Input Price Discrimination by Resale Market*

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Abstract

This paper analyzes supply tariffs that discriminate between resale in different markets. In a setting with competing retailers that operate in multiple (independent or interdependent) markets, we show that, all else equal, a monopolist supplier wants to discriminate against resale in the market with the higher aggregate cross-seller diversion ratio. We find that discrimination can improve allocative efficiency and present sufficient conditions, involving the pass-through rates and the inverse market demand curvatures in the different markets, under which discrimination has positive effects on output and welfare. Our insights are relevant for the policy treatment of vertical restraints on online sales.

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1 Introduction

Starting with Pigou's (1920) seminal work, a large literature in economics has studied the effects of allowing sellers to charge different prices to different consumers (see, e.g., Robinson 1933; Schmalensee 1981; Varian 1985; Schwartz 1990; Aguirre et al. 2010; Bergemann at al. 2015). Recognizing that many instances of price discrimination involve intermediate-goods markets, the literature has also looked at the effects of allowing sellers to discriminate between different downstream firms rather than final consumers (see, e.g., Katz 1987; DeGraba 1990; Yoshida 2000; Inderst and Shaffer 2009). In all cases, the defining feature of what constitutes price discrimination has been that a seller discriminates across its buyers.

This paper considers a different form of price discrimination, one that is prevalent in intermediate-goods markets but has not received much attention in the economics literature: price discrimination *across resale markets*, rather than across buyers. This form of price discrimination occurs, for instance, when a supplier charges different wholesale prices for goods and services resold in different geographic locations by the same downstream firms. A prominent example in the pharmaceutical industry are contracts between manufacturers and wholesalers in which the wholesale price of a drug depends on the country in which the wholesaler resells the drug.[1] Other examples include TV content providers charging fees to downstream cable companies that depend on the subscriber's location,[2] and consumer-goods manufacturers offering trade promotions to grocery retailers that are available in only some of the regions in which a retailer operates.[3]

Discrimination across resale markets also encompasses situations in which multiple outputs are produced from a common input and input prices depend on which final output is produced. For example, the licensing fees of many standards-essential patents depend on the price or the type of the end product for which the patents are used, resulting in

[1] This practice has been challenged in several cases in Europe involving manufacturers setting two wholesale prices when selling to wholesalers in Spain, one for reselling through the Spanish health care system, and another, higher wholesale price for exporting. See Commission Decision 2001/79 1/EC; GlaxoSmithKline Services v Comission, T- 168/01, ECR, EU:T:2006:265; ECJ Judgment of 6 October 2009, cases C-501, 513, 515, 519/06 P; Spanish Competition Authority, Pfizer/Cofares, S/DC/0546/15, 19 January 2017.

[2] See Singer and Sidak (2007, p.387). We thank Ali Yurukoglu for bringing this example to our attention.

[3] Trade promotions involve discounts off invoice that are granted to retailers for a certain time period (Coughlan et al. 2013). Reimbursements are typically made contingent on how much the retailer actually sells in each market—verified through store level scanner data—as opposed to how much it orders.

different per-unit fees for different products sold by a downstream firm.[4] Another example is Du Pont's "value-in-use pricing" system, which specifies more than a dozen different price points for the *Kevlar* fiber depending on end use.[5] Input price discrimination across resale markets also arises when supply contracts specify different terms for products or services that are resold to different customer segments. An example of this are patent licensing fees that depend on whether the end product is sold to a commercial-use customer or a home-use customer.[6]

An example of particular policy relevance are input supply contracts that specify different terms for products resold in different distribution channels, especially when such contracts discriminate against resale on the internet. In the European Union, charging a higher wholesale price for products that are intended to be resold by a distributor online than for products that are intended to be resold off-line, a practice also known as "dual pricing," is a hardcore restriction of competition according to the 2010 Guidelines on Vertical Restraints. The related practice of restricting the share of online sales in the total sales made by a distributor, i.e., from "agreeing that the distributor shall limit its proportion in overall sales made over the internet," is also treated as a hardcore restriction.[7] Cases include *Dornbracht*, 2011, in which the German Federal Cartel Office prohibited contracts that required wholesalers to pay higher input prices for products resold to online retailers as opposed to traditional offline retailers; and *Bosch-Siemens*, 2013, in which the German

[4] Licensing fees are often set as a fixed percentage of the final product's selling price. While such fees have generally been upheld in the U.S. (see *Automatic Radio Manufacturing Co. v. Hazeltine Research, Inc.*, 339 U.S. 827, 834 (1950)), the Indian Competition Commission recently challenged basing licensing on a percentage of the sales price as being "prima facie discriminatory" (see Teece et al. 2017).

[5] A downstream firm that puts the input to a different end use than the one it was initially purchased for is required to pay Du Pont an amount representing the difference between the initial purchase price and the price for the ultimate end use. See *Akzo NV v. USITC*, 808 F.2d 1471 (1986). See also *Carter-Wallace, Inc. v. United States* 449 F.2d 1374 (1971), where the manufacturer of the chemical compound meprobamate charged a lower wholesale price for meprobamate used in combination drugs than for meprobamate resold as a stand-alone drug.

[6] The license fees for the MPEG Surround standard, for instance, are higher for professional PC software than for consumer PC software. See http://www.via-corp.com/us/en/licensing/mpegsurround/licensefees.html (accessed on September 6, 2018).

[7] See paragraphs 52 (c) and 52 (d) of the *Commission notice of 10 May 2010: Guidelines on vertical restraints* [SEC (2010) 411 final]. Hardcore restrictions give rise to the presumption that the agreement is prohibited under Article 101(1) of the Treaty on the Functioning of the European Union. Although firms can try to claim that such restrictions have pro-competitive effects under Article 101(3), it is considered "unlikely that vertical agreements containing such hardcore restrictions fulfil the conditions of Article 101(3)" (Commission Regulation (EU) No 330/2010).

Federal Cartel Office prohibited Bosch-Siemens's practice of granting wholesale price discounts for household appliances that were proportional to a retailer's offline versus online sales.[8]

To analyze the effects of this type of price discrimination, we consider a setting in which a monopolist supplier offers input supply tariffs to competing downstream firms that operate in multiple downstream markets (e.g., online and offline).[9] The downstream firms (henceforth, retailers) then engage in resale to final consumers. Allowing the final products sold to consumers to be substitutes or independent across markets, we address two sets of questions. First, why does the supplier want to discriminate across resale in different markets, and what determines *which* of the downstream markets the supplier wants to discriminate against? Second, what are the effects of such discrimination on allocative efficiency, total output, and welfare?

We first show that when the downstream markets are asymmetric (in terms of how much market power the retailers have, for instance), supply tariffs that depend on the total amount ordered by a retailer will *not* allow the supplier to achieve its first-best outcome, even if the retailers themselves are symmetric and general non-linear tariffs are feasible. Intuitively, although the supplier can control each retailer's total quantity through the tariff it offers, this is not enough because each retailer ignores the externalities on the profits generated by rival retailers when deciding how to *allocate* its total quantity across the different downstream markets. To correct for these externalities and induce industry profit maximization, the supplier must therefore discriminate between resale in different markets, e.g., by making its wholesale prices depend on the intended resale market.[10]

We find that three conceptually distinct factors determine *which* downstream market the supplier wants to discriminate against at the wholesale level: the intensity of competition

[8]For full case summaries, see Bundeskartellamt, "Fallbericht: Bundeskartellamt stellt sicher: Hochwertige Sanitärarmaturen im Internet erhältlich," B5-100/10, Dec. 12, 2011, for the *Dornbracht* case, and Bundeskartellamt, "Fallbericht: Rabattsystem der Bosch Siemens Hausgeräte GmbH, München," B7-11/13, Dec. 23, 2013, for the *Bosch Siemens* case. See also Bundeskartellamt, "Vertical Restraints in the Internet Economy," Background Paper – Working Group on Competition Law, October 10, 2013.

[9]With single-market downstream firms, the distinction between discrimination across markets versus across buyers is blurred. Nevertheless, our analysis also yields insights about discrimination by resale market in settings with single-market downstream firms, as will be described in more detail below.

[10]Because our focus is on price discrimination in the input market, we assume that resale price maintenance, which under some conditions can achieve similar effects (see Chen 1999), is infeasible.

between retailers in each market (which, as we will show, can be measured by "market power parameters" that depend on the product diversion ratios within and across markets), the retail prices that the supplier seeks to induce in the different markets, and marginal production costs. Between two resale markets, the supplier wants to discriminate against resale in the more competitive market, provided the monopoly prices in the two markets are similar or the monopoly price is higher in the more competitive market.[11] Conversely, if the intensity of competition is similar in the two markets or competition is fiercer in the market with the higher monopoly price, the supplier wants to discriminate against the market with the higher monopoly price.

Consistent with the examples on patent licensing and pharmaceuticals mentioned previously, these results suggest that we would expect to see suppliers charging higher licensing fees and wholesale prices for resale in downstream markets that support higher prices (all else equal). The results also suggest that contracts that discriminate against resale on the internet may simply be a result of retailers having greater market power offline than online, which can be explained by the lower importance of geographical differentiation in an online setting, or by asymmetries in consumer search costs between online and offline markets.[12,13]

In response to our second set of questions, we establish conditions under which input price discrimination by resale market has positive or negative effects on allocative efficiency, total output sold, and social welfare defined as the sum of profits and consumer surplus. As in the case of monopolistic third-degree price discrimination in final-goods markets, which arises as a limiting case of our setting, the welfare effect of discrimination can be decomposed into an *allocation effect* and an *output effect*. But whereas the allocation effect

[11] By monopoly prices we mean the downstream prices that maximize total industry profits, i.e., the sum of the supplier's and the retailers' profits.

[12] See Lieber and Syverson (2012) for a detailed discussion of online versus offline competition.

[13] An alternative explanation for vertical restrictions on internet sales is that suppliers seek to mitigate potential free-riding on the investment efforts of their retailers' brick-and-mortar stores–see Iacobucci and Winter (2015) for a nice summary of this theory and a criticism of the EU Guidelines on its basis. Our explanation does not rely on investment incentives to explain vertical restraints on online sales and thus also applies to products for which free-riding on services may be less of a concern. Moreover, it also applies to markets in which the same retailers operate both online and offline, so that no clear "victim" of free-riding can be identified in the downstream market. Helfrich and Herweg (2017) and Dertwinkel-Kalt and Köster (2017) offer explanations for why a supplier may want to ban online distribution of its products, building on the assumption that consumer decisions are distorted by salient thinking as in Bordalo et al. (2013). Our theory assumes that consumers are fully rational and applies to hybrid retailers.

is always negative in the case involving final-goods markets,[14] we find that it can be positive in our setting. The reason is that the supplier's optimal choice of wholesale prices in the different markets can push the final-goods prices closer together, paradoxically resulting in *less* discrimination to final consumers than in the absence of input price discrimination.[15] It follows that welfare can rise with discrimination even if total output decreases.

Another fundamental difference between the kind of price discrimination we consider and price discrimination across buyers in final-goods markets is that in our setting, the sign of the output effect depends not just on the curvatures of the market demand functions, but also on the extent to which wholesale price changes are passed through to final consumers.[16] The pass-through rates, in turn, depend on the relative intensities of competition in the downstream markets and on demand curvatures. This gives rise to a previously unrecognized functional dependence, one that can generate a positive output effect even in settings where, given existing results, one might have expected that there would be no, or a negative, output change. With linear demands (and zero or symmetric cross-market price effects), for instance, we find that discrimination against resale in the more competitive downstream market has a positive output effect. This result contrasts sharply with Pigou's (1920) famous insight that price discrimination in final-goods markets has no effect on output when demand functions are linear.[17]

For the case of independent markets, we establish general sufficient conditions under which discrimination has a positive or negative output effect by combining the marginal effects technique commonly used in the literature on third-degree price discrimination in final-goods markets (e.g., Aguirre et al. 2010) with the exogenous quantity method proposed by Weyl and Fabinger (2013). Total output sold increases with discrimination if the pass-through rate from wholesale to retail prices is lower in the market where prices fall than

[14]Price discrimination generates a misallocation of output across consumers relative to uniform pricing in the case of final-goods markets, because output is not allocated to the consumers who value it the most.

[15]An example is when the intensity of competition differs across markets, but the monopoly prices are the same. Allowing discrimination at the wholesale level then eliminates discrimination at the retail level.

[16]Aguirre et al. (2010) offer a comprehensive analysis of the role played by demand curvature for the output and welfare effects of monopolistic third-degree price discrimination in final-goods markets. In Section 5, we will contrast the effects of input price discrimination by resale market with their results.

[17]This holds unless price discrimination leads to new markets being served. See, e.g., Tirole's (1988, p. 139) textbook treatment of third-degree price discrimination. Layson (1998) shows that Pigou's result carries over to the case of interdependent markets with symmetric cross-market price effects.

in the market where prices rise, and if the inverse market demand function is at least as convex in the market where prices fall as in the market where prices rise. Conversely, if the pass-through rate is lower in the market where prices rise and the inverse market demand function is at least as concave in the market where prices fall as in the market where prices rise, total output falls with discrimination. Regarding total welfare, we use the same method to establish a sufficient condition—involving the monopoly prices, the pass-through rates, and the curvatures of the inverse market demand functions in the two markets—that guarantees a positive welfare effect when the market demand functions satisfy the (commonly met) increasing ratio condition of Aguirre et al. (2010). Given the latter condition and log-concave market demand functions, so that more intense competition in a market is associated with a higher pass-through rate, discrimination against resale in the more competitive market raises welfare if the monopoly prices are not too far apart (or the monopoly price is higher in the market where price falls) and the convexities of the inverse market demand functions at the monopoly prices are sufficiently similar (or inverse market demand is more convex in the market where price falls).

