Pricing and fees in auction platforms with two-sided entry

[ Job market paper ]

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Abstract

Auction platforms are increasingly popular marketplaces that generate revenues from fees charged to users. The platform faces a “two-sided market” with network effects; increased seller entry raises its value to bidders, and vice versa. This means that both the platform revenue-maximizing fee structure and welfare impacts of these fees are ambiguous. I examine these issues with a new data set of wine auctions using a model with endogenous bidder and seller entry, seller selection, and costly listing inspection. I show that relevant model primitives are identified from observed variation in reserve prices, transaction prices, and the number of bidders. My estimation strategy combines methods from the auction and discrete choice literatures. Model estimates reveal significant network effects, which can be harnessed to improve both platform profitability and user surplus. Decreasing (increasing) the buyer premium (seller commission) by 15 percentage points and increasing the listing fee increases platform revenues by about 30 percent. It is striking that, in the face of such fee changes, even sellers are better off as additional bidder entry drives up transaction prices. I also estimate that welfare impacts from increasing fees individually are about twice as high as when abstracting from endogenous entry and that 70-90 percent of the loss falls on sellers.

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1 Introduction

Auction platforms provide increasingly popular marketplaces for trading goods and services, ranging from freelance jobs to vehicles to oil and gas drilling rights. Examples include: eBay.com, BidforWine.co.uk, ClassicCarAuctions.co.uk, CarsontheWeb.com, EnergyNet.com, Upwork.com, uShip.com and ComparetheManwithVan.co.uk. These auction platforms generate revenues from fees charged to buyers and sellers: eBay generated over 2 billion dollars in fee revenues in Q3 2018.1 The platform faces a “two-sided market” with network effects given that it is more valuable to potential bidders when more sellers enter, and vice versa. This generates complications for the platform or a benevolent planner when determining how to optimally allocate fees among users. In fact, the two-sided market literature highlights that both 1) the platform revenue-maximizing fee structure, and 2) welfare impacts of those fees are theoretically ambiguous and depend on the magnitude of network effects.2

To study these two issues, I exploit a new data set of wine auctions and develop a structural model in which network effects arise from endogenous bidder and seller entry. A key innovation is that I leverage the transparency of payoffs in the auction game to characterize network effects in this setting.3 This allows me to provide a tight quantitative analysis of how fee changes affect both platform profitability and user welfare. My wine auction data is representative of auction platforms for idiosyncratic goods for which bidders and sellers have private information about their willingness to pay.4 As storage conditions and provenance of these “fine, rare, and vintage wines” are important descriptors of their quality, it is costly for bidders to inspect each listing. Empirical patterns in the data, including thin markets and independent listings, are consistent with listing inspection cost and set this environment apart from previously studied auction platforms for more homogeneous goods.5

While a significant literature examines implications of costly bid preparation or value discovery in auctions, it addresses markets in which a single seller can influence bidder entry through optimal auction design.6 My emphasis on selective seller entry is novel to the empirical auction literature. It generates an additional trade-off that is relevant for answering the key questions in this paper. Bidders expect lower (reservation) prices when lower-value sellers are attracted to the platform, so

1 https://investors.ebayinc.com/fast-facts/default.aspx
3 Previous studies that introduced a two-sided market pricing question in an auction framework are Athey and Ellison (2011) and Gomes (2014), studying position auctions.
4 This also motivates the use of auctions rather than more convenient posted prices as the selling mechanisms on the platform. See Milgrom (1989) and Wang (1993) on auctions versus posted prices. Some auction platforms offer both auctions and posted price listings. Einav et al. (2018) find that on eBay.com, where sellers can choose between the two mechanisms, auctions are typically selected by less experienced sellers and for goods that are used or more idiosyncratic. This motivates my use of this term.
5 Previous auction platform models include: Anwar et al. (2006), Peters and Severinov (1997) (see also Albrecht et al. (2012)), Nekipelov (2007), Backus and Lewis (2016) and Bodoh-Creed et al. (2013, 2016). None of these evaluate the impact of fees.
bidder entry depends both on the expected number and type of sellers that enter. The importance of this dynamic was first postulated in Ellison et al. (2004) but never implemented in practice. The authors hypothesize that a major reason why Yahoo! and Amazon were unsuccessful as auction platforms was their zero listing fee policy: this attracted many nonserious sellers with high reservation prices that in turn shunned bidders from their platforms. My structural model addresses this mechanism and allows me to incorporate the seller selection channel in evaluating the role of fees.

In line with the wider empirical auction literature, I exploit the relatively controlled auction environment where strategic interactions and resulting payoffs are accurately described by Bayes-Nash equilibrium properties of an incomplete information game. The observed distributions of reserve prices, transaction prices and number of bidders are endogenous to the fee structure through optimal entry, bidding and reserve pricing strategies. Variation in outcomes allows for the estimation of model primitives needed to answer how fees affect user welfare in this market. As such, the wine auctions provide an opportune setting to understand the otherwise hard to quantify network effects by tracing fees through the auction platform game.

The introduction of seller selection in the auction platform model does introduce empirical challenges regarding nonparametric identification and estimation of the population distribution of seller valuations. I demonstrate that using the first order optimality condition of equilibrium reserve prices the relevant distribution of seller valuations is identified for any counterfactual fee policy that reduces expected seller surplus. Every reserve price maps to a valuation for all sellers that entered the platform, given identification of the distribution of bidder valuations from the observed second highest bid and number of bidders according to Athey and Haile (2002). Only a subset of sellers currently listing on the platform would enter for any counterfactual world in which platform entry is less profitable for sellers. The positive identification result does not apply for fee structures that make seller entry more profitable because a typical sample selection problem causes observables to be uninformative about valuations among sellers that did not enter.

The entry equilibrium is the unique solution to a fixed point problem in seller valuation space with a nested zero profit entry condition on the bidder side. This complicates estimation of the distribution of seller valuations because: 1) the support of the distribution of reserve prices depends on parameters and 2) the equilibrium is costly to compute for each set of candidate parameters. To address these issues I first obtain an initial estimate based on a concentrated likelihood using a consistent estimate of the entry threshold, suggested in Donald and Paarsch (1993, Footnote 4) for a similar support problem. I then solve the game once and re-estimate seller parameters

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8My identification result requires the assumption that all relevant auction-level heterogeneity is observed. This is plausible given that the web scraping algorithm arguably delivered the same observables that bidders get to see when bidding on the wine. Related are Roberts (2013) and Freyberger and Larsen (2017) who use the reverse approach: variation in reserve prices traces out unobserved heterogeneity assuming that reserve prices and bids respond to common factors unobserved to the econometrician. To do so, Roberts (2013) assumes that sellers are homogeneous. Freyberger and Larsen (2017) do have heterogeneous sellers; the reserve price is additive in the common unobserved factor and an idiosyncratic seller-specific term and they use deconvolution to separately identify the two components.
based on the updated entry threshold. This algorithm is based on the Aguirregabiria and Mira (2002, 2007) nested pseudo likelihood method to solve estimation problems involving fixed point characterizations in (dynamic) games. In my case, the algorithm uses the auction structure to obtain seller parameters from a first order condition. The estimation of bidder parameters uses standard methods from the empirical auction literature, involving a first stage that controls for auction heterogeneity following Haile et al. (2003) and maximum likelihood estimation of parameters from the homogenized bidder valuation distribution as in e.g. Donald and Paarsch (1993) and Paarsch (1997).

Model estimates reveal significant network effects in this platform, which can be harnessed to improve both platform profitability and user welfare. I estimate that platform revenues can increase by up to 80 percent without reducing sale volume when implementing fee structures that subsidize buyers (more) by reducing the buyer premium while at the same time increasing the seller commission.\(^9\) As the buyer premium is currently zero it requires providing winning bidders with a discount on the transaction price. This fully agrees with the idea that businesses in two-sided markets should subsidize the side that contributes most to profits, even if this results in negative fees.\(^{10}\) Counterfactual experiments also demonstrate that all parties benefit with the adoption of some of these fee structures. For example, combining a 15 percent buyer discount with a 15 percentage point increase in seller commission and an increase in listing fee from 1.75 to 5 pounds increases platform revenues by about 30 percent. But even sellers are about 20 percent better off in this scenario despite a significant increase in seller fees. This is because the buyer discount attracts additional bidders, driving up transaction prices in the auction mechanism.

In a second set of counterfactual exercises, I focus on welfare impacts from isolated increases in buyer or seller commission. A key finding is that sellers are better off if their seller commission is increased by 5 percentage points than in the case when the buyer commission increases by the same amount. This feature of the platform setting, driven by network effects, would be missed if bidder and seller participation is considered exogenous. The magnitude of welfare impacts is also striking. For example, a 5 percentage point increase in buyer (seller) commission reduces expected surplus for winning bidders by 7 (4) percent and for sellers by 17 (15) percent. These results demonstrate that abstracting from endogenous entry and strategic interactions between platform users, as has been the norm in antitrust policy, significantly biases estimated welfare impacts of changes in the fee structure.

Wine auctions are a particularly relevant market in this context because two mayor players, auction houses Sotheby’s and Christie’s, have been found guilty of commission fixing in the mid-90s. Using this case for context, I estimate that it is plausible that the true antitrust injury to both parties would have been about double the estimated damages underlying the settlement of 512 million dollars (roughly 729 million dollars in 2018 prices). Especially sellers would be

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9 I solve for fee revenues with my static game and impose volume constraints to capture that current volume likely affects future revenues through e.g. brand familiarity or word of mouth. This approach avoids having to make stronger assumptions about the exact dynamic platform objective function.

undercompensated: while they received only one sixth of the total settlement, about 70-90 percent of estimated damages falls on sellers regardless of which side the commission increase is charged to.

My empirical findings underscore the idea that economic principles underlying regulation in traditional markets do not necessarily apply to two-sided markets and that both sides should be evaluated in tandem. A competitive auction platform could combine high fees on one side of the market with below marginal cost prices on the other side. Both practices could be considered predatory when evaluated in isolation but they prove to be socially optimal in the two-sided market in this paper. In recent years also competition authorities and courts recognize that regulation of platform markets requires different tools and tailored solutions, but the perceived difficulty to quantify user interactions has been a bottleneck for practical application of these ideas.\textsuperscript{11}

The rest of this paper is organized as follows. Section 2 provides institutional details about online wine auctions, presents the data and empirical patterns that distinguish it from previously studied homogeneous good auctions. Section 3 sets out the theoretical auction model and solves for equilibrium entry, bidding, and reserve price strategies. Section 4 explains how to identify model primitives from available data. Details about the estimation approach are presented in 5 and results in 6. Section 7 presents results from counterfactual fee policies that shed light on network effects, the economic incidence of fees, and platform profitability. Concluding remarks are offered in Section 8.

2 Wine auctions

Fine wine is sold at auction in secondary markets, run by online wine platforms as well as brick-and-mortar auction houses.\textsuperscript{12} Auction data for the empirical analysis in this paper comes from online auction platform: www.Bidforwine.co.uk (BW). It offers a marketplace for buyers and sellers to trade, akin to the eBay consumer-to-consumer format.\textsuperscript{13} When sellers create a listing they choose the auction duration, whether or not to increase the minimum bid amount or to set a secret reserve price. They also provide wine characteristics and description, and information on delivery and insurance. When the sale is successful, they receive payment from the winning bidder, ship the wine, and receive an invoice for the amount of seller commission due. For these seller-managed lots, BW charges no buyer premium and maintains a seller commission on a sliding scale between 8.5-5.5 percent of the sale price (see Table 1). Upfront charges to sellers are: a 1.75 pounds listing charge.


\textsuperscript{12}The major platforms sold for 338 million dollars of wine in 2016, and have also been burgeoning in 2017 and 2018 (Wine Spectator (2017a,b, 2018)) The biggest players in 2016 were: Sotheby’s (74 million), Zachys (66 million) and Acker, Merrall & Condit (59 million).

\textsuperscript{13}Such seller-managed listings are the focus of this paper. They are distinct from wines consigned to the platform and sold on behalf of the seller, especially because they do not undergo quality control by the platform. BW only offers consignment services when selling a “large collection”, roughly exceeding five cases, and charges higher fees for these auctions.
Table 1: Fee structure in wine auction data

<table>
<thead>
<tr>
<th></th>
<th>Notation</th>
<th>Amount / rate</th>
<th>Conditional on selling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bidders:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buyer premium</td>
<td>$c_B$</td>
<td>0</td>
<td>✓</td>
</tr>
<tr>
<td>Entry fee</td>
<td>$e_B$</td>
<td>£0</td>
<td>✓</td>
</tr>
<tr>
<td>Opportunity cost of time</td>
<td>$e_B^o$</td>
<td>estimated</td>
<td></td>
</tr>
<tr>
<td><strong>Sellers:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seller commission</td>
<td>$c_S$</td>
<td>0.085</td>
<td>£200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.075</td>
<td>£200.01-£1500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.066</td>
<td>£1500.01-£2500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.055</td>
<td>≥£2500.01</td>
</tr>
<tr>
<td>Listing fee</td>
<td>$e_S$</td>
<td>£1.75</td>
<td></td>
</tr>
<tr>
<td>Reserve price fees</td>
<td>$e_R$</td>
<td>£0.75</td>
<td></td>
</tr>
<tr>
<td>Opportunity cost of time</td>
<td>$e_R^o$</td>
<td>estimated</td>
<td></td>
</tr>
</tbody>
</table>

Source: www.bidforwine.co.uk. Displayed fees exclude 20 percent VAT, which are included in estimation. Opportunity cost $e_B^o$ and $e_S^o$ are added for reference but fall outside the platform fee structure $f = \{c_B, e_B, c_S, e_S, e_R\}$. As described in the text, the reserve price fee is made up of 0.50 pounds for raising the minimum bid and 0.25 pounds for adding a secret reserve price. Different fees apply to lots consigned to and sold by the platform on behalf of sellers.

fee, a 0.50 pounds minimum bid fee (optional, if increased), and a 0.25 pounds reserve price fee (optional, if set).

