Betting on Exports: Trade and Endogenous Heterogeneity*

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Abstract

We study the equilibrium determinants of firm-level heterogeneity in a model in which firms can choose between different probability distributions when drawing productivity at the entry stage and explore the implications in closed and open economy. One novel result is that export opportunities, by increasing payoffs in the tail, induce firms to draw technology from riskier distributions. When more productive firms also pay higher wages, trade amplifies wage dispersion by inducing firms to take more risk *ex-ante* and hence making them more unequal *ex-post*. Our model is consistent with new evidence on how firm-level heterogeneity varies across U.S. industries.

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1 Introduction

Current research in international trade puts firm-level heterogeneity at a center stage. As documented by a growing empirical literature, firms differ in size and productivity even within narrowly defined industries (e.g., Syverson, 2004a,b) and these differences vary systematically with trade participation (e.g., Bernard et al. 2012). In particular, exporters are bigger and more productive than nonexporters, and they pay higher wages. Firm heterogeneity also has crucial implications for other macroeconomic outcomes, such as aggregate efficiency (e.g., Hopenhayn, 2014). Yet, despite the growing attention that firm-level productivity differences are attracting, we still have a limited understanding of the theoretical and empirical underpinnings of this heterogeneity.

Although the distribution of the entire population of existing firms has some common characteristics that have been documented extensively (e.g., Axtell, 2001), these aggregate statistics mask significant heterogeneity across sectors and even between countries. For example, Helpman, Melitz and Yeaple (2004) show that cross-sector variation in measures of firm heterogeneity has important effects on firm strategies. Rossi-Hansberg and Wright (2007) find that the standard deviation of establishment size is smaller in sectors with larger capital shares. Poschke (2014) and Bartelsman, Haltiwanger and Scarpetta (2009) document instead differences in the firm-size distribution across countries. For instance, firm size is typically found to be larger but also more dispersed in the United States than in Spain or Italy. Moreover, given that more productive firms pay higher wages, firm heterogeneity is likely to map into wage dispersion, and wage inequality varies significantly across countries.[1] Besides these scant observations, systematic evidence and theoretical explanations for differences in firm heterogeneity are scarce. The primary goal of this paper is to take a step towards filling this gap.

We start our analysis by documenting some little-known facts regarding how a synthetic measure of firm heterogeneity, the standard deviation of the log of sales, varies across sectors and time in the U.S. economy. We find that this measure of dispersion can differ by a factor of ten between 6-digit NAICS industries, that it has increased on average by 11.8 per cent between 1997 and 2007, and that it correlates with industry characteristics such as average sales and export intensity. We also explore the robustness of these findings and argue that

[1] Dunne et al. (2004) and Faggio, Salvanes and Van Reenen (2010) show that a large fraction of the observed wage dispersion is between firms.

they are not easy to reconcile with existing models. Motivated by this evidence, we propose one possible explanation based on the idea that the observed heterogeneity stems from the endogenous choice of risk in the innovation strategies of entering firms.[2]

More precisely, we propose a model where firms can choose between different probability distributions when drawing productivity at the entry stage and we explore the implications for the equilibrium distribution of firms and wages in closed and open economies. Leading models of heterogeneous firms take the probability distribution from which firms draw productivity as given and characterize the resulting distribution of firm-level characteristics through the dynamics of entry, exit and possibly growth. Prominent examples are Melitz (2003), Rossi-Hansberg and Wright (2007), Luttmer (2010) and more recently Jones and Kim (2014). The aim of this project is to take a complementary approach, namely, to recognize that firms can affect directly the expected dispersion of productivity by choosing the riskiness of their entry investment.[3]

Although the success in starting a new enterprise is inherently uncertain, firms can deliberately choose between small projects with relatively safe returns and large projects with risky payoffs. Such a trade off is very familiar to anyone pursuing academic research, but is also common in the world of business. For instance, designing and assembling a new variety of laptop PCs, which mostly requires the use of established technologies, is safer and less costly than developing an entirely new product, such as tablet computers. In fact, the first tablet-like products date back to the 1980s, but did not reach success until the release of the iPad in 2010.[4] After decades of research, Apple's investment was rewarded with the sale of more than 250 million units over a period of five years only.

We formalize these ideas in a multi-industry model à la Melitz (2003) in which firms can draw a random productivity level upon paying a fixed entry cost and there is a fixed export

[2]Doraszelski and Jaumandreu (2009) show that the outcome of R&D investment is subject to a high degree of uncertainty and that R&D expenditure plays a key role in determining the differences in productivity across firms. Moreover, Coles, Daniel and Naveen (2006) show empirically that firm risk is endogenous.

[3]Note however that the logic of our argument applied to any innovation strategy (especially product innovation) of existing firms, not just of new entrants. Hence, although our static model does not distinguish between entrants and incumbents, it should be understood that the results do not rely exclusively on the new firm margin.

[4]In a 1983 speech, Steve Jobs said: "Apple's strategy is really simple. What we want to do is we want to put an incredibly great computer in a book that you can carry around with you and learn how to use in 20 minutes... and we really want to do it with a radio link in it so you don't have to hook up to anything and you're in communication with all of these larger databases and other computers." Yet, inventing the iPad required 27 years of investment constellated with failures and unforseen spin-offs, including the development of the iPhone.

cost. We modify this setup by allowing firms to choose the variance of the probability distribution from which to draw their productivity. Since expected profits are increasing in the dispersion of productivity, assuming that draws from riskier distributions are more costly delivers a well-defined trade-off. A first result of the paper is to show how the optimal point in this trade-off depends on industry-level characteristics in a way consistent with the patterns found in the data.

A second key result is that export opportunities induce firms to draw technology from a riskier distribution. The reason is that trade reallocates profits in favor of the most productive firms, thereby increasing the payoffs in the right tail. Thus, export opportunities increase the returns to risky investment.[5] Returning to the example of the invention of the iPad, our model suggests that globalization is what made Apple strategy so much more rewarding: without international markets, Apple's revenue could not have increased by a factor 13 between 2005 and 2014. On the contrary, during the same period, the sales of less-innovative manufacturers of traditional computers, such as Dell, stagnated. There is also ample anecdotal evidence that more firms are following Apple's strategy. To name just one prominent example, in 2010 Google started to invest in "Google X" project, a semi-secret lab dedicated to making major, high-risk, technological advancements.[6]

Finally, we extend the model to show how firm heterogeneity can map into income and wage inequality, as in Helpman, Itskhoki and Redding (2010). When more productive firms pay higher wages, we obtain a third novel result: trade amplifies wage dispersion by inducing firms to take on more risk *ex-ante* and hence making them more unequal *ex-post*.

At a broad level, this paper is related to the vast literature aimed at explaining productivity differences across firms (see Syverson, 2011, for a survey). More specifically, to the best of our knowledge, the choice of the riskiness of innovation, and its aggregate implications, has received little attention. Papers on technological change sometimes consider the distinction between radical and incremental innovation (e.g., Acemoglu and Cao, 2015). But these types of innovations differ more in the degree to which they replace or complement existing technologies, rather than in the variance of the potential outcomes. Some exceptions are Gabler

[5] This result is consistent with the evidence that greater trade openness leads to higher macroeconomic volatility in economies with a discrete number of firms, as in di Giovanni and Levchenko (2012). The endogenous choice of risk would amplify the mechanism based on the idea that trade makes large firms more important.

[6] To date, among Google X projects that have been revealed are the development of the self-driving car and Google Glass.

and Poschke (2013) and Bartelsman, Gautier and de Wind (2015), who study respectively how distortions and employment protection affect the choice between risky technologies. In any case, studying how different types of innovations affect the distribution of firms and wages is an underexplored and promising area of research.

The large literature on trade with heterogeneous firms started by Melitz (2003) does study the implications of export opportunities for the distribution of existing firms.[7] As it is well-known, trade can make firms more unequal by reallocating profits and workers from the least to the most productive firms. This effect is however very different from the one we emphasize, in that it abstracts from the possibility that trade changes the fundamental reason why firms are different, i.e., the unconditional productivity distribution.[8] Moreover, the focus of our paper is on measures of dispersion of firms' attributes, such as the log of sales, that are scale invariant rather than other characteristics, such as average size or the productivity cutoff for exit, that have been studied more extensively. In this respect, our paper is close in spirit to a nascent strand of the literature aimed at exploring the effect of trade on higher moments of the distribution of firm characteristics (e.g., Mayer, Ottaviano and Melitz, 2015)

Some recent papers study the impact of trade productivity via *ex-post* decisions on product scope or innovation. These include Bernard, Redding and Schott (2011), Dhingra (2013), Bustos (2011), Lileeva and Trefler (2010) and Atkeson and Burstein (2010), among others. All these models propose different channels through which trade liberalization can raise firm-level productivity, but do not focus on its dispersion. This literature has also shown that trade can help to overcome the fixed cost of technology adoption through a scale effect, a result that is however very different from our finding that trade induces firms to invest in riskier technologies.[9] Yet, combining our *ex-ante* choice of innovation risk with *ex-post* decisions that can affect an initial realization of productivity seems a natural step forward to develop a comprehensive theory of how productivity differences emerge and evolve.

Finally, several papers have shown, both theoretically and empirically, that trade impacts

[7] See Melitz and Redding (2014) for an excellent survey.

[8] Some papers, including Yeaple (2005) and more recently Grossman and Helpman (2014), trace productivity differences across firms to heterogeneity in ability across workers and managers. We follow the complementary approach that emphasizes the role of difference in technology rather than ability.

[9] Vannoorenberghe (2014) argues that trade can induce excessive risk taking, but in a very different model without firm heterogeneity. He also shows some supportive evidence using cross-country data. Besides this paper, there is little work on trade and risk-taking.

wage inequality because exporters pay higher wages.[10] In our model, however, the effect of trade works not only through the exporters' wage premium, but also by making the entire wage schedule steeper, with different implications. For instance, our mechanism predicts that more export opportunities will increase wage dispersion even among the group of non-exporting firms. This may help explain why the rise in inequality is often found to be an ubiquitous, within-group, phenomenon.

The remainder of the paper is organized as follows. In Section 2, we document some stylized facts regarding how the dispersion of log-sales varies across sectors and time in the U.S. economy. Motivated by these empirical observations, in Section 3 we propose a closed-economy model where differences in the variance of firm-level outcomes stem from the possibility of choosing the probability distribution from which to draw productivity at the entry stage. Section 4 adds costly trade and shows that more export opportunities induce firms to draw their productivity from riskier distributions, thereby generating more heterogeneity in equilibrium. In Section 5 we consider the implications of the model for income and wage inequality. Section 6 concludes.

2 The Evidence: Sale Dispersion and Trade

In this section, we document how the dispersion of sales of U.S. firms varies across sectors and time, and how it correlates with a number of sector characteristics. First, we show that the dispersion of sales differs significantly across sectors and that it has increased over time. Second, we report panel regressions suggesting that higher dispersion at the industry level is systematically associated with larger scale in terms of average sales and with higher export intensity. Finally, we provide additional evidence suggestive of a causal effect of export intensity on the dispersion of sales.[11]

[10] See, for example, Helpman, Itskhoki and Redding (2010), Helpman et al. (2014) and other papers surveyed in section ten of Melitz and Redding (2014).

