Trade and Informality in the Presence of Labor Market Frictions and Regulations*

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1 Introduction

Over the past 30 years, most developing countries have entered the world market. This process was most remarkable in Latin America: during the 1980's and early 1990's, many countries drastically cut tariff and non-tariff barriers and substantially increased their participation in international trade. These changes are often celebrated as contributing to economic growth, efficiency and overall welfare. However, those who oppose globalization argue that its benefits are not evenly distributed and that it may generate adverse effects on inequality and labor market performance in these countries. Amongst the potential adverse effects from globalization, the increase in the size of the informal sector is often pointed as a particularly important one (Goldberg and Pavcnik, 2007; Harrison et al., 2003; Harrison and Scorse, 2010).

On the firm side, informality implies that firms do not comply with taxes nor the relevant regulations (e.g. labor laws). This can be harmful to the economy for two main reasons. First, it implies substantial tax evasion thus hindering fiscal capacity and the provision of public goods. Second, it might entail substantial misallocation of resources

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and hamper growth, as non-productive firms can survive by evading taxes and avoiding compliance with labor market regulations. On the worker side, one can broadly define informality in two ways: The first defines a worker as informal if she does not have permanent and stable employment associated with benefits such as health and social security. The second defines a worker as informal if, in addition to not receiving benefits, she is invisible to the tax authorities and her employer illegally evades labor market regulations (including minimum wages and firing rules). The first definition has become relevant even in developed countries in recent years with the emergence of companies such as Uber, Taskrabbit or Airbnb. The second definition applies primarily to developing countries where informality, and the tax evasion associated with it, is a first-order issue and has been shown to be associated with low productivity and a barrier to growth.

Although a substantial share of the labor force in developing countries is employed informally (for example, in Latin America, this share falls between 35% in Chile and 80% in Peru), trade models have typically abstracted from informality. Recent work has shown, in different contexts, that shifts into or out of informality and non-employment constitute important margins of labor market adjustment to trade (Dix-Carneiro and Kovak (2017a) and McCaig and Pavcnik (2018)). In addition, there is evidence that the magnitude of these effects depends on the intensity with which labor market regulations are enforced (Ponczek and Ulyssea (2018)). These facts imply that understanding and measuring the labor market and welfare effects of globalization within a model of trade with informality, unemployment and regulations is a first order question.

This paper studies the labor market and welfare effects of trade in an environment with burdensome regulations but with imperfect enforcement or monitoring of these regulations. The imperfect enforcement of regulations gives incentives for firms to operate in the informal sector. We anticipate that a trade-induced reallocation of resources towards the informal sector can have opposing effects on welfare. On one hand, it would constitute a reallocation of resources towards a less productive sector that does not contribute to the provision of public goods. This suggests that an expansion of the informal sector hurts welfare. On the other hand, recent work by Dix-Carneiro and Kovak (2017a) and Ponczek and Ulyssea (2018) suggests that the informal sector served as a fallback sector to trade-displaced workers. These papers focused on the Brazilian trade liberalization of the 1990s and exploited a difference-in-difference framework. Their results suggest that had the enforcement of labor market regulations been stricter in Brazil, the effect of import competition on trade-displaced workers' employment outcomes could have been much more adverse than it actually was. These facts motivate the set of questions we

address in this paper: How do labor market regulations and policies directed towards the informal sector influence the labor market effects of globalization? More specifically, how are the labor market effects of trade shaped by the constellation of firing costs, minimum wages and enforcement of labor market regulations? What is the impact of the "costs of formality", such as payroll and sales taxes and the bureaucratic cost that comes with being a formal firm?

To shed light on these questions, we build on Cosar et al. (2016) and develop a structural equilibrium model with heterogeneous firms that choose whether to operate in the formal or in the informal sector. The model features a rich institutional setting, where formal firms must comply with minimum wages, and are subject to firing costs as well as payroll and revenue taxes. Taxes and labor market regulations are imperfectly enforced by the government, giving rise to incentives for some firms to be informal. The labor market is characterized by labor market frictions and costs of hiring, features leading to unemployment. The economy is composed by tradable and non-tradable sectors, and tradable sector firms are able to export. We estimate the model using several data sources, including matched employer-employee data from formal and informal firms and workers in Brazil.

Brazil constitutes a relevant case study for several reasons. First, it has strict and burdensome labor regulations that are imperfectly enforced and a large informal sector: nearly two thirds of businesses, 40% of GDP and 35% of employees are informal (Ulyssea, 2018). Second, the Brazilian case is typical of developing countries, especially in Latin America, where the urban labor force employed informally averages over 50 percent, with this number varying from 35 percent in Chile to 80 percent in Peru (Perry et al., 2007). Third, it has unique data availability and quality, allowing the direct observation of informality for workers and firms. We define as informal workers those employees who do not hold a formal labor contract, which in Brazil is sharply defined as having a booklet (*carteira de trabalho*) that registers workers' entire employment history in the formal sector. We define as informal firms those not registered with the tax authorities, which means that they do not possess the tax identification number required for Brazilian firms (*Cadastro Nacional de Pessoa Juridica* – CNPJ). We can observe both definitions directly from the data available (more details are provided in the Data section). Finally, even though Brazil experienced a relatively fast and intense trade liberalization episode in early 1990's (e.g. Dix-Carneiro and Kovak, 2017b), it remains a relatively closed economy. Therefore, our analyses in this paper are of great policy relevance.

This paper contributes to three different literatures. First, it contributes to the litera-

ture that seeks to identify the impact of globalization on labor market outcomes and welfare in developing countries. Several papers in this literature have empirically examined the effects of trade on informality using different countries, sectors and methodologies, yielding mixed conclusions(Goldberg and Pavcnik, 2003; Bosch et al., 2012; Menezes-Filho and Muendler, 2011; Dix-Carneiro and Kovak, 2017b). In particular, Dix-Carneiro and Kovak (2017a) focus on the dynamic behavior of unemployment and informality in the aftermath of the Brazilian trade liberalization. They document that in the short run, the reforms had large effects on unemployment and small effects on informality. However, in the long run, this pattern is reversed, suggesting a potentially important role of the informal sector in smoothing out the labor market trajectories of displaced workers. Second, this project contributes to an extensive literature on the causes and consequences of informality by developing a new framework to analyze firms' and workers' decisions regarding informality (De Soto, 1989; Perry et al., 2007; La Porta and Shleifer, 2008; Bacchetta et al., 2009; Ulyssea, 2010, 2018; Meghir et al., 2015). Third, it contributes to the literature on misallocation and the role of size-dependent distortions (e.g. Hsieh and Klenow, 2009; Guner et al., 2008; Adamopoulos and Restuccia, 2014; Garicano et al., 2016).

2 Model

2.1 Set Up

The economy is populated by homogeneous, infinitely-lived workers-consumers. Individuals derive utility from the consumption of a composite good of differentiated, tradable sector goods C and from the consumption of a composite good of differentiated, non-tradable sector goods S. Preferences are given by

$$U = \sum_{t=1}^{\infty} \frac{C_t^{\zeta} S_t^{1-\zeta}}{(1+r)^t}, \tag{1}$$

where

$$C_t = \left(\int_0^{N_{Ct}} c_t(n)^{\frac{\sigma-1}{\sigma}} dn \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

$$S_t = \left(\int_0^{N_{St}} s_t(n)^{\frac{\sigma-1}{\sigma}} dn \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

and $\zeta \in (0,1)$ is the fraction of expenditure on tradable sector goods, $\sigma > 1$ is the elasticity of substitution across varieties within sectors, N_{kt} denotes the measure of varieties available in sector $k = C, S$ at time t, and $n \in (0, N_{kt})$ indexes varieties. As we will focus on steady state equilibria, we henceforth drop the time subscript for notational convenience.

2.2 Firms

There is a continuum of firms in both tradable and non-tradable sectors. Formal and informal firms coexist in both sectors, and each firm produces a unique variety $n \in (0, N_k)$. Firms use labor as the single input in a constant returns to scale production function: $q(z, \ell) = z\ell$, where ℓ denotes firm's employment size. Firms' idiosyncratic productivity evolves over time following the AR(1) process below:

$$\ln z' = \rho_k \ln z + \sigma_k^z \varepsilon, \tag{4}$$

where $\rho_k \in (0,1)$, $\varepsilon \sim N(0,1)$ and σ_k^z is the standard deviation of the shocks. It will be convenient to denote $G_k(z'|z)$ the cumulative distribution function of z' conditional on z and $g_k(z'|z)$ its density.[1]

Monopolistic competition implies that revenues in sector $k = C, S$ are given by:

$$R_k(z, \ell) = \left(\frac{X_k}{P_k^{1-\sigma}}\right)^{\frac{1}{\sigma}} (z\ell)^{\frac{\sigma-1}{\sigma}} \tag{5}$$

where X_k is total expenditure in sector k goods, and $P_k = \left(\int_0^{N_k} p_k(n)^{1-\sigma} dn\right)^{\frac{1}{1-\sigma}}$ is the price index for sector $k = C, S$. For the tradable sector, $X_C = \zeta I$, where I is aggregate income. For the non-tradable sector, $X_S = (1-\zeta) I + R$, where R represents expenditures on service sector goods made by firms in order to cover hiring, fixed and export costs (which we discuss below). Aggregate income is determined by total wages, government transfers and aggregate firms' profits.

Timing

Every period, formal incumbent firms must choose whether to stay or exit their industry. If the firm decides to stay, it draws its new productivity shock and must decide

[1] This process is imposed to be the same across formal and informal firms within tradable and non-tradable sectors. Unfortunately, we do not have longitudinal data on firms in the informal sector, so that this process cannot be separately identified for formal and informal firms.

to adjust or not its labor force. Informal firms face a similar problem but also have one additional option, which is to formalize their businesses. If they decide to formalize, they will then be subject to all regulatory costs faced by formal firms, namely the payroll and revenue taxes, firing costs and minimum wages. After the informal firm decides to stay informal or migrate to the formal sector, it draws its new productivity shock and must also decide whether to adjust its labor force.

The timing of events and firms' behavior is illustrated in Figure 1. Consider an informal firm which starts period t with state (z, ℓ, i). There are three initial possibilities: (i) the firm decides to stay informal and draws a new shock z'; (ii) the firm exits because it decides to, or because it is hit with an exogenous death shock (with probability α_{ki}); or (iii) the firm registers with the authorities, becomes formal, and draws a new shock z'. If the firm decides to stay active (as informal or formal), it must choose how to adjust its workforce in response to the shock z'. To do so, it posts vacancies or fires workers and ends period t with ℓ' workers. At that point, it realizes profits and starts period $t+1$ with state (z', ℓ', i), if it decided to remain informal or with state (z', ℓ', f), if it decided to become formal.

Now, consider a formal firm which starts period t with state (z, ℓ, f). The timing and sequence of events is the same as for informal firms. The only difference is that we do not allow for formal firms to become informal, and the exogenous death shock arrives with probability α_{kf}.

Hiring and Firing Costs

When deciding employment levels, both formal and informal firms in tradable and non-tradable sectors face hiring costs. These are defined by the costs of posting vacancies, which are given by the following function:

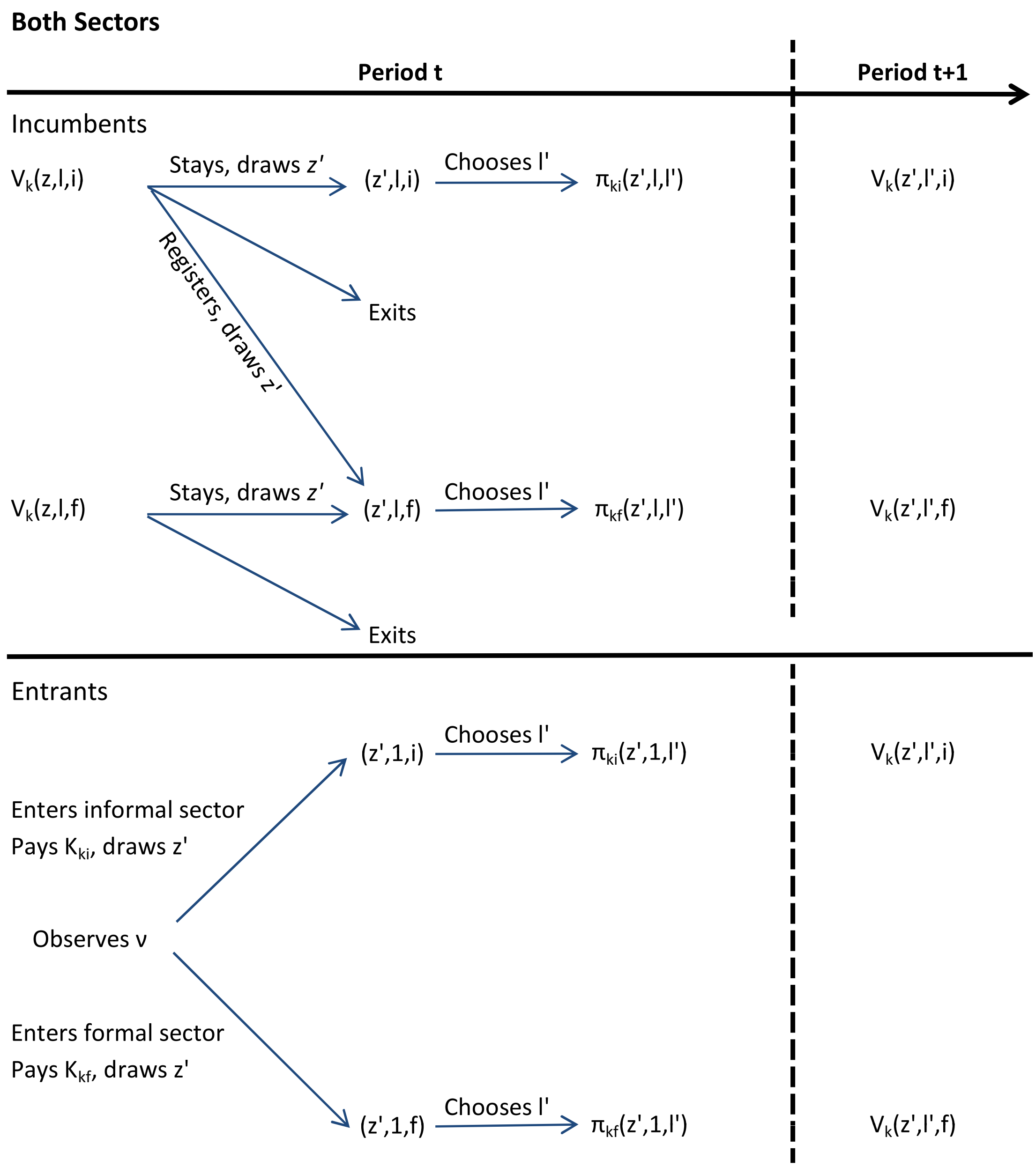
$$C_{kj}^{h}(\ell, v) = \left(\frac{h_{kj}}{\gamma_{k1}}\right)\left(\frac{v}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}} \tag{6}$$

where h_{kj}, γ_{k1} and $\gamma_{k2} \in [0,1]$ for $k = C, S$ and $j = i, f$, are parameters. The γ_{kj1} determines the convexity in hiring costs and γ_{k2} captures economies of scale in hiring.

Expanding from ℓ to ℓ' therefore requires posting $v = \frac{\ell' - \ell}{\mu_{kj}^{v}}$ vacancies, where μ_{kj}^{v} is the probability of filling a vacancy faced by a firm of type j in sector k. The cost of expanding from ℓ to ℓ' workers for a formal firm is therefore given by:

$$H_{kj}(\ell, \ell') = \left(\mu_{kj}^{v}\right)^{-\gamma_{k1}}\left(\frac{h_{kj}}{\gamma_{k1}}\right)\left(\frac{\ell' - \ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}} \tag{7}$$

Figure 1: Diagram of Firms' Behavior



The functional form of the hiring cost function is important for a couple of reasons. First, depending on the estimate of the scale parameter γ_{k2} , it is possible to generate the stylized fact that firm-level growth rates in employment decline with size. To obtain some intuition, suppose that $\gamma_{k2} = 0$. In that case, all firms posting v vacancies face the same hiring costs, irrespective of their size. On the other hand, if $\gamma_{k2} = 1$, then all firms face the same cost of a given employment growth rate. For values of γ_{k2} between 0 and 1, larger firms face a higher cost if they want to grow their employment by a particular rate. So, in the event of a positive shock, larger firms will grow less, and in the event of a negative shock, they will also downsize less (as they anticipate large hiring costs if they are hit with a positive shock in the future). Second, the parameter γ_{k1} governs the convexity of the hiring function. If $\gamma_{k1} > 1$, then hiring costs are convex. Allowing for convexity is important for the model to be able to generate wage dispersion. In this type of model, linear hiring costs lead to no wage dispersion. This is because, as we show later when we discuss the wage determination process, in our framework wages are proportional to average revenue per worker, which is – by virtue of our assumptions - proportional to marginal worker revenue. Optimizing firms set marginal revenue equal to marginal cost of an additional worker. But with linear hiring costs, the marginal cost is constant and equal across firms, so that wages will also be equalized across firms. In contrast, with convex hiring costs, the marginal cost of an additional worker is increasing in the growth of employment, so that expanding firms will pay higher wages.

