

Business Cycles during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective

Kjetil Storesletten
University of Oslo

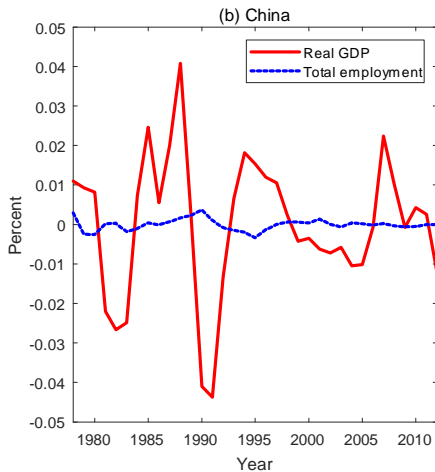
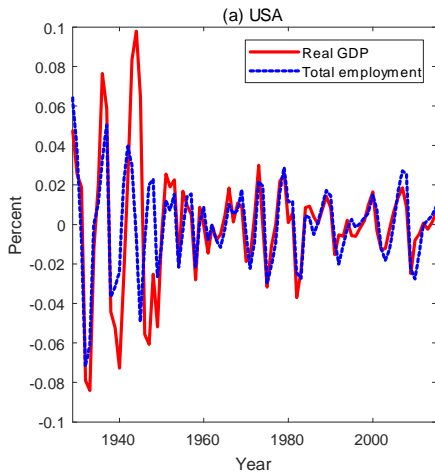
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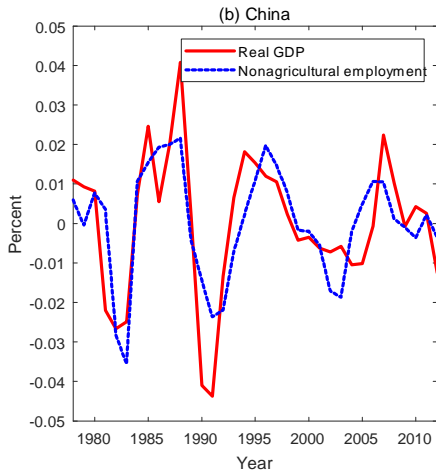
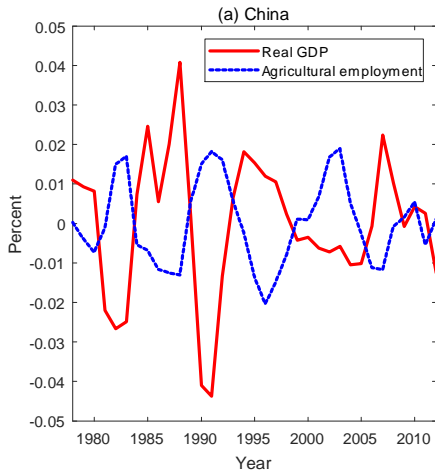
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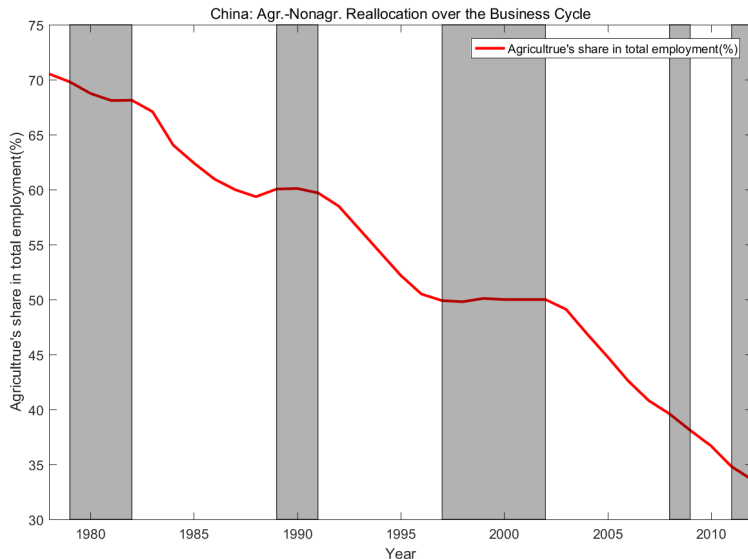
US (left) & China (right): GDP vs. Total Employment



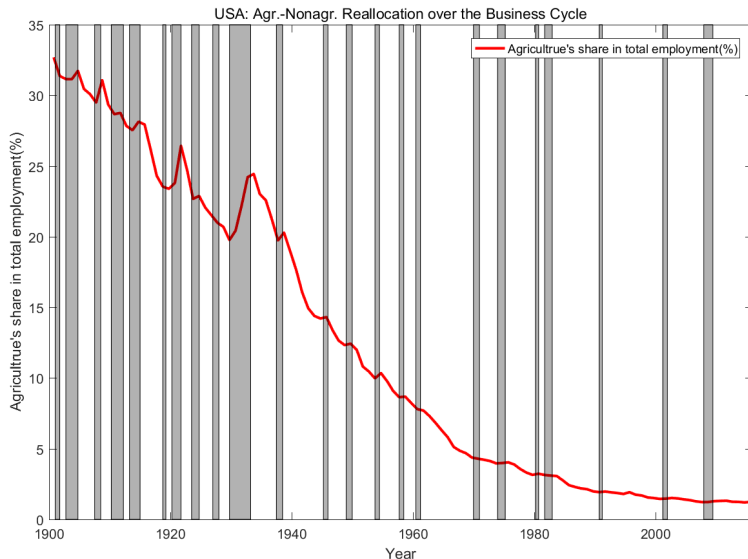
China: GDP vs. Agric. empl & Non-Agric. empl



China: Agr-NonAgr Reallocation over the Business Cycle



US: Agr-NonAgr Reallocation over the Business Cycle

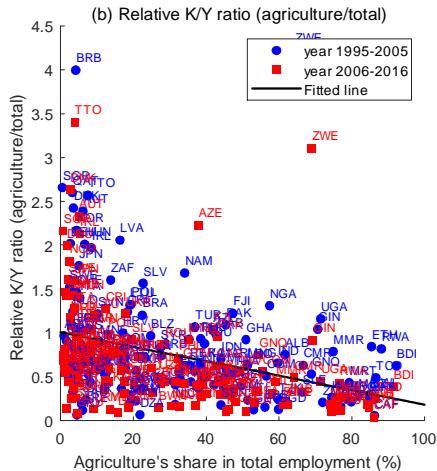
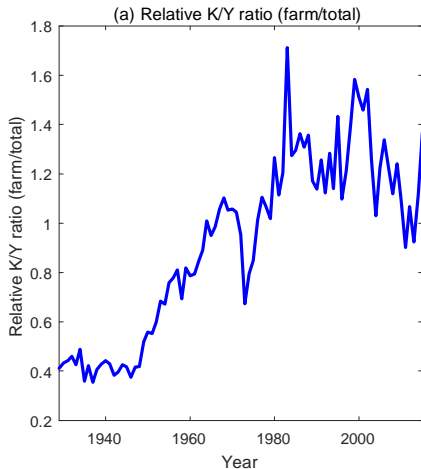


Purpose of the Paper

- Unified theory of business cycles and structural transformation
- Structural transformation:
 - ① Reallocation of labor away of Agr as capital accumulates
 - ② Modernization of Agr: As workers leave Agr,
labor productivity gap NonAgr-vs-Agr ↓ and labor share in Agr ↓
- Business cycles change during structural change. Poor countries have:
 - ① Acyclical and smooth labor supply
 - ② Strong labor reallocation between Agr and NonAgr
 - ③ Labor productivity in Agr ↑ in booms (also relative to NonAgr)
- Goals:
 - ① Propose a theory quantitatively consistent with both structural transformation and business cycles
 - ② Match China-US (and cross-country) patterns
 - ③ Novel framework to analyze fluctuations during transition