[11] An antecedent of this analysis is Syverson (2004b), who studies how various measures of productivity dispersion covary with industry characteristics in the U.S. manufacturing sector. Yet, his evidence is limited to the 1977 cross-section.

2.1 Sales Dispersion Across Sectors and Over Time

Our main measure of dispersion is the standard deviation of the logarithm of sales at establishment level.[12] We focus on sales because they are an easy-to-observe, synthetic measure of overall size, and we take the log to make the variance scale invariant. We compute this variable using data from the U.S. Census of Manufacturing for years 1997, 2002 and 2007. Data on (receipts of) sales, number of firms and establishments, and number of employees are available at 6-digit NAICS industry level for the universe of U.S. firms, aggregated into sales-size categories.[13] Since we do not have access to firm-level data, we follow Helpman, Melitz and Yeaple (2004) in assuming that all establishments falling within the same sales-bin have the same value as the group mean, and using the number of establishments in each category as weights. In particular, we consider each bin in a 6-digit NAICS industry as a single observation, and compute the standard deviation of log-sales weighting observations appropriately. Helpman, Melitz and Yeaple (2004) show that this methodology to compute dispersions approximates well measures based on the entire population. As an additional check, we also computed the variance of log-sales using firm-level data from Compustat, which only includes listed firms. Although the Compustat sample is too restricted for the purpose of our analysis, we find that the variance computed on it is highly correlated with the one we obtain from Census data.

Table 1 reports some descriptive statistics. For 3-digit manufacturing sectors, it shows the average standard deviation of log-sales in 2007, its minimum and maximum in each 6-digit subindustry, and its average percentage change over the previous ten years.[14] For convenience, sectors are ordered by increasing dispersion. The first columns show that dispersion varies significantly across sectors, ranging from a minimum of 1.686 (in Paper Manufacturing) to a maximum of 2.712 (in Transportation Equipment Manufacturing). The second and third columns show however that the main source of heterogeneity is within the 3-digit sectors: among all the 6-digit industries, the dispersion of sales varies almost by a factor of ten. The last column also reports the average number of establishments in the

[12] In what follows, we will refer indifferently to establishments and plants.

[13] Census data by sales-size category are released every five years and are currently available for Census years 1997, 2002 and 2007. The raw data are disaggregated into ten bins in 1997, eight bins in 2002 and eighteen bins in 2007. The lowest bin contains firms with revenues below 50 thousand US$ and the highest bin contains firms with revenues above 100 millions US$. In most of the analysis, we aggregate the data into six bins consistently observed throughout the period. In a robustness check, we show that our results are unchanged when using the raw bins.

[14] All reported averages are simple averages. Weighting by sales does not affect the qualitative results.

6-digit industries. Comparing the first and the last columns reassures that dispersion in an industry is not mechanically driven by sample size. Finally, the fourth column shows that sales dispersion has increased remarkably in the vast majority of sectors, on average by 11.8 per cent, between 1997 and 2007. This increase grows to a remarkable 28.5 per cent if we weight industries by sales.

2.2 Exploring the Data: Correlations

To further explore the data, we now exploit the variation across 6-digit industries and over time to study how dispersion in revenues correlates with a number of industry characteristics. Among the covariates, we consider average log-sales per establishment, export intensity, total employment, the number of establishments, the intensites in skill, physical capital and raw materials, and the mean and standard deviation of the log of workers' educational attainment. Export intensity is the ratio of exports to total shipments, constructed with export data from Schott (2008) and shipment data from the NBER-CES Manufacturing Industry Database. Total employment is expressed in terms of workers and is constructed with data from the U.S. Census. Skill, capital and material intensities are computed as in Romalis (2004) with data from the NBER-CES Manufacturing Industry Database, and are equal to the ratios of non-production workers' wage bill, capital compensation and material expenditure, respectively, over the sum of value added and material costs. The mean and standard deviation of workers' education are computed with data from the CPS Merged Outgoing Rotation Groups.[15]

We estimate OLS regressions with the following form:

$$SD_{it} = \alpha + \beta \mathbf{X}_{it} + \eta_i + \varepsilon_{it}, \tag{1}$$

where SD_{it} is the standard deviation of log-sales in sector i at time $t \in \{1997, 2002, 2007\}$, $\mathbf{X}_{it}$ is a vector of controls, η_i is an industry fixed effect, and ε_{it} is an error term.[16] We first estimate equation (1) with and without industry fixed effects. Next, to account for the overall business cycle, we also control for the growth rate of nominal GDP over the two years prior to each observation.[17] Finally, we also use a specification in first differences and

[15]See the Appendix for more detail on variable definitions and data sources.
[16]All variables except for GDP growth and standard deviations are expressed in logarithms.
[17]The growth rate of nominal GDP is computed over the two years prior to each observation (i.e., 1995 to 1997 for the 1997 observation) using data from the World Development Indicators.

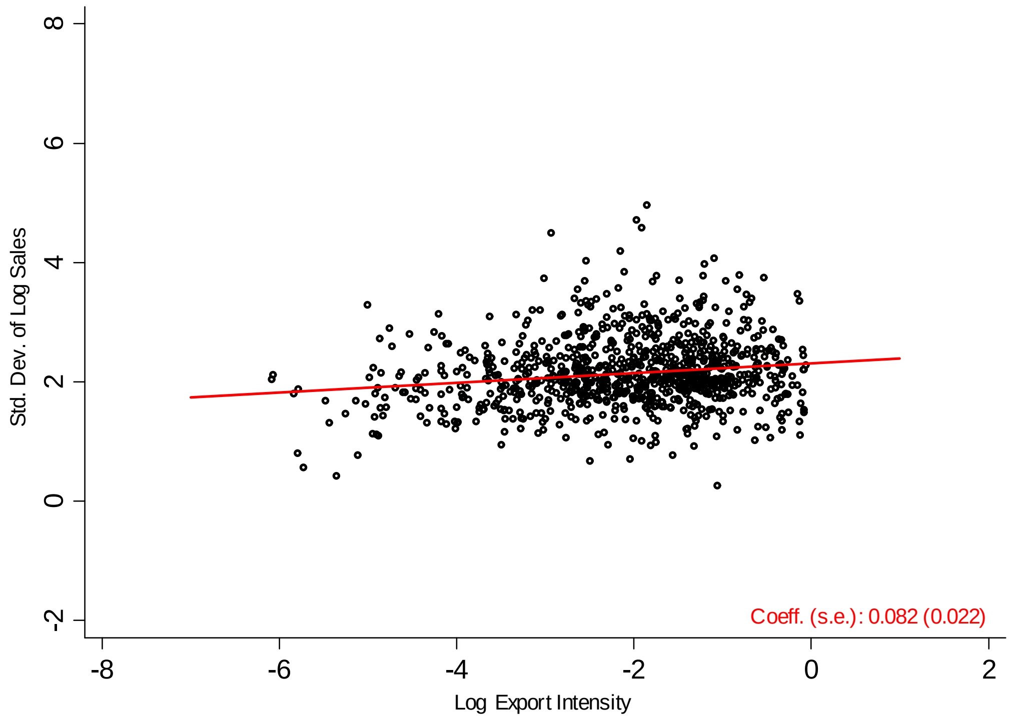

Figure 1: Exports and Sales Dispersion

estimate the following equation:

$$\Delta SD_{it} = \tilde{\alpha} + \tilde{\beta}\Delta\mathbf{X}_{it} + \tilde{\eta}_i + \tilde{\varepsilon}_{it}, \tag{2}$$

where the industry fixed effect $\tilde{\eta}_i$ now captures an industry-specific trend.

Table 2 reports the baseline results. Columns (1)-(4) estimate equation (1) by pooled OLS, columns (5) and (6) control for industry fixed effects, and column (7) estimate the first-differenced specification in equation (2). We work with a consistent sample of observations for which we have data on all covariates and correct the standard errors for clustering within 6-digit industries. In columns (1) and (2) we regress SD_{it} on establishment sales and export intensity, respectively. Dispersion of sales is positively correlated with average plant size and exports across industries and over time. Figure 1 plots the relationship between SD_{it} and export intensity and shows that the positive correlation found in column (2) is not driven by outliers.

In column (3) we include establishment sales and export intensity jointly with other industry characteristics. The correlation of SD_{it} with plant size and export intensity is

unchanged. Moreover, SD_{it} is negatively correlated with skill intensity and positively correlated with the dispersion of workers' education, but these correlations are not robust across specifications as shown below. In column (4) we add GDP growth. The coefficient on this variable is positive and precisely estimated, implying that dispersion of sales is higher in periods of expansion. SD_{it} is still positively correlated with average plant sales and export intensity.

In columns (5) and (6) we add industry fixed effects, so as to identify the coefficients using within-industry variation over time. Note that the coefficients on establishment sales and export intensity are still positive and precisely estimated. Both coefficients are larger than before, suggesting that the raw correlations are stronger within-industry. Furthermore, dispersion in sales is still positively correlated with GDP growth. Finally, in column (7) we estimate equation (2), by expressing all variables in first differences and adding industry fixed effects, so as to control for industry-specific trends. Sales dispersion is positively correlated with establishment sales and export intensity also in this very rich specification.

In Table 3, we perform some robustness checks using the specification in first differences. First, we add further controls, namely, import penetration in column (1) and the high-tech share of investment in column (2). The results confirm that sales dispersion strongly correlates with average plant size and export intensity. In columns (3) and (4), we compute the dependent variable on two subsets of establishments, excluding the smallest and largest ones respectively, and re-estimate our baseline specification to assess whether the correlations found in Table 2 are driven by large or small plants. The main results hold in both cases. As a further robustness check, in column (5) we re-compute standard deviations using all the sales-size bins available in each period, in order to make sure that using homogeneous bins does not bias our results. The coefficients are similar to those in column (7) of Table 2, suggesting that the number of bins does not affect our results. Finally, we estimate equations (1) and (2) using the standard deviation of log-sales per worker as the dependent variable. The coefficients in columns (6)-(8) show that also labor productivity is more dispersed in industries with larger plant size and higher export intensity, and is positively correlated with the business cycle.

2.3 Identifying the Effect of Trade

We now focus on the positive correlation between export intensity and the standard deviation of log-sales found in the OLS analysis, which may be consistent with causality in both directions. On the one hand, industries with higher dispersion of sales may have a higher export intensity because they are more likely to have large firms which are known to participate more in the export market (see Bernard et al., 2012). On the other hand, better export opportunities may induce firms to take riskier strategies, which generate more dispersion of sales. In this section, we investigate whether causality goes from exports to dispersion. To do so, we first follow an instrumental variables approach to identify the effect of exogenous export shocks on the standard deviation of log-sales. Next, following Hummels (2007) and Hummels et al. (2014), we adopt an alternative, difference-in-differences, strategy based on changes in transportation costs.

