the safe interest rate $r_K$ takes place. This reduction in the safe interest rate effectively limits the growth of Knightian wealth so that the safe asset market remains in equilibrium. If the safe interest rate is against the zero lower bound $r_K = 0$, then this reduction in the safe interest rate cannot take place. With full employment and $r_K = 0$, the growth rate of Knightian wealth would be too high and an excess demand for safe assets would develop over time. Instead, a recession takes place (a reduction in $\xi$) which reduces the income of Knightians (newborns) and hence the growth of Knightian wealth.

Note that the recession drags down the whole economy, reducing not only the income of Knightians, but also that of Neutrals (the dividends on risky assets and the income of Neutrals newborns) and hence the wealth of Neutrals. Of course, the flip side of this reduction in Neutral wealth is a reduction in the value of risky assets, which occurs through a reduction in dividends (and despite a decrease in the risky interest rate $r$). The reduction in Neutral wealth in turn reduces demand in the goods market, thereby justifying the recession.\footnote{Note that the adjustment in Knightian wealth is the same whether the safe interest rate $r_K$ is or is not at the zero lower bound. What is different is the adjustment in Neutral wealth. In response to a negative shock to the value of safe assets, Neutral wealth ends up at a lower level when the safe interest rate is against the lower bound than when it can freely adjust downwards.}

A similar logic applies if we raise the share of Knightian agents $\alpha$ instead of reducing $\rho \mu$, in which case the recession factor is

$$\xi = \frac{\alpha}{\bar{\alpha}} < 1,$$

where $\alpha = \frac{\rho \mu}{1 - \delta}$ corresponds to the value of this parameter for which zero is the Wicksellian equilibrium safe interest rate. This interpretation resembles the Keynesian paradox of thrift. Combining both, asset supply and demand factors, we have that the severity of the recession
is determined by the sufficient statistic $\frac{\rho \mu - \alpha}{\alpha}$ according to the simple equation:

$$\xi = \frac{\alpha}{\rho \mu - \alpha},$$

where $\frac{\rho \mu}{\alpha} = 1 - \delta$ corresponds to the value of these combined parameters for which zero is the Wicksellian equilibrium safe interest rate.

### 2.2.2 A New Keynesian Cash-In-Advance Microfoundation

In this section we develop a New Keynesian Cash-In-Advance microfoundation. When we add these ingredients to the model, it has a unique equilibrium, which exhibits the safety trap feature and mechanics described above. Again, we do it in two steps. The first step consists of making output demand determined and to associate real to nominal safe rates by adding standard New Keynesian features. The second step adds money and its transaction role, which introduces a lower bound for safe rates and links the use of money as a store of value (as opposed to transaction services) to the severity of the recession.

We show that the outcome of the disequilibrium model presented above is exactly the cashless limit of the unique equilibrium of the New-Keynesian Cash-In-Advance model. Throughout the rest of the paper, we therefore work with the disequilibrium model, seen as the cashless limit of the New Keynesian Cash-In-Advance model.

**Demand determined output** Let us incorporate the traditional ingredients of New Keynesian economics: imperfect competition, sticky prices, and a monetary authority.

In this setting, in every period, non-traded inputs are used to produce differentiated varieties of goods $x_k$ indexed by $k \in [0, 1]$ where each variety is produced using a different variety of non-traded good also indexed by $k \in [0, 1]$. We index trees by $i \in [0, \delta]$, where each tree $i$ yields a dividend of $X$ non-traded goods. Similarly, we index newborns by $j \in [\delta, 1]$ where each newborn $j$ is endowed with $X$ non-traded goods. Goods with indices $k \in [0, \delta]$ are produced with the non-traded inputs from the dividends of trees indexed by $k$, and goods with indices $k \in [\delta, 1]$ are produced with the non-traded inputs from the endowments of the newborns indexed by $k$. Each variety is sold by a monopolistic firm. Firms post prices $p_k$ in units of the numeraire. These differentiated varieties of goods are valued by consumers according to a standard Dixit-Stiglitz aggregator $\xi X = \left( \int_0^1 x_k^{\alpha-1} dk \right)^{\frac{1}{\alpha-1}}$, and consumption
expenditure is \( P \xi X = \int_0^1 p_k x_k dk \) where the price index is defined as \( P = \left( \int_0^1 p_k^{1-\sigma} dk \right)^{\frac{1}{1-\sigma}} \).

The resulting demand for each variety is given by \( x_k = \left( \frac{p_k}{P} \right)^{-\sigma} \xi X \).

The prices of different varieties are entirely fixed (an extreme form of sticky prices) and equal to each other, \( p_k = P \). Firms accommodate demand at the posted price, and firm profits accrue to the agent owning and supplying the corresponding non-traded input. Without loss of generality, we use the normalization \( P = 1 \). Note that because the prices of all varieties are identical, the demand for all varieties is the same. Output is demand-determined, and as a result, capacity utilization rate \( \xi \) is the same for all firms (the recession is economywide) so that \( x_k = \xi X \) for all \( k \).

Finally, a monetary authority sets a safe nominal interest rate \( i \). Because prices are rigid, this determines the real interest rates \( r^K = i \). The equilibrium of the resulting model yields exactly the same equations as those used in the previous section. The advantage of this modelling approach is that assumptions pertaining to disequilibrium concepts—such as the focus on the Keynesian regime—can be dispensed with since they are implied by equilibrium of the microfounded New-Keynesian Cash-In-Advance model.

**Money, the Zero Lower Bound and the Cashless Limit** To justify a zero lower \( r^K \geq 0 \), we introduce money into the model. We then define a cashless limit (see e.g. Woodford 2003) and show that in that limit, the economy converges to our basic model.