STYLIZED FACTS: STRUCTURAL CHANGE

Modernization of Agriculture: KY ratio

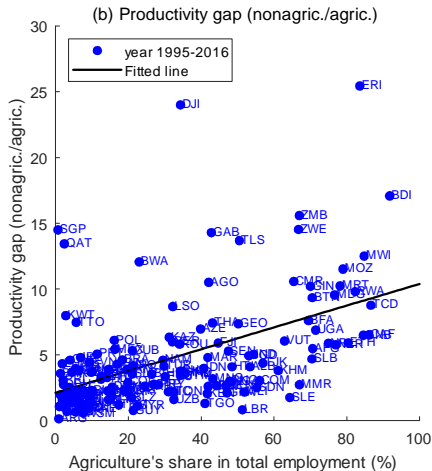
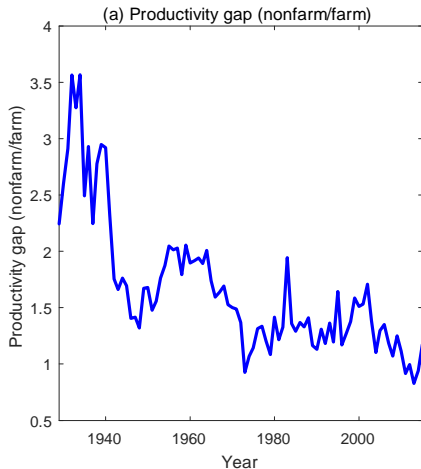


Labor Productivity Gap

- Define Productivity Gap as the ratio of the Average Productivity of Labor (APL) in NonAgr vs. Agr

$$\text{Prod. Gap} \equiv \frac{\text{Value Added per Worker in NonAgr}}{\text{Value Added per Worker in Agr}}$$

Modernization of Agric.: Productivity Gap



STYLIZED FACTS: BUSINESS CYCLE

Business Cycle Over Structural Change

- Modernization is accompanied by five transformations in the nature of business cycle fluctuations.
- Consider HP Filtered or First-Differenced data:

	Large Agriculture (poor country)	Small Agriculture (rich country)
Employment-GDP correlation	acyclical	procyclical
Employment volatility	low	high
corr(agr. empl., nonagr empl.)	negative	≈ 0
Labor productivity gap	countercyclical	acyclical
Consumption volatility	high	low

- US time-series and US-China contrasting evidence in line with cross-country evidence.

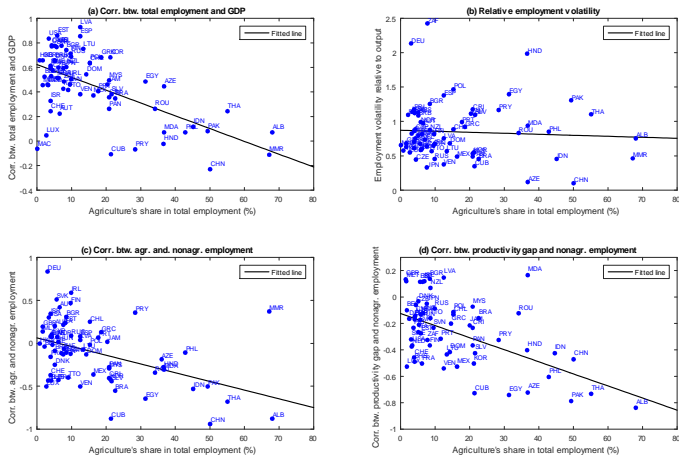


Figure: X-axis: avg. agr. empl. share. Y-axis: (a) corr. empl.-GDP; (b) rel. empl. volatility; (c) corr. nonag-ag employment; (d) cyclicality APL ratio.

DO WE NEED A NEW THEORY?

Theory: Why Hansen-Prescott Does Not Work

- Consider a two-sector neoclassical benchmark
 - cf. Hansen and Prescott (2002)
- Cobb-Douglas production function in each sector

$$Y^M = Z^M \times (K^M)^{1-\alpha} (L^M)^\alpha \text{ and } Y^G = Z^G \times (K^G)^{1-\beta} (L^G)^\beta$$

which implies constant factor shares:

$$\frac{wL^M}{P^M Y^M} = \alpha \text{ and } \frac{wL^G}{P^G Y^G} = \beta$$

- So, the productivity gap is

$$\frac{P^M Y^M}{L^M} / \frac{P^G Y^G}{L^G} = \frac{\beta}{\alpha}$$

- Counterfactual!

Our Model: Traditional vs. Modern Agr Sector

- Introduce business cycles in a transition model *à la* Acemoglu-Guerreri (2008) with Agr and NonAgr sector.
- Structural transformation is driven by two forces:
 - exogenous differential technical progress,
 - endogenous capital deepening.
- Extend Acemoglu-Guerreri to incorporate a “rural Lewis sector.”
- Agr goods are produced using two different technologies
 - 1 Modern (neoclassical) sector using labor, capital, and land;
 - 2 Traditional sector with no capital.

- Structural Changes

- Baumol (1967), Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007; 2008), Acemoglu and Guerrieri (2008)
- Alvarez-Cuadrado and Poschke (2011), Herrendorf, Rogerson and Valentinyi (2013, 2015), Alvarez-Cuadrado et al (2017)

- Business Cycle

- Cross-country: Rogerson (1991), Da-Rocha and Restuccia (2006), Aguiar and Gopinath (2007)
- Zhang, Rozelle, and Huang (2001), Brandt and Zhu (2000; 2001), Yao and Zhu (2017)

- Development

- Lewis (1954); Harris and Todaro (1970); Hansen-Prescott (2002); Parente, Rogerson, and Wright (2000); Gollin, Lagakos, and Waugh (2014)

THE MODEL

Production: Final Good

- The final good is produced competitively
- It combines Agr and NonAgr goods, with elast. of subst. ε

$$Y = F(Y^G, Y^M) = \left[\gamma (Y^G)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) (Y^M)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Can be interpreted as a preference aggregator.
- Extension: nonhomothetic (Stone Geary) preferences:
 - Agr good as “necessity” in consumption.
- Study both $\varepsilon > 1$ and $\varepsilon < 1$
 - (although estimation suggests $\varepsilon > 1$)

Production: NonAgr and Agr Sector

- Production Function in NonAgr sector:

$$Y^M = (K^M)^{1-\alpha} (Z^M N^M)^\alpha$$

- Agr is produced in two ways: modern (AM) and traditional (S) technology with an elasticity of substitution $\omega > 1$:

$$Y^G = \left[\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}} + (1-\varsigma) (Y^S)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

where

$$\begin{aligned} Y^{AM} &= (K^{AM})^{1-\beta} (Z^{AM} N^{AM})^\beta, \\ Y^S &= Z^S N^S. \end{aligned}$$

- Assume $\beta > \alpha$ (M more capital intensive than AM)
- Later (estimation): allow land in agriculture

TFP Growth and Urban-Rural Wedge

- TFP grows at a constant rate in each sector
- Only one friction:
 - an exogenous time-invariant wedge (a "tax" on nonagr employment) that keeps marginal productivity higher in urban than in rural sector;
 - stand-in for a variety of institutional frictions inducing rural overpopulation;
 - does not matter for the theory, matters for quantitative results.