In Table 4, we estimate the main versions of equations (1) and (2) instrumenting U.S. exports in industry i at time t with the exports of all non-U.S. countries to the destination markets of the U.S. in industry i at time t. By doing so we aim to identify variation in exports generated by foreign demand shocks, while cleaning out the variation due to U.S. industry-specific technological shocks, which may induce reverse causality. To be valid, this instrument requires technological shocks originated in U.S. industries to be uncorrelated with those originated in other countries' industries. Because the majority of countries included in the instrument are low- or middle-income economies, it is unlikely that their shocks be strongly correlated with the U.S. ones. In any case, we show below that our conclusions are unchanged when using a complementary strategy that does not rest on this identifying assumption.

The bottom panel of the table reports the first-stage results, which show that the instrument has strong power for predicting U.S. exports. Indeed, the first-stage coefficient is always precisely estimated and large, ranging between 0.35 and 0.55 across specifications, and the F-statistics for excluded instruments is remarkably high, ranging from 45.2 and 69.5. The second-stage results show that the coefficient on export intensity is positive and precisely estimated across the board, which is consistent with a causal effect of exports on the standard deviations of log-sales and labor productivity. At the same time, the positive correlations of SD_{it} with average plant size and GDP growth are preserved.

As an alternative to the IV strategy described above, we now identify the effect of exports

on sales dispersion by estimating with OLS the following specification:

$$SD_{it} = \alpha + \beta \mathbf{X}_{it} + \gamma OIL_t + \delta BW_i * OIL_t + \eta_i + \varepsilon_{it}, \tag{3}$$

where OIL_t is the log price of crude oil (Brent) at time t and BW_i is the bulk weight, in Kg per dollar, of U.S. shipments in industry i. The bulk weight of each industry is the export-weighted average of the bulk weights of constituent products. In turn, the product-level bulk weights are computed as averages between air and vessel transportation in 1995, in order to make sure that the choice of transport mode does not react to changes in oil prices. The interaction between oil price and bulk weight captures the differential change in export (transport) costs entailed by a oil price shock at time t in industries that produce heavier goods, and are thus subject to higher transportation costs per dollar shipped. Accordingly, the coefficient δ is identified by the differential response to a common oil price change across industries characterized by a different importance of transportation costs. Hence, we refer to this approach as a difference-in-differences strategy. A negative estimate for δ would imply that, when hit by a reduction in oil price, industries shipping heavier goods, which see a larger drop in trade costs (increase in export opportunities), experience a larger increase in sales dispersion.

Table 5 reports coefficient estimates for various specifications of equation (3). Note that the coefficient δ is always negative and precisely estimated, which also points in the direction of a causal effect of exports on the dispersion of sales and labor productivity. At the same time, sales dispersion is still positively correlated with average plant size and GDP growth.

2.4 Discussion

The analysis in this section has unveiled a number of facts. First, the dispersion of sales at the establishment level varies considerably, both across narrowly defined sectors and over time. Second, the dispersion of sales is systematically and positively correlated with average sales at the establishment level and export intensity. Third, these correlations hold across the whole distribution of establishment sales-size. Fourth, an increase in export intensity at the industry level causes an increase in the dispersion of sales. Fifth, the dispersion of sales is also positively correlated with the business cycle.

These stylized facts raise a number of important questions. Why has dispersion of sale increased so much in recent times? Why is higher export intensity associated to more

heterogeneity across establishments? Before proposing our novel explanation, we now pause to discuss why we think that existing models do not provide complete and fully satisfactory answers.

Natural candidates for explaining the rise in dispersion could be "granularity" in the data and misallocation across firms. As documented by a recent literature, the law of large numbers may fail at the industry level, especially if the distribution of sales is very fat tailed. Hence, sale differences across sectors and time can be simply due to granularity. There is also a growing literature suggesting that firm heterogeneity partly reflects misallocation, i.e., the presence of frictions allowing inefficient firms to survive. If economic booms were associated to a rise in misallocation, for example due to excessive entry of unproductive firms, this might indeed contribute to explain the increase in sale dispersion and the significant association with past GDP growth. Although these explanations certainly have some appeal, they do not seem fully consistent with the observations that the number of establishments (and changes in the number of establishments) is not robustly correlated with sale dispersion and that the rise in heterogeneity is driven neither by small firms (which may reflect more inefficient entry), nor by large firms (which should matter more under the granular hypothesis). Furthermore, these theories do not provide a clear answer for the statistically significant effect of trade.

Turning to the correlation between the standard deviation of log-sales and export intensity, candidate explanations could be reverse causality, selection effects and reallocations towards exporters. Once again, even if all these mechanisms can be important, they do not seem to provide a compete account of the patterns in the data. Although more dispersion in productivity can increase the fraction of exporters when the latter are in the tail of the distribution, the IV and diff-in-diff results suggest that reverse causality is not the whole story. The effect of export may also be due to the fact that trade opening forces the least productive firms to exit and reallocates market shares towards larger firms. Yet, the impact of this reallocation is theoretically ambiguous, in that it depends on the characteristics of the firms it affects. Instead, we have found that trade is associated to more dispersion even when removing smaller firms, which are more likely to exit, or bigger firms, which are more likely to export. Moreover, trade raises also the dispersions of labor productivity, not just sales. Finally, the effect of trade seems to go beyond a mere scale effect, because all regressions control for average firm size.

Motivated by this evidence, in the remainder of the paper we will show that a parsimonious extension of the workhorse model of trade with heterogenous firms is consistent with

all the stylized facts presented in this section. In our model, export opportunities will induce firms to draw productivity from riskier distribution, a prediction that seems in line with the finding by di Giovanni and Levchenko (2009) that volatility is higher in sectors that are more open to trade. Compared to alternative mechanisms, a key advantage of our theory will be to show why trade may raise the dispersion of sales and productivity across the whole size distribution.

3 Closed-Economy Model

We now build a multi-sector, one factor, model of monopolistic competition between heterogeneous firms along the lines of Melitz and Redding (2014). After paying a fixed entry cost, firms draw their productivity from a distribution and exit if they cannot profitably cover a fixed cost of production. Differently from Melitz (2003), we allow firms to affect the "riskiness" of their entry investment by choosing the distribution from which to draw their productivity. In this section, we study the determinants of entry risk and how it affects the equilibrium distributions in a closed economy. We defer to the next section the case in which firms can engage in costly trade. For simplicity, we consider a static model in which entry and production decisions are all simultaneous.

3.1 Preferences

Consider an economy populated by a unit measure of identical households of size L with quasi-linear preferences over consumption of a homogenous good q_0 and differentiated goods produced in I industries:

$$U = q_0 + \sum_{i=1}^{I} \frac{\alpha_i X_i^{\zeta_i}}{\zeta_i}, \quad \zeta_i \in (0,1) \quad \alpha_i > 0$$

Each industry $i \in \{1, ..., I\}$ produces differentiated varieties and preferences over these varieties take the constant elasticity of substitution form:

$$X_i = \left[\int_{\omega \in \Omega_i} x_i (\omega)^{\frac{\sigma_i - 1}{\sigma_i}} \mathrm{d}\omega \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1$$

where $x_i (\omega)$ is consumption of variety ω, Ω_i denotes the set of varieties produced in sector i and σ_i is the elasticity of substitution between varieties within an industry. We denote by

$p_i(\omega)$ the price of variety ω in industry i and by P_i the ideal price of the consumption basket X_i :

$$P_i = \left[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma_i}\,\mathrm{d}\omega\right]^{1/(1-\sigma_i)}.$$

The demand for the differentiated basket X_i is $X_i = (\alpha_i/P_i)^{1/(1-\zeta_i)}$ and the demand for each individual variety is

$$x_i(\omega) = X_i\left(\frac{P_i}{p_i(\omega)}\right)^{\sigma_i}. \tag{4}$$

The demand for the homogenous good q_0 is residual. We assume that income of each household is sufficiently high to always guarantee a positive consumption of the homogenous goods, which is chosen as the numeraire. In the remainder of the paper, we focus on a single sector and derive results that do not depend on general equilibrium effects. For this reason, and to save notation, from now on we remove the index i with the understanding that all parameters can potentially vary across sectors.

3.2 Problem of the Firm

Recall that we now focus on the industry equilibrium of a single sector $i \in \{1, ..., I\}$. Within each sector, every variety ω is produced by monopolistically competitive firms that are heterogeneous in their labor productivity, φ. Since all firms with the same productivity behave symmetrically, we index firms by φ. There are fixed costs of production, f, and of entry, λF, in units of the numeraire good. Upon entry, a firm can choose to randomly draw its productivity from a menu of distributions differing in the riskiness of their realizations and pays the corresponding entry cost. Next, the firm faces standard production and pricing decisions. We solve the problem backwards: first, we describe the strategy of a firm with a given productivity and then solve the problem of choosing the productivity distribution given rational expectations on the industry equilibrium. Note also that, as it is customary, we follow the convention of identifying firms with varieties.

Given a productivity φ and a marginal cost of w/φ, where w is the wage, the firm will choose its price and whether to exit so as maximize profit, $\pi(\varphi)$, subject to a downward-sloping demand curve with elasticity σ. The first-order conditions for this problem imply that firms set prices equal to a constant markup over the marginal cost,

$$p(\varphi) = \frac{\sigma}{\sigma-1}\frac{w}{\varphi}, \tag{5}$$

and exit if $\pi(\varphi) < 0$. Using (4) and (5), we can express profit as a function of productivity:

$$\pi(\varphi) = A\varphi^{\sigma-1} - f \tag{6}$$

where $A = \left(\frac{\sigma w}{\sigma-1}\right)^{1-\sigma} \frac{XP^{\sigma}}{\sigma}$. Since profits are increasing in φ, the firm will exit whenever its productivity is below the cutoff $\varphi^* = (f/A)^{1/(\sigma-1)}$.

Combining the pricing and exit decision, we can write *ex-ante* expected profit of a firm drawing its productivity from a distribution with cumulative distribution $G(\varphi)$ as:

$$\mathbb{E}[\pi] = \int_0^{\infty} \pi(\varphi)\, \mathrm{d}G(\varphi) = \int_{\varphi^*}^{\infty} \left(A\varphi^{\sigma-1} - f\right) \mathrm{d}G(\varphi). \tag{7}$$

To study the incentives for firms to choose the "riskiness" of their entry investment, we assume that $G(\varphi)$ belongs to a family of Pareto distributions with different shape parameters:

$$G(\varphi) = 1 - (\varphi_{\min}/\varphi)^{1/v}, \quad v \in [\underline{v}, \bar{v}]$$

where $\varphi_{\min} > 0$ is the lower bound of the support and the shape parameter is $1/v$. Written in this way, v can be interpreted as an index of the variance of the distribution. More precisely, v is equal to the standard deviation of the log of φ and will be one of the key determinants of the equilibrium distributions of the log of firm characteristics, such as sales. The bounds $\underline{v} > 0$ and $\bar{v} < 1$ rule out the possibility of a degenerate distribution and ensure a finite mean for v. Note also that the mean of φ is $\varphi_{\min}(1-v)^{-1}$ which increases with v. Thus, our assumption that firms can choose between distributions with different v embeds the notion that high expected payoffs are associated to more risk. As we will show shortly, this property is not strictly needed for many of the results of the paper. Yet, it seems a very natural property and we conform to it. Nonetheless, we report in the Appendix the complete characterization of the case in which firms can choose between mean-preserving spreads.

