Consistent Flexibility: Enforcement of Fiscal Rules Through Political Incentives

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Abstract

We study a fiscal policy model in which the government is present-biased towards public deficit but faces shocks to tax revenues. We study the optimal fiscal rule to trade off the benefit of committing the government to not overspend against the benefit of granting it flexibility to react to shocks. The rule consists of two parameters: a level of expected deficit and a degree of flexibility of the rules to respond to the shock. Unlike prior work, we characterize a rule that is enforced through political incentives: the punishment for a violation consists in a reduction of the politician's payoff from being in office during the following period, for example, triggered by reporting of an independent fiscal institution. We show that the optimal fiscal rule prescribes zero structural deficit. Moreover - and somewhat surprising - countries with a stronger present bias should be granted a higher degree of fiscal flexibility. We discuss the results in the context of the debate on the reform of the EU's Stability and Growth Pact.

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1 Introduction

Fiscal rules are widely used to constrain a government's fiscal policy and aim for moderate levels of budget deficits, debt or expenditure levels. They have become increasingly prevalent, in place in 97 countries in 2013, compared to only seven countries in 1990[1]. Governments however do not always respect these rules. For example, in 2003, the governments of France and Germany violated the terms of the European Union's Stability and Growth Pact by running deficits above the allowed limit, without facing any formal sanction (see Schuknecht et al. 2011). Out of 28 EU countries 25 have been subject to an Excessive Deficit Procedure (EDP) at some point in the past, an indication that compliance with fiscal rules in the EU cannot been taken for granted. Credible enforcement mechanisms are critical to the institution of fiscal rules, for governments only abide to rules if the ensuing penalties for breaching them are severe enough. Nevertheless, a monetary punishment is often politically unfeasible, as the case of the European Union's Stability and Growth Pact illustrates.[2]

In this paper we ask how an optimal fiscal rule should look like when the political process leads to a deficit bias and monetary punishment mechanisms are absent. More specifically, we ask how restrictive in terms of a maximum deficit limit and how flexible in terms of accommodating shocks to public finances a fiscal rule should be. We are not the first to discuss the optimal design of fiscal rules. Our approach differs in some important elements from the previous literature, however, and has the following three key features.

i) A shock to tax revenues makes compliance with a fiscal rule uncertain. Uncertainty about government revenues and expenditures is a central feature

[1]See IMF Fiscal Rules Data Set, 2013 and Budina *et al.* (2012).

[2]In the preventive arm of the Stability and Growth Pact a deposit of 0.2% of GDP for Euro area countries is mandated in case of violation, but it has never been implemented, despite of multiple and repeated violations occurred in several EU member countries. In addition, in the corrective arm financial sanctions regarding in the European Structural and Investment Fund are foreseen. For details see European Commission (2018).

of budgetary planning and forecasting. We assume that the realization of the shock is observable to policy makers and households. We believe that the latter assumption is a reasonable approximation in many countries in which independent fiscal institutions ("fiscal watchdogs") either need to endorse government budget projections or do their own projections, and in addition assess compliance with fiscal objectives ex post (Beetsma and Debrun, 2018, Horvath, 2017). Often there are also other institutions that provide forecasts like central banks, which together limit the private information of governments. Our assumption is in contrast to Halac and Yared (2014, 2017, 2018) who assume that a shock to the value of public spending is observable to the government but not to the public and any mechanism to limit the deficit bias must obey this information asymmetry. Our approach does not negate the importance of private information, but rather we believe that even when there is no asymmetric information an interesting issue arises due to the next two features of our setup.

ii) A fiscal rule consists of two parameters: the level of the maximum (structural) deficit, that is, the highest deficit level allowed under the rule if the tax shock takes its expected value of zero, and secondly, the degree to which the tax shock modifies the maximum deficit level. The latter captures the degree to which fiscal rules should accommodate macroeconomic circumstances, one offs and other temporary measures. While the first generation fiscal rules like the Maastricht criteria (inter alia: headline deficit no more than 3% of GDP) did not account for these circumstances, second generation rules like the Fiscal Compact do exactly this (medium term objective for structural deficit 0.5% of GDP, see European Commission, 2018), and are more flexible. Our focus on a fiscal rule with two parameters is admittedly restrictive, but the main insights of this analysis hold true in even if this assumption is relaxed.

Halac and Yared (2014) have shown that under persistent shocks optimal mechanisms depend not only on payoff relevant variables of that period but also on the entire history of shocks. Our assumption is guided by the relative simplistic nature of fiscal rules in practice. Even the second generation fiscal rules that take call for cyclical adjustments for the target variable are sometimes considered too complex . Various forms of balanced budget rules are also used among US states, see Bohn and Inman (1996) for a classic treatment. Our approach is more in line with Azzimonti et al. (2016) who consider the effects of a balanced budget rule that is not adjusted for shocks.

iii) Monetary punishments to rule violations are absent, perhaps because they are not credible, or because politically they are not enforceable.For instance, the violation of a deficit rule is typically more likely during a recession. Thus, a monetary punishment tends to have pro-cyclical effects on the government budget, and to reduce in turn the ability of policy-makers of smoothing public consumption over time. This may generate credibility issues, because the fiscal authority may step back on the commitment to punish violations during a recession. This means that a monetary punishment may not be ex-post incentive compatible for an independent fiscal authority with a welfare-improving target.

Instead we assume that the violation of a fiscal rule leads to a loss in the rent of holding office in the next period, which (partially) disciplines politicians. Such enforcement mechanism has the advantage - relative to the one based on monetary punishments - that the problem of the *credibility of commitment* to punish is typically less severe, at least as long as the fiscal authority is independent of the government Moreover, because the punishment affects politicians rather than citizens, this mechanism is less prone to induce disruptive or normatively worrying consequences, e.g. it does not have direct pro-cyclical fiscal effects.[3] Of course, if a violation of a fiscal rule is considered a plus in the view

[3]See Beetsma et al. 2017, Horvath 2017. Reputation effects are also likely to play a role

of voters, as may have been the case in the recent struggle between the Italian government and the European Commission, then fiscal rules may become ineffective in disciplining politicians. In such a case compliance with fiscal rules may be impossible to achieve. In that sense our assumption is crucial.

The loss in the rent of holding office may be in the form of negative reputation effects when "neutral" institutions like fiscal councils or the European Commission assess non-compliance. Compliance with a fiscal rule is a stochastic outcome, as the government sets fiscal policy that includes a planned debt/deficit level, but the actual deficit/debt level emerges only after the shock is realized. Unlike most of the existing literature that either assumes perfect enforcement of fiscal rules or self-enforcing rules, our setup allows for occasional violation, which seems better in line with the evidence reported above.

Our setup contributes to the political debate regarding the design of fiscal rules in the European Union, which has centered on the *degree of flexibility* of such rules with respect to macroeconomic shocks. On one hand, there is support among academics and policy makers for the claim that fiscal rules should be more flexible in order to ensure smooth provision of governmental services and to avoid welfare losses in case of large negative shocks. For example, the European Commission has introduced more flexible interpretations in the handling of the Stability and Growth Pact (European Commission, 2015). A prominent group of French and German economists (Benassy-Quere et al. 2018) has also advocated more flexibility. On the other hand, others are concerned that more flexible rules would be less effective in disciplining politicians that are biased towards excessive spending, resulting in larger sovereign debt and high risks of default. Critical positions are often found in Northern European countries such as Germany, see for instance Burret and Schnellenbach (2013) and Deutsche

in the context of the Greek fiscal crisis. The enforcement of a delegation of EU, ECB, and IMF experts - the so-called Troika - to verify the implementation of certain reforms in Greece starting in 2010 can be seen as an example of non-monetary punishment.

Bundesbank (2017). In this paper we show that there is not necessarily a trade off between fiscal discipline and flexibility.

To derive the optimal design of the fiscal rule, we consider an environment in which the government is present-biased towards public spending. Current generations of voters do not fully internalize the harm that public debt imposes on future generations. This preference structure results naturally from the aggregation of heterogeneous, time-consistent citizen's preferences in an overlapping-generation setting. At the beginning of each period, a politician is chosen in a two-candidate contest. Politicians are purely office seekers and voting is probabilistic. The government in office chooses in each period a linear tax rate on labor income, the level of expected public debt, and in period 2 default on public debt. Together these policies jointly determine the amount of public spending on a public good. An i.i.d. shock affecting the tax revenue is observed in period 1 after the policy choice and is public information. A social planner chooses the parameters of a fiscal rule to trade off the benefit of committing a government to not run excessive debt against the benefit of granting it flexibility to react to shocks. Violation of a fiscal rule is costly to the policymaker in office in the next period.

We derive the following main results. First, the benchmark policy of the social planner can be implemented via an optimally designed fiscal rule if the tax shock and the taste shock of voters over candidates have enough variance. The implementation of the benchmark policy is possible if the politician's cost of violating the rule is large enough. This is ensured if the distribution of the voters' taste shock has enough variance. The distribution of the shock on tax revenues must also have enough variance, which ensures that the politician is not induced to choose a suboptimally low level of expected deficit.

Second, if the above conditions are satisfied, we characterize the parameters of the fiscal rule that implements the benchmark. Interestingly, the optimal rule prescribes a *zero structural deficit*, which is close to the requirement of a balanced or surplus structural deficit laid down in Art. 3(1) of the Fiscal Compact, as well as the requirements in the German and Swiss debt brakes. The intuition that underpins this result is simple. Politicians' tax choices affect output by distorting labor supply decisions. The fiscal rule is in the form of a threshold on the deficit/output ratio. This implies that the maximum level of deficit allowed by the rule is increasing in output, at a rate equal to the value of the threshold itself. Thus, if the latter is set to zero, then there is no impact of output on the probability that a violation of the rule occurs, and therefore imposing zero structural deficit is sufficient to ensure that tax choices are not distorted. Moreover, we show that typically the optimal rule accounts only partially for the tax shock, that is, the maximum deficit under the rule is the target level minus a fraction less than one of the tax shock (relative to GDP). A full consideration of tax shocks under the target of a balanced structural budget is typically not optimal because the marginal cost of increasing public debt in terms of expected cost of rule violationmay becometoo large and hence induce a debt level that is too small.

Third, the optimal fiscal rule prescribes *more flexibility* to countries that have - *ceteris paribus* - stronger incentives to run excessive deficit in the first period. The intuition is the following: Because the shock is not observed in the moment in which the fiscal policy is chosen in the first period, the policy maker faces a probability of being punished in the next period (if he gets to be reelected). The more flexible the rule is, the larger is the *marginal effect* of increasing the planned deficit on the probability of being punished. In other words, a more flexible rule is more effective in disciplining the politician because it implies a stronger link between current fiscal policies and probability of a future punishment. At the extreme opposite of the spectrum, under a very inflexible rule the *marginal effect* of increasing expected deficit on the probability of being punished is very small, because the probability of being punished depends heavily on the realization of the macroeconomic shock, i.e. on luck rather than on the chosen fiscal policies.

Fourth, we characterize the optimal fiscal rule when the benchmark policy is not implementable, that is at least one of the variance conditions is not satisfied. In particular, we show that a positive structural deficit target is optimalif the tax shock has too little variance relative to the expected deficit.. This may be the case, for instance, if the current output level is very low relatively to the one expected in the future, e.g. if the country is facing a recession.

Our analysis of fiscal rules shares several similarities with the approach used in Amador, Werning, and Angeletos (2006) and Halac and Yared (2014), in particular the role of a government that is present-biased towards public spending because of the overlapping-generation nature of the voter's problem. Jackson and Yariv (2015, 2014) propose a model that exhibits similar properties. Other papers obtain an analogue result as a consequence of political turnover (e.g., Aguiar and Amador, 2011).[4] We depart from prior work by relaxing the assumption that rules can be perfectly enforced, or that rules are enforced through monetary punishments (as in Halac and Yared 2017). We posit that fiscal rules must be self-enforcing: the politician in office must prefer to comply with the rule rather than face the punishment that follows a breach. Such punishment must also be self-enforcing, meaning that the independent fiscal authority must not have strict incentives to step back on the commitment to punish violations. Punishment may come also from reputation effects, for example, when independent fiscal institutions, often referred to as fiscal watchdog, assess government's

[4]See also Persson and Svensson (1989), Alesina and Tabellini (1990), Alesina and Perotti (1995), Lizzeri (1999), Tornell and Lane (1999), Battaglini and Coate (2008), and Caballero and Yared (2010).

(non-)compliance with fiscal rules. Independent fiscal institutions have been mandated by the Fiscal Compact and have spread widely in recent years also in many countries around the world (Beetsma and Debrun, 2016).

The remainder of the paper is organized as follows. In section 2 we describe the model and solve for the first best rule in the absence of political economy considerations. In section 3 we then introduce voting over candidates which leads to a present bias in government spending. We solve for the optimal fiscal rule in this environment and compare it to the first best. We discuss our results and consider several extensions in section 4. Section 5 concludes.

2 Model

We begin with a short overview of the model. We study a small open economy that lasts for two periods $b = 1, 2$. The population of consumers-voters is a continuum of size 1 in each period. A share θ_1 of the population is of type $T = Y$ and cares both about the current period and about the next period, while a share $(1 - \theta_1)$ is of type $T = O$ and only cares about the current period. One can think about the two types to be "young" vs. "old" voters (an alternative interpretation could be "forward looking" and "myopic" voters). A young voter survives to period 2 with probability equal to π. Thus, a share $\pi\theta_1$ of the population lives for two periods. The individuals born at the beginning of period 2 represent a share $\theta_2 = 1 - \pi\theta_1$ of the total population in that period.

All individuals work and consume a consumption good in both periods. There are no savings.[5] The government collects taxes on labor income and provides public goods in periods 1 and 2. Tax revenues are stochastic in period 1.