We represent the demand for real money balances for transactional services using a Cash-In-Advance constraint that stipulates that individuals with wealth \( w_t \) and money holdings \( m_t \) can only consume \( \min(w_t, \frac{m_t}{\varepsilon}) \). When \( i > 0 \), money is held only for transaction services. When \( i = 0 \) money is also held as a safe store of value, which competes with its transaction services. This model has no equilibrium with \( i < 0 \), because then money would dominate other safe assets. Hence there is a zero lower bound \( i \geq 0 \). The model becomes isomorphic to our basic model in the cashless limit as \( \varepsilon \to 0 \). We develop this setup next.

The demand for real money balances for transactional services is \( \varepsilon W^K_t \) and \( \varepsilon W^N_t \) for Knightians and Neutrals respectively. We assume that the money supply is \( \varepsilon M^\varepsilon \) with \( M^\varepsilon = \frac{X}{\varepsilon} \). If the bad Poisson shock hits, the government buys back part of the money stock so that the money supply is \( \varepsilon M^{\varepsilon^-} \) with \( M^{\varepsilon^-} = \mu^{-\frac{X}{\varepsilon}} \). If the good Poisson shock hits, the government issues more money so that the money supply is \( \varepsilon M^{\varepsilon^+} \) with \( M^{\varepsilon^+} = \mu^{+\frac{X}{\varepsilon}} \).

This ensures that money is adequate and output is at potential after the Poisson shock.
In order to finance this purchase, we let the government issue short term debt, the principal of which is rolled over and the interest of which is paid using a tax on the dividends of risky assets. Importantly, the ability to retire the extra money after the Poisson shock requires the government to have the fiscal capacity to raise these taxes, a key concept that we analyze at length in Section 3.

After the Poisson shock, the value $V^S$ of the safe tranches of trees is a fraction $\rho$ of the total value of assets excluding money (trees and government debt). And we therefore have

$$\theta \left( \frac{1}{\rho} V^S + \varepsilon M^\varepsilon \right) = \mu^- X,$$

i.e.

$$V^S = \rho \mu^- \frac{X}{\theta}.$$

Denoting the real money supply as $M = \varepsilon M^\varepsilon$, the equilibrium equations are now,

$$r^K V^S = \delta^S \xi X,$$

$$r V^R = (\delta - \delta^S) \xi X,$$

$$\dot{W}^K_t = -\theta W^K_t + \alpha (1 - \delta) \xi X + r^K (1 - \varepsilon) W^K_t,$$

$$\dot{W}^N_t = -\theta W^N_t + (1 - \alpha) (1 - \delta) \xi X + r (1 - \varepsilon) W^N_t,$$

$$\varepsilon (W^K_t + W^N_t) \leq \varepsilon M^\varepsilon \quad \text{with equality if} \quad r^K > 0$$

$$W^K_t + \varepsilon W^N_t \leq V^S + \varepsilon M^\varepsilon,$$

$$W^K_t + W^N_t = V^S + V^R + \varepsilon M^\varepsilon,$$

and the requirement that

$$r^K \geq 0.$$

When $r^K > 0$, we always have $\xi = 1$ as long as money is adequate $M^\varepsilon = \frac{X}{\theta}$, which we assume throughout.\(^{10}\) The interesting case for us is when $r^K = 0$, for then $\xi$ is determined from equilibrium in the safe asset market, in which part of money is used for store of value.

\(^{10}\)We can have $\xi < 1$ even when $r^K > 0$ if money is scarce $M^\varepsilon < \frac{X}{\theta}$. These effects are standard in Keynesian models and are not our focus here.
At $r^K = 0$ the supply for safe assets (safe tranches and money) is

$$\rho \mu^- (1 - \varepsilon) \frac{X}{\theta} + \varepsilon M^\varepsilon.$$

The demand for safe assets (safe tranches and money) is

$$\varepsilon (W^K + W^N) + (1 - \varepsilon) W^K,$$

which can be written as:

$$\varepsilon \frac{\xi}{\theta} X (1 - \varepsilon) W^K.$$

Replacing Knightian wealth $W^K = \alpha (1 - \delta) \frac{\xi X}{\theta}$ into this expression yields equilibrium output:

$$\xi = \frac{\rho \mu^- + \varepsilon M^\varepsilon}{\frac{\varepsilon}{1 - \varepsilon} + \alpha (1 - \delta)}.$$

which converges to the expression in the basic model in the cashless limit as $\varepsilon \to 0$. Hence the cashless limit of the New-Keynesian Cash-In-Advance model is exactly the disequilibrium model developed in Section 2.2.1.

### 2.3 Discussion of Equilibrium

It is possible to understand how a recession necessarily comes about through the double lens of the safe asset market and of the money market in the full-fledged New-Keynesian Cash-In-Advance model (and away from the stylized cashless limit). The key observation is that at the zero lower bound, there is both a transactional demand for money and a demand for money as a safe asset (store of value).

When the zero lower bound binds, there is an excess demand for safe assets and an excess demand for money at full capacity utilization. As output decreases below potential, Neutral wealth and Knightian wealth decline. This lowers the transactional demand for money as well as the demand for money as a safe asset. The excess demand for money subsides. Moreover, as money is reallocated from Neutrals to Knightians, a larger share of the money supply is used to satisfy the demand for safe assets rather than the transactional demand. Together with the reduction in Knightian wealth, this reduces the excess demand for safe assets. An equilibrium is reached when output is sufficiently below potential, that both the excess demand for money and the excess demand for safe assets are eliminated.
3 Public Debt and Quantitative Easing

Could government policy and instruments reduce the severity of the safety trap? In particular, could the government affect the supply and demand for safe assets in productive ways? In this section we show that as long as the government has fiscal capacity, in the sense of having the resources to repay its debt in the bad state of the world, the answer to the above questions is affirmative, and that the so called Quantitative Easing (QE) policies can be effective.

3.1 Public Debt and Fiscal Capacity

We start by introducing public debt and discussing the role of public purchases and sales of such debt. To isolate the insights of this section we assume for now that private trees cannot be tranched at all ($\rho = 0$), and hence cannot produce safe assets by themselves.