There are several reasons for focusing on Pareto distributions. Besides being tractable and widely used, Pareto distributions have been shown to approximate well some observed firm-level characteristics (especially in the right tail). Thus, it is an empirically reasonable assumption. Moreover, the Pareto distribution has the useful property that power functions of φ are also Pareto distributed, although with a different shape parameter. This helps to map the model to the data because it will allow us to obtain closed-form solutions for the measure of dispersion computed in Section 2.

Substituting $A\left(\varphi^*\right)^{\sigma-1} = f$ into (7), assuming $\varphi^* > \varphi_{\min}$ (so that there is selection) and using $G\left(\varphi\right)$, we can solve for expected profits:

$$\mathbb{E}\left[\pi\right] = f\int_{\varphi^*}^{\infty}\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right]\mathrm{d}G\left(\varphi\right) = \frac{f\varsigma}{1/\upsilon - \varsigma}\left(\frac{\varphi_{\min}}{\varphi^*}\right)^{1/\upsilon}. \tag{8}$$

where it proves convenient to define $\varsigma \equiv \sigma - 1$ and we assume $\upsilon < 1/\varsigma$ for $\mathbb{E}\left[\pi\right]$ to be finite. It is easy to see that expected *ex-ante* profits are increasing in υ with elasticity equal to:

$$\frac{\partial \ln \mathbb{E}\left[\pi\right]}{\partial \ln \upsilon} = \frac{1}{1-\upsilon\varsigma} + \ln\left(\frac{\varphi^*}{\varphi_{\min}}\right)^{1/\upsilon} > 0. \tag{9}$$

There are three reasons why a higher υ, and hence more dispersion in the distribution of productivity draws, implies higher expected profits. First, as already seen, a higher υ raises average productivity directly. Second, given the shape of the Pareto distribution, it increases the probability of drawing a productivity above the exit cutoff φ^*.[18] Third, even in the absence of the previous effects, more dispersion increases expected profits whenever the profit function is convex in prices and hence in φ. As equation (6) shows, this is the case when $\sigma > 2$ (i.e., for $\varsigma > 1$). To see this, suppose now that $\varphi_{\min} = \bar{\varphi}\left(1-\upsilon\right)$ so that the mean of the distribution is constant at $\bar{\varphi}$ and an increase in υ corresponds to a mean-preserving spread. Then:

$$\frac{\partial \ln \mathbb{E}\left[\pi\right]}{\partial \ln \upsilon} = \frac{1}{1-\upsilon\varsigma} - \frac{1}{1-\upsilon} + \ln\left(\frac{\varphi^*}{\varphi_{\min}}\right)^{1/\upsilon}$$

which is necessarily positive when $\varsigma > 1$ ($\sigma > 2$), even in the absence of selection effects (i.e., when $\varphi^* \to \varphi_{\min}$). The intuition is that firms can expand to take advantage of good realizations of productivity and contract to insure against bad realizations, making them potentially risk loving. This is a well-known result, sometimes referred to as the Hartman (1972) and Abel (1983) effect.

Having characterized the value of drawing productivity from riskier distributions, we need to specify its cost. In order to have a well defined trade-off, we assume that the entry cost λF is an increasing and convex function of υ. Formally, $F'\left(\upsilon\right) > 0$ and $F''\left(\upsilon\right) > 0$, while $\lambda > 0$ is positive constant parametrizing all the costs of financing the entry investment F. The fact that the entry cost is increasing in υ (at a sufficient rate) is not just needed to prevent

[18] An increase in υ raises the density at any $\varphi > \varphi^*$. As shown below, this is true even holding constant the mean of the Pareto distribution.

firms from choosing trivially the distribution with the highest dispersion; it also captures the sensible notion that risky projects with high expected payoffs require bigger investments. All in all, although our description of the entry risk faced by firms is admittedly stylized, it accords well with common sense.[19] For the interested reader, we show in the Appendix how our assumptions on entry can be derived from a simple model in which firms choose how much to invest in projects with an exogenous random quality. Besides providing a possible microfoundation, this extension helps interpreting how risk may be chosen in practice: it illustrates that investing in (few) large projects of uncertain outcome is more risky than investing in (many) smaller ones.

We are now in the position to solve the entry stage. The problem is greatly simplified by the fact that all firms in a given sector are *ex-ante* identical and therefore face the same problem of choosing v so as to maximize expected profits minus the entry cost:

$$\max_{v \in [\underline{v}, \bar{v}]} \left\{ \mathbb{E}\left[\pi\right] - \lambda F\left(v\right) \right\}.$$

To ensure that the maximand is concave, we impose $\partial^2 \mathbb{E}\left[\pi\right] / \partial v^2 < F''\left(v\right)$. Then, the first-order condition for v is

$$\frac{\mathbb{E}\left[\pi\right]}{v} \left[\frac{1}{1 - v\varsigma} + \ln \left(\frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} \right] \geq \lambda F'\left(v\right). \tag{10}$$

Concavity and implicit differentiation allow us to sign the comparative statics for v. If interior, the unique equilibrium choice of v is increasing in the elasticity of substitution, ς, average profit, $\mathbb{E}\left[\pi\right]$, and the exit cutoff, $\varphi^*/\varphi_{\min}$. However, both $\mathbb{E}\left[\pi\right]$ and $\varphi^*/\varphi_{\min}$ are endogenous and to solve for them we now turn to the industry equilibrium.

3.3 Industry Equilibrium

Free entry implies that *ex-ante* expected profits must be equal to the entry cost: $\mathbb{E}\left[\pi\right] = \lambda F\left(v\right)$. Substituting (8) into this condition, we can solve for the exit cutoff:

$$\left(\frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} = \frac{f}{\lambda F\left(v\right)} \frac{\varsigma}{1/v - \varsigma}. \tag{11}$$

[19] Equivalently, we could have assumed that firms can choose some other variable s which in turn affects positively v. If $v(s)$ is a sufficiently convex function, we would obtain a non-increasing marginal benefit of raising s.

To make sure that $\varphi^*/\varphi_{\min} > 1$, we impose $f > \max_{v\in[\underline{v},\bar{v}]}\{\lambda F(v)(1/\varsigma v - 1)\}$. Next, using $\mathbb{E}[\pi] = \lambda F(v)$ and (11), we can rewrite the first-order condition for v in the case of an interior solution (10) as:

$$\ln\left(\frac{f}{\lambda F(v)}\frac{\varsigma}{1/v-\varsigma}\right) + \frac{1}{1-v\varsigma} = \frac{vF'(v)}{F(v)} \tag{12}$$

Under regularity conditions that we take for granted, equation (12) has an interior solution over the relevant range $v \in [\underline{v}, \bar{v}]$.[20] Moreover, provided that $F(v)$ is sufficiently convex, the solution will be unique. Although the possibility of multiple equilibria is interesting, exploring it goes outside the scope of this paper and we therefore disregard this possibility.[21]

We can now study the equilibrium determinants of v. A higher fixed cost of production, f, increases the exit cutoff and hence raises the benefit of choosing a more dispersed distribution. A higher elasticity of substitution raises the value of v by making profits more convex in productivity and by increasing the exit cutoff. Moreover, since of $\mathbb{E}[\pi] = \lambda F(v)$ and $F'(v) > 0$, the model predicts a positive association between average profits (or sales) and the variance of productivity.

The choice of v affects the equilibrium distribution of firm characteristics. Consider the distribution of revenues, which matches closely the variable documented in Section 2. It is easy to show that revenues are a power function of productivity: $r(\varphi) = r(\varphi^*)(\varphi/\varphi^*)^{\varsigma}$. Then, from the properties of the Pareto distribution, $r(\varphi)$ is also Pareto distributed with c.d.f. $G_r(r) = 1 - (r_{\min}/r)^{1/v\varsigma}$, for $r > r_{\min} = \sigma f$.[22] Hence, the log of revenue is exponential with a standard deviation equal to $v\varsigma$. This immediately implies that differences in the choice of entry risk across sectors will translate into differences in the equilibrium distributions of firm characteristics as summarized in the following Proposition.

Proposition 1 *Assume that the solution to (12) is unique and interior. Then, the equilibrium dispersion of firm productivity and revenue, as measured by the variance of the log of*

[20] Sufficient conditions are $F'(\underline{v}) = 0$ with $F(\underline{v}) > 0$ and $F'(\bar{v}) \to \infty$ with $\bar{v} < 1/\varsigma$.

[21] One can imagine a situation in which the expectation of high profits in equilibrium induces firms to choose a high initial investment, which in turn confirms the initial expectation that firms be large. Similarly to Bonfiglioli and Gancia (2014), this multiplicity may help explain cross-country differences in the prevalence of small and large firms and other outcomes.

[22] If φ follows a Pareto(φ^*, z), then $x \equiv \ln(\varphi/\varphi^*)$ is distributed as an exponential with parameter z. Then, any power function of φ of the type $A\varphi^B$, with A and B constant, is distributed as a Pareto$(A(\varphi^*)^B, z/B)$, since $A\varphi^B = A(\varphi^*)^B e^{Bx}$ with $Bx \sim Exp(z/B)$, by the properties of the exponential distribution.

φ and $r(\varphi)$, is larger in sectors with a higher fixed cost, f, higher elasticity of substitution between varieties, ς, and a lower entry cost as parametrized by λ.

These results are broadly consistent with the empirical evidence documented in Section 2. As long as economic expansions are associated to lower entry costs, for instance by lowering financing costs, the effect of λ is compatible with the finding that faster economic growth correlates to a rise in dispersion. Finally, given that average revenue is equal to $\sigma f/(1-\upsilon\varsigma)$, which is increasing in σ, f and υ, the model reproduces the positive correlation between the variance and the mean of revenues observed in the data.[23]

4 Trade and Equilibrium Firm Heterogeneity

We now extend the model by adding the possibility for firms to export their varieties subject to fixed and variable costs. This will lead to the familiar results that only the most productive firms export and that trade forces the least productive firms out. This *ex-post* reallocation of revenues will have new implications for the *ex-ante* entry stage: by increasing the payoffs in the tail, trade will induce firms to take on more risk and draw their productivity from more dispersed distributions.

Consider a world economy composed, for simplicity, of two symmetric countries. To serve the foreign market, firms must incur a fixed cost f_x in units of the numeraire and an iceberg variable cost such that $\tau > 1$ units must be shipped for one unit to arrive at destination. The presence of a fixed trade cost implies that only the most productive firms choose to serve the foreign market. Formally, notice that, in analogy to (6), profits from exporting are $\pi_x(\varphi) = A(\varphi/\tau)^{\sigma-1} - f_x$. These profits would be negative for firms with productivity $\varphi < \varphi_x^* = \tau(f_x/A)^{1/\varsigma}$. As usual, we restrict attention to the space of parameters such that $\varphi_x^*/\varphi^* = \tau(f_x/f)^{1/\varsigma} > 1$, so that there is a range of firms with $\varphi \in [\varphi^*, \varphi_x^*]$ operating in the domestic market only, while the most productive firms also export.