At the beginning of period $b = 1$ two candidates run for elections. Each of

[5] In an extension we show that all the results qualitatively hold for an economy with savings.

them fully commits to a policy platform consisting of a linear income tax rate on labor income and a level of planned debt. At the beginning of period $b = 2$ the same two candidates run for elections. Each of them fully commits to a policy platform consisting of a linear income tax rate on labor income. There is no default, thus all debt must be repaid in period 2.[6] An elected candidate always implements the platform he/she proposes before the elections.

A common deficit rule for all countries can be imposed in period 1, whose violation carries cost for the government in period 2. The stochastic nature of tax revenues makes compliance with the deficit rule uncertain ex ante.

2.1 Private sector

Consumers in each period $b \in \{1, 2\}$ derive utility from consumption of a private good c_b and of a public good g_b. In each period $b \in \{1, 2\}$ individuals supply labor $l_b \in [0, \bar{l}]$ and are compensated at wage rate w_b (equal to their productivity). They face a stictly covex cost of labor $v(l_b)$. The wage at time 2 is assumed to be $w_2 \geq v'(\bar{l})$, which implies that the labor supply in period 2 is fully inelastic.

Income is taxed at a linear rate t_b, such that $c_b = (1 - t_b)w_b l_b$. Thus, the within-period utility of any type of consumer for $b \in \{1, 2\}$ is given by $U(c_b, l_b, g_b) = c_b - v(l_b) + u(g_b)$, where u is strictly concave. The lifetime utility of a young household born in period 1 is therefore

$$U(c_1, l_1, g_1) + \beta \pi E \left[U(c_2, l_2, g_2) \right], \tag{1}$$

where β is the discount factor. Individuals born in period 2 live for one period only. Thus, the young generation born in period 2 enjoys utility $U(c_2, l_2, g_2) =$

[6]These simplifying assumptions about public finances are imposed only for ease of exposition. All the results go through in a closed economy with endogenous interest rate and default.

$c_2 - v(l_2) + u(g_2)$.

Notice that the wage rate w_b and the cost of labor $v(\cdot)$ are identical across young and old citizens in any given period. Thus, the two types face the same trade-off between utility consumption and cost of labor. As a result, the optimal labor supply is the same across types. Because of that, for ease of notation we denote with l_b the labour supply of a citizen of any type in period b.

2.2 Government sector

The government faces different decisions over time. In period 1 tax revenue has two components:

$$T_1 = t_1 w_1 l_1 + \epsilon, \tag{2}$$

where t_1 is the tax rate, w_1 is the wage rate, and l_1 is the (endogenous) labor supply. The second component ϵ is the realization of a i.i.d. shock with support $\epsilon \in [-a, a]$, and such that $E[\epsilon] = 0$. The shock ϵ is distributed continuously with c.d.f. F and p.d.f. f. Specifically, we assume that the shock on tax revenues ϵ is distributed as a two-sided symmetrically truncated normal, i.e. it possesses c.d.f. F as follows.

$$F(\epsilon) = \begin{cases} 0 & \epsilon < -a \\ \Phi\left(\frac{\epsilon}{\sigma_\epsilon^2}\right) \Big/ \left[\Phi\left(\frac{a}{\sigma_\epsilon^2}\right) - \Phi\left(-\frac{a}{\sigma_\epsilon^2}\right)\right] & -a \leq \epsilon \leq a \\ 1 & \epsilon > a \end{cases} \tag{3}$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution, $a > 0$ is a threshold.[7]

The government can borrow from abroad at a fixed interest rate r. Let D_1^{act} denote the stock of debt at the end of period 1, after the tax shock has realized.

[7] One could assume $\sigma_\epsilon = \sigma_\nu \lambda y_1$ where $y_1 = w_1 \bar{l}_1$ is the expected output calculated at the optimal tax rate t_1.

The intended debt level D_1 is the one planned prior to the realization of the tax shock. Thus, $D_1 = D_1^{act} - \epsilon$. In period 1, by assumption the government repays its existing debt inherited from the past. The government budget constraint in period 1 is therefore

$$g_1 \leq t_1 w_1 l_1 + \epsilon - D_0(1+r) + D_1^{act} = t_1 w_1 l_1 - D_0(1+r) + D_1. \quad (4)$$

We assume in the following that the budget constraint holds with equality and write public consumption good as function of tax and intended debt level $g_1(t_1, D_1)$.

The government budget constraint in period 2 has formula:

$$g_2 \leq t_2 w_2 l_2 - (D_1 - \epsilon)(1+r) \quad (5)$$

Similarly to period 1, we construct $g_2(t_2, \epsilon)$, using the budget constraint in period 2.

We assume that the values of productivity w_2 is large enough to ensure that the repayment of debt in period 2 can be always fully satisfied. Specifically, we impose $\overline{D}_1 < w_2 \bar{l}(1+r) - a$, where $\overline{D}_1$ represents the maximum values of the intended debt in period 1. Moreover, we assume that the choice of planned debt level D_1 lies in the range $[D_0, \bar{D}_1]$, i.e. in expectation a government chooses a level of deficit that is weakly positive. Lastly, the upper bound $\overline{D}_1$ satisfies $u'(\overline{D}_1 - D_0(1+r)) \leq \beta(1+r)$.[8].

2.3 Normative Benchmark: Social Planner's Problem

For this analysis we introduce a benevolent social planner who can set D_1 and t_1 optimally in period 1, from which the public good level in period 1 follows

[8]This assumption ensures that the socially optimal planned debt level in period 1 is an interior solution.

immediately from (4). Thus, the planner in period 1chooses a policy $(t_1, D_1) \in X$ with $X = [0,1] \times [D_0, \bar{D}_1]$. In period 2, based on the actual debt level of period 1, the planner chooses the labor tax $t_2 \in [0,1]$. The public good level follows from budget constraint (2).

Denote with $u^Y(t_1, D_1)$ the indirect expected lifetime utility enjoyed by a young voter in period 1 under policy (t_1, D_1), and with $u^O(t_1, D_1)$ the one enjoyed by a old voter. The former writes:

$$u_1^Y(t_1, D_1) = \begin{array}{c} (1-t_1)w_1 l_1(t_1) - v(l_1(t_1^i)) + u(g_1(t_1, D_1)) \\ +\beta\pi E\left[(1-t_2)w_2 l_2 - v(l_2) + u(g_2(t_2, \epsilon) \mid t_1, D_1\right] \end{array} \tag{6}$$

where expectation are rational given history. The latter is given by:

$$u_1^O(t_1, D_1) = (1-t_1)w_1 l_1(t_1)) - v(l_1(t_1)) + u(g_1(t_1, D_1)) \tag{7}$$

Lastly, the indirect utility of a young or old individual individual in period 2 writes:[9]

$$u_2^Y(t_2, \epsilon) = u_2^O(t_2, \epsilon) = (1-t_2)w_2 l_2 - v(l_2) + u(g_2(t_2, \epsilon)) \tag{8}$$

The social planner maximizes the sum of the utilities of all individials over both periods. He/she discounts the utility of the future generation at rate β.[10] Thus, her objective function writes

$$\theta_1 u_1^Y(t_1^i, D_1) + (1-\theta_1)u_1^O(t_1, D_1) + \beta\theta_2 E\left[u_2^Y(t_2, \epsilon) \mid t_1, D_1\right] \tag{9}$$

It is easy to show that the social planner's objective function is strictly concave. Substituting the formulas from (6)-(8) for $u^Y(t_1, D_1)$, $u^Y(t_1, D_1)$, and $u_2^Y(t_2, \epsilon)$

[9]Notice that the indirect utility of a young individual in period 2 is identical to the one of an old individual in the same period.

[10]Alternatively, one could assume that the social planner discounts utility of future generation at a different rate relative to the ones of young individuals in period 1.

into the above, we derive the planner's problem

$$\max_{(t_1,D_1)\in X} \begin{array}{c} (1-t_1)w_1l_1(t_1) - v(l_1) + u(g_1(t_1,D_1)) + \\ +\beta E\left[(1-t_2^*)w_2l_2(t_2^*) - v(l_2) + u(g_2(t_2,\epsilon)) \mid t_1, D_1\right] \end{array} \tag{10}$$

The solution (t_1^*, D_1^*) is called the *optimal policy*. Notice that the social planner's objective function is independent of θ_1, θ_2, and π. Rational expectations imply that in period t_2 is chosen optimally given D_1 and ϵ. Thus, the first order conditions are:

$$[t_1] := \; w_1l_1(t_1)\left[u'\left(g_1(t_1,D_1^*)\right)(1+\eta_1(t_1)) - 1\right] = 0 \tag{11}$$

$$[D_1] := \; u'\left(g_1(t_1,D_1^*)\right) - \beta(1+r) = 0 \tag{12}$$

where $\eta_1(t_1)$ is the tax elasticity of labor supply at tax rate t_1.[11]

2.4 Deficit rule

In section 3 we will assume that fiscal policy in any given period is not chosen by a social planner but by a policymaker who won the election in that period. Because policymakers focus on current voters, the well-being of future generations is ignored. This generates a present bias and leads to excessive deficit, against which a deficit rule may be put in place. In the remainder of section 2 we describe the structure of the fiscal rule that will be considered in section 3.

A deficit rule is in place consisting in two parameters: $k \in \left[0, \bar{k}\right]$ and $\delta \in \left[0, \bar{\delta}\right]$ with $\bar{\delta} < 1$. The parameter k is the maximum level of the deficit-to-output ratio that is allowed at the end of period 1 if $\epsilon = 0$, and describes the *tightness* of the rule. The parameter δ represents the degree of *flexibility* of the rule relative to the magnitude of the shock on tax revenues as a share of output. Flexibility

[11]Notice that because $\eta_1(t_1) \leq 0$ an interior solution for t_1 requires $\beta(1+r) > 1$. This assumption is not needed in amodel with default and endogenous interest rate.

is defined as the marginal effect of a change in $\epsilon/w_1 l_1$ on the maximum level of the deficit-to-output ratio that is allowed by the rule. The government is compliant with the rule if and only if

$$\frac{deficit_1}{output_1} \leq \underbrace{k}_{level} + \underbrace{\delta}_{flexibility} \times \left(-\frac{shock}{output_1}\right) \tag{13}$$

The above implies that an economy facing a shock ϵ is allowed a deficit/output ratio equal to $k - \frac{\delta\epsilon}{w_1 l_1}$, where output is given by $w_1 l_1$.

Notice that the deficit is by definition total spending plus total cost of interest on debt minus total tax revenue. Thus, the deficit-to-output ratio and the deficit rule write

$$\frac{deficit_1}{output_1} = \frac{g_1 + rD_0 - t_1 w_1 l_1 - \epsilon}{w_1 l_1} = \frac{D_1 - D_0 - \epsilon}{w_1 l_1} \leq k - \frac{\delta\epsilon}{w_1 l_1}, \tag{14}$$

where we have made use of budget constraint (4). Alternatively, one could rearrange the formula above to derive a debt-to-output rule. The two kinds of rules deliver similar predictions, thus we focus on the deficit rule.

Given a rule (k, δ), we define a threshold of the shock $\tilde{\epsilon}$, based on (7), below which the politician gets punished as follows[12]

$$\tilde{\epsilon} = \frac{D_1 - D_0 - kw_1 l_1}{1 - \delta} \tag{15}$$

Lastly, we assume $a \geq \frac{1}{1-\delta} \frac{\bar{D}_1 - D_0}{w_1 l_1(\bar{t})}$.[13]

[12] Notice that $\tilde{\epsilon}$ may assume values such that $f(\tilde{\epsilon}) = 0$.

[13] This assumption ensures that, whenever a fiscal rule is present, the probability of a violation of the rule is always strictly larger than 0.

3 Political Equilibrium

We now turn to a positive model of fiscal policy choices. In each period two candidates compete for the support of voters, and the elected winner implements her preferred choice. We use a probabilistic voting model in the tradition of Lindbeck and Weibull (1987) and Banks and Duggan (2005).

3.1 Timing of events and choices

At the beginning of period 1 two candidates denoted by superscript $I \in \{A, B\}$ run for elections. Each of them fully commits to a policy platform (t_1^I, D_1^I) consisting of a linear income tax rate and a level of planned debt. The (planned) level of public good follows from this policy proposal via the government budget constraint (4). The winner of the election implements her proposed platform. Voters observe the policy and choose their labor supply l_1 and consumption c_1. The government collects taxes and provides a public good g_1. At the end of period 1 a shock on tax revenues is realized and it is publicly observable. Such realization determines the actual level of debt accumulated D_1^{act}.

At the beginning of the second period a new election takes place between the same two candidates. Each of them fully commits to a policy platform consisting solely of a linear income tax rate t_2^I, which via the government budget constraint defines public consumption, as there is by assumption no tax shock in the second period. Then - if a deficit rule is in place - the supranatural authority or a independent fiscal institutions verifies if a violation of the rule has occurred in period 1 and, if so, imposes a punishment to the politician in power. The punishment is thus a cost to the policy maker in that country regardless of who was in power in the previous period. The winner of the elections implements her proposed platform. The government collects taxes, provides a public good

g_2, and repays debt.[14]

The politician that holds the office in period $s \in \{1, 2\}$ enjoys an exogenous rent W_s. If a violation of the fiscal rule has occurred in period 1, then the rent enjoyed by the politician that holds the office in period 2 - whether incumbent or not - is reduced by an exogenous amount $C < W_2$.

Each politician wishes to maximize the weighted expected return of being in office in the two periods. In period 2 politicians take as given the actual debt level inherited from period 1, and chooses tax rate and default. For example, politician A in period 1 maximizes

$$\begin{aligned} \Pi_1^A = & Pr(win_1^A | t_1^A, D_1^A, t_1^B, D_1^B) \times W_1 + Pr(win_2^A | D_1^A, t_1^B, D_1^B) \times W_2 \\ & -Pr(win_2, nc | D_1^A, t_1^B, D_1^B) \times C \end{aligned} \tag{16}$$

where win_s^A denotes the event corresponding to a victory of candidate A in the election at time s, and nc denotes the event of non-compliance with the fiscal rule in period 2.