Lots are sold through an English auction mechanism with proxy bidding. Bidders submit a maximum bid and the algorithm places bids to keep the current price one increment above the second-highest bid.\(^{14}\) A soft closing rule extends the end time of the auction by two minutes whenever a bid is placed in the final two minutes of the auction. Therefore, there is no opportunity for a bid sniping strategy (bidding in the last few seconds, potentially aided by sniping software) on the BW platform.\(^{15}\)

### 2.1 Data collection and description

I constructed a dataset of wine auctions by web-scraping all open auctions on BW at 30-minute intervals between January 2017 and May 2018.\(^{16}\) At these intervals, I observe everything that bidders observe as well. This data collection effort resulted in a wealth of data, including: the number of bidders and bids, the current standing price, the identity of the seller, and feedback from earlier transactions. Only a quarter of listings is created by a seller with feedback, pointing to the consumer-to-consumer nature of the platform.

The repetitive recording of bids for ongoing auctions was necessary to approximate the reserve price distribution. When the seller sets a reserve price without making it public in the form of a

\(^{14}\)When the highest bid is less than one increment above the second highest bid, the transaction price remains the second highest bid. This is different from the rule at eBay, where the standing price in that case would increase to the highest bid. Engelberg and Williams (2009), Hickman (2010) and Hickman et al. (2017) assess implications of this alternative bidding rule that is practically a mix between a first-price and second-price auction.

\(^{15}\)See Ockenfels and Roth (2006) on strategic behaviour in auctions with these two types of closing rules and Harker and Sickles (2010) and Bajari and Hortacsu (2004) for an overview of various explanations for bid sniping evaluated in the literature.

\(^{16}\)The exact data collection times depended on when the scraping job got scheduled on the cluster, also affected by computing node failures. An example listing page is provided in Figure 9.
minimum bid amount, the notifications “reserve not met” or “reserve almost met” accompany any standing price that does not exceed the reserve. I approximate the reserve price as the average between the highest standing price for which the reserve price is not met and the lowest for which it is met. While only 26 percent of listings has an increased minimum bid amount, 44 percent has a (secret) reserve price, and 3 percent has both. The use of secret reserve prices in auction platforms remains a puzzle in the empirical auction literature and solving that puzzle is beyond the scope of this paper. In the rest of this paper I group them together and refer to the “reserve price” as the maximum of: the minimum bid amount and the approximated secret reserve price. Of larger consequence is the choice made by a third of sellers to refrain from setting any form of reserve. This is observable to bidders by a “no reserve price” button - even before they enter the listing. The BW website encourages sellers to set no reserve price with the following argumentation: “Bid for Wine’s own statistics show that lots listed without reserve prices typically attract 50-75% more bidders and sell for up to 40% more than those with reserves.” My model therefore incorporates higher (optimal) bidder entry into no-reserve listings.

I also observe wine characteristics such as the type of wine, grape, vintage, region of origin; plus the textual description, delivery and payment information. Basic summary statistics are reported in Table 2. While there is a significant range in sale prices, 84 percent of all sales in the sample do not exceed the 200 pounds over which sellers pay a higher marginal seller commission. The sample includes 3,487 auctions after excluding auctions with a “buy-now” option, that are consigned, sell spirits, or sell multiple lots at once.

### 2.2 Why listing inspection and seller selection matter

Wine sold at auction is often described as fine, rare, and vintage wine. A key difference with retail wines is that they are sold by individual collectors who stored the bottles either in professional warehouses or in private cellars - sometimes for decades. Sellers therefore know how much the wine is worth to them and they have their own idiosyncratic value (taste) for it. When the platform changes its fee structure, it therefore affects both the number and the type of sellers that enter. Moreover, this feeds back on how attractive the platform is for potential bidders given that more serious sellers with lower valuations set lower reserve prices. This is the first paper to estimate an auction platform model with (selective) seller entry.

Listing inspection cost arise in this context because all offered wines are different. This has to do with why there is a flourishing secondary market in the first place. The paramount influence of weather and harvesting conditions results in some vintages outperforming others in terms of quality. Older wines can be valuable as increased scarcity of these star vintages drives up prices.

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17 If all bids would be recorded in real time, this approximation would be accurate up to half a bidding increment due to the proxy bidding system. Appendix B presents suggestive evidence that also the 30-minute scraping interval result in a good approximation of the reserve price distribution.

18 See e.g. Jehiel and Lamy (2015) and Hasker and Sickles (2010).

19 Ellison et al. (2004) hypothesize that seller selection was likely a main driver for why auction sites of Amazon and Yahoo! struggled: their zero listing fees attracted non-serious sellers with high reserve prices, shunning bidders.

20 Ashenfelter et al. (1995) and Ashenfelter (2008) predict with surprising accuracy the value of high-end Bordeaux wines.
Table 2: Descriptive statistics: selected auction characteristics

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction price</td>
<td>3.487</td>
<td>140.56</td>
<td>239.94</td>
<td>1.00</td>
<td>82.50</td>
<td>6,000.00</td>
</tr>
<tr>
<td>Is sold</td>
<td>3.487</td>
<td>0.64</td>
<td>0.48</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number bottles</td>
<td>3.487</td>
<td>3.70</td>
<td>4.22</td>
<td>1</td>
<td>2</td>
<td>72</td>
</tr>
<tr>
<td>Price per bottle if sold</td>
<td>2,230</td>
<td>74.84</td>
<td>124.52</td>
<td>0.50</td>
<td>35.00</td>
<td>2,200.00</td>
</tr>
<tr>
<td>Number of bidders</td>
<td>3.487</td>
<td>3.10</td>
<td>2.52</td>
<td>0</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>Seller has feedback</td>
<td>3.487</td>
<td>0.29</td>
<td>0.46</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Has reserve price</td>
<td>3.487</td>
<td>0.44</td>
<td>0.50</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Has increased minimum bid</td>
<td>3.487</td>
<td>0.26</td>
<td>0.44</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Textual description:
- related to storage conditions 3.487 0.17 0.38 0 0 1
- related to delivery 3.487 0.58 0.49 0 1 1
- related to en primeur 3.487 0.17 0.38 0 0 1
- related to expert opinion 3.487 0.51 0.50 0 1 1
Number of words in description 3.487 84.22 79.11 1 65 851

Textual description statistics are obtained using text mining with count-based evaluation. The dummy variables equal one if it contains a word that is associated with respectively “stored”, “delivery”, “primeur”, or “parker” (referring to wine advocate Robert Parker who maintains a 50-100 point scale for fine wines); the minimum association threshold is a Pearson correlation of at least 0.3.

given that fewer of them remain uncorked over time. Moreover, certain high-tannin wines such as red Bordeaux age well and are thought to reach their full potential only after many years. But the commodities are also perishable so that humidity and temperature control are key to deliver this potential quality. As such, assessing the wine’s idiosyncratic storage conditions, provenance, ullage and other indicators of wine quality make it costly for bidders to bid in every auction they enter.21

Conceptually, also auctions of other idiosyncratic products such as second hand cars, freelance jobs or house moving trips likely involve costly listing inspection by bidders. While previous empirical studies investigate auction platforms for more homogeneous goods, this is the first to focus on goods with listing inspection cost.22

2.3 Descriptive evidence

Here, I document four empirical patterns related to the idiosyncratic nature of the goods.

1) Thin markets. The data reveals a strikingly low number of identical products per market, even when using conservative product / market definitions. All listings are active for at most 31 days, and most sellers pick the pre-set 5, 7 or 10 day duration. In this paragraph, I therefore use conservative one month periods to define a market. The BW site has filters for high level characteristics corresponding to the idea that potential bidders enter the site with at least a rough

21 Ullage describes the unfilled space in a container; in wine auctions it refers to visible oxidation of the wine. For example, a “Base of Neck” fill level is better than “Top Shoulder”. These apply to wines in Bordeaux-style bottles with a visible neck and shoulders; a metric classification is used for Burgundy-style bottles (see Figure 8).

22In previous literature, auction platform models are estimated using data from Kindle e-readers (in Bodoh-Creed et al. (2013, 2016)), indistinguishable CPU’s (in Anwar et al. (2006)) and pop CD’s (in Nekipelov (2007)).
Table 3: Descriptive statistics: thin markets

<table>
<thead>
<tr>
<th>Number of times a product is listed</th>
<th>Per market, percentiles</th>
<th>Total over 15 months, percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

This table is based on conservative product-market specifications. In this table, products are combinations of: region of origin, wine type, and vintage decade; markets are 4 week intervals. The Independent listings section on page 10 describes other specifications used.

The idea of the product they are looking for. As product specification, I take the combination of three high level filters: i) region of origin, ii) vintage decade and iii) wine type. For example, a red Bordeaux from the 1980s and a non-vintage Champagne are distinct products by that definition. Even with these relatively coarse product-market specifications, for 50 percent of listings this is the only one of that product offered in that market and for another 20 percent there are only two of these products available (see Table 3). Half of the products have been listed only 28 times during the full 15 months spanning my data, conditional on having been offered at least once.

2) Non-selective bidder entry. Whether bidders do or don’t know their valuation for the listed products before they enter the platform is crucial in the way bidder entry affects outcomes. Which case is likely to describe my data generating process can be tested; in a selective entry model valuations are lower in the first order stochastic dominance sense when more bidders enter the platform. Estimates presented in Figure 1 contest such a selective bidder entry process. It shows that estimated distributions of second-highest bids are similar for above-median and below-median bidder platform entry, evaluated separately for auctions with 2-9 bidders. The same conclusion can be drawn from a reduced-form OLS regression of transaction prices on the number of bidders in the auction and total number of bidders on all comparable listings in the same market, also when controlling for product fixed effects in Table 4. Reported patterns are consistent with non-selective bidder entry and suggest that an extra bidder in an auction is associated with a transaction price that is on average 26-27 pounds higher. These documented empirical patterns are consistent with bidders needing to inspect a listing before learning specifics of the wine and how much they value these specifics.

23 If they are fully informed before they enter, as in the Samuelson (1985) selective entry model, every additional bidder has a lower valuation so entry affects prices less than when they don’t know their valuation when entering, as in Levin and Smith (1994). Roberts and Sweeting (2010) and Gentry and Li (2014) capture both scenario’s as polar cases in their flexible entry models.

24 Distributions are estimated from auctions without a reserve price. They are obtained with fitting nonparametric epanechnikov Kernels with optimal cross-validated bandwidths on transaction prices from auctions with below or above median bidder participation. For example, it compares transaction prices in months where non-vintage Champagne is more popular - in the sense of attracting more total bidders on this type of wine - with months where there is lower bidder interest for these wines. Not observing the pool of potential bidders precludes me from testing selection
Table 4: Descriptive statistics: non-selective bidder entry

<table>
<thead>
<tr>
<th>Dependent variable: Transaction price (OLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number bidders in auction</td>
</tr>
<tr>
<td>Total number bidders product/market</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product fixed effects</th>
<th>Observations</th>
<th>Adj. R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>1,218</td>
<td>0.163</td>
</tr>
<tr>
<td>Yes</td>
<td>1,218</td>
<td>0.143</td>
</tr>
</tbody>
</table>

**∗∗∗**: Significant at the 1% level, standard errors in square parenthesis. In this preliminary analysis, product fixed effects here are high-level observables: wine type, region of origin, decade of production; markets are 4 week intervals.

Figure 1: Estimated CDF transaction prices; x-axis: value. y-axis: probability

Black dotted line: estimated second-highest bid distribution for below-median total number bidders per product/market.
Cyan solid line: estimated second-highest bid distribution for above-median total number bidders per product/market.

In these graphs, products are combinations of: region of origin, wine type, and vintage decade; markets are 4 week intervals.

3) Independent listings. Previous papers show that transaction and reserve prices in homogeneous good auctions can be affected by the number of competing listings. An ordinary least squares regression analysis, detailed in Appendix C suggests that, in the wine auction data, listings are not systematically related despite ending in close proximity of each other and offering similar

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25 Peters and Severinov (1997) and Anwar et al. (2006) consider cross-bidding, motivated by the absence of listing-specific entry cost in auctions for homogeneous products. The incremental cross-bidding strategy requires bidders to always submit a bid on the auction with the lowest standing price, and only one increment above the standing price (e.g., not submit a bid once equal to their valuation). As I don’t observe bidder identities, I cannot examine the incremental cross-bidding strategy directly, but a strong clue for the absence of it is that on average all bidders place only 1.7 bid (median: 1.5) so at least it cannot be a very prevalent strategy. To wit, not every bidder can be placing two bids or more in the same auction (but they could still bid once or twice in many competing auctions - if available). Another suggestion is that the majority of winning bidders that left feedback has only won an auction (and left feedback on it) once or twice (58 percent) over the entire 15 months period.
items. This conclusion is robust to using different product and market specifications. Dependent variables analyzed are: i) the number of bidders per listing, ii) the number of bids per bidder, iii) the transaction price and iv) the reserve price. The results rely on cross-market variation in the number of listings of a certain product. The different market specifications considered are all auctions ending within a rolling window of: i) 30 days, ii) 7 days, and iii) 2 days of each other. Product specifications also vary. The coefficient on competing listings is in 68 out of the resulting 72 regressions statistically insignificant at the 10 percent level. To rule out that there are non-linear effects, the absence of a clear structural relation between the number of (competing) listings and these outcomes of interest is also confirmed with data visualizations.