Under these assumptions, *ex-ante* expected profits are:

$$\mathbb{E}[\pi] = f\int_{\varphi^*}^{\infty}\left[\left(\frac{\varphi}{\varphi^*}\right)^{\varsigma} - 1\right]\mathrm{d}G(\varphi) + f_x\int_{\varphi_x^*}^{\infty}\left[\left(\frac{\varphi}{\varphi_x^*}\right)^{\varsigma} - 1\right]\mathrm{d}G(\varphi), \tag{13}$$

[23] Although in this version of the model, revenue-based labor productivity is constant across firms, it can easily be made an increasing function of φ by assuming that the fixed cost of production, f, is in units of labor. Alternatively, the model in Section 5 generates variation in revenue per worker across firms through another channel.

where the two terms represent expected profits from the domestic and the foreign market. Solving the integrals yields:

$$\mathbb{E}[\pi] = \frac{\varsigma}{1/v - \varsigma}\left[f\left(\frac{\varphi_{\min}}{\varphi^*}\right)^{1/v} + f_x\left(\frac{\varphi_{\min}}{\varphi_x^*}\right)^{1/v}\right].$$

To study how export opportunities affect the value of drawing productivity from a riskier distribution, we compute again the elasticity of expected profits to v:

$$\frac{\partial \ln \mathbb{E}[\pi]}{\partial \ln v} = \frac{1}{1 - v\varsigma} + \ln\left(\frac{\varphi^*}{\varphi_{\min}}\right)^{1/v} + \frac{\ln\left(\varphi_x^*/\varphi^*\right)^{1/v}}{\left(\varphi_x^*/\varphi^*\right)^{1/v} f/f_x + 1} \tag{14}$$

Comparing this derivative to (9), we see that choosing a riskier distribution yields now a new advantage: conditional on surviving, it increases the probability of reaching the export cutoff, φ_x^*. Moreover, as it is well known and we show next, $\varphi^*/\varphi_{\min}$ is higher with trade.

As in autarky, we solve for the equilibrium v by imposing the free-entry condition, $\mathbb{E}[\pi] = \lambda F(v)$. This condition allows us to find the exit cutoff:

$$\left(\frac{\varphi^*}{\varphi_{\min}}\right)^{1/v} = \frac{\varsigma}{1/v - \varsigma}\frac{f + f_x\left(\varphi_x^*/\varphi^*\right)^{-1/v}}{\lambda F(v)}. \tag{15}$$

As expected, the exit cutoff is higher than in autarky and is increasing in the barriers to export. For convenience, we now define $\rho \equiv \varphi^*/\varphi_x^* = \left(f/f_x\right)^{1/\varsigma}/\tau$ and use it as a synthetic measure of trade openness. This index, which varies between zero and one, only depends on exogenous parameters and determines the fraction of exporting firms, which is equal to $\rho^{1/v}$. Using this notation and (15) into (14), we can show how trade affects the elasticity of expected profits to v, and hence the incentive to draw productivity from riskier distribution:

$$\frac{\partial^2 \ln \mathbb{E}[\pi]}{\partial \ln v \partial \rho} = \frac{f/f_x}{\rho^{1+1/v}}\frac{\ln \rho^{-1/v}}{v\left(\rho^{-1/v} f/f_x + 1\right)^2} > 0. \tag{16}$$

In words, more openness raises *unambiguously* the return from riskier productivity draws. This result is intuitive: trade offers new profitable opportunities, but only to the most productive firms and hence reallocates profits to the right tail of the distribution. In turn, a higher v (a lower shape parameter of the distribution of productivity) increases the probability mass in that tail. This is one of the main results of the paper: the chance of winning the extra prize of exporting induces firms to take a riskier bet at the entry stage. Notice

also that even in the extreme case in which all firms export ($\varphi^*/\varphi_x^* \to 1$), υ will be higher in the trading equilibrium than in autarky because of the higher fixed cost faced by each firm ($f + f_x$ instead of f).[24]

Following the same steps as in autarky, the equilibrium υ is implicitly determined by:

$$\frac{1}{1-\upsilon\varsigma} + \ln\left(\frac{\varsigma}{1/\upsilon - \varsigma}\frac{f + f_x\rho^{1/\upsilon}}{\lambda F(\upsilon)}\right) + \frac{\ln \rho^{-1/\upsilon}}{\rho^{-1/\upsilon} f/f_x + 1} = \frac{\upsilon F'(\upsilon)}{F(\upsilon)} \tag{17}$$

Since the left-hand side is increasing in openness (this follows from 16), and assuming again the solution to be unique and interior, more openness leads to a higher equilibrium υ and hence more productivity dispersion.

These results are summarized in the following Proposition.

Proposition 2 *An increase in openness triggered by a fall in the variable cost of trade, τ, induces firms to choose riskier productivity draws (higher υ) and raises the equilibrium dispersion of firm productivity, as measured by the variance of the log of φ.*

An additional interesting implication of this model is that trade has a new effect on productivity. Since a higher υ also raises the unconditional mean of φ, export opportunities induce firms to choose riskier technologies with higher expected returns. As a result, in an equilibrium with trade firms will be more productive, not just because of the usual selection effect, but also because firms choose more costly, but on average more efficient, technologies. This prediction is also of intuitive appeal: the higher premium for success in the global economy makes firms more "ambitious" by choosing a bigger (and riskier) investment in the entry stage.

Of course, the analytical results derived in this section partly hinge on functional form assumptions and on the convenient properties of Pareto distributions. Yet, we expect the main mechanism to hold more in general. In particular, as long as trade reallocates profits in favor of exporters and exporting firms are a minority, trade will increase the payoff in the tails and hence raise the return from taking a riskier bet on productivity.

[24] This result is true as long as trade is costly. Of course, the case of costless trade (e.g., if f is not destination specific, $f_x = 0$ and $\tau = 1$) would be equal to autarky.

5 From Firm Heterogeneity to Income Inequality

We now explore the implications of our theory for income and wage inequality. This as a natural step: the distribution of productivity is likely to be a major determinant of the distribution of wages because in the data more productive firms pay higher wages. We therefore extend the model to allow for differences in wages across firms. This will yield two main results: first, it will highlight a new channel through which trade can increase wage inequality and, second, it will identify some additional variables affecting the choice of risk at the entry stage.

In principle, our theory can be used to study top-income inequality. An immediate way of doing this is to draw a link between profits and entrepreneurial income. For example, one could assume that there is a class of agents, entrepreneurs, who are the only ones who can enter and start new firms. These agents may be able to finance part of the entry cost externally and will be the residual claimants on a share of profits. Recent models along these lines include Jones and Kim (2014) or Grossman and Helpman (2014). Since trade increases the dispersion of profits, it will also make entrepreneurial income more unequal. Several contributions in corporate finance, such as Gabaix and Landier (2008), have indeed shown that CEO compensations are proportional to firm size and that this can explain why they have increased so much in recent decades. Our theory can then help rationalize some of the changes in the firm size distribution behind this phenomenon.

Another possibility, that we consider more in detail, is to extend the model to study implications for wage dispersion. In the literature, there are several ways of linking firm productivity to wages. With competitive labor markets, wages can vary because of differences in workforce composition across firms (e.g., Sampson, 2014, Monte, 2011, Yeaple, 2005). Alternatively, workers could be paid different wages due to labor market frictions (e.g., Helpman et al. 2010, Amiti and Davis, 2012, Egger and Kreickemeier, 2009, Felbermayr, Impullitti and Prat, 2014). For example, in Helpman, Itskhoki, and Redding (2010, HIR henceforth) workers matched randomly with heterogeneous firms draw a match-specific ability which is not observed and firms can invest in costly screening. In equilibrium, more productive firms screen workers more intensively to exclude those with lower ability. As a result, they have workforces of higher average ability and pay higher wages. These models yield an exporter wage premium and have been found to have considerable empirical support (e.g., Helpman et al. 2014). We therefore now borrow the framework of HIR to study the implications of

our theory for wage dispersion. One key advantage of HIR is that it preserves the main equations of the basic Melitz model, thereby allowing us to apply our previous results in a relatively straightforward manner.

We briefly derive the equations of HIR that are relevant for our purpose and refer the reader to the original article for more details. For ease of comparison, we try to follow the original notation whenever possible. Production depends on the productivity of the firm, φ, the measure of hired workers, h, and the average ability of these workers, $\bar{a}$:

$$y = \varphi h^{\gamma} \bar{a},$$

where $\gamma \in (0, 1)$ implies diminishing returns to hired workers. Two important properties of this production function are the complementarity between firm productivity and average worker ability and a trade-off between the quantity and quality of hired workers. Workers' ability is assumed to be independently distributed and drawn form a Pareto distribution with shape parameter $k > 1$ and c.d.f. $G_a(a) = 1 - (a_{\min}/a)^{-k}$. Search frictions in the labor market imply that a firm has to pay bn units of the numeraire to be matched randomly with a measure n of workers. Ability is unknown. However, once the match is formed, the firm can use a screening technology to identify workers with ability below a_c at the cost of ca_c^{δ}/δ units of the numeraire, with $c > 0$, $\delta > k$. Given the distribution of ability, a firm matched with n workers and screening at the cutoff a_c will hire a measure $h = n\left(a_{\min}/a_c\right)^k$ of workers with an average ability of $\bar{a} = a_c k/(k-1)$. Following the notation in HIR, we define $\beta \equiv 1 - 1/\sigma$. Then, total revenue of a firm with productivity φ can be written as

$$r(\varphi) = (1 + \mathbb{I}\tau^{1-\sigma})^{1-\beta} P X^{1-\beta} (\varphi \bar{a})^{\beta} h^{\beta\gamma}$$

where $\mathbb{I}$ is an indicator function taking value 1 if the firm decides to export and zero otherwise.

Wages are determined through strategic bargaining between the firm and workers, after the firm has paid all the costs. HIR show that the outcome is that the firm retains a fraction of revenues equal to the Shapley value, $1/(1 + \beta\gamma)$, and pays the rest to the workers. Thus, the profit maximization problem of the firm is:

$$\pi(\varphi) = \max_{n, a_c, \mathbb{I}} \left\{ \frac{r(\varphi)}{1 + \beta\gamma} - bn - \frac{ca_c^{\delta}}{\delta} - f - \mathbb{I} f_x \right\},$$

and the first-order conditions for n and a_c are

$$\begin{aligned}\frac{\beta\gamma}{1+\beta\gamma}r(\varphi) &= bn(\varphi) \\ \frac{\beta(1-\gamma k)}{1+\beta\gamma}r(\varphi) &= ba_c(\varphi)^{\delta}.\end{aligned}$$

Inspection reveals immediately that firms with higher revenue sample more workers (higher n) and screen more intensively (higher a_c). Assuming $\delta > k$ also ensures that firms with higher revenue hire more workers.