Social Planner's Problem

- The Recursive Competitive Equilibrium is equivalent to the solution to the following distorted social planner's problem

$$\max_{K^M, K^{AM}, N^M, N^{AM}, N^S, c} \int_0^\infty e^{-(\rho-n)t} \times \log(c_t) dt$$

subject to the resource constraints

$$\begin{aligned}\dot{K}_t &= F(Y_t^M, Y_t^G) - \delta K_t - cN_t - \tau \bar{W} N_t^M + Tr_t, \\ K_t &= K_t^M + K_t^{AM}, \\ N_t &= N_t^M + N_t^{AM} + N_t^S,\end{aligned}$$

given exogenous law of motions for TFPs, and initial conditions.

- We later augment it with endogenous labor supply and shocks.

Static Equilibrium

- Static efficiency: equate MPL and MPK across sectors.
- Let:

$$\chi \equiv \frac{K}{L} \text{ (endogenous state variable)}$$

$$\kappa \equiv K^M / K \text{ (share of capital in Nonagr)}$$

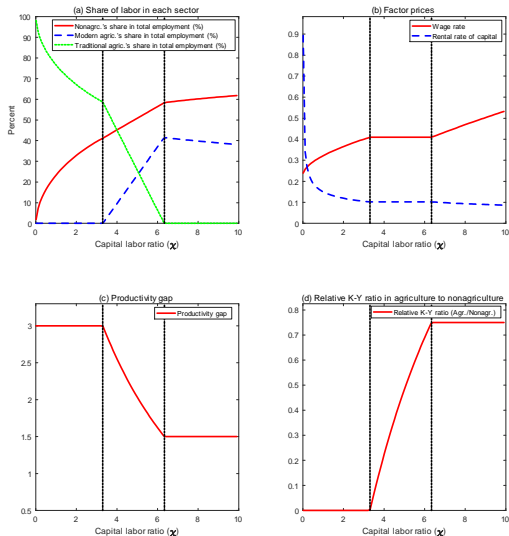
$$v \equiv \frac{\varsigma(Y^{AM})^{\frac{\omega-1}{\omega}}}{\varsigma(Y^{AM})^{\frac{\omega-1}{\omega}} + (1-\varsigma)(Y^S)^{\frac{\omega-1}{\omega}}} \text{ (Agr modernization).}$$

- $\kappa(\chi, \mathbf{Z})$ and $v(\chi, \mathbf{Z})$ are sufficient for characterization
 - pin down employment in the three sectors.
- RESULT: for ω close to $\varepsilon > 1$: $\partial\kappa/\partial\chi > 0$ and $\partial v/\partial\chi > 0$

Static Equilibrium (Lewis)

- Monotone dynamics is not a robust feature.
- Consider a “Lewis model” ($\omega \rightarrow \infty$ and $\varepsilon > 1$) driven by capital accumulation.
- Three stages of economic growth:
 - 1 Early Lewis: no modern agriculture ($v = 0, \kappa = 1$);
 - 2 Advanced Lewis: modernization of agriculture ($v \uparrow, \kappa \downarrow, N^S \downarrow$).
 - 3 Neoclassical: demise of agriculture ($\kappa \uparrow$ and $\kappa \rightarrow 1$) and further modernization of agriculture ($v \rightarrow 1$).

Static Equilibrium (Lewis)



Asymptotic Balanced Growth Path (ABGP)

- Sufficient conditions: suppose $\omega > 1$ and

$$\varepsilon > 1, \quad g^M \geq g^{AM} \geq g^S.$$

[or, alternatively, $\varepsilon < 1$ and $g^{AM} \geq g^M \geq g^S$]

- ... then the dynamic equilibrium converges to unique ABGP where

$$\begin{aligned} \kappa_t &\rightarrow 1, & v &\rightarrow 1, \\ \frac{\dot{c}_t}{c_t} &\rightarrow g^M, & \frac{\dot{\chi}_t}{\chi_t} &\rightarrow g^M. \end{aligned}$$

- Note: labor and capital accumulation in agriculture can be positive in the ABGP, but it goes to zero as a share of total GDP.

ELASTICITIES OF SUBSTITUTION

Elasticity of Substitution btw. Agr/Nonagr Goods

- What should we believe about elasticities ω and ε ?
- Large ω seems plausible (in Lewis, $\omega \rightarrow \infty$).
- What about ε ?
 - Herrendorf et al. (2013), Comin et al. (2018), etc. argue $\varepsilon < 1$
 - Foster and Rosenzweig (2004), Macona (2018) argue $\varepsilon > 1$

Elasticity of Substitution btw. Agr/Nonagr Goods

- Herrendorf et al. (2013) assume three goods (agriculture, manufacturing, services) and a constant elasticity ϵ_h between them:

$$Y = \left[x_m \left(Y^{Manuf} \right)^{\frac{\epsilon_h - 1}{\epsilon_h}} + x_s \left(Y^{Serv} + \bar{s} \right)^{\frac{\epsilon_h - 1}{\epsilon_h}} + x_g \left(Y^G + \bar{c} \right)^{\frac{\epsilon_h - 1}{\epsilon_h}} \right]^{\frac{\epsilon_h}{\epsilon_h - 1}}$$

- Find $\epsilon_h \approx 0$ for US data 1950-2010
- Generalize their model to nested CES w/elasticity ϵ_{ms} between manuf. and services, elasticity ϵ between agriculture and non-agriculture:

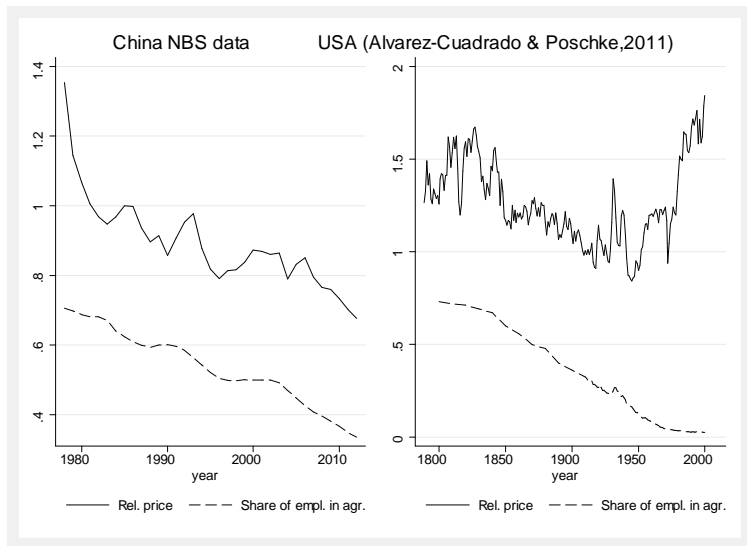
$$Y = \left[\gamma \left(Y^M \right)^{\frac{\epsilon - 1}{\epsilon}} + (1 - \gamma) \left(Y^G + \bar{c} \right)^{\frac{\epsilon - 1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon - 1}}$$
$$Y^M = \left[\hat{\gamma} \left(Y^{Manuf} \right)^{\frac{\epsilon_{ms} - 1}{\epsilon_{ms}}} + (1 - \hat{\gamma}) \left(Y^{Serv} + \bar{s} \right)^{\frac{\epsilon_{ms} - 1}{\epsilon_{ms}}} \right]^{\frac{\epsilon_{ms}}{\epsilon_{ms} - 1}}$$

- Estimate model for 3 largest countries: China, USA, Japan
- Find $\epsilon > 1$ for all countries ($\epsilon \in [1.3, 6]$)