Regarding firing costs, since they are entirely driven by labor market regulation, we assume that only formal firms are subject to them and they are determined as follows:

$$F\left(\ell, \ell'\right) = \kappa\left(\ell - \ell'\right) \tag{8}$$

where $\kappa > 0$ is the parameter governing the firing cost function. We assume that firing costs are equal across the C and S sectors, which is consistent with the Brazilian labor regulation. We also assume that firing costs are collected by the government and are rebated back to consumers, while the hiring costs are incurred in terms of the service sector composite good.

Profit and Value Functions

Formal firms are subject to payroll and revenue taxes, firing costs and the minimum wage regulation. The profit function of a formal firm in sector $k = C, S$ is given by:

$$\pi_{kf}(z', \ell, \ell') = (1 - \tau_y) R_k(z', \ell') - C_{kf}(z', \ell, \ell') - \bar{c}_{kf}, \tag{9}$$

where $R_k(z', \ell')$ denotes the revenue function; $\bar{c}_{kf}$ denotes a per-period, fixed cost of operation, which we assume that it is incurred in terms of the service sector composite good; τ_y is a sales/revenue tax, collected by the government and rebated to consumers.

Due to hiring and firing costs, the total cost function for a formal firm adjusting from ℓ to ℓ' workers is given by the following expression:

$$C_{kf}(z', \ell, \ell') = \begin{cases} (1 + \tau_w) \max\{w_{kf}(z', \ell'), \underline{w}\} \ell' + H_{kf}(\ell, \ell') & \text{if } \ell' > \ell \\ (1 + \tau_w) \max\{w_{kf}(z', \ell'), \underline{w}\} \ell' + \kappa(\ell - \ell') & \text{if } \ell' \leq \ell \end{cases} \tag{10}$$

where $w_{kf}(z', \ell')$ denotes the wage of workers in a formal firm with productivity z' and size ℓ', $\underline{w}$ denotes the minimum wage and τ_w is the payroll tax, which is assumed to be collected by the government and rebated to consumers. The wage schedule $w_{kf}(z', \ell')$ is the result of a bargaining problem between the firm and its workers that will be detailed in section 2.4.

Since formal firms have to choose to stay or leave their industry, their value function is given by:

$$V_k(z, \ell, f) = (1 - \alpha_{kf}) \max\left\{0, E_{z'|z} \max_{\ell'} \left\{\pi_{kf}(z', \ell, \ell') + \frac{1}{1+r} V_k(z', \ell', f)\right\}\right\} \tag{11}$$

where α_{kf} denotes the exogenous death probability that firms face every period for $k = C, S$. The solution of (11) leads to the employment policy function $\ell' = L_k(z', \ell, f)$ and to the vacancy posting policy function $v_{kf}(z', \ell) = \frac{L_k(z', \ell, f) - \ell}{\mu^v_{kj}}$ (as well as to other policies such as exit and stay-active decisions).

Even though informal firms do not have to incur any of the regulatory costs (taxes, minimum wages, firing costs), they face a probability of detection by the government, which is (presumably) increasing in their number of employees. Therefore, we allow the expected cost of being informal to depend on firm size, which is a common formulation in the literature (see Ulyssea, 2018, and the references therein). The intuition for this assumption is that as firms grow larger, they become more visible to the government and therefore are inspected with higher probability, which entails costs in the form of fines and

bribes, or can lead to the firm shutting down its operations. Similarly, this assumption captures the idea that the opportunity costs of informality increase as the firm becomes larger because it might want to access the formal financial market (e.g. credit lines), issue invoices and expand its costumers base. Informal firm's profit function is thus given by:

$$\pi_{ki}\left(z',\ell,\ell'\right) = \left(1 - p_{ki}\left(\ell'\right)\right) R_k\left(z',\ell'\right) - C_{ki}\left(z',\ell,\ell'\right) - \bar{c}_{ki}, \tag{12}$$

where $p_{ki}\left(\ell'\right)$ summarizes the costs associated to informality, which are assumed to be proportional to firm's revenues. We impose that

$$p_{ki}\left(\ell'\right) = \max\left\{\min\left\{a_k + b_k\left(\ell'\right)^{c_k}, 1\right\}, 0\right\}. \tag{13}$$

Since informal firms are not subject to firing costs, their cost function is given by:

$$C_{ki}\left(z',\ell,\ell'\right) = \begin{cases} w_{ki}\left(z',\ell'\right)\ell' + H_{ki}\left(\ell,\ell'\right) & \text{if } \ell' > \ell \\ w_{ki}\left(z',\ell'\right)\ell' & \text{if } \ell' \leq \ell \end{cases} \tag{14}$$

where $w_{ki}\left(z',\ell'\right)$ denotes the wage of workers in an informal firm with productivity z' and size ℓ'. This wage schedule will be determined by a bargaining problem between the firm and its workers as we describe in section 2.4.

Informal firms' value functions are similar to formal firms', except that they have the additional option to formalize their businesses. The informal value functions are therefore given by:

$$V_k\left(z,\ell,i\right) = \left(1-\alpha_{ki}\right)\max\left\{\begin{array}{c} 0, E_{z'|z}\max\limits_{\ell'}\left\{\pi_{ki}\left(z',\ell,\ell'\right) + \frac{1}{1+r}V_k\left(z',\ell',i\right)\right\}, \\ E_{z'|z}\max\limits_{\ell'}\left\{\pi_{kf}\left(z',\ell,\ell'\right) + \frac{1}{1+r}V_k\left(z',\ell',f\right)\right\} \end{array}\right\}. \tag{15}$$

The solution of (15) leads to the employment policy function $\ell' = L_k(z',\ell,i)$ and to the vacancy posting policy function $v_{ki}(z',\ell) = \frac{L_k(z',\ell,i)-\ell}{\mu^v_{kj}}$ (as well as to other policies such as exit, change to formal and stay informal decisions).

Entry

Firm entry is illustrated in the lower panel of Figure 1. Every period there is a pool of potential entrants into the tradable and non-tradable sectors. These potential entrants observe a pre-entry signal of how productive they will be if they decide to enter, denoted by ν, which is drawn from the ergodic distribution of z'. They can choose to enter as a

formal or an informal firm, and the decision to enter is made solely based on ν. Once they enter, they draw their actual productivity, z', from:

$$\ln z' = \rho_k \ln \nu + \sigma_k^z \varepsilon.$$

which is analog to incumbents' productivity process, described in expression (4). We adopt this structure to allow for the fact that there may be an overlap of productivity of entrants in both the formal and informal sectors (Meghir et al., 2015).

Once entry occurs and entrants draw their actual productivity, z', they start behaving as incumbents. Formal and informal entrants start their first period with workforce 1 and we assume that the recruitment costs of these initial workforces are included in the fixed entry costs. The value functions for entrants in either sector are given by:

$$V_k^e(\nu, j) = E_{z'|\nu} \max_{\ell' \geq 1} \left\{ \pi_{kj}(z', 1, \ell') + \frac{1}{1+r} V_k(z', \ell', j) \right\} \tag{16}$$

where $j = i, f$. The entry conditions into the informal and formal sectors are given by the following inequalities, respectively:

$$V_k^e(\nu, i) - K_{ki} \geq \max\{0, V_k^e(\nu, f) - K_{kf}\} \tag{17}$$

$$V_k^e(\nu, f) - K_{kf} \geq \max\{0, V_k^e(\nu, i) - K_{ki}\} \tag{18}$$

The solution to equations (17) and (18) lead to policy entry functions $I^{informal}(\nu)$ and $I^{formal}(\nu)$. The equilibrium mass M_k of entrants in each sector $k = C, S$ is pinned down by the free entry condition below.

$$V_k^e = \int \left[\left(V_k^e(\nu, i) - K_{ki}\right) I^{informal}(\nu) + \left(V_k^e(\nu, f) I^{formal}(\nu) - K_{kf}\right) \right] g_k^e(\nu) \, d\nu \leq c_{e,k}, \tag{19}$$

where $c_{e,k}$ is the cost of entry in sector k – the cost of drawing a ν signal. If entry in sector k is positive, then (19) holds with equality.

2.3 Labor Market Frictions

Formal and informal labor markets are characterized by search and matching frictions, which prevent unemployed workers to immediately find open vacancies. We assume undirected search, and therefore unemployed workers form a unique pool of individuals who are randomly matched with formal or informal firms in one of the sectors $k = C, S$. Thus, formal and informal firms operating in tradable and non-tradable sectors compete

for workers in the labor market. Given the total number of vacancies posted in each sector and type of firm $(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si})$, and the mass of unemployed workers searching for jobs, L_u, the total number of matches that are formed is given by:[2]

$$m\left(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si}, L_u\right) = \frac{\widetilde{V} L_u}{\left(\widetilde{V}^{\theta} + L_u^{\theta}\right)^{1/\theta}}, \tag{20}$$

Where $\widetilde{V} = \xi_{Cf} V_{Cf} + \xi_{Ci} V_{Ci} + \xi_{Sf} V_{Sf} + \xi_{Si} V_{Si}$ aggregates vacancies across sectors and types of firms, and $\xi_{kj} > 0$ is a mesure of efficiency/visibility of vacancies posted by firms of type j in sector k. ξ_{Cf} is normalized to 1. Matches are split across sectors according to the following proportionality rule:

$$m_{kj} = \frac{\xi_{kj} V_{kj}}{\widetilde{V}} m\left(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si}, L_u\right). \tag{21}$$

This implies that firms of type j in sector k face probability of filling a vacancy given by:

$$\mu_{kj}^{v} = \frac{m_{kj}}{V_{kj}} = \frac{\xi_{kj} L_u}{\left(\widetilde{V}^{\theta} + L_u^{\theta}\right)^{1/\theta}} = \xi_{kj} \mu^{v}, \tag{22}$$

where $\mu^{v} \equiv \frac{L_u}{\left(\widetilde{V}^{\theta} + U^{\theta}\right)^{1/\theta}}$. Equation (22) highlights that formal firms directly compete with informal firms in the labor market. Finally, unemployed workers face job finding probabilities given by:

$$\mu_{kj}^{e} = \frac{m_{kj}}{L_u} = \frac{\xi_{kj} V_{kj}}{\left(\widetilde{V}^{\theta} + L_u^{\theta}\right)^{1/\theta}} = \xi_{kj} \frac{V_{kj}}{L_u} \mu^{v}. \tag{23}$$

2.4 Wages

We assume that workers collectively bargain with their employer, after hiring costs are sunk and matching has taken place. More concretely, we assume that workers collectively bargain with their firms in a "all in or all out" fashion. To simplify exposition, we refer to workers as "unions". The surpluses of a formal firm in sector k, and the union it faces

[2] In principle, and depending on ξ_{kj}, the probability of filling a vacancy by firms in sector k of type j can be greater than 1. Our estimation procedure ensures that we only search for parameters leading to $\mu_{kj}^{v} \leq 1$. On the other hand, μ_{kj}^{e} in equation (23) is always bounded by 1.

are given by, respectively:

$$S_{kf}^{e}(z,\ell) = (1-\tau_y) R_k(z,\ell) - (1+\tau_w) w_{kf}(z,\ell)\ell - \bar{c}_{kf} + \frac{1}{1+r} V_k(z,\ell,f) \quad (24)$$

$$S_{kf}^{u}(z,\ell) = \left[w_{kf}(z,\ell) + \frac{1}{1+r} J_k^e(z,\ell,f) - \left(b + b^u + \frac{1}{1+r} J^u \right) \right] \ell \quad (25)$$

where b denotes the utility flow from being unemployed and b^u the value of unemployment benefits, which are only received by formal workers, and $w_{kf}(z,\ell)$ is the unrestricted wage for formal workers (who nevertheless cannot receive a lower wage than the minimum wage).

We assume that if all workers leave, the firm exits, and that fixed operating costs are incurred after the bargaining process. Let β_f be the bargaining power of workers in the formal sector, the outcome of bargaining is given by:

$$\beta_f S_{kf}^{e}(z,\ell) = (1-\beta_f) S_{kf}^{u}(z,\ell) \quad (26)$$

Substituting expressions (24) and (25) into (26), and assuming that the current surplus is shared the same way as future surpluses (as in, for example, Bertola and Garibaldi, 2001; Cosar et al., 2016), one obtains the following (unrestricted) wage functions for formal workers:

$$w_{kf}(z,\ell) = \frac{(1-\beta_f)(b+b^u)}{1+\beta_f \tau_w} + \frac{\beta_f (1-\tau_y)}{1+\beta_f \tau_w} \frac{R_k(z,\ell)}{\ell} - \frac{\beta_f}{1+\beta_f \tau_w} \frac{\bar{c}_{kf}}{\ell} \quad (27)$$

and we again note that formal workers always receive the maximum between the unrestricted wage, $w_{kf}(z,\ell)$, and the minimum wage, $\underline{w}$.

Wages in the informal sector are determined in a similar way. Let the bargaining power parameter be denoted by β_i, where we allow the bargaining power of formal and informal workers to be different. These could differ due to institutional reasons, such as the existence of a centralized union or labour courts, or because informal workers and firms have greater flexibility to negotiate wages. Since these will be directly estimated, the question of whether these bargaining power parameters are indeed different is an empirical one. Following the same steps as above, it is straightforward to obtain:

$$w_{ki}(z,\ell) = (1-\beta_i) b + \beta_i (1-p_{ki}(\ell)) \frac{R_k(z,\ell)}{\ell} - \beta_i \frac{\bar{c}_{ki}}{\ell} \quad (28)$$

where the major differences relatively to expression (27) are the absence of unemployment

benefits (b^u), payroll and revenue taxes (τ_w and τ_y, respectively); and the presence of the cost of informality function, $p_{ki}(\ell)$.

Expressions (27) and (28) are intuitive: wages are directly increasing with sales per worker, and the slope is larger if bargaining power is larger. An alternative to this wage setting would be to assume a somewhat more common structure *a la* Stole and Zwiebel (1996), where firms bargain with all of their workers simultaneously and continuously in a one-to-one basis, treating each worker as the marginal one. However, the present formulation generates a richer wage distribution that fits much better the degree of wage dispersion found in the data. Frameworks *a la* Stole and Zwiebel (1996) tend to generate less realistic distributions, as they imply that, for example, all firms that are willing to downsize pay the same wage to all workers (which is equal to workers' reservation wage). Additionally, the present wage setting framework implies wage schedules that are very close to those in the rent sharing literature (e.g. Card et al., 2018) and commonly found in trade models, such as Helpman and Itskhoki (2010).

2.5 Open Economy

We now extend the model to the open economy case. We assume that the home country is small relative to the rest of the world and therefore foreign conditions do not react to its policies. In the following analysis, we drop the formal/informal qualifier in order to simplify notation, as we assume throughout that informal firms cannot export.[3] In what follows, it will be convenient to re-write domestic revenues (Equation (5)) as $R_k(z,\ell) = D_{H,k}^{\frac{1}{\sigma}} q(z,\ell)^{\frac{\sigma-1}{\sigma}}$, where $k = C, S$, $q(z,\ell) = z\ell$, and $D_{H,k} = \frac{X_k}{P_k^{1-\sigma}}$. Since the focus in this section lies on the tradable sector only, and for the sake of notation simplicity, we drop the subscript $k = C, S$ for the reminder of this subsection.

Price Indices and Aggregates

The price index in the non-tradable sector remains the same, but in the tradable sector it is modified to account for trade. First, we characterize the price index of imports denominated in home-currency:

$$P_F = \epsilon\tau_a\tau_c \left(\int_0^{N_F} p^*(n)^{1-\sigma} \, dn\right)^{\frac{1}{1-\sigma}} = \epsilon\tau_a\tau_c \tag{29}$$

[3] This assumption comes from the fact that firms that are not registered cannot undertake the necessary legal and bureaucratic procedures to export.

where $p^*(n)$ is the free on board (FOB) price of imported variety n, denominated in foreign currency; N_F denotes the mass of imported varieties; ϵ is the exchange rate, $\tau_a - 1 > 0$ is the ad-valorem tariff and $\tau_c > 1$ the iceberg trade cost. The second equality in the above expression comes from the normalization $\left(\int_0^{N_F} p^*(n)^{1-\sigma}\, dn\right)^{\frac{1}{1-\sigma}} \equiv 1$. This is without loss of generality, as this term is exogenous to our model given the small open economy assumption. The price index of domestically produced varieties $n \in (N_F, N]$ is given by:

$$P_H = \left(\int_{N_F}^{N} p(n)^{1-\sigma}\, dn\right)^{\frac{1}{1-\sigma}} \tag{30}$$

and the price index for the composite tradable sector good is given by

$$P = \left[P_H^{1-\sigma} + P_F^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\int_{N_F}^{N} p(n)^{1-\sigma}\, dn + (\epsilon\tau_a\tau_c)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{31}$$

The foreign market price index for exported goods, denominated in foreign currency, is given by $P_x^* = \left(\int_{N_F}^{N} \mathcal{I}^x(n)\, p_x^*(n)^{1-\sigma}\, dn\right)^{\frac{1}{1-\sigma}}$, where $p_x^*(n)$ is the price of domestic variety n in the foreign country, denominated in foreign currency, and $\mathcal{I}^x(n)$ is an indicator function that equals one if variety n is exported.