Substituting the first-order conditions for n and a_c into the profit function yields $\pi(\varphi) = \frac{\Gamma r(\varphi)}{1+\beta\gamma} - f - \mathbb{I}f_x$, with $\Gamma \equiv 1-\beta\gamma-(1-\gamma k)\beta/\delta$. Since revenues are increasing in productivity, the fixed costs implies that firms with $\varphi < \varphi^*$ exit (where $\pi_{\mathbb{I}=0}(\varphi^*) = 0$) and firms with $\varphi > \varphi_x^*$ export (where $\pi_{\mathbb{I}=0}(\varphi_x^*) = \pi_{\mathbb{I}=1}(\varphi_x^*)$). Moreover, the relative revenue of any two firms only depends on their relative productivity and export status:

$$\frac{r(\varphi)}{r(\varphi^*)} = \left(1+\mathbb{I}\tau^{1-\sigma}\right)^{(1-\beta)/\Gamma}\left(\frac{\varphi}{\varphi^*}\right)^{\beta/\Gamma}.$$

Combining these results, we find an expression for *ex-ante* expected profits, $\mathbb{E}[\pi]$, which turns out to be identical to the one in the previous section (equation 13) after the redefinition of the parameter $\varsigma = \beta/\Gamma$ (instead of $\sigma - 1$). The ratio of the cutoffs is now:

$$\rho = \frac{\varphi^*}{\varphi_x^*} = (f/f_x)^{1/\varsigma}\left[\left(1+\tau^{1-\sigma}\right)^{\varsigma(1-\beta)/\beta} - 1\right]^{1/\varsigma}.$$

which is still increasing in ς.

The equilibrium v depends on ς, f and ρ as implied by equation (17) and, in particular, it is increasing in ς. The difference, however, is that ς corresponds now to a combination of more parameters, $\varsigma = \left[\beta^{-1} - \gamma - (1-\gamma k)/\delta\right]^{-1}$, so that in this extended version of the model there are more determinants of v. In particular, through their impact on ς, an increase in γ or a fall in k and δ leads firms to draw from more dispersed distributions. These results are intuitive. As already discussed, more risk taking is optimal for the firm when profits are more convex in productivity. In the simpler version of the model, convexity only depends on σ. Now, instead, the profit function is more convex also when there are weaker diminishing returns (high γ) and when screening - which is disproportionately beneficial to more productive firms - is more effective, i.e., when worker ability is more dispersed and the

screening cost not too elastic.

Proposition 3 *The dispersion of firm productivity, as measured by the variance of the log of φ, is larger in sectors with more ability dispersion and weaker decreasing returns to scale.*

These results may also contribute at explaining why the dispersion of firm productivity varies across countries and over time. For example, it suggests that firms in countries with a more heterogeneous labor force will choose riskier investments and hence be more unequal in equilibrium. Likewise, the growing evidence on the "flattening of the firm" may indicate a rise in the span-of-control parameter and this may help explain the generalized increase in productivity dispersion documented in Section 2.

What are the implications for wages? Using the definition of wages as a share of revenue per hired worker yields:

$$w(\varphi) \equiv \frac{\beta\gamma}{1+\beta\gamma} \frac{r(\varphi)}{h(\varphi)} = b\left[\frac{a_c(\varphi)}{a_{\min}}\right]^k.$$

Since $a_c(\varphi)$ is increasing in productivity, more productive firms pay higher wages. Due to the complementarity in production between average worker ability and productivity, more productive firms have a stronger incentive to be more selective, hire workers with higher ability and pay them higher wages. Moreover, since wages are proportional to revenue, which jumps at the export cutoff $\varphi = \varphi_x^*$, the model implies an exporter wage premium. More precisely, the wage paid by firms with productivity φ can be written as

$$w(\varphi) = \left(1 + \mathbb{I}\tau^{1-\sigma}\right)^{\frac{k(1-\beta)}{\delta\Gamma}} \varphi^{\frac{\beta k}{\delta\Gamma}} w(\varphi^*).$$

Finally, since employment, $h(\varphi)$, is also a power function of productivity, the wages of workers employed by domestic firms and exporters are Pareto distributed with shape parameter:

$$1 + \delta[(\upsilon\varsigma)^{-1} - 1]/k,$$

which is decreasing in υ. Thus, heterogeneity in productivity maps into wage dispersion. This allows us to state the following proposition on the impact of trade on wage inequality.

Proposition 4 *More openness raises unambiguously sectoral wage dispersion among workers employed by domestic firms and among workers employed by exporters. Conditional on*

not changing export status, more openness increases wage inequality between workers employed by any pair of firms with different productivity.

Before concluding, it is important to highlight the qualitative and quantitative differences between our result and HIR. In HIR and some other existing models, trade affects wage dispersion through the exporter wage premium. The sign of the effect then depends on the fraction of exporters. As long as exporters are a minority, trade increases wage dispersion by raising the share of firms paying high wages. Once exporters are a majority, instead, trade decreases wage dispersion by pushing low-wage domestic firms to exit and making the surviving firms more equal. Thus, the overall effect of trade on inequality is inverted-U shaped. This effect is present also in our model. But there is now another, potentially more powerful, force: by making all firms more unequal, trade is changing the slope of the entire wage schedule. This second effect, which is absent in HIR, implies that trade now increases wage inequality within exporters, within nonexporters, and also between the two groups of firms. It follows that, as stated in Proposition 4, openness raises unambiguously some measures of wage inequality. Other measures of inequality, such as the Gini coefficient, will instead depend on the combination of the exporter wage premium, as in HIR, and the steeper wage schedule. For those measures, depending on which effect dominates, the effect of trade on inequality may or may not be ambiguous.

6 Conclusions

In this paper, we made several contributions to the literature. First, we started documenting some little-known facts regarding how the distribution of firms varies across sectors and over time. We have found that the extent of heterogeneity, measured by the standard deviation of log sales, changes systematically with industry characteristics and has increased significantly over time. Second, we have proposed one possible explanation, based on the idea that firms can choose the risk of their random productivity draw at the entry stage. The model formalizes the hypothesis that firms can choose between larger and riskier projects with high expected payoffs, and smaller but safer projects with lower expected returns. Third, we have found that export opportunities, by reallocating profits to the most productive firms, increase the return to risk. Finally, we have explored the implication for wage inequality and found a new channel through which trade liberalization can affect the entire wage distribution and increase its dispersion: export opportunities induce more risk taking and this translates into

a higher equilibrium heterogeneity both in productivity and wages. As we discussed, this mechanism differs in important respects from those already emphasized in existing models.

In many ways, however, this paper raises more questions than answers. Our theory explores only one out the many forces shaping the equilibrium distribution of firms. For example, to focus on one mechanism and preserve tractability, we left firm dynamics and innovation by incumbent firms out of the analysis. Yet, it is clear that our approach to the choice of entry risk would apply immediately also to product innovation of existing firms. Hence, making the model dynamic, for instance along the lines of Arkolakis (2015) or Gabler and Poschke (2013), seems a desirable extension. Within our theory, we also restricted the attention to positive implications. Yet, the model suggests interesting normative questions: do firms take too much or too little risk, especially if workers are risk averse and insurance markets are imperfect? Does international trade introduce new externalities in the technology choice at the entry stage?

Finally, our look at the data is just a first scratch on the surface. Much remains to be done to document extensively how firm heterogeneity varies across sectors, countries and time, for instance using alternative measures including estimates of firm-level productivity, and how it affects wage inequality. Moreover, since various moments of the observed distribution of firms can be shaped by many factors, identifying empirically different mechanisms remain and important challenge. We hope that the last contribution of this paper will be to stimulate more research on these important questions.

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7 Appendix

7.1 Data Appendix

Here we discuss in more detail data construction and variables definitions.

Dispersion measures To compute the standard deviation of log sales at the establishment level, we use the Statistics of U.S. Businesses released by the U.S. Census. These data contain information on receipts of sales, number of firms and establishments, and number of employees by sale-size category and industry in Census years 1997, 2002 and 2007. The publicly available data are obtained by aggregating confidential establishment-level data from the Business Registry, which covers the universe of establishments with paid employees in the U.S..

The data are available for ten sale-size bins in 1997, eight bins in 2002 and eighteen bins in 2007. The lowest bin contains firms with revenues smaller than 50 thousand US$ and the highest bin firms with revenues larger than 100 millions US$. In our main analysis, we aggregate the data into six bins consistently observed over the sample period. These refer to firms with US$ revenues in the following intervals: (1) less than 100,000; (2) 100,000-499,999; (3) 500,000-999,999; (4) 1,000,000-4,999,999; (5) 5,000,000-99,999,999; (6) 100 millions or more. As for the industry classification, the data are reported at the 4-digit level of SIC-1987 in 1997, and at the 6-digit level of NAICS-1997 and NAICS-2002 in 2002 and 2007, respectively. We convert the data in a consistent number of 460 6-digit industries, using the correspondence table between SIC-1987 and NAICS-1997 from the NBER, and crosswalks between different versions of the NAICS classification from the U.S. Census.

Finally, we compute the dispersion of sales in each industry and year as the standard deviation of log average sales per establishment across the six bins, weighting each observation by the number of establishments in the bin. The standard deviation of log sales per worker used in some robustness checks is computed analogously.

Export intensity We construct export intensity in each industry and year as the ratio of exports to total shipments. Shipment data come from the NBER-CES Manufacturing Industry Database. Export data are sourced from Schott (2008). For 1997 and 2002, we use data by 6-digit NAICS industry and destination country. For 2007, we compute bilateral exports in each 6-digit industry by aggregating exports across all products—defined at the

10-digit level of the Harmonized System (HS) classification—belonging to that industry. After excluding 37 inconsistent observations with exports greater than total shipments, our final data set contains export intensity for 373 6-digit industries over the three years.

Other variables The number of establishments and workers in each industry and year comes from the Statistics of U.S. Businesses. Factor intensities are computed as in Romalis (2004) using data from the NBER-CES Manufacturing Industry Database. Material intensity is equal to material costs divided by the sum of material costs and value added. Capital intensity is computed as 1 minus material intensity, times the non-labor share of valued added. Skill intensity is computed as 1 minus material intensity, times the product of the labor share of value added and the employment share of non-production workers.

The mean and standard deviation of the log of workers' educational attainment are constructed with data from the CPS Merged Outgoing Rotation Groups. Education is a discrete variable ranging from 1 to 16, with higher values corresponding to higher educational attainment. We restrict to individuals aged 18-64, and compute the mean and standard deviation of log education in each industry by weighting the individual observations with full-time equivalent hours of labor supply, as in Autor et al. (2003). We then convert the data from the CPS industry classification to the 6-digit NAICS classification using correspondence tables from the U.S. Census.