While our primary focus is on multi-market retailers, our insights also apply to the case of discrimination across markets with single-market retailers. This expands our theory's applicability to any setting in which the focus is on understanding why firms in one market may have received lower input prices than firms in another market, whether or not the firms are the same across markets. What distinguishes our work from previous research in this respect is that the supplier can deal with multiple retailers in each downstream market. In contrast, existing work on input price discrimination has considered settings in which there is either a single downstream market or a single retailer in each market (see, e.g., Katz 1987; DeGraba 1990; Inderst and Shaffer 2009; Herweg and Müller 2013).

In summary, our theory shows that input price discrimination across resale markets is a natural consequence of asymmetries between the various markets in which the downstream firms compete. Furthermore, our results suggest that if asymmetries in the intensity of competition across markets are the primary driver of discrimination, it is reasonable to expect that discrimination will have a positive effect on social welfare. In light of these findings, the current treatment of vertical restraints on online sales in the EU appears to be overly aggressive, because it severely restricts suppliers' ability to practice a form of

input price discrimination that may well improve welfare. Our results are also relevant for the U.S. policy on input price discrimination, especially primary-line Robinson-Patman Act cases in which (actual or potential) rival sellers allege that they have been injured by a seller's discriminatory prices across markets, and potentially also for import policy in the U.S. involving alleged dumping whereby products manufacturerd overseas are sold in the U.S. at "less than fair value".[18] In such cases, information about the downstream firms' market power in different markets could help to identify drivers of input price discrimination that are unrelated to whether other suppliers are also present in the market, and thus help to establish the credibility (or lack thereof) of any alleged anti-competitive conduct.[19]

The rest of this paper proceeds as follows. In the next section, we describe the framework. Section 3 derives market power parameters that measure the intensity of competition in each market allowing for cross-market demand interdependencies. These measures will be used in the subsequent analysis, and they may also prove to be independently useful in other applications. Section 4 explains why the supplier wants to discriminate and characterizes the supplier's optimal discriminatory wholesale prices. Section 5 analyzes the allocation, output, and welfare effects of input price discrimination by resale market. Section 6 concludes. Proofs of the propositions and lemmas are relegated to Appendix A.

2 The model

We consider a setting with one supplier and n competing retailers. Each retailer operates in the same two markets. The retailers are indexed by $i = 1, 2, ..., n$, and the markets are indexed by $s = A, B$.[20] The retailers transform the supplier's inputs into final outputs at zero marginal cost, using one unit of input to make each unit of output. The supplier has a constant marginal cost $c \geq 0$.

[18] See the Antidumping and Countervailing Duty Laws under the Tariff Act of 1930.

[19] The focus in these cases is typically on whether the seller's discriminatory prices were below-cost—and therefore predatory—an assessment that is often difficult to measure by direct means.

[20] Allowing firms to compete in more than one market distinguishes our model from much of the vertical contracting literature, which typically assumes that the downstream firms compete in only one market. An exception is Arya and Mittendorf (2010), who consider a setting with one multi-market and one single-market retailer. In contrast to our paper, however, they consider only discrimination *across buyers* and not across resale markets. Further distinguishing our work from theirs, we allow for interdependent demands within and between all downstream markets and use general functional forms.

Each retailer sets two prices, one for market A and one for market B. The demand for retailer i in market $s \in \{A, B\}$ is given by

$$D_{si}\left(p_{s1}, p_{s2}, ..., p_{sn}, p_{-s1}, p_{-s2}, ..., p_{-sn}\right),$$

where p_{sj} denotes j's price in market s and p_{-sj} denotes j's price in market $-s \neq s$. The $n \times 2$ final products are (weak) substitutes, and own-price effects dominate cross-price effects. That is, for all $i \neq j$ and $s \neq -s$,

$$\begin{aligned} \frac{\partial D_{-si}}{\partial p_{si}}, \frac{\partial D_{sj}}{\partial p_{si}}, \frac{\partial D_{-sj}}{\partial p_{si}} &\geq 0, \\ -\frac{\partial D_{si}}{\partial p_{si}} &> \sum_{j \neq i} \frac{\partial D_{sj}}{\partial p_{si}} + \sum_{k=1,2,...,n} \frac{\partial D_{-sk}}{\partial p_{si}}. \end{aligned}$$

This formulation allows for the possibility of demand interactions within and across markets. Note that the special case of independent markets obtains if and only if there are no cross-market price effects, i.e., $\frac{\partial D_{-si}}{\partial p_{si}} = \frac{\partial D_{-sj}}{\partial p_{si}} = 0$ for all $i \neq j$ and $s \neq -s$.[21]

To focus on asymmetries between markets, we assume that the retailers are symmetric, so that in each market s,

$$D_{si}\left(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s}\right) = D_{sj}\left(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s}\right)$$

for any (p_s, p_{-s}) and $i \neq j \in \{1, 2, ..., n\}$. This then allows us to denote the (per-retailer) quantity demanded in market s at symmetric prices by the *market demand function*[22]

$$Q_s\left(p_s, p_{-s}\right) = D_{s1}\left(p_s, p_s, ..., p_s, p_{-s}, p_{-s}, ..., p_{-s}\right).$$

The marginal change in the (per-retailer) quantity demanded in market s when all retailers' prices move in tandem is thus

[21] To exclude the trivial case in which all $n \times 2$ final products are independent, we assume that $\frac{\partial D_{sj}}{\partial p_{si}} > 0$ for at least one s when the markets are independent.

[22] In the case of independent markets ($\frac{\partial Q_s}{\partial p_{-s}} = 0$), Q_s corresponds to Chamberlin's (1933) *DD* demand curve. See also Anderson, de Palma and Kreider (2001).

$$
\begin{aligned}
\frac{\partial Q_s}{\partial p_s} &= \frac{\partial D_{si}}{\partial p_{si}} + \sum\nolimits_{j\neq i} \frac{\partial D_{si}}{\partial p_{sj}} = \frac{\partial D_{si}}{\partial p_{si}} + \sum\nolimits_{j\neq i} \frac{\partial D_{sj}}{\partial p_{si}}, \\
\frac{\partial Q_s}{\partial p_{-s}} &= \frac{\partial D_{si}}{\partial p_{-si}} + \sum\nolimits_{j\neq i} \frac{\partial D_{si}}{\partial p_{-sj}} = \frac{\partial D_{si}}{\partial p_{-si}} + \sum\nolimits_{j\neq i} \frac{\partial D_{sj}}{\partial p_{-si}},
\end{aligned}
$$

where the second equality in each line follows from retailer symmetry.

Industry profits are equal to the sum of profits across the $n \times 2$ final products:

$$
\Pi\left(p_{A1}, ..., p_{An}, p_{B1}, ..., p_{Bn}\right) = \sum_{i=1,2,...,n} \sum_{s=A,B} \left(p_{si} - c\right) D_{si}\left(p_{s1}, ..., p_{sn}, p_{-s1}, ..., p_{-sn}\right).
$$

We assume the Hessian matrix of Π is negative definite, which implies that industry profits are strictly concave in the retail prices. This and symmetry imply that there is a unique symmetric pair of prices that maximize industry profits, which we denote by (p_A^*, p_B^*). In what follows, we will also refer to (p_A^*, p_B^*) as the *monopoly prices*. To economize on notation, we let $D_{si}^* = D_{si}\left(p_s^*, ..., p_s^*, p_{-s}^*, ..., p_{-s}^*\right)$ and $Q_s^* = Q_s\left(p_s^*, p_{-s}^*\right)$.

The game that we analyze proceeds in two stages:

1. The supplier offers a (common) tariff T to the retailers. We consider two classes of tariffs: (i) tariffs of the form $T(q_i)$, where q_i is the total quantity ordered by retailer i, and (ii) tariffs of the form $T(q_{Ai}, q_{Bi})$, where q_{si} is the quantity ordered by retailer i for resale in market s. The retailers make simultaneous offer acceptance decisions.[23]

2. The retailers simultaneously set their downstream prices in the two markets.[24] Each retailer then orders sufficient quantities to serve its demands and pays the supplier according to the tariff T.[25]

[23] Indifferences are broken in favor of contract acceptance.

[24] A retailer that rejected the supplier's tariff offer sets both prices above some choke price level at which its demands are zero.

[25] We assume that retailers do not ration demand, which implies that they need access to sufficient quantities of the input after any history of prices. The latter can be guaranteed by imposing either that tariffs permit retailers to order as many units as they wish, although possibly with lump-sum penalities or high marginal prices for large quantities, or that retailers have access to an outside source of supply in case their demand exceeds the maximum amount specified in the tariff. This assumption allows us to circumvent the technical difficulties associated with price competition under capacity constraints (Kreps and Scheinkman 1983, Montez and Schutz 2018) and focus on the supplier's optimal input price discrimination instead.

Although we allow for general non-linear tariffs, it will prove useful to denote i's variable profit given some constant marginal cost c_A in market A and c_B in market B by

$$\pi_i(p_{A1},...,p_{An},p_{B1},...,p_{Bn};c_A,c_B)=\sum_{s=A,B}(p_{si}-c_s)\,D_{si}(p_{s1},...,p_{sn},p_{-s1},...,p_{-sn}).$$

We assume that for any vector (c_A, c_B) (below some choke level), the simultaneous-move game in which each retailer i sets its prices (p_{Ai}, p_{Bi}) in order to maximize its variable profit π_i has a symmetric and stable Nash equilibrium.

We will say that *full channel coordination* can be achieved if the two-stage game (of input supply contracting in stage one followed by downstream pricing decisions in stage two) has a subgame-perfect equilibrium in which all retailers charge p_A^* in market A and p_B^* in market B.

3 Preliminaries

Our goal in this section is to obtain measures of where the retailers' market power in each downstream market falls in the realm between perfect competition and monopoly. These measures, which are developed by analyzing the downstream pricing game in isolation, will be used in the next section when we characterize optimal input supply tariffs. Our results in this section may also prove to be independently useful in other applications that involve price competition between multi-product firms.

In models of price competition with differentiated goods and only a *single* product per firm, between-product diversion ratios are widely used to measure market power.[26] Using the analog of our notation, i.e., letting $D_i(p_1, p_2, ..., p_n)$ denote the demand of i as a function of the prices of the n sellers and $Q(p)$ the market demand function when all sellers charge p, the first-order equilibrium condition can be written as

$$(p-c)\frac{-\frac{\partial Q}{\partial p}}{Q}=1-d \tag{1}$$

in this case, where $d=-\sum_{j\neq i}\frac{\partial D_j}{\partial p_i}/\frac{\partial D_i}{\partial p_i}$ denotes the aggregate diversion ratio between

[26]Diversion ratios play a particularly prominent role in horizontal merger analysis—see Shapiro (1996), Katz and Shapiro (2003), and Conlon and Mortimer (2018), among others.

products. The left-hand side of (1) is the elasticity-adjusted Lerner index, which is equal to the price-cost margin multiplied by the semi-elasticity of the market demand function. The right-hand side measures the firms' market power: As $1-d$ rises from zero to one, market power varies between perfect competition and monopoly.[27]

In what follows, we first derive the analogs of the single-product firm oligopoly market power parameter $1-d$ for our more complex setting with multi-market firms and asymmetries across markets, and then show how the relative market power parameters in the downstream markets are linked to the aggregate *cross-seller* demand diversion ratios of price changes in the two markets.

Hypothetical downstream monopolist We begin by deriving implicit conditions for the retail prices that a hypothetical downstream monopolist would set. Because demands in each market are symmetric, the profit-maximizing prices in each market will also be symmetric (i.e., $p_{s1} = ... = p_{sn} = p_s$ for each s). Given constant marginal costs c_A and c_B for sales in markets A and B, respectively, it follows that the monopolist's first-order conditions can be reduced to

$$Q_A + (p_A - c_A)\frac{\partial Q_A}{\partial p_A} + (p_B - c_B)\frac{\partial Q_B}{\partial p_A} = 0, \tag{2}$$
$$Q_B + (p_A - c_A)\frac{\partial Q_A}{\partial p_B} + (p_B - c_B)\frac{\partial Q_B}{\partial p_B} = 0. \tag{3}$$

Solving the system of equations in (2) and (3) for the markups in the two markets yields

$$p_s - c_s = \frac{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s}\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - R_{AB}R_{BA}} \tag{4}$$

[27] The expression $1-d$ is also sometimes referred to as the *conduct parameter* of the symmetric differentiated price-competition model—see, for instance, Weyl and Fabinger (2013), who, following Genesove and Mullin's (1998) variation on Breshnahan (1989), set the elasticity-adjusted Lerner index equal to a conduct parameter and then derive the expressions for the conduct parameter in a variety of different models of oligopoly behavior, including differentiated price competition. We will use the term "market power parameter" instead, so as to distinguish our approach, which measures the seller market power induced by the underlying demand functions for a given model of oligopoly behavior, from empirical studies that seek to estimate the type of oligopoly behavior or conduct in a particular industry.

for each $s \in \{A, B\}$, where the cross-market diversion ratio

$$R_{s-s}(p_s, p_{-s}) \equiv \frac{\partial Q_{-s}/\partial p_s}{-\partial Q_s/\partial p_s} \in [0, 1)$$

can be thought of as the share of sales lost in market s that is captured by market $-s$ when p_s increases. In the special case of independent markets ($R_{AB} = R_{BA} = 0$), (4) simplifies to the standard monopoly pricing formula, $p_s - c_s = -\frac{Q_s}{\partial Q_s/\partial p_s}$.