The outcome of elections is probabilistic and shaped by voters' preferences.

In each period, each voter - given the platform proposed by both candidates - casts her vote for candidate A if the utility difference from electing A vs. B is positive. The utility difference depends upon a deterministic and a stocastic component. Recall that $u^T(t_1, D_1)$ represents the indirect expected lifetime utility enjoyed by a type T voter under policy (t_1, D_1). The deterministic part consists in the difference between the utility induced by the policy platforms that each politician has proposed, i.e. $u^T(t_1^A, D_1^A) - u^T(t_1^B, D_1^B)$. The stochastic part is simply a common preference shock ν_1, which is assumed to be i.i.d. across time independent of the tax shock ϵ, and normally distributed with mean μ and

[14] As mentioned, one may allow for the possibility of default in period 2. In such a case, the policy platform offered by the politician at $t = 2$ is two-dimensional, as it includes the binary default choice.

variance σ_ν^2. Thus, a voter of type $T \in \{Y, O\}$ casts her vote for candidate A in period 1 if and only if

$$u^T(t_1^A, D_1^A) - u^T(t_1^B, D_1^B) + \nu_1 \geq 0 \qquad (17)$$

Similarly, in period 2 a voter of type $T \in \{Y, O\}$ casts her vote for candidate A if and only if $u^T(t_2^A) - u^T(t_d^B) + \nu_2 \geq 0$.

3.2 Voting Equilibrium

It is well known that in a large class of probabilistic voting model the equilibrium policy outcome corresponds to the platform that maximizes a weighted average of the voters' expected utilities (Lindbeck and Weibull, 1987; Banks and Duggan, 2005).

In our setting, we can prove a similar result. Namely, both politicians in equilibrium propose the same platform. Such aplatform maximizes a weighted average of the expected utility of period 1's voters, *corrected* by a factor $cF(\tilde{\epsilon})$, where c captures the expected reputational cost that the politician must face in period 2 as a consequence of the punishment that is imposed if a violation of the deficit rule occurs, and $F(\tilde{\epsilon})$ is the probability that the rule is violated. The expected reputational cost c is itself a function of the exogenous rent loss C, and of the endogenous probability of reelection faced by each politician. It is increasing in the variance of the aggregate taste shock ν_1, which affects voters' electoral choices.

The probabilistic nature of the voting process, together with the presence of the fiscal rule, imply that the candidates' equilibrium platforms are identical to the policy that a partially benevolent social planner would choose. We will refer to this fictive agent as the *representative politician* or, more simply, the *politician.*

The politician's problem in period 1 writes:

$$\max_{(t_1,D_1)\in X} \theta_1 u^Y(t_1,D_1) + (1-\theta_1)u^O(t_1,D_1) - \beta\pi c F(\tilde{\epsilon})$$

Similarly, in period 2, the equilibrium platform maximizes the weighted expected utility of period 2's voters. Formal proofsof these results are provided in Appendix A1.

Using the formulas for $u_1^Y(t_1,D_1)$, $u_1^O(t_1,D_1)$, and and abstracting from the parts that do not affect the optimal outcome, one can rewrite the politician's problem as follows:

$$\max_{(t_1,D_1)\in X} \begin{array}{c} (1-t_1)w_1 l_1(t_1) - v(l_1) + u(g_1(t_1,D_1)) - \beta c F(\tilde{\epsilon}) + \\ +\beta\pi\theta_1 E\left[(1-t_2)w_2 l_2(t_2) - v(l_2) + u(t_2,d_2) \mid t_1, D_1\right] \end{array} \tag{18}$$

Comparing the above with the social planner's problem in formula (10), it is immediately evident that the two objective functions differ solely in two aspects. First, the politician discounts future utility at rate $\beta\pi\theta_1$ while the social planner does so at rate β. Because of that, we call $1-\pi\theta_1$ the *political present bias*. Second, the politician's objective function includes a cost c to be paid if the fiscal rule is violated. In the rest of the paper, we use the objective function of the representive politician to characterize the policy choices in equilibrium. This characterization, together with the normative benchmark described in section 2.4, allows us to derive the main results of this paper, which are stated in the next section.

3.3 Preliminary Insights

The first result simply states that a politician facing a more present-biased electorate tends to run a larger government deficit. This result holds both in the presence of a fiscal rule, and with no fiscal rule.

Proposition 1. *The expected deficit is increasing in the political present bias* $(1 - \pi\theta_1)$.

Proof. See Appendix.

The result is not surprising given the nature of the political process. It is in line with...

The next result follows from Proposition 1 and implies that without a fiscal rule the political process leads to an inefficient outcome, because the voters, on average, do not care about the future as much as a benevolent social planner does.

Proposition 2. *In the absence of a fiscal rule the equilibrium level of deficit in period 1 is weakly larger than the optimal level.*

Proof. See Appendix.

An implication of Proposition 2 is that the period 1 tax rate is lower than the optimal one.

4 The Design of the Fiscal Rule

In this section we characterize the optimal fiscal rule parameters (k, δ) when fiscal policy is chosen via the political process described in the previous section. Proposition 2 gives room for a fiscal rule to improve the outcome. However, it is far from clear whether a fiscal rule can implement the optimal allocation that would be induced by a social planner who chooses the tax rate and the debt level in period 1 directly (which we denoted by t_1^*, D_1^*). The case is not hopeless because the two parameters of the fiscal rule match the number of instruments available to the social planner in the first period. In period 2 there is no political bias, thus the politician's policy choice is the same as the one of the social planner. But this choice is affected by the level of debt accumulated in period 1, thus it is tyically different from the one that would prevail if the social planner had chosen the policy in period 1. In this sense, the equilibrium policy in period 1 spills over into period 2, even though there is no further shock in that period, a fiscal rule does not need to be satisfied, and taxation is lump sum as labor supply is exogenous.

4.1 Characterization of Optimal Policy

Our first main result establishes conditions under which the optimal policy is implementable.

Proposition 3. *The optimal policy* (t_1^*, D_1^*) *is implementable if the following conditions hold:*
(i) The taste shock has enough variance: $\sigma_\nu \geq \bar{\sigma}_\nu$ *for some* $\bar{\sigma}_\nu \in (0, \infty)$*;*
(ii) The tax shock has enough variance: $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ *for some* $\bar{\sigma}_\epsilon \in (0, \infty)$*.*

Proof. See Appendix A2.

Proposition 3 delivers some important intuition. First, the implementation of the optimal policy is possible if the politician's cost of violating the rule $c(\sigma_\eta)$ is large enough. This is ensured if the distribution of the voters' taste shock has enough variance. Second, the distribution of the shock on tax revenues must also have enough variance. Such condition ensures that the politician is not induced to choose a suboptimally low level of expected deficit. Define a threshold $\tilde{\delta} = 1 - \frac{\bar{D}_1 - D_0}{w_1 l_1(t_1^*)\sigma_\epsilon}$ if $\frac{\bar{D}_1 - D_0}{w_1 l_1(t_1^*)\sigma_\epsilon} \leq 1$, and $\tilde{\delta} = 0$ otherwise.

Let us now assume that the conditions of Proposition 3 are satisfied. The following result characterizes the fiscal rule parameters that achieve the benchmark policy outcome.

Proposition 4. *If the optimal policy* (t_1^*, D_1^*) *is implementable, then the implementation occurs at* $k^* = 0$ *and* $\delta^* \in \left[0, \tilde{\delta}\right]$.

Proof. See Appendix.

Proposition 4 states that the optimal rule prescribes *zero structural deficit*, i.e. zero deficit in expectation. Notice that this does not necessarily mean that the *actual* expected deficit is going to be equal to zero, as the realization of the tax shock determines the actual deficit. The result also implies that under the optimal rule tax shocks are not fully, but only partially accounted for, as $\tilde{\delta}$ is alwaysbelow one. To understand Proposition 4 intuitively, it is helpful to recall the threshold level of the tax shock that just leads to fiscal rule compliance (13). If the tightness parameter of the fiscal rule k is non-zero, the tax rate influences the threshold via the effect on labor supply, that is $d\widetilde{\varepsilon}/dt_1 \neq 0$. By contrast, $k = 0$ induces a tax setting by the policy maker that is optimal in the sense of balancing the marginal cost and benefits of taxation in that period given D_1. From this it becomes clear that the flexibility parameter δ is driving the debt decision of the policy maker. To see that a full consideration of tax shocks ($\delta = 1$) under the assumption $k = 0$ is never optimal, note that the marginal cost of increasing public debt in terms of expected cost of rule violation reaches a peak in the interior of the interval $(0, 1)$ and is decreasing in δ if the latter parameter is close enough to 1.

Proposition 5. *If the optimal policy is implementable for all values of θ_1 in a range $[\theta_1', \theta_1'']$, then the optimal degree of flexibility δ^* is weakly increasing in the political present bias θ_1 within such range.*

Proof. See Appendix A2.

This result suggests that flexibility may actually *encourage fiscal discipline*, rather than jeopardizing it. The intuition is the following. A more flexible fiscal rule reduces the weight of the shock on tax revenues in determining the probability of punishment, and increases the weight of the actual policy choices made by the politician. Therefore, the *marginal effect* of running a larger expected deficit on the probability of punishment increases with the degree of flexibility. As a result, a more flexible fiscal rule tends to be more effective in disciplining the politician. Therefore a trade-off between *fiscal discipline* and *flexibility* - often discussed among EU policy makers in recent times - may not always exist.

The result in Proposition 5 captures the key mechanism that shapes the politician's choices. This result holds true even if a less restrictive theoretical environment is assumed. For instance, it is easy to show that Proposition 5 holds even if (i) the fiscal rule is not restricted to belong to a linear family, and (ii) the tax shock is not normally distributed.[15]

4.2 Non-Implementable Case

Suppose that the optimal policy described in Proposition 3 is not implementable through a linearfiscal rule $(0, \delta^*)$, because at least one of the conditions for implementability is not satisfied. We characterize the optimal rule in such case.

[15]The proofs to these extended results are available in the online supplementary material.

Assume $u'(\bar{D}_1 - D_0) \geq 1$.[16] Consider $\hat{\delta} \in [0,1]$ (if it exists) that solves

$$\hat{\delta} = 1 - \frac{D_1^*(0,\hat{\delta}) - D_0}{\sigma_\epsilon}$$

where $(t_1^*, D_1^*)\,(0,\hat{\delta})$ is the optimal policy platform chosen by the politician under fiscal rule $(0,\hat{\delta})$. Notice that

$$\hat{\delta} = 1 - \frac{D_1^*(0,\hat{\delta}) - D_0}{y_1} \Big/ \frac{\sigma_\epsilon}{y_1}$$

i.e. $\hat{\delta}$ is equal to 1 minus the ratio of the intended deficit o the standard deviation of the budget shock (both expressed in terms of ratio to the potential GDP). Define δ^{max} as follows.

$$\delta^{max} = \begin{cases} \hat{\delta} & if \quad exists \\ 0 & otherwise \end{cases}$$

i.e. δ^{max} represents the level of δ that maximizes the expected marginal cost of punishment for the politician at $k = 0$.

Proposition 7. *If the optimal policy is not implementable, then (i) there exists threshold* $\mathring{\sigma}_\epsilon > 0$ *such that if (i)* $\sigma_\epsilon < \mathring{\sigma}_\epsilon$*, then the optimal rule has* $k > 0$*. Conversely, if (ii)* $\sigma_\epsilon \geq \mathring{\sigma}_\epsilon$ *and* $D_1^*(0,\delta^{max})$ *is interior, then the optimal rule is*$(k,\delta) = (0,\delta^{max})$.

Proposition 7 provide a characterization that is far from complete. It suggests that if the benchmark policy is not implementable and the standard deviation of the tax shock is low, then there is room for a fiscal rule that has a deficit target *level* greater than 0 (case (i)). The reason is that in a neighborhood of $k = 0$, a marginal increase in k implies a higher marginal cost of

[16]This assumption ensures that the optimal t_1 is never a corner solution with $t_1 = 0$.

punishment from increasing the level of intended debt for the politician, helping in turn to disciplining her debt choices. This follows the factthat if theintended deficit $D_1 - D_0$ is relatively large, the the probability of violating the rule is very high, and therefore the marginal probability is low (i.e. the point in which the politician is indifferent between compliance and non-compliance islocated on a tail of the probability distribution of the tax shock). Thus, an increase in k helps to reduce the overall probability of punishment, implying in turn a higher marginal probability. The intuition is the following. If the politician is "almost sure" to be punished in any case, then he/she finds optimal to please voters in period 1 as much as possible – i.e. the fiscal rule does not discipline the politician's behavior. This provides a rationale for a "less tight" fiscal rule in presence of a persistent and anticipated bad economic condition. For instance, if a country faces a long-lasting recession, it is conceivable that the politician in charge may expect that the country won't be able to run balanced budget, unless an extremely positive unexpected shock is realized. In such situation, setting $k = 0$ may mean that the politician, being aware of facing an almost-impossible target, may just choose to accept the punishment and maximize the probability of being elected today, by promising very large spending and low taxes. Conversely, a slightly higher k may induce him/her to try to be compliant. Case (ii) instead represents an extreme case of the optimal rule in Proposition 3. Namely, this is a case with "maximum flexibility", i.e. the amount above which further flexibility starts to encourage weak fiscal discipline. Because this level is not enough to achieve the benchmark policy, the outcome induced is going to induce a policy that implies a welfare loss.