The fact that reserve prices are not affected by competing listings is intuitive since most of them are kept secret. As bidders cannot select on what they cannot observe, there is no motive for sellers to compete on that margin. The absence of a cross-bidding strategy, as suggested by the constant number of bids per bidder, can be explained by the accumulation of listing inspection cost associated with that strategy. Overall, the fact that transaction prices do not decrease with the number of competing listings points to the absence of a “business stealing” effect and is also consistent with bidders entering and bidding in one listing at a time.

4) Network effects. Network effects describe that a product is more valuable to a group of users when it is more widely adopted by another group. In our auction platform setting, network effects arise mechanically from the fact that transaction prices are endogenous to the number of bidders per listing. As bidders sort over available listings, a platform with more listings is more attractive to potential bidders c.p., and vice versa. This positive feedback effect is observed from the positive correlation between the number of total bidders and the availability of listings after controlling for product fixed effects (left-hand panel of Figure 2). The pattern also persists when controlling for a time trend.

An equilibrium prediction from a model in which (reserve) prices are unaffected by the number of listings, as shown in the next section, is that the mean number of bidders per listing is also independent of the number of listings. The right-hand panel of Figure 2 supports this pattern in the BW auction data. Given that the fee structure is fixed in the data, additional listings are not associated with higher cost sellers populating the platform. Network effects are such that potential bidders enter to the point of keeping the mean number of bidders per listing constant. Reported coefficients in Appendix C confirm that this result is robust to different product/market specifications (Table 11 Column 1).

Implications for structural model. Informed by these empirical patterns, the structural model considers a platform where bidders have a constant cost of inspecting a listing and therefore bid in one listing at a time. The game is static in correspondence with the empirical pattern of a low

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26 In contrast, Newberry (2015) show that in eBay auctions for Corvettes more listings result in the thinning of bidders per listing. The constant mean number of bidders per listing in my data disproves bidder thinning.

27 See Katz and Shapiro (1985) and Rochet and Tirole (2006).
Figure 2: Patterns suggesting that additional listings attract additional bidders but the mean number of bidders per listing remains constant.

Figures are based on data from auctions with no reserve price in which the number of bidders is directly observed. The blue solid lines represent the estimated coefficients in OLS regressions: on the left a slope of 0.7 (statistically significant at the 1 percent level) and on the right an insignificant 0. The residual total bidders in a) is obtained from a linear regression of this outcome in market $m$ on product dummies and the residual bidders per listing in b) is obtained from a linear regression of this outcome for product $p$ in market $m$ on product dummies. The left-hand graph shows that, for example, markets with more listings of non-vintage champagne attract more bidders on non-vintage champagne listings while the right-hand graph suggests that bidders enter only to keep the mean number of bidders on non-vintage champagnes constant across markets. In these graphs, products are combinations of: region of origin, wine type, and vintage decade; markets are 4 week intervals.

The presence of other auction platforms for wine besides BW is captured by the opportunity cost of entering and trading on BW. Hence, the (partial) equilibrium analysis is based on an implicit assumption that competing platforms keep their fee structure unchanged.29

3 A model of an auction platform for idiosyncratic goods

In this section, I model bidder and seller behavior on the platform as a static multi-stage game and study its equilibrium properties.

3.1 Model assumptions and game structure

Risk-neutral potential bidders and sellers consider trading on a monopoly platform with a given fee structure and furthermore have opportunity cost of doing so. Bidders have unit demands. The

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28Figure 10 in the Appendix.
29This is justified by BW being a small platform; the assumption would be more restrictive using eBay data unless you consider it to be a monopolist in the relevant market.
allocation mechanism in each listing is an English auction with flexible end time and proxy bidding. Assumptions on the matching process and model primitives that are maintained throughout are:

**Assumption 1.** Bidders bid in one listing at a time and enter available listings with equal probability.

This assumption can be justified on the basis that bidders learn about the wine’s details only after they enter the product page and spend time inspecting it. Reserve prices are secret. In estimation, I implement the uniform allocation assumption conditional on the observed “no reserve price button”, allowing bidders to enter such auctions more numerously.

While the valuations of bidders and sellers may be correlated by their common appreciation of certain wine characteristics, their individual tastes are the basis of the following assumption:\(^{30}\)

**Assumption 2.** Conditional on the vector of observed wine attributes, variation in valuations across buyers and sellers is of a purely idiosyncratic -private values- nature. In addition, the idiosyncratic variation is independent.

Independence is needed for identification of the distribution of idiosyncratic bidder valuations, but on the seller side it can be relaxed to unrestricted private values. The two conditional distribution functions are assumed to satisfy standard regularity conditions:

**Assumption 3.** The distribution functions of idiosyncratic buyer and seller values are: i) absolutely continuous, ii) defined on a bounded support, and iii) characterized by an increasing failure rate (IFR).

Continuity is needed for identification of the distribution of bidder valuations, but it could be omitted on the seller side. IFR is a standard restriction that guarantees uniqueness of the optimal reserve price, and is also not needed on the seller side.\(^ {31}\)

Zero reserve price auctions attract more bidders, but the benefit of setting a positive reserve price increases in the seller valuation. Combined with a positive reserve price fee, the set of sellers that sets a zero reserve price is determined by a threshold-crossing problem. I chose not to endogenize this threshold (which I refer to as “screening value \(v_{0,r=0}\)) in the baseline model. Doing so significantly complicates the estimation of the game. Instead, \(v_{0,r=0}\) is assumed fixed.

The valuation distributions, allocation mechanism, population sizes, and all cost (fees and opportunity cost) are common knowledge. While relatively parsimonious, this model captures the main features of an auction platform for idiosyncratic goods detailed in Section 2. The incomplete

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\(^{30}\)The assumption is also justified by the elaborate data collection effort. In particular, it rules out the existence of wine features observable to bidders and sellers that are not excluded from their valuations and that are unobserved to the econometrician. Such features would render the private valuations of bidders and sellers affiliated in the sense of Milgrom and Weber (1982) and Aradillas-López (2016), even after controlling for observed wine attributes.

\(^{31}\)With no reason to assume that their taste distributions differ, in estimation I use the same parametric restrictions on both sides (although I estimate parameters for bidders and sellers separately) and therefore I use a model with an identical set of restrictions on these distributions.
information structure and strategic interaction makes this suitable to study with the usual game-theoretic tools.

**Timing of the game.**

- **Entry stage** ($t=1$): Potential sellers learn their valuation and decide whether to enter and simultaneously, bidders decide whether to enter.
- **Auction stage** ($t=2$): Sellers set a reserve price.
- **$t=3$: Bidders learn their valuation and bid.

My analysis uses only one bid order statistic: the second-highest; and its relation to the second-highest valuation. English auctions generally allow for bidding strategies that complicate a tight mapping between observed bids and unobserved valuations (Haile and Tamer (2003)), and I rely on the following assumption to restrict bidder behavior:\footnote{Aradillas-López et al. (2013) previously adopt this assumption in an English auction model.}

**Assumption 4.** The transaction price in each auction is the greater of the second-highest bidder’s willingness to pay and the reserve price.

This assumption can be justified in different ways. It is the exact outcome of a model that imposes only the behavioral assumptions of Haile and Tamer (2003) that: 1) bidders never bid more than their valuation, and 2) never let someone else win at a price they are willing to beat, in the case of infinitesimal bidding increments and accounting for buyer’s premium. With these intuitive behavioral assumptions, Assumption 4 therefore holds approximately to within one bidding increment. The assumed transaction price could also be derived from the more restrictive behavioral “button auction model” of Milgrom and Weber (1982) in the independent private values case.

**Notation.**

Let fee structure $f = \{c_B, c_S, e_B, e_S\}$ consist of respectively, buyer’s premium, seller commission, buyer entry cost, seller entry cost (listing fee). Opportunity cost of time equal ($e_{B,r=0}^o, e_{B,r>0}^o$) for potential bidders in respectively zero reserve and regular positive reserve price auctions (allowing for the inspection cost to differ) and $e_S^o$ for potential sellers. Random vector $Z$ contains auction covariates observed at the product-page. $N^B, N^S, M, \text{ and } T$ respectively denote the number of: potential bidders, potential sellers, bidders (on the platform), and sellers (listings) on the platform. $F_{V|Z}$ and $F_{V'|Z}$ respectively denote the conditional valuation distributions for potential sellers and bidders, defined on bounded supports $V_0 \in [v_0, \bar{v}_0]$ and $V \in [v, \bar{v}]$. Random variables are denoted in upper case and their realizations in lower case. Furthermore, $v_0$ is the realized valuation of a generic seller while $v_{0k}$ indicates the realized valuation for (potential) seller $k$. Similarly, $v$ denotes the realized valuation for a generic bidder while $v_i$ is used to denote the valuation of bidder $i$. Order statistics are useful as well: $X_{(n,n)}$ and $X_{(n-1,n)}$ denote respectively the highest and second-highest draw from a sample of size $n$ from random variable $X$. $\mathbb{I}$ denotes the indicator function and stars...
denote equilibrium values. Additional notation will be introduced where necessary.

**Payoffs.**

The payoff for bidder $i$ is simply his valuation $v_i$ minus the transaction price increased with buyer premium if he wins the auction. On top of that, regardless of whether he wins, by entering he foregoes entry and opportunity cost. The transaction price is the maximum of the second-highest bid and reserve price $r$ denoted by placeholder $H$ here:

$$
\pi_b(v_i, H) = \begin{cases} 
  v_i - H(1 + c_B) - e_B - e^o_B & : \text{for a bidder with valuation } v_i \text{ who wins the auction} \\
  -e_B - e^o_B & : \text{for a bidder with valuation } v_i \text{ who fails to win} \\
  0 & : \text{otherwise}
\end{cases}
$$

The payoff for a seller is the transaction price decreased with seller commission minus entry and opportunity cost if he sells. If he lists the good for sale but it does not sell, he only foregoes entry and opportunity cost but he does enjoy his valuation $v_{0k}$:

$$
\pi_s(v_{0k}, H) = \begin{cases} 
  H(1 - c_S) - e_S - e^o_S & : \text{for a seller who sells his lot} \\
  v_{0k} - e_S - e^o_S & : \text{for a seller with valuation } v_{0k} \text{ who fails to sell} \\
  v_{0k} & : \text{otherwise}
\end{cases}
$$

### 3.2 Equilibrium strategies

In this section, I solve for players equilibrium strategies. Considering two distinct stages of entry and auction, and given symmetry up to players’ private valuations, I restrict attention to symmetric Perfect Bayesian-Nash Equilibria (PBE) in weakly undominated strategies. This equilibrium concept requires that strategies are best responses given competitors’ strategies, and that beliefs are consistent with those strategies in equilibrium.

#### 3.2.1 Auction stage

Conditional on entry decisions and the matching of bidders to listings, the idiosyncratic-good auction platform is made up of independent English auctions. I therefore derive standard reserve pricing (as in: Riley and Samuelson (1981)) and bidding (as in: Vickrey (1961)) strategies, up to the impact of buyer premium and seller commission.

**Lemma 1.** It is a weakly undominated strategy for a bidder with valuation $v$ to bid:

$$
b^*(v, f) \equiv \frac{v}{1 + c_B} \tag{1}
$$

**Proof.** This follows directly from Vickrey (1961), as bidding more than $\frac{v}{1 + c_B}$ may result in negative utility and bidding less than $\frac{v}{1 + c_B}$ decreases the probability of winning without affecting the
transaction price in that case.

Only the second-highest bid is relevant for this game and while other strategies are allowed to have been played by bidders with lower valuations (by Assumption 4), those strategies are not dominated by the strategy in Lemma 1 as they would lead to the same payoffs.

For sellers who set a positive reserve price, the optimal reserve price strategy is described by the familiar Riley and Samuelson (1981) formula:

**Lemma 2.** For sellers with valuation $v_0 \geq v_{0,r=0}^*$, it is a weakly undominated strategy to set a secret reserve price that solves:

$$r^*(v_0, f) = \frac{v_0}{1 - c_S} + \frac{1 - F_{V|Z}(1 + c_B)^{r^*(v_0, f)}}{(1 + c_B)f_{V|Z}((1 + c_B)r^*(v_0, f))}$$  \hspace{1cm} (2)

$r^*(v_0, f)$ is increasing in $c_S$ and decreasing in $c_B$.