Competitive pricing Now consider the pricing decisions of n independent symmetric retailers. In a symmetric equilibrium, $p_{s1} = ... = p_{sn} = p_s$ for each s. It follows that the first-order conditions, given marginal costs c_A and c_B, can be written as

$$Q_A + (p_A - c_A)\frac{\partial D_{Ai}}{\partial p_{Ai}} + (p_B - c_B)\frac{\partial D_{Bi}}{\partial p_{Ai}} = 0, \tag{5}$$

$$Q_B + (p_A - c_A)\frac{\partial D_{Ai}}{\partial p_{Bi}} + (p_B - c_B)\frac{\partial D_{Bi}}{\partial p_{Bi}} = 0. \tag{6}$$

Solving the system of equations in (5) and (6) for the markups in the two markets yields

$$\begin{bmatrix} p_A - c_A \\ p_B - c_B \end{bmatrix} = -\begin{bmatrix} \frac{1}{1-d_A}\frac{\partial Q_A}{\partial p_A} & \frac{1}{1+d_{AB}}\frac{\partial Q_B}{\partial p_A} \\ \frac{1}{1+d_{BA}}\frac{\partial Q_A}{\partial p_B} & \frac{1}{1-d_B}\frac{\partial Q_B}{\partial p_B} \end{bmatrix}^{-1} \begin{bmatrix} Q_A \\ Q_B \end{bmatrix}, \tag{7}$$

where

$$d_s(p_s, p_{-s}) \equiv \frac{\sum_{j \neq i} \frac{\partial D_{sj}}{\partial p_{si}}}{-\frac{\partial D_{si}}{\partial p_{si}}} \tag{8}$$

measures the diversion of sales to the rival retailers in market s when p_{si} changes, and

$$d_{s-s}(p_s, p_{-s}) \equiv \frac{\sum_{j \neq i} \frac{\partial D_{-sj}}{\partial p_{si}}}{\frac{\partial D_{-si}}{\partial p_{si}}} \tag{9}$$

measures the diversion of sales to the rival retailers in market $-s$ versus to the same retailer in market $-s$ when p_{si} changes. One can think of the diversion ratios d_A and d_B as capturing the strength of the within-market (intra-market) competitive externalities, and of d_{AB} and d_{BA} as capturing the strength of the cross-market (inter-market) competitive externalities.

Writing the markups in non-matrix form, we obtain, for each $s \neq -s \in \{A, B\}$,

$$p_s - c_s = \frac{(1-d_s)\frac{Q_s}{-\partial Q_s/\partial p_s} + \frac{(1-d_A)(1-d_B)}{1+d_{s-s}} R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - \frac{(1-d_A)(1-d_B)}{(1+d_{AB})(1+d_{BA})} R_{AB} R_{BA}}. \tag{10}$$

Market power parameters We are now in a position to assess the relative intensity of competition in the two markets. Let $\xi_s(p_s, p_{-s})$ denote the right-hand side of (4), so that the implicit conditions for the monopoly prices can be expressed as

$$p_s - c_s = \xi_s(p_s, p_{-s}). \tag{11}$$

We can then define *market power parameters* for the two markets under competition by rewriting (10) as

$$p_s - c_s = \theta_s\left(p_s, p_{-s}\right) \xi_s(p_s, p_{-s}), \tag{12}$$

where the market power parameters are

$$\theta_s\left(p_s, p_{-s}\right) \equiv \kappa\left(p_A, p_B\right) \frac{(1-d_s)\frac{Q_s}{-\partial Q_s/\partial p_s} + \frac{(1-d_A)(1-d_B)}{1+d_{AB}} R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}, \tag{13}$$

with

$$\kappa\left(p_A, p_B\right) \equiv \frac{1 - R_{AB} R_{BA}}{1 - \frac{(1-d_A)(1-d_B)}{(1+d_{AB})(1+d_{BA})} R_{AB} R_{BA}} \in (0, 1]. \tag{14}$$

Our assumption that the final products are substitutes and that own-price effects dominate cross-price effects imply that $\theta_s\left(p_s, p_{-s}\right) \in (0, 1]$. The case $\theta_s = 1$, which holds if and only if the markets are independent ($R_{AB} = R_{BA} = 0$) and $d_s = 0$, corresponds to monopoly power in market s. As θ_s falls, the divergence between competitive pricing and monopoly pricing rises. The limit case $\theta_s \to 0$ corresponds to perfect competition in market s.[28]

The next lemma summarizes the main insights thus far:

Lemma 1 *(i) The prices that maximize industry profits given a marginal cost c_s in market*

[28] All else equal, θ_s is weakly decreasing in d_s and d_{s-s} because stronger competitive (within or cross-market) externalities of price changes in market s put downward pressure on the prices in market s. θ_s is also weakly decreasing in d_{-s} and d_{-ss}, all else equal, because the profit margins in market $-s$ are weakly decreasing in d_{-s} and d_{-ss}, which in turn makes raising prices in market s less attractive when some of the demand lost in market s is diverted to the same retailer in market $-s$.

s satisfy, for each $s \neq -s$,

$$p_s - c_s = \xi_s(p_s, p_{-s}) \equiv \frac{\frac{Q_s}{-\partial Q_s/\partial p_s} + R_{s-s}\frac{Q_{-s}}{-\partial Q_{-s}/\partial p_{-s}}}{1 - R_{AB}R_{BA}}. \tag{15}$$

(ii) There exist market power parameters $\theta_s(p_s, p_{-s}) \in (0, 1]$ *such that the equilibrium retail prices of competing multi-market retailers given a marginal cost* c_s *in market* s *satisfy, for each* $s \neq -s$,

$$p_s - c_s = \theta_s(p_s, p_{-s})\, \xi_s(p_s, p_{-s}). \tag{16}$$

With independent markets, the cross-market diversion ratios are all zero, and the market power parameters are then simply $\theta_s = 1 - d_s$. With interdependent markets, however, the expressions for the market power parameters incorporate the semi-elasticities of demand in both markets and the demand diversions between markets. This is because the profit impact of a price change in one market depends on the profit margins in both markets, and the latter can be asymmetric. The semi-elasticities of the market demand functions thus do not cancel out of the market power parameter formulae, resulting in relatively long and harder-to-interpret expressions than in single-product firm settings. In spite of this, as our next result shows, the comparison of market power parameters across markets boils down to a comparison of readily interpretable *aggregate cross-seller diversion ratios*:

Lemma 2 *Competition is locally less (more) intense in market* A *than in market* B, *i.e.,* $\theta_A(p_A, p_B) > (<)\theta_B(p_B, p_A)$, *if and only if* $DR_A(p_B, p_A) < (>) DR_B(p_B, p_A)$, *where*

$$DR_s(p_s, p_{-s}) \equiv \frac{\sum_{j \neq i}\left(\frac{\partial D_{Aj}}{\partial p_{si}} + \frac{\partial D_{Bj}}{\partial p_{si}}\right)}{-\left(\frac{\partial D_{Ai}}{\partial p_{si}} + \frac{\partial D_{Bi}}{\partial p_{si}}\right)}.$$

The aggregate cross-seller diversion ratio DR_s can be thought of as the share of the *net* loss in retailer i's quantity sold (taking into account that i sells a substitute product in market $-s$) that goes to the rival retailers (in either market) when p_{si} increases. As shown in Lemma 2, the market with the higher aggregate cross-seller diversion ratio features more intense competition as measured by the market power parameters defined in Lemma 1.

4 Optimal input price discrimination by resale market

We now turn to the two-stage game in which the supplier can offer either a tariff of the form $T(q_i)$, which does not discriminate by resale market, or of the form $T(q_{Ai}, q_{Bi})$, which does discriminate by resale market, in stage one. Since we do not impose any other functional-form restrictions on the tariffs, in both cases the supplier can extract all downstream variable profits (by means of fixed fees, for instance) without affecting the retailers' pricing incentives. The supplier thus obtains the entire industry profit in equilibrium, and hence achieves its first-best outcome if it can induce the retailers to set the monopoly prices.

Our first result is that the supplier faces a fundamental channel coordination problem when selling to multi-market retailers, one that cannot (generically) be solved by a tariff of the form $T(q_i)$. No restrictions on $T(q_i)$ are required to establish this result, which would also continue to hold if the supplier could make buyer-specific offers of the form $T_i(q_i)$.

Proposition 1 *If the supplier is restricted to offering a tariff of the form $T(q_i)$ and the downstream markets differ in competitiveness ($\theta_A^* \neq \theta_B^*$) or in their monopoly prices ($p_A^* \neq p_B^*$),[29] then generically full channel coordination cannot be achieved.*

The channel coordination problem arises because each retailer allocates its total quantity across the two markets with the goal of maximizing its own profit rather than industry profit, ignoring externalities on the profits generated by its rivals. Even if the supply tariff induces each retailer to order and sell a total quantity of $D_{Ai}^* + D_{Bi}^*$, full channel coordination will thus fail because the total quantity will not be allocated optimally across the two markets from an industry profit standpoint. Full channel coordination therefore requires a tariff that distinguishes between resale in A and B—sales in one market must be discriminated against relative to sales in the other market.

One way for the supplier to achieve full channel coordination is to offer each retailer a "dual two-part tariff" that combines a fixed fee with two different (per-unit) wholesale prices, one for resale in market A and another for resale in market B. Our next proposition characterizes the optimal dual wholesale prices in such a tariff:[30]

[29] In line with our previous notation, $\theta_s^* \equiv \theta_s(p_s^*, p_{-s}^*)$.

[30] There are other tariffs of the form $T(q_{Ai}, q_{Bi})$ that allow the supplier to achieve its first-best outcome. For example, if the optimal wholesale prices from Proposition 2 satisfy $w_B^* > w_A^*$, the supplier can also

Proposition 2 *If the supplier can offer a tariff of the form $T(q_{Ai}, q_{Bi})$, the game has an equilibrium with full channel coordination in which the supplier offers the tariff*

$$T(q_{Ai}, q_{Bi}) = \pi_i(p_A^*, ..., p_A^*, p_B^*, ..., p_B^*; w_A^*, w_B^*) + w_A^* q_{Ai} + w_B^* q_{Bi}, \tag{17}$$

where, for each $s \in \{A, B\}$,

$$w_s^* = \theta_s^* c + (1 - \theta_s^*) p_s^*. \tag{18}$$

As shown in (18), the optimal wholesale price for resale in market s is a weighted average of the monopoly price in that market and the supplier's marginal cost, where the weight on the supplier's marginal cost equals the retailers' market power parameter in s evaluated at the monopoly prices.[31]

The polar cases of monopoly and perfect competition are easily understood. In the case of monopoly power ($\theta_s^* = 1$) in market s, the supplier optimally sets $w_s^* = c$. The retailers' pricing decisions in market s have no vertical externality on the supplier when the upstream margin is zero, which, given the absence of any horizontal externalities in the monopoly case, implies that each retailer's incentives are aligned with collective incentives. At the other extreme, as pricing in market s approaches perfect competition ($\theta_s^* \to 0$), the retailers pass on whatever wholesale price they receive; hence, the supplier can induce industry profit maximization by setting its wholesale price equal to the monopoly price.

Our characterization of the optimal dual wholesale prices shows that—between the polar cases of monopoly and perfect competition—the optimal dual wholesale price in market s is increasing in the (local) intensity of competition in market s, for a given monopoly price p_s^*.[32] Intuitively, as retailer competition intensifies in a market, the supplier needs to raise its wholesale price in that market in order to offset the erosion in the retailers' downstream margins that would otherwise cause the retail prices to fall.

achieve its first-best outcome by combining a (single) two-part tariff with wholesale price $w = \frac{Q_A^*}{Q_A^*+Q_B^*} w_A^* + \frac{Q_B^*}{Q_A^*+Q_B^*} w_B^*$ with an upper limit $\frac{Q_B^*}{Q_A^*+Q_B^*}$ on the *proportion* of the retailer's sales in market B.

[31] Positive downstream costs can be incoporated easily. If the retailers face a constant marginal cost $k > 0$, then $w_s^* = \theta_s^* c + (1 - \theta_s^*)(p_s^* - k)$.

[32] For an example of a demand specification for which the market power parameters can vary independently of the monopoly prices, see Section 5.2, which considers linear demand functions derived from a representative-consumer quadratic-utility model. Changes in the parameters measuring retailer substitutability affect the market power parameters without affecting the monopoly prices in this case.

The characterization of wholesale prices in Proposition 2 extends the literature on vertical control in two directions. First, it shows that intuitions on how downstream competition affects the optimal wholesale price in a two-part tariff, which previously were obtained by considering polar cases or specific functional forms in settings with just a single downstream market,[33] hold for general demand functions and extend to multi-markets settings. Second, it explains how the optimal *dual* wholesale prices in our multi-market setting depend on the retailers' relative market power and the monopoly prices in the two markets.

It follows from equation (18) in Proposition 2 that $w_A^* < w_B^*$ if and only if

$$\left(1-\theta_A^*\right)\left(p_A^*-c\right) < \left(1-\theta_B^*\right)\left(p_B^*-c\right), \tag{19}$$

which implies that the supplier optimally discriminates against resale in the (locally) more competitive market if the monopoly prices in the two markets are sufficiently close or the monopoly price is higher in the more competitive market. Moreover, if the downstream markets differ in competitiveness, discrimination against resale in the market with the lower monopoly price can be optimal. In the case of monopoly power in one market but less than monopoly power in the other market (e.g., $\theta_B^* < \theta_A^* = 1$),[34] for instance, the supplier optimally discriminates against resale in the more competitive market *regardless* of the monopoly prices.[35]

Remark: Single-market retailers While our analysis focuses on multi-market retailers, the characterization of the optimal wholesale prices in Proposition 2 is also relevant for settings with single-market retailers. To see this, suppose that there are n symmetric

[33] As far as the optimal wholesale price in a two-part tariff is concerned, single-market settings with symmetric firms correspond to the special case of independent markets of our framework.