Estimating a nested CES implies $\epsilon > 1$

IFGNLS Estimates of Nested CES Elasticities					
	Elasticity	Cons. value added		GGDC	
		3-sector	2-sector	3-sector	2-sector
USA	ϵ	2.49*** (0.28)	2.32*** (0.32)	1.36*** (0.13)	1.53*** (0.48)
	ϵ_{ms}	0	- -	0.48*** (0.005)	- -
Japan	ϵ	1.58*** (0.053)	2.24*** (0.36)	5.46*** (0.80)	6.06*** (0.91)
	ϵ_{ms}	0.79*** (0.003)	- -	1.66*** (0.13)	- -
China	ϵ	1.70*** (0.22)	1.34*** (0.25)	1.30*** (0.13)	1.75*** (0.08)
	ϵ_{ms}	0.007 (0.05)	- -	0.65*** (0.04)	- -

Relative price (non-agr./agr.) vs. agricultural employment share: CHINA vs. USA



Causal identification: Green Revolution

- Omitted variable bias in regression analysis: factors (other than exogenous supply shocks) could influence P_t^G / P_t^M and Y_t^G / Y_t^M simultaneously
- Potential solution: study response of P_t^G / P_t^M and Y_t^G / Y_t^M to exogenous and observable productivity shock: Green Revolution
 - Introduction of high-yield cereals induced sharp increase in agricultural TFP in developing countries.
 - TFP increase differed across areas
- Result: in areas with larger agric TFP increase, agric employment expanded and nonagric growth slowed down
 - across regions in India (Foster and Rosenzweig, 2004)
 - across countries (Macona, 2018)
- **Conclusion: causal evidence for $\varepsilon > 1$**

QUANTITATIVE ANALYSIS

Quantitative Model

- Discrete time.
- Persistent shocks to the three TFPs.
- Endogenous labor supply (pref. for leisure).
- Land in (modern) agriculture.
- First estimate the deterministic model to match structural change.
- Then, estimate stochastic processes for TFPs.
- Finally, simulate the stochastic model and compare business cycle statistics.

Model Estimation: SMM (for China)

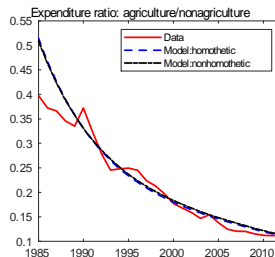
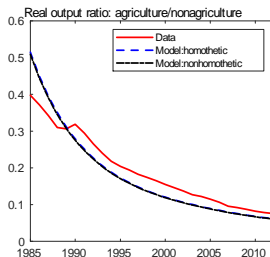
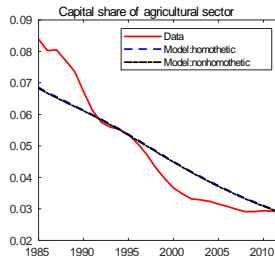
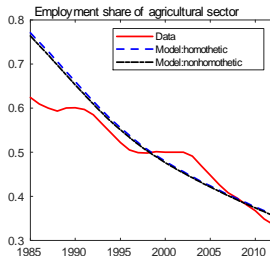
- 5 parameters are calibrated outside the model
 - $n = 1.5\%$, $\delta = 5\%$, $(1 + \rho)^{-1} = 0.96$, $\alpha = 0.50$, $Y_{1985} = 1$
- 14 parameters are estimated by SMM to match 226 moments (China 1985-2012):
 - Capital stock (in current price) share in agr. sector
 - Employment share in agr. sector
 - Aggregate GDP growth and K/Y ratio
 - Productivity Gap during 1985-2012
 - Ratio of (real) agr to total output
 - Expenditure share in agr products
 - Hours worked in the long run (1/3)

Estimated Parameters (SMM)

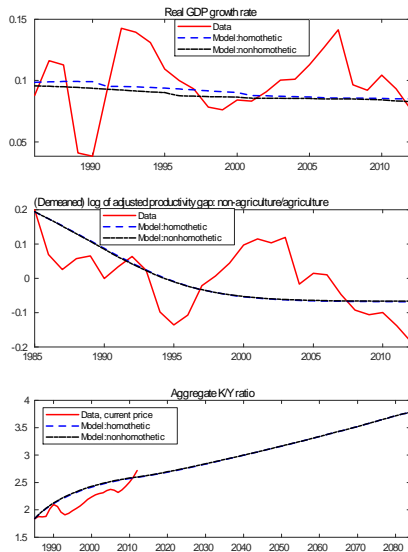
	Estimated Parameter	Homoth	NonHom	NoTargPG
\bar{C}^G / Y	Subsistence level Agr	0	0.05	0.05
ε	ES btw Nonagr and Agr	3.60	3.36	4.00
ω	ES btw Modern and Trad. Agr	9.00	9.00	8.22
τ	labor wedge	0.76	0.75	0.73
θ	pref. weight on consumption	0.73	0.73	0.71
γ	weight on Agr output	0.61	0.60	0.54
ζ	weight on Modern Agr output	0.40	0.39	0.50
ξ	capital inc. share in Modern Agr	0.14	0.13	0.21
β	labor inc. share in Modern Agr	0.61	0.60	0.68
g^M	TFP growth rate Nonagr	6.5%	6.5%	6.5%
g^{AM}	TFP growth rate Modern Agr	6.1%	6.1%	5.9%
g^S	TFP growth rate Trad Agr	0.9%	0.9%	1.0%
Z_{1985}^M	initial TFP level Nonagr	4.33	4.45	3.42
Z_{1985}^{AM}	initial TFP level Modern Agr	2.26	2.25	2.42
Z_{1985}^S	initial TFP level Trad Agr	1.23	1.18	1.35

QUANTITATIVE ANALYSIS: FITTING STRUCTURAL CHANGE

Model Fit 1: Decline of Agricultural Sector



Model Fit 2: GDP growth, Prod. Gap, K/Y Ratio



Trajectories: Traditional Agr Share in Agr



QUANTITATIVE ANALYSIS: BUSINESS CYCLE DURING STRUCTURAL CHANGE

Estimate joint TFP process

- TFP has cyclical and trend components: $\ln Z_t^j = (1 + g^j) + z_t^j$
- Cyclical component is simple VAR(1),

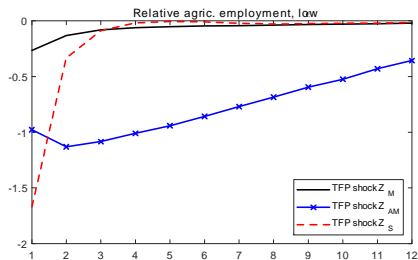
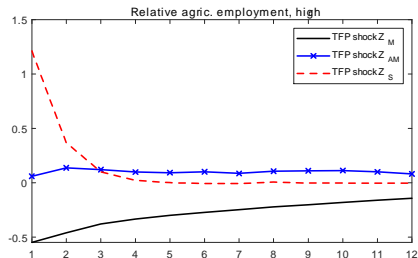
$$\begin{bmatrix} z_{t+1}^M \\ z_{t+1}^{AM} \\ z_{t+1}^S \end{bmatrix} = \begin{bmatrix} \phi^M & 0 & 0 \\ 0 & \phi^{AM} & 0 \\ 0 & 0 & \phi^S \end{bmatrix} \cdot \begin{bmatrix} z_t^M \\ z_t^{AM} \\ z_t^S \end{bmatrix} + \epsilon_t,$$