Import penetration equals c.i.f. imports over apparent consumption, defined as production plus imports minus exports. Import data come from Schott (2008) and are constructed analogously to the export data described before. Production (shipment) data come from the NBER-CES Manufacturing Industry Database. The high-tech share of capital investment is constructed with data on investment in private non-residential fixed assets by 3-digit NAICS industry from the Bureau of Economic Analysis. High-tech capital includes mainframes, PCs, DASDs, printers, terminals, tape drives, storage devices, system integrators, communications, photocopy and related equipment, office and accounting equipment, prepackaged software, custom software and own account software.

In our IV regressions, we instrument for export intensity using non-U.S. countries' exports to the destination markets of U.S. exports in each industry and year. To construct this variable, we map BACI's bilateral trade data at the product (HS 6-digit) level into the 6-digit NAICS industry level, using correspondence tables from the World Integrated Trade Solution and the U.S. Census. Finally, in our diff-in-diff strategy, we use oil (Brent) prices

from FRED (Federal Reserve of St. Louis) and industry-level bulk weights expressed in Kg per US$. The bulk weights are computed with export data from Schott (2008). They are defined as the export-weighted average of individual products' bulk weights across air and vessel shipments in 1995.

7.2 Mean-Preserving Spreads

We now solve the model under the assumption that $\varphi_{\min} = \bar{\varphi}(1-v)$ so that the mean of the distribution is constant at $\bar{\varphi}$ and an increase in v corresponds to a mean-preserving spread.

Using the expression for $\frac{\partial \ln \mathbb{E}[\pi]}{\partial \ln v}$ derived in the text, the first-order condition for the problem

$$\max_{v \in [\underline{v}, \bar{v}]} \{\mathbb{E}[\pi] - \lambda F(v)\}$$

becomes:

$$\frac{\partial \mathbb{E}[\pi]}{\partial v} = \frac{\mathbb{E}[\pi]}{v} \left[\frac{1}{1-v\varsigma} - \frac{1}{1-v} + \ln \left(\frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} \right] = \lambda F'(v).$$

Imposing free-entry, $\mathbb{E}[\pi] = \lambda F(v)$, yields the same expression for the exit cutoff, $\varphi^*/\varphi_{\min}$. Using these results in the new first-order condition yields:

$$\frac{1}{1-v\varsigma} - \frac{1}{1-v} + \ln \left(\frac{f}{\lambda F(v)} \frac{\varsigma}{1/v - \varsigma} \right) = \frac{vF'(v)}{F(v)}.$$

This equation, which pins down implicitly the equilibrium v, is identical to (12), except for the new term $-1/(1-v)$ on the left-hand side. Intuitively, the fact that higher risk is not associated to higher expected productivity draw lowers the value of v. This immediately implies that firms will choose a lower equilibrium level of entry risk. However, the comparative static for all the parameters are unchanged. Moreover, revenue, $r(\varphi)$, is still Pareto distributed with c.d.f. $G_r(r) = 1 - (r_{\min}/r)^{1/v\varsigma}$, for $r > r_{\min} = \sigma f$ as in the benchmark case. Hence, all the results in Proposition 1 still hold.

Consider now the model with trade. Deriving expected profit (13) with respect to v when $\varphi_{\min} = \bar{\varphi}(1-v)$ yields:

$$\frac{\partial \ln \mathbb{E}[\pi]}{\partial \ln v} = \frac{1}{1-v\varsigma} - \frac{1}{1-v} + \ln \left(\frac{\varphi^*}{\varphi_{\min}} \right)^{1/v} + \frac{\ln (\varphi_x^*/\varphi^*)^{1/v}}{(\varphi_x^*/\varphi^*)^{1/v} f/f_x + 1}.$$

This expression for the return to risk is again, is identical to (14), except for the new term

$-1/(1-v)$ on the left-hand side.

Imposing free-entry, $\mathbb{E}[\pi] = \lambda F(v)$, yields the same expression (15) for the exit cutoff, $\varphi^*/\varphi_{\min}$. Then, the effect of openness, ρ, on the value of risk, as captured by $\frac{\partial^2 \ln \mathbb{E}[\pi]}{\partial \ln v \partial \rho}$ is identical to (16). Following the same steps as in autarky, the equilibrium v is implicitly determined by:

$$\frac{1}{1-v\varsigma} - \frac{1}{1-v} + \ln\left(\frac{\varsigma}{1/v-\varsigma}\frac{f+f_x\rho^{1/v}}{\lambda F(v)}\right) + \frac{\ln \rho^{-1/v}}{\rho^{-1/v}f/f_x+1} = \frac{vF'(v)}{F(v)}.$$

Once again, the lower value of risk (the new term $-1/(1-v)$ on the left-hand side) implies the firms will draw from less dispersed distributions, but the effect of openness and other parameters on the choice of v is qualitatively unchanged. However, the result that trade induces firms to choose technologies with higher expected productivity is now absent by construction.

7.3 A Simple Model of Risk and Technology Choice

We now show how the assumptions regarding the choice of risk at the entry stage can be derived from a model of investment in "projects" of uncertain quality. The goal of this exercise is to illustrate one simple and intuitive way in which firms can affect entry risk and to show that the assumptions on v and $F(v)$ made in the text are not difficult to justify.

Suppose a firm can invest resources in innovation projects. The outcome of the project is a technology allowing the firm to produce a new product with a productivity φ, and the final realization of this productivity depends on both on the quality of the project, which is uncertain, and the amount of resources invested on it.

Assume that quality q of projects is random and exponentially distributed:

$$\Pr[q > x] = e^{-\delta x},$$

with support $x \in [0, \infty)$. Notice that quality is inherently uncertain and its realization is beyond control of the firm. In reality, firms might be able to distinguish between "riskier" and "safer" ideas. Yet, our purpose here is to show that exposure to innovation risk can be controlled even when the deep source of uncertainty is exogenous. To this end it is enough to let the firm choose the size of the project, s. Assume next that productivity, φ, depends

on both the quality and size of the project according to the following equation:

$$\ln \varphi = sq + \ln \varphi_{\min}.$$

This equation embeds a complementarity between quality and size: resources invested in a bad project ($q = 0$) are wasted, while even a great idea is useless without some investment to implement it. Then, φ is Pareto distributed, as can be seen from:

$$1 - G(\varphi) = \Pr\left[q > \frac{\ln(\varphi/\varphi_{\min})}{s}\right] = \left(\frac{\varphi}{\varphi_{\min}}\right)^{-\frac{\delta}{s}}.$$

The inverse of the shape parameter of the productivity distribution, which we denoted as v, is equal to s/δ.

In conclusion, by simply choosing the size of the project, the firm is effectively choosing both the average and the variance of its productivity draw. If we finally assume that there are diminishing returns to the size the project, then we obtain that the cost of investing in a project is increasing and convex in $v = s/\delta$, as postulated in the main text of the paper.

Table 1 - Descriptive Statistics on the Dispersion of Sales

NAICS code	Industry Description	SD mean	SD min	SD max	SD % change	# of est. mean
322	Paper Manufacturing	1,686	0,964	2,269	0,067	252
327	Nonmetallic Mineral Product Manufacturing	1,885	0,972	3,556	0,033	728
333	Machinery Manufacturing	1,968	0,483	3,212	-0,029	498
313	Textile Mills	1,996	1,174	2,967	-0,113	258
331	Primary Metal Manufacturing	2,070	1,443	3,096	0,094	203
326	Plastics and Rubber Products Manufacturing	2,072	1,359	3,077	0,080	1024
315	Apparel Manufacturing	2,089	1,000	2,941	0,045	546
332	Fabricated Metal Product Manufacturing	2,119	1,217	3,331	0,099	1387
324	Petroleum and Coal Products Manufacturing	2,171	0,750	4,477	0,466	482
316	Leather and Allied Product Manufacturing	2,203	0,891	3,256	0,056	139
339	Miscellaneous Manufacturing	2,237	0,947	3,670	0,235	1571
325	Chemical Manufacturing	2,242	0,554	4,015	0,061	394
337	Furniture and Related Product Manufacturing	2,262	0,921	3,182	0,183	1753
334	Computer and Electronic Product Manufacturing	2,265	0,881	3,761	-0,014	499
321	Wood Product Manufacturing	2,427	1,460	3,253	0,322	1187
335	Electrical Equipment, Appliance, and Component Manufacturing	2,468	1,791	3,662	0,049	279
312	Beverage and Tobacco Product Manufacturing	2,523	1,976	3,116	0,313	452
323	Printing and Related Support Activities	2,545	1,332	3,309	0,365	2773
311	Food Manufacturing	2,586	1,075	4,567	0,410	549
314	Textile Product Mills	2,595	1,447	3,337	0,317	649
336	Transportation Equipment Manufacturing	2,712	1,723	4,221	0,019	404
	Total	2,230	0,483	4,567	0,118	720

Note: SD is the standard deviation of log sales, computed on 6 sales-size bins, homogeneous for all years, for each 6-digit industry. Percentage changes refer to the period between 1997 and 2007. Statistics are computed on data at 6-digit industry level.

Table 2 - OLS Regressions

Dep. var.: Standard deviation of log establishment-level sales

	Pooled OLS				Industry Fixed Effects		
	Establishment Sales	Export Intensity	Industry Controls	GDP Growth	Industry Controls	GDP Growth	First Differences
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log establishment-level sales	0.203***		0.194***	0.210***	0.587***	0.676***	0.519**
	[0.020]		[0.056]	[0.057]	[0.110]	[0.111]	[0.248]
Log export intensity		0.076***	0.091***	0.089***	0.130***	0.074*	0.171**
		[0.015]	[0.015]	[0.015]	[0.047]	[0.046]	[0.070]
Log number of establishments			0.036	0.053	0.449**	0.554***	0.959**
			[0.070]	[0.071]	[0.176]	[0.175]	[0.391]
Log employment			0.130*	0.110	-0.159	-0.318***	-0.252
			[0.069]	[0.070]	[0.108]	[0.110]	[0.336]
Log cap. intensity			0.064	0.041	-0.037	-0.182	0.074
			[0.108]	[0.108]	[0.220]	[0.222]	[0.192]
Log sk. intensity			-0.094*	-0.082	0.202	0.339***	-0.172
			[0.054]	[0.054]	[0.124]	[0.126]	[0.199]
Log mat. intensity			-0.218	-0.248	-0.420	-0.760**	0.190
			[0.156]	[0.157]	[0.344]	[0.354]	[0.422]
Log av. education			0.283	0.258	-0.626	-0.448	-0.853
			[0.431]	[0.432]	[0.907]	[0.874]	[1.222]
S.D. log education			1.032***	1.021***	0.205	0.127	-0.314
			[0.363]	[0.362]	[0.538]	[0.515]	[0.658]
GDP growth				2.104***		3.448***	
				[0.461]		[0.495]	
Obs.	1,036	1,036	1,036	1,036	1,036	1,036	670
R-squared	0,17	0,03	0,29	0,29	0,67	0,69	0,46

Note: The dependent variable and all controls but GDP growth are observed at 6-digit NAICS industry level in years 1997, 2002 and 2007. All industry-level controls are contemporaneous to the dependent variable. The dependent variable is computed on 6 sales-size bins, homogeneous for all years, for each 6-digit industry. GDP growth is computed over the two years prior to each observation. All specifications are estimated with Ordinary Least Squares. Standard errors, clustered by 6-digit indutries, are reported in brackets. ***, **, and * denote significance at 1, 5, and 10 per cent respectively.