5 Conclusion

Appendix

A1 Political Process

Description of the two-candidate electoral competition. In periods 1 and 2. In period 2, all voters have preferences:

$$u_2(t_2, \epsilon) = (1 - t_2) w_2 \bar{l}_2 - v\left(\bar{l}_2\right) + u(g_2(t_2, \epsilon)$$

The government uniformly provides the public good g_b. . Using the definition above, the expected indirect utility of a young individual in period 1 is given by:

$$u_1^Y(t_1, D_1) = \begin{array}{c} (1 - t_1) w_1 l_1(t_1) - v\left(l_1(t_1)\right) + u(g_1(t_1, D_1)) + \\ +\beta \pi E\left[(1 - t_2) w_2 \bar{l}_2 - v\left(\bar{l}_2\right) + u(g_2(t_2, \epsilon) \mid t_1 . D_1\right] \end{array}$$

in which the expectation is taken over ϵ, and expectations are rational. The one of an old individual is given by:

$$u_1^O(t_1, D_1) = (1 - t_1) w_1 l_1(t_1) - v\left(l_1(t_1)\right) + u(g_1(t_1, D_1))$$

I.e. an old voter cares only about consumption of public goods, leisure, and private goods in the current period. [17]

[17]Notice that such features of the population can be easily generated in a model with an exogenous birth process and positive probability of death. Namely, suppose that an additional generation N of size $s^i\sigma$ is born in period 2 and that a share λ^i of the past period young generation survives and becomes the old generation in the following period. This implies that in the second period there are $\lambda^i \vartheta_1^i s^i$ old voters and $s^i \vartheta_1^i \sigma^i$ young voters. Lastly, assume $\sigma^i = \frac{1-\lambda^i \vartheta_1^i}{\vartheta_1^i}$, such that the total size of the population remains constant in the two periods.

Consider now a modified version of Lindbeck and Weibull's (1987) and Banks and Duggan's (2005) probabilistic voting model. In each period and in each country two purely office-seeker politicians - denoted with A and B - compete in elections and wish to maximize the weighted expected return of being in office in the two periods. Elections take place at the beginning of period 1 and 2, and politicians derive a positive payoff from being in office. Namely, in period 1 the politician in office enjoys a rent $W_1 > 0$. In period 2, the politician in office enjoys a rent W_2^{nv}, where the subcript stands for no violation. If a deficit rule is in place, and a violation of the rule occurs, then the politician's rent is W_2^v, with $W_2^v < W_2^{nv}$. The idea is that the punishment imposed to the politician in case of a violation of the deficit rule is a form of control on her actions that constrains her ability to extract rent from her office.

Define the share of type $T \in \{Y, O\}$ voters that vote for candidate A in period 1 as:

$$\Pi_1^{A,T}(t_1^A, D_1^A, t_1^B, D_1^B) = H_1^T\left(u^T(t_1^A, D_1^A) - u^T(t_1^B, D_1^B) + \nu_1\right),$$

where H_1^T is increasing, $\frac{\partial H_1^T(x)}{\partial x} = h_1^T(x)$ and $h_1^T(0) = \bar{h}_1$ for any $T \in \{Y, O\}$[18].

The share of young voters in period 2 is given by

$$\vartheta_2^i = \frac{\vartheta_1^i \sigma^i}{\vartheta_1^i \sigma^i + \lambda^i \vartheta_1^i} = \frac{\sigma^i}{\sigma^i + \lambda^i}$$

This means that the share of elderly voters in the economy increases between the two periods if $\lambda^i \geq \frac{1-\vartheta_1^i}{\vartheta_1^i}$, and decreases otherwise.

[18]Notice that, as in Duggan 2005, $H_1^T(\bar{x}_1 + \nu_1)$ can be interpreted also as the probability of a voter of type T to vote for politician A conditional on ν_1 and $x = \bar{x}_1$. This is true because, if the number of voters is arbitrarily large, then the law of large numbers implies that the share of votes for candidate B denoted with $H_1^T(\bar{x}_1 + \nu_1)$ becomes *exactly* equal to such probability. To see this, define an i.i.d. random variable ξ_j with c.d.f. $H_1^T(\cdot)$ and p.d.f. $h_1^T(\cdot)$. Say an individual j vote for B if $-\bar{x}_1 - \nu_1 + \xi_j \geq 0$, which means that $Pr(B|\,\bar{x}_1 + \nu_1) = \int_{x+\nu}^{\infty} h_1^T(\xi)d\xi = 1 - H_1^T(\bar{x}_1 + \nu_1)$ and therefore $Pr(A|\,\bar{x}_1 + \nu_1) = H_t^T(\bar{x}_t + \nu_1)$. The share of votes for candidate A for a voting population of size n is given by $\frac{\sum_{j=1}^n \mathbf{1}(\xi_j \leq \bar{x} + \nu)}{n}$. The law of large numbersimplies that $\lim_{n\to\infty} \frac{\sum_{j=1}^n \mathbf{1}(\xi_j \leq \bar{x} + \nu)}{n} = E_\xi\left[\mathbf{1}(\xi_j \leq \bar{x} + \nu)\right] = \int_{-\infty}^{\bar{x}+\nu} h_1^T(\xi)d\xi = H_1^T(\bar{x} + \nu)$, implying that, for an arbitrarily large number of voters, $H_1^T(\bar{x} + \nu)$ is both the share of votes for A and the probability of A winning given ν. As a consequence, the uncertainty in the electoral outcome is entirely due to the common shock ν_1. This implies in turn that for a large electorate

The first assumption implies that the share of votes for candidate A is weakly increasing in the utility difference induced by the policies proposed by candidate A and candidate B (standard). The second assumption states the two types of voters do not have ex-ante asymmetric preferences for the two candidates (for instance, one could assume $\bar{h} = .5$, implying that if the two candidates propose platforms that induce the same utility in both type of voters, then the expected share of votes for each candidate is .5 for each type of voter). Moreover, ν_b is a continuous i.i.d. random variable with c.d.f. $G_b(a)$. The random variable ν_b represents a random realization of some shift in voters' behavior due to circumstances that cannot be foreseen by the candidates and it is common to all voters. Thus, the share of vote for candidate A in the whole population of voters in period 1 is

$$\Pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = \begin{array}{c} \vartheta_1 H_1^Y \left(u^Y(t_1^A, D_1^A) - u^Y(t_1^B, D_1^B) + \nu_1\right) + \\ +(1-\vartheta_1) H_1^O \left(u^O(t_1^A, D_1^A) - u^O(t_1^B, D_1^B) + \nu_1\right). \end{array}$$

The probability of victory for candidate A vs B in period 1 is:

$$\pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = \begin{array}{c} Pr\left[\vartheta H_1^Y \left(u^Y(t_1^A, D_1^A) - u^Y(t_1^B, D_1^B) + \nu_1\right)\right. \\ \left. +(1-\vartheta) H_1^O \left(u^O(t_1^A, D_1^A) - u^O(t_1^B, D_1^B) + \nu_1\right) \geq .5\right], \end{array}$$

while the probability of victory for candidate B in period 1 is simply $\pi_1^B(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = 1 - \pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta)$. Similarly, the probability of victory for candidate A vs B in period 2 is denoted with $\pi_2^A(t_2^A, t_2^B, t_1, D_1, \epsilon)$. Politician A in period 1 maximizes her expected payoff, which is given by:

the presence of a common shock ν_1 is necessary to have probabilistic voting. Without ν_1 the electoral outcome would be deterministic for all values of x, except the exact point in which $H_1^T(\bar{x}) = .5$ and the two candidates win with equal probability.

$$P\left(t_1^A, D_1^A, t_1^B, D_1^E, \vartheta_1, W_1, W_2^{nv}, W_2^v\right) = W_1 \pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) +$$
$$\beta W_2^v \int_{\underline{\epsilon}}^{\tilde{\epsilon}} \pi_2^A(\hat{t}_2^A, \hat{t}_2^B, t_1, D_1, \epsilon) f(\epsilon)\, d\epsilon$$
$$+\beta W_2^{nv} \int_{\tilde{\epsilon}}^{\infty} \pi_2^A(\hat{t}_2^A, \hat{t}_2^B, t_1, D_1, \epsilon^i) f(\epsilon)\, d\epsilon,$$

where $\tilde{\epsilon}$ is the threshold below which the politician gets "punished" (see previous section) and $\hat{t}_2^A, \hat{t}_2^B$ are the rational expectation values of t_2^A, t_2^B conditional on t_1, D_1, ϵ. H_1^Y, H_1^O and H_2 represent the probability that voters of type T vote for candidate A. One can easily derive the expected payoff of candidate B by noticing that the share of voters of type T that vote for candidate C is $1 - H^T$. Lastly, define the weight θ_2 as follows:

$$\theta_2 = \frac{\vartheta_2 h_2^Y(0)}{\vartheta_2 h_2^Y(0) + (1-\vartheta_2) h_2^O(0)}$$

Now we can state the following proposition.

Proposition A1. *In period 2 both candidates propose the same platform. Such platform is the same as the policy chosen by a social planner that assigns weight θ_2 to voters of type Y and $(1-\theta_2)$ to voters of type O. If $h_2^Y(0) = h_2^O(0) = \bar{h}_2$, then $\theta_2^i = \vartheta_2^i$, i.e. the policy proposed in equilibrium is the one that would be chosen by a social planner that maximizes the expected utility of voters in period 2* [19].

[19] Notice that this setting is different from an utilitarian social planner, because there is a potential loss from defaults of foreign lenders which is not kept into account. Moroever, because the default choice is discrete, the identity between the equilibrium condition of such kind of planner and the one of the politician can be proved only under a second-order Taylor approximation. With the introduction of a normal distribution for the shock in section 3, the identity becomes *exactly* true.

Proof. The probability of victory for candidate A vs B in period 1 is:

$$\pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = \begin{array}{c} Pr\left[\vartheta H_1^Y\left(u^Y(t_1^A, D_1^A) - u^Y(t_1^B, D_1^B) + \nu_1\right)\right. \\ \left.+(1-\vartheta)H_1^O\left(u^O(t_1^A, D_1^{E,A}) - u^O(t_1^B, D_1^B) + \nu_1\right) \geq .5\right] \end{array}$$

which writes:

$$\pi_1^A(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta) = 1 - G\left[\tilde{\eta}_1(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta)\right]$$

where $\tilde{\eta}_1(t_1^A, D_1^A, t_1^B, D_1^B, \theta)$ solves:

$$\vartheta H_1^Y\left(u^Y(t_1^A, D_1^A) - u^Y(t_1^B, D_1^B) + \tilde{\nu}_1\right) + (1-\vartheta)H_1^O\left(u^O(t_1^A, D_1^A) - u^O(t_1^B, D_1^B) + \tilde{\nu}_1\right) - .5 = 0$$

Notice that such $\tilde{\nu}_1(t_1^A, D_1^A, t_1^B, D_1^B, \vartheta)$ that solves the above always exists as long as $u^T(t_1^A, D_1^A) - u^T(t_1^B, D_1^B)$ only takes finite values. Similarly, the share of vote for candidate A in period 2 is:

$$\Pi_2^A(t_2^A, D_2^A, t_2^B, D_2^B, t_1, D_1^E, \epsilon^i) = \begin{array}{c} \vartheta_2 H_2^Y\left(u_2^Y(t_2^A, D_2^A, t_1, D_1, \epsilon) - u_2^Y(t_2^B, D_2^B, t_1, D_1, \epsilon) + \nu_2\right) + \\ (1-\vartheta_2)H_2^O\left(u_2^O(t_2^A, D_2^A, t_1, D_1, \epsilon) - u_2^O(t_2^B, D_2^B, t_1, D_1, \epsilon) + \nu_2\right) \end{array}$$

The probability of victory for A vs B in period 2 (conditional on previous choices t_1, D_1 and on the realization of the shock ϵ) is:

$$\pi_2^A(t_2^A, t_2^B, t_1, D_1, \epsilon) = 1 - G\left[\tilde{\eta}_2(t_2^A, t_2^B, t_1, D_1, \epsilon)\right]$$

where $\tilde{\eta}_2(t_2^A, t_2^B, t_1, D_1, \epsilon^1)$ solves

$$\begin{array}{c} \vartheta_2 H_2^Y\left(u_2^Y(t_2^A, t_1, D_1, \epsilon) - u_2^Y(t_2^B, , t_1, D_1, \epsilon) + \tilde{\eta}_2\right) + \\ (1-\vartheta_2)H_2^O\left(u_2^O(t_2^A, t_1, D_1, \epsilon) - u_2^O(t_2^B, t_1, D_1, \epsilon) + \tilde{\eta}_2\right) - .5 = 0 \end{array}$$

In period 2, politician A solves (independently of a previous violation of the rule, i.e. $s \in \{v, nv\}$):

$$\max_{t_2,d_2} W_2^s \left\{1 - G_2\left[\tilde{\nu}_2(t_2, \bar{t}_2^B, t_1, D_1, \epsilon)\right]\right\}$$

while politician B solves

$$\max_{t_2,d_2} W_2^s G_2\left[\tilde{\nu}_2(\bar{t}_2^A, , t_2, t_1, D_1, \epsilon)\right]$$

Because it is a zero-sum game, at any equilibrium each player must reach payoff of exaclty $\overline{V} = W_2^s \{1 - G_2[\tilde{\nu}_2(\bar{t}_2, \bar{t}_2, t_1, D_1, \epsilon)]\}$ and $1-\overline{V} = W_2^s G[\tilde{\nu}_2(\bar{t}_2, \bar{t}_2, t_1, D_1, \epsilon)]$ respectively, where $\tilde{\nu}_2(\bar{t}_2, \bar{t}_2, t_1, D_1, \epsilon)$ is the $\tilde{\nu}_2$ if the two politicians propose the exact same platform. The proof is simple. Suppose politiican A gets a payoff lower than $\overline{V}$ in an equilibrium. Then he could gain a strictly higher payoff $\overline{V}$ simply switching to a platform $(t_2) = (\bar{t}_2^B)$. Similarly, if A gets a payoff higher than $\overline{V}$, then politician B is getting less than $1 - \overline{V}$, which implies that he could obtain a strictly higher payoff $1 - \overline{V}$ swithcing to a platform $(t_2) = (\bar{t}_2^A)$. Lastly, suppose there is an action profile such that $(t_2^A) \neq (t_2^B)$, and such that the payoff of politician A is exactly $\overline{V}$. Recall that he could obtain payoff $\overline{V}$ also with a platform $(t_2) = (\bar{t}_2^B)$. This implies that the maximum of politician A's objective function given $(\bar{t}_2^B)$ is not unique. But this is impossible because we have assumed a strictly concave objective function that is maximized over a compact set. Thus, in any equilibrium it must be true that $(t_2^A) = (t_2^B)$. Now we characterize the equilibrium outcome.