The proof is provided in Appendix G. Note that, if $c_S = c_B = 0$, the optimal reserve price is identical to the Riley and Samuelson (1981) public reserve price in auctions with a fixed number of bidders. It is easy to see why this is the case. Because $r^*(v_0, f)$ is secret, it does not affect the number of bidders in the seller’s listing. This is true for any reserve price strategy of competing sellers.\(^{33}\)

### 3.2.2 Entry stage

With their valuations materialising in the auction stage, $N^B$ identical potential bidders adopt a mixed strategy to enter with a probability that in equilibrium leaves their opponents indifferent between entering and staying out, as in Levin and Smith (1994). Technically, the auction platform model with both zero and positive reserve prices demands splitting $N^B$ into a population of potential bidders for zero and one for positive reserve price auctions ($N^B_{r=0}, N^B_{r>0}$) and deriving the two relevant mixing probabilities ($p_{r=0}, p_{r>0}$). By contrast, the $N^S$ potential sellers know their valuation $v_0$ so they enter selectively. Their expected surplus decreases in $v_0$, so they adopt the pure strategy to enter only if their valuation is below a threshold value that in equilibrium makes the marginal seller indifferent between entering and staying out given that his opponents adopt the same threshold strategy. In what follows, I denote the sellers’ entry strategy by that equilibrium threshold value, $v_0^*$. Note that unless it is optimal for all sellers on the platform to set a zero reserve price, the seller who is indifferent between entering and staying out will set a positive reserve price: $\bar{v}_0 \leq v_{0,r=0} \leq v_0^* \leq \bar{v}_0$. In this section I restrict attention to the case where all these inequalities are strict. First, because in my data about one third of sellers (not none, not everyone) sets a zero reserve price so the marginal entrant finds it optimal to set a positive reserve price. Second, entry

\(^{33}\)Sellers could be better off if they would collectively adopt (e.g., find a way to enforce) a different reserve price rule. In particular, a rule that results in a lower reserve price for any $(v_0, f)$ increases bidder entry.
The following proposition summarizes key results about the entry equilibrium in this game.

**Proposition 1.** The entry stage of the game results in a unique equilibrium for any fee structure. It is characterized by: i) a bidder entry probability (for positive reserve price auctions), ii) a seller entry threshold \((p^*_r > 0(f, v_0^*(f)), v_0^*(f))\) that jointly solve: 1) potential bidders’ zero profit condition in positive reserve price auctions, and 2) the marginal seller’s zero profit condition, and iii) a bidder entry probability (for no reserve price auctions, \(p^*_r = 0(f)\)) that solves potential bidders’ zero profit condition in zero reserve price auctions.

The remainder of this section derives the entry equilibrium. I first show that any candidate seller entry threshold, \(\bar{v}_0\), maps to an equilibrium bidder entry probability in positive reserve price auctions, \(p^*(f, \bar{v}_0), r > 0\). Higher seller values increase the expected reserve price, so \(p^*(f, \bar{v}_0), r > 0\) is strictly decreasing in \(\bar{v}_0\). As a result, sellers are strategic substitutes and the entry game reduces to a single agent discrete choice problem. Because the seller with \(v_0 = v_0^\ast\) sets a positive reserve price, this describes entry into positive reserve price auctions. By contrast, bidder entry into auctions with a zero reserve price does not depend on the expected value of those seller’s valuations, which don’t affect the reserve price. Hence \(p^*_r = 0(f)\) is easily obtained as the value that solves a breakeven entry condition for those auctions independent of seller side behavior. I first derive \(p^*_r > 0(f, v_0^*(f))\) and \(v_0^*(f)\), before turning to \(p^*_r = 0(f)\). For additional intuition behind the entry equilibrium, additional material including specification of listing-level surplus for bidders and sellers is presented in Appendix D.

**Bidder entry in auctions with a positive reserve price.**

Let \(\pi_b(n, f, v_0), r > 0\) denote the ex-ante expected surplus for a bidder arriving in a listing with \(n - 1\) other bidders, fee structure \(f\), and seller valuation \(v_0\). The seller valuation enters \(\pi_b\) through optimal reserve price \(r^*(v_0, f)\). In fact, this is why **seller selection** matters to bidders: \(\pi_b(n, f, v_0)\) is strictly decreasing in \(v_0\) in positive reserve price auctions. Being unobserved to bidders, they form an expectation over \(V_0\) using \(\bar{v}_0\): \(\mathbb{E}[\pi_b(n + 1, f, v_0)|V_0 \in [v_0, r > 0, \bar{v}_0]\). They also form an expectation over the number of competing bidders in their listing, using its compound Binomial distribution, \(f_{N, r > 0}(n ; p, \bar{v}_0)\). From the perspective of a bidder who enters the platform, \(f_{N, r > 0}(n ; p, \bar{v}_0)\) combines uncertainty about: 1) the stochastic number of listings \(T\) (with realization \(t\)) given entry threshold \(\bar{v}_0\), and 2) how many of \(N^B - 1\) competing bidders end up in his listing when they enter the platform with probability \(p\) and sort uniformly over available listings. Implementing part 1), let \(F_{V_0|Z, v_0, r = 0}\) denote the left-censored distribution of seller valuations:

\[
F_{V_0|Z, v_0, r = 0}(v_0) = \frac{F_{V_0|Z}(v_0) - F_{V_0|Z}(v_0, r = 0)}{F_{V_0|Z}(v_0, r = 0)},
\]

\(^{34}\)I exclude the no-trade equilibrium also in counterfactual analysis, but I do allow for some alternative fee structures to result in all sellers setting zero reserve price. This happens only when the entry probability drops so drastically the marginal seller has a valuation equal to \(v_0, r = 0\). Fee structures that result in even fewer listings are likely unattractive for the platform as fewer listings are not compensated by increased bidder entry into those listings.
∀v_0 ∈ [v_0, r = 0, v_0]. Combined with entry and opportunity cost, Π_b(f, \bar{v}_0; p) denotes potential bidders’ expected surplus from entering the platform:

\[
\Pi_b(f, \bar{v}_0; p) = N^B \sum_{n=0}^{N^B-1} E[\pi_0(n + 1, f, v_0)|V_0 ∈ [v_0, r = 0, v_0]f_{N,r>0}(n; p, \bar{v}_0) − e_B − e^0_{B,r>0}]
\]

f_{N,r>0}(n; p, \bar{v}_0) = N^B \sum_{t=0}^{N^S} \left( \frac{N^B-1}{n} \right) \left( p \right)^n \left( 1 - \frac{p}{t} \right)^{N^B-1-n} \left( \frac{N^S}{t} \right) F_{V_0|Z,v_0,r=0}(\bar{v}_0)^f(1 - F_{V_0|Z,v_0,r=0}(\bar{v}_0))^{N^S-t}

Lemma 3. Given candidate seller entry threshold \bar{v}_0 and fee structure f, the equilibrium bidder entry probability solves the zero profit condition:

\[
p^*_r>0(f, \bar{v}_0) \equiv \arg_{p ∈ (0,1)} \{\Pi_b(f, \bar{v}_0; p) = 0\}
\]

Equilibrium properties are:

i) p^*_r>0(f, \bar{v}_0) is unique ∀(f, \bar{v}_0)

ii) p^*_r>0(f, \bar{v}_0) is strictly decreasing in (\bar{v}_0, c_B, c_S, e_B, e^0_{B,r>0}) so also f_{N,r>0}(n; p^*_r>0, \bar{v}_0) decreases in the first-order stochastic dominance sense in (\bar{v}_0, c_B, c_S, e_B, e^0_{B,r>0})

iii) f_{N,r>0}(n; p^*_r>0, \bar{v}_0) is invariant to changes in N^B or N^S

Proof is relegated to Appendix G. Crucial is that the selection of less “serious” sellers, through an increase in \bar{v}_0, reduces expected bidder listing-level surplus. That decreases their equilibrium entry probability so that f_{N,r>0} places more weight on lower realizations of the number of bidders per listing. The same holds for increases in buyer premium, seller commission, and bidder entry and opportunity cost; since they all decrease expected listing-level surplus. Population sizes on the other hand do not directly affect bidder surplus, so the zero profit condition dictates that in equilibrium f_{N,r>0}(n; p^*_r>0, \bar{v}_0) is not affected by it. This also relates to the equilibrium prediction that is referred to in the network effects section on page 11: without affecting seller selection, more listings increase the number of bidders, but only to keep the mean number of bidders per listing constant.

Seller entry.
Potential sellers’ expected surplus from entering the platform involves: 1) their listing-level expected surplus, and 2) an expectation over the number of bidders per listing, N, r > 0, given \bar{v}_0 and bidders’ equilibrium best-response to this threshold. Let Π_s(f, v_0; p^*_r>0(f, \bar{v}_0), \bar{v}_0) denote expected surplus for a seller with valuation v_0 when N^S − 1 competing sellers enter the platform if and only if their
valuation is less than threshold \( \bar{v}_0 \):\(^{35}\)

\[
\Pi_s(f, v_0; p^*_r > 0(f, \bar{v}_0), \bar{v}_0) = \sum_{n=0}^{N^B} \pi_s(n, f, v_0) f_{N,r=0}(n; p^*_r > 0(f, \bar{v}_0), \bar{v}_0) - e_S - e_S^0 \tag{7}
\]

Lemma 4. Given fee structure \( f \), the equilibrium seller entry threshold solves the marginal seller’s zero profit condition:

\[
v^*_0(f) \equiv \arg_{\bar{v}_0 \in (0, 1)} \left\{ \Pi_s(f, \bar{v}_0; p^*_r > 0(f, \bar{v}_0)) = 0 \right\} , \text{ with } p^*_r > 0(f, \bar{v}_0) \text{ solving } (6) \tag{8}
\]

Equilibrium properties are:

i) \( v^*_0(f) \) is unique \( \forall f \)

ii) \( v^*_0(f) \) is strictly decreasing in \( e_B \)

iii) The impact of \( (c_B, c_S, e_S, e_S^0) \) is ambiguous

Proof is relegated to Appendix G. Key take-aways are the following. Sellers have a unique best response for any competing seller entry (candidate) threshold, because sellers expected surplus \( (\Pi_s(f, v_0; p^*_r > 0(f, \bar{v}_0)) \) is strictly decreasing in \( v_0 \). Crucially, given that 1) \( p^*_r > 0(f, \bar{v}_0) \) is strictly decreasing in \( \bar{v}_0 \), and 2) entry of competing sellers does not affect seller surplus in other ways, the best response function is strictly decreasing in competing sellers entry threshold. Symmetry then delivers a unique equilibrium threshold, \( v^*_0(f) \), that is the fixed point in seller value space solving equation 8 i.e., making the marginal seller indifferent between entering and staying out.

Bidder entry in auctions with no reserve price.

Let \( \pi_b(n, f)_{r=0} \) denote the ex-ante expected surplus for a bidder arriving in a listing without reserve price, with \( n - 1 \) other bidders and fee structure \( f \). Crucially, the seller valuation does not affect \( \pi_b(n, f)_{r=0} \) because the reserve price is fixed at zero. As such, \( N^B_{r=0} \) potential bidders only form an expectation over the number of competing bidders in their listing if all enter with probability \( p \), using its compound Binomial distribution, \( f_{N,r=0}(n; p) \). From the perspective of a bidder who enters the platform, \( f_{N,r=0}(n; p) \) combines uncertainty about: 1) the stochastic number of listings \( T \) (with realization \( t \)) given screening value \( v_0,r=0 \) and entry threshold \( v^*_0 \), and 2) how many of \( N^B_{r=0} - 1 \) competing bidders end up in his listing when they enter the platform with probability \( p \) and sort uniformly over available listings with zero reserve. To implement the expectation in 1), let \( F_{V_0|Z,v^*_0}(v_0,r=0) \) denote the share of sellers on the platform with a value less than the screening value:

\[
F_{V_0|Z,v^*_0}(v_0,r=0) \equiv \frac{F_{V_0|Z}(v_0,r=0)}{F_{V_0|Z}(v^*_0)} \tag{9}
\]

\(^{35}\)A slight abuse of notation is that expectation involves \( f_{N,r=0}(n; p^*_r > 0, \bar{v}_0) \) (characterizing entry among \( N^B_{r=0} - 1 \) potential bidders) as defined in (5) instead of the distribution based on the full bidder population \( N^B_{r=0} \). This avoids introduction of additional notation and the \(-1\) will be irrelevant in the large-\( N^B_{r=0} \) approximation adopted for empirical tractability (page 20). In that world, the two distributions are identical by the \textit{environmental equivalence} property of the Poisson distribution (Myerson (1998)).
Combined with entry and opportunity cost, $\Pi_{b,r=0}(f;p)$ denotes the expected surplus from entering the platform for these potential bidders:

$$\Pi_{b,r=0}(f;p) = \sum_{n=0}^{N_{B,r=0}^B - 1} \pi_b(n, f) r_{N_{r=0}^B}(n; p) - e_B - e_{B,r=0}^o$$

$$f_{N,r=0}(n;p) = \sum_{t=0}^{N_{B,r=0}^S} \left( \frac{N_{B,r=0}^B - 1}{n} \right) \left( \frac{p}{t} \right)^n \left( 1 - \frac{p}{t} \right)^{N_{B,r=0}^B - 1 - n} \left( \frac{N_{S,t}^S}{t} \right) F_{V_0|Z,v_0^r(0,r=0)}(1 - F_{V_0|Z,v_0^r(0,r=0)})(N_{S,t}^S - t)$$

**Lemma 5.** Given fee structure $f$, the equilibrium bidder entry probability solves the zero profit condition:

$$p^*_r(f) \equiv \arg_{p \in (0,1)} \{ \Pi_{b,r=0}(f;p) = 0 \}$$

**Equilibrium properties are:**

i) $p^*_r(f)$ is unique $\forall(f)$

ii) $p^*_r(f)$ is strictly decreasing in $(c_S, c_B, e_B, e_{B,r=0}^o)$ so also $f_{N,r=0}(n;p^*_r=0)$ decreases in the first-order stochastic dominance sense in $(c_S, c_B, e_B, e_{r=0}^o)$

iii) $f_{N,r=0}(n;p^*_r=0)$ is invariant to changes in $N_{B,r=0}^B$ or $N_{S}^S$

iv) $f_{N,r=0}(n;p^*_r=0)$ is invariant to changes in $v_0^r$

Proof is relegated to Appendix G.