The government taxes dividends, $\delta X$. The tax rate is $\tau^+$ after the good Poisson shock occurs, $\tau^-$ after the bad Poisson shock occurs, while the tax rate before the Poisson event is set to a value $\tau$ that satisfies the government flow budget constraint. The government issues a fixed amount of risk-free bonds that capitalize future tax revenues and pays a variable rate $r^K_t$. The proceeds of the sales of these bonds are rebated lump-sum to agents at date 0. Hence in this model government debt acts exactly like tranching, with $\tau^-$ playing the role of $\rho$.

Let the value of public debt be given by $D$, then we have

$$D = \tau^- \mu^\delta X.$$

The equilibrium is described by the following equations:

$$r^K D = \tau \delta X,$$

$$r V = \delta (1 - \tau) X,$$

$$\dot{W}^K_t = -\theta W^K_t + \alpha (1 - \delta) X + r^K W^K_t.$$

\footnote{It is the latter feature that makes this debt “short-term,” since its value remains constant over time as its coupons vary with the riskless rate. In the previous version of this paper (Caballero and Farhi 2012), we introduce long-term public debt and study Operation Twist (OT) policies that swap long-term debt for short-term debt.}
\[ \dot{W}_t^N = -\theta W_t^N + (1 - \alpha) (1 - \delta) X + r W_t^N. \]

\[ W_t^K + W_t^N = D + V, \]

\[ W_t^K \leq D \text{ and } r^K \leq r. \]

At a steady state of the constrained regime we have

\[ W^K = D = \tau^- \mu^- \frac{X}{\theta}, \quad W^N = V = (1 - \tau^- \mu^-) \frac{X}{\theta} \]

and

\[ \delta \tau = \tau^- \mu^- - \alpha (1 - \delta), \]

\[ r^K = \delta \theta - (1 - \delta) \theta \frac{\alpha - \tau^- \mu^-}{\tau^- \mu^-}, \]

\[ r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \tau^- \mu^-}{1 - \tau^- \mu^-}. \]

The economy is in the constrained regime if and only if \( \alpha > \tau^- \mu^- \), which we assume. The safety premium is then given by

\[ r - r^K = \theta (1 - \delta) \frac{\alpha - \tau^- \mu^-}{\tau^- \mu^- (1 - \tau^- \mu^-)} \geq 0. \]

The notion of fiscal capacity is crucial. In the model, we necessarily have \( \tau^+ \mu^+ = \tau^- \mu^- \), which implies that \( \tau^+ < \tau^- \). For this reason, it is natural to expect fiscal constraints to be more binding after the bad Poisson shock than after the good Poisson shock. This is why we adopt \( \tau^- \), a measure of the ability of the government to raise tax revenues after the bad Poisson shock, as our measure of fiscal capacity.

The supply of safe assets comes entirely in the form of short-term public debt. The supply of the latter is determined by a notion of fiscal capacity, as measured by \( \tau^- \). The larger fiscal capacity, the more short-term debt the government can issue, the larger the supply of safe assets and the lower the safety premium.

If the economy is in a safety trap where the safe interest rate is fixed at zero and output is below potential with \( \xi < 1 \), then in increasing public debt from \( D \) to \( \hat{D} > D \) stimulates output, increasing \( \xi \) to \( \hat{\xi} \) where

\[ \hat{\xi} = \frac{\hat{D}}{D} \xi > \xi. \]

Increasing the supply of public debt to \( \hat{D} \) requires the government to have spare fiscal
capacity, that is to have the ability to raise more taxes after the bad Poisson shock

\[ \hat{\tau}^- = \frac{\hat{D}}{D} \tau^- > \tau^- . \]

The government’s ability to expand this supply either because it has excess fiscal capacity or because it can implicitly tranche assets in a way the private sector cannot, which gives it a comparative advantage in the production of safe assets. This result does not require the extreme assumption \( \rho = 0 \) that we have made solely to simplify the exposition. Indeed, the comparative advantage of the government in the production of safe assets is present as long as there are some limits to the tranche of private assets (\( \rho < 1 \)). It is only when there are no limits to the tranche of private assets (\( \rho = 1 \)) that this comparative advantage disappears and that the supply of public debt becomes irrelevant—a form of Ricardian equivalence.\(^{12}\)

**Fiscal capacity limits.** In the rest of the paper, we investigate policy options for the government when it is against its long-run fiscal capacity, with limited ability to increase future taxes. For this reason, we fix \( \tau^- \) and treat it as a hard fiscal capacity constraint.

**Remark 2** Issuing money while at the zero bound is equivalent to issuing short-term bonds, and both are constrained by the long-term fiscal capacity of the government. Indeed, after the bad Poisson shock, the government must raise taxes to retire the additional money that it has issued before the Poisson event. See the appendix for a detailed exposition of these arguments.

\(^{12}\)This mechanism has some commonality with the idea in Holmström and Tirole (1998) that the government has a comparative advantage in providing liquidity. In their model this result arises from the assumption that some agents (consumers in their model) lack commitment and hence cannot borrow because they cannot issue securities that pledge their future endowments. This can result in a scarcity of stores of value. The government can alleviate this scarcity by issuing public debt and repaying this debt by taxing consumers. The proceeds of the debt issuance can actually be rebated to consumers. At the aggregate level, this essentially relaxes the borrowing constraint of consumers: They borrow indirectly through the government. The comparative advantage of the government in providing liquidity arises from its unique regalian taxation power: It is essentially better than private lenders at collecting revenues from consumers. In the case where consumers face no commitment problems in the securitization of their future income, there are no borrowing constraints, public debt is irrelevant, Ricardian equivalence is recovered and the comparative advantage of the government disappears. Hence the imperfect ability of consumers to securitize their future income plays a similar role in the theory of Holmström and Tirole (1998) as the assumption of imperfect trancheability in ours.
3.2 Quantitative Easing

We remind the reader that we are using the term QE loosely to encompass policies that swap risky assets for safe assets such as QE1, LTRO, and many other lender of last resort central bank interventions. We model QE as follows. The government purchases trees and issues additional short-term debt. Let $\hat{\beta}^g$ be the fraction of the trees purchased by the government. Let $\hat{D}$ be the value of government debt and let $\hat{\tau}^-$ be the new value of taxes after the bad Poisson shock (which must satisfy $\hat{\tau}^- \leq \tau^-$). We continue to assume that the stock of short-term debt is unchanged before and after the Poisson event. We have

$$\hat{D} = \hat{\tau}^-(1 - \hat{\beta}^g)\mu\frac{X}{\theta} + \hat{\beta}^g\mu\frac{X}{\theta}.$$

As long as

$$\hat{\tau}^-(1 - \hat{\beta}^g) + \hat{\beta}^g > \tau^-,$$

the safe asset shortage is alleviated by this policy: $r^K$ increases, $r$ decreases, and the safety premium shrinks.