The problem is

$$\max_{t_2} W_2^s \left\{1 - G_2\left[\tilde{\nu}_2(t_2, \bar{t}_2^B, t_1, D_1, \epsilon)\right]\right\}$$

Notice that

$$\frac{\partial \tilde{\nu}_2^A}{\partial t_2} = -\frac{\vartheta_2 h_2^Y (u_2^Y - \bar{u}_2^{Y,B}) \frac{\partial}{\partial t_2}\left[u_2^Y\right] + (1-\vartheta_2) h_2^O (u_2^O - \bar{u}_2^{O,B}) \frac{\partial}{\partial t_2}\left[u_2^O\right]}{\vartheta_2 h_2^Y (u_2^Y - \bar{u}_2^{Y,B}) + (1-\vartheta_2) h_2^O (u_2^O - \bar{u}_2^{O,B})}$$

and

$$\frac{\partial \tilde{\nu}_2^A}{\partial D_2} = -\frac{\vartheta_2 h_2^Y (u_2^Y - \bar{u}_2^{Y,B}) \frac{\partial}{\partial D_2}\left[u_2^Y\right] + (1-\vartheta_2) h_2^O (u_2^O - \bar{u}_2^{O,B}) \frac{\partial}{\partial D_2}\left[u_2^O\right]}{\vartheta_2 h_2^Y (u_2^Y - \bar{u}_2^{Y,B}) + (1-\vartheta_2) h_2^O (u_2^O - \bar{u}_2^{O,B})}$$

hence the FOC is:

$$[t_2]: \; -W_2^k g_2(\tilde{\nu}_2^A) \frac{\partial \tilde{\nu}_2^A}{\partial t_2} = 0$$

where $\tilde{\nu}_2^A = \tilde{\nu}_2(t_2, \bar{t}_2^B, t_1, D_1, \epsilon)$. Similarly candidate B solves:

$$\max_{t_2} W_2^k G_2 \left[\tilde{\nu}_2(\bar{t}_2^A, t_2, t_1, D_1, \epsilon)\right]$$

hence

$$[t_2]: \; W_2^k g_2(\tilde{\nu}_2^B) \frac{\partial \tilde{\nu}_2^B}{\partial t_2} = 0$$

where

$$\frac{\partial \tilde{\nu}_2^B}{\partial t_2} = \frac{\vartheta_2 h_2^Y (\bar{u}_2^{Y,A} - u_2^Y) \frac{\partial}{\partial t_2}\left[u_2^Y\right] + (1-\vartheta_2) h_2^O (\bar{u}_2^{O,A} - u_2^O) \frac{\partial}{\partial t_2}\left[u_2^O\right]}{\vartheta_2 h_2^Y (\bar{u}_2^{Y,A} - u_2^Y) + (1-\vartheta_2) h_2^O (\bar{u}_2^{O,A} - u_2^O)}$$

$$\frac{\partial \tilde{\nu}_2^B}{\partial D_2} = \frac{\vartheta_2 h_2^Y (\bar{u}_2^{Y,A} - u_2^Y) \frac{\partial}{\partial D_2}\left[u_2^Y\right] + (1-\vartheta_2) h_2^O (\bar{u}_2^{O,A} - u_2^O) \frac{\partial}{\partial D_2}\left[u_2^O\right]}{\vartheta_2 h_2^Y (\bar{u}_2^{Y,A} - u_2^Y) + (1-\vartheta_2) h_2^O (\bar{u}_2^{O,A} - u_2^O)}$$

Notice that the second period voting game is a standard Probabilistic voting game, and exhibits the usual well-known result. Specifically, in equilibrium $\tilde{\nu}_2^A = \tilde{\nu}_2^B = \tilde{\nu}_2^*$ and $\bar{u}_2^{Y,A} = u_2^{Y,A}$, $\bar{u}_2^{O,A} = u_2^{O,A}$, $\bar{u}_2^{Y,B} = u_2^{Y,B}$, $\bar{u}_2^{O,B} = u_2^{O,B}$, which implies in turn that

$$\frac{\partial \tilde{\nu}_2^A}{\partial t_2^A} = -\frac{\partial \tilde{\nu}_2^B}{\partial t_2^B}$$

and

$$\frac{\partial \tilde{\nu}_2^A}{\partial D_2^A} = -\frac{\partial \tilde{\nu}_2^B}{\partial D_2^B}$$

Using this results in the FOCs of the two candidates, this implies that the FOCs are identical. Because the objective function is strictl concave, there is a unique equilibrium in which both candidates propose the same policy, i.e. $t_2^A = t_2^B = t_2^*$. Therefore, as long as the function g_2 is everywhere positive, the FOC of both candidates are satisfied for

$$[t_2]: \ -W_2^k g_2(\tilde{\nu}_2^J)\left\{\theta_2 \frac{\partial}{\partial t_2}\left[u_2^Y\right] + (1-\theta_2)\frac{\partial}{\partial t_2}\left[u_2^O\right]\right\} = 0$$

with $J \in \{A, B\}$ and

$$\theta_2 = \frac{\vartheta_2 h_2^Y(0)}{\vartheta_2 h_2^Y(0) + (1-\vartheta_2)h_2^O(0)}$$

But this condition is the same as for the one one gets for a planner that maximizes an utilitarian social welfare function, namely:

$$\max_{t_2} \theta_2 u_2^Y(t_2, t_1, D_1, \epsilon) + (1-\theta_2)u_2^O(t_2, t_1, D_1, \epsilon)$$

Also notice that whenever $h_2^Y(0) = h_2^O(0)$ (i.e. young and old voters do not have different preferences toward sthe candidates), then $\theta_2 = \vartheta_2$.This means that in such case the politicians maximize the utilitarian social welfare function. Therefore, the level of t_2 chosen by both candidates - both in presence of a reputational cost or not - is the one that maximize a weighted average of the population utility conditional on previous choices t_1, D_1. This result is true for all values of t_1, D_1 and ϵ. Therefore:

$$t_2^A = t_2^B = t_2^*(t_1, D_1, \epsilon) = \arg\max_{t_2} \theta_2 u_2^Y(t_2, t_1, D_1, \epsilon) + (1-\theta_2)u_2^Y(t_2, t_1, D_1, \epsilon)$$

and this proves the proposition. Q.E.D.

Notice that, because $C^i = W_2^{nv} - W_2^v$ is a cost for the politician only, it will not affect the normative analysis in the following sections. Now define a cost c^i as follows:

$$c^i = \frac{0.5C^i}{W_1 g_1(\bar{\nu}_1)}$$

where $\bar{\eta}_1$ solves

$$\vartheta_1 H_1^Y(\bar{\eta}_1) + (1-\vartheta_1) H_1^O(\bar{\eta}_1) - .5 = 0$$

i.e. it is the value of ν_1 that implies a tie between the two candidates if they both propose the same platform. For instance, if $H_1^Y(0) = H_1^O(0) = .5$, then $\bar{\nu}_1 = 0$.

Proposition A2. *In period 1 both candidates propose the same platform. Such platform maximizes the weighted expected utility of period 1 voters - with weight θ_1 to voters of type Y and $(1-\theta_1)$ to voters of type O -, minus the expected cost $\beta c F(\tilde{\epsilon})$ of violating the deficit rule in the following period. If $h_1^Y(0) = h_1^O(0) = \bar{h}_1$, then $\theta_1 = \vartheta_1$, i.e. the policy proposed in equilibrium is the one that would be chosen by a social planner that maximizes the expected utility of country i's period* 1 *voters facing a cost c in the event in which a deficit rule is violated in the following period.*

Proof. Both politicians and voters can fully anticipate the outcome in period 2 conditional on the choices made in period 1 and the realization of the shock. Recall $\tilde{\nu}_2^* = \tilde{\nu}_2(g_2^{A*}, g_2^{B*}, t_1, D_1, \epsilon^1) = \bar{\nu}_2$. Lastly, use the assumption $G_2[H_2^{-1}(.5)] = .5$ and notice that it implies $G_2(\tilde{\nu}_2(\bar{g}_2, \bar{g}_2, t_1, D_1, \epsilon)) = .5$. Thus the objective function of a politician simplifies and the problem of a politician is:

$$\max_{t_1,D_1} \left\{1 - G_1\left[\tilde{\nu}_1(t_1, D_1, \bar{t}_1^B, \bar{D}_1^B, \vartheta)\right]\right\} W_1 + 0.5\beta\left[W_2^{nv} - CF\left(\tilde{\epsilon}\right)\right]$$

FOCs are:

$$[t_1]: \quad \begin{array}{c} W_1 g_1(\tilde{\nu}_1^A)\left[\frac{\vartheta h_1^Y\left(u^Y - \bar{u}^Y\right)\frac{\partial}{\partial t_1}\left[u^Y(t_1,D_1)\right] + (1-\vartheta)h_1^O\left(u^Y - \bar{u}^Y\right)\frac{\partial}{\partial t_1}\left[u^O(t_1,D_1)\right]}{\vartheta h_1^Y(u^Y - \bar{u}^Y) + (1-\vartheta)h_1^O(u^Y - \bar{u}^Y)}\right] + \\ -0.5\beta C f\left(\tilde{\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial t_1} = 0 \end{array}$$

Similarly:

$$[D_1]: \quad \begin{array}{c} W_1 g_1(\tilde{\nu}_1^A)\left[\frac{\vartheta h_1^Y\left(u^Y - \bar{u}^Y\right)\frac{\partial}{\partial D_1}\left[u^Y(t_1,D_1)\right] + (1-\vartheta)h_1^O\left(u^Y - \bar{u}^Y\right)\frac{\partial}{\partial D_1}\left[u^O(t_1,D_1)\right]}{\vartheta h_1^Y(u^Y - \bar{u}^Y) + (1-\vartheta)h_1^O(u^Y - \bar{u}^Y)}\right] + \\ -0.5\beta C^i f\left(\tilde{\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial D_1} = 0 \end{array}$$

Similarly, candidate B solves

$$\max_{t_1,D_1} \left\{G_1\left[\tilde{\nu}_1(\bar{t}_1^A, \bar{D}_1^A, t_1, D_1, \theta)\right]\right\} W_1 + 0.5\beta\left[W_2^{nv} - CF\left(\tilde{\epsilon}\right)\right]$$

FOCs are:

$$[t_1]: \quad \begin{array}{c} W_1 g_1(\tilde{\nu}_1^B)\left[\frac{\vartheta h_1^Y\left(\bar{u}^Y - u^Y\right)\frac{\partial}{\partial t_1}\left[u^Y(t_1,D_1)\right] + (1-\vartheta)h_1^O\left(\bar{u}^Y - u^Y\right)\frac{\partial}{\partial t_1}\left[u^O(t_1,D_1)\right]}{\vartheta h_1^Y(\bar{u}^Y - u^Y) + (1-\vartheta)h_1^O(\bar{u}^Y - u^Y)}\right] + \\ -0.5\beta C^i f\left(\tilde{\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial t_1} = 0 \end{array}$$

Similarly:

$$[D_1]: \quad \begin{array}{c} W_1 g_1(\tilde{\nu}_1^B)\left[\frac{\vartheta h_1^Y\left(\bar{u}^Y - u^Y\right)\frac{\partial}{\partial D_E}\left[u^Y(t_1,D_1)\right] + (1-\vartheta)h_1^O\left(\bar{u}^Y - u^Y\right)\frac{\partial}{\partial D_E}\left[u^O(t_1,D_1)\right]}{\vartheta h_1^Y(\bar{u}^Y - u^Y) + (1-\vartheta)h_1^O(\bar{u}^Y - u^Y)}\right] + \\ -0.5\beta C f\left(\tilde{\epsilon}\right)\frac{\partial \tilde{\epsilon}}{\partial D_1} = 0 \end{array}$$

If the objective function is strictly concave, then there is a unique equilibrium (see Duggan 2005). I will show later under which assumption this condition is satisfied. Notice that in any equilibrium it must be that $\tilde{\nu}_1^A = \tilde{\nu}_1^B = \tilde{\nu}_1^*$. Thus, there is asymmetric equilibrium in which both candidates propose the exact same policy (t_1^*, D_1^*). To see that, suppose both candidates propose the same policy vector. In such case, $\tilde{\nu}_1^A = \tilde{\nu}_1^B$ and, defining

$$\theta_1 = \frac{\vartheta_1 h_1^Y(0)}{\vartheta_1 h_1^Y(0) + (1-\vartheta_1) h_1^O(0))}$$

Then the FOCS simplify to:

$$[t_1]: \; W_1 g_1(\tilde{\nu}_1^T) \left[\theta_1 \frac{\partial}{\partial t_1}\left[u^Y(t_1, D_1)\right] + (1-\theta_1)\frac{\partial}{\partial t_1}\left[u^O(t_1, D_1)\right]\right] - 0.5\beta C f(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial t_1} = 0$$

and:

$$[D_1]: \; W_1 g_1(\tilde{\nu}_1^T) \left[\theta_1 \frac{\partial}{\partial D_1}\left[u^Y(t_1, D_1)\right] + (1-\theta_1)\frac{\partial}{\partial D_1}\left[u^O(t_1, D_1)\right]\right] - 0.5\beta C f(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial D_E} = 0$$

For candidate $T \in \{A, B\}$. If the equilibrium is unique (see before) , then this is the only policy observed. Using $\tilde{\eta}_1^A = \tilde{\eta}_1^B = \tilde{\eta}_1^* = \bar{\eta}_1$ one can rewrite the equilibrium conditions as follows:

$$[t_1]: \; \left[\theta_1 \frac{\partial}{\partial t_1}\left[u^Y(t_1, D_1)\right] + (1-\theta_1)\frac{\partial}{\partial t_1}\left[u^O(t_1, D_1)\right]\right] + \beta \underbrace{\frac{0.5C}{W_1 g_1(\bar{\nu}_1)}}_{c^i} f(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial t_1} = 0$$

and

$$[D_1]: \; \left[\theta_1 \frac{\partial}{\partial D_1}\left[u^Y(t_1, D_1)\right] + (1-\theta_1)\frac{\partial}{\partial D_1}\left[u^O(t_1, D_1)\right]\right] - \beta \underbrace{\frac{0.5C}{W_1 g_1(\bar{\nu}_1)}}_{c} f(\tilde{\epsilon}) \frac{\partial \tilde{\epsilon}}{\partial D_1} = 0$$

Notice that the FOCs above are the same as the one of an individual maximizing:

$$\max_{t_1, D_1} \theta_1 u^Y(t_1, D_1) + (1-\theta_1) u^Y(t_1, D_1) - \beta c F(\tilde{\epsilon}) \tag{19}$$

Also notice that, if $h_1^Y(0) = h_1^O(0)$, which would be the case for instance if

$H_1^Y(\cdot) = H_1^O(\cdot)$, then $\theta_1 = \vartheta_1$, i.e. the politician maximize the expected utility of her voters corrected for a cost associated to the probability of violating the rule. Q.E.D.