[34] The case of $\theta_B^* < \theta_A^* = 1$ holds if the markets are independent, $d_A = 0$, and $d_B > 0$.

[35] In this context, it is worth noting that the market with the higher monopoly price need not be the market with the lower elasticity of market demand, because a multi-market monopolist internalizes demand diversion to its products in the other market when setting prices. Using the formulae for the monopoly margins in (4), we obtain that $p_A^* < p_B^*$ if and only if

$$\frac{-\partial Q_B^*/\partial p_B}{Q_B^*}\left(1-R_{BA}\right) < \frac{-\partial Q_A^*/\partial p_A}{Q_A^*}\left(1-R_{AB}\right). \tag{20}$$

In the presence of demand interdependencies between the two markets ($R_{s-s} \neq 0$), the market demand elasticities (20) are weighted by (what can be thought of as) the share of demand that would be lost *altogether* (i.e., that would not be captured in market $-s$) if the prices in market s increase slightly.

retailers in each market, making for a total of $n \times 2$ retailers. We first derive the market power parameters for this setting. Given marginal costs c_A and c_B in markets A and B, the competitive markups become, for each $s \in \{A, B\}$,

$$p_s - c_s = (1 - d_s) \frac{Q_s}{-\partial Q_s / \partial p_s}. \tag{21}$$

The markups of a hypothetical downstream monopolist remain unchanged, as given in (4). The market power parameters thus become, for each $s \neq -s \in \{A, B\}$,

$$\widetilde{\theta}_s \left(p_s, p_{-s} \right) = \left(1 - R_{AB} R_{BA} \right) \frac{\left(1 - d_s \right) \frac{Q_s}{-\partial Q_s / \partial p_s}}{\frac{Q_s}{-\partial Q_s / \partial p_s} + R_{s-s} \frac{Q_{-s}}{-\partial Q_{-s} / \partial p_{-s}}}. \tag{22}$$

With independent markets ($R_{AB} = R_{BA} = 0$), $\widetilde{\theta}_s = \theta_s = 1 - d_s$. With interdependent markets, $\widetilde{\theta}_s \left(p_s, p_{-s} \right) < \theta_s \left(p_s, p_{-s} \right)$, because single-market retailers do not take into account any of the competitive externalities on products in the other market.[36]

Given the single-market market power parameters $\widetilde{\theta}_A$ and $\widetilde{\theta}_B$, the supplier can then achieve its first-best outcome by means of discriminatory two-part tariffs, with the retailers in market s paying a fixed fee $F_s = \left(p_s^* - w_s^* \right) Q_s \left(p_s^*, p_{-s}^* \right)$ and a wholesale price

$$w_s^* = \widetilde{\theta}_s^* c + \left(1 - \widetilde{\theta}_s^* \right) p_s^*,$$

where $\widetilde{\theta}_s^* = \widetilde{\theta}_s \left(p_s^*, p_{-s}^* \right)$. As in the case of multi-market retailers, the optimal wholesale price in market s is a weighted average of the supplier's marginal cost and the monopoly price in market s, where the weight on the marginal cost equals the (single-market-firm) market power parameter in s evaluated at the monopoly prices.

[36] Comparing across markets, we obtain a result analogous to Lemma 2: $\widetilde{\theta}_A \left(p_A, p_B \right) > \widetilde{\theta}_B \left(p_A, p_B \right)$ if and only if

$$\frac{\sum_{j \neq i} \frac{\partial D_{Aj}}{\partial p_{Ai}} + \sum_{j=1}^{n} \frac{\partial D_{Bj}}{\partial p_{Ai}}}{-\frac{\partial D_{Ai}}{\partial p_{Ai}}} < \frac{\sum_{j \neq i} \frac{\partial D_{Bj}}{\partial p_{Bi}} + \sum_{j=1}^{n} \frac{\partial D_{Aj}}{\partial p_{Bi}}}{-\frac{\partial D_{Bi}}{\partial p_{Bi}}},$$

where the aggregate diversion ratio is now the sum of the diversion ratios to the $(n \times 2) - 1$ retailers.

5 Welfare analysis

We now turn to the effects on social welfare of allowing tariffs that discriminate between resale in A and B. To this end, we will compare welfare when the supplier offers the optimal dual two-part tariff to welfare when the supplier is restricted to offering a single two-part tariff of the form $T(q_i) = F + wq_i$. Restricting attention to two-part tariffs is without loss of generality in both cases, as shown in Appendix B.

The first part of this section will focus on the case of independent markets. This enables us to derive results that hold for general functional forms, and it facilitates comparison to previous results on monopolistic third-degree price discrimination in final-goods markets. Later, in Section 5.2, we will show that key qualitative insights carry over to the case of interdependent markets.

5.1 Independent markets

5.1.1 Model and additional notation

With independent markets, the market demand function $Q_s(p_s, p_{-s})$ is independent of p_{-s}. Hence, to simplify the notation, we will let $Q_s(p_s) = Q_s(p_s, p_{-s})$. The curvature of the inverse market demand function in market s (as a function of price) is then given by $\sigma_s(p) \equiv Q_s(p)\frac{Q_s''(p)}{(Q_s'(p))^2}$. Note that $Q_s(p_s)$ is (locally) log-concave if and only if $\sigma_s(p_s) < 1$.

In the downstream pricing game, the equilibrium retail price in market s given the wholesale price w_s for resale in s is implicitly defined by the first-order condition $p_s - w_s = \theta_s(p_s)\frac{-Q_s(p_s)}{Q_s'(p_s)}$ where $\theta_s = 1 - d_s$ (see Lemma 1(ii)), and will be denoted by $p_s^e(w_s)$. By the definition of w_s^*, $p_s^e(w_s^*) = p_s^*$. The pass-through rates, which will play crucial roles in the analysis, are given by:[37]

$$\rho_s(p_s) \equiv \frac{\partial p_s^e}{\partial w_s} = \frac{1}{1 + \theta_s(p_s)(1 - \sigma_s(p_s)) - \frac{\partial \theta_s(p_s)}{\partial p_s}\frac{Q_s(p_s)}{-Q_s'(p_s)}} > 0.$$

The pass-through rate ρ_s depends on the retailers' market power and the curvature of the market demand function in market s. In the case of monopoly power ($\theta_s = 1$), $\rho_s(p_s) =$

[37] Applying the implicit function theorem to the first-order condition $p_s - w_s = \theta_s(p_s)\frac{-Q_s(p_s)}{Q_s'(p_s)}$ yields the expression for the pass-through rate. Positivity of the pass-through rate is implied by the stability condition for the downstream pricing game (see also Anderson, de Palma and Kreider 2001).

$\frac{1}{2-\sigma_s(p_s)}$, and in the limiting case of perfect competition, $\rho_s = 1$. The pass-through rate under monopoly is less than the pass-through rate under perfect competition if and only if $Q_s(p_s)$ is locally log-concave.

The industry profit generated in market s given a common price p_s is denoted by $\Pi_s(p_s) = n(p_s - c) Q_s(p_s)$. Our assumptions of symmetry between retailers and negative definiteness of the Hessian matrix of $\Pi(p_{A1}, ..., p_{An}, p_{B1}, ..., p_{Bn})$ imply that $\Pi_s'' < 0$. Profits as a function of wholesale rather than retail prices will be denoted by

$$\pi_s(w_s) = \Pi_s(p_s^e(w_s)),$$

with first and second derivatives

$$\begin{aligned} \pi_s'(w_s) &= \rho_s(p_s^e(w_s)) \Pi_s'(p_s^e(w_s)), \\ \pi_s''(w_s) &= \rho_s(p_s^e(w_s)) [\rho_s(p_s^e(w_s)) \Pi_s''(p_s^e(w_s)) + \rho_s'(p_s^e(w_s)) \Pi_s'(p_s^e(w_s))]. \end{aligned}$$

We assume that $\pi_s''(w_s) < 0$, which ensures that the second-order conditions hold for the supplier's maximization problems. Given that $\Pi_s'' < 0$, a sufficient (but not necessary) condition for $\pi_s'' < 0$ is a constant pass-through rate in market s.

When the supplier can discriminate between resale in A versus B, it sets the wholesale prices (w_A^*, w_B^*) at which $\pi_s'(w_s^*) = 0$ for each s. For the remainder of the analysis, we focus on cases in which

$$w_A^* < w_B^*.$$

As mentioned previously, a sufficient condition for $w_A^* < w_B^*$, regardless of p_A^* and p_B^*, is monopoly power in A and some competition in B.

When the supplier is constrained to setting a uniform wholesale price, it sets the wholesale price $\overline{w}$ such that[38]

$$\pi_A'(\overline{w}) + \pi_B'(\overline{w}) = \rho_A \Pi_A'(p_A^e(\overline{w})) + \rho_B \Pi_B'(p_B^e(\overline{w})) = 0.$$

Concavity of each profit function and the first-order condition imply that $\overline{w} \in (w_A^*, w_B^*)$.

[38] We assume that both markets are served at the supplier's optimal uniform wholesale price.

5.1.2 Analysis

Adopting a technique commonly used in the context of third-degree price discrimination in final-goods markets,[39] we assume that the supplier chooses its wholesale prices subject to the constraint that $w_B - w_A \leq r$ where $r \in [0, w_B^* - w_A^*]$ is the degree of discrimination allowed. The supplier's objective function is $\pi_A(w_A) + \pi_B(w_A + r)$ and the first-order condition is $\pi'_A(w_A) + \pi'_B(w_A + r) = 0$ when the constraint binds. When $r = 0$, the supplier sets the non-discriminatory wholesale price $\overline{w}$. As r rises between 0 and $r^* \equiv w_B^* - w_A^*$, the wholesale price in market A falls and the wholesale price in market B rises:

$$w'_A(r) = \frac{-\pi''_B}{\pi''_A + \pi''_B} < 0; \; w'_B(r) = \frac{\pi''_A}{\pi''_A + \pi''_B} > 0, \tag{23}$$

which implies that the retail price falls in market A falls and rises in market B:

$$\frac{dp_A^e}{dr} = \rho_A w'_A(r) < 0; \; \frac{dp_B^e}{dr} = \rho_B w'_B(r) > 0.$$

The marginal change in social welfare as more discrimination is allowed is

$$\begin{aligned} W'(r) &= (p_A^e(w_A(r)) - c) Q'_A(p_A^e(w_A(r))) \rho_A(w_A(r)) w'_A(r) \\ &\quad + (p_B^e(w_B(r)) - c) Q'_B(p_B^e(w_B(r))) \rho_B(w_B(r)) w'_B(r). \end{aligned} \tag{24}$$

Denoting the marginal effect of discrimination on total output by $Q'(r) = Q'_A \rho_A w'_A(r) + Q'_B \rho_B w'_B(r)$, (24) can be rewritten as

$$W'(r) = \underbrace{(p_A^e - p_B^e) Q'_A \frac{dp_A^e}{dr}}_{\text{allocation effect}} + \underbrace{(p_B^e - c) Q'(r)}_{\text{(value of) output effect}}. \tag{25}$$

The first term represents the marginal allocation effect, and the second term the marginal output effect. Integrating (25) over $r \in [0, r^*]$ gives the welfare effect of input price discrimination by resale market. In what follows, we will first analyze the allocation and the output effect separately and then turn to the total welfare impact of discrimination.

[39] See Schmalensee (1981), Holmes (1989), and Aguirre et al. (2010), among others.

Allocation effect Since the retail price in market A falls with r and the retail price in market B rises with r, discrimination has a positive marginal allocation effect if and only if $p_A^e(w_A(r)) > p_B^e(w_B(r))$, in which case a marginal increase in r brings the retail prices closer together. It follows that if $p_A^* \geq p_B^*$, then the marginal allocation effect is positive for all r and hence the total allocation effect, obtained by integrating the first term in (25) over $r \in [0, r^*]$, is positive. If $p_A^e(\overline{w}) > p_B^e(\overline{w})$ but $p_A^* < p_B^*$, then the marginal allocation effect is first positive and then negative as r rises, making the sign of the total allocation effect ambiguous. Finally, if $p_A^e(\overline{w}) \leq p_B^e(\overline{w})$, then $p_A^e(w_A(r)) \leq p_B^e(w_B(r))$ for all $r \in [0, r^*]$; hence, the marginal allocation effect is negative throughout and the total allocation effect is negative. The next lemma summarizes these observations:

Lemma 3 *Suppose $w_A^* < w_B^*$. (i) Input price discrimination by resale market has a positive allocation effect if $p_A^* \geq p_B^*$. (ii) Input price discrimination by resale market has a negative allocation effect if $p_A^e(\overline{w}) \leq p_B^e(\overline{w})$. (iii) Input price discrimination by resale market can have a positive or a negative allocation effect if $p_A^e(\overline{w}) > p_B^e(\overline{w})$ and $p_A^* < p_B^*$.*

The relative market power parameters in the two markets play important roles both for the ranking of the optimal dual wholesale prices and for the ranking of the equilibrium retail prices when the supplier cannot discriminate. If the retailers have (close enough to) monopoly power in market A but there is (close enough to) perfect competition in market B, then $w_A^* < w_B^*$ and $p_A^e(\overline{w}) > p_B^e(\overline{w})$; hence, either case (i) or case (iii) of Lemma 3 applies: either the marginal allocation effect will be positive throughout, or the marginal allocation effect will be first positive and then negative as more discrimination is allowed.