Here QE works not so much by removing risky private assets from private balance sheets, but rather by injecting public assets into private balance sheets. In other words, QE works by increasing the supply of safe assets. The government can expand this supply even when it does not have excess fiscal capacity because it can implicitly tranch assets in a way the private sector cannot, which gives it a comparative advantage in safety transformation. The key difference between QE and simply issuing more public debt is what the government does with the proceeds from the debt issuance. In QE, the government uses the proceeds to purchase private risky assets instead of simply rebating them to private agents. By doing so, it is able to run up its debt without stretching its future fiscal capacity.

If the economy is in a safety trap where the safe interest rate is fixed at zero and output is below potential with $\xi < 1$, then QE acts by stimulating output, increasing the value of $\xi$.
Another major policy tool advocated in the context of zero lower bound of interest rates is Forward Guidance (commitment to low future interest rates once the economy recovers). However, in this section we show that when the reason for this low interest rate is a shortage of safe assets, the policy is ineffective.

The reason is that only policy commitments that support future bad states work in safety traps. This is a higher level of requirement than in the standard New-Keynesian liquidity-trap mechanism where any future wealth increase has the potential to stimulate the economy, including wealth created after the recovery is completed.

We illustrate this point with an example of forward guidance policy that would work in a New-Keynesian liquidity trap environment but not in a Safety Trap. Since public debt is not key to our main concern here, we temporarily revert to our model in Section 2 where there are only private assets.

We introduce two modifications to that model. First, we temporarily (only for this section) assume a non-zero intensity of the good Poisson shock $\lambda^+ > 0$. Second, we allow agents to produce $\zeta > 1$ units of output per unit of input. However, we imagine that there is a large utility loss from doing so.

This model functions similar to that in Section 2. It features the possibility of a safety

\[ \hat{\xi} = \frac{\hat{D}}{D} \xi > \xi. \]

4 Forward Guidance

In general, QE might require a transition phase where the government raises taxes before the Poisson event in order to gradually acquire those assets. The analysis in the main text assumes that this adjustment has taken place and examines the consequences of the eventual buildup of such a portfolio. After this portfolio buildup phase, we assume that the government rebates the excess of the dividends that it perceives on risky assets over and above its needs to finance the interest on debt to the Neutrals (either to the newborns in the form of a lump-sum rebate or in the form or a subsidy on the dividends on risky assets).

In certain circumstances, it is possible to design QE policies that do not require a buildup phase with increased taxes before the Poisson event, i.e. such that the debt issuance more than covers the asset purchases. The condition is $\hat{\beta}^g \hat{\xi} X^g \leq \hat{D} - D$, where $\hat{\xi} = \frac{\hat{D}}{D} \xi$, which can be shown after some manipulation to boil down to

\[ \xi \leq \frac{\tau^-}{\hat{\tau}^- (1 - \hat{\beta}^g) + \hat{\beta}^g} \frac{\hat{D} \xi - \hat{\tau}^- (1 - \hat{\beta}^g) + \hat{\beta}^g - \tau^-}{\beta^g} \mu^- . \]

Hence if the economy is depressed enough, then it is possible to build up a QE portfolio without immediately raising taxes, that stimulates output.
trap with \( r^K = 0 \) and a recession. Indeed \( \xi \) is determined by the exact same equation. The only difference is in the (risky) interest rate \( r \). The interest rate \( r \) is now determined by the following set of equations (and \( \lambda^+ \) only enters the last of these equations):

\[
V^R = \left( \xi - \rho \mu^- \right) \frac{X}{\theta},
\]

\[
V^S = \rho \mu \frac{X}{\theta},
\]

\[
rV^R = \xi \delta X + \lambda^+ \left( \mu^+ - \xi \right) \frac{X}{\theta}.
\]

This yields

\[
r = \frac{\xi \delta \theta + \lambda^+ \left( \mu^+ - \xi \right)}{\xi - \rho \mu^-}.
\]

In New-Keynesian models of the liquidity trap (see e.g. Krugman 1998, Eggertsson and Woodford 2003, and Werning 2012), committing to keep the interest rate low in the future once the economy recovers (after the good Poisson shock) stimulates the economy—a policy often referred to as forward guidance. The latter works by creating a boom in the future, which raises current demand through a combination of a wealth effect (higher income in the future) and substitution effect (lower real interest rates because of inflation). Our model shuts down the latter mechanism, rendering the former ineffective since what matters is the perceived wealth of Knightians, not that of Neutrals.