- Estimates of persistence: $\hat{\phi}^M = 0.63$, $\hat{\phi}^{AM} = 0.9$, and $\hat{\phi}^S = 0.42$
- Implied volatility of innovations: $\sigma(\epsilon_t^M) = 0.042$, $\sigma(\epsilon_t^{AM}) = 0.036$, and $\sigma(\epsilon_t^S) = 0.053$

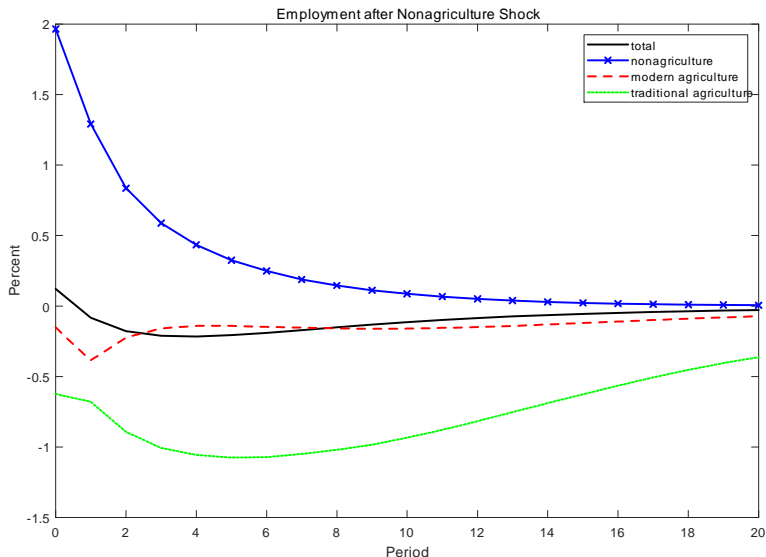
Impulse Response to Agriculture TFP Shock

- Consider increase in agricultural TFP: $Z^{AM} \uparrow$ and $Z^S \uparrow$
- $\varepsilon > 1$: shock $Z^{AM} \uparrow$ and $Z^S \uparrow$ *reverses* structural change: $N^M \downarrow$
(workers go from manufacturing to agriculture when agriculture becomes more productive)
- $\varepsilon < 1$: shock $Z^{AM} \uparrow$ or $Z^S \uparrow$ *accelerates* structural change: $N^M \uparrow$
(workers leave countryside and move to manufacturing when agriculture becomes more productive)

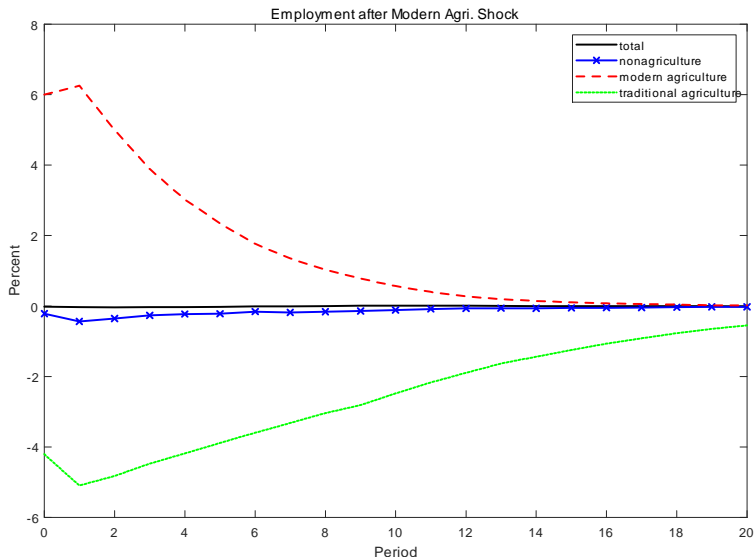
TFP Shock & Structural Change: High vs. Low epsilon



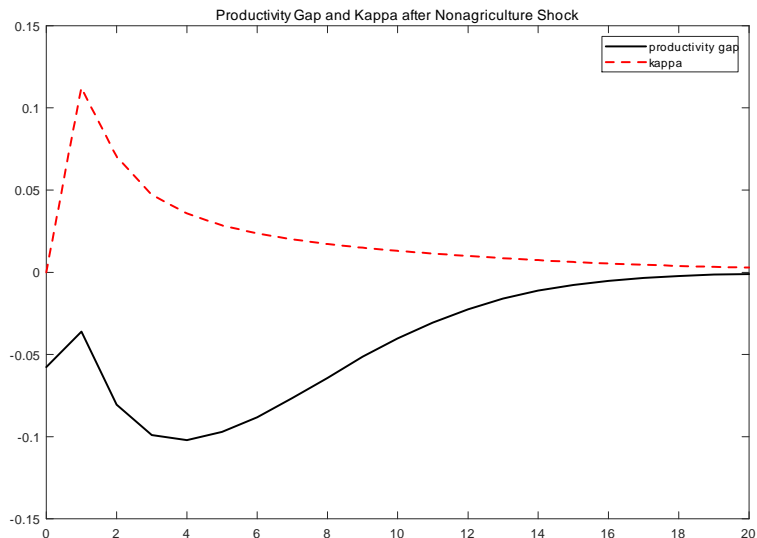
Impulse-Resp. of Employment to NonAgr TFP Shock



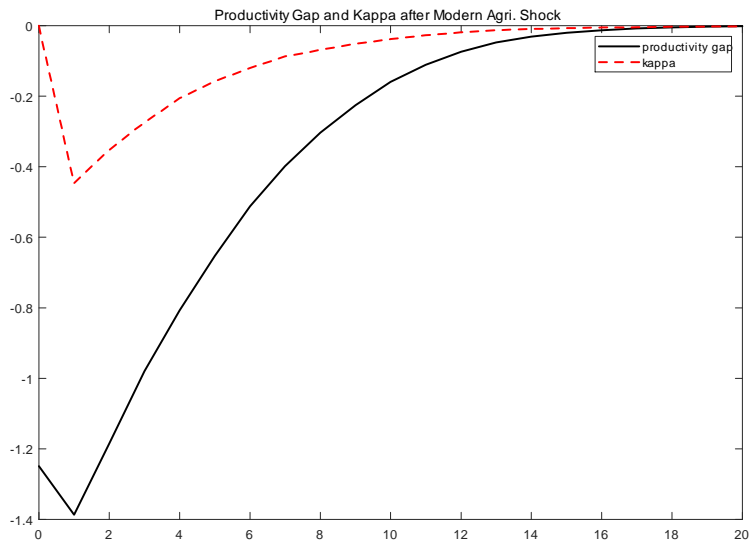
Imp.-Resp. of Employment to Modern Agr TFP Shock



Imp.-Resp. of Prod. Gap to Nonagr TFP Shock



Imp.-Resp. of Prod. Gap to Modern Agr TFP Shock



Business Cycle Statistics: China data vs. model

Table: Business Cycle Statistics: Model vs Data

HP FILTER	$x =$							
HOMOTH.	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	PrGap	n^G	n^M	n
A. HP-filtered China Data: $std(y) = 1.7\%$								
$\frac{std(x)}{std(y)}$	0.99	3.53	1.63	1.34	2.04	0.64	0.73	0.10
$corr(x, y)$	0.70	0.65	0.06	0.95	-0.17	-0.69	0.73	-0.23
$corr(x, n^G)$	-0.60	-0.31	-0.37	-0.55	0.48	1.00	-0.94	0.48
$corr(x, n^M)$	0.60	0.37	0.41	0.57	-0.54	-0.94	1.00	0.04
B. HP-filtered Model: $std(y) = 1.6\%$								
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr(x, n^G)$	-0.08	-0.25	0.78	-0.43	0.72	1	-0.75	0.69
$corr(x, n^M)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21