Table 3 - Robustness Checks

Dependent variables indicated in columns' headings

	Dep. Var.: S.D. of Log Establishment-Level Sales					Dep. Var.: S.D. of Log Labor Productivity		
	Industry Fixed Effects and First Differences							
	Import Penetration	High-Tech Capital	No Small Bins	No Large Bin	Raw Bins	Pooled OLS	Industry Fixed Effects	Industry FE and First Differences
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log establishment-level sales	0.519**	0.298	0.367**	0.106	0.171***	0.281***	0.262***	0.544**
	[0.249]	[0.210]	[0.175]	[0.177]	[0.029]	[0.067]	[0.095]	[0.230]
Log export intensity	0.169**	0.168**	0.094*	0.142*	0.028***	0.110***	0.149***	0.157**
	[0.069]	[0.070]	[0.049]	[0.079]	[0.008]	[0.033]	[0.038]	[0.068]
Log number of establishments	0.955**	0.574*	0.356	0.811**	0.251***	0.399***	0.758***	0.867**
	[0.393]	[0.340]	[0.268]	[0.383]	[0.035]	[0.107]	[0.223]	[0.354]
Log employment	-0.252	-0.168	-0.164	-0.067	-0.206***	-0.283***	-0.432***	-0.295
	[0.336]	[0.294]	[0.231]	[0.285]	[0.032]	[0.073]	[0.144]	[0.322]
Log cap. intensity	0.076	0.035	-0.191	0.429**	-0.009	-0.009	0.112	0.044
	[0.193]	[0.201]	[0.143]	[0.205]	[0.063]	[0.148]	[0.175]	[0.169]
Log sk. intensity	-0.178	-0.361*	-0.189	-0.101	-0.084***	0.036	0.007	-0.081
	[0.202]	[0.184]	[0.132]	[0.195]	[0.030]	[0.087]	[0.087]	[0.184]
Log mat. intensity	0.191	0.239	-0.224	0.817**	-0.098	-0.250	0.150	0.069
	[0.426]	[0.459]	[0.287]	[0.414]	[0.093]	[0.273]	[0.257]	[0.390]
Log av. education	-0.818	-2.360*	-0.023	0.127	0.119	0.106	0.077	-1.065
	[1.231]	[1.314]	[0.586]	[1.288]	[0.215]	[0.559]	[0.619]	[1.133]
S.D. log education	-0.293	-0.952	-0.154	-0.134	0.420**	0.026	0.060	-0.492
	[0.657]	[0.646]	[0.347]	[0.630]	[0.207]	[0.311]	[0.316]	[0.615]
GDP growth					0.419**	0.661**		
					[0.209]	[0.307]		
Log import penetration	0.026							
	[0.082]							
High-tech share of investment		-0.204						
		[0.981]						
Obs.	670	591	660	646	1,029	1,029	664	670
R-squared	0,46	0,48	0,54	0,48	0,46	0,17	0,48	0,49

Notes: The dependent variables and all controls but GDP growth are observed at 6-digit NAICS industry level in years 1997, 2002 and 2007. All industry-level controls are contemporaneous to the dependent variable. Unless otherwise specified, the dependent variable is computed on 6 sales-size bins, homogeneous for all years, for each 6-digit industry. GDP growth is computed over the two years prior to each observation. In column (3), the dependent variable is computed excluding observations for the two smallest sales-size bins for each industry. In column (4), the dependent variable is computed excluding observations for the largest sales-size bin for each industry. In column (5), the dependent variable is computed on all sales-size bins available for each year and industry. In colums (6)-(8), the dependent variable is the dispersion of log labor productivity, defined as sales per worker. All specifications are estimated with Ordinary Least Squares. Standard errors, clustered by 6-digit indutries, are reported in brackets. ***, **, and * denote significance at 1, 5, and 10 per cent respectively.

Table 4 - Instrumental Variables Regressions

Dependent variables indicated in columns' headings

	Dep. Var.: S.D. of Log Establishment-Level Sales				Dep. Var.: S.D. of Log Labor Productivity		
	Pooled OLS	Pooled OLS	Industry Fixed Effects	Industry FE and First Differences	Pooled OLS	Industry Fixed Effects	Industry FE and First Differences
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log establishment-level sales	0.213***	0.227***	0.699***	0.812***	0.172***	0.267***	0.301***
	[0.057]	[0.058]	[0.103]	[0.229]	[0.030]	[0.064]	[0.102]
Log export intensity	0.142***	0.141***	0.295**	0.707***	0.050***	0.180*	0.316***
	[0.030]	[0.030]	[0.137]	[0.167]	[0.017]	[0.094]	[0.095]
Log number of establishments	0.073	0.089	0.597***	1.129***	0.259***	0.407***	0.743***
	[0.071]	[0.072]	[0.170]	[0.424]	[0.036]	[0.107]	[0.240]
Log employment	0.110	0.092	-0.220*	-0.486	-0.205***	-0.249***	-0.440***
	[0.070]	[0.071]	[0.133]	[0.322]	[0.033]	[0.084]	[0.152]
Log cap. intensity	0.074	0.053	-0.155	0.039	0.003	0.006	0.096
	[0.111]	[0.111]	[0.227]	[0.216]	[0.066]	[0.147]	[0.171]
Log sk. intensity	-0.117**	-0.106*	0.368***	0.007	-0.099***	0.025	0.021
	[0.058]	[0.058]	[0.121]	[0.205]	[0.032]	[0.086]	[0.091]
Log mat. intensity	-0.256	-0.283*	-0.838**	-0.314	-0.109	-0.266	0.043
	[0.163]	[0.163]	[0.347]	[0.464]	[0.098]	[0.272]	[0.266]
Log av. education	-0.015	-0.034	-0.909	-0.880	-0.018	0.073	0.176
	[0.456]	[0.455]	[0.875]	[1.308]	[0.237]	[0.572]	[0.630]
S.D. log education	0.872**	0.863**	0.008	-0.279	0.317	0.003	0.120
	[0.379]	[0.378]	[0.505]	[0.710]	[0.221]	[0.311]	[0.327]
GDP growth		1.936***	2.927***		0.386*	0.553*	
		[0.468]	[0.555]		[0.208]	[0.334]	
Obs.	1,015	1,015	1,001	630	1,008	992	620
First-Stage results							
Log world exports	0.448***	0.447***	0.397***	0.546***	0.447***	0.398***	0.568***
	[0.068]	[0.068]	[0.044]	[0.068]	[0.068]	[0.045]	[0.064]
Kleibergen-Paap F-Stat	44,0	43,8	80,5	65,3	43,0	79,5	77,7

Notes: The dependent variable and all controls but GDP growth are observed at 6-digit NAICS industry level in years 1997, 2002 and 2007. All industry-level controls are contemporaneous to the dependent variable. The dependent variable is computed on 6 sales-size bins, homogeneous for all years, for each 6-digit industry. GDP growth is computed over the two years prior to each observation. Exports are instrumented with non-US exports to US destination markets for each industry and year. All specifications are estimated with Two-Stages Least Squares. Standard errors, clustered by 6-digit indutries, are reported in brackets. F-statistics are reported for the Kleibergen-Paap test for weak instruments. ***, **, and * denote significance at 1, 5, and 10 per cent respectively.

Table 5 - Difference-in-Differences

Dependent variables indicated in columns' headings

	Dep. Var.: S.D. of Log Establishment-Level Sales				Dep. Var.: S.D. of Log Labor Productivity		
	Pooled OLS	Pooled OLS	Industry Fixed Effects	Industry FE and Time Dummies	Pooled OLS	Industry Fixed Effects	Industry FE and Time Dummies
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log establishment-level sales	0.154**	0.170***	0.677***	0.677***	0.150***	0.119	0.119
	[0.061]	[0.062]	[0.158]	[0.158]	[0.036]	[0.074]	[0.074]
Bulk weight * Log oil price	-0.074**	-0.071**	-0.097**	-0.097**	-0.027**	-0.032**	-0.032**
	[0.029]	[0.030]	[0.040]	[0.040]	[0.012]	[0.014]	[0.014]
Bulk weight	0.093	0.081			0.062		
	[0.103]	[0.109]			[0.040]		
Log oil price	0.147***	0.142***	0.051		0.058**	0.093**	
	[0.034]	[0.034]	[0.056]		[0.026]	[0.040]	
Log number of establishments	-0.040	-0.020	0.680***	0.680***	0.228***	0.324**	0.324**
	[0.071]	[0.074]	[0.226]	[0.226]	[0.041]	[0.127]	[0.127]
Log employment	0.167**	0.144*	-0.401**	-0.401**	-0.200***	-0.201**	-0.201**
	[0.073]	[0.076]	[0.162]	[0.162]	[0.041]	[0.099]	[0.099]
Log cap. intensity	0.008	-0.014	-0.331	-0.331	-0.008	-0.016	-0.016
	[0.121]	[0.121]	[0.216]	[0.216]	[0.079]	[0.172]	[0.172]
Log sk. intensity	-0.120**	-0.111*	0.280*	0.280*	-0.104***	0.079	0.079
	[0.060]	[0.060]	[0.155]	[0.155]	[0.035]	[0.108]	[0.108]
Log mat. intensity	-0.173	-0.199	-0.874**	-0.874**	-0.051	-0.211	-0.211
	[0.170]	[0.171]	[0.376]	[0.376]	[0.111]	[0.317]	[0.317]
Log av. education	0.528	0.498	-1.090	-1.090	0.252	-0.115	-0.115
	[0.455]	[0.457]	[1.107]	[1.107]	[0.246]	[0.731]	[0.731]
S.D. log education	1.084***	1.056***	-0.444	-0.444	0.495**	-0.116	-0.116
	[0.392]	[0.392]	[0.627]	[0.627]	[0.236]	[0.389]	[0.389]
GDP growth		1.626***	3.291***		0.276	0.504	
		[0.518]	[0.554]		[0.230]	[0.364]	
Obs.	776	776	776	776	771	771	771
R-squared	0,31	0,32	0,66	0,66	0,18	0,48	0,48

Notes: The dependent variables and all controls, but GDP growth and oil price, are observed at 6-digit NAICS industry level in years 1997, 2002 and 2007. Bulk weight is expressed in Kg per US$ shipped by air and/or vessel, and refers to year 1995. All other industry-level controls are contemporaneous to the dependent variable. The dependent variables are computed on 6 sales-size bins, homogeneous for all years, for each 6-digit industry. GDP growth is computed over the two years prior to each observation. All specifications are estimated with Ordinary Least Squares. Standard errors, clustered by 6-digit indutries, are reported in brackets. ***, **, and * denote significance at 1, 5, and 10 per cent respectively.