In our model a commitment to low interest rates after the good Poisson shock ends up being a failed attempt to stimulate the economy by increasing the value of risky assets. In fact, it leaves economic activity and the values of safe and risky assets unchanged. Its only effect is to increase the risky interest rate \( r \). Consider the following policy: Suppose that the good Poisson shock occurs at \( \tau \). After the good Poisson shock, the central bank stimulates the economy by setting the interest rate \( i_t \) below the natural interest rate \( \delta \theta \) until \( \tau + T \), at which point it reverts to setting the nominal interest rate equal to the natural interest rate \( i = \delta \theta \). For \( t > \tau + T \), output is equal to potential so that \( \zeta_t = 1 \). For \( \tau \leq t \leq t + T \), output is above potential, and capacity utilization satisfies a simple differential equation

\[
\frac{\dot{\zeta_t}}{\zeta_t} = i_t - \delta \theta \leq 0,
\]

with terminal condition

\[
\zeta_{\tau+T} = 1.
\]
The solution is
\[ \zeta_t = e^{\int_{t}^{t+T} (\delta \theta - i_s) ds}. \]

By lowering interest rates, the central bank creates a temporary boom after the Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from
\[ \mu^+ \frac{X}{\theta} \]
to
\[ \mu^+ \zeta_T \frac{X}{\theta} > \mu^+ \frac{X}{\theta}. \]

Let us now work backwards to understand the effects of this policy before the Poisson event, while the economy is in a safety trap. The only effect of this policy is to increase the interest rate \( r \) during the safety trap to
\[ r = \frac{\xi \delta \theta + \lambda^+ (\mu^+ \zeta_T - \xi)}{\xi - \rho \mu^-}. \]

This increase in the interest rate is such that the value of risky assets (and hence the wealth of Neutrals) is unchanged, despite the fact that their value after a good Poisson shock has increased. Importantly, the increase in \( r \) is orthogonal to the safe-asset shortage problem. Since the policy leaves the supply of safe assets unchanged, it does not expand output, which remains depressed by exactly the same factor \( \xi \).

A safety trap is addressed more directly by committing to provide support during bad rather than good times, as would be the case of a commitment to lower interest \( i_t \) rates after the bad Poisson shock. By setting the nominal interest rate \( i_t \) below the natural interest rate \( \delta \theta \) after the bad Poisson shock, monetary authorities stimulate the economy and inflate the value of safe assets to
\[ \hat{V}^S = \rho \mu^- \frac{\zeta_T}{\theta} X, \]

There is one caveat to this conclusion. We have assumed that prices are entirely rigid. If prices could adjust gradually over time, then forward guidance could regain some kick: A commitment to lower interest rates after the good Poisson shock could increase inflation while the economy is in a safety trap. This would lower the safe interest rate \( r^K \) and mitigate the recession.

The same comments apply to the unconventional tax policies considered by Correia, Farhi, Nicolini and Teles (2012), which here could simply take the form of an increasing path of sales taxes—say through a sales tax holiday—which would create inflation in consumer prices and hence reduce \( r^K \).

Another example is the OMT (outright monetary transactions) program established by the ECB in late 2012, which had an immediate impact on the Eurozone risk perception.
where
\[ \zeta_T = e^{\int_T^{t+T}(\delta\theta - i_s)ds}. \]

This mitigates the recession in the safety trap by raising \( \xi \) to \( \xi \zeta_T > \xi \) (the analysis is almost identical to that of a monetary stimulus after the good Poisson shock explained above).\(^{17,18}\)

However, it is natural to question whether monetary authorities would have the ability to lower interest rates in that state. If indeed the bad state happens to coincide with yet another safety or liquidity trap, monetary authorities could find themselves unable to deliver a lower interest rate. Perhaps a more realistic policy option would be a commitment by the authorities to buy up safe assets at an inflated price after the Poisson shocks—a form of government (central bank?) put. A commitment to buy up safe private assets at an inflated value \( \sigma \rho \mu^{-\frac{X}{\theta}} > \rho \mu^{-\frac{X}{\theta}} \) would mitigate the recession and increase the value of \( \xi \) to \( \hat{\xi} \) where
\[ \hat{\xi} = \sigma \xi > \xi. \]

It could be carried out by monetary authorities but it does require spare fiscal capacity (in the form of taxes or seigniorage). This kind of public insurance policy can potentially play a crucial role in a safety trap.\(^{19}\)

5 Bubbles

The very low interest rates that characterize a safety trap raises the issue of whether speculative bubbles may emerge, and whether these can play a useful role through their wealth effect, as it may happen in economies experiencing a liquidity trap (see Section 6).

We show that bubbles can indeed arise in safety traps, but that their emergence has no or little impact on economic activity. This is simply because bubbles are risky assets, and hence do not alleviate safe asset shortages.

\(^{17}\)Note we could just as well have used the model with public debt. The central banker’s put works by increasing both the public and private sectors’ ability to provide safe assets.

\(^{18}\)Just like in New Keynesian models of the liquidity trap, and to the extent that they are possible at all, these forms of policy commitments raise time-consistency issues: Their efficacy hinges on the ability of monetary authorities to carry out credible commitments.

\(^{19}\)See, e.g., Caballero and Kurlat (2010) for a proposal to increase the resilience of the financial system in a shortage of safe assets environment. Also, see Brunnermeir et al (2012) for a related proposal in the context of the current Euro crisis.
5.1 Growth

It is well understood in the rational bubbles literature that the growth rate of the economy is a key determinant of the possibility and size of bubbles. We therefore generalize our model by allowing for an arbitrary growth rate \( g > 0 \).

At every point in time, there is a mass \( X_t \) of trees. A mass \( \dot{X}_t = gX_t \) of new trees are created, which are claims to a dividend of \( \delta \) units of goods at every future date until a Poisson event occurs, at which point the dividend jumps permanently to \( \delta \mu^+ \) if the good Poisson shock takes place and to \( \delta \mu^- \) if the bad Poisson shock takes place. For reasons that will appear clear below, we assume that new trees are initially endowed to Neutral newborns.\(^{20}\)

A fraction \( \rho \) of these new trees can be tranched into a safe and a risky component.

We some abuse of notation, we suppress time indices throughout. Hence we write \( X, V^S, V^R, W^K, W^N, W \) for \( X_t, V^S_t, V^R_t, W^K_t, W^N_t, W_t \). All these variables grow at rate \( g \) in equilibrium. We also write \( r^K, r, \delta^S \) for \( r^K_t, r_t, \delta^S_t \). All these variables are constant in equilibrium.