Output effect The method we use to analyze the output effect combines the "marginal effects" technique used thus far with the "exogenous quantity" approach proposed by Weyl and Fabinger (2013). Later, we use the same method to analyze the welfare effect directly. For each degree of discrimination allowed and associated retail prices, we determine the amounts of exogenous quantity in each market that would make the supplier want to induce the same retail prices if it could discriminate freely. The total output effect is then found by aggregating the marginal effects of changes in the exogenous quantities on the total quantity sold in each market. This method will allow us to derive sufficient conditions for

a positive or negative output effect that involve straightforward comparisons of the inverse market demand curvatures and the pass-through rates in the two markets.[40]

To begin, note that given any $r \in [0, r^*]$, the supplier's optimal wholesale prices $w_A(r)$ and $w_B(r) = w_A(r) + r$ satisfy

$$\rho_A\left(p_A^e\left(w_A(r)\right)\right)\Pi_A'\left(p_A^e\left(w_A(r)\right)\right) + \rho_B\left(p_B^e\left(w_B(r)\right)\right)\Pi_B'\left(p_B^e\left(w_B(r)\right)\right) = 0, \tag{26}$$

which is equivalent, for each $s \neq -s \in \{A, B\}$, to

$$\left(p_s^e\left(w_s(r)\right) - c\right)Q_s'\left(p_s^e\left(w_s(r)\right)\right) + Q_s\left(p_s^e\left(w_s(r)\right)\right) + \frac{\rho_{-s}\left(p_{-s}^e\left(w_{-s}(r)\right)\right)}{\rho_s\left(p_s^e\left(w_s(r)\right)\right)}\frac{\Pi_{-s}'\left(p_{-s}^e\left(w_{-s}(r)\right)\right)}{n} = 0. \tag{27}$$

Now suppose that the supplier can discriminate freely but that an exogenous per-retailer quantity $\widetilde{q}_s$ enters market s. The profit that the supplier can extract in market s becomes $n(p_s - c)(Q_s - \widetilde{q}_s)$, where Q_s denotes the total quantity sold in market s—a positive exogenous quantity can be thought of as competition in the form of a capacity-constrained competitive fringe that reduces the demand of each retailer by $\widetilde{q}_s$, while a negative exogenous quantity corresponds to additional per-retailer demand of $-\widetilde{q}_s$ at any price p_s. Facing an exogenous per-retailer quantity $\widetilde{q}_s$ but no limits on discrimination, the supplier optimally chooses its wholesale price in market s to induce the retail price $p_s(\widetilde{q}_s)$ that solves

$$\left(p_s(\widetilde{q}_s) - c\right)Q_s'\left(p_s(\widetilde{q}_s)\right) + Q_s\left(p_s(\widetilde{q}_s)\right) - \widetilde{q}_s = 0. \tag{28}$$

Comparing (27) and (28) shows that

$$p_s^e\left(w_s(r)\right) = p_s\left(\widetilde{q}_s(r)\right)$$

for

$$\widetilde{q}_s(r) \equiv -\frac{\rho_{-s}\left(p_{-s}^e\left(w_{-s}(r)\right)\right)}{\rho_s\left(p_s^e\left(w_s(r)\right)\right)}\frac{\Pi_{-s}'\left(p_{-s}^e\left(w_{-s}(r)\right)\right)}{n} = \frac{\Pi_s'\left(p_s^e\left(w_s(r)\right)\right)}{n}. \tag{29}$$

The supplier's choice of $w_s(r)$ thus induces the same retail price in market s that the

[40] In contrast, if in our setting we were to use the main method used by Aguirre et al. (2010), which would involve substituting the expressions for the marginal price changes into $Q'(r)$ and then establishing conditions under which the marginal effect is either positive or negative throughout, we would obtain less general conditions that also depend on the slopes of the pass-through rates.

supplier would induce if there were no limits on discrimination but an exogenous per-retailer quantity of $\widetilde{q}_s(r)$ were present in market s.

Full discrimination corresponds to zero exogenous quantity in each market, i.e., $\widetilde{q}_s(r^*) = 0$ for all s. When the scope for discrimination is limited (i.e., $r < r^*$), then

$$\widetilde{q}_A(r) < 0 < \widetilde{q}_B(r). \tag{30}$$

A limit on discrimination is thus equivalent to adding exogenous quantity to market B, which pushes price down in that market, while removing exogenous quantity from market A, which pushes price up in that market.

Applying the implicit function theorem to (28) yields the marginal effect of exogenous quantity on the retail price in a market:

$$\frac{dp_s}{d\widetilde{q}_s} = \frac{1}{2Q'_s + (p_s(\widetilde{q}_s) - c)\, Q''_s} = \frac{1}{2Q'_s - (Q_s - \widetilde{q}_s)\frac{Q''_s}{Q'_s}} < 0. \tag{31}$$

The total effect of discrimination on the output sold in market s is equal to[41]

$$\int_{\widetilde{q}_s(0)}^{\widetilde{q}_s(r^*)} Q'_s \frac{dp_s}{d\widetilde{q}_s} d\widetilde{q}_s = \int_{\widetilde{q}_s(0)}^{\widetilde{q}_s(r^*)} \frac{1}{2 - \frac{Q_s(p_s(\widetilde{q}_s)) - \widetilde{q}_s}{Q_s(p_s(\widetilde{q}_s))}\sigma_s(p_s(\widetilde{q}_s))} d\widetilde{q}_s. \tag{32}$$

Adding up the output effects in the two markets after substituting for the limits of the exogenous quantities, and noting that $\Pi'_B(p^e_B(\overline{w})) > 0$, yields the following expression for the effect of discrimination on the total output sold in the two markets:

$$\begin{aligned} \int_0^{r^*} Q'(r)\, dr \quad = & \int_{-\frac{\rho_B\left(p^e_B(\overline{w})\right)}{\rho_A\left(p^e_A(\overline{w})\right)}\frac{\Pi'_B\left(p^e_B(\overline{w})\right)}{n}}^{0} \frac{1}{2 - \frac{Q_A(p_A(\widetilde{q}_A)) - \widetilde{q}_A}{Q_A(p_A(\widetilde{q}_A))}\sigma_A(p_A(\widetilde{q}_A))} d\widetilde{q}_A \\ & - \int_0^{\frac{\Pi'_B(p^e_B(\overline{w}))}{n}} \frac{1}{2 - \frac{Q_B(p_B(\widetilde{q}_B)) - \widetilde{q}_B}{Q_B(p_B(\widetilde{q}_B))}\sigma_B(p_B(\widetilde{q}_B))} d\widetilde{q}_B. \end{aligned} \tag{33}$$

[41] Weyl and Fabinger (2013) state that the marginal effect of exogenous quantity on the total output sold in a market is equal to the monopoly pass-through rate, i.e., that $Q'_s \frac{dp_s}{d\widetilde{q}_s} = \frac{1}{2-\sigma_s}$. However, as (32) shows, their expression holds only for marginal changes starting at zero. In Miklós-Thal and Shaffer (2019), we study the implications of this insight for applications of pass-through analysis discussed in Weyl and Fabinger (2013).

Our next proposition follows from the fact that (33) is positive if the following two conditions hold: (i) $\rho_B(p_B^e(\overline{w})) \geq \rho_A(p_A^e(\overline{w}))$, so that discrimination corresponds to a larger absolute change in exogenous quantity in market A than in market B; and (ii) $\sigma_A \geq \sigma_B \geq 0$ throughout the relevant range of prices, so that the integrand in the first term of (33) exceeds that in the second term for all $\widetilde{q}_A \in [\widetilde{q}_A(0), 0]$ and $\widetilde{q}_B \in [0, \widetilde{q}_B(0)]$, meaning that a marginal change in exogenous quantity has a greater impact on the quantity sold in market A than in market B throughout the relevant range. Conversely, if the inequalities in (i) and (ii) are reversed, the total output effect is negative.

Proposition 3 *Suppose $w_A^* < w_B^*$.*

(a) *Output increases with input price discrimination by resale market if (i) the inverse market demands are convex and more so in market A ($\sigma_A \geq \sigma_B \geq 0$ throughout the relevant range of prices), and (ii) the pass-through rate is higher in market B than in market A ($\rho_B(p_B^e(\overline{w})) \geq \rho_A(p_A^e(\overline{w}))$).*

(b) *Output decreases with input price discrimination by resale market if (i) the inverse market demands are concave and more so in market A ($0 \geq \sigma_B \geq \sigma_A$ throughout the relevant range of prices), and (ii) the pass-through rate is lower in market B than in market A ($\rho_B(p_B^e(\overline{w})) \leq \rho_A(p_A^e(\overline{w}))$).*

Proposition 3 encompasses previous results in the literature on the output effect of price discrimination in final-goods markets. In the limiting case of perfect competition in each market, the pass-through rates are $\rho_A = \rho_B = 1$, and input price discrimination by resale market is equivalent to monopolistic third-degree price discrimination where A is the 'weak' market featuring a monopoly price $p_A^* = w_A^*$ and B the 'strong' market featuring a monopoly price $p_B^* = w_B^* > p_A^*$. By Proposition 3, output thus increases with discrimination if $\sigma_A \geq \sigma_B \geq 0$ and falls with discrimination if $0 \geq \sigma_B \geq \sigma_A$, which replicates results by Aguirre et al. (2010, Proposition 4).[42] The case of linear demand functions in both markets lies at the boundary of cases (a) and (b), which confirms Pigou's (1920) famous insight that

[42] Intuitively, when price falls in a market with a relatively convex inverse demand function, it leads to a large increase in output in that market, whereas when price rises in a market with a relatively concave inverse demand function, it leads to small decrease in output in that market.

the output effect of monopolistic third-degree price discrimination is zero when the demand functions are linear.

When the competition between retailers is imperfect in one or both markets, however, the output effect of input price discrimination by resale market depends not just on the inverse market demand curvatures, but also on the comparison of the pass-through rates in the two markets. The latter, in turn, depend on the retailers' market power (in addition to the demand curvatures). In the case of linear market demand functions in each market, Proposition 3 implies that the output effect is positive if and only if $\rho_B > \rho_A$, which, in turn, holds if and only if competition between retailers is less intense in market A than in market B ($\theta_B < \theta_A$). Discrimination against resale in the more competitive downstream market *increases* the total output sold when the market demand functions are linear. More generally, if $1 > \sigma_A \geq \sigma_B \geq 0$, part (a) of Proposition 3 implies that the output effect is positive when there is (close enough to) monopoly power in market A and (close enough to) perfect competition in market B.

The intuition for why the retailers' relative market power in the two markets matters for the output effect is as follows. Suppose that the retailers' market power rises in market A, without any change in the market demand functions. The direct effect of greater retailer market power in A is a higher retail price, and thus less output sold, in market A for any given wholesale price w_A. If the supplier can discriminate freely, however, it will adjust its wholesale price in market A downwards to fully compensate for the rise in market power in A—hence, total quantity sold will be unaffected. If there are limits to the supplier's ability to discriminate, on the other hand, adjustments in w_A will necessarily have repercussions in market B, which implies that the change in market power in market A will affect quantities. And with log-concave market demand functions, greater market power in A makes adjusting the wholesale price downwards relatively *less* attractive (for a given p_A) because it leads to a lower pass-through rate from wholesale to retail price in market A.[43] The direct effect dominates in these cases and total output in the non-discriminatory regime falls. By the same reasoning, given log-concave market demand functions, more intense competition in B

[43]It is straightforward to construct numerical examples in which the supplier *raises* the non-discriminatory wholesale price when market power in A rises, which means that outputs in both markets fall as the supplier adjusts its non-discriminatory wholesale price to account for the greater market power in A.

implies a higher pass-through rate in B, which (for a given p_B) makes *upward* adjustments in the non-discriminatory wholesale price *more* attractive for the supplier.

Welfare effect So far, we have analyzed the welfare effect indirectly by establishing conditions under which the allocation and the output effect are positive or negative. If both effects can be signed as positive (negative), welfare rises (falls). We now use the same technique as in the analysis of the output effect to analyze the welfare effect *directly.*

Our analysis makes use of the following assumption, which was introduced by Aguirre et al. (2010) in the context of monopolistic third-degree price discrimination:

Increasing ratio condition: $z_s(p_s) \equiv \frac{(p_s-c)Q_s'}{2Q_s'+(p_s-c)Q_s''}$ is increasing in p_s in each market s.

As discussed by Aguirre et al. (2010), the increasing ratio condition (henceforth IRC) is met by a large class of demand functions (in our setting, market demand functions). It holds if the inverse market demand functions have constant positive curvature, which includes linear, exponential, and constant-elasticity functions. It also holds if the direct market demand functions have constant curvature (of either sign), or if the market demand functions are derived from the normal, the logistic, or the lognormal distribution.