Appendix B

Proposition 1. *The expected deficit of each country i is increasing in the political present bias $(1 - \pi\theta_1)$.*

Proof. First we need to consider the problem of the elected politician in period 2. From the previous section we know that in period 2, the problem is equivalent to the one of social planner that maximize voters' expected utility. Consider first the problem of a politician that chooses not to default. The problem is:

$$\max_{t_2 \in [0,1]} \left[(1 - t_2) w_2 \bar{l}_2 - v(\bar{l}_2) + u\left(g_2(t_2, \epsilon)\right) \right]$$

where

$$g_2(t_2, \epsilon) = t_2 w_2 \bar{l}_2 - (D_1 - \epsilon)(1 + r)$$

Define the tax elasticity of labor supply as:

$$\eta_b(t_b) = \frac{\partial l_b^*}{\partial t_b} \frac{t_b}{l_b^*} = -\frac{w_b t_b}{v''(l_b^*) l_b^*}$$

The FOC implies:

$$[t_2]: \; w_2 \bar{l}_2 \left\{ -1 + u'\left(g_2\right) \right\} = 0$$

Notice that the above equations imply:

$$g_2 = u'^{-1}\left(1\right) = \bar{g}_2$$

which implies that g_2 is independent of D_1 (this is a consequence of linearity). And therefore the problem in period 1 can be rewritten as follows:

$$\max_{(t_1,D_1)\in X} \begin{array}{l} (1-t_1)w_1l_1^* - v(l_1^*) + u(g_1) - \beta cF\left(\frac{D_1-D_0-kw_1l_1^*}{1-\delta}\right) + \\ +\beta\pi\theta_1\left\{w_2l_2^* - \bar{g}_2 - v(l_2^*) + u\left(\bar{g}_2\right) - D_1(1+r)f(\epsilon)d\epsilon\right\} \end{array}$$

Calculate the FOCs w.r.t. t_1 and D_1:

$$[t_1]:\ -w_1l_1 + u'(g_1)w_1l_1[1+\eta_1(t_1)] + \beta\frac{ckw_1l_1}{(1-\delta)t_1}\eta_1(t_1)f\left(\frac{D_1-D_0-kw_1l_1^*}{1-\delta}\right) = 0$$

Notice that the punishment c implies a distortion on t_1 unless $k=0$, hence we will seek solution that do not distort taxation. If t_1 and D_1 are set optimally, this means the last addendum must be made equal to zero by setting $k=0$.

$$[D_1]:\ u'(g_1(t_1,D_1)) - \beta\pi\theta_1(1+r) - \frac{\beta c}{1-\delta}f\left(\frac{D_1-D_0-kw_1l_1^*}{1-\delta}\right) = 0$$

Notice that $\hat{\epsilon}$ is a function of D_1, R, r. Use Monotone Comparative Statics:

$$[t_1,\theta_1]:=0$$

$$[D_1,\theta_1]:=-\beta\pi\theta_1(1+r)\leq 0$$

$$[t_1,D_1]:=u''(g_1(t_1,D_1))w_1l_1[1+\eta_1(t_1)] + \frac{\beta ckw_1l_1}{(1-\delta)^2t_1}\eta_1(t_1)f'\left(\frac{D_1-D_0-kw_1l_1^*}{1-\delta}\right)$$

Notice that the sign of $[t_1,D_1]$ is ambiguous. Neverthless, because $[t_1,\theta_1]:=0$, the sign of the comparative statics is unambiguous: D_1 is weakly decreasing in θ_1. Q.E.D.

$$[t_1,\delta] := \begin{array}{c} \frac{\beta c k w_1 l_1}{(1-\delta)^2 t_1}\eta_1(t_1) f\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right) + \\ -\frac{\beta c k w_1 l_1}{(1-\delta)^3 t_1}\eta_1(t_1) f'\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right)(D_1-D_0-kw_1 l_1^*) \leq 0 \end{array}$$

negative under the assumption $-\frac{f'(\epsilon)\epsilon}{f(\epsilon)} \leq 1$, and

$$[D_1,\delta] := -\frac{\beta c}{(1-\delta)^2} f\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right) - \frac{\beta c}{(1-\delta)^3} f'\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right)(D_1-D_0-kw_1 l_1^*) \leq 0$$

negative under the assumption $-\frac{f'(\epsilon)\epsilon}{f(\epsilon)} \leq 1$. Traditional Comparative Statics w.r.t. D_1, δ in case of uniform distribution

$$-\frac{1}{[D_1,D_1]}\left\{-\frac{\beta c}{(1-\delta)^2} f(\hat{\epsilon}) + u''(g_1(t_1,D_1)) w_1 l_1 [1+\eta_1(t_1)] \frac{\beta c k w_1 l_1}{(1-\delta)^2 t_1}\eta_1(t_1) f(\hat{\epsilon})\right\}$$

Notice that as $k \to 0$ the above is negative. Increasing flexibility reduces expected debt. The intuition is that an increase in flexibility makes the effect of a marginal increase in D_1 on the expected punishment larger. This traditional comparative statics will be useful to achieve the normative benchamrk in the next session. Comparative statics w.r.t. k

$$[t_1,k]: \frac{\beta c w_1 l_1}{(1-\delta) t_1}\eta_1(t_1) f\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right) - \beta \frac{c k w_1^2 l_1^2}{(1-\delta)^2 t_1}\eta_1(t_1) f'\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right)$$

negative in the proximity of $k = 0$.

$$[D_1,k]: \frac{\beta c w_1 l_1^*}{(1-\delta)^2} f'\left(\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}\right)$$

Ambiguous sign: increasing for low $\frac{D_1-D_0-kw_1 l_1^*}{1-\delta}$. Idea, k is not a good instrument to adjust D_1. It should just be set to $k = 0$ to ensure that taxes are not distorted, and then use the paramenter δ to adjust the level of expected debt. Q.E.D.

Proposition 3. *The optimal policy* (t_1^*, D_1^*) *is implementable if the following conditions hold:*

(i) The taste shock has enough variance: $\sigma_\nu \geq \bar{\sigma}_\nu$ *for some* $\bar{\sigma}_\nu \in (0, \infty)$*;*

(ii) The tax shock has enough variance: $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ *for some* $\bar{\sigma}_\epsilon \in (0, \infty)$*.*

Proof. First I must derive the condition for the optimal choice of the social planner. This planner can decide D_1 and t_1 optimally (no need of the deficit rule). From formula (1) , the social planner's problem writes:

$$\max_{(t_1,D_1)\in X} \begin{array}{c} (1-t_1)w_1 l_1(t_1) - v(l_1) + u(g_1(t_1,D_1)) + \\ +\beta E\left[(1-t_2)w_2 l_2(t_2) - v(l_2) + u(t_2,d_2) \mid t_1, D_1\right] \end{array}$$

subject to

$$t_1 w_1 l_1 - g_1 - D_0(1+r) + D_1 \geq 0$$

In period 2, the platform chosen is the same as the one of the politician, which corresponds to the one of a planner that maximizes the sum of voters utilities. Thus, the problem in period 1 can be rewritten as follows:

$$\max_{(t_1,D_1)\in X} \begin{array}{c} (1-t_1)w_1 l_1(t_1) - v(l_1) + u(g_1(t_1,D_1)) + \\ +\beta\left\{w_2 l_2^* - \bar{g}_2 - v(l_2^*) + u\left(\bar{g}_2\right) - (1+r)\int_{-a}^{a}(D_1-\epsilon)f(\epsilon)d\epsilon\right\} \end{array}$$

Calculate the FOCs:

$$\left[t_1^{SP}\right]: \ -w_1 l_1 + u'(g_1)w_1 l_1[1+\eta_1(t_1)] = 0$$

$$\left[D_1^{SP}\right]: \quad u'(g_1(t_1,D_1)) - \beta(1+r) = 0$$

Notice that the punishment c implies a distortion on $t_1$unless $k = 0$ (and D_1 is set optimally), hence we will seek solution that do not distort taxation. If t_1 and D_1 are set optimally, then g_1 also is: this means the last addendum must be made equal to zero by setting $k = 0$. Now recall that the necessary condition for optimality of the politicians are:

$$[t_1]: \; -w_1 l_1 + u'(g_1) w_1 l_1 [1 + \eta_1(t_1)] + \beta \frac{ckw_1 l_1}{(1-\delta)t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)$$

$$[D_1]: \; u'(g_1(t_1, D_1)) - \beta\pi\theta_1(1+r) - \frac{\beta c}{1-\delta} f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right) = 0$$

For a convex objective finction a necessary condition for implementability is that both sets of FOCs are satisfied at (t_1^*, D_1^*). Thus, one can just verify if the difference betweeneach pair of FOCs is zero for some set of paramenters k, δ. If the objective function is strictly concave, this ensures that the desired outcome is implemented. Thus:

$$[D_1] - \left[D_1^{SP}\right] \; = \beta(1 - \pi\theta_1)(1+r) - \tfrac{\beta c}{1-\delta} f\left(\tfrac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)$$

$$[t_{1,}] - \left[t_1^{SP}\right] = \beta \frac{ckw_1 l_1}{(1-\delta)t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)$$

I.e. if no punishment is implemented, $c = 0$, then the politician chooses excessive debt because $[D_1] - \left[D_1^{SP}\right] > 0$ and $[t_1] - \left[t_1^{SP}\right] = 0$. If a cost c is introduced, $[D_1, c] - [D_1, 0] < 0$ and $[t_1, c] - [t_1, 0] < 0$ unless $k = 0$. By setting $k = 0$ one can offset $[t_1] - \left[t_1^{SP}\right]$ (this does not mean t_1 is for sure optimal, because the F.O.C. w.r.t. t_1 is a function of D_1). The benchmark can be reached if $\beta(1 - \pi\theta_1)(1+r) - \frac{\beta c}{1-\delta} f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right) = 0$ for some $\delta \in [0, 1)$.

Then, for sufficiency we need to establish whether the objective function is concave (at $k = 0$).

$$[D_1 D_1]: \; u''(g_1(t_1, D_1)) - \frac{\beta c}{(1-\delta)^2}\left[f'\left(\frac{D_1 - D_0}{1-\delta}\right)\right] < 0$$

$$u''(g_1(t_1, D_1)) - \frac{\beta c}{(1-\delta)^2}\left[f'\left(\frac{D_1 - D_0}{1-\delta}\right)\right] < 0$$

$$u''(g_1(t_1, D_1)) + \frac{\beta c(D_1 - D_0)}{(1-\delta)^3\sqrt{2\pi}\sigma_\epsilon^3}\left[\exp\left(-\frac{1}{2\sigma_\epsilon^2}\left(\frac{D_1 - D_0}{1-\delta}\right)^2\right)\right] < 0$$

Lastly, recall that $u''(g) < 0$ for all $g \geq 0$. Thus, for any finite $D_1 - D_0$ and threshold $\bar{\delta} < 1$, there exists threshold $\widehat{c(\sigma_\nu)/\sigma_\epsilon^3}$ such that if $c(\sigma_\nu)/\sigma_\epsilon^3 \leq \widehat{c(\sigma_\nu)/\sigma_\epsilon^3}$ then the objective function is convex.