We focus on the constrained regime where \( W^K = V^S = \frac{\rho \mu^- X}{\theta} \) and \( r > r^K \). This occurs as long as

\[
\frac{\alpha - \rho \mu^-}{\rho \mu^-} > \frac{g}{(1 - \delta) \theta}.
\]

The equilibrium equations in the constrained regime are

\[
\begin{align*}
r^K V^S &= \delta^S X, \\
r V^R &= (\delta - \delta^S) X, \\
g W^K &= -\theta W^K + \alpha (1 - \delta) X + r^K W^K, \\
g W^N &= -\theta W^N + (1 - \alpha) (1 - \delta) X + g V + r W^N, \\
W^K + W^N &= V^S + V^R, \\
W^K &= V^S = \frac{\rho \mu^- X}{\theta}.
\end{align*}
\]

\(^{20}\)If new trees are endowed in equal proportions to Knightians and Newborns, then bubbles do stimulate the economy in a safety trap because they reduce the value of the new trees endowed to Knightian newborns and hence reduce the growth rate of Knightian wealth. Endowing the new trees exclusively to Neutrals shuts down this somewhat artificial effect of bubble on safe asset demand.
We then have
\[\delta S = g\rho\mu - \delta\rho\mu - (1 - \delta)\left(\alpha - \rho\mu\right),\]
\[r^K = g + \delta\theta - \theta \left(1 - \delta\right)\frac{\alpha - \rho\mu}{\rho\mu},\]
\[r = g + \delta\theta - \frac{g}{1 - \rho\mu} + (1 - \delta)\theta\frac{\alpha - \rho\mu}{1 - \rho\mu}.
\]

Now suppose that we are in a safety trap where
\[g + \delta\theta - \theta \left(1 - \delta\right)\frac{\alpha - \rho\mu}{\rho\mu} < 0,\]
then as in Section 2, we have a recession determined by
\[0 = g + \delta\theta - (1 - \delta)\theta\frac{\alpha - \rho\mu}{\rho\mu},\]

5.2 Bubbles

We now extend the model to allow for bubbles. We assume that there is a bubble \(B_t\) which grows at rate \(\dot{B}_t = gB_t\) until the bad Poisson shock occurs, at which point the bubble crashes to zero.\(^{21}\) Whether the bubble crashes or not after the good Poisson shock is irrelevant for our analysis. Again, we suppress the dependence on time and write \(B\) for \(B_t\).

As above, we focus on the constrained regime where \(W^K = V^S = \frac{\rho\mu - X}{\theta}\) and \(r > r^K\) (which occurs under the same conditions on parameters as in the bubbleless equilibrium analyzed in Section 5.1 above). Bubbles can arise as long as the risky interest rate \(r\) is less

\(^{21}\)We can also analyze a model where bubbles do not crash to zero after the bad Poisson shock. This requires that \(\delta\theta \leq g\). After the bad Poisson shock, the (stationary) bubble \(B^{post} > 0\) is given by \(B^{post} = \frac{\mu - X}{\rho} \left(1 - \frac{\delta\theta}{\theta}\right)\), and the value of assets is given by \(V^{post} = \frac{\mu - X}{\rho} \frac{\delta\theta}{\xi}\).

We assume that the same fraction \(\rho\) of trees and bubbles can be tranch into a safe and a risky part. The supply of safe assets is still \(V^S = \frac{\rho\mu - X}{\theta}\), and the equilibrium equations are unchanged. After the bad Poisson shock, the bubble perfectly crowds out the value of trees, so that the total supply of safe assets is unchanged. As a result, the conclusion that bubbles are irrelevant for economic activity is robust to this variant of the model.

Things would be different if a larger fraction of bubbles than trees could be tranch. Then bubbles would expand the supply of safe assets and stimulate economic activity in a safety trap.
than $g$ in the bubbleless equilibrium:

$$ g > g + \delta \theta - \frac{g}{1 - \rho \mu^-} + (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{1 - \rho \mu^-}. $$

The equilibrium equations in the constrained regime in the presence of bubbles are then

$$ r^K V^S = \delta^S X, $$

$$ r V^R = (\delta - \delta^S) X, $$

$$ r = g, $$

$$ g W^K = -\theta W^K + \alpha (1 - \delta) X + r^K W^K, $$

$$ g W^N = -\theta W^N + (1 - \alpha) (1 - \delta) X + g V + r W^N, $$

$$ W^K + W^N = V^S + V^R + B, $$

$$ W^K = V^S = \frac{\rho \mu^- X}{\theta}. $$

The solutions for $\delta^S$ and $r^K$ are exactly the same as in the bubbleless equilibrium analyzed in Section 5.1 above. The value of $r$ is now higher at $g$, and the equilibrium bubble is given by

$$ B \frac{X}{\theta} = \frac{1 - \rho \mu^-}{g} \left[ g - \left( g + \delta \theta - \frac{g}{1 - \rho \mu^-} + (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{1 - \rho \mu^-} \right) \right]. $$

Now suppose that we are in a safety trap. Then we have a recession with $\xi < 1$ determined exactly as in the bubbleless equilibrium analyzed in Section 5.1 above. Because bubbles are risky, they do not increase the supply of safe assets which is at the root of the safety trap. Actually, bubbles even fail to increase the total supply of assets since they perfectly crowd out other risky assets through an increase in the risky interest rate $r$.