Noting that, by (31),

$$z_s\left(p_s\left(\widetilde{q}_s\right)\right) = \left(p_s\left(\widetilde{q}_s\right) - c\right) Q_s'\left(p_s\left(\widetilde{q}_s\right)\right) \frac{dp_s}{d\widetilde{q}_s}\left(p_s\left(\widetilde{q}_s\right)\right),$$

we can use the exogenous quantity method to write the welfare effect as

$$\int_0^{r^*} W'(r)\, dr = \int_{-\frac{\rho_B\left(p_B^e(\overline{w})\right)}{\rho_A\left(p_A^e(\overline{w})\right)}\frac{\Pi_B'\left(p_B^e(\overline{w})\right)}{n}}^{0} z_A\left(p_A\left(\widetilde{q}_A\right)\right) d\widetilde{q}_A - \int_0^{\frac{\Pi_B'\left(p_B^e(\overline{w})\right)}{n}} z_B\left(p_B\left(\widetilde{q}_B\right)\right) d\widetilde{q}_B. \quad (34)$$

Since p_s is decreasing in $\widetilde{q}_s$ and z_s is increasing in p_s, $z_A\left(p_A\left(\widetilde{q}_A\right)\right) \geq z_A\left(p_A(0)\right) = z_A\left(p_A^*\right)$ and $z_B\left(p_B\left(\widetilde{q}_B\right)\right) \leq z_B\left(p_B(0)\right) = z_B\left(p_B^*\right)$ for all $\widetilde{q}_A \leq 0 \leq \widetilde{q}_B$, which implies that

$$\int_0^{r^*} W'(r)\, dr \geq \left(\frac{\rho_B\left(p_B^e\left(\overline{w}\right)\right)}{\rho_A\left(p_A^e\left(\overline{w}\right)\right)} z_A\left(p_A^*\right) - z_B\left(p_B^*\right)\right) \frac{\Pi_B'\left(p_B^e\left(\overline{w}\right)\right)}{n}. \quad (35)$$

The right-hand-side of (35) has the same sign as

$$\frac{z_A(p_A^*)}{\rho_A(p_A^e(\overline{w}))} - \frac{z_B(p_B^*)}{\rho_B(p_B^e(\overline{w}))} = \frac{p_A^* - c}{\rho_A(p_A^e(\overline{w}))(2-\sigma_A(p_A^*))} - \frac{p_B^* - c}{\rho_B(p_B^e(\overline{w}))(2-\sigma_B(p_B^*))}, \quad (36)$$

where the equality follows from (31) and $\widetilde{q}_s(r^*) = 0$. Our main result on the welfare effect of input price discrimination follows from this analysis:

Proposition 4 *Suppose $w_A^* < w_B^*$. Given IRC, welfare rises with input price discrimination by resale market if*

$$\frac{p_A^* - c}{\rho_A(p_A^e(\overline{w}))(2-\sigma_A(p_A^*))} \geq \frac{p_B^* - c}{\rho_B(p_B^e(\overline{w}))(2-\sigma_B(p_B^*))}. \quad (37)$$

Condition (37) holds for a larger set of monopoly prices and pass-through rates if inverse market demand is more convex in the market where price falls than in the market where price rises ($\sigma_A(p_A^*) > \sigma_B(p_B^*)$), which is consistent with insights from the literature on monopolistic third-degree price discrimination in final-goods markets.

A first fundamental difference to discrimination in final-goods markets, however, is that the market in which price rises when input price discrimination by resale market is allowed need *not* be the market with the higher monopoly price. For instance, it can be that $w_A^* < w_B^*$ although $p_A^* \geq p_B^*$, if competition is sufficiently more intense in market B than in A. It follows that (37) can hold even if inverse market demand in the market where price falls is *less* convex than in the market where price rises ($\sigma_A(p_A^*) \leq \sigma_B(p_B^*)$).[44] Input price discrimination by resale market has a positive allocation effect in this case (see Lemma 3(i)), which implies that welfare can rise with discrimination even if total output falls.

The second fundamental difference to monopolistic third-degree price discrimination in final-goods markets is that the pass-through rates play important roles in our setting. Even when $p_A^* < p_B^*$ and $\sigma_A(p_A^*) \leq \sigma_B(p_B^*)$, so that the allocation effect is potentially negative and inverse market demand is less convex in the market where price falls, condition (37) still

[44]In comparison, the analog to (37) in Aguirre et al. (2010, Proposition 2) is the condition

$$\frac{p_A^* - c}{2-\sigma_A(p_A^*)} \geq \frac{p_B^* - c}{2-\sigma_B(p_B^*)}, \quad (38)$$

which ensures monopolistic third-degree price discrimination in the final-goods market raises welfare when $p_A^* < p_B^*$ and IRC holds. Because $p_A^* < p_B^*$, (38) can only be satisfied if $\sigma_A(p_A^*) > \sigma_B(p_B^*)$.

holds if the pass-through rate in the market where prices rises is sufficiently large relative to the pass-through rate in the market where price falls.

The pass-through rates, in turn, depend on the retailers' market power and on the inverse market demand curvatures in the two market. To fix ideas, suppose that the market power parameters and inverse market demand curvatures are constant.[45] Condition (37) can then be rewritten as

$$\frac{1+\theta_A\left(1-\sigma_A\right)}{2-\sigma_A}\left(p_A^*-c\right) \geq \frac{1+\theta_B\left(1-\sigma_B\right)}{2-\sigma_B}\left(p_B^*-c\right), \tag{39}$$

which depends on the market power parameters, the inverse market demand curvatures, and the monopoly margins in the two markets. If the market demand functions are log-concave ($\sigma_s < 1$ in each market s), more intense competition in a market is associated with a higher pass-through rate. Because of this, the left-hand side of (39) is higher for less intense competition in market A (larger θ_A) while the right-hand side of (39) is lower for more intense competition in market B (smaller θ_B). All else equal, given log-concave market demand functions, the sufficient condition for a positive welfare effect is thus more likely hold for more intense competition in the market where price rises (market B) and less intense competition in the market where price falls (market A).

5.2 Interdependent markets

With interdependent markets, we denote the equilibrium retail prices given the wholesale prices (w_A, w_B) by $p_A^e(w_A, w_B)$ and $p_B^e(w_A, w_B)$. The pass-through rate from the wholesale price w_t ($t = A, B$) on the equilibrium retail price in market s is denoted by ρ_{ts}:

$$\rho_{AA} = \frac{\partial p_A^e}{\partial w_A}, \rho_{BA} = \frac{\partial p_A^e}{\partial w_B}, \rho_{AB} = \frac{\partial p_B^e}{\partial w_A}, \rho_{BB} = \frac{\partial p_B^e}{\partial w_B}.$$

Total industry profits as a function of the wholesale prices are denoted by

$$\pi^w\left(w_A, w_B\right) = \Pi\left(p_A^e\left(w_A, w_B\right), ..., p_A^e\left(w_A, w_B\right), p_B^e\left(w_A, w_B\right), ..., p_B^e\left(w_A, w_B\right)\right).$$

[45] Many common functional forms in industrial organization, including the linear, exponential, and constant-elasticity forms in Bulow and Pfleiderer (1983)'s seminal paper about pass-through in monopoly, satisfy the constant curvature assumption.

Subscripts will be used to denote derivatives, e.g., $\pi_A^w = \frac{\partial \pi}{\partial w_A}$ and $\pi_{AB}^w = \frac{\partial^2 \pi}{\partial w_A \partial w_B}$.

In what follows, continue to assume that $w_A^* < w_B^*$. Adopting the same method of analyzing marginal effects as in the case of independent markets, we assume that the supplier chooses its wholesale prices subject to the constraint that $w_B - w_A \leq r$ where $r \in [0, w_B^* - w_A^*]$ is the degree of discrimination allowed. The objective function is $\pi^w(w_A, w_A + r)$ and the first-order condition is $\pi_A^w(w_A, w_A + r) + \pi_B^w(w_A, w_A + r) = 0$ when the constraint is binding. We assume that the second-order condition, $\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w < 0$, is satisfied.[46] For $r \in (0, r^*)$,

$$w_A'(r) = -\frac{\pi_{AB}^w + \pi_{BB}^w}{\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w},\ w_B'(r) = \frac{\pi_{AA}^w + \pi_{AB}^w}{\pi_{AA}^w + 2\pi_{AB}^w + \pi_{BB}^w}.$$

The marginal effects on the retail prices follow from the marginal effects on the wholesale prices and the pass-through rates:

$$\begin{aligned} \frac{dp_A^e}{dr} &= \rho_A^A w_A'(r) + \rho_B^A w_B'(r), \\ \frac{dp_B^e}{dr} &= \rho_A^B w_A'(r) + \rho_B^B w_B'(r). \end{aligned}$$

The marginal change in welfare as more wholesale price discrimination is allowed is

$$W'(r) = (p_A^e(w_A(r), w_B(r)) - c)\, Q_A'(r) + (p_B^e(w_A(r), w_B(r)) - c)\, Q_B'(r),$$

where $Q_s'(r)$ is the marginal effect on total output in market $s \in \{A, B\}$. Denoting the marginal effect on total output by $Q'(r) = Q_A'(r) + Q_B'(r)$, the marginal effect on welfare can again be decomposed into two effects, an allocation and an output effect:

$$W'(r) = \underbrace{(p_A^e - p_B^e)\, Q_A'(r)}_{\text{allocation effect}} + \underbrace{(p_B^e - c)\, Q'(r)}_{\text{(value of) output effect}}.$$

The sign of the first term is determined by the change in output in market A and the relative retail prices. If $\pi_{AB}^w + \pi_{BB}^w < 0$ and $\pi_{AA}^w + \pi_{AB}^w < 0$, so that $\frac{dp_A^e}{dr} < 0 < \frac{dp_B^e}{dr}$ for $r \in (0, r^*)$, then $Q_A'(r) > 0$.[47] As in the case of independent markets, the marginal

[46] The second-order condition is satisfied in the linear-demand case.

[47] Given $w_A^* < w_B^*$, the wholesale price in market A decreases and that in market B increases as r rises

consumption allocation effect is then positive if and only if $p_A^e > p_B^e$, and the insights from Lemma 3 about the sign of the allocation effect apply.

Further insights into the signs of the output and welfare effects can be gained by assuming linear demands and symmetric cross-market effects:

Proposition 5 *Suppose $w_A^* < w_B^*$ and that the demand functions are linear with symmetric cross-market effects ($\frac{\partial Q_A}{\partial p_B} = \frac{\partial Q_B}{\partial p_A}$).*

(i) *Input price discrimination by resale market raises total output if and only if*

$$-\left(\rho_A^B - \rho_A^A\right) < \rho_B^B - \rho_B^A. \tag{40}$$

(ii) *Input price discrimination by resale market raises welfare if*

$$\frac{p_A^* - c}{\rho_A^A + \rho_B^A} > \frac{p_B^* - c}{\rho_B^B + \rho_A^B}. \tag{41}$$

Condition (40) can be interpreted as requiring that an increase in w_B has a greater positive impact on $p_B^e - p_A^e$ than a decrease in w_A of the same magnitude. Condition (41) generalizes the sufficient condition for a welfare increase in the case of independent markets and linear demands, $\frac{p_A^* - c}{\rho_A} > \frac{p_B^* - c}{\rho_B}$ (see Proposition 4), to the case of interdependent markets where each wholesale price affects the retail prices in both markets.

Quadratic-utility linear-demand example To illustrate how asymmetries in the intensity of competition between the two markets affect the pass-through rates and the welfare effects of input price discrimination by resale market in the case of interdependent markets, suppose that there are two retailers and that the representative consumer has the following

if and only if $\pi_{AB}^w + \pi_{BB}^w < 0$ and $\pi_{AA}^w + \pi_{AB}^w < 0$. If the markets are asymmetric and $\pi_{AB}^w > 0$, then one of these conditions may be violated, in which case either both wholesale prices increase or both wholesale prices decrease. (See Layson 1998 for a related discussion of the direction of price changes in the context of third-degree price discrimination in final-goods markets with interdependent markets.) Our discussion focuses on cases in which $w_A'(r) < 0 < w_B'(r)$ for $r \in (0, r^*)$.

quadratic-utility function:

$$\begin{aligned} U\left(q_{A1}, q_{A2}, q_{B1}, q_{B2}\right) &= \alpha_A\left(q_{A1}+q_{A2}\right)-\frac{\beta_A}{2}\left(q_{A1}^2+q_{A2}^2\right)-\gamma_A q_{A1} q_{A2} \\ &\quad +\alpha_B\left(q_{B1}+q_{B2}\right)-\frac{\beta_B}{2}\left(q_{B1}^2+q_{B2}^2\right)-\gamma_B q_{B1} q_{B2} \\ &\quad -\gamma_C\left(q_{A1} q_{B1}+q_{A2} q_{B2}\right)-\gamma_D\left(q_{A1} q_{B2}+q_{A2} q_{B1}\right). \end{aligned}$$

The parameter γ_s measures the degree of intra-market inter-retailer substitutability in market s, γ_C measures the degree of inter-market intra-retailer substitutability, and γ_D measures the degree of inter-market inter-retailer substitutability. We assume that $\beta_A > \gamma_A > \gamma_C$, $\beta_B > \gamma_B > \gamma_C$, $\gamma_C \geq \gamma_D \geq 0$, and $(\beta_A - \gamma_A)(\beta_B - \gamma_B) > (\gamma_C - \gamma_D)^2$. These assumptions are sufficient to ensure that the Hessian of the utility functions is (strictly) negative definite. The special case of independent markets corresponds to $\gamma_C = \gamma_D = 0$.