The domestic demand for domestically produced goods is given by $Q_H(n) = D_H p(n)^{-\sigma}$, for $n \in (N_F, N]$; and the domestic demand for foreign produced goods is given by $Q_H(n) = D_H(\epsilon\tau_a\tau_c p^*(n))^{-\sigma}$, for $n \in [0, N_F]$. Finally, foreign demand for domestically produced goods is given by $Q_F(n) = D_F^*(p_x^*(n))^{-\sigma}$, for $n \in (N_F, N]$. Therefore, we have that the value of aggregate imports (before import tariffs) and exports are given by the following expressions:

$$Imports = \frac{D_H}{\tau_a}\int_0^{N_F} (\epsilon\tau_a\tau_c p^*(n))^{1-\sigma}\, dn = \frac{D_H P_F^{1-\sigma}}{\tau_a} = \frac{D_H(\epsilon\tau_a\tau_c)^{1-\sigma}}{\tau_a} \tag{32}$$

$$Exports = D_F^*\epsilon\int_{N_F}^{N} \mathcal{I}^x(n)\, p_x^*(n)^{1-\sigma}\, dn = \epsilon D_F^* P_x^{*1-\sigma} \tag{33}$$

Exporters

Given the expression of foreign demand for home variety n just described, $Q_F(n)$, revenues from exports are given by $\epsilon D_F^{*\frac{1}{\sigma}}(q_x/\tau_c)^{\frac{\sigma-1}{\sigma}}$, where q_x is the total quantity exported. If a firm exports, it must decide which fraction η of its product to sell abroad.

Conditional on being an exporter, total gross revenue is given by

$$\begin{aligned} R^x(z,\ell,\eta) &= D_H^{\frac{1}{\sigma}}\left[(1-\eta)q(z,\ell)\right]^{\frac{\sigma-1}{\sigma}} + \epsilon D_F^{*\frac{1}{\sigma}}\left(\frac{\eta q\left(z,\ell\right)}{\tau_c}\right)^{\frac{\sigma-1}{\sigma}} \\ &= q(z,\ell)^{\frac{\sigma-1}{\sigma}}\exp\left(d_H + d_F(\eta)\right) \end{aligned} \tag{34}$$

where $d_H = \ln\left(D_H^{\frac{1}{\sigma}}\right)$ and $d_F(\eta) = \ln\left((1-\eta)^{\frac{\sigma-1}{\sigma}} + \epsilon\left(\frac{D_F^*}{D_H}\right)^{\frac{1}{\sigma}}\left(\frac{\eta}{\tau_c}\right)^{\frac{\sigma-1}{\sigma}}\right)$.

The optimal share of exports is given by:

$$\eta^o = \arg\max_{\eta} d_F(\eta) = \left(1 + \frac{\tau_c^{\sigma-1}}{\epsilon^\sigma}\frac{D_H}{D_F^*}\right)^{-1} \tag{35}$$

which shows that, conditional on exporting, all firms choose to export the same share of their output. The revenue functions for non-exporters and exporters are then given by, respectively:

$$\begin{aligned} R^d(z,\ell) &= (z\ell)^{\frac{\sigma-1}{\sigma}}\exp\left(d_H\right) \\ R^x(z,\ell) &= R^d(z,\ell)\Delta(z,\ell) \end{aligned}$$

where $\Delta(z,\ell) = \exp\left(d_F(\eta^o)\right)$, and $d_F(\eta^o)$ is obtained by substituting the expression of the optimal η^o into $d_F(\eta)$.[4] The export policy is then given by:

$$I_C^x\left(z,\ell\right) = \left\{\begin{array}{ll} 1 & \text{if } R_C^x\left(z,\ell\right) - f_x > R_C^d\left(z,\ell\right) \\ 0 & \text{otherwise} \end{array}\right\} \tag{36}$$

where $f_x > 0$ denotes the fixed cost of exporting, which is denominated in terms of the service composite good.

Since $\Delta(z,\ell) > 1$, being an exporter magnifies firms' revenues and also makes them more sensitive to productivity shocks, for any given state (z,ℓ). Thus, as in Cosar et al. (2016), reducing trade costs will produce two opposing forces: (i) there will be a reallocation of workers toward larger and higher productivity firms, which tend to be more stable and have lower worker turnover (e.g. they face larger costs of growing the workforce); (ii) due to the term $\Delta(z,\ell)$, both new and old exporters become more sensitive to idiosyncratic shocks, which tends to increase turnover. We follow Cosar et al. (2016) and refer to these two forces as the "distribution effect" and "sensitivity

[4] When one substitutes η^o into $d_F(\eta)$, one obtains $d_F = \log\left(\left(1 + \frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$.

effect", respectively. Turnover is tightly linked to unemployment, as workers who are fired must spend at least one period in unemployment. In turn, workers transition from unemployment to formal and informal sector jobs. In addition to these forces, we also have "Melitz effects", where trade liberalization affects the "productivity/size threshold" for firms to export, but it will also affect the thresholds for operating formally, informally and exit. An attractive feature of this model is that it can accommodate both a increase or a decrease in informality. The net effect of the forces in the model is ultimately an empirical question.

2.6 Equilibrium

- Firms act optimally and make entry and exit decisions and post vacancies according to equations (11), (15), (16), (17) and (18). If entry is positive in sector k, the free entry condition (19) holds with equality.

- Wages solve the bargaining problem between workers and the firm, as in equations (27) and (28).

- Labor markets clear, that is, the sum of employment levels across sectors and the number of unemployed workers must be equal to the total labor force.

- The government runs a balanced budget, the tradable and non-tradable markets must clear, and trade is balanced. Government's revenues come from tax collection and firing costs, while it pays unemployment benefits to all unemployed who come from formal employment. We assume that any surplus is directly rebated to consumers.

- Aggregate income is given by the sum of wages, unemployment benefits, profits and government transfers. Expenditure on nontradable goods is divided between final goods expenditure – given by $(1 - \zeta)$ – and intermediate goods expenditure R (hiring costs, fixed operational costs and the fixed costs of exporting).

- We focus on steady state equilibria, where all aggregates remain constant. In particular, no sector can be expanding or contracting, which implies that: (i) the flow of workers out of unemployment and into the formal/informal and tradable/non-tradable sectors must be the same as the flow out of these sectors and into unemployment; (ii) the mass of firms entering the informal sector must be equal to the mass of informal firms that decide to exit or to formalize their businesses in either

sector $k = C, S$; and (iii) the sum of the number of firms entering the formal sector and those formalizing their businesses must be equal to the mass of formal firms that decide to exit either sector $k = C, S$.

Appendix A.1 details all of the equilibrium conditions.

3 Background: The cost of labor regulations in Brazil

The relevant laws and regulations that apply to formal labor relations in Brazil are contained in the the Brazilian Labor Code (*Consolidação das Leis Trabalhistas* – CLT), which dates back to 1943. In 1988, the new Federal Constitution was enacted and extended the range of labor regulations and workers' benefits, which substantially increased both the variable labor costs associated to formal employment and firing costs (De Barros and Corseuil, 2004).[5] As a result of the changes in 1988, the regulatory framework of the Brazilian labor market became quite burdensome and costly, and that has remained unaltered since then. According to the employment index in Botero et al. (2004), the cost of labor regulations in Brazil is around 20 percent above the mean and median of 85 countries and more than 2.5 times as large as in the United States.

The main aspects of the labor regulations in Brazil, in terms of their magnitude and potential impacts on labor market functioning, are the following: the presence of a national minimum wage, sizeable payroll taxes, unemployment insurance that is only available to formal workers, and substantial firing costs. Since these play an important role in our model and counterfactuals, we provide a brief background discussion on each of them individually and refer the reader to existing studies that provide a more in depth analysis of these different institutional aspects.

Starting by the national minimum wage, since 1995 (with the end of hyper-inflation) its nominal value is determined by the federal government once a year and is typically quite binding. In 2003, for example, the minimum wage corresponded to 49 percent of the national average wage and 81.3 percent of the national median wage.[6] As for the unemployment insurance, its rules remained unaltered from 1994 to 2015 but substantial

[5]Among the changes introduced by the new Constitution, one can highlight the following: regular working hours went from 48 to 44 hours per week; overtime premium increased from 20 to 50 percent; maternity leave increased from three to four months; and the value of paid vacations increased from one to, at least, 4/3 of the regular monthly wage (see De Barros and Corseuil, 2004, for a more detailed description of the changes).

[6]The mean and the median wages are computed using micro data from the National Household Survey (PNAD) and pooling together all formal and informal employees who are between 18 and 64 years old and worked at least 20 hours per week.

changes have been implemented since then. Since our empirical analysis focuses on the period prior to the UI reforms, we discuss the rules in place until 2015.[7] In terms of eligibility, generally a formal worker who is laid off and who has at least 6 months of job tenure is eligible to receive UI benefits for up to 5 months.[8] The actual duration of the benefit depends on the worker's accumulated tenure across her formal jobs in the 36 months prior to layoff. In practice, most workers receive between 4 and 5 months of UI benefits, with the mean and median number of monthly payments per UI spell equal to 4.3 and 4.7 months, respectively. Finally, the value of the benefit depends on the worker's average wage in the three months prior to layoff and the replacement rate is 100% for individuals who earn one minimum wage, with an average replacement rate of 64 percent (all data comes from Gerard and Gonzaga, 2018).

As for the firing costs, the Brazilian labor regulation states that all formal workers "dismissed with no just cause" should receive a monetary compensation paid by the employer. Since labour courts are extremely favourable to workers, *de facto* all workers are entitled to receive this compensation upon an involuntary separation. The magnitude of this compensation is determined proportionally to the funds accumulated in the worker's *Fundo de Garantia por Tempo de Serviço* (FGTS), which is a job security fund accumulated while the worker remains employed at a given firm. This is a private and individual fund that is specific to the worker, and to which employers must contribute, every month, the equivalent of 8 percent of worker's monthly wage. Hence, the worker's FGTS funds are proportional to her tenure and accumulate at a rate of roughly one monthly wage per year. Although these resources are owned by the worker, the fund is run by the government and the real return rates are typically below market rates, when not negative. Moreover, workers only have access to their own fund when they are laid off or upon retirement. In addition to the totality of their fund, workers who are laid off also receive a penalty, paid by their employer, which amounts to 40 percent of total resources accumulated in their fund during the duration of the job they are being laid off from. Firms must also pay an additional 10 percent of the FGTS in fines, which go directly to the federal government. In addition to this severance payment of 50 percent (40 plus 10 percent) of the FGTS, firms must provide a one-month advance notice, which *de facto* means that workers receive an additional monthly wage and are dismissed immediately.[9]

[7]See Carvalho et al. (2018) for a discussion of the reform, which substantially changed the eligibility criteria of unemployment benefits, and its impacts on layoffs in Brazil.

[8]There are some nuances to eligibility that depend upon the elapsed time since worker's last successful application to UI benefits. See Gerard and Gonzaga (2018) for a more detailed discussion of the UI program in Brazil.

[9]Gonzaga et al. (2003) provide an in depth discussion of the legislation on dismissal costs in Brazil.

Finally, Brazil has a burdensome tax system, which is not only characterized by high tax rates but also by a complex structure that implies large compliance costs. For example, the estimated cost in terms of time required to comply with the tax system in Brazil is 2,600 hours, which is the highest in the world, and more than 8 times larger than the cost that a firm faces in the U.S. Even though a substantial part of this cost is not due to the payment of labor taxes, the time required to comply with labor taxes in Brazil is almost 5 times higher than in the U.S. (491 and 100 hours, respectively).[10] In terms of the tax rate, even though we use the statutory values for both payroll and revenue taxes in our estimation, it is useful to provide a comparison to other countries, which is done in Doing Business (2007): The labor tax computed as a share of commercial profits amounts to 42.1 percent in Brazil, while it is 12.9 percent in Canada and 10 percent in the U.S. Hence, not only labor taxes seem to be quite high in Brazil, but also they imply substantial compliance costs.

4 Data and Facts

4.1 Firms

In this paper we make use of 6 datasets that contain information on formal and informal firms and their workers. The first is the *Relação Anual de Informações Sociais* (RAIS), which is a matched employer-employee dataset assembled by the Brazilian Ministry of Labor every year since 1976. RAIS is a high quality panel that contains the universe of formal firms and workers.[11] It provides information on firms' 5-digit industry, location and ownership (i.e public vs. private enterprises), among others. At the worker level, the main variables are gender, age, level of education, monthly wage, number of hours in the contract, tenure at the firm, occupation, month of accession into the job (if accession occurred during the current year), and month of separation (if any). We use the matched employer-employee structure to compute firm size and firm-level average wages over time.

We also use three economic surveys that cover the *formal* manufacturing, retail and service sectors: *Pesquisa Industrial Anual* (PIA), *Pesquisa Anual de Comércio* (PAC), and *Pesquisa Anual de Serviços* (PAS), respectively. These surveys collect detailed infor-

[10] These data come from Doing Business (2007), which is the earliest report available on paying taxes in the Doing Business Initiative that provides comparability across a comprehensive set of countries.

[11] The RAIS data set has been increasingly used in different applications. For recent examples see Dix-Carneiro (2014), Helpman et al. (2017), Alvarez et al. (2018), Ulyssea (2018), among others.

mation about firms' inputs, output and revenues, and are a combination of a census for larger firms and a representative sample for smaller firms. In the manufacturing sector (PIA), all firms with at least 30 employees are part of the census and are surveyed every year, while firms with 5 to 29 employees are randomly sampled.[12] The PAC (retail sector) and PAS (services) follow similar designs, although they have lower size thresholds for firms to be included in the census: firms with 20 employees or more are part of the census, while firms with up to 19 employees are randomly sampled.

The fifth data source used is Customs data from *Secretaria de Comércio Exterior* (SECEX), which give us the list of every export and import transaction (and values) made from and by Brazilian firms every year since 1990 and until 2007. Importantly for this study, there is a unique firm identifier across these 5 data sets, which allows us to merge the production information from PIA, PAS and PAC with the information about firms' labor and wages coming from RAIS, and the customs data from SECEX.

These 5 data sets provide a comprehensive coverage of the formal sector, but are completely silent about the informal sector (by design). We therefore use a sixth data source, which is the *Pesquisa de Economia Informal Urbana* (ECINF). This survey was collected by the Brazilian Bureau of Statistics (IBGE) in 1997 and 2003, and was designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal). It is a matched employer-employee data set that contains information on entrepreneurs, their businesses and employees. Firms are directly asked whether they are registered with the tax authorities and whether each of their workers has a formal labor contract. Thus, it is possible to directly observe both firms' and workers' formal status. Given that the formality/informality statuses are self-reported, one could have concerns about measurement error and under-reporting. However, the IBGE has a long tradition of accurately measuring labor informality, and it has very strict confidentiality clauses, so the information cannot be used for auditing purposes by other government branches, in particular those responsible for enforcing the relevant laws and regulations. These characteristics, associated to the high levels of informality observed in the data, make us confident that respondents are not systematically underreporting their informality status.[13]

In all 6 data sets we exclude public sector firms and those in agriculture, mining, coal,

[12]The main source of information used by IBGE to design its sample is the RAIS data set described above.

[13]Additionally, Ulyssea (2018) shows that the ECINF reproduces very well the RAIS in all the dimensions that are common to both data sets (e.g. size and sectoral distributions), which is reassuring of ECINF's quality.

oil and gas industries. We do so because our focus lies on private sector, urban firms. Moreover, our model is not well suited to describe sectors with very large economies of scale and dominated by few very large firms, such as oil and gas. In the data, as well as in the model, the tradable sector is comprised by manufacturing firms and the non-tradable sector is comprised by services and retail firms. In sum, information on the formal tradable sector comes from RAIS, PIA and SECEX; on the non-tradable formal sector comes from RAIS, PAS and PAC; and data on both tradable and non-tradable informal sectors come from the ECINF survey.

Since 2003 is the last year available for the ECINF survey, we use it as the reference year for all other data sets. Table 1 shows the size distribution (measured as number of employees) in the tradable and non-tradable sectors for formal and informal firms. As expected, the number of observations is much larger for formal firms, as these come from a census (the RAIS data). Nevertheless, the share of tradable sector firms is quite similar in the formal and informal sectors (13.1 and 14.2 percent, respectively). The size difference between formal exporters and non-exporters in the tradable sector is quite remarkable, with exporting firms being more than 8 times larger than non-exporting firms, on average. Figure 2 shows this fact from a different angle, as the share of exporters increases steeply moving up in the size distribution.

Formal firms in the tradable sector are also larger on average than those in the non-tradable sector and the distribution is more skewed to the left. The size difference between informal firms in tradable and non-tradable sectors is almost null, which is expected: the ECINF survey has a size cap, which mechanically limits the size differential. More substantially, informal firms cannot grow much without becoming too visible to the authorities and cannot export either, which limits their ability to grow.