### 6 Safety Traps versus Liquidity Traps

As we have argued above, the shortage of safe assets environment and its safety trap shares many features in common with the Keynesian liquidity trap, however there are also important differences between them. In this section we highlight some of these differences by comparing the impact of several macroeconomic policies in these environments.
We consider the following simple model of a liquidity trap. It is a version of our model where the economy is in the unconstrained regime so that \( r = r^K \) (which means the distinction between risky and riskless assets is irrelevant). We make one modification: The possibility of the bad shock is \( \lambda^- > 0 \) rather than studying the limit \( \lambda^- \to 0 \). This is necessary for the interest rate \( r \) to reach zero (and it cannot go below zero because of the zero lower bound on nominal interest rates). We maintain our focus on the limit \( \lambda^+ \to 0 \) for now.\(^{22}\)

The equilibrium equations are

\[
    rV = \delta X + \lambda^- \left( \frac{\mu^- X}{\theta} - V \right),
\]

\[
    0 = -\theta W + (1 - \delta) X + rW - \lambda^- \left( \frac{\mu^- X}{\theta} - V \right) + gV,
\]

\[
    V = W.
\]

As long as the zero bound is not binding, we have \( V = W = \frac{X}{\theta} \) and

\[
    r = \delta \theta - \lambda^- (1 - \mu^-) > 0.
\]

When the zero bound \( r = 0 \) binds, the economy enters a recession \((\xi < 1)\) where \( \xi \) is determined by the requirement that \( r = 0 \):

\[
    0 = \delta \theta - \lambda^- \left(1 - \frac{\mu^-}{\xi}\right),
\]

i.e.

\[
    \xi = \frac{\mu^-}{1 - \frac{\delta \theta}{\lambda^-}}.
\]

The recession originates from a scarcity of assets (stores of value). It is more severe, the worse the expected bad shock (the lower is \( \mu^- \)), the more likely is the bad shock (the higher \( \lambda^- \)), the higher the propensity to save (the lower \( \theta \)), and the lower is the ability of the economy to create assets that capitalize future income (the lower is \( \delta \)).

We can use this model to examine the effects of the same policies that we have considered in the context of the safety trap: balance sheet policies (QE), fiscal policy (redistribution and government spending), and monetary policy commitments (forward guidance). We can

\(^{22}\)We relax this assumption later when we analyze forward guidance in Section 6.2.
also examine the possibility and the consequences of bubbles.

6.1 Public Debt and QE

We start with public debt and QE. We introduce public debt in the model exactly as in Section 3. The key point is that public debt issuances and QE have no effect at all on the recession $\xi$ in the liquidity trap model. This precise irrelevance result relies on our assumption that dividends are taxed while the endowment of newborns (wages) is not. As a result, public debt issuances and QE simply reshuffle the fraction of dividends that accrues to private asset holders and the fraction of dividends that is absorbed by taxes to pay interest on debt of various maturities. This assumption essentially renders our framework Ricardian, despite the fact that we have overlapping generations of agents.\footnote{If we allowed the endowments of newborns to be taxed, then public debt issuances and QE could have some non-Ricardian effects, depending on exactly how these taxes are levied, and hence affect economic activity in a liquidity trap. For example, Kocherlakota (2013) studies a non-Ricardian environment where issuing public debt can stimulate the economy in a liquidity trap.} These conclusions about the irrelevance of public debt issuances and QE in liquidity traps must be contrasted with those reached in Section 3 for safety traps. The effects of public debt issuances and QE in safety traps rely entirely on the (assumed) superior ability of the government to address a form of market incompleteness—the difficulty to isolate safe from risky assets.

6.2 Forward Guidance

We now turn to monetary policy commitments. To do so, we introduce the possibility of a good shock as in Section 4. As in Section 4, we temporarily (only for this section) assume that $\lambda^+ > 0$. In a liquidity trap $r = 0$, the recession is now determined by

$$0 = \delta \theta - \lambda^- \left( 1 - \frac{\mu^-}{\xi} \right) - \lambda^+ \left( 1 - \frac{\mu^+}{\xi} \right),$$

i.e.

$$\xi = \frac{\lambda^- \mu^- + \lambda^+ \mu^+}{1 - \frac{\delta \theta}{\lambda^- + \lambda^+}}.$$

Consider the following policy: Suppose that the good Poisson shock occurs at $\tau$. After the good Poisson shock, the central bank stimulates the economy by setting the interest rate $i_t$ below the natural interest rate $\delta \theta$ until $\tau + T$, at which point it reverts to setting the
nominal interest rate equal to the natural interest rate \( i = \delta \theta \). For \( t > \tau + T \), output is equal to potential so that \( \zeta_t = 1 \). For \( \tau \leq t \leq t + T \), output is above potential, and capacity utilization satisfies a simple differential equation

\[
\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta \theta \leq 0,
\]

with terminal condition

\[
\zeta_{\tau+T} = 1.
\]

The solution is

\[
\zeta_t = e^{\int_{\tau + T}^{t} (\delta \theta - i_s) ds}.
\]

By lowering interest rates, the central bank creates a temporary boom after the good Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from

\[
\mu^+ \frac{X}{\theta}
\]

to

\[
\mu^+ \zeta_\tau \frac{X}{\theta} > \mu^+ \frac{X}{\theta}.
\]

This policy alleviates the recession while the economy is in a liquidity trap, pushing \( \xi \) to \( \hat{\xi} \) where

\[
\hat{\xi} = \xi \left( \frac{\lambda^-}{\lambda^- + \lambda^+} \mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+} \zeta_\tau \mu^+ \right) > \xi.
\]

Basically, committing to low interest rates after the good Poisson shock increases the value of assets while the economy is in the liquidity trap. This wealth effect increases demand and mitigates the recession. Forward guidance works by alleviating the asset shortage that is at the root of the recession.\(^{24}\)

Forward guidance trades off a future boom against a mitigation of the current recession and hence raises time-consistency issues. Because of the utility loss that comes with the boom, monetary authorities might be tempted to renege on their commitment to keep interest rates low when the time comes to deliver on this promise. Nevertheless, our main point here is that the effectiveness of forward guidance in liquidity traps is to be contrasted with its

\(^{24}\)We should emphasize that one channel through which forward guidance works in traditional New Keynesian models of the liquidity trap—reducing real interest rates by creating inflation—is absent from this model because we have assumed that prices are rigid. Instead, forward guidance works entirely through a wealth effect by boosting asset values, wealth, and hence spending.
relative ineffectiveness in safety traps.