Business Cycle Statistics: China data vs. model

Table: Business Cycle Statistics: Model vs Data

HP FILTER	$x =$							
HOMOTH.	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	PrGap	n^G	n^M	n
A. HP-filtered China Data: $std(y) = 1.7\%$								
$\frac{std(x)}{std(y)}$	0.99	3.53	1.63	1.34	2.04	0.64	0.73	0.10
$corr(x, y)$	0.70	0.65	0.06	0.95	-0.17	-0.69	0.73	-0.23
$corr(x, n^G)$	-0.60	-0.31	-0.37	-0.55	0.48	1.00	-0.94	0.48
$corr(x, n^M)$	0.60	0.37	0.41	0.57	-0.54	-0.94	1.00	0.04
B. HP-filtered Model, $std(y) = 1.6\%$								
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr(x, n^G)$	-0.08	-0.25	0.78	-0.43	0.72	1	-0.75	0.69
$corr(x, n^M)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21

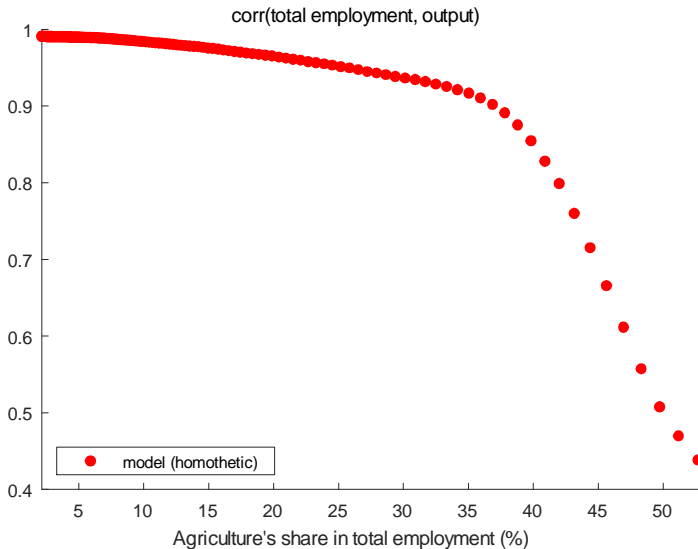
Business Cycle Statistics: China data vs. model

Table: Business Cycle Statistics: Model vs Data

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HOMOTH.	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	PrGap	n^G	n^M	n
A. HP-filtered China Data , $std(y) = 1.7\%$								
$\frac{std(x)}{std(y)}$	<u>0.99</u>	<u>3.53</u>	1.63	1.34	<u>2.04</u>	<u>0.64</u>	<u>0.73</u>	<u>0.10</u>
$corr(x, y)$	<u>0.70</u>	<u>0.65</u>	0.06	0.95	<u>-0.17</u>	<u>-0.69</u>	<u>0.73</u>	<u>-0.23</u>
$corr(x, n^G)$	-0.60	-0.31	-0.37	-0.55	<u>0.48</u>	1.00	<u>-0.94</u>	0.48
$corr(x, n^M)$	0.60	0.37	0.41	0.57	<u>-0.54</u>	<u>-0.94</u>	1.00	0.04
B. HP-filtered Model , $std(y) = 1.6\%$								
$\frac{std(x)}{std(y)}$	<u>0.27</u>	<u>2.39</u>	1.09	1.18	<u>0.62</u>	<u>1.03</u>	<u>1.07</u>	<u>0.42</u>
$corr(x, y)$	<u>0.81</u>	<u>0.99</u>	0.30	0.97	<u>-0.38</u>	<u>-0.25</u>	<u>0.73</u>	<u>0.43</u>
$corr(x, n^G)$	-0.08	-0.25	0.78	-0.43	<u>0.72</u>	1	<u>-0.75</u>	0.69
$corr(x, n^M)$	0.45	0.75	-0.31	0.87	<u>-0.74</u>	<u>-0.75</u>	1	-0.21

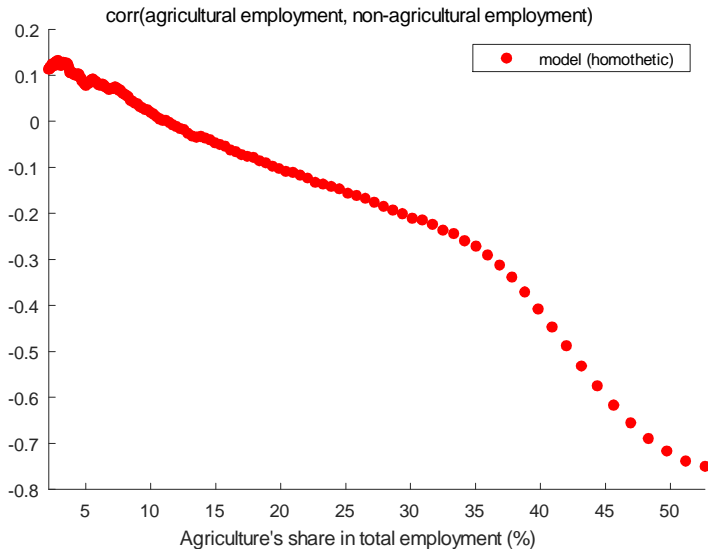
Employment: From Acyclical to Procyclical

Richer countries (lower share of employment in agriculture) to the left



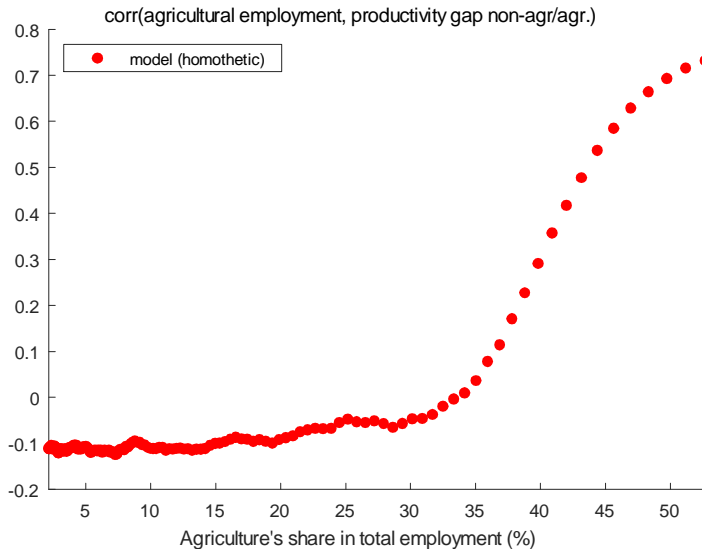
Employment Agr-NonAgr Turns Less (Neg.) Correlated

Richer countries (lower share of employment in agriculture) to the left



Prod Gap (Nonag/Ag) Becomes Less Countercyclical

Richer countries (lower share of employment in agriculture) to the left



- Modify TFP process for traditional sector
 - assume same persistence, $\phi^S = \phi^{AM}$
 - common shock to entire agric. sector
- Capital adjustment costs
- Cobb-Douglas preferences ($\varepsilon = 1$ and large subsistence level in food)

- We document how business cycle features changes throughout development
 - China vs. US
 - A cross section of countries
- Provide unified theoretical framework to account for business cycles and structural change
- Estimate model to match structural transformation in China
 - Model is broadly consistent with business cycle properties of China
- As productivity grows and capital accumulates, business cycles become more similar to those of the US