Maximizing the utility function subject to a budget constraint yields the following linear inverse demand functions:[48]

$$P_{si}\left(q_{si}, q_{s-i}, q_{-si}, q_{-s-i}\right) = \frac{\partial U}{\partial q_{si}} = \alpha_s - \beta_s q_{si} - \gamma_s q_{s-i} - \gamma_C q_{-si} - \gamma_D q_{-s-i}.$$

The monopoly prices depend on the intercepts of the inverse demand functions, but not on the parameters that measures inter- and intra-retailer substitutability:

$$p_s^* = \frac{\alpha_s + c}{2}. \tag{42}$$

To isolate the role played by asymmetries in intra-market inter-retailer substitutability between the two markets, focus on cases in which

$$\gamma_B > \gamma_A,$$

[48] Direct demands in the region where all four demands are positive are given by

$$D_{si} = a_s - b_s p_{si} + c_s p_{s-i} + h p_{-si} + g p_{-s-i},$$

where $a_s = \frac{\alpha_s(\beta_{-s}+\gamma_{-s})+\alpha_{-s}(\gamma_C+\gamma_D)}{\Delta_+}$, $b_s = \frac{1}{2}\left(\frac{\beta_{-s}-\gamma_{-s}}{\Delta_-} + \frac{\beta_{-s}+\gamma_{-s}}{\Delta_+}\right)$, $c_s = \frac{1}{2}\left(\frac{\beta_{-s}-\gamma_{-s}}{\Delta_-} - \frac{\beta_{-s}+\gamma_{-s}}{\Delta_+}\right)$, $h = \frac{1}{2}\left(\frac{\gamma_C-\gamma_D}{\Delta_-} + \frac{\gamma_C+\gamma_D}{\Delta_+}\right)$, $g = \frac{1}{2}\left(-\frac{\gamma_C-\gamma_D}{\Delta_-} + \frac{\gamma_C+\gamma_D}{\Delta_+}\right)$, $\Delta_- = (\beta_A - \gamma_A)(\beta_B - \gamma_B) - (\gamma_C - \gamma_D)^2$, and $\Delta_+ = (\beta_A + \gamma_A)(\beta_B + \gamma_B) - (\gamma_C + \gamma_D)^2$. Our parameter assumptions imply that $b_s > c_s > 0$ and $h > 0$. Moreover, there exists some $\widehat{\gamma} \in (0, \gamma_C)$ such that $g > 0$ if and only if $\gamma_D > \widehat{\gamma}$.

but the markets are symmetric otherwise: $\alpha_A = \alpha_B = \alpha$ and $\beta_A = \beta_B = \beta$. We then have that $p_A^* = p_B^*$ and

$$\theta_A = \frac{\beta + \gamma_C - (\gamma_A + \gamma_D)}{\beta + \gamma_C} > \theta_B = \frac{\beta + \gamma_C - (\gamma_B + \gamma_D)}{\beta + \gamma_C},$$

which together imply that

$$w_A^* < w_B^*.$$

Moreover, discrimination leads to a lower p_A and a higher p_B, as is easy to check. Together, these observations imply that input price discrimination that is motivated by asymmetries in the intra-market substitution parameters has a positive allocation effect.

To evaluate the output effect, we compute the pass-through rates:

$$\rho_s^s = \frac{\beta\left(2\beta - \gamma_{-s}\right) - \gamma_C\left(2\gamma_C - \gamma_D\right)}{\left(2\beta - \gamma_A\right)\left(2\beta - \gamma_B\right) - \left(2\gamma_C - \gamma_D\right)^2}, \tag{43}$$

$$\rho_t^s = \frac{\gamma_s \gamma_C - \beta\gamma_D}{\left(2\beta - \gamma_A\right)\left(2\beta - \gamma_B\right) - \left(2\gamma_C - \gamma_D\right)^2}. \tag{44}$$

It is easy to check that $\gamma_B > \gamma_A$ implies $\rho_B^B + \rho_A^B > \rho_A^A + \rho_B^A$. By Proposition 5(i), input price discrimination by resale market motivated by asymmetries in intra-market substitutability therefore has a positive output effect. Given that the allocation effect is positive as well, discrimination thus raises welfare. The next proposition summarizes these insights:

Proposition 6 *In the quadratic-utility linear-demand setting, input price discrimination by resale market that is motivated by asymmetries in intra-market substitutability has positive allocation and output effects and thus raises welfare.*

Figure 1 provides a graphical illustration of the output effect, welfare effect, and consumer surplus effect of discrimination in our linear-demand specification. Market B features greater retailer substitutability than market A if and only if $\gamma_B > \gamma_A = 0.4$. The parameter α_A captures the "size" of market A, which affects the monopoly retail price: $p_A^* > p_B^*$ if and only if $\alpha_A > \alpha_B = 2$. The region of interest lies above the black area (otherwise $w_A^* \geq w_B^*$). In the dark-shaded region in the north-east corner, discrimination has positive effects on output, welfare, and consumer surplus. In the medium-shaded region, discrimination raises output and welfare, but decreases consumer surplus. In the light-shaded region, the output

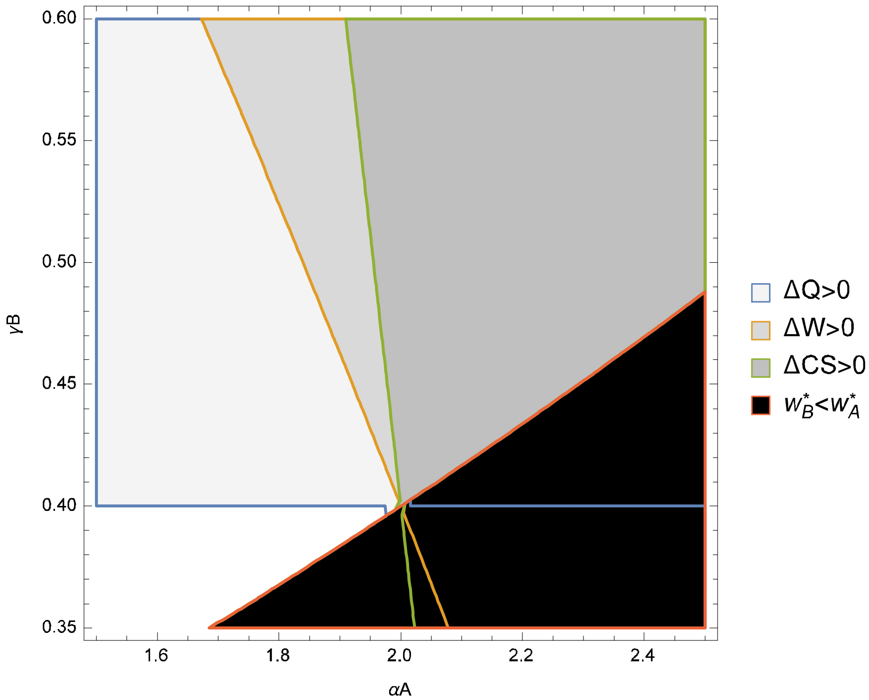

Figure 1: Output, welfare and consumer surplus effects of dual pricing ($\alpha_B = 2, \gamma_A = 0.4, \gamma_C = 0.2, \gamma_D = 0.18, c = 0$)

effect is positive, but discrimination lowers welfare and consumer surplus. In the unshaded region, all three effects are negative.

When the markets differ only in the intra-market inter-retailer substitutability, i.e., for $\alpha_A = \alpha_B = 2$, the output effect and welfare effect are both positive, illustrating the insight of Proposition 6. Discrimination also has a positive effect on consumer surplus in this case. For $\alpha_A > \alpha_B = 2$, discrimination raises output and welfare provided that γ_B is high enough to ensure $w^*_B > w^*_A$ although $p^*_A > p^*_B$. In this region, discrimination has a positive allocation effect (because $p^*_A > p^*_B$) and a positive output effect (because $\gamma_B > \gamma_A$). If $\alpha_A < \alpha_B$, the welfare effect can be negative due to a negative allocation effect. In particular, there exists an area in which the output effect is positive but welfare falls nonetheless because discrimination worsens the allocation of output across markets. However, if γ_B exceeds γ_A sufficiently, the positive output effect more than compensates for the negative allocation effect and welfare rises with discrimination.

6 Conclusion

We conclude by discussing some policy implications of our findings and by highlighting some directions for future research. As discussed previously, our findings are relevant for the current policy treatment of vertical restraints on online sales in Europe. The expressed view in the current EU Guidelines on Vertical Restraints is that such restraints, which are treated as hardcore restrictions of competition, are tantamount to output restrictions, because they limit distributors' ability "to reach more and different consumers than will be reached when only more traditional sales methods are used."[49] According to this view, removing restraints on online sales, by banning practices like dual wholesale pricing and relative quantity restrictions, will necessarily result in more sales. However, this view implicitly assumes that the wholesale terms in the online markets would effectively be forced to match the existing terms in the offline markets, which ignores the fact that the suppliers' wholesale terms in both markets would be expected to adjust depending on the policy regime in place. Further, it fails to consider why a supplier might specifically want to limit only the *online* sales of its distributors. If restricting output were the goal, the supplier could presumably achieve this goal by imposing high wholesale prices on all of its retailers' sales, not just on the retailers' sales that are intended for the online market.

The explanation for vertical restraints on online sales that we propose follows from the fact that online and offline markets typically differ in their demand characteristics, in particular, the intensity of competition between retailers. Viewed as input price discrimination, vertical restraints on online sales have ambiguous welfare effects a priori, with discrimination leading to higher prices online, but at the same time also lower prices offline. And, as our analysis shows, discrimination that is motivated mainly by asymmetries in retailer market power across markets tends to increase output and welfare. In summary, our results suggest that suppliers may impose vertical restraints on online sales for price discrimination reasons, and that the welfare effects of such restraints may well be positive. The current policy treatment of such restraints in the EU therefore appears to be overly aggressive.

Our findings also have implications for U.S. policy on input price discrimination. Cases involving discrimination across resale markets, where the downstream firms in each market may differ, often hinge on whether the supplier's discriminatory terms are found to be

[49] See Paragraph 52 of the Guidelines, which opines about the internet as a powerful sales tool.

predatory."[50] Typically, making this assessment entails determining whether the supplier's prices to the retailers in the favored market are below some appropriate measure of costs. Since these costs may be difficult for courts to determine, it is useful to consider whether the supplier would have wanted to discriminate in the alleged manner even in the absence of any upstream competitors to predate. Our analysis suggests some of the factors that would be relevant for such consideration, including the intensity of competition in each market and a sense of the relative retail prices that the supplier would want to support.

This paper offers the first analysis of what we have called input price discrimination by resale market, and as such it uses a relatively stylized model in order to capture some of the key drivers of this type of discrimination. Future research could extend our model in various directions. For example, throughout the paper we have assumed that the supplier is able to monitor resale and prevent arbitrage across resale markets. In practice, suppliers do rely on various strategies to monitor resale and enforce contracts that discriminate across resale markets, for instance, by requiring retailers to use supplier-managed inventory systems or by asking them to provide scanner-data evidence of their sales in different markets. Nonetheless, such monitoring can be costly and imperfect, which may affect optimal input pricing. Another promising direction for future research could be to extend the model to allow for competition between multiple suppliers or for a mix of multi-market and single-market firms downstream. Yet another direction would be to analyze input price discrimination by resale market in the presence of contracting inefficiencies that could arise due to restrictions on the class of available tariffs or due to information asymmetries.

[50] Primary-line cases (alleged injury to rivals of the seller offering the discriminatory price) turn on showing predatory conduct by the defendant. See, e.g., *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.*, 113 U.S. 2578 (1993), where the Court ruled that a claim of primary-line injury under the Robinson-Patman Act is of the same general character as a predatory pricing claim under Section 2 of the Sherman Act.

Appendix

A Proofs

Proof of Lemma 2. Given the definition of $\xi_s(\cdot)$, $\xi_A(p_A, p_B) < \xi_B(p_B, p_A)$ if and only if

$$\frac{Q_A}{-\partial Q_A/\partial p_A}(1 - R_{BA}) < \frac{Q_B}{-\partial Q_B/\partial p_B}(1 - R_{AB}),$$

which can be rewritten as

$$e_B \frac{1 - d_B - d_B^O - d_B^C}{1 - d_B} < e_A \frac{1 - d_A - d_A^O - d_A^C}{1 - d_A}, \tag{45}$$

where $e_s \equiv \frac{-\partial Q_s/\partial p_s}{Q_s}$ and, for each $s \neq -s$,

$$d_s^O \equiv \frac{\frac{\partial D_{-si}}{\partial p_{si}}}{-\frac{\partial D_{si}}{\partial p_{si}}}, d_B^C \equiv \frac{\sum_{j \neq i} \frac{\partial D_{-sj}}{\partial p_{si}}}{-\frac{\partial D_{si}}{\partial p_{si}}}.$$

Using the fact that $DR_s = \frac{d_s + d_s^C}{1 - d_s^O}$ to rewrite (45) and rearranging yields that

$$1 < \frac{\xi_B(p_B, p_A)}{\xi_A(p_A, p_B)} \tag{46}$$

$$\Longleftrightarrow$$

$$\frac{1 - DR_B}{1 - DR_A} < \frac{e_A(1 - d_B)\left(1 - d_A^O\right)}{e_B(1 - d_A)(1 - d_B^O)}. \tag{47}$$

Next, using the definitions of θ_s, ξ_s, and d_s^O, we obtain that

$$\frac{\theta_A(p_A, p_B)}{\theta_B(p_B, p_A)} < \frac{\xi_B(p_B, p_A)}{\xi_A(p_A, p_B)} \tag{48}$$

$$\Longleftrightarrow$$

$$1 < \frac{e_A(1 - d_B)\left(1 - d_A^O\right)}{e_B(1 - d_A)(1 - d_B^O)}. \tag{49}$$

If $\theta_A = \theta_B$, then (46) and (48) coincide, hence (47) and (49) must be equivalent, which implies that $DR_B = DR_A$. Similarly, if $DR_B = DR_A$, then (47) and (49) coincide, hence (46) and (48) must be equivalent, which implies that $\theta_A = \theta_B$. In summary, $\theta_A = \theta_B$ if and only if $DR_B = DR_A$.