**Corollary 1.** The entry equilibrium of the auction platform game is characterized by the pair of $(v_0^r(f), p^*_r>0(f), v_0^s(f)))$ that solves equation (8) and $p^*_r=0(f)$ that solves equation (12), which are unique for any fee structure.

**Large population approximation.**

The remainder of this section discusses an approximation of the entry equilibrium that is adopted for empirical tractability. A second reason is that the approximation relaxes the requirement that players know population sizes $N_{B,r=0}^B$, $N_{B,r=0}^B$, and $N_{S}^S$, which indeed are likely to be unobserved by potential bidders and sellers.\(^\text{36}\)

**Assumption 5.** The population of potential bidders is large relative to the number of bidders on the platform: $(N_{r>0}^B, N_{r=0}^B) \to \infty$ and $(p^*_r>0, p^*_r=0) \to 0$.\(^\text{37}\)

Under this assumption, the number of bidders per listing $(N_{r>0}, N_{r=0})$ in (5) and (11) are approximately Poisson distributed, and approximation error relative to the Binomial distribution

---

\(^{36}\)Relatedly, given that the population of potential bidders is likely to be large relative to the actual number of bidders, the Poisson approximation of the binomial distribution is a natural one, also adopted previously in similar settings by e.g. Engelbrecht-Wiggans (2001), Bajari and Hortacsu (2003) and Jehiel and Lamy (2015).

\(^{37}\)To avoid any misinterpretation (with $p^*$ endogenous), the population is assumed to be large and the entry probability is assumed to be small and it is not a statement about letting the population grow large or the entry probability go to 0.
Athey and Haile (2002, Theorem 1) prove identification of \( F_{V|Z} \) in an English auction model that

\[ f_N(k; \lambda^*_r, R^0) = \frac{\exp(-[\lambda^*_{\lambda>0}(1-R^0) + \lambda^*_{\lambda=0}(R^0)])([\lambda^*_{\lambda>0}(1-R^0)]^k}{k!}, \forall k \in \mathbb{Z}^+ \]  

where \( R^0 \) is a dummy variable equal to one in the event of a zero reserve price.

---

38 The number of bidders in any type of auction follows a conditional Poisson distribution:

\[ f_N(k; \lambda^*_r, R^0) = \frac{\exp(-([\lambda^*_{\lambda>0}(1-R^0) + \lambda^*_{\lambda=0}(R^0)](1-R^0) + \lambda^*_{\lambda=0}(R^0)^k)}{k!}, \forall k \in \mathbb{Z}^+ \]
places identical restrictions on this distribution up to the presence of binding reserve prices. Their proof relies on the relationship between the distribution of the second-highest draw (valuation) in a sample of known size (number of bidders) from its parent distribution and that parent distribution (see also e.g., Arnold et al. (1992)):

\[ F_{V(n-1:n)}(v) = n(n-1) \int_{v}^{v} F_{V(t)}^{n-2}(1 - F_{V}(t))f_{V}(t)dt \equiv \phi(F_{V}(v); n) \]  

(15)

When \( n \) is known, given that \( \phi(F_{V}(v); n) \) is strictly increasing in \( F_{V}(v) \), \( F_{V} \) is identified whenever \( F_{V(n-1:n)} \) is. In particular, \( F_{V}(v) \) is identified point-wise \( \forall v \in [\underline{v}, \bar{v}] \) by inverting \( \phi(\cdot; n) \):

\[ F_{V}(v) = \phi^{-1}(F_{V(n-1:n)}(v); n) \]  

(16)

This argument also applies conditional on observed \( Z \), so identification of \( F_{V|Z} \) follows. What remains to be shown is identification of \( F_{V(n-1:n)} \), which is slightly different from Athey and Haile (2002) due to the presence of binding reserve prices. In auctions without a reserve price, an event that is known, order statistic \( V(n-1:n) \) equals the transaction price and \( n \) is observed. Hence, replacing \( F_{V(n-1:n)} \) with the empirical distribution \( F_{B} \) from auctions without a reserve price in (16) completes the proof.

4.2 The distribution of seller valuations

Given that \( F_{V|Z} \) is identified, in all auctions with a positive reserve price the reserve price identifies the seller’s valuation in that listing. In particular, re-arranging the equilibrium reserve price strategy in Lemma 2:

\[ v_{0} = (1 - c_{S}) \left( r - \frac{1 - F_{V|Z}(r(1 + c_{B}))}{(1 + c_{B})f_{V|Z}(r(1 + c_{B}))} \right) = \bar{r}, \]  

(17)

where \( \bar{r} \) denotes the observed scalar-valued right-hand side. Its distribution function, \( F_{R} \), trivially identifies the distribution of valuations among sellers who enter the platform and set a positive reserve price, point-wise \( \forall v \in [v_{0,r=0}, v_{0}^{*}(f)] \):

\[ F_{R}(v) = \frac{F_{V_{0}|Z,v_{0,r=0}}(v)}{F_{V_{0}|Z,v_{0,r=0}}(v_{0}^{*}(f))} \]  

(18)

Dividing the number of listings, \( T \), by the population of potential sellers, \( N^{S} \), delivers the seller entry probability in the denominator of (18). The distribution of valuations in the population of potential sellers is identified pointwise \( \forall v \in [v_{0,r=0}, v_{0}^{*}(f)] \) as:

\[ F_{V_{0}|Z,v_{0,r=0}}(v) = F_{R}(v)\frac{T}{N^{S}} \]  

(19)

\( v_{0}^{*}(f) \) is the equilibrium seller entry threshold defined in (8). Previously, also Elyakime et al. (1994) identify seller cost using a first order optimality condition on the secret reserve price in first price auctions (in which case, the secret reserve price is equal to the seller’s valuation).
Without identifying variation in $v_0^r(f)$ and unless $v_0^s(f) = \bar{v}_0$, the population distribution $F_{V_0|z,v_0,r=0}(v)$ is not identified on the part of its support exceeding $v_0^s(f)$. It is worthwhile to point out that non-parametric identification of the right-truncated distribution of potential seller valuations in (18) is sufficient for any counterfactual that reduces $v_0^s(f)$, i.e. any scenario that reduces expected seller surplus ($\Pi_s(f,v_0; p_{r>0}^r(f,v_0),\bar{v}_0)$ defined in (7)). In any such scenario, only a subset of sellers currently trading on the platform will find it optimal to enter. As such, the distribution of valuations among sellers currently trading on the platform would be the relevant latent distribution necessary to characterize the counterfactual entry equilibrium and distributions of endogenous outcomes.

### 4.3 Opportunity cost

Opportunity cost are identified from the three zero profit conditions. In auctions with a zero reserve price the number of bidders is not truncated; observables from those auctions render the equilibrium distribution $f_N; p_{r=0}^r(f,v_0), v_0^s(f)$ identified. Other components of expected bidder surplus $\Pi_{b,r=0}(f,v_0^r(f); p_{r=0}^r(f,v_0^s(f)))$ (defined in (4), here referring to its equilibrium value) are: the distribution of bidder valuations (used in the definition of $\pi_s(n+1,f,v_0)$) and the right-truncated distribution of seller valuations (to take expectations of $\pi_s(n+1,f,v_0)$ over realizations of $V_0$), which are both identified, and observed $f$. $\Pi_{b,r>0}(f,v_0^r(f); p_{r>0}^r(f,v_0^s(f)))$ is strictly decreasing in the last remaining unobservable, opportunity cost $c_{r>0}^o$. Hence $c_{r>0}^o$ is identified as the value that solves the zero profit bidder entry condition in (6), setting $\Pi_{b,r>0}(f,v_0^r(f); p_{r>0}^r(f,v_0^s(f))) = 0$.

Similarly, the surplus for a marginal seller must by equilibrium play and the zero profit condition in (8) correspond to opportunity cost $c_{r>0}^o$. Surplus for the marginal seller, in equilibrium, $\Pi_s(f,v_0^s; p^r(f,v_0^s(f)), v_0^s(f)))$ is defined in (7). Besides $c_{r>0}^o$, it depends on: the identified distribution of bidder valuations (used in the definition of $\pi_s(n,f,v_0)$), observed fees $f$, the identified $f_N; p^r(f,v_0^s(f))$, and the value of $v_0^s(f)$. The latter is observed as the maximum implied seller valuation in (17). This is a valid basis for identification of $v_0^s(f)$ since identification analysis concerns a hypothetical environment with infinite data.\(^{40}\) As $\Pi_s(f,v_0^s; p^r(f,v_0^s(f)), v_0^s(f))$ is strictly decreasing in the seller opportunity cost, which is the last remaining unknown, $c_{r>0}^o$ is identified as the value that solves the marginal seller’s zero profit entry condition in (18), setting $\Pi_s(f,v_0^s; p^r(f,v_0^s(f)), v_0^s(f)) = 0$.

While the distribution of (potential) bidders is not immediately pinned down from observables with a positive reserve price, the joint distribution of reserve prices and number of potential bidders is. Intuitively, the distribution of the number of actual bidders $A$ in auctions with a given reserve price value pins down the distribution of $N_{r>0}$ given that the distribution of bidder valuations is identified. Technically, it is required that there exists one reserve price in the data that delivers variation in the observed number of actual bidders. $\Pi_{b,r=0}(f;p_{r=0}^r(f))$ is strictly decreasing in the last remaining unobservable, opportunity cost $c_{r=0}^o$. Hence $c_{r=0}^o$ is identified as the value that solves the zero profit bidder entry condition in (12), setting $\Pi_{b,r=0}(f;p_{r=0}^r(f)) = 0$.

\(^{40}\)By contrast, the finite-sample sample maximum of a noisy estimator may be far removed from the true entry threshold, as discussed in more detail in the estimation section.
Corollary 3. Given exogenous observables $X$ and endogenous observables $(A, B, R)$, the idiosyncratic-good auction platform model $M$ identifies $[F_{V|Z}, e_{S,r>0}^0, e_{B,r=0}^0]$ and identifies $F_{V_0|Z,v_0,r=0}$ right-truncated at $v_0^*(f)$.

These positive identification results are not altered when $(N_{r>0}^B, N_{r=0}^B, N^S)$ are unobserved and the large population assumption (Assumption 5) is added to the model. This is because: i) $f_{N,r=0}(p_{r=0}^*(f))$ is identified from observables in auctions without a reserve price, ii) $f_{N,r>0}(p_{r>0}^*(f,v_0^r), v_0^*(f))$ is identified from variation in the number of actual bidders in auctions with a positive reserve price (for any reserve price that delivers variation in $A$), iii) the expectations over values of $N_{r>0}$ in (4) and $N_{r=0}$ in (10) are then over an infinite support, and iv) the results don’t rely on population sizes otherwise.

5 Estimation method

I estimate a parametric specification of the model, allowing me on the seller side to extrapolate beyond the support on which $F_{V_0|Z,v_0,r=0}$ is identified. Parameters from the distributions of idiosyncratic bidder and seller values are estimated separately; I refer to these as bidder parameters ($\theta_b$) and seller parameters ($\theta_s$). Even when assuming that $F_{V_0|Z,v_0,r=0}(\theta_s)$ and $F_{V|Z}(\theta_b)$ are known up to finite-dimensional parameters, the fact that the entry equilibrium depends on those parameters complicates estimation. The equilibrium $v_0^*(f, \theta_s, \theta_b)$ is the solution to a fixed point problem that itself depends on a threshold-crossing problem on the bidder side, $\lambda_{r>0}^*(f,v_0^r(\theta_s, \theta_b), \theta_b)$. This equilibrium is computationally costly to compute for each set of candidate parameters, making full maximum likelihood estimation of all parameters infeasible. I adopt a multi-step estimation method that is based on:

1) controlling for auction heterogeneity $Z$ (using the homogenization step in Haile et al. (2003), also used for ascending auctions in e.g., Bajari and Hortaçsu (2003) and Freyberger and Larsen (2017))

2) estimating $\theta_b$ by maximum likelihood (as in e.g. Donald and Paarsch (1996) and Paarsch (1997)), using homogenized bids

3) estimating $\theta_s$ by maximum concentrated likelihood (mentioned in Donald and Paarsch (1993, Footnote 4) to overcome a support problem in first price auctions), using homogenized reserve prices Small sample estimation error from steps 1 and 2 affect the estimation of $\theta_s$, especially because it involves the sample maximum of estimated seller values in equation 17. I therefore add the following steps:

4) solving for the entry equilibrium given estimated parameters

5) re-estimating seller parameters at the updated entry equilibrium

These last two steps can be iterated on until convergence, but for any number of iterations this method delivers a consistent estimate of $\theta_s$ (shown in Aguirregabiria and Mira (2002)).\footnote{To see why I adopt this algorithm, notice that the seller entry problem resembles a discrete choice programming problem and that the three referenced estimators based on maximum likelihood, maximum concentrated likelihood}
Monte Carlo simulations, the estimation includes only one iteration on steps 4-5 (see Appendix E for Monte Carlo results and Appendix F for details about the necessary numerical approximation of the entry equilibrium). The rest of this section provides estimation details.