ADDITIONAL MATERIAL

- Business cycles in developing countries
 - Sectoral comovement
 - Hornstein and Praschnik (1997), Horvath (2000), Boldrin et. al. (2001), Kim and Kim (2006)
 - Cross-country business cycle differences
 - Rogerson (1991): movement out of Agriculture in the US has been concentrated during upturns in economic activity, whereas the movement of workers out of manufacturing has been concentrated during downturns.
 - Da-Rocha and Restuccia (2006) focus on the role of Agriculture. We provide new evidence and a model with structural change
 - Aguiar and Gopinath (2007): emphasize trend shocks
 - China:
 - Zhang, Rozelle, and Huang (2001); in the early 1990's the layoffs increased and hiring slowed. Those who lost their jobs returned to the Agricultural sector.
 - Brandt and Zhu (2000; 2001), Yao and Zhu (2017)

- Structural change
 - Driving force: differential technical change and capital deepening Baumol (1967), Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007; 2008), Acemoglu and Guerrieri (2008)
 - China: Cheremukhin et. al. (2015),
- Dual labor market:
 - Lewis (1954), Harris and Todaro (1970)

Deterministic Dynamic Systems (Constant h)

- In absence of shocks, the deterministic equilibrium is characterized by the following systems of differential equations w.r.t. (c, v^A, κ^M, χ) where

$$\kappa^M \equiv \frac{K^M}{K}, v^A \equiv \frac{\varsigma (Y^{AM})^{\frac{\omega-1}{\omega}}}{(Y^A)^{\frac{\omega-1}{\omega}}}, \chi \equiv \frac{K}{N},$$

$$\begin{aligned} \frac{\dot{c}}{c} &= \frac{1}{1 + \theta(\sigma - 1)} \times \left[\eta_t^{\frac{1}{\varepsilon}} (1 - \gamma) (1 - \alpha_M) \times \right. \\ &\quad \left. (\kappa_t^M)^{-\alpha_M} (Z_t^M v_t^M)^{\alpha_M} \chi_t^{-\alpha_M} - \delta - \rho \right] \\ \frac{\dot{\chi}_t}{\chi_t} &= \eta_t (Z_t^M)^{\alpha_M} (\kappa_t^M)^{1-\alpha_M} (v_t^M)^{\alpha_M} \chi_t^{-\alpha_M} - \delta - c \chi_t^{-1} - n, \end{aligned}$$

Deterministic Dynamic Systems (Constant h)

$$\frac{\dot{\kappa}_t^M}{\kappa_t^M} = (1 - \kappa_t^M) \frac{\left(\left(\alpha_M g^M - \alpha_A g^A + (\alpha_A - \alpha_M) \frac{\dot{\chi}_t}{\chi_t} \right) + \left(\frac{1}{\omega-1} - \frac{(\alpha_A - \alpha_M)(1 - v_t^M)}{\alpha_A v_t^A + 1 - v_t^A} \right) \frac{\dot{v}_t^A}{v_t^A} \right)}{\frac{1}{\varepsilon-1} + (\alpha_A - \alpha_M) (\kappa_t^M - v_t^M)},$$

$$\frac{\dot{v}_t^A}{v_t^A} = \frac{(1 - v_t^A) \left(\alpha_A g^A - g^S + (1 - \alpha_A) \left(\frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\kappa}_t^M}{\kappa_t^M} \frac{\kappa_t^M - v_t^M}{1 - \kappa_t^M} \right) \right)}{\frac{1}{\omega-1} + \frac{(1 - v_t^A)(1 - \alpha_A)(1 - v_t^M)}{\alpha_A v_t^A + 1 - v_t^A}},$$

$$\frac{Z_t^M}{Z_t^M} = g^M, \quad \frac{Z_t^A}{Z_t^A} = g^A, \quad \frac{Z_t^S}{Z_t^S} = g^S,$$

Deterministic Dynamic Systems (Constant h)

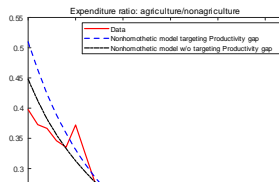
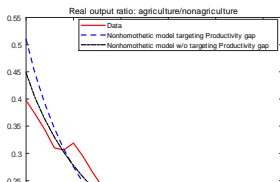
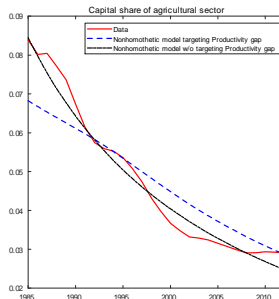
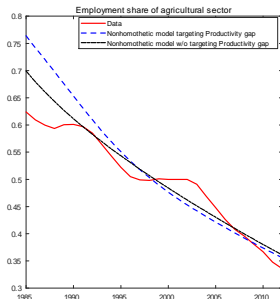
...where

$$\eta_t \equiv (1 - \gamma)^{\frac{\varepsilon}{\varepsilon-1}} \left(1 + \frac{1 - \alpha_M}{1 - \alpha_A} \frac{1 - \kappa_t^M}{\kappa_t^M} \frac{1}{v_t^A} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

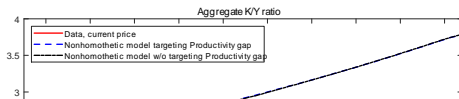
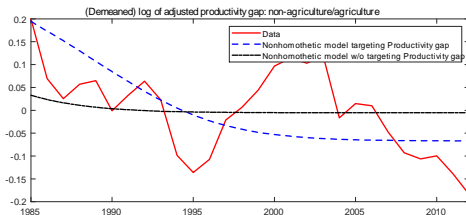
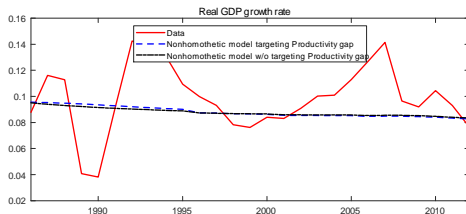
$$v_t^M = \left(1 + \frac{1 - \kappa_t^M}{\kappa_t^M} \frac{1 - \alpha_M}{1 - \alpha_A} \left(\frac{\alpha_A}{\alpha_M} + \frac{1}{\alpha_M} \frac{1 - v_t^A}{v_t^A} \right) \right)^{-1},$$

$$v_t^A = \frac{1}{1 - \tau} \frac{1 - \kappa_t^M}{\kappa_t^M} \frac{1 - \alpha_M}{1 - \alpha_A} \frac{\alpha_A}{\alpha_M} v_t^M.$$

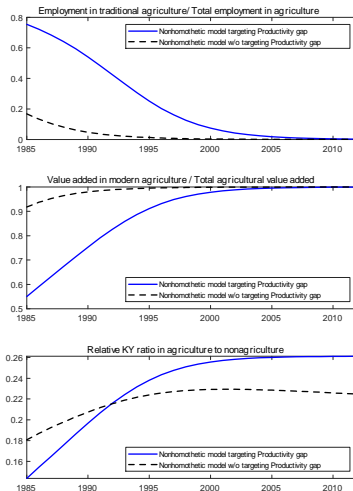
Comparing Two Versions of Non-homothetic Model