Table 2 shows the same information as in Table 1 but focusing on firms' revenues. The same patterns found in Table 1 arise, but it is worth noting that the size differences across percentiles are much larger when one uses revenues instead of employment as the size measure. For example, the 99th percentile of the size distribution measured as number of employees is nearly three times larger in the formal tradable than in the formal non-tradable. The same ratio is more than 30 when one uses revenues. Interestingly, this relationship is inverted in the informal sector, where firms in the non-tradable sector earn higher revenues than firms in the tradable sector. This is intuitive, as one would expect that the penalty for remaining small (and informal) is lower in the non-tradable sector.

Figure 3 shows that there is a substantial size-wage premium in both tradable and non-tradable formal sectors, but the same is not true for informal firms. This is somewhat

Table 1: Firm Size Distribution in Number of Employees

	Formal		Informal	
	Sector C	Sector S	Sector C	Sector S
All Firms				
Mean (Log-Employment)	1.78	1.18	0.10	0.10
Variance (Log-Employment)	1.82	1.26	0.09	0.08
Exporters				
Mean (Log-Employment)	3.9	–	–	–
Variance (Log-Employment)	2.7	–	–	–
Employment Distribution				
Pct. 20	2	1	1	1
Pct. 40	4	2	1	1
Pct. 60	7	4	1	1
Pct. 80	17	8	1	1
Pct. 90	35	14	2	2
Pct. 95	67	25	2	2
Pct. 99	298	109	4	3
# Observations	216,467	1,430,633	1,069	6,192

Notes: To compute the moments for the formal tradable sector, we use the PIA; for non-tradable formal sector, the PAS and PAC data sets; and for both tradable and non-tradable informal sectors, we use the ECINF survey.

Figure 2: Share of Exporters by Firm Size Percentiles

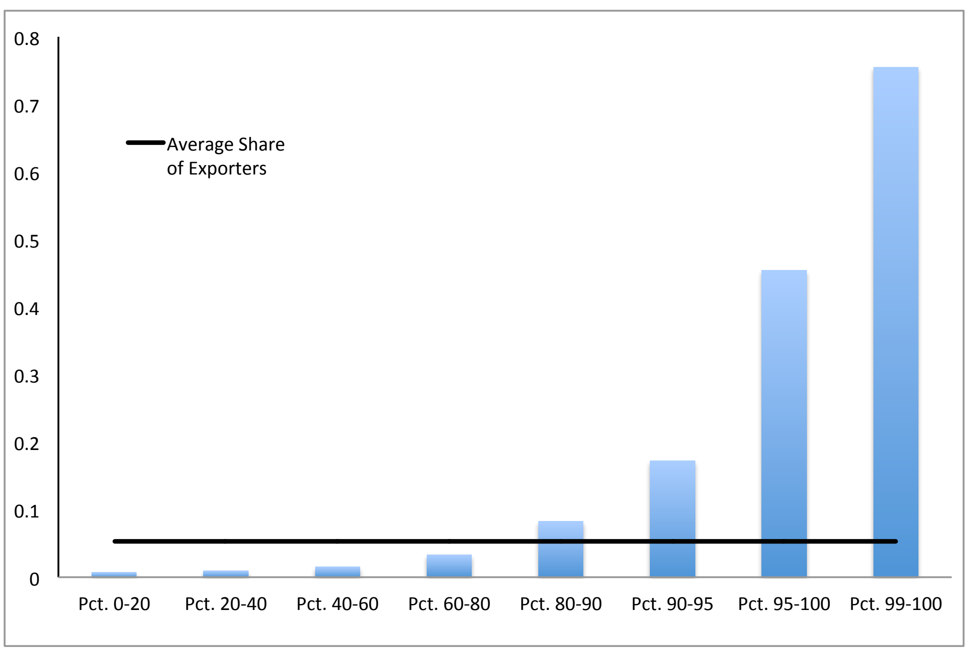


Table 2: Revenue Distribution

	Formal		Informal	
	Tradable	Non-Tradable	Tradable	Non-Tradable
All Firms				
Mean (Log-Revenue)	12.73	10.81	8.53	8.95
Variance (Log-Revenue)	3.51	2.07	1.44	1.30
Exporters				
Mean	15.46	–	–	–
Variance	4.45	–	–	–
Revenue Distribution (in 2003 R$)				
Pct. 20	77,962	15,897	1,920	3,600
Pct. 40	166,110	31,102	4,200	6,000
Pct. 60	407,595	59,492	6,600	9,600
Pct. 80	1,143,359	137,162	13,428	19,200
Pct. 90	4,038,112	288,717	24,000	32,160
Pct. 95	12,494,325	558,989	36,000	49,200
Pct. 99	103,287,792	3,229,837	72,000	108,000

Notes: To compute the moments for the formal tradable sector, we use the PIA; for non-tradable formal sector, the PAS and PAC data sets; and for both tradable and non-tradable informal sectors, we use the ECINF survey.

mechanical, as most informal firms have only one employee. As for employment, wage and revenue growth, Tables 3 and 4 show different patterns moving up the firm size distribution. Table 3 shows that, on average, expanding firms tend to present higher wage growth, but this relationship is not constant across different percentiles of the size distribution (for none of the groups considered in the table). On the contrary, Table 4 shows a clear pattern that is in line with other available evidence in the literature: yearly employment and revenues growth rates decrease with size, except at the very top of the distribution (top 5 and one percent of the size distribution).

4.2 Workers

In order to complement the information on firms, we use the Monthly Employment Survey (PME – *Pesquisa Mensal de Emprego*) to obtain information on worker allocations and labor market flows. This is a rotating panel with a similar design to that of the Current Population Survey in the U.S.: individuals in a given household are interviewed for 4 consecutive months, they "rest" for 8 months and are then re-interviewed for additional 4 consecutive months, which implies a maximum panel length of 16 months. This employment survey covers the six main metropolitan areas in Brazil and contains detailed information on individuals' socio-demographic characteristics and labor market

Figure 3: Average Log-Wages by Firm Size Percentiles

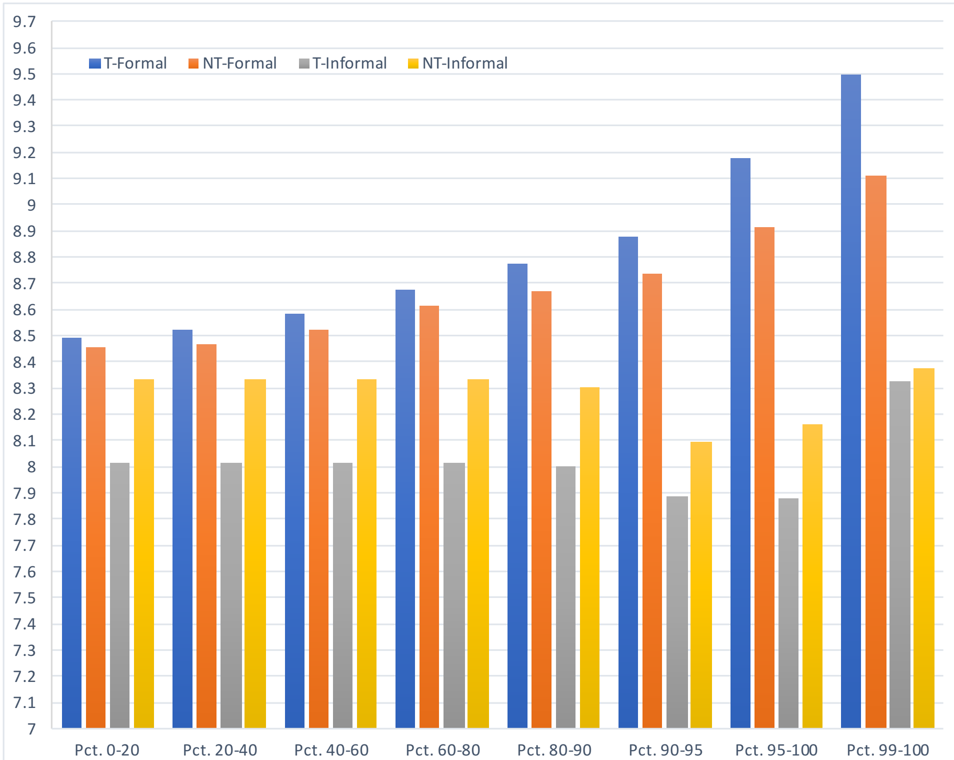


Table 3: Formal Firms' Average Wage Growth

	Surviving Firms		Expanding Firms		Contracting Firms	
	T	N-T	T	N-T	T	N-T
All Firms	0.051	0.044	0.063	0.059	0.042	0.037
Pct. 0-20	0.043	0.032	0.065	–	0.040	0.032
Pct. 20-40	0.055	0.042	0.072	0.075	0.048	0.038
Pct. 40-60	0.050	0.049	0.066	0.064	0.040	0.042
Pct. 60-80	0.051	0.042	0.062	0.057	0.039	0.032
Pct. 80-90	0.048	0.042	0.060	0.053	0.033	0.031
Pct. 90-95	0.055	0.047	0.072	0.056	0.032	0.036
Pct. 95-100	0.074	0.060	0.050	0.061	0.109	0.059
Pct. 99-100	0.059	0.123	0.029	0.094	0.110	0.168

Notes: We compute firm-level yearly average growth using the RAIS data for the years of 2002 and 2003. Surviving firms are those that are alive in 2002 and 2003. Expanding firms are those for which employment in 2003 is strictly larger than 2002. Contracting firms are those whose labor force remains constant or decreases between 2002 and 2003. T and N-T denote the tradable and non-tradable sectors, respectively.

outcomes, including informal employment and self employment.[14]

[14] See Meghir et al. (2015) for a more detailed description of the PME data.

Table 4: Formal Firms' Employment and Revenue Growth

	Employment Growth		Revenue Growth	
	Tradable	Non-Tradable	Tradable	Non-Tradable
All Firms	0.156	0.121	0.201	0.229
Pct. 0-20	0.362	0.318	0.216	0.242
Pct. 20-40	0.155	0.231	0.212	0.227
Pct. 40-60	0.096	0.073	0.210	0.235
Pct. 60-80	0.072	0.036	0.201	0.236
Pct. 80-90	0.072	0.031	0.178	0.215
Pct. 90-95	0.071	0.036	0.169	0.212
Pct. 95-100	0.088	0.046	0.146	0.149
Pct. 99-100	0.101	0.060	0.148	0.112

Notes: We compute firm-level yearly employment and revenue growth using the years of 2003 and 2004. We compute employment growth using the RAIS data set. For revenue growth in the formal tradable sector, we use the PIA data set; for the non-tradable formal sector, we use the PAS and PAC data sets.

We exploit the panel structure of PME to estimate one-year labor market transitions between formal employment, informal employment (in both tradable and non-tradable sectors) and unemployment statuses.[15] As in the firm-level data, we exclude individuals employed in the public sector, agriculture, mining, coal, oil and gas industries. In addition to these filters, we also exclude individuals younger than 17 and older than 65 years old.

Panel A of Table 5 shows worker allocations in 2003. It is noteworthy that 15% of the working age population is unemployed (or more precisely, not employed), and that approximately 20% of employed workers are in the C-sector. These numbers also indicate that 48% of the labor force is employed in the informal sector. In addition, 35% of C-sector workers are informal, whereas 51% of S-sector workers are. Panel B of Table 5 shows the relevant transition matrix for the purposes of our model. Even though we estimate the full transition matrix in the data, we only report the transitions that are accounted for in our model, while the remaining ones are omitted (such as from the formal tradable sector to the informal non-tradable sector). We start by noting that the table confirms two well-known facts: (i) most of the labor force is in the non-tradable sector (69.3 percent); and (ii) informality is very high in Brazil, accounting for 41 percent of the labor force. As for the probabilities of transition, the rate of retention (main diagonal in the transition matrix) is highest in the non-tradable formal sector (68.5 percent) and is lowest in the informal tradable sector (27 percent). Unemployed workers are most likely

[15] A worker is defined to be unemployed if she is not working – regardless of whether she is searching for a job or not.

to exit to a non-tradable informal sector (38 percent), while the formal tradable sector is the least likely destination of those who are unemployed.

Table 5: Sectoral Shares and 12-month Transition Rates

Panel A: Workers Allocation

	Unemp.	Tradable Inf.	Tradable Form.	Non-Trad. Inf.	Non-Trad. Form.
Shares†	0.137	0.058	0.112	0.352	0.341

Panel B: 12-month Transition Rates

	Unemp.	Tradable Inf.	Tradable Form.	Non-Trad. Inf.	Non-Trad. Form.
Unemp.	0.335	0.062	0.049	0.381	0.159
Tradable - Inf.	0.087	0.270	0.135	0.369	0.119
Tradable - Form.	0.039	0.044	0.530	0.074	0.300
Non-Tradable - Inf	0.086	0.062	0.035	0.610	0.177
Non-Tradable - Form.	0.044	0.016	0.100	0.132	0.685

Notes: Authors' own calculations from the Monthly Employment Survey (PME), years 2003 and 2004. We use the first and 4th interviews to compute 4-months transition rates for the full transition matrix, M. We then annualize M by computing M^3. † We use sampling weights to compute these shares using the entire sample.

5 Estimation

Our estimation procedure follows two steps. First, we fix a subset of parameters using a combination of aggregate data, estimates from previous papers and the statutory value of institutional parameters, such as revenue and payroll taxes. Then, we estimate the remaining parameters of the model using an Indirect Inference estimator, which allow us to combine information from the different data sources discussed in the previous section.

As discussed in Section 3, labor regulations are quite costly and cumbersome in Brazil, so we need to make a few simplifying assumptions. We follow Ulyssea (2018) and set τ_w so that it reflects the main taxes that are proportional to firms' wage bill, namely, employer's social security contribution (20 percent), payroll tax (9 percent), and severance contributions to FGTS (8.5 percent). τ_y includes only the federal VAT taxes, IPI (20 percent) and PIS/COFINS. We exclude state level value added taxes because these vary greatly across states and there is a cumbersome system of tax substitution across the

production chain, which would be impossible to properly capture.[16]

Firing costs are set following Heckman et al. (2000), which compute the expected discounted cost of dismissing a worker for several Latin American countries, including Brazil. This is done taking into account the main characteristics of dismissal costs in Brazil, as discussed in Section 3, and the expected cost is expressed as a multiple of the monthly wage. To make this parameter compatible with our model, we convert it to a fixed monetary value using the average formal wage found in the data in 2003. The minimum wage corresponds to the annualized value of the national monthly minimum wage in 2003. Finally, the unemployment benefit is set assuming that all workers receive the maximum number of benefits (5 monthly payments), which is very close to both the mean and median number of benefits (Section 3), while we use the mean monthly value paid in 2003 reported by the Ministry of Labor, which is denominated in multiples of the minimum wage. Table 6 shows these parameter values and their sources.

Table 6: Calibrated Parameters

Parameter	Description	Source	Value
τ_c	Iceberg Trade Costs	Cosar, Guner and Tybout (2016)	2.50
ζ	Share of expend. C	World Input-Output Database	0.283
r	Interest rate	Ulyssea (2010)	0.08
τ_y	Sales Tax	Ulyssea (2017)	0.293
τ_w	Payroll Tax	Ulyssea (2017)	0.375
τ_a	Import Tariff	TRAINS	1.12
κ	Firing Costs	Heckman and Pages (2000) (in R$)	1,956.7
$\underline{w}$	Min. Wage	Annualized 2003 value (in R$)	2,880
b_u	Unemp. Benefit	$1.37 \times 5 = 6.85$ monthly MW	1,644

In a second step, we take the parameters described in Table 6 as given and estimate the remaining parameters using an Indirect Inference estimator with equilibrium constraints.[17] The estimation algorithm is described in details in Appendix B.1. In this step we estimate 35 parameters using 139 data moments and auxiliary parameters. This version of the model imposes $xi_{kj} = 1$ and $K_{kj} = 0$.

[16] As discussed in Ulyssea (2018), these taxes can be large in some states, which would imply that we underestimate the overall tax burden that firms face. However, we do not include intermediate inputs, which implies that we might be overestimating the actual tax burden faced by some firms. The net effect of these forces is *a priori* unclear.

[17] This is the usual Indirect Inference estimator (e.g. Gourieroux and Monfort, 1996; Smith, 2008), but we also penalize deviations from the model's equilibrium constraints in the objective function.

5.1 Identification

The choice of parameters from auxiliary regressions (and moments) to be matched by the model is crucial to achieve identification. Given the high non-linearity and dimension of the model, it is not possible to provide a direct proof of identification. Nevertheless, we provide a heuristic discussion of which variation in the data provides information on different sets of parameters to be estimated.

We start by noting that even though one can directly use micro data to estimate the parameters of the AR(1) processes for productivity (ρ_k and σ_k^z), we estimate them within our Indirect Inference procedure and use the persistence and volatility of firm revenues and labor force sizes to obtain information about these parameters. This information is obtained with PIA (Manufacturing Survey), PAS (Services Survey), PAC (Retail-trade Survey) and RAIS (all sectors). We choose to proceed in this way because the production functions typically assumed by the existing estimators (e.g. Olley and Pakes, 1996; Ackerberg et al., 2007, 2015) are not compatible with our setting where firms use labor as their only input and there is no investment decision.[18] The CES parameters σ_k are identified by the coefficient on log-employment in a regression of log-revenues on log-employment.