5.1 Auction heterogeneity: homogenizing bids and reserve prices

Considering that valuations for wines auctioned at the BW platform consist of both a common value element (due to the importance of provenance, ullage, the expected quality of wines from different vintages or regions) and a private “taste”, and that valuations are plausibly non-negative, bidder- and potential seller valuations are taken to satisfy the following log-linear single-index structure:

\[
\ln(V) = g(Z) + U
\]

\[
\ln(V_0) = g(Z) + U_0,
\]

with \(\ln\) the natural logarithm and \((U, U_0, Z)\) mutually independent. The common \(g(Z)\) term is interpreted as “quality”. For example, it is commonly accepted that the 1961 Bordeaux vintage is better than most other vintages as a result of favourable weather conditions and that low fill levels relative to the age of the wine are bad.\(^{42}\) By additivity of the idiosyncratic taste component, for all bidders \(i\):

\[
V_i = g(Z) + U_i
\]

so that also:

\[
V_{(n-1:n)} = g(Z) + U_{(n-1:n)}
\]

Quality is then estimated by regressing the transaction price on auction characteristics, using only data from auctions without a reserve price and with more than one bidder in which the transaction price equals the second-highest valuation.\(^{43}\) Residuals from this regression (plus the intercept) are the homogenized second-highest bids used for estimation of \(\theta_b\) in (22). On the seller side, the residualized implied seller tastes are used for the estimation of \(\theta_s\) in (26).

\(^{42}\)It would also be feasible to adopt an alternative specification that estimates a separate \(g_0(Z)\) for sellers. In that case, a sample of \(u_0\) is obtained as the residual plus intercept from a regression of the implied seller valuation given \(g(Z)\) and \(\theta_b\) on \(Z\).

\(^{43}\)The term refers to the homogenization step in Haile et al. (2003). This first stage regression is standard in the analysis of ascending auctions and used in e.g., Bajari and Hortaçsu (2003) and Freyberger and Larsen (2017).
5.2 Bidder valuations

Both $U$ and $U_0$ in (20) are assumed to be normally distributed.\footnote{The lognormal distribution is commonly used to analyze bidding data in the empirical auction literature (adopted in a variety of settings, e.g. Paarsch (1992), Laffont et al. (1995), Haile (2001), Hong and Shum (2002)). Another common specification, the loglogistic distribution, is also considered but its heavier tails provide a slightly worse fit to nonparametric bidder values (details below).} Following the identification argument, the mean and variance of $U$, $(\mu_b, \sigma_b \in \theta_b)$, are estimated by maximum likelihood estimation in auctions with a zero reserve price. Let $\mathcal{T}$, $\mathcal{T}_0$ and $\mathcal{T}_{r>0}$ denote the set of listings, listings with a zero reserve price, and listings with a positive reserve. Let $h(b_t|n_t, z_t, f; \theta_b)$ denote the density of transaction prices given the number of bidders $n_t$, characteristics $z_t$ and fees $f$. For all auctions with a zero reserve price it is simply the probability that the homogenized transaction price / second-highest bid $b_t$ is the second-highest among $n_t$ draws from $F_{V|Z}$. Hence $\forall t \in \mathcal{T}_0$:

$$h(b_t|n_t, z_t, f; \theta_b) = n_t(n_t - 1) F_{V|Z}(b_t; \theta_b)^{n_t - 2} [1 - F_{V|Z}(b_t; \theta_b)] f_{V|Z}(b_t; \theta_b) \tag{22}$$

Note the tight mapping between the identification result and the estimating equation. The log likelihood of bidder parameters given data is specified as:

$$\mathcal{L}(\theta_b; \{n_t, z_t, b_t, r_t\}_{t \in \mathcal{T}_0}, f) = \sum_{t \in \mathcal{T}_0} \ln(\{h(b_t|n_t, z_t, f; \theta_b)\}) \tag{23}$$

$$\hat{\theta}_b = \arg\max \mathcal{L}(\theta_b; \{n_t, z_t, b_t, r_t\}_{t \in \mathcal{T}_0}, f)$$

Bidder parameters are thus estimated using data from auctions with no reserve price. I use observations from positive reserve price auctions to obtain a maximum likelihood estimate of the (unobserved) mean number of bidders in those auctions. In particular, I estimate that mean from variation in bids and number of observed bidders in positive reserve price auctions together with estimated bidder parameters from equation (23) and the Poisson structure. Details are provided in Section 5.4.

5.3 Seller valuations

Given estimated $\hat{\theta}_b$ and $g(Z)$, a sample of implied sellers’ valuations as in (17) is obtained, $\forall t \in \mathcal{T}_{r>0}$:\footnote{$P[V \leq r|Z = z] = P[\exp(g(z) + U) \leq r] = P[U \leq \ln(r) - g(z)] = F_{V|Z}(\ln(r) - g(z))$}

$$\hat{v}_{0,t} = (1 - c_S) \left( r_t - \frac{1 - F_{V|Z}(\ln(\tilde{r}_t) - g(\tilde{z}_t); \hat{\theta}_b)}{(1 + c_B) f_{V|Z}(\ln(\tilde{r}_t) - g(\tilde{z}_t); \hat{\theta}_b)} \right), \text{ and hence:} \tag{24}$$

$$\hat{u}_{0,t} = \ln \left( (1 - c_S) \left( r_t - \frac{1 - F_{V|Z}(\ln(\tilde{r}_t) - g(\tilde{z}_t); \hat{\theta}_b)}{(1 + c_B) f_{V|Z}(\ln(\tilde{r}_t) - g(\tilde{z}_t); \hat{\theta}_b)} \right) \right) - g(\tilde{z}_t), \tag{25}$$
with \( \tilde{r}_t = r_t(1 + c_B) \) denoting the buyer premium-adjusted reserve price and \( \tilde{u}_{0,t} \) the homogenized idiosyncratic part of the implied seller value in auction \( t \). The sample maximum of implied residual seller valuations, \( \hat{v}_T = \max(\{\tilde{u}_{0,t}\}_{t \in T_{>0}}) \), is a consistent estimator of the seller entry threshold. Intuitively, sellers with higher residual value draws than \( v_0^*(f) \) will never list so \( \hat{v}_T - v_0^*(f) \) is always negative (at population values of \( \theta_b \) and \( g(\mathbf{Z}) \)) and the more sellers that do list the larger the probability that the marginal seller has a valuation equal to the threshold.\(^{46}\) The same holds when observed iterations of the game tend to infinity but the number of listings in each game stays constant.

Let \( h(\tilde{u}_{0,t}\left|\mathbf{z}_t, f, v_0^*(f, \theta_s, \theta_b), \theta_b; \theta_s \right) \) denote the density of \( \tilde{u}_{0,t} \) given bidder parameters and given the true seller equilibrium entry threshold, which follows from the relevant identification equation (18):

\[
\begin{align*}
    &h(\tilde{u}_{0,t}\left|\mathbf{z}_t, v_0^*(f, \theta_s, \theta_b), f; \theta_s \right) = \frac{f_{V_0|Z,0,v_0,r=0}(\tilde{u}_{0,t}; \theta_s)}{F_{V_0|Z,0,v_0,r=0}(v_0^*(f, \theta_s, \theta_b); \theta_s)} \mathbb{I}\{\tilde{u}_{0,t} \in [v_{0,r=0}, v_0^*(f, \theta_s, \theta_b)]\} \\
\end{align*}
\]

A complication is that the support of the implied valuations (and reserve prices) observed in the data depends on \( \theta_s \) through its effect on \( v_0^*(f, \theta_s, \theta_b) \), so that standard regularity conditions demonstrating consistency and asymptotic normality of the maximum likelihood estimate of \( \theta_s \) don’t apply.\(^{47}\)

To address the support problem, I estimate seller parameters by maximizing a concentrated likelihood that includes the consistent estimate \( \hat{v}_T \) in place of \( v_0^*(f, \theta_s, \theta_b) \):\(^{48}\)

\[
\begin{align*}
    &\mathcal{L}(\theta_s; \{\tilde{u}_{0,t}, \mathbf{z}_t\}_{t \in T_{>0}}, f, \hat{v}_T) = \sum_{t \in T_{>0}} \ln(h(\tilde{r}_t|\mathbf{z}_t, \hat{v}_T, f; \theta_s)) \\
    &\hat{\theta}_s^0 = \arg \max \mathcal{L}(\theta_s; \{\tilde{u}_{0,t}, \mathbf{z}_t\}_{t \in T_{>0}}, f, \hat{v}_T) \\
\end{align*}
\]

The first order condition of the concentrated likelihood with respect to \( (\mu_s, \sigma_s \in \theta_s) \) does not depend on \( \hat{v}_T \). However, the fact that \( \tilde{u}_{0,t} \) depends on estimated \( \hat{\theta}_b \) and \( g(\mathbf{Z}) \) makes it likely that in finite samples \( \hat{v}_T \) is biased. In particular, because it is the maximum of a noisily estimated sample of homogenized idiosyncratic seller valuations it likely an overestimate of the true \( v_0^*(f) \). Relatedly, it introduces the possibility that the largest values of \( \tilde{u}_{0,t} \) incorporate the highest bias.

Monte Carlo simulations show that a noisy first stage especially affects the standard deviation \( \sigma_b \), and in the expected direction: the initial \( \sigma_s \) overestimates the truth as the sample of implied seller values appears more disperse. Correspondingly, the initial estimate of the seller entry threshold is also too high. Updating this threshold by solving the entry game once and then re-estimating seller parameters from a sample that excludes observations exceeding the threshold addresses that

\(^{46}\)A more precise statement given that valuation distributions are continuous is that the probability that the marginal seller has a valuation within a fixed small interval around the threshold increases.

\(^{47}\)This has been pointed out by Donald and Paarsch (1993) in the context of first-price auctions and addressed by Jofre-bonet and Pesendorfer (2003) for dynamic first price auctions with a Weibull specification of bidder valuations.

\(^{48}\)This has been suggested e.g. in Donald and Paarsch (1993, Footnote 4) in the context of a support problem in first-price auctions.
In particular, it involves numerical approximation of the entry equilibrium given estimated \((\hat{\theta}_b, \hat{\theta}_s^0)\) as detailed in Appendix F, resulting in equilibrium entry threshold \(v^*_0(f, \hat{\theta}_s^0, \hat{\theta}_b)\). Then, seller parameters are estimated by maximizing:

\[
L(\theta_s; \{\hat{u}_{0,t}, z_t\}_{t \in T_r>0}, f, v^*_0(f, \hat{\theta}_s^0, \hat{\theta}_b))
\]

(29)

This describes steps 4 and 5 in the outline at the beginning of this section. Based on results from Monte Carlo simulations, I use only one update (see Appendix E). The 0 superscript in \(\hat{\theta}_s^0\) in (27) indicates that this is the initial estimate of seller parameters before solving the game for entry parameters (next section) and updating parameter estimates; the final estimated seller parameters are denoted by \(\hat{\theta}_s\).

### 5.4 Entry parameters

The mean number of bidders in no reserve auctions is a consistent estimate of \(\lambda^*_{t=0}\):

\[
\hat{\lambda}^*_{t=0} = \frac{1}{|T_r=0|} \sum_{t \in T_r=0} n_t
\]

(30)

A consistent estimate of \(\lambda^*_{t>0}\) equals the value that maximizes the likelihood of transaction prices, \(b_t\), and number of actual bidders, \(a_t\), in positive reserve auctions given estimated bidder valuation parameters. In particular, the joint density of \(b_t, a_t\) if the number of potential bidders \(n_t\) would be known, in auctions with a positive reserve price:

\[
\begin{align*}
&h(b_t, a_t|n_t, r_t > 0, z_t, f, \hat{\theta}_b) = \{F_{V|Z}(\tilde{r}_t; \hat{\theta}_b)^{n_t}\} \mathbb{I}\{a_t = 0\} \\
&\{n_t F_{V|Z}(\tilde{r}_t; \hat{\theta}_b)^{n_t-1}[1 - F_{V|Z}(\tilde{r}_t; \hat{\theta}_b)]\} \mathbb{I}\{a_t = 1\} \\
&\{\binom{n_t}{a_t} F_{V|Z}(\tilde{r}_t; \hat{\theta}_b)^{n_t-a_t}[1 - F_{V|Z}(\tilde{r}_t; \hat{\theta}_b)]^{a_t} \}
\end{align*}
\]

(31)

(\(a_t(a_t - 1) F_{V|Z}(\tilde{b}_t; \hat{\theta}_b)^{a_t-2}[1 - F_{V|Z}(\tilde{b}_t; \hat{\theta}_b)] f_{V|Z}(\tilde{b}_t; \hat{\theta}_b)\}) \mathbb{I}\{a_t \geq 2\}

Note that \(h(b_t, a_t|n_t, r_t > 0, z_t, c_B, \hat{\theta}_b) = 0\) when \(n_t = 0\). The first line covers the probability that all \(n_t\) bidders draw a valuation below the reserve price, the second line the probability that one out of \(n_t\) draw a valuation exceeding \(\tilde{r}\) while the others don’t (in which case \(b_t = r_t\) with certainty), and the final two lines capture the probability that \(a_t\) out of \(n_t\) draw a valuation exceeding the reserve and that the second-highest out of them draws a valuation equal to \(\tilde{b}_t = b_t(1 + c_B)\).