Next, we show that $\theta_A > \theta_B$ implies $DR_B > DR_A$. Suppose first that $\theta_A > \theta_B$ and (48) holds. Given $\theta_A > \theta_B$, (48) implies (46), hence (49) must imply (47). Since $DR_B = DR_A$ if and only if $\theta_A = \theta_B$, the latter in turn implies that $DR_B > DR_A$. Now suppose that $\theta_A > \theta_B$ and (48) is violated (i.e., $\frac{\theta_A}{\theta_B} \geq \frac{\xi_B}{\xi_A}$). There are two subcases to consider, depending on whether (46) holds or not. If (46) is violated (i.e., $1 \geq \frac{\xi_B}{\xi_A}$), then (46) implies that $\frac{\theta_A}{\theta_B} \geq \frac{\xi_B}{\xi_A}$ for any $\theta_A > \theta_B$, hence $\frac{1-DR_B}{1-DR_A} \geq \frac{e_A(1-d_B)\left(1-d_A^O\right)}{e_B(1-d_A)\left(1-d_B^O\right)}$ must imply that $1 \geq \frac{e_A(1-d_B)\left(1-d_A^O\right)}{e_B(1-d_A)\left(1-d_B^O\right)}$. From the latter, it follows that $DR_A \leq DR_B$. Since $DR_B = DR_A$ if and only if $\theta_A = \theta_B$, we can conclude that $DR_B > DR_A$ if $\theta_A > \theta_B$ and both (46) and (48) are violated. If (48) is violated but (46) holds, then it must be that

$$\frac{1-DR_B}{1-DR_A} < \frac{e_A\left(1-d_B\right)\left(1-d_A^O\right)}{e_B\left(1-d_A\right)\left(1-d_B^O\right)} \leq 1,$$

which again implies that $DR_B > DR_A$. In summary, if $\theta_A > \theta_B$ then $DR_B > DR_A$.

The proofs of the "only if" part of the statement and the case $\theta_A < \theta_B$ proceed in the same manner and are therefore omitted. ■

Proof of Proposition 1. Suppose (in negation) that, following a tariff offer of the form $T(q_i)$, the game has a symmetric continuation equilibrium in which all retailers set the monopoly prices (p_A^*, p_B^*). A necessary condition for the prices (p_A^*, p_B^*) to arise in equilibrium is that no retailer can gain by adjusting its retail prices so as to sell more in one market and less in the other market, keeping its total quantity sold constant.[51] That is, it must be that[52]

$$(p_A^*, p_B^*) \in \arg\max_{(p_{Ai}, p_{Bi})} p_{Ai} D_{Ai}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) + p_{Bi} D_{Bi}(p_{Bi}, p_{Ai}; p_B^*, p_A^*),$$

$$\text{s.t. } D_{Ai}(p_{Ai}, p_{Bi}; p_A^*, p_B^*) + D_{Bi}(p_{Bi}, p_{Ai}; p_B^*, p_A^*) = D_{Ai}(p_A^*, p_B^*; p_A^*, p_B^*) + D_{Bi}(p_B^*, p_A^*; p_B^*, p_A^*),$$

[51] Notice that because we consider deviations in which a retailer changes its allocation of quantity across the two markets without changing how much quantity it orders in total, no restrictions on the functional form of the tariff T are required to make this argument.

[52] In what follows, $D_{si}\left(p_{si}, p_{-si}; p_A^*, p_B^*\right)$ denotes the demand of retailer i in market s when i sets the prices (p_{Ai}, p_{Bi}) and all other retailers charge (p_A^*, p_B^*). Moreover, $D_{si}^* = D_{si}\left(p_A^*, p_B^*; p_A^*, p_B^*\right)$.

which leads to the first-order conditions

$$
\begin{aligned}
D^*_{Ai} + (p^*_A - \lambda)\frac{\partial D_{Ai}(p^*_A, p^*_B; p^*_A, p^*_B)}{\partial p_{Ai}} + (p^*_B - \lambda)\frac{\partial D_{Bi}(p^*_B, p^*_A; p^*_B, p^*_A)}{\partial p_{Ai}} &= 0, \quad (50)\\
D^*_{Bi} + (p^*_A - \lambda)\frac{\partial D_{Ai}(p^*_A, p^*_B; p^*_A, p^*_B)}{\partial p_{Bi}} + (p^*_B - \lambda)\frac{\partial D_{Bi}(p^*_B, p^*_A; p^*_B, p^*_A)}{\partial p_{Bi}} &= 0, \quad (51)
\end{aligned}
$$

Since the Lagrange multiplier λ plays the role of a marginal cost in each market, by using the insights from the preliminaries section these conditions can be expressed as

$$
\begin{aligned}
p^*_A - \lambda &= \theta_A(p^*_A, p^*_B)\,\xi_A(p^*_A, p^*_B), \quad (52)\\
p^*_B - \lambda &= \theta_B(p^*_B, p^*_A)\,\xi_B(p^*_B, p^*_A), \quad (53)
\end{aligned}
$$

which, using Lemma 1(i), can be rewritten as

$$
\begin{aligned}
p^*_A - \lambda &= \theta_A(p^*_A, p^*_B)\,(p^*_A - c), \quad (54)\\
p^*_B - \lambda &= \theta_B(p^*_B, p^*_A)\,(p^*_B - c). \quad (55)
\end{aligned}
$$

It follows that the supplier is unable to achieve its first-best outcome with a tariff of the form $T(q_i)$ if $\theta_A(p^*_A, p^*_B) \neq \theta_B(p^*_B, p^*_A)$ or $p^*_A \neq p^*_B$, because there does not (generically) exist a λ such that (54) and (55) both hold. ■

Proof of Proposition 2. Suppose the supplier offers the retailers a tariff of the form $T(q_{Ai}, q_{Bi}) = F + w_A q_{Ai} + w_B q_{Bi}$. If all retailers accept the offer, the downstream price competition game has an equilibrium in which the retailers charge the monopoly prices (p^*_A, p^*_B) if and only if the wholesale prices w_A and w_B are such that

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial p_{Ai}}(p^*_A, ..., p^*_A, p^*_B, ..., p^*_B; w_A, w_B) &= 0, \quad (56)\\
\frac{\partial \pi_i}{\partial p_{Bi}}(p^*_A, ..., p^*_A, p^*_B, ..., p^*_B; w_A, w_B) &= 0. \quad (57)
\end{aligned}
$$

Using Lemma 1(ii), (56) and (57) are equivalent to $p^*_s - w_s = \theta^*_s(p^*_s - c)$ for each s, which is true if and only if $w_s = w^*_s \equiv \theta^*_s c + (1 - \theta^*_s)\,p^*_s$ for each s. It follows that the game has an equilibrium in which the supplier offers a dual two-part tariff with wholesale prices (w^*_A, w^*_B) and a fixed fee $F = \pi_i(p^*_A, ..., p^*_A, p^*_B, ..., p^*_B; w^*_A, w^*_B)$, and earns the industry monopoly profit

$\Pi\left(p_A^*,...,p_A^*,p_B^*,...,p_B^*\right)$. ■

Proof of Proposition 5. With constant pass-through rates, as is the case when the demand functions are linear, the marginal output effect can be written as follows:

$$\begin{aligned} Q'(r) &= \left[\left(\frac{\partial Q_A}{\partial p_A}+\frac{\partial Q_B}{\partial p_A}\right)\rho_A^A+\left(\frac{\partial Q_A}{\partial p_B}+\frac{\partial Q_B}{\partial p_B}\right)\rho_A^B\right]w_A'(r) \\ &+\left[\left(\frac{\partial Q_A}{\partial p_A}+\frac{\partial Q_B}{\partial p_A}\right)\rho_B^A+\left(\frac{\partial Q_A}{\partial p_B}+\frac{\partial Q_B}{\partial p_B}\right)\rho_B^B\right]w_B'(r). \end{aligned} \tag{58}$$

Using the expression for $w_A'(r)$ and $w_B'(r)$ and assuming that demands are linear with symmetric cross-market effects ($\frac{\partial Q_A}{\partial p_B}=\frac{\partial Q_B}{\partial p_A}$), $Q'(r)$ simplifies to

$$Q'(r)=\underbrace{\frac{\left(\rho_A^A\rho_B^B-\rho_A^B\rho_B^A\right)\left(\frac{\partial Q_A}{\partial p_A}\frac{\partial Q_B}{\partial p_B}-\frac{\partial Q_A}{\partial p_B}\frac{\partial Q_B}{\partial p_A}\right)}{\Pi_{AA}^w+2\Pi_{AB}^w+\Pi_{BB}^w}}_{<0}\left[\left(\rho_A^A+\rho_B^A\right)-\left(\rho_B^B+\rho_A^B\right)\right]. \tag{59}$$

Since the first term in (59) is negative, $Q'(r)>0$ if and only if

$$-\left(\rho_A^B-\rho_A^A\right)<\rho_B^B-\rho_B^A. \tag{60}$$

Similarly, the marginal welfare effect can be written as

$$\begin{aligned} W'(r) &= \left[\left(p_A^e-c\right)\left(\frac{\partial Q_A}{\partial p_A}\rho_A^A+\frac{\partial Q_A}{\partial p_B}\rho_A^B\right)+\left(p_B^e-c\right)\left(\frac{\partial Q_B}{\partial p_A}\rho_A^A+\frac{\partial Q_B}{\partial p_B}\rho_A^B\right)\right]w_A'(r) \\ &+\left[\left(p_A^e-c\right)\left(\frac{\partial Q_A}{\partial p_A}\rho_B^A+\frac{\partial Q_A}{\partial p_B}\rho_B^B\right)+\left(p_B^e-c\right)\left(\frac{\partial Q_B}{\partial p_A}\rho_B^A+\frac{\partial Q_B}{\partial p_B}\rho_B^B\right)\right]w_B'(r), \end{aligned}$$

when the pass-through rates are constant. In the case of linear demand with symmetric cross-market effects ($\frac{\partial Q_A}{\partial p_B}=\frac{\partial Q_B}{\partial p_A}$), this further simplifies to

$$W'(r)=\underbrace{\frac{\left(\rho_A^A\rho_B^B-\rho_A^B\rho_B^A\right)\left(\frac{\partial Q_A}{\partial p_A}\frac{\partial Q_B}{\partial p_B}-\frac{\partial Q_A}{\partial p_B}\frac{\partial Q_B}{\partial p_A}\right)}{\pi_{AA}^w+2\pi_{AB}^w+\pi_{BB}^w}}_{<0}\left[\left(\rho_A^A+\rho_B^A\right)\left(p_B^e-c\right)-\left(\rho_B^B+\rho_A^B\right)\left(p_A^e-c\right)\right], \tag{61}$$

which has the sign of

$$\frac{p_A^e-c}{\rho_A^A+\rho_B^A}-\frac{p_B^e-c}{\rho_B^B+\rho_A^B}. \tag{62}$$

If (62) holds for r^*, that is, if[53]

$$\frac{p_A^* - c}{\rho_A^A + \rho_B^A} > \frac{p_B^* - c}{\rho_B^B + \rho_A^B}, \tag{63}$$

then $W'(r) > 0$ for all r. ■

B Additional results

Two-part tariffs are without loss of generality If tariff of the form $T(q_{Ai}, q_{Bi})$ are permitted, the supplier can achieve its first-best by means of the dual two-part tariff in Proposition 2, hence there is no loss of generality in restricting attention to two-part tariffs.

If only tariffs of the form $T(q_i)$ are permitted, then there must exist a λ such that the equilibrium retail prices (p_A^e, p_B^e) satisfy

$$\frac{\partial \pi_i \left(p_A^e, ..., p_A^e, p_B^e, ..., p_B^e; \lambda, \lambda\right)}{\partial p_{Ai}} = \frac{\partial \pi_i \left(p_A^e, ..., p_A^e, p_B^e, ..., p_B^e; \lambda, \lambda\right)}{\partial p_{Bi}} = 0, \tag{64}$$

otherwise, as discussed in Section 4.1, each retailer would have a profitable unilateral deviation to different prices. Now suppose that instead of $T(q_i)$, the supplier offers the retailers a two-part tariff with wholesale price $w = \lambda$ and fixed fee

$$F = T(q_i^e) - w\left(Q_A\left(p_A^e, p_B^e\right) + Q_B\left(p_A^e, p_B^e\right)\right).$$

The retailers are willing to accept this two-part tariff if they were willing to accept the tariff $T(q_i)$, and each will order $Q_A\left(p_A^e, p_B^e\right) + Q_B\left(p_A^e, p_B^e\right)$ units. The supplier's profit with the two-part tariff becomes

$$n\left(T(q_i^e) - c\left(Q_A\left(p_A^e, p_B^e\right) + Q_B\left(p_A^e, p_B^e\right)\right)\right),$$

which is (at least) as much as the profit it would have earned by offering $T(q_i)$.[54]

---

[53] This follows because with linear demand functions p_A^e decreases and p_B^e increases with r.

[54] The two profits are the same if the quantity ordered $q^e = Q_A(p_A^e, p_B^e) + Q_B(p_B^e, p_A^e)$. If $q^e > Q_A(p_A^e, p_B^e) + Q_B(p_B^e, p_A^e)$, the two-part tariff yields a higher profit for the supplier because it implies lower upstream variable costs.

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