The parameters of the hiring costs function (h, γ_1 and γ_2 in equation 6) are identified using information on growth rates of formal firms, and how these depend on firm size. The convexity in hiring is also important for the model to generate dispersion in wages across firms. Therefore, the relationship between wages and size provide useful information on γ_1. The matching function's parameters are identified from worker transitions out of unemployment and into formal and informal employment in the tradable and non-tradable sectors. We estimate those from the monthly employment survey (PME) and annualize the transitions to make them compatible with the model's period.

The exogenous death shocks for formal firms can be identified off the exit rates of very large firms. Because we cannot observe exit rates of informal firms, we set the exogenous death shocks to be equal for formal and informal firms within sectors. The fixed costs of operation of formal firms ($\bar{c}_{kf}$) are disciplined by how exit rates decline with firm size. In addition, average firm-level revenues help the identification of fxed operating costs of formal firms but also those of informal firms ($\bar{c}_{ki}$). Average firm-level revenues are available from PIA, PAS and PAC for formal firms and ECINF (for informal firms). Larger fixed costs force low-revenue firms to exit, and thereby increase average revenues

[18] As a cross-check, we use Olley and Pakes (1996)'s estimator to obtain a measure of firm-level productivity for manufacturing firms and use it to estimate a simple AR(1) process. The estimate for the persistence parameter,ρ_C, is remarkably close to the one we obtain in our Indirect Inference estimator. These results are available upon request.

among survivors.

The fixed cost of exporting, f_x, is identified by the fraction of exporters among formal firms, which is available merging information from RAIS and SECEX. The foreign demand shifter d_F is identified using information on the average size and revenue of exporters (RAIS, PIA and SECEX), fraction of revenues in the tradable sector coming from exports (PIA and SECEX), and the log-wage premium (regression of firm-level log-wages on log-size and exporter indicator, using RAIS and SECEX).

The identification of bargaining power parameters β_f and β_i is straightforward in light of equations (27) and (28). We target the coefficients of a linear regression of firm-level log-wages on revenue per worker (for both the formal and informal sectors, using data from RAIS and ECINF respectively). Lastly, the cost of informality function is identified off the size distribution of firms in the informal sector (ECINF), and the share of informal firms by employment size (ECINF).

5.2 Estimates

Table 7 shows our preliminary estimation results. We now discuss the magitude and plausibility of some of these estimates. First, note that our estimate of θ, the matching function parameter, is larger than CGT's estimate of 1.8 for Colombia and a bit closer to the estimate of 2.16 in Cosar (2013), who focuses on Brazil. Next, notice that the value of leisure is estimated at $b = 4,515$, quite a bit over the annualized value of the minimum wage of $R\$$ 2,880. This relatively high value is necessary for a good fit of wages. Also important for the fit of wages are the bargaining parameters $\beta_f = 0.355$ and $\beta_i = 0.999$. Our estimates point toward a very large bargaining power of informal sector workers. Perhaps surprising, this result is not necessarily unreasonable. Firm-level revenues in the informal sector are typically very low – the geometric mean in the informal C-sector is of \$5,080 – so the model needs to assign a very large bargaining power to informal-sector workers in order to be able to match wages in that sector. Revenues (and revenues per worker) in the informal sector are much lower in the informal sector. The only way the model can assign wages in the informal sector that are closer to those in the data is by assigning a high value of the bargaining power parameter of informal workers. In the formal sector, the value of $\beta_f = 0.355$ is quite close to the value CGT estimate in Colombia, which amounts to 0.4.

The expected cost of informality – as a share of revenue – is different across sectors. For informal firms in the C-sector, we estimate that firms of size 1 face an expected cost of about half their revenue, but this cost does not increase much with size. On the other

hand, the expected cost of informality is negligible for firms of size 1 in the S-sector, but it increases quite steeply with size.

The hiring cost function presents quite a bit of convexity in both sectors ($\gamma_{1C} = 3.28$ and $\gamma_{1S} = 1.81$) and a fair degree of scale economies ($\gamma_{2C} = 0.38$ and $\gamma_{2S} = 0.26$). For comparison, CGT obtain estimates of $\gamma_1 = 3.1$ and $\gamma_2 = 0.39$. To illustrate the magnitude of hiring costs, consider a firm of size 10 in the C-sector. It will cost this firm $R\$394$ to expand to 11 employees, or 0.03 times the annual average wage in the formal sector (which is of $R\$12,230$), $R\$3,830$ to expand to 12, and $R\$77,360$ (or 6 times the annual average wage) to expand to 15. On the other hand, it will cost $R\$36,000$ – or 3 times the annual average wage – for a firm with size 50 to expand to 60 employees.

We note that the fixed costs of operation in the formal sector ($\bar{c}_{Cf} = 3,730$ and $\bar{c}_{Sf} = 2,806$) are much larger than the fixed costs of operation in the informal sector ($\bar{c}_{Ci} = 53$ and $\bar{c}_{Si} = 880$). This is expected, given the large perceived costs of operating a firm in the formal sector (compliance with regulations, bureaucracy, bribes, etc.).

Finally, remember that μ_v, d_{HC}, d_{HS} are actually endogenous objects and not parameters. As we discuss in Appendix B.1, we treat these objects as parameters in the estimation procedure, but penalize the deviations from their equilibrium values in the objective function. What is noteworthy is that, in equilibrium, the vacancy filling rate is $\mu_v = 0.88$, so that firms need to post, on average, 1.13 vacancies to be able to hire 1 worker.

5.3 Model Fit

Tables 8 through 13 compare the moments and statistical relationships generated by the model (under the parameterization described in Tables 6 and 7) with those found in the data. Several features of the data are well matched by the model.

Table 8 shows the allocation of workers across sectors and unemployment, worker transitions from unemployment, and the distribution of firm size for firms in the C and S sectors. The model is able to match all of these moments reasonably well.

Table 9 shows trade-related moments, as well as firm- exit and turnover moments. The model matches well the share of exports in exporters' revenue and the ratio between total exports and total C-sector revenue. On the other hand, the model overestimates a bit the share of sector-C firms that export and the correlation between log-employment and export status. The model matches well average firm exit rates and the fact that exit rates are much larger among smaller firms. However, the model exclusively attributes exit to death shocks for firms that are above the 20^{th} percentile of the size distribution

Table 7: Parameter Estimates

μ^v	Vacancy Filling Rate	0.883
b	Value of Leisure	4,515
β_f	Bargaining Power in the Formal Sector	0.355
β_i	Bargaining Power in the Informal Sector	0.999
θ	Matching Function Parameter	2.949
a_C	Cost of Informality: Intercept, C sector	0.539
b_C	Cost of Informality: Slope, C sector	0.033
a_S	Cost of Informality: Intercept, S sector	0.017
b_S	Cost of Informality: Slope, S sector	0.390
h_{Cf}	Hiring Cost Function, level	14,828
h_{Ci}	Hiring Cost Function, level	20,733
γ_{1C}	Hiring Cost Function, convexity	3.280
γ_{2C}	Hiring Cost Function, scale	0.377
h_{Sf}	Hiring Cost Function, level	5,081
h_{Si}	Hiring Cost Function, level	261.0
γ_{1S}	Hiring Cost Function, convexity	1.809
γ_{2S}	Hiring Cost Function, scale	0.260
d_F	Foreign demand shifter	0.063
f_x	Fixed costs of exporting	69,348
$d_{H,C}$	Domestic demand shifter	7.794
σ_C	CES parameter	4.908
ρ_C^Z	AR(1) process, persistence	0.972
σ_C^Z	AR(1) process, volatility	0.367
α_{Cf}	Death Shocks	0.058
$\bar{c}_{Cf}$	Fixed operating costs	3,730
α_{Ci}	Death Shocks	0.039
$\bar{c}_{Ci}$	fixed operating costs	53.32
$d_{H,S}$	Domestic demand shifter	8.047
σ_S	CES parameter	5.234
ρ_S^Z	AR(1) process, persistence	0.949
σ_S^Z	AR(1) process, volatility	0.350
α_{Sf}	Death Shocks	0.022
$\bar{c}_{Sf}$	Fixed operating costs	2,806
α_{Si}	Death Shocks	0.071
$\bar{c}_{Si}$	fixed operating costs	880.4

(that is, firms of size 3 or larger). The firm-exit rates in the S-sector are generally better matched. Firm-level turnover is measured as the absolute value of the employment growth rates popularized by Davis and Haltiwanger (1990) – so it takes firm exit inton account.[19] The model matches well the fact that larger firms tend to experience lower employment turnover, but that conditional on size, exporters experience larger turnover. The model matches well the pattern of turnover even when we condition on employment expansions

[19]The growth rate in Davis and Haltiwanger (1990) is defined as $g = \frac{x_{t+1}-x_t}{0.5x_{t+1}+0.5x_t}$, so it is well defined when $x_{t+1} = 0$.

or contractions.

Table 10 shows moments related to wages in the formal sector and to the firm-size distribution in the informal sector. The wage moments are generally well matched. Perhaps the exception is the exporter-wage premium, which is underestimated by the model. However, that may be explained by the fact thatour model does not allow for skill heterogeneity and capital. However, in the data exporters tend to be more skill and capital intensive. The distribution of firm-size in the informal sector is very well matched by the model.

Tables 11 and 12 show remaining moments related to firm-level revenues, informal sector wages and firm-level serial correlation of employment and revenues. The model replicates the fact that transitions from unempoyment to the informal sector are much more likely than transitions from unemployment to the formal sector.

Finally, Table 13 shows the fraction of informal firms conditional on firm size, fosr size varying from 1 to 5. The model is able to replicate the fact that this fraction is very large for small firm sizes, but decreases steeply with size.

Table 8: Model Fit – Panel A

Employment Allocations	Dataset	Model	Data
Share of Workers in informal-C	PME	0.045	0.059
Share of Workers in formal-C	PME	0.105	0.106
Share of Workers in informal-S	PME	0.355	0.351
Share of Workers in formal-S	PME	0.319	0.334
Share of Workers in Unemployment	PME	0.177	0.150

Worker Yearly Transition Rates	Dataset	Model	Data
From Unemployment to informal-C	PME	0.027	0.063
From Unemployment to formal-C	PME	0.070	0.050
From Unemployment to informal-S	PME	0.380	0.387
From Unemployment to formal-S	PME	0.194	0.161
From Unemployment to Unemployment	PME	0.329	0.338

Distribution of Employment across formal-C Firms	Dataset	Model	Data
20th Percentile Employment Distribution	RAIS	2	2
40th Percentile Employment Distribution	RAIS	5	4
60th Percentile Employment Distribution	RAIS	9	7
80th Percentile Employment Distribution	RAIS	21	17
90th Percentile Employment Distribution	RAIS	36	35
95th Percentile Employment Distribution	RAIS	61	67
Mean log-employment	RAIS	1.990	1.779
Variance log-employment	RAIS	1.420	1.821
Mean log-employment formal-C | Exporter	RAIS + SECEX	4.285	3.936

Distribution of Employment across formal-S Firms	Dataset	Model	Data
20th Percentile Employment Distribution	RAIS	1	1
40th Percentile Employment Distribution	RAIS	3	2
60th Percentile Employment Distribution	RAIS	4	4
80th Percentile Employment Distribution	RAIS	9	8
90th Percentile Employment Distribution	RAIS	14	14
95th Percentile Employment Distribution	RAIS	24	25
Mean log-employment	RAIS	1.271	1.178
Variance log-employment	RAIS	0.988	1.262

Table 9: Model Fit – Panel B

Trade-Related Moments formal-C	Dataset	Model	Data
Correlation log-employment and Exporter Status	RAIS + SECEX	0.529	0.378
Fraction of formal-C firms that Export	RAIS + SECEX	0.070	0.053
Mean Exports / Revenue | Exporter (firm-level)	SECEX + IBGE	0.264	0.264
Total Exports / Total Revenue formal-C	SECEX + IBGE	0.131	0.136
Firm Exit - formal-C	Dataset	Model	Data
Mean Firm Exit Rate	RAIS	0.101	0.096
Firm Exit Rate Employment $\leq$ 20th Percentile	RAIS	0.235	0.208
Firm Exit Rate Employment 20th-40th Percentile	RAIS	0.058	0.108
Firm Exit Rate Employment 40th-60th Percentile	RAIS	0.058	0.063
Firm Exit Rate Employment 60th-80th Percentile	RAIS	0.058	0.041
Firm Exit Rate Employment 80th-90th Percentile	RAIS	0.058	0.026
Firm Exit Rate Employment 90th-95th Percentile	RAIS	0.058	0.021
Firm Exit Rate Employment $\geq$ 95th Percentile	RAIS	0.058	0.020
Firm Exit - formal-S	Dataset	Model	Data
Mean Firm Exit Rate	RAIS	0.138	0.113
Firm Exit Rate Employment $\leq$ 20th Percentile	RAIS	0.280	0.218
Firm Exit Rate Employment 20th-40th Percentile	RAIS	0.197	0.181
Firm Exit Rate Employment 40th-60th Percentile	RAIS	0.037	0.092
Firm Exit Rate Employment 60th-80th Percentile	RAIS	0.085	0.053
Firm Exit Rate Employment 80th-90th Percentile	RAIS	0.022	0.043
Firm Exit Rate Employment 90th-95th Percentile	RAIS	0.022	0.037
Firm Exit Rate Employment $\geq$ 95th Percentile	RAIS	0.034	0.030
Turnover formal-C Firms	Dataset	Model	Data
Mean Turnover	RAIS	0.343	0.499
Regression Turnover: Constant	RAIS	0.622	0.731
Regression Turnover: log-Employment	RAIS	-0.145	-0.134
Regression Turnover: Exporter Status	RAIS	0.147	0.108
Regression Turnover | Expansion: Constant	RAIS	0.502	0.712
Regression Turnover | Expansion: log-Employment	RAIS	-0.118	-0.146
Regression Turnover | Expansion: Exporter Status	RAIS	0.125	0.141
Regression Turnover | Contraction: Constant	RAIS	0.673	0.730
Regression Turnover | Contraction: log-Employment	RAIS	-0.129	-0.117
Regression Turnover | Contraction: Exporter Status	RAIS	0.285	0.099
Turnover Formal-S Firms	Dataset	Model	Data
Mean Turnover	RAIS	0.437	0.504
Regression Turnover: Constant	RAIS	0.599	0.642
Regression Turnover: log-Employment	RAIS	-0.127	-0.117
Regression Turnover | Expansion: Constant	RAIS	0.596	0.695
Regression Turnover | Expansion: log-Employment	RAIS	-0.153	-0.156
Regression Turnover | Contraction: Constant	RAIS	0.591	0.622
Regression Turnover | Contraction: log-Employment	RAIS	-0.106	-0.095

Table 10: Model Fit – Panel C

Firm-level Wages Formal-C	Dataset	Model	Data
Mean log-Wages	RAIS	8.880	8.637
Mean log-Wages | Exporter	RAIS + SECEX	9.269	9.276
Regression 1 log-Wages: Constant	RAIS	8.671	8.443
Regression 1 log-Wages: log-Employment	RAIS	0.099	0.094
Regression 1 log-Wages: Exporter Status	RAIS	0.174	0.462
Regression 2 log-Wage: Constant	IBGE	2.826	6.334
Regression 2 log-Wage: log-Revenue/Worker	IBGE	0.620	0.235

Firm-level Wages Formal-S	Dataset	Model	Data
Mean log-Wages	RAIS	8.761	8.562
Regression 1 log-Wages: Constant	RAIS	8.601	8.434
Regression 1 log-Wages: log-Employment	RAIS	0.126	0.108
Regression 2 log-Wage: Constant	IBGE	3.143	7.417
Regression 2 log-Wage: log-Revenue/Worker	IBGE	0.586	0.109

Distribution of Employment across Informal-C Firms (truncated at size 5)	Dataset	Model	Data
20th Percentile Employment Distribution	ECINF	1	1
40th Percentile Employment Distribution	ECINF	1	1
60th Percentile Employment Distribution	ECINF	1	1
80th Percentile Employment Distribution	ECINF	1	1
90th Percentile Employment Distribution	ECINF	2	2
95th Percentile Employment Distribution	ECINF	2	2
99th Percentile Employment Distribution	ECINF	4	4
Mean log-Employment	ECINF	0.096	0.105
Variance log-Employment	ECINF	0.093	0.092

Distribution of Employment across Informal-S Firms (truncated at size 5)	Dataset	Model	Data
20th Percentile Employment Distribution	ECINF	1	1
40th Percentile Employment Distribution	ECINF	1	1
60th Percentile Employment Distribution	ECINF	1	1
80th Percentile Employment Distribution	ECINF	1	1
90th Percentile Employment Distribution	ECINF	2	2
95th Percentile Employment Distribution	ECINF	2	2
99th Percentile Employment Distribution	ECINF	3	3
Mean log-Employment	ECINF	0.086	0.097
Variance log-Employment	ECINF	0.061	0.075