---

\(^{49}\)This describes the Nested Pseudo Likelihood estimator in Aguirregabiria and Mira (2002, 2007) used in discrete choice games. Roberts and Sweeting (2010) are the first to apply this algorithm to the auction literature to study auctions with selective bidder entry. Studies by Pesendorfer and Schmidt-Dengler (2010), Kasahara and Shimotsu (2012) and Egesdal et al. (2015) provide conditions under which NPL does (not) converge to the true equilibrium. A best-response stable equilibrium is a sufficient condition for the NPL algorithm to converge to it and this is certainly guaranteed (Proposition 1) by the game reducing to a single agent discrete choice problem with unique equilibrium. Aguirregabiria and Mira (2002) find that in single-agent games asymptotic efficiency is independent of the number of iterations.
observing $n_t$, a feasible specification takes the expectation over realizations of random variable $N \sim \text{Pois}(\lambda_{r>0}^*)$. This is the basis of the likelihood function that $\hat{\lambda}_{r>0}^*$ maximizes:

$$g(b_t, a_t | r_t > 0, z_t, f, \hat{\theta}_b; \lambda_{r>0}^*) = \sum_{n_t = a_t}^{\infty} h(b_t, a_t | k, r_t > 0, z_t, f, \hat{\theta}_b) f_{N|N \geq A}(k; \lambda_{r>0}^*)$$ (32)

$$\mathcal{L}(\lambda_{r>0}^*; \{b_t, a_t, r_t, z_t\}_{t \in T_{r>0}}, f) = \sum_{t \in T_{r>0}} \ln(g(b_t, a_t | r_t > 0, z_t, f, \hat{\theta}_b; \lambda_{r>0}^*))$$ (33)

$$\hat{\lambda}_{r>0}^* = \arg \max L(\lambda_{r>0}^*; \{b_t, a_t, r_t, z_t\}_{t \in T_{r>0}}, f)$$ (34)

Bidder opportunity cost $\hat{e}_{B,r>0}^0$ and $\hat{e}_{B,r=0}^0$ are estimated as the values that equal expected surplus from entering, estimated by computing the values in equations (4) and (10) at the estimated $(\hat{\theta}_b, \hat{\theta}_s, \hat{\lambda}_{r>0}^*, \hat{\lambda}_{r=0}^*)$. As the seller opportunity cost are identified off only one data point, the expected surplus of the marginal seller, I instead estimate $\hat{e}_S^0$ as the average between $\hat{e}_{B,r>0}^0$ and $\hat{e}_{B,r=0}^0$. Monte Carlo simulations in Appendix F confirm that the seller opportunity cost are truly a normalisation for the estimation of $\hat{\theta}_s$. The equilibrium seller entry threshold is calculated as the value that makes the marginal seller indifferent between entering and staying out, i.e. by solving equation (8) at the estimated parameters. Further details on the computation of equilibrium values are provided in Appendix F.

6 Estimation results

6.1 Impact of observed wine characteristics

The homogenization (step 1) is done separately for auctions with transaction prices of at most 200 pounds, referred to as the “main sample” as they contain 80.93 percent of observations, and the remaining auctions that is referred to as the “high-value” sample. Estimation is done per bottle-equivalent which delivers a better fit than the lot level. Results from this step are presented in Tables 9 and 10 in the Appendix. The estimated (sign of) coefficients for various key variables are as expected. Among other findings, results show that prices are higher for bottles sold in a case of 6 or 12, but conditional on this case effect the price is lower the more bottles are included in the lot. For bottles stored in a specialized warehouse (with optimal temperature and humidity control) and for wines in special format bottles (e.g. magnums) prices are higher. This corresponds to the idea that these wines are expected to be of higher quality. The omitted ullage category in the Tables is “Into Neck”, the best fill level, so logically all other levels deliver lower prices (some coefficients are insignificant). The tables furthermore report estimated relative values for a host of wine regions, grapes, and shipping options. These observables explain a large share of total price

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50 Seller commissions are tiered on BW (Table 1): for realized transaction prices exceeding 200 pounds, sellers pay one percentage point less in seller commission on the excess than for goods selling below 200 pounds. The benefit of this approach is that it generates a relevant dimension of product variety along which the welfare impacts of fee changes can be assessed, even after homogenizing bids and values.

51 While larger bottles may also be attractive for their fun factor, their smaller surface area conditional on wine content is also associated with a lower oxidation rate.
### Table 5: Estimation results: idiosyncratic valuations and entry

| Parameters of $F_{V|Z}$, $F_{V_0|Z}$ | Entry equilibrium |
|---------------------------------------|-------------------|
|                                       | **Main** | **High-value** | **Main** | **High-value** |
| Bidders ($\theta_b$)                  |          |                |          |                |
| $\mu_b$                               | 3.1736   | 5.376          | 3.835    | 4.651          |
| [0.029]                               | [0.034]  |                | [0.007]  | [0.033]        |
| $\sigma_b$                            | 0.903    | 0.564          | 5.238    | 7.271          |
| [0.001]                               | [0.022]  |                | [0.004]  | [0.011]        |
| Sellers ($\theta_s$)                  |          |                |          |                |
| $\mu_s$                               | 4.475    | 5.857          | 5.111    | 5.011          |
| [0.084]                               | [0.093]  |                | [0.002]  | [0.003]        |
| $\sigma_s$                            | 4.141    | 0.741          | 13.048   | 13.285         |
| [0.165]                               | [0.022]  |                | [0.165]  | [0.642]        |
| Bidders per listing                   |          |                |          |                |
| $\lambda^*_{r>0}$                     |          |                | 4.991    | 13.048         |
| [0.002]                               | [0.003]  |                | [0.165]  | [0.642]        |
| Seller entry probability $F_{V_0|Z}(v^*_0)$ |         |                | 4.782    | 13.285         |
| [0.159]                               | [0.641]  |                | [0.165]  | [0.642]        |
| Opportunity cost                      |          |                | 5.200    | 14.412         |
| $e^o_{B,r>0}$                         |          |                | [0.171]  | [0.661]        |
| [0.165]                               | [0.093]  |                | [0.165]  | [0.642]        |

Standard errors are reported in square brackets and are obtained with 250 bootstrap repetitions.

Variation, even without controlling for the number of bidders. The adjusted R-squared is 0.530 for the main sample and 0.855 for the smaller high-value sample.

### 6.2 Estimates of idiosyncratic valuations

Estimated parameters from the distributions of idiosyncratic (potential) bidder and seller valuations are presented in Table 5. Standard errors are obtained from 250 bootstrap samples (see Horowitz (2001)) and include variability from the first-stage regressions. The conditional valuation distributions for these two populations are allowed to be different, and parameter estimates reveal that the two user groups do indeed have different value distributions. While the population distribution of seller valuations is more dispersed than that of bidders, the distribution of bidder values (at least) second-order stochastically dominates the distribution of values among sellers on the platform who set a positive reserve price. While the auction characteristics manage to explain the majority of price variation in the sample, there is still significant variation in the idiosyncratic tastes for the fine wine offered at the platform. For example, at the median estimated quality (−0.33) the mean bidder value is estimated to be 26 pounds and the interquartile range 9-32 pounds. Sellers are estimated to have an average value of 20 pounds for that item, with an interquartile range of 9-31 pounds. Real gains from trade come from some bidders drawing a much higher value, with the 95th percentile of estimated bidder values at 75 pounds and the same statistic for sellers at 45 pounds. Estimated taste distributions have a higher mean but lower dispersion, which is also related to auction observables explaining an even larger share of observed price variation in the high-value sample.

### Fit and validation.

While a log-normal specification is a common choice to parameterize value distributions in the auction literature, suitability of this distribution has yet to be evaluated. I compare estimation results...
with those obtained from a different distribution. The Log-logistic distribution has a similar shape but heavier tails, which can deliver significantly different results in auction models where bidding and reserve prices depend on these tails. Table 6 compares model fit of estimation results obtained with both parameterizations in the main sample and shows that while both distributions deliver a similarly fit of second-highest bids, the reserve price distribution is much better approximated with the log-normal distribution (p-value of 0.488 compared to 0.0425).

For the distribution of idiosyncratic valuations, my measure of model fit is the mean deviation between the predicted (including the predicted quality level $g(\mathbf{Z})$ from the homogenization step) and observed second-highest bid as a share of the observed value. Table 6 provides this statistic separately for auctions with 2-10 bidders and in expectation over all number of bidders exceeding 1. Both are reported in auctions without reserve prices to focus only on the fit of the distribution of idiosyncratic bidder valuations. Results show that estimated parameters replicate the second-highest bid well, even when slicing the data in bins with 2-10 bidders. The two plots on the left-hand side in figure 3 support this statement visually. The top-left compares the estimated idiosyncratic value distribution with its empirical distribution estimated separately for 2-8 bidders. Nonparametric estimation follows directly from the identification argument of $F_{V|Z}$ in (15) by applying $\phi^{-1}(\cdot; n)$ to the empirical probability that the second-highest bid in auctions with $n$ bidders is less than $v$ (given that $c_B = 0$ on the BW platform). The bottom-left figure combines the estimated bidder parameters with draws from estimated quality, the estimated bidder arrival process, and the optimal bidding strategy to compare predicted against observed second-highest bids.

Estimation of seller parameters is evaluated by comparing the distribution of predicted values (including estimated quality) with the distribution of observed reserve prices. I report the p-value from a two-sample Kolmogorov-Smirnov test (see equation (35) for details) with null hypothesis that observed reserve prices and predicted reserve prices are drawn from the same population distribution. This statistic suggests that the log-normal distribution has a better fit than the log-logistic. With a p-value of 0.448, I cannot reject the null at any reasonable level. The top plot on the right-hand side of Figure 3 plots observed and predicted reserve prices. Values are non-homogenized so the predictions include an expectation over draws from the empirical distribution of $g(\mathbf{Z})$, displaying the great fit.

However, the implied seller value (given $\hat{\theta}_b, g(\hat{\mathbf{Z}})$) according to (17) is for 4.17 percent of sellers estimated to be negative. If we were to assume that seller valuations are in fact non-negative, this could be driven by: i) a portion of sellers setting reserve prices below the optimal levels, ii) small-sample estimation bias stemming from first-stage estimates $\hat{\theta}_b$ and $g(\mathbf{Z})$, or potentially iii) approximation error in the reserve price (see Appendix B). It is also conceivable that other sellers with low valuations set sub-optimal reserve prices (note that the equilibrium mark-up is highest for sellers with the lowest valuations). Estimation of $\theta_s$ excludes the 4.17 percent of positive reserve price auctions that violate non-negativity of $(r - \text{estimated mark-up})$. 

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Figure 3: Model fit / validation. Top-left: comparing the estimated idiosyncratic bidder valuation distribution (thick orange dashed line) in auctions with no reserve price against its empirical CDF in auctions with \( n = 2, \ldots, 8 \) bidders. Bottom-left: observed vs. predicted second-highest bids in auctions with no reserve, including draws from estimated quality, \( \hat{g}(Z) \). Top-right: comparing estimated idiosyncratic values of sellers in positive reserve auctions against its empirical CDF. Bottom-right: comparing observed reserve prices and predicted values including estimated quality. As quality is estimated in the sample of auctions with no reserve price, the reserve price fit reflects out-of-sample predictions.
Table 6: Model fit and validation statistics

| Parameters of $F_{V|Z}$, $F_{V|0}$ (and $g(Z)$) | Entry equilibrium |
|-----------------------------------------------|--------------------|
| Mean deviation as share of observed           | Probability $N = n$ (when $r = 0$) |
| Mean deviation as share of observed           | Absolute difference with Poisson |
| - 2 bidders                                   | -n=0               |
| - 3 bidders                                   | -n=1               |
| - 4 bidders                                   | -n=2               |
| - 5 bidders                                   | -n=3               |
| - 6 bidders                                   | -n=4               |
| - 7 bidders                                   | -n=5               |
| - 8 bidders                                   | -n=6               |
| - 9 bidders                                   | -n=7               |
| - 10 bidders                                  | -n=8               |
| - Expectation over $N$                        | -n=9               |
| -n=10                                         | 0.001              |
| -n=0                                          | 0.011              |
| -n=1                                          | 0.013              |
| -n=2                                          | 0.002              |
| -n=3                                          | 0.022              |
| -n=4                                          | 0.008              |
| -n=5                                          | 0.010              |
| -n=6                                          | 0.002              |
| -n=7                                          | 0.001              |
| -n=8                                          | 0.002              |
| -n=9                                          | 0.001              |
| -n=10                                         |                    |

Results for auctions in the main sample. * p-value from a two-sample Kolmogorov-Smirnov test, as explained in relation to equation (35). Both predicted reserve prices and predicted highest bids include draws from the estimated quality in the data, $g(Z)$, and estimated valuation parameters, $(\hat{\theta}_b, \hat{\theta}_s)$. The mean deviation statistics report the mean (difference between predicted second-highest bid and its observed value in auctions without a reserve price, as a share of the observed value). The absolute difference with Poisson statistic compares observed and predicted number of bidders $N = n$, drawn from the sample with no reserves.