Comparing Two Versions of Non-homothetic Model



Comparing Two Versions of Non-homothetic Mode



Business Cycle Statistics: China data vs. model

Table: Business Cycle Statistics: Model vs Data

FIRST DIFF HOMOTH	$x =$							
	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	PrGap	n^G	n^M	n
A. FD- Filtered China Data: $std(y) = 2.4\%$								
$\frac{std(x)}{std(y)}$	1.27	3.34	1.82	1.31	2.32	1.00	0.76	0.30
$corr(x, y)$	0.57	0.63	0.12	0.93	-0.09	-0.57	0.66	-0.25
$corr(x, n^G)$	-0.74	-0.34	-0.38	-0.38	0.35	1.00	-0.50	0.71
$corr(x, n^M)$	0.32	0.37	0.40	0.53	-0.52	-0.50	1	0.19
B. FD- Filtered Model, $std(y) = 2.6\%$								
$\frac{std(x)}{std(y)}$	0.30	2.36	1.11	1.25	0.72	1.10	1.27	0.49
$corr(x, y)$	0.80	0.99	0.24	0.95	-0.42	-0.30	0.69	0.18
$corr(x, n^G)$	-0.22	-0.27	0.80	-0.51	0.79	1	-0.78	0.75
$corr(x, n^M)$	0.55	0.66	-0.40	0.88	-0.81	-0.78	1	-0.52

Labor's Income Share in non-farm/farm sector

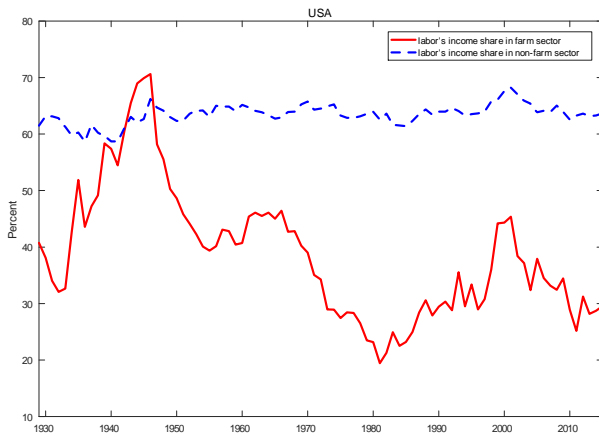


Figure: The Figure plots the labor's income share in farm/non-farm sectors in the USA. The labor's income share is defined as the compensation of employees divided by the value-added output minus proprietor's income. Source:

Compensation of employees by farm/non-farm come from NIPA Table 6.2A, 6.2B, 6.2C, and 6.2D. Proprietor's income by farm/non-farm come from NIPA Table

Rel. Price of non-farm/farm output in the US

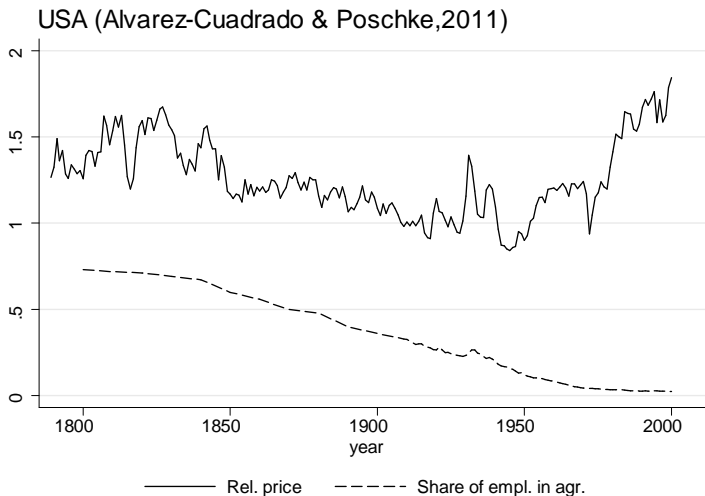


Figure: The Figure is from the Figure 1 in Alvarez-Cuadrado and Poschke (2011, [link](#))

Rel. Price of non-farm/farm output in CHINA

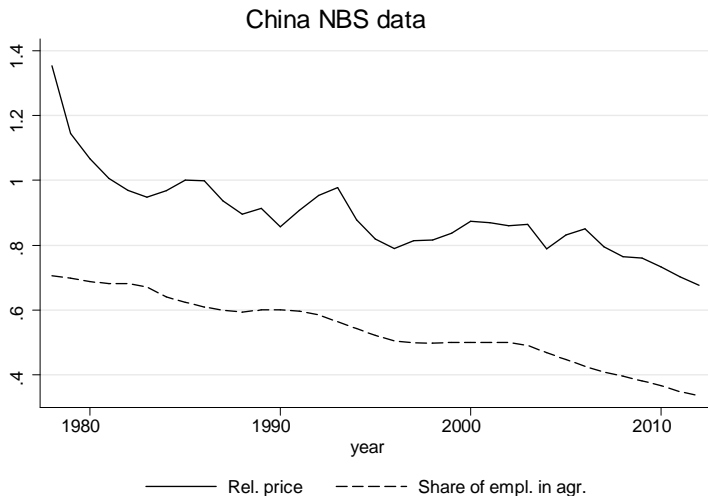


Figure: The Figure plots the share of employment in agriculture and the relative

Rel. Price of non-farm/farm output in Other Countries

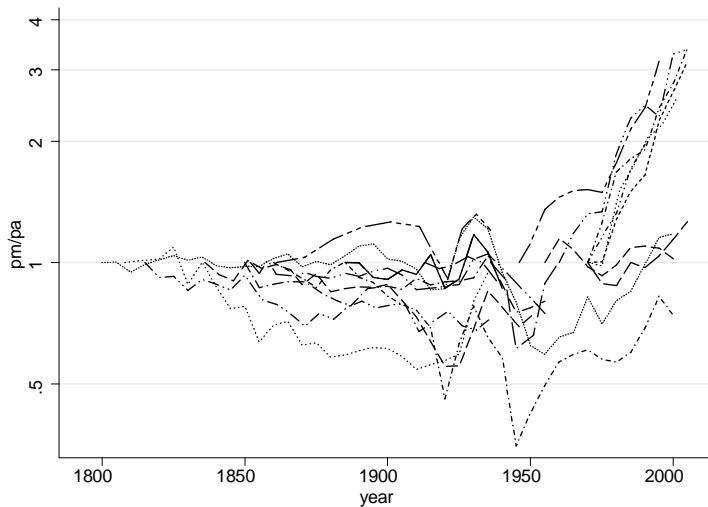


Figure: The Figure is from the Left panel of Figure 4 in Alvarez-Cuadrado and [SZZ \(Sciences-Po\)](#)

Rel. Price of non-farm/farm output (Pre-WWII)

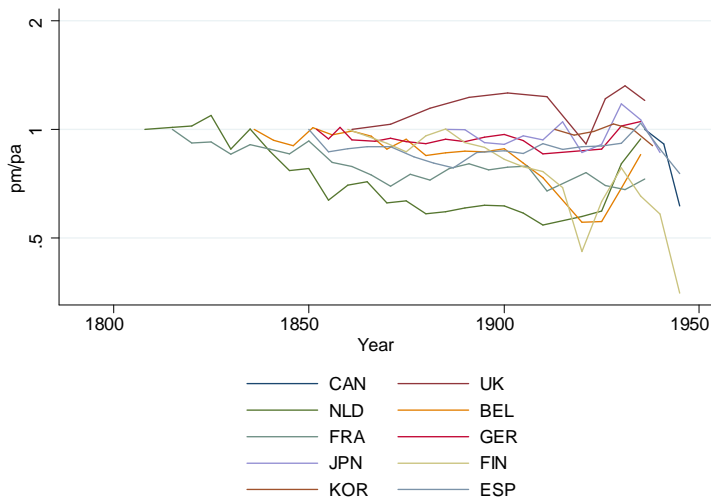


Figure: The Figure is based on the Left panel of Figure 4 in [Alvarez-Cuadrado](#)