Now consider the case in which the preference shocks ν_1 and the shocks on tax revenues ϵ are i.i.d. and distributed as follows: $\nu_1 \sim N(\mu_\nu, \sigma_\nu^2)$, and ϵ is truncated-normal as described in section 4.1. Denote with $A = \frac{D_1 - D_0}{y(t_1^*)}$ the expected deficit/output ratio and set $x = \frac{A}{1-\delta}$. With normal distribution, the formula for the cost of violating the rule becomes:

$$c(\sigma_\nu) = \frac{0.5\,(W_2^{nv} - W_2^v)}{W_1 g(\tilde{\nu})} = \frac{0.5\,(W_2^{nv} - W_2^v)\,\sigma_\nu}{W_1} \Big/ \phi\left(\frac{\tilde{\nu} - \mu_\nu}{\sigma_\nu}\right)$$

The assumption $a \geq \frac{1}{1-\bar{\delta}}\frac{\bar{D}_1 - D_0}{w_1 l_1(t)}$ ensures that the p.d.f. is non-zero at any possible choce of $t_1$and D_1. Notice that

$$\frac{\partial}{\partial \delta}\left[\frac{1}{1-\delta} f\left(\frac{A}{1-\delta}\right)\right] = \frac{1}{(1-\delta)^2}\left[f(x) + f'(x)x\right] = \frac{1}{(1-\delta)^2}\frac{1}{\sqrt{2\pi}\sigma_\epsilon}\exp\left(-\frac{(x)^2}{2\sigma_\epsilon^2}\right)\left[1 - \frac{(x)^2}{\sigma_\epsilon^2}\right]$$

which is weakly positive for $x \leq \sigma_\epsilon$, and strictly positive for all $x < \sigma_\epsilon$. So the maximum cost is reached at $x = \sigma_\epsilon$. Define a threshold (if it exists) $\bar{\delta} = 1 - \frac{\bar{A}}{\sigma_\epsilon}$, with $\bar{A} = \frac{\bar{D}_1 - D^0}{\hat{y}_1}$ and $\hat{y}_1 = \min_{t_1 \in [0,t]} y_1^*(t_1)$. This threshold ensures that the derivative above is weakly positive for all $\delta \leq \bar{\delta}$, and for all $A \in [0, \bar{A}]$. Because $A^* \in [0, \bar{A}]$, a sufficient condition for implementability is

$$\frac{c(\sigma_\nu)}{1-\bar{\delta}} f\left(\frac{A}{1-\bar{\delta}}\right) \geq \beta(1-\pi\theta_1)(1+r) \geq c(\sigma_\nu) f\left(A\right)$$

for all $A \in [0, \bar{A}]$. The condition above - thanks to the continuity of f and the assumption $A \in [0, \bar{A}]$ - ensures that there exists $\delta^* \in [0, \bar{\delta}]$ such that

$$\frac{c(\sigma_\nu)}{1-\delta^*} f\left(\frac{A}{1-\delta^*}\right) = \beta(1-\pi\theta_1)(1+r)$$

at $A = A^* = \frac{D_1^* - D_0}{y_1(t_1^*)}$. Thus, at $(k, \delta) = (0, \delta^*)$ the FOCs of the politician in period 1 becomes identical to the one of the social planner, which ensures $(t_1^*, D_1^*) = (t_1^{SP}, D_1^{SP})$. Notice that, because $x = \frac{A}{1-\delta}$, then the maximum value of the cost is schieved at $\bar{x}(A) = \frac{A\sigma_\epsilon}{\bar{A}}$, which makes:

$$c(\sigma_\nu) f(x) \frac{x}{A} = \frac{c(\sigma_\nu)}{\bar{A}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(A/\bar{A})^2\right)$$

Also notice that

$$c(\sigma_\nu) f(x) \frac{x}{A} \geq \frac{c(\sigma_\nu)}{\bar{A}} \frac{1}{\sqrt{2\pi e}}$$

And the minum cost is reached at $\delta = 0$, which is equivalent to $x = A$, and makes:

$$c(\sigma_\nu) f(A) = \frac{c(\sigma_\eta)}{\sqrt{2\pi}\sigma_\epsilon} \exp\left(-\frac{(A)^2}{2\sigma_\epsilon^2}\right)$$

Also notice that

$$\frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon} \geq \frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon} \exp\left(-\frac{A^2}{2\sigma_\epsilon^2}\right) = c(\sigma_\nu) f(A)$$

Thus, using $f(\hat{\epsilon}) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp\left(-\frac{\hat{\epsilon}^2}{2\sigma_\epsilon^2}\right)$ a sufficient condition for implementability writes

$$\frac{c(\sigma_\nu)}{\bar{A}} \frac{1}{\sqrt{2\pi e}} \geq \beta(1-\pi\theta_1)(1+r) \geq \frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon}$$

Sufficient for the first inequality to hold:

$$\frac{c(\sigma_\nu)}{\bar{A}}\frac{1}{\sqrt{2\pi e}} \geq \beta(1-\pi\theta_1)(1+r)$$

Sufficient for the second inequality to hold:

$$\beta(1-\pi\theta_1)(1+r) \geq \frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon}$$

First of all, notice that

$$c(\sigma_\nu) = \frac{0.5\,(W_2^{nv}-W_2^{v})\,\sigma_\nu}{W_1} \Big/ \phi\left(\frac{\tilde{\nu}-\mu_\nu}{\sigma_\nu}\right)$$

is finite, continuous and increasing in σ_ν, with $\lim_{\sigma_\nu\to\infty}\frac{c(\sigma_\nu)}{\bar{A}}\frac{1}{\sqrt{2\pi e}} = +\infty$. Thus, for any positive scalar $\bar{c}$, there exists finite threshold $\bar{\sigma}_\nu$ such that $\frac{c(\sigma_\nu)}{\bar{A}}\frac{1}{\sqrt{2\pi e}} \geq \bar{c}$ for all $\sigma_\nu \geq \bar{\sigma}_\nu$. In particular, set $\bar{c} = \beta(1-\pi\theta_1)(1+r)$ which is positive under the assumption $\overline{D}_1 < a$ (also, notice that $\bar{c}$ is unaffected by changes in the distribution of ν). This ensures that the first part of the inequality is satisfied for all finite $\sigma_\nu \geq \bar{\sigma}_\nu$. Now suppose that σ_ν is finite and satisfies $\sigma_\nu \geq \bar{\sigma}_\nu$. Notice that $\frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon}$ is continuous and decreasing in σ_ϵ, with $\lim_{\sigma_\epsilon\to\infty}\frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon} = 0$. Thus, for any positive scalar $\underline{c}$, there exists a finite threshold $\bar{\sigma}_\epsilon$ such that $\frac{c(\sigma_\nu)}{\sqrt{2\pi}\sigma_\epsilon} \leq \underline{c}$ for all $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$. In particular, set $\underline{c} = \beta(1-\pi\theta_1)(1+r)$. This ensures that the second part of the inequality is satisfied for all $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$. Thus, for $\sigma_\epsilon \geq \bar{\sigma}_\epsilon$ and $\sigma_\nu \geq \bar{\sigma}_\nu$, the optimal deficit rule is implementable. Thus, the planner can induce the same platform that he will choose if he could control directly the policies implemented. Q.E.D.

Proposition 2. *In the absence of a fiscal rule the equilibrium level of deficit in period 1 is weakly larger than the optimal level.*

Proof. For $c = 0$, then $[t_1] - [t_1^{SP}] = 0$, and $[D_1] - [D_1^{SP}] = \beta(1-\pi\theta_1)(1+r) \geq 0$, thus it must be true that $D_1^* \geq D_1^{SP}$. Q.E.D.

Proposition 4. *If the optimal policy (t_1^*, D_1^*) is implementable, then the implementation occurs at $k^* = 0$ and $\delta^* \in [0, \delta^{max}]$.*

Proof. Follows the proof for Proposition 3. Namely, notice that

$$[t_1] - [t_1^{SP}] = \beta \frac{ckw_1 l_1}{(1-\delta)t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)$$

is always satisfied for $k = 0$. The second part $\delta^* \in \left[0, 1 - \frac{\bar{D}_1 - D_0}{w_1 l_1 (t_1^*)\sigma_\epsilon}\right]$ is simply stating that δ^* must be such that the argument of f lies within the range in which the cost of violating the rule is weakly increasing in δ. This is ensured under the assumptions $D_1 \in [D_0, \bar{D}_1]$ and $\delta \leq \bar{\delta} = 1 - \frac{\bar{A}}{\sigma_\epsilon}$. Q.E.D.

Proposition 5. *If the optimal policy is implementable for all values of θ_1 in a range $[\theta_1', \theta_1'']$, then the optimal degree of flexibility δ^* is weakly increasing in the political present bias $\pi\theta_1$ within such range.*

Proof. Consider the partial derivative of $[t_1]$, $[D_1]$w.r.t. δ and the cross derivative $[t_1, D_1]$:

$$[t_1, \delta] := \begin{array}{c} \frac{\beta ckw_1 l_1}{(1-\delta)^2 t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right) + \\ -\frac{\beta ckw_1 l_1}{(1-\delta)^3 t_1} \eta_1(t_1) f'\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)(D_1 - D_0 - kw_1 l_1^*) \end{array}$$

negative under the assumption $-\frac{f'(\epsilon)\epsilon}{f(\epsilon)} \leq 1$, and

$$[D_1, \delta] := -\frac{\beta c}{(1-\delta)^2} f\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right) - \frac{\beta c}{(1-\delta)^3} f'\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)(D_1 - D_0 - kw_1 l_1^*)$$

$$[t_1, D_1] := u''(g_1(t_1, D_1))w_1 l_1[1+\eta_1(t_1)] + \frac{\beta ckw_1 l_1}{(1-\delta)^2 t_1}\eta_1(t_1) f'\left(\frac{D_1 - D_0 - kw_1 l_1^*}{1-\delta}\right)$$

Notice that at the optimal fiscal rule - if it is attainable - prescribes $k = 0$ from Proposition 4. Thus the partial derivatives above - at $k = 0$ - become:

$$[t_1, \delta] := 0$$

$$[D_1, \delta] := -\frac{\beta c}{(1-\delta)^2} f\left(\frac{D_1 - D_0}{1-\delta^i}\right) - \frac{\beta c}{(1-\delta)^3} f'\left(\frac{D_1 - D_0}{1-\delta^i}\right)(D_1 - D_0)$$

$$[t_1, D_1] := u''(g_1(t_1, D_1))w_1 l_1[1 + \eta_1(t_1)] < 0$$

Lastly, using the uniform distribution one gets:

$$[D_1, \delta] := -\frac{\beta c}{(1-\delta)^2}\left\{\frac{1}{\sqrt{2\pi}\sigma_\epsilon}\exp\left(-\frac{A^2}{2\sigma_\epsilon^2(1-\delta^i)^2}\right)\left[1 - \left(\frac{A}{\sigma_\epsilon(1-\delta)}\right)^2\right]\right\} \leq 0$$

Recall that $\delta \leq 1 - \frac{\bar{A}}{\sigma_\epsilon}$. This implies that $\frac{A}{\sigma_\epsilon(1-\delta)} \leq \frac{A}{\bar{A}} \leq 1$. In turn, this means $[D_1, \delta] \leq 0$. Notice that

$$\frac{\partial D_1^*}{\partial \delta} = -\frac{[D_1, \delta] - [t_1, \delta] * [t_1, D_1]}{[D_1, D_1]} = -\frac{[D_1, \delta]}{[D_1, D_1]} \leq 0$$

and because the objective function is strictly concave $[D_1, D_1] < 0$, which means that $\frac{\partial D_1^*}{\partial \delta}$ has the same sign of $[D_1, \delta]$, namely it is weakly negative. This means that the optimal level of expected debt for the politician D_1^*, at $k = 0$, is locally decreasing in the flexibility paramenter δ. Now consider two different degrees of political present bias given by $\theta_1' < \theta_1''$. Both can implement the benchmark policy by assumption, thus for both cases $k^* = 0$. Also notice that: (1.) the

FOC w.r.t. t_1 are identical for the two countries; (2.) the FOCs for the Social Planner are identical in the two cases:

$$\left[t_1^{SP}\right]: \ -w_1 l_1 + u'(g_1) w_1 l_1 [1 + \eta_1(t_1)] = 0$$

$$\left[D_1^{SP}\right]: \quad u'(g_1(t_1, D_1)) - \beta(1+r) = 0$$

which means that the benchmark policies must also be identical. Thus, $(t_1, D_1)' = (t_1, D_1)'' = (t_1^*, D_1^*)$ at $k' = k'' = 0$ and at $\delta' < \delta''$, where (k', δ') and (k', δ') denote the optimal fiscal rule under parameter θ_1' and θ_1'', respectively. Using the FOC w.r.t. D_1 this implies

$$[\theta_1'' - \theta_1'] \, \beta(1+r) = \frac{\beta c}{1-\delta'} f\left(\frac{D_1^* - D_0}{1-\delta'}\right) - \frac{\beta c}{1-\delta''} f\left(\frac{D_1^* - D_0}{1-\delta''}\right)$$

But the LHS of the above is strictly positive while the RHS is strictly negative (see proof of Proposition 3), thus the above is never satisfied. This lead to a contradiction, thus it must be true $\delta' \geq \delta''$. Q.E.D.

Proposition 7. *If the optimal policy is not implementable, then (i) there exists threshold $\mathring{\sigma}_\epsilon > 0$ such that if (i) $\sigma_\epsilon < \mathring{\sigma}_\epsilon$, then the optimal rule has $k > 0$. Conversely, if (ii) $\sigma_\epsilon \geq \mathring{\sigma}_\epsilon$ and $D_1^*(0, \delta^{max})$ is interior, then the rule $(k, \delta) = (0, \delta^{max})$ is optimal.*

Proof. Assume the objective function is still convex in (t_1, D_1) in a neighborhood of $k = 0$. This means that $c(\sigma_\nu)/\sigma_\epsilon^3 \leq \widehat{c(\sigma_\nu)}/\sigma_\epsilon^3$, i.e. the variance of the taste shock is small relative to the variance of the macroeconomic shock. Step 1. Fix $\delta = \delta^{max}$. Suppose $D_0 \leq D_1^*(0, \delta^{max}) \leq D_1^*$. Then, if (i) $D_1^*(0, \delta^{max}) = D_1^*$, $k = 0$ ensures that $t_1 = t_1^*$. Thus, the social welfare benchmark is implemented by $(0, \delta^{max})$, which leads to a contradiction. If (ii) $D_1^*(0, \delta^{max}) < D_1^*$, then there exists $\delta \in [0,1]$ that implements the social benchmark. To prove this, notice that $D_1^*(0, \delta^{max}) < D_{1,}^*$ is true only if at $(0, \delta^{max})$ and $(t_1^*, D_1^*)\,(0, \delta^{max})$ one has $[D_1] - \left[D_1^{SP}\right] < 0$. Notice that at $k = 0$, $[D_1] - \left[D_1^{SP}\right] > 0$ as $\delta \to 1$ for all (t_1, D_1). Because of continuity, there exists $\check{\delta} \in [0,1]$ that solves $[D_1] - \left[D_1^{SP}\right] = 0$ at (t_1^*, D_1^*). Moreover, $k = 0$ ensures that $[t_1] - \left[t_1^{SP}\right] = 0$ at all (t_1, D_1). The consequence is that the politician optimal choice at $\left(0, \check{\delta}\right)$ is $\left(t_1^*, D_{1,}^*\right)$. Thus, the benchmark policy is implementable, which leads to a contradiction. I have proved that neither $D_1^*(0, \delta^{max}) = D_1^*$ nor $D_1^*(0, \delta^{max}) < D_1^*$ is true. Thus, it must be true that $D_1^*(0, \delta^{max}) > D_1^*$. This result also implies $[D_1] - \left[D_1^{SP}\right] > 0$ at $(t_1, D_1)\,(0, \delta^{max})$. Lastly, notice that $[D_1] - \left[D_1^{SP}\right]$ is independent of t_1 at $k = 0$. This implies in turn that $[D_1] - \left[D_1^{SP}\right] > 0$ at $(t_1, D_1)\,(0, \delta^{max})$ for all $\delta \in [0,1]$. Notice that if $\bar{D}_1 - D_0$ is small enough relative to $c(\sigma_\eta)$, then there exists at least one value of δ such that the solution for (t_1^*, D_1^*) under rule $(0, \delta)$ is interior.