6.3 Bubbles

To consider the possibility and consequences of bubbles, we generalize the environment to allow for growth. As in Section 5, we assume that the number of trees $X_t$ grows over time at rate $\dot{X}_t = gX_t$, and that the new trees are initially endowed to newborns. In the bubbleless equilibrium, the value of the interest rate $r$ is actually independent of $g$. It is still given by

$$r = \delta \theta - \lambda^- (1 - \mu^-),$$

and in a liquidity trap where $\delta \theta - \lambda^- (1 - \mu^-) < 0$, the recession is characterized by the same equation

$$\xi = \frac{\mu^-}{1 - \frac{\delta \theta}{\lambda^+}}.$$

We now assume that there is a bubble $B_t$ which grows at rate $\dot{B}_t = gB_t$ until the bad Poisson shock occurs, at which point the bubble crashes to zero. Whether or not the bubble crashes after the good Poisson shock is irrelevant for our analysis. Such bubbles are possible as long as

$$\delta \theta + \lambda^- \mu^- < g.$$

The interest rate is given by

$$r = g - \lambda^-,$$

and the value of the bubble is given by

$$B = \frac{g - (\delta \theta + \lambda^- \mu^-) X}{\theta}.$$

Now suppose that we are in a liquidity trap in the bubbleless equilibrium ($\delta \theta - \lambda^- (1 - \mu^-) < 0$) with a recession given by $\xi = \frac{\mu^-}{1 - \frac{\delta \theta}{\lambda^+}} < 1$. Assume in addition that

$$g - \lambda^- > 0.$$

Then we automatically have $\delta \theta + \lambda^- \mu^- < g$, so that bubbles are possible. Moreover, there is no recession in the bubbly equilibrium. The bubble increases the total supply of assets, and increases the interest rate $r$ from zero to $g - \lambda^- > 0$. 31
This potency of bubbles in stimulating the economy in liquidity traps is in sharp contrast with their ineffectiveness in safety traps we emphasized in Section 5.

7 Final Remarks

In this paper we provided a model that captures some of the most salient macroeconomic consequences and policy implications of a safety trap. We highlighted the similarities and differences with liquidity traps. However, as we mentioned in the introduction, we do not see safety and liquidity traps as mutually exclusive options. On the contrary, one way of thinking about a safety trap is as a more severe form of a liquidity trap, as the bottleneck is concentrated in a set of assets that are naturally more difficult for economies to produce (safe assets). Under this perspective, it is reasonable to expect for safety trap aspects to dominate during the most severe phases of crises and deep recessions, and to gradually mutate into a liquidity crisis as the (slow) recovery evolves.

Given the faster growth of safe-asset-consumer economies than that of safe-asset-producer economies, absent major financial innovations, the shortage of safe assets is only likely to worsen over time, perhaps as a latent factor during booms but reemerging in full force during contractions. It is our conjecture that the shortage of safe assets will remain as a structural drag, lowering safe rates, increasing safety spreads, straining the financial system, and weakening the effectiveness of conventional monetary policy during contractions.
References


A Appendix: Helicopter Money and Fiscal Capacity

One may wonder why not directly addressing the shortage of safe assets directly by printing money. Here we show that this is entirely equivalent to issuing public debt and hence it is subject to the same fiscal constraints.

Let us start backwards. In order to buy back the money stock after the bad Poisson shock, the government undertakes an open market operation immediately after the realization of the shock, swapping the extra supply of money $M^\varepsilon - M^{\varepsilon-}$ for debt $D$ where

$$D = M^\varepsilon - M^{\varepsilon-},$$

the interest on which it finances by a tax $\tau^-$ on the dividends of trees, where

$$D = \tau^- \mu^- \frac{X}{\theta}.$$ 

Consider what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but maintains an adequate supply of money $M^{\varepsilon-} = \mu^- X$ after the bad Poisson shock. This stimulates output to

$$\hat{\xi} = \rho \mu^- + \varepsilon \hat{M}^\varepsilon \frac{\theta}{X} \xi > \xi.$$ 

This is exactly the same effect as that which would be achieved by issuing additional short-term debt in the amount $\varepsilon(\hat{M}^\varepsilon - M^\varepsilon)$, which is intuitive given that money and short-term debt are perfect substitutes at the zero lower bound. And exactly like this debt issuance policy, it requires that the government be able to increase taxes $\hat{\tau}^- > \tau^-$ after the bad Poisson shock where

$$(\hat{\tau}^- - \tau^-) \mu^- \frac{X}{\theta} = \varepsilon(\hat{M}^\varepsilon - M^\varepsilon).$$

Consider next what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but keeps an excessive supply of money $\hat{M}^{\varepsilon-} > \mu^- X$ after the Poisson shock occurs (perhaps because it doesn’t have the fiscal capacity to retire the extra money), while maintaining an interest rate of $\delta \theta$. In this case output is above potential at $\zeta \mu^- X$ where

$$\zeta = \hat{M}^{\varepsilon-} \frac{\theta}{\mu^- X}.$$
Hence the value of safe assets is increased to
\[ \hat{V}^S = \frac{\rho \mu^- \zeta X (1 - \varepsilon)}{\theta}, \]
resulting in a mitigation of the recession before the Poisson shock when the economy is in a safety trap, increasing the value of \( \xi \) to \( \hat{\xi} \) where
\[
\hat{\xi} = \frac{\rho \mu^- \hat{M}^\varepsilon - \frac{\theta}{\mu^- X} + \frac{\varepsilon}{1-\varepsilon} \hat{M}^\varepsilon \frac{\theta}{X}}{\rho \mu^- + \frac{\varepsilon}{1-\varepsilon}} \xi > \frac{\rho \mu^- + \frac{\varepsilon}{1-\varepsilon} \hat{M}^\varepsilon \frac{\theta}{X}}{\rho \mu^- + \frac{\varepsilon}{1-\varepsilon}} \xi > \xi.
\]
Thus issuing money while the economy is in a safety trap and not taking it away when the economy exits the safety trap further mitigates the recession associated with the safety trap. However, this extra effectiveness is no free lunch, as it comes with the important cost of excessively stimulating the economy when it exits the safety trap.