Table 11: Model Fit – Panel D

Firm-level Revenues informal-C	Dataset	Model	Data
Mean log-Revenue	ECINF	8.720	8.533
Variance log-Revenue	ECINF	0.585	1.444
Correlation log-Revenue and log-Employment	ECINF	0.641	0.339
Firm-level Revenues informal-S	Dataset	Model	Data
Mean log-Revenue	ECINF	8.733	8.952
Variance log-Revenue	ECINF	0.409	1.298
Correlation log-Revenue and log-Employment	ECINF	0.747	0.318
Firm-level Wages Informal-C	Dataset	Model	Data
Mean log-Wages	ECINF	7.850	8.014
Regression 1 log-Wage: Constant	ECINF	7.809	8.006
Regression 1 log-Wage: log-Employment	ECINF	0.427	0.079
Regression 2 log-Wage: Constant	ECINF	0.147	3.777
Regression 2 log-Wage: log-Revenue/Worker	ECINF	0.893	0.397
Firm-level Wages Informal-S	Dataset	Model	Data
Mean log-Wages	ECINF	8.357	8.415
Regression 1 log-Wage: Constant	ECINF	8.348	8.413
Regression 1 log-Wage: log-Employment	ECINF	0.103	0.020
Regression 2 log-Wage: Constant	ECINF	0.122	3.912
Regression 2 log-Wage: log-Revenue/Worker	ECINF	0.952	0.379

Table 12: Model Fit – Panel E

Firm-level Revenues formal-C	Dataset	Model	Data
Mean log-Revenue	IBGE	11.747	12.726
Variance log-Revenue	IBGE	2.084	3.511
Mean log-Revenue | Exporter	IBGE	14.640	15.465
Variance log-Revenue | Exporter	IBGE	0.223	4.448
Regression log-Revenue: Constant	IBGE	9.501	10.118
Regression log-Revenue: log-Employment	IBGE	1.116	1.000
Regression log-Revenue: Exporter	IBGE	0.356	1.462
Firm-level Revenues formal-S	Dataset	Model	Data
Mean log-Revenue	IBGE	10.851	10.814
Variance log-Revenue	IBGE	1.387	2.074
Regression log-Revenue: Constant	IBGE	9.382	10.004
Regression log-Revenue: log-Employment	IBGE	1.156	0.872
Longitudinal Relationships	Dataset	Model	Data
Correlation log-Emp(t) and log-Emp($t+1$) – formal-C	RAIS	0.922	0.918
Correlation log-Emp(t) and log-Emp($t+1$) – formal-S	RAIS	0.906	0.908
Correlation log-Revenue(t) and log-Revenue($t+1$) – formal-C	IBGE	0.884	0.929
Correlation log-Revenue(t) and log-Revenue($t+1$) – formal-S	IBGE	0.673	0.845
Miscellaneous	Dataset	Model	Data
Transitions from U to Informal / Transitions from U to formal	PME	1.541	2.130
Share of Informal Employed Workers	PME	0.485	0.482

Table 13: Model Fit – Panel F

Fraction of Informal Firms – C and S Sectors Pooled	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.956	0.933
Among Firms with 2 Workers	ECINF	0.877	0.711
Among Firms with 3 Workers	ECINF	0.340	0.491
Among Firms with 4 Workers	ECINF	0.117	0.261
Among Firms with 5 Workers	ECINF	0.055	0.372
Fraction of Informal Firms – C Sector Firms Only	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.985	0.962
Among Firms with 2 Workers	ECINF	0.641	0.755
Among Firms with 3 Workers	ECINF	0.544	0.411
Among Firms with 4 Workers	ECINF	0.568	0.508
Among Firms with 5 Workers	ECINF	0.539	0.497
Fraction of Informal Firms – S Sector Firms Only	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.954	0.929
Among Firms with 2 Workers	ECINF	0.901	0.705
Among Firms with 3 Workers	ECINF	0.330	0.509
Among Firms with 4 Workers	ECINF	0	0.218
Among Firms with 5 Workers	ECINF	0	0.341

6 Counterfactual Simulations

TBD

7 Final Remarks

TBD

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Appendix

A Model Appendix

A.1 Equilibrium Conditions

A.1.1 Steady-state distribution of states

Let $\widetilde{\widetilde{\psi}}_{ki}(z',\ell)$ be the **mass** distribution of informal firms across the (z',ℓ) spectrum.

$$\begin{aligned}\widetilde{\widetilde{\psi}}_{ki}(z',\ell) &= \mathbf{1}[\ell=1]\times M_{ki}\times\psi^{e}_{ki}(z') \\ &\quad+\mathbf{1}[\ell\geq 1]\times(1-\alpha_{ki})\,N_{ki}\times\left(\int_{z}\psi_{ki}(z,\ell)\,I_{k}^{stay}(z,\ell,i)\,g_{k}(z'|z)dz\right)\end{aligned} \tag{A.1}$$

where $\psi^{e}_{ki}(z')=\frac{\int_{\nu}I_{k}^{informal}(\nu)g_{k}(z'|\nu)g_{k}^{e}(\nu)d\nu}{\int_{z'}\int_{\nu}I_{k}^{informal}(\nu)g_{k}(z'|\nu)g_{k}^{e}(\nu)d\nu dz'}$ is the density of z' among entrants in the k informal sector; M_{ki} is the mass of entrants into the k informal sector; N_{ki} is the mass of sector k informal incumbents who started the period and $g_{k}^{e}(.)$ is the ergodic distribution of z. M_{ki} and N_{ki} are also equilibrium objects.

For expositional purposes, it is useful to define an interim period, which corresponds to the intra-period adjustment window, when the firm has already drawn its new productivity, z', but has not yet adjusted its labor force. In steady state, the total mass of firms is equal to N_{ki} in the beginning of the period as well as in the interim stage and end of period, so the integral of $\widetilde{\widetilde{\psi}}_{ki}$ sums to N_{ki}. The interim distribution is given by:

$$\widetilde{\psi}_{ki}(z',\ell)=\frac{\widetilde{\widetilde{\psi}}_{ki}(z',\ell)}{\int_{z'}\int_{\ell}\widetilde{\widetilde{\psi}}_{ki}(z',\ell)\,dz'd\ell}=\frac{\widetilde{\widetilde{\psi}}_{ki}(z',\ell)}{N_{ki}} \tag{A.2}$$

$$\begin{aligned}&=\mathbf{1}[\ell=1]\times\frac{M_{ki}}{N_{ki}}\times\psi^{e}_{ki}(z') \\ &\quad+\mathbf{1}[\ell\geq 1]\times(1-\alpha_{ki})\times\left(\int_{z}\psi_{ki}(z,\ell)\,I_{k}^{stay}(z,\ell,i)\,g_{k}(z'|z)dz\right)\end{aligned}$$

The end-of-period distribution reproduces the start-of-period distribution:

$$\psi_{ki}(z',\ell')=\frac{\int_{\ell}\widetilde{\psi}_{ki}(z',\ell)\,I\left(L_{k}(z',\ell,i)=\ell'\right)d\ell}{\int_{z'}\int_{\ell}\widetilde{\psi}_{ki}(z',\ell)\,I\left(L_{k}(z',\ell,i)=\ell'\right)d\ell dz'} \tag{A.3}$$

$$=\int_{\ell}\widetilde{\psi}_{ki}(z',\ell)\,I\left(L_{k}(z',\ell,i)=\ell'\right)d\ell$$

where the last equality follows from $\int_{z'}\int_{\ell}\widetilde{\psi}_{ki}(z',\ell)\,I\left(L_{k}(z',\ell,i)=\ell'\right)d\ell dz'=1$.

When considering the distribution of states across formal firms, we need to take into account that some informal firms change their status to formal. Let $\widetilde{\widetilde{\psi}}_{kf}(z', \ell)$ be the **mass** distribution of formal firms across the (z', ℓ) spectrum.

$$\widetilde{\widetilde{\psi}}_{kf}(z', \ell) = \begin{array}{c} \mathbf{1}[\ell = 1] \times M_{kf} \times \psi^e_{kf}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1 - \alpha_{kf}) N_{kf} \times \left(\int_z \psi_{kf}(z, \ell) I^{stay}_k(z, \ell, f) g_k(z'|z) dz\right) + \\ (1 - \alpha_{ki}) N_{ki} \times \left(\int_z \psi_{ki}(z, \ell) I^{change}_k(z, \ell, i) g_k(z'|z) dz\right) \end{array} \right) \end{array} \tag{A.4}$$

Where $\psi^e_{kf}(z') = \frac{\int_\nu I^{formal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu}{\int_{z'} \int_\nu I^{formal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu dz'}$ is the density of z' among entrants in the k formal sector; M_{kf} is the mass of entrants into the k formal sector; and N_{ki} is the mass of incumbents who started the period. M_{kf} and N_{kf} are also equilibrium objects. In steady state, the integral of $\widetilde{\widetilde{\psi}}_{kf}$ sums to N_{kf}. The interim distribution is given by:

$$\widetilde{\psi}_{kf}(z', \ell) = \frac{\widetilde{\widetilde{\psi}}_{kf}(z', \ell)}{\int_{z'} \int_\ell \widetilde{\widetilde{\psi}}_{kf}(z', \ell) dz' d\ell} = \frac{\widetilde{\widetilde{\psi}}_{kf}(z', \ell)}{N_{kf}}$$

$$\widetilde{\psi}_{kf}(z', \ell) = \begin{array}{c} \mathbf{1}[\ell = 1] \times \frac{M_{kf}}{N_{kf}} \times \psi^e_{kf}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1 - \alpha_{kf}) \times \left(\int_z \psi_{kf}(z, \ell) I^{stay}_k(z, \ell, f) g_k(z'|z) dz\right) + \\ (1 - \alpha_{ki}) \frac{N_{ki}}{N_{kf}} \times \left(\int_z \psi_{ki}(z, \ell) I^{change}_k(z, \ell, i) g_k(z'|z) dz\right) \end{array} \right) \end{array} \tag{A.5}$$

The end-of-period distribution reproduces the start-of-period distribution:

$$\psi_{kf}(z', \ell') = \frac{\int_\ell \widetilde{\psi}_{kf}(z', \ell) I(L_k(z', \ell, f) = \ell') d\ell}{\int_{z'} \int_\ell \widetilde{\psi}_{kf}(z', \ell) I(L_k(z', \ell, f) = \ell') d\ell dz'} \tag{A.6}$$

$$= \int_\ell \widetilde{\psi}_{kf}(z', \ell) I(L_k(z', \ell, f) = \ell') d\ell,$$

where the last equatlity follows from $\int_{z'} \int_\ell \widetilde{\psi}_{kf}(z', \ell) I(L_k(z', \ell, f) = \ell') d\ell dz' = 1$.

A.1.2 Entry

Let M_k denote the mass of entrants in sector $k = C, S$. The fraction of entrants into the formal and informal sectors are given respectively by ω_{kf} and ω_{ki}:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(\nu) = 1\right) = \int g^e_k(\nu) d\nu \tag{A.7}$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(\nu) = 1\right) = \int g^e_k(\nu) d\nu \tag{A.8}$$

Therefore, the masses of entrants in the formal and informal sectors are given by:

$$M_{ki} = \omega_{ki} M_k \tag{A.9}$$
$$M_{kf} = \omega_{kf} M_k \tag{A.10}$$

The masses of entrants into each sector, M_k, are pinned down by the free entry condition (assuming positive entry in both sectors):

$$c_{e,k} = V_k^e = \int \left[V_k^e(\nu, i) I(\overline{\nu}_{kf} > \nu \geq \overline{\nu}_{ki}) + V_k^e(\nu, f) I(\nu \geq \overline{\nu}_{kf}) \right] g_k^e(\nu) d\nu \tag{A.11}$$

A.1.3 Flow conditions for workers and firms

In order to write the labor market clearing conditions, we first define the following quantities:

- Number of workers in the **beginning** of the period in sector k, working in formal or informal firms (T stands for "total"):

$$W_{kj}^T = N_{kj} \underbrace{\int_z \int_\ell \ell \psi_{kj}(z, \ell) d\ell dz}_{\text{avg. \# of workers per firm}} = L_{kj} \tag{A.12}$$

  for $j = f, i$ and $k = C, S$.

- Number of workers in sector (k, j) who are fired because their firms receive a death shock:

$$W_{kj}^{DS} = \alpha_{kj} N_{kj} \int_z \int_\ell \ell \psi_{kj}(z, \ell) d\ell dz = \alpha_{kj} L_{kj} \tag{A.13}$$

- Number of workers in sector (k, j) who are fired due to **endogenous** firm exit:

$$W_{kj}^{EE} = (1 - \alpha_{kj}) N_{kj} \times \int_z \int_\ell \ell \psi_{kj}(z, \ell) I_k^{exit}(z, \ell, j) d\ell dz \tag{A.14}$$

  where $(1 - \alpha_{kj}) N_{kj}$ is the mass of firms that survive after the death shock hits.

- Number (mass) of **surviving incumbent** firms in sector (k, j) in the interim period:

$$N'_{kj} = (1 - \alpha_{kj}) N_{kj} \int_z \int_\ell \psi_{kj}(z, \ell) I_k^{stay}(z, \ell, j) d\ell dz \tag{A.15}$$

- Number of workers initially in sector (k, j) who are fired due to downsizing at the interim stage:

$$W_{kj}^D = N'_{kj} \int_{z'} \int_\ell \widetilde{\psi}_{kj}^{incumbent}(z', \ell) \left(1 - I_k^{hire}(z', \ell, j)\right) (\ell - L_k(z', \ell, j)) dz' d\ell \tag{A.16}$$

where $\widetilde{\psi}_{kj}^{incumbent}(z',\ell)$ is the distribution of states in the interim stage among **surviving incumbents**. Note that this is not the same distribution as $\widetilde{\psi}_{kj}(z',\ell)$ as it does not include entrants. It is obtained as follows:

$$\begin{aligned}\widetilde{\psi}_{kj}^{incumbent}(z',\ell) &= \frac{(1-\alpha_{kj})N_{kj}}{N'_{kj}}\int_z \psi_{kj}(z,\ell)\,I_k^{stay}(z,\ell,j)\,g_k(z'|z)\,dz \\ &= \frac{\int_z \psi_{kj}(z,\ell)\,I_k^{stay}(z,\ell,j)\,g_k(z'|z)\,dz}{\int_z\int_\ell \psi_{kj}(z,\ell)\,I_k^{stay}(z,\ell,j)\,d\ell dz}\end{aligned} \tag{A.17}$$

- Total fraction of workers in the formal sector of sector k who are laid off, conditional on starting the period in a formal firm in sector k :

$$\begin{aligned}\chi_{kf}^{layoff} &= \frac{W_{kf}^{DS}+W_{kf}^{EE}+W_{kf}^{D}}{W_{kf}^{T}} \\ &= \alpha_{kf} + \frac{\left(\begin{array}{c}(1-\alpha_{kf})\int_z\int_\ell \ell\psi_{kf}(z,\ell)\,I_k^{exit}(z,\ell,f)\,d\ell dz \\ (1-\alpha_{kf})\left(\int_z\int_\ell \psi_{kf}(z,\ell)\,I_k^{stay}(z,\ell,f)\,d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{kf}^{incumbent}(z',\ell)\left(1-I_k^{hire}(z',\ell,f)\right)(\ell-L_k(z',\ell,f))\,dz'd\ell\end{array}\right)}{\int_z\int_\ell \ell\psi_{kf}(z,\ell)\,d\ell d}\end{aligned} \tag{A.18}$$

- Number of firms that start the period as informal firms, but end the period as formal firms (because they formalized).

$$N'_{ki\to f} = (1-\alpha_{ki})N_{ki}\int_z\int_\ell \psi_{ki}(z,\ell)\,I_k^{change}(z,\ell,i)\,d\ell dz \tag{A.19}$$

where $(1-\alpha_{ki})N_{ki}$ is the mass of firms that survive after the death shock hits.