6.3 Entry estimates

The estimated Poisson parameters are $\lambda_{r>0}^* = 3.8$ and $\lambda_{r=0}^* = 5.2$, so setting no reserve price attracts on average more than one additional bidder into the listing (Table 6). It also makes intuitive sense that this participation differential is larger in the high-value sample; the probability of being the sole entrant and winning the more expensive bottle for the 1 pound opening bid contributes more to expected surplus. Indeed, in high-value auctions $\lambda_{r>0}^* = 4.7$ and $\lambda_{r=0}^* = 7.3$. The estimated opportunity cost are also significantly different; roughly three times as high in the high-value sample. But as a percent of the average second-highest bid, estimated opportunity cost are higher in the main sample (6 - 7 percent, versus 4 percent in the high-value sample). Estimates do in both cases correspond to the idea that listing inspection cost are significant in this idiosyncratic goods environment.

Fit and validation.

The right-hand side panel in Table 6 examines the fit of the assumed Poisson distribution with the estimated $\lambda_{r=0}^*$ and the observed Binomial distribution of the number of bidders. It reveals a good fit for all $n = 1 – 10$. Figure 4 support this visually. More formally, a chi-square goodness of fit test fails to reject at the ten percent level that $N$ is generated by a Poisson distribution, based on auctions with no reserve price in which the number of bidders is not censored (p-value 0.146).

It is of particular interest that the data does not reveal any overdispersion relative to the Poisson distribution. That would point to an entry process in which bidders enter significantly more numerously into auctions with certain characteristics - conditional on having no reserve price.
Figure 4: The number of bidders per listing is approximately Poisson distributed. A chi-square goodness of fit test fails to reject that $N$ is generated by a Poisson distribution at reasonable confidence levels, with a $p$-value of 0.146. The test uses data from auctions with no reserve price in which the number of bidders is not censored.

Some characteristics may be slightly more popular than others, but a model with uniform sorting over listings captures the first order effects of entry behavior in the BW data.\footnote{By contrast, estimating a model of entry in Kindle e-reader auctions on eBay, Bodoh-Creed et al. (2013) need to incorporate auction observables in their single-index conditional Poisson distribution to explain the observed pattern of a higher number of bidders in listings with certain characteristics. While I may, for the sake of studying differential entry by wine types, condition on additional characteristics as well; it is not necessary to fit the data.}

Another source of model validation comes from comparing the estimated bidder opportunity cost for entry in no reserve auctions and positive reserve auctions. While they are allowed to be different, there is no reason to suspect that it is significantly more time-intensive to inspect listings with or without a reserve price if the reserve price does itself not reveal any information about the quality of the item. Supportive of this argument are my opportunity cost estimates in both the main and high-value samples, with the 95 percent confidence intervals of $\hat{e}_{B,r=0}$ and $\hat{e}_{B,r>0}$ overlapping. They are computed in two cuts of the data as the values that justify $N_{r=0} \sim \text{Pois}(\hat{\lambda}_{r=0})$ and $N_{r>0} \sim \text{Pois}(\hat{\lambda}_{r>0})$ given observables and estimated taste distributions. The fact that the estimated opportunity cost are statistically insignificant suggests that also the parsimonious model is a good description of bidder behavior on this platform - and that the computation of expected surplus is solid.\footnote{In a previous version of this paper it turned out to be complicated to explain both the share of sellers that sets a zero reserve price and additional entry of bidders in no-reserve auctions, assuming these choices are optimal to maximize both bidder and seller expected surplus. My early calculations suggested that the true screening value $(v_{0,r=0})$ should be higher than the observed one; more sellers should be setting a zero reserve price given the estimated difference in $\hat{\lambda}_{r=0}$ and $\hat{\lambda}_{r>0}$. One potential explanation for that inconsistency could be that the platform explicitly encourages sellers not to set a reserve price in order to attract “50-75 percent” more bidders and a “40 percent” higher transaction price. If these rules of thumb don’t correspond with the actual benefit for sellers to set a zero reserve price, that may explain part of the inconsistency if sellers do act on those rules of thumb. My results reveal that no-reserve auctions attract on average about 37 percent (main sample) to 56 percent (high-value sample) more bidders. Overall, I do consider explaining seller motives to set no reserve price better to be an interesting next step.}
7 Counterfactual policies

This section uses model estimates to address two key indeterminacy’s of two-sided markets in the context of the wine auction platform. First, how should the platform allocate fees between different platform users? Second, how do increases in fees affect users on both sides? The latter is an open question in antitrust policy that is crucial to better understand the unusual economic relationships in platform markets.\footnote{See e.g., Evans and Schmalensee (2013).}

7.1 Harnessing network effects to increase platform profitability

In this counterfactual, I simulate the game for a host of alternative fee structures. At each fee combination, I compute platform revenue, bidder and seller surplus, and the volume of sales.\footnote{Results expressed as changes with respect to baseline values and are computed in homogenized value space.}\footnote{This is consistent with a model of network growth with myopic users who can terminate their participation at no cost, as provided in Evans and Schmalensee (2010) to explain the emphasis of platforms such as eBay, Facebook and MySpace on network growth in their early years. The idea that sale volume is relevant is also consistent with an alternative fee structure that the platform discussed with me in general terms. I estimate that that fee structure would reduce expected static platform revenue by as much as 22 percent while increasing the sale volume by 6 percent. As such, for this to be a smart policy they would need to place considerable weight on the volume of sales.} Many changes in fees result in a trade-off between the volume of sales and platform fee revenue. Higher listing fees, for example, makes it less attractive for sellers to enter and as a result fewer listings depresses the volume of sales. Even if higher fee revenues are beneficial in a static world, if the volume of sales affects future revenues through (say) word of mouth or brand awareness, a forward-looking platform will include this statistic in their objective function.\footnote{The presence of network effects is detected in the contour plot from the fact that only changing the allocation of commissions between buyers and sellers, while keeping their total levels constant, has significant effects on platform profitability. In particular, this is a requirement for the market to be two-sided by its standard definition in Rochet and Tirole (2006).}

I therefore estimate both the impact of alternative fee structures on static fee revenues as well as the volume of sales, and consider the problem of maximizing current volume-constrained fee revenues. This approach avoids having to impose further restrictions on the exact platform objective function.

Results show that there are significant network effects in this market that can be harnessed to improve platform profitability.\footnote{The presence of network effects is detected in the contour plot from the fact that only changing the allocation of commissions between buyers and sellers, while keeping their total levels constant, has significant effects on platform profitability. In particular, this is a requirement for the market to be two-sided by its standard definition in Rochet and Tirole (2006).} Contour plots in Figure 5 support this visually; with the levels referring to the share of current fee revenues. The top panel varies buyer and seller commission but holds flat fees at their current levels. For example, adopting any \((c_B, c_S)\) pair that intersects on an area of the plot with level 1 (the turquoise-green colour) would result in the same platform revenue as generated by the current fee structure. The bold red line indicates the baseline volume of sales. Any fee combination to the south west of this line increases the sale volume, and any fee combination to the north east decreases volume. Note that it is not a coincidence that baseline volume and revenue (level 1) intersect at the current commission allocation, the point indicated in the Figure with a red marker.

in the understanding of the impacts of platform fees, despite the complexity in capturing this choice perfectly in a two-sided market setting. I am revealing this information here to perhaps spark interest in this issue.
A key take-away is that the BW platform can increase fee revenues with up to 80 percent without reducing volume by increasing \( c_S \) and decreasing \( c_B \) when holding current entry fees fixed (top panel in Figure 5). For example, increasing the seller commission by 15 percentage points and reducing the buyer premium by the same amount increases platform revenues by about 30 percent. As the buyer premium is currently zero, this would require charging a negative buyer premium, e.g. to give winning bidders a 15 percent discount on the transaction price. While negative fees may seem unintuitive, it fully agrees with the idea that businesses in two-sided markets subsidize the side that contributes most to profits even to the point of charging that side below marginal cost. Commissions, which users are familiar with in wine auction platforms, can feasibly be set to negative values as they only apply to successful sales.

I also consider the problem with additional (self-imposed) non-negativity constraints on \( c_B \) and \( c_S \), which is reflected in the top panel of Figure 5 by the two grey lines. The contour plot shows clearly that, when keeping other fees at their current levels, the platform cannot increase its revenues without reducing volume if it does not want to set a negative buyer commission. All commission combinations that do increase volume reduce platform revenue. Furthermore, even a small reduction in seller commission without additional fee changes is risky as it can lead to significantly lower platform revenue.

Figure 6 displays platform revenues for different combinations of seller commission and listing fee, keeping other fees at their current levels. The non-negativity constraint on the listing fee is in this case crucial to avoid sellers listing unsellable items to collect the fee. The current volume constraint is displayed in red. The plot shows that there is significant scope to increase revenues without reducing volume by increasing the listing fee. Sellers must be rather inelastic with respect to the listing fee to generate the contours observed.

Combining the above findings, the bottom panel in 5 displays revenue for combinations of commissions while increasing the listing fee to 5 pounds and keeping the bidder entry fee at the current level of 0. It paints a remarkable picture. Even when considering only positive commissions, platform revenues increase by roughly 20 percent without affecting volume when keeping commissions at their current levels. Alternatively, they can increase volume by 7.5 percent without affecting revenues by combining the listing fee increase with a reduction in seller commission of about three percentage points to 0.07.

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59 This economic reality does not seem lost on current BW management. One change to the fee structure they consider making is to increase the listing fee from 1.75 to 4 pounds. They also consider waiving this fee for sellers that set a zero reserve price. In my assessment of the discussed fee structure I find that this policy would increase the share of zero reserve price listings by 19 percent. However, I estimate that the impact of this significant change in listing composition does by itself not affect platform revenue or volume by much. An interesting question that I leave for future work is whether and how a platform should optimally nudge sellers in their pricing decisions. This is related to the theoretical observations that when \( v_0 > 0 \), sellers will set too high reserve prices from the platform’s perspective as sellers trade-off expected sale revenues with the value of keeping the item.
a) Holding listing fee at current level of 1.75 pounds

b) Increasing listing fee to 5 pounds

Figure 5: Platform revenue at counterfactual commissions, with volume constraint

Contour plots of counterfactual platform revenue as a share of current revenue, for combinations of \( c_B \) and \( c_S \), holding entry fees \((c_S, c_B)\) at current level (1.75,0) (top) and increasing \( c_S \) to 5 pounds (bottom). The red line indicates current volume of sales; fee combinations to the south west of this line increase volume.

7.2 Users are better off with alternative fee structures

Figure 7 provides contour plots for expected seller and winning bidder surplus as a share of their current levels. The dark blue and red lines tie these values in with the platform objective function; the dark blue swirl indicates the current level of platform revenues (any fee combination to the north east increases it), and the red curve indicates the current level of sale volume (any fee combination to the south west increases it). Estimates refer to the case that increases the listing fee to 5 pounds
Figure 6: Platform revenue at counterfactual seller fees, with volume constraint

Contour plots of counterfactual platform revenue as a share of current revenue for combinations of $e_S$ and $c_S$, holding $(c_B, e_B)$ at current and constrained-optimal level $(0,0)$. The red line indicates the current volume of sales; fee combinations to the south west of this line increase volume.

while keeping other fees at baseline levels. A key take-away from this picture is that for a region of combinations of $c_B$ and $c_S$, implementing the negative buyer premium policy leaves both sellers and winning bidders better off. Consider the fee structure highlighted above for example, increasing the seller commission with 15 percentage points (to 25 percent) and providing winning bidders with a 15 percent discount. While it is intuitive that this fee structure increases winning bidder surplus, it also is estimated to increase seller surplus by about 20 percent. Especially the beneficial impact on sellers is striking because their fees go up significantly in this scenario. This result resonates with the idea that bidder participation is very valuable to sellers in many auction settings, as additional bidders drive up transaction prices.\textsuperscript{60}

7.3 Ignoring entry significantly biases welfare estimates

In this counterfactual, I evaluate the impact of isolated changes in seller commission and buyer premium on user welfare and compare it to results obtained when using a model without entry. As this issue is relevant for antitrust policy, I use a prominent commission-fixing case involving auction giants Sotheby’s and Christie’s (SC) to provide context. After the conspiracy came to light, they settled with buyers and sellers for a total of 512 million dollars (roughly 729 million dollars in 2018 prices) and five sixths of this amount went to buyers. Civic case litigation makes clear that damage estimates are based on direct (alleged) overcharges and not pass through or

\textsuperscript{60}For example, Bulow and Klemperer (1996) show that setting no reserve and attracting one additional bidder is more profitable than negotiating with fewer participants and using a reservation price. The impact of entry in the case where bidders enter selectively is theoretically ambiguous, but Roberts and Sweeting (2010) show that for in USFS timber auctions the value of increasing the pool of (potential) entrants also outweighs the value of setting a reserve price.
Figure 7: The negative buyer premium policy could be implemented to make both sellers and winning bidders better off.

Levels indicate the surplus as a share of current surplus, for combinations of buyer and seller commission when increasing the listing fee to 5 pounds and keeping the bidder entry fee constant at 0. The red line is the platform volume constraint: combinations to the south west increase volume. The blue swirl is the platform revenue constraint: combinations to the east increase revenue. An example of a fee structure that makes all parties better off is a negative 10 percent buyer premium and a 20 percent seller commission combined with the 5 pound listing fee.

I estimate the welfare impacts of a five percentage point increase in buyer commission, which is most likely what the SC case settlement is based on, and compare results with the economic incidence and therefore abstracted from any interconnectedness between users or