Step (2). Having established that if the social welfare benchmark is not implementable, then it must be true that $D_1^*(0, \delta^{max}) > D_1^*$, I need to study the sign of the comparative statics of (t_1^*, D_1^*) w.r.t. marginal changes in k and δ in a neighborhood of $k = 0$. Using traditional comparative statics methods, one gets the following results:

1. for interior D_1 and interior t_1 (a)

$$\frac{dD_1^*}{dk} = -\frac{V_{D,k} \times V_{t,t} - V_{D,t} \times V_{t,k}}{V_{D,D} V_{t,t} - V_{D,t}^2}$$

2. for interior D_1 and corner t_1(b):

$$\frac{dD_1^*}{dk} = -\frac{V_{D,k}}{V_{D,D}}$$

3. for corner D_1^* (c):

$$\frac{dD_1^*}{dk} = 0$$

Similarly, one gets:

$$\frac{dt_1^*}{dk} = \begin{cases} -\frac{V_{t,k} \times V_{D,D} - V_{D,t} \times V_{D,k}}{V_{D,D} V_{t,t} - V_{D,t}^2} & if \quad both \quad interior \\ -\frac{V_{t,k}}{V_{t,t}} & if \quad D_1 \quad corner \\ 0 & if \quad t_1 \quad corner \end{cases}$$

Let me start by studying case (a):

$$\frac{dD_1^*}{dk} = -\frac{V_{D,k} \times V_{t,t} - V_{D,t} \times V_{t,k}}{V_{D,D} V_{t,t} - V_{D,t}^2} < 0$$

because the objective function is strictly concave in (t_1, D_1), the above is true if and only if

$$V_{D,k} \times V_{tt} + V_{D,t} \times V_{t,k} < 0$$

The formulas for the cross derivatives of the politician's objective function $V_{t,k}$, $V_{D,k}$, $V_{D,t}$, and $V_{t,t}$ are

$$V_{t,k} = \beta \frac{c w_1 l_1}{(1-\delta^i) t_1} \eta_1(t_1) \left[f\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta} \right) - \frac{k y_1}{1-\delta} f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta} \right) \right] \Bigg|_{k=0} < 0$$

$$V_{D,k} = \frac{\beta c y_1}{(1-\delta)^2} f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta} \right) \Bigg|_{k=0} < 0$$

$$V_{D,t} = u''(g_1(t_1, D_1)) w_1 l_1 [1 + \eta_1(t_1)] + \frac{\beta c k w_1 l_1}{(1-\delta)^2 t_1} \eta_1(t_1) f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta} \right) \Bigg|_{k=0} < 0$$

$$V_{t,t} = \frac{\partial y_1}{\partial t_1} \left\{ -1 + u'(g_1(t_1, D_1))[1 + \eta_1(t_1)] + \beta \frac{ck}{(1-\delta) t_1} \eta_1(t_1) f\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta} \right) \right\} +$$

$$+ y_1 \left\{ u''(g_1(t_1, D^E))[1+\eta_1(t_1)] + u'(g_1(t_1, D_1))\frac{\partial \eta_1(t_1)}{\partial t_1} \right\} + \frac{\beta c k w_1 l_1}{(1-\delta)^2 t_1}\eta_1(t_1) f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta}\right)\Bigg|_{k=0}$$

Recall that the assumptions $u'(\bar{D}_1 - D_0) \geq 1$ and $v'(l) > 0$ imply that the first part of the formula for $V_{t,t}$ is always equal to 0 at the optimal platform $(t_1, D_1)\,(0, \delta)$[20]. Thus, using traditional comparative statics methods in a neighborhood of the optimal platform one gets:

$$V_{D,k} \times V_{tt} + V_{D,t} \times V_{t,k}|_{k=0,\delta=\delta^*} = -\frac{\beta c y_1^2}{(1-\delta^*)^2} f'\left(\frac{D_1 - D_0}{1-\delta^*}\right)[u''(g_1(t_1, D_1))[1+\eta_1(t_1)] +$$

$$+ u''(g_1(t_1, D_1))\frac{\partial \eta_1(t_1)}{\partial t_1}\Bigg] + u''(g_1(t_1, D_1)) y_1^2 [1+\eta_1(t_1)]\beta \frac{c}{(1-\delta^*) t_1}\eta_1(t_1) f\left(\frac{D_1 - D_0}{1-\delta^*}\right)$$

$$V_{D,k} \times V_{tt} + V_{D,t} \times V_{t,k}|_{k=0,\delta=\delta^*} = -\frac{\beta c y_1^2}{(1-\delta^*)} f\left(\frac{D_1 - D_0}{1-\delta^*}\right)\left\{\frac{f'}{(1-\delta) f}[u''(g_1(t_1, D_1))[1+\eta_1(t_1)] + \right.$$

$$\left. + u'(g_1(t_1, D_1))\frac{\partial \eta_1(t_1)}{\partial t_1}\Bigg] - u''(g_1(t_1, D_1))[1+\eta_1(t_1)]\frac{\eta_1(t_1)}{t_1}\right\}$$

which rewrites:

$$V_{D,k} \times V_{tt} + V_{D,t} \times V_{t,k}|_{k=0,\delta=\delta^{i*}} = \frac{\beta c y_1^2}{(1-\delta^*)} f\left(\frac{D_1 - D_0}{1-\delta^*}\right)\left\{\frac{D_1 - D_0}{(1-\delta)^2 \sigma_i^2}[u''(g_1(t_1, D_1))[1+\eta_1(t_1)] + \right.$$

$$\left. + u'(g_1(t_1, D)_1)\frac{\partial \eta_1(t_1)}{\partial t_1}\Bigg] + u''(g_1(t_1, D_1))[1+\eta_1(t_1)]\frac{\eta_1(t_1)}{t_1}\right\}$$

Notice that the above is either always negative, or if $\sigma_\epsilon \to 0$ the above is negative at any given D_1 (recall that $D_1 \geq D_1^* > D_0$, see proof to Step (1)). Because of continuity, this means that in that case there exists $\mathring{\sigma}_\epsilon > 0$ such that at given

[20] This is true because the assumptions $u'(\bar{D}_1 - D_0) \geq 1$ and $v'(0) > 0$ imply that the optimal t_1 is always interior.

D_1 the above is negative for any $\sigma_i < \mathring{\sigma}_i$. Case (b) is easy:

$$\frac{dD_1^*}{dk} = -\frac{V_{D,k}}{V_{D,D}} < 0$$

because

$$V_{D,k} = \frac{\beta c y_1}{(1-\delta)^2} f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta}\right)\Bigg|_{k=0} < 0$$

and case (c) is trivial. Regarding $\frac{dt_1^*}{dk}$, I am going to study only the case in which D_1 is a corner solution (the reason will become clear in the next paragraph). In this case one gets:

$$\frac{dt_1^*}{dk} = -\frac{V_{t,k}}{V_{t,t}} < 0$$

because

$$V_{t,k} = \beta \frac{c w_1 l_1}{(1-\delta) t_1} \eta_1(t_1) \left[f\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta}\right) - \frac{k y_1}{1-\delta} f'\left(\frac{D_1 - D_0 - k w_1 l_1^*}{1-\delta}\right)\right]\Bigg|_{k=0} < 0$$

In the next paragraph, I use these results to study the optimal rule.

Step (3). Recall that step (1) implies $D_1^*(0, \delta^{max}) > D_1^*$. Say the optimal rule has form (k^*, δ^*) and suppose that $k^* = 0$. There are two possible cases. Case 1. Say $D_1^*(0, \delta^{max}) = \bar{D}_1$ with $[D_1] > 0$ (corner solution). Notice that in this at $k = 0$ case $D_1^*(0, \delta^*) = \bar{D}_1$ for all $\delta \in [0, 1]$. Thus, at $k = 0$, one gets $\frac{dD_1^*}{dk} = 0$ at all $\delta \in [0, 1]$. To prove this, set δ at the (unknown) globally optimal level, i.e. $\delta = \delta^*$. Notice that the formula for the change in social welfare (defined as in the social planner's objective function in the paper) induced by a marginal increase in k is the following:

$$\frac{dW}{dk} = \underbrace{\frac{\partial W}{\partial D_1}}_{(-)} \underbrace{\frac{dD_1^*}{dk}}_{(0)} + \underbrace{\frac{\partial W}{\partial t_1}}_{(0)} \underbrace{\frac{dt_1^*}{dk}}_{(\pm)} + \underbrace{\frac{\partial W}{\partial k}}_{(0)} = 0$$

where the sign of $\frac{\partial W}{\partial D_1}$ is obtained as it follows. Recall from step (1) that

$[D_1] - \left[D_1^{SP}\right] > 0$ at $D_1^*(0, \delta^{max})$ for all $\delta \in [0,1]$ and all t_1. Because in this case $D_1^*(0,\delta^{max}) = D_1^*(0,\delta^*) = \bar{D}_1$, it must be also true that $[D_1] - \left[D_1^{SP}\right] > 0$ at $D_1^*(0,\delta^*)$. Secondly, $\frac{\partial W}{\partial t_1} = 0$ is always true at $k = 0$ thanks to the assumptions that ensure that t_1^* is always interior. Then, one must check the second derivative. Because W is strictly concave in t_1, D_1, one gets:

$$\frac{d^2W}{dk^2} = \underbrace{\frac{\partial W}{\partial D_1}}_{(-)}\underbrace{\frac{d^2 D_1^*}{dk^2}}_{(0)} + \underbrace{\frac{\partial^2 W}{\partial D_1^2}}_{(-)}\underbrace{\frac{dD_1^*}{dk}}_{(0)} + \underbrace{\frac{\partial^2 W}{\partial t_1^2}}_{(-)}\underbrace{\frac{dt_1^*}{dk}}_{(-)} + \underbrace{\frac{\partial W}{\partial t_1}}_{(0)}\underbrace{\frac{d^2 t_1^*}{dk^2}}_{(\pm)} + \underbrace{\frac{\partial^2 W}{\partial k^2}}_{(0)} > 0$$

Thus, at fixed $\delta = \delta^*$ W is convex in k, and $k = 0$ is a local minimum. Because $k \in [0, \bar{k}]$ and (k^*, δ^*) is optimal by definition, it must be true that $k^* > 0$. This leads to a contradiction.

Case 2. $D_1^*(0,\delta^{max}) \leq \bar{D}_1$ with $[D_1] = 0$ (interior solution). If $\sigma_\epsilon < \mathring{\sigma}_\epsilon$, then either: (i) $D_1^*(0,\delta^*) = \bar{D}_1$ with $[D_1] > 0$, in such case the proof is identical to Case 1; or (ii) one gets $D_1^*(0,\delta^{max}) \leq D_1^*(0,\delta^*) \leq \bar{D}_1$ with $[D_1] = 0$. Notice that the convexity of W in D_1 implies that if $[D_1] - \left[D_1^{SP}\right] > 0$ is true at $D_1^*(0,\delta^{max})$ under rule $(0,\delta^*)$, then it is also true at $D_1^*(0,\delta^*) \geq D_1^*(0,\delta^{max})$ under the same fiscal rule. The reason is that - given that in this particular case we are assuming interior solution both for $(0,\delta^{max})$ and for $(0,\delta^*)$ - $[D_1] - \left[D_1^{SP}\right] \leq 0$ would imply $\frac{\partial^2 W}{\partial D_1^2} > 0$ at some value in the interval $[D_1^*(0,\delta^{max}), D_1^*(0,\delta^*)]$, which would represent a violation of strict concavity. Thus, it must be true that $\frac{\partial W}{\partial D_1} > 0$ in the neighborhood of the equilibrium platform. In this case (ii) $\frac{dD_1^*}{dk} < 0$, thus $\frac{dW}{dk} > 0$ and the globally optimal rule has $k > 0$. This leads to a contradiction.

Part (ii). Recall that at $k = 0$, it must be true (see Step (1) in part (i)) that $D_1^*(0,\delta^{max}) > D_1^*$ and therefore $D_1^*(0,\delta) > D_1^*$. Notice that the additional condition in the proposition states that $D_1^*(0,\delta)$ is interior. In such case, we have shown that $\frac{\partial W}{\partial D_1} > 0$ in the neighborhood of the equilibrium platform. (see

Step (3) in part (i)). Suppose $\delta^* < \delta^{max}$. Then a marginal increase in δ implies:

$$\frac{dW}{d\delta} = \underbrace{\frac{\partial W}{\partial D_1}}_{(-)} \underbrace{\frac{dD_1^*}{d\delta}}_{(-)} + \underbrace{\frac{\partial W}{\partial t_1}}_{(0)} \underbrace{\frac{dt_1^*}{d\delta}}_{(\pm)} + \underbrace{\frac{\partial W}{\partial \delta}}_{(0)} > 0$$

thus, δ^* cannot be optimal at $k = 0$. Similarly, one can show that $\delta^* > \delta^{max}$ is also sub-optimal. These two results lead to a contradiction. Q.E.D.