- Distribution of states among firms that switched from informal to formal, in the interim period

$$\begin{aligned}\widetilde{\psi}_{ki\to f}(z',\ell) &= \frac{(1-\alpha_{ki})N_{ki}}{N'_{ki\to f}}\int_z \psi_{ki}(z,\ell)\,I_k^{change}(z,\ell,i)\,g_k(z'|z)\,dz \\ &= \frac{\int_z \psi_{ki}(z,\ell)\,I_k^{change}(z,\ell,i)\,g_k(z'|z)\,dz}{\int_z\int_\ell \psi_{ki}(z,\ell)\,I_k^{change}(z,\ell,i)\,d\ell dz}\end{aligned} \tag{A.20}$$

- Number of workers who started the period in informal firms, but end the period in formal firms (their employers switched to formal, and they were not fired after the interim productivity was realized):

$$W_{k,i\to f} = N'_{ki\to f}\int_{z'}\int_\ell \widetilde{\psi}_{ki\to f}(z',\ell)\left(\begin{array}{c}\ell\times I^{hire}(z',\ell,f)+ \\ L_k(z',\ell,f)\times\left(1-I^{hire}(z',\ell,f)\right)\end{array}\right)d\ell dz' \tag{A.21}$$

- Fraction of workers who start the period in informal firms, but end the period in

formal firms:

$$\chi_{ki\to f}^{change} = \frac{W_{k,i\to f}}{W_{ki}^{T}}$$
$$= \frac{\left(\begin{array}{c} (1-\alpha_{ki})\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{ki\to f}(z',\ell)\left(\begin{array}{c} \ell\times I^{hire}(z',\ell,f)+ \\ L_k(z',\ell,f)\times\left(1-I^{hire}(z',\ell,f)\right)\end{array}\right) d\ell dz' \end{array}\right)}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, d\ell dz} \quad \text{(A.22)}$$

- Number of workers who start the period in informal firms, but their employers switched to formal status:

$$W_{ki}^{SF} = (1-\alpha_{ki})\, N_{ki}\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz \quad \text{(A.23)}$$

- Fraction of workers who start employed in the informal sector and leave it in the interim period (became unemployed or employer switched to formal):

$$\chi_{ki}^{leave} = \frac{W_{ki}^{DS}+W_{ki}^{EE}+W_{ki}^{SF}+W_{ki}^{D}}{W_{ki}^{T}}$$
$$= \alpha_{ki} + \frac{\left(\begin{array}{c} (1-\alpha_{ki})\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{exit}(z,\ell,i)\, d\ell dz+ \\ (1-\alpha_{ki})\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz+ \\ (1-\alpha_{ki})\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{stay}(z,\ell,i)\, d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{ki}^{incumbent}(z',\ell)\left(1-I_k^{hire}(z',\ell,i)\right)\left(\ell - L_k(z',\ell,i)\right) dz' d\ell \end{array}\right)}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, d\ell dz} \quad \text{(A.24)}$$

With these objects, we can define the equilibrium conditions that refer to labor market flows:

$$\chi_{ki}^{leave} L_{ki} = L_u \mu_{ki}^{e} \quad \text{(A.25)}$$

$$\chi_{kf}^{layoff} L_{kf} = L_u \mu_{kf}^{e} + L_{ki}\chi_{ki\to f}^{change} \quad \text{(A.26)}$$

$$\sum_{k,j}\mu_{kj}^{e} L_u = \sum_{k,j}\left(W_{kj}^{DS}+W_{kj}^{EE}+W_{kj}^{D}\right)+\left(W_{ki}^{SF}-W_{k,i\to f}\right), \quad \text{(A.27)}$$

where $L_u = U$ is the mass of workers who start the period unemployed, and the last term in equation (A.27) represents the informal switchers who are fired after their employers decide to formalize.

These conditions state that the mass of workers in each sector (k, j) cannot be contracting or expanding in equilibrium (expressions (A.25) and (A.26)), while expression (A.27) states that the mass of unemployed workers must be constant in steady state.

Finally, the sum of unemployment and employment levels across sectors equals the total labor force $\overline{L}$:

$$L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u = \overline{L}. \tag{A.28}$$

We can proceed in a similar way to define the equilibrium flow conditions for firms. The relevant objects are the following:

- Fraction of formal firms exiting sector k:

$$\varrho_{kf}^{exit} = \alpha_{kf} + (1 - \alpha_{kf}) \int_z \int_\ell I_k^{exit}(z, \ell, f)\, \psi_{kf}(z, \ell)\, d\ell dz \tag{A.29}$$

- Fraction of informal firms exiting sector k:

$$\varrho_{ki}^{exit} = \alpha_{ki} + (1 - \alpha_{ki}) \int_z \int_\ell \Big(I_k^{exit}(z, \ell, i) + I_k^{change}(z, \ell, i) \Big)\, \psi_{ki}(z, \ell)\, d\ell dz \tag{A.30}$$

- Fraction of informal firms changing status in sector k:

$$\varrho_{ki}^{change} = (1 - \alpha_{ki}) \int_z \int_\ell I_k^{change}(z, \ell, i)\, \psi_{ki}(z, \ell)\, d\ell dz \tag{A.31}$$

Similarly to workers, the mass of firms in each sector (k, j) must be constant in steady state. This means that the inflow of firms must equal the outflow, which can be written as:

$$\varrho_{kf}^{exit} N_{kf} = M_{kf} + \varrho_{ki}^{change} N_{ki} \tag{A.32}$$

$$\varrho_{ki}^{exit} N_{ki} = M_{ki}. \tag{A.33}$$

A.1.4 Vacancies

Aggregate vacancies in sector kj are given by:

$$V_{kj} = N_{kj} \int_{z'} \int_\ell v_{kj}(z', \ell)\, \widetilde{\psi}_{kj}(z', \ell)\, d\ell dz' + \frac{M_{kj}}{\mu_{kj}^v}, \tag{A.34}$$

where $v_{kj}(z', \ell)$ is the number of vacancies a firm with shock z' and labor force ℓ posts.

A.1.5 Unemployment Benefits / Tax Collection / Transfers

- We assume that all government revenue G (taxes and firing costs) that are not spent in unemployment benefits – T – are rebated to consumers.

$$G - \underbrace{\left(b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D} \right)}_{\text{mass of } \textbf{formal} \text{ workers who transition to unemployment}} \right)}_{\text{Total Expenditure with Unemployment Benefits}} \tag{A.35}$$

$$= T$$

$$\begin{aligned} G = & \sum_k N_{kf} \tau_y \int_z \int_\ell R_k (z, \ell) \, \psi_{kf} (z, \ell) \, d\ell dz \\ & + \sum_k N_{kf} \tau_w \int_z \int_\ell \max \left\{ w_{kf} (z, \ell), \underline{w} \right\} \ell \psi_{kf} (z, \ell) \, d\ell dz \\ & + \sum_k N_{ki} \int_z \int_\ell p_{ki} (\ell) \, R_k (z, \ell) \, \psi_{ki} (z, \ell) \, d\ell dz \\ & + \sum_k N_{kf} \kappa \int_{z'} \int_\ell \widetilde{\psi}_{kf} (z', \ell) \left(\ell - L_k (z', \ell, f) \right) \left(1 - I^{hire} (z', \ell, f) \right) d\ell dz' \\ & + (\tau_a - 1) \frac{D_{H,C} \left(\epsilon \tau_a \tau_c \right)^{1-\sigma}}{\tau_a} \end{aligned} \tag{A.36}$$

A.1.6 Aggregate Income

Aggregate income is given by total wages, government transfers and total profits:

$$\begin{aligned}
I = & \sum_k N_{ki} \int_z \int_\ell w_{ki}(z,\ell)\,\ell\psi_{ki}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf} \int_z \int_\ell \max\left\{w_{kf}(z,\ell), \underline{w}\right\}\ell\psi_{kf}(z,\ell)\,d\ell dz \\
& + G \\
& + \sum_k N_{ki} \int_z \int_\ell \widetilde{\pi}_{ki}(z,\ell)\,\psi_{ki}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf} \int_z \int_\ell \widetilde{\pi}_{kf}(z,\ell)\,\psi_{kf}(z,\ell)\,d\ell dz \\
& - \sum_k N_{kf}\kappa \int_{z'} \int_\ell \widetilde{\psi}_{kf}(z',\ell)\left(\ell - L_k(z',\ell,f)\right)\left(1 - I^{hire}(z',\ell,f)\right) d\ell dz' \\
& - \sum_{k=C,S\ ;\ j=i,f} \left(N_{kj}\overline{H}_{kj} + M_{kj}K_{kj}\right) - \sum_{k=C,S} \varrho_{ki}^{change} N_{ki}\left(K_{kf} - K_{ki}\right) \\
& - \sum_{k=C,S} M_k c_{e,k},
\end{aligned} \tag{A.37}$$

where profits $\widetilde{\pi}$ are computed before subtracting hiring costs. For the equations above, note that the number of firms in sector (k,j) in the interim stage is given by $N'_{ki} + M_{ki} = N_{ki}$ and $N'_{kf} + M_{kf} + \varrho_{ki}^{change} N_{ki} = N_{kf}$ (in steady state).

A.1.7 Service Sector Market Clearing

Service sector goods are used for final consumption (consumers spend $(1-\zeta)\,I$ on it), and as inputs for hiring costs, fixed costs and entry costs (and fixed costs of exporting). The average hiring costs in sector (k,j):

$$\overline{H}_{kj} = \int_{z'} \int_\ell H_{kj}\left(\ell, L_k(z',\ell,j)\right) I^{hire}(z',\ell,j)\,\widetilde{\psi}_{kj}(z',\ell)\,d\ell dz', \tag{A.38}$$

and the fraction of tradable-sector goods firms that export is given by

$$\mu_x = \int_z \int_\ell \psi(z,\ell)\, I^x(z,\ell)\,d\ell dz \tag{A.39}$$

Therefore, we can write the total expenditure on service sector goods as follows:

$$\begin{aligned}
X_S = (1-\zeta)\,I + & \sum_{k=C,S\ ;\ j=i,f} \left(N_{kj}\left(\overline{H}_{kj} + \overline{c}_{kj}\right) + M_{kj}K_{kj}\right) + \\
& \sum_{k=C,S} \varrho_{ki}^{change} N_{ki}\left(K_{kf} - K_{ki}\right) + N_{Cf}\mu_x f_x + \sum_{k=C,S} M_k c_{e,k}
\end{aligned} \tag{A.40}$$

A.1.8 Trade Balance

Trade balance implies that total imports must equal total exports, which is given by:

$$\frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma}}{\tau_a} = \epsilon D_F^* P_{C,X}^{*1-\sigma} \tag{A.41}$$

B Estimation Appendix

B.1 Estimation Algorithm

In this section we describe the estimation algorithm in detail, which we break down into several steps for expositional clarity.

$d_{H,C}$, $d_{H,S}$, and μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} are treated as parameters to be estimated along with the remaining ones, but these are endogenous variables. The procedure makes sure that the value guessed for $d_{H,C}$ is the outcome of the equilibrium (see Step 7 for details). Deviations from remaining equilibrium conditions will be penalized, forcing the outcome of the optimization algorithm to choose values of $d_{H,S}$, and μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} consistent with the equilibrium. We will treat μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} as separate parameters to be estimated, but they are all tied by $\mu^v_{kj} = \xi_{kj}\mu^v$. This approach guarantees that we will never end up with a probability of filling a vacancy μ^v_{kj} that is larger than 1.

Step 1: Start with a parameter guess Θ, including values for $d_{H,C}$, $d_{H,S}$, μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf} and μ^v_{Si}, with $0 \leq \mu^v_{kj} \leq 1$. Obtain d_F using

$$\frac{R^{exports}\left(z,\ell\right)}{R\left(z,\ell\right)} = \left(1-\exp\left(-\sigma d_F\right)\right)$$

$$\Rightarrow d_F = \frac{1}{\sigma_C}\log\left(1 - E\left(\frac{R^{exports}}{R}\right)_{Data}\right)$$

$E\left(\frac{R^{exports}}{R}\right)_{Data}$ is the average fraction of revenues coming from exports in the data, at the firm level. In other words, we match that moment exactly.

Step 2: Given that $\xi_{Cf} = 1$, set

$$\mu^v = \mu^v_{Cf},$$

and recover the matching function parameter

$$\xi_{kj} = \frac{\mu^v_{kj}}{\mu^v}$$

for $kj \in \{Ci, Sf, Si\}$.

Step 3: Compute revenue functions $R_k\left(z,\ell\right)$, and compute wage schedules.

$$w_{kf}\left(z,\ell\right) = \frac{\left(1-\beta_f\right)\left(b+b^u\right)}{1+\beta_f\tau_w} + \frac{\beta_f\left(1-\tau_y\right)}{1+\beta_f\tau_w}\frac{R_k\left(z,\ell\right)}{\ell} - \frac{\beta_f}{1+\beta_f\tau_w}\frac{\bar{c}_{kf}}{\ell}$$

$$
\begin{aligned}
w_{kf}\left(z,\ell;\underline{w}_f\right) &= \max\left\{w_{kf}\left(z,\ell\right),\underline{w}_f\right\}\\
w_{ki}\left(z,\ell\right) &= \left(1-\beta_i\right)b+\beta_i\left(1-p_{ki}\left(\ell\right)\right)\frac{R_k\left(z,\ell\right)}{\ell}-\beta_i\frac{\overline{c}_{ki}}{\ell}\\
w_{ki}\left(z,\ell;\underline{w}_i\right) &= \max\left\{w_{ki}\left(z,\ell\right),\underline{w}_i\right\}
\end{aligned}
$$

Where $\underline{w}_f$ is the minimum wage in the formal sector (which is observed and fixed throughout estimation). $\underline{w}_i$ is the first percentile of the distribution of informal wages in PME and fixed throughout the estimation procedure. This is to avoid zero or negative informal wages.

Step 4: Compute firms' value functions. Obtain firms' policy functions. Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$
\begin{aligned}
\omega_{kf} &\equiv \Pr\left(I_k^{formal}\left(\nu\right)=1\right) = \int I_k^{formal}\left(\nu\right)g_k^e\left(\nu\right)d\nu\\
\omega_{ki} &\equiv \Pr\left(I_k^{informal}\left(\nu\right)=1\right) = \int I_k^{informal}\left(\nu\right)g_k^e\left(\nu\right)d\nu
\end{aligned}
$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$
\begin{aligned}
M_{ki} &= \omega_{ki}M_k\\
M_{kf} &= \omega_{kf}M_k
\end{aligned}
$$

Step 5: Obtain the entry costs $c_{e,k}$ $(k=C,S)$:

$$
c_{e,k}=V_k^e=\int\left[\left(V_k^e\left(\nu,i\right)-K_{Ci}\right)I_k^{informal}\left(\nu\right)+\left(V_k^e\left(\nu,f\right)-K_{Cf}\right)I_k^{formal}\left(\nu\right)\right]g_k^e\left(\nu\right)d\nu
$$

These costs will be subtracted from aggregate income, and will be added to the expenditure on S-sector goods.

Step 6: Compute the steady state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$
\psi_{ki}^e\left(z'\right)=\frac{\int g_k\left(z'|\nu\right)g_k^e\left(\nu\right)I_k^{informal}\left(\nu\right)d\nu}{\int_{\widetilde{z}}\int_\nu g_k\left(\widetilde{z}|\nu\right)g_k^e\left(\nu\right)I_k^{informal}\left(\nu\right)d\nu d\widetilde{z}}
$$

$$
\varrho_{ki}^{exit}=\alpha_{ki}+\left(1-\alpha_{ki}\right)\int_z\int_\ell\left(I_k^{exit}\left(z,\ell,i\right)+I_k^{change}\left(z,\ell,i\right)\right)\psi_{ki}\left(z,\ell\right)d\ell dz
$$

In steady state $N_{ki}=\left(1-\varrho_{ki}^{exit}\right)N_{ki}+M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$
\boxed{\frac{M_{ki}}{N_{ki}}=\varrho_{ki}^{exit}=\frac{\omega_{ki}M_k}{N_{ki}}.}
$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\widetilde{\psi}_{ki}\left(z', \ell\right) = \mathbf{1}\left[\ell = 1\right] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}\left(z'\right) \\ + \mathbf{1}\left[\ell \geq 1\right] \times \left(1 - \alpha_{ki}\right) \times \left(\int_{z} \psi_{ki}\left(z, \ell\right) I_{k}^{stay}\left(z, \ell, i\right) g_{k}(z'|z) dz\right)$$

Update ψ_{ki} with

$$\psi_{ki}\left(z', \ell'\right) = \frac{\int_{\ell} \widetilde{\psi}_{ki}\left(z', \ell\right) I\left(L_{k}\left(z', \ell, i\right) = \ell'\right) d\ell}{\int_{\widetilde{z}} \int_{\ell} \widetilde{\psi}_{ki}\left(\widetilde{z}, \ell\right) I\left(L_{k}\left(\widetilde{z}, \ell, i\right) = \ell'\right) d\ell d\widetilde{z}}$$

And repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute

$$\psi_{kf}^{e}\left(z'\right) = \frac{\int g_{k}\left(z'|\nu\right) g_{k}^{e}\left(\nu\right) I_{k}^{formal}\left(\nu\right) d\nu}{\int_{\widetilde{z}} \int_{\nu} g_{k}\left(\widetilde{z}|\nu\right) g_{k}^{e}\left(\nu\right) I_{k}^{formal}\left(\nu\right) d\nu d\widetilde{z}}$$

$$\varrho_{kf}^{exit} = \alpha_{kf} + \left(1 - \alpha_{kf}\right) \int_{z} \int_{\ell} I_{k}^{exit}\left(z, \ell, f\right) \psi_{kf}\left(z, \ell\right) d\ell dz$$

$$\varrho_{ki}^{change} = \left(1 - \alpha_{ki}\right) \int_{z} \int_{\ell} I_{k}^{change}\left(z, \ell, i\right) \psi_{ki}\left(z, \ell\right) d\ell dz$$

In steady state

$$\varrho_{kf}^{exit} N_{kf} = \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_{k}}{\varrho_{ki}^{exit}}} + \omega_{kf} M_{k} \\ = M_{k} \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}\right)$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_{k} \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}}=\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki}+\varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\boxed{\widetilde{\psi}_{kf}\left(z',\ell\right)=\begin{array}{c}\mathbf{1}\left[\ell=1\right]\times\underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki}+\omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}}\times\psi_{kf}^{e}\left(z'\right)+\\ \mathbf{1}\left[\ell\geq 1\right]\times\left(\begin{array}{c}\left(1-\alpha_{kf}\right)\times\left(\int_{z}\psi_{kf}\left(z,\ell\right)I_{k}^{stay}\left(z,\ell,f\right)g_{k}(z'|z)dz\right)+\\ \left(1-\alpha_{ki}\right)\underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki}+\varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}}\times\\ \left(\int_{z}\psi_{ki}\left(z,\ell\right)I_{k}^{change}\left(z,\ell,i\right)g_{k}(z'|z)dz\right)\end{array}\right)\end{array}}$$

Update ψ_{kf} with:

$$\psi_{kf}\left(z',\ell'\right)=\frac{\int_{\ell}\widetilde{\psi}_{kf}\left(z',\ell\right)I\left(L_{k}\left(z',\ell,f\right)=\ell'\right)d\ell}{\int_{\widetilde{z}}\int_{\ell}\widetilde{\psi}_{kf}\left(\widetilde{z},\ell\right)I\left(L_{k}\left(\widetilde{z},\ell,f\right)=\ell'\right)d\ell d\widetilde{z}}$$

$$=\int_{\ell}\widetilde{\psi}_{kf}\left(z',\ell\right)I\left(L_{k}\left(z',\ell,f\right)=\ell'\right)d\ell$$

And repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.18), (A.22) and (A.24)).

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.
Step 7a: Write aggregate income I, price indices $P_C^{1-\sigma}$and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C and M_S.
Step 7b: Solve for $\frac{M_S}{M_C}$ that matches $d_{H,C}$.
Step 7c: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L}-L_u=\sum\limits_{k=C,S,j=i,f}L_{kj}$. Express M_C and M_S as functions of L_u.
Step 7d: Express masses of firms N_{kj} as functions of L_u.
Step 7e: Express aggregate posted vacancies V_{kj} as functions of L_u.
Step 7f: Use equation for μ^v (and the value initially guessed in Step 1 for μ^v) to obtain L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.
Step 7g: Go back and obtain masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s.

Step 8: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Step 6.

$$\mu^e_{kj} = \frac{\xi_{kj} V_{kj}}{\left((\xi_{Cf} V_{Cf} + \xi_{Ci} V_{Ci} + \xi_{Sf} V_{Sf} + \xi_{Si} V_{Si})^{\theta} + L_u^{\theta} \right)^{\frac{1}{\theta}}}$$

Step 9: Use equations (A.24)-(A.25) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si} as the solution to a system of linear equations.

$$\begin{aligned}
L_{Cf} &= \frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} \overline{L} + \frac{\chi^{change}_{Ci \to f} - \mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} L_{Ci} - \frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} L_{Sf} - \frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} L_{Si} \\
L_{Ci} &= \frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} \overline{L} - \frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} L_{Cf} - \frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} L_{Sf} - \frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} L_{Si} \\
L_{Sf} &= \frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} \overline{L} + \frac{\chi^{change}_{Si \to f} - \mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} L_{Si} - \frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} L_{Cf} - \frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} L_{Ci} \\
L_{Si} &= \frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} \overline{L} - \frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} L_{Cf} - \frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} L_{Ci} - \frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} L_{Sf}
\end{aligned}$$

Defining

$$\mathbf{L} = \begin{pmatrix} L_{Cf} \\ L_{Ci} \\ L_{Sf} \\ L_{Si} \end{pmatrix}, \ b = \begin{pmatrix} \frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} \overline{L} \\ \frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} \overline{L} \\ \frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} \overline{L} \\ \frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} \overline{L} \end{pmatrix} \text{ and}$$

$$A = \begin{pmatrix} 0 & \frac{\chi^{change}_{Ci \to f} - \mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} & -\frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} & -\frac{\mu^e_{Cf}}{\chi^{layoff}_{Cf} + \mu^e_{Cf}} \\ -\frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} & 0 & -\frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} & -\frac{\mu^e_{Ci}}{\chi^{leave}_{Ci} + \mu^e_{Ci}} \\ -\frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} & -\frac{\mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} & 0 & \frac{\chi^{change}_{Si \to f} - \mu^e_{Sf}}{\chi^{layoff}_{Sf} + \mu^e_{Sf}} \\ -\frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} & -\frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} & -\frac{\mu^e_{Si}}{\chi^{leave}_{Si} + \mu^e_{Si}} & 0 \end{pmatrix}$$

We obtain allocations

$$\mathbf{L} = (I - A)^{-1} b$$

Compute L_u^{new}

$$L_u^{new} = \max \left\{ \overline{L} - L_{Cf} - L_{Ci} - L_{Sf} - L_{Si}, \underline{L_u} \right\}$$

Where, $\underline{L_u}$ is a small positive number (to avoid zero unemployment).

Step 10: Compute deviation between L_u^{new} and L_u:

$$Dev_U = abs \left(\frac{L_u^{new} - L_u}{L_u} \right)$$

Step 11: Compute deviations from the C- and S-sector market clearing equations:

$$Dev_C = abs\left(\frac{\zeta I - (Revenue_C + (\tau_a - 1)\, Exports)}{(Revenue_C + (\tau_a - 1)\, Exports)}\right)$$

$$Dev_S = abs\left(\frac{[(1-\zeta)\, I + R] - Revenue_S}{Revenue_S}\right)$$

Where

$$Revenue_k = N_{kf}\int_\ell\int_z R_k(z,\ell)\,\psi_{kf}(z,\ell)\,dzd\ell + N_{ki}\int_\ell\int_z R_k(z,\ell)\,\psi_{ki}(z,\ell)\,dzd\ell$$

And $Revenue_C + (\tau_a - 1)\, Exports$ is total expenditure with goods from the C-sector (after imposing trade balance). In equilibrium:

$$\begin{aligned}\zeta I &= Revenue_C + \tau_a Imports - Exports \\ &= Revenue_C + (\tau_a - 1)\, Exports\end{aligned}$$

Note: Given the procedure outlined above, Dev_C should always be zero, unless we are not able to match $d_{H,C}$ with a positive value of $\frac{M_S}{M_C}$.

Step 12: Compute all moments to be matched with those in the data.

Step 13: Compute Loss Function. Add Model/Data deviations to equilibrium penalty $EQ\_Penalty$. The objective function is therefore given by

$$L = L_{mom} + EQ\_Penalty$$

Where L_{mom} penalizes deviations between moments in the data and $EQ\_Penalty$ penalizes deviations from equilibrium restrictions:

$$EQ\_Penalty = W_1 Dev_U + W_2 Dev_C + W_3 Dev_S,$$

With W_1, W_2 and W_3 denoting large weights.

Step 14: Optimization routine picks new parameter vector Θ. Go back to Step 2 until convergence.

Step 15 (Post-estimation): Obtain D_F^* (this is the deep parameter that we need for the counterfactuals as d_F is endogenous):

$$D_F^* = \frac{(\exp(\sigma_C d_F) - 1)\, D_H}{\epsilon^{\sigma_C}\tau_c^{1-\sigma_C}}$$

Where ϵ is the exchange rate value that balances trade:

$$\epsilon = \frac{1}{\tau_a\tau_c}\frac{(\tau_a Exports)^{\frac{1}{1-\sigma_C}}}{\exp\left(\frac{\sigma_C}{1-\sigma_C}d_{H,C}\right)}$$

C Simulation Appendix

C.1 Simulation Algorithm

In this section we describe the simulation algorithm in detail, which we break down into several steps for expositional clarity.

Step 1: Start with guesses of $d_{H,C}$, $d_{H,S}$, μ^v, and ϵ.

Step 2: Compute d_F implied by the guesses of $d_{H,C}$ and ϵ.

$$d_F = \log\left(\left(1 + \frac{D_F^*}{\exp\left(\sigma \times d_{H,C}\right)}\epsilon^\sigma \tau_c^{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$$

Step 3: Compute revenue functions $R_k\left(z,\ell\right)$, and compute wage schedules.

$$w_{kf}\left(z,\ell\right) = \frac{\left(1-\beta_f\right)\left(b+b^u\right)}{1+\beta_f\tau_w} + \frac{\beta_f\left(1-\tau_y\right)}{1+\beta_f\tau_w}\frac{R_k\left(z,\ell\right)}{\ell} - \frac{\beta_f}{1+\beta_f\tau_w}\frac{\overline{c}_{kf}}{\ell}$$

$$w_{kf}\left(z,\ell;\underline{w}_f\right) = \max\left\{w_{kf}\left(z,\ell\right),\underline{w}_f\right\}$$

$$w_{ki}\left(z,\ell\right) = \left(1-\beta_i\right)b + \beta_i\left(1-p_{ki}\left(\ell\right)\right)\frac{R_k\left(z,\ell\right)}{\ell} - \beta_i\frac{\overline{c}_{ki}}{\ell}$$

$$w_{ki}\left(z,\ell;\underline{w}_i\right) = \max\left\{w_{ki}\left(z,\ell\right),\underline{w}_i\right\}$$

Where $\underline{w}_f$ is the minimum wage in the formal sector (which is observed and fixed throughout estimation). $\underline{w}_i$ is the first percentile of the distribution of informal wages in PME and fixed throughout the estimation procedure. This is to avoid zero or negative informal wages.

Step 4: Compute firms' value functions. Obtain firms' policy functions. Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I_k^{formal}\left(\nu\right)=1\right) = \int I_k^{formal}\left(\nu\right)g_k^e\left(\nu\right)d\nu$$

$$\omega_{ki} \equiv \Pr\left(I_k^{informal}\left(\nu\right)=1\right) = \int I_k^{informal}\left(\nu\right)g_k^e\left(\nu\right)d\nu$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki}M_k$$
$$M_{kf} = \omega_{kf}M_k$$

Step 5: Compute the expected value of entry in each sector $k = C, S$.

$$V_k^e = \int \left[\left(V_k^e(\nu, i) - K_{Ci}\right) I_k^{informal}(\nu) + \left(V_k^e(\nu, f) - K_{Cf}\right) I_k^{formal}(\nu) \right] g_k^e(\nu)\, d\nu$$

Step 6: Compute the steady state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\psi_{ki}^e(z') = \frac{\int g_k(z'|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu}{\int_{\widetilde{z}} \int_\nu g_k(\widetilde{z}|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu d\widetilde{z}}$$

$$\varrho_{ki}^{exit} = \alpha_{ki} + (1 - \alpha_{ki}) \int_z \int_\ell \left(I_k^{exit}(z, \ell, i) + I_k^{change}(z, \ell, i) \right) \psi_{ki}(z, \ell)\, d\ell dz$$

In steady state $N_{ki} = (1 - \varrho_{ki}^{exit})\, N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned} \widetilde{\psi}_{ki}(z', \ell) = &\ \mathbf{1}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^e(z') \\ & + \mathbf{1}[\ell \geq 1] \times (1 - \alpha_{ki}) \times \left(\int_z \psi_{ki}(z, \ell)\, I_k^{stay}(z, \ell, i)\, g_k(z'|z) dz \right) \end{aligned}$$

Update ψ_{ki} with

$$\psi_{ki}(z', \ell') = \frac{\int_\ell \widetilde{\psi}_{ki}(z', \ell)\, I\left(L_k(z', \ell, i) = \ell'\right) d\ell}{\int_{\widetilde{z}} \int_\ell \widetilde{\psi}_{ki}(\widetilde{z}, \ell)\, I\left(L_k(\widetilde{z}, \ell, i) = \ell'\right) d\ell d\widetilde{z}}$$

And repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute

$$\psi_{kf}^e(z') = \frac{\int g_k(z'|\nu)\, g_k^e(\nu)\, I_k^{formal}(\nu)\, d\nu}{\int_{\widetilde{z}} \int_\nu g_k(\widetilde{z}|\nu)\, g_k^e(\nu)\, I_k^{formal}(\nu)\, d\nu d\widetilde{z}}$$

$$\varrho_{kf}^{exit} = \alpha_{kf} + (1 - \alpha_{kf}) \int_z \int_\ell I_k^{exit}(z, \ell, f)\, \psi_{kf}(z, \ell)\, d\ell dz$$

$$\varrho_{ki}^{change} = (1 - \alpha_{ki}) \int_z \int_\ell I_k^{change}(z, \ell, i)\, \psi_{ki}(z, \ell)\, d\ell dz$$

In steady state

$$\begin{aligned}
\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\
&= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)
\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{ki}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\boxed{\widetilde{\psi}_{kf}(z', \ell) = \begin{array}{c} \mathbf{1}[\ell = 1] \times \underbrace{\frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \times \psi_{kf}^{e}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1 - \alpha_{kf}) \times \left(\int_z \psi_{kf}(z, \ell) I_k^{stay}(z, \ell, f) g_k(z'|z) dz \right) + \\ (1 - \alpha_{ki}) \underbrace{\frac{\varrho_{kf}^{exit} \omega_{ki}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \times \\ \left(\int_z \psi_{ki}(z, \ell) I_k^{change}(z, \ell, i) g_k(z'|z) dz \right) \end{array} \right) \end{array}}$$

Update ψ_{kf} with:

$$\psi_{kf}\left(z', \ell'\right) = \frac{\int_{\ell} \widetilde{\psi}_{kf}\left(z', \ell\right) I\left(L_k\left(z', \ell, f\right) = \ell'\right) d\ell}{\int_{\widetilde{z}} \int_{\ell} \widetilde{\psi}_{kf}\left(\widetilde{z}, \ell\right) I\left(L_k\left(\widetilde{z}, \ell, f\right) = \ell'\right) d\ell d\widetilde{z}}$$

$$= \int_{\ell} \widetilde{\psi}_{kf}\left(z', \ell\right) I\left(L_k\left(z', \ell, f\right) = \ell'\right) d\ell$$

And repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki \to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.18), (A.22) and (A.24)).

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with $d_{H,C}$, $d_{H,S}$, μ^v, ϵ and $\overline{P}_S$.
Step 7a: The price index for the service sector is fixed at the value implied in estimation. $P_S = \overline{P}_S$.
Step 7b: Write aggregate income I, price indices $P_C^{1-\sigma}$and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C, M_S and exchange rate ϵ.
Step 7c: Solve for M_S given $\overline{P}_S$ and solve for M_C that perfectly matches the guess for $d_{H,C}$.
Step 7d: Obtain masses of firms N_{kj}.
Step 7e: Obtain aggregate vacancies V_{kj}.

Step 8: Obtain the value of L_u consitent with the guesses for μ^v, $d_{H,C}$, $d_{H,S}$, and ϵ:

$$L_u = \frac{\mu^v \widetilde{V}}{\left(1 - \left(\mu^v\right)^{\theta}\right)^{1/\theta}}$$

Step 9: Obtain job finding rates μ_{kj}^e. Note that:

$$\mu_{kj}^e = \xi_{kj} \frac{V_{kj}}{\widetilde{V}} \left(1 - \left(\mu^v\right)^{\theta}\right)^{1/\theta}$$

Step 10: Use equations (A.24)-(A.25) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$L_{Ci} = \frac{\mu_{Ci}^{e} L_u}{\chi_{Ci}^{leave}}$$

$$L_{Si} = \frac{\mu_{Si}^{e} L_u}{\chi_{Si}^{leave}}$$

$$L_{Cf} = \frac{\mu_{Cf}^{e} L_u + \chi_{Ci\to f}^{change} L_{Ci}}{\chi_{Cf}^{layoff}}$$

$$L_{Sf} = \frac{\mu_{Sf}^{e} L_u + \chi_{Si\to f}^{change} L_{Si}}{\chi_{Sf}^{layoff}}$$

Step 11: Compute L_u^{new} and $\mu^{v,new}$

$$L_u^{new} = \overline{L} - \sum_{k,j} L_{kj}$$

$$\mu^{v,new} = \frac{L_u^{new}}{\left(\widetilde{V}^{\theta} + \left(L_u^{new}\right)^{\theta}\right)^{1/\theta}}$$

Step 12: Compute Imports and Exports

$$Exports = N_{Cf} \times Avg\_Exports_{Cf}$$

$$Imports = \frac{\exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma}}{\tau_a}$$

Step 13: Compute deviations

$$Dev_{\mu} = abs\left(\mu^{v,new} - \mu^{v}\right)/\mu^{v}$$

$$Dev_{Tr} = abs\left(Exports - Imports\right)/Imports$$

$$Dev_{FE,C} = abs\left(V_C^e - c_C^e\right)/c_C^e$$

$$Dev_{FE,S} = abs\left(V_S^e - c_S^e\right)/c_S^e$$

Step 14: Compute Loss Function.

$$L = norm\left(\mathbf{Dev}\right)$$

$$\mathbf{Dev} = \begin{pmatrix} Dev_{\mu} \\ Dev_{Tr} \\ Dev_{FE,C} \\ Dev_{FE,S} \end{pmatrix}$$

Step 15: Optimization routine picks new guesses of $d_{H,C}$, $d_{H,S}$, μ^v, and ϵ . Go back to Step 2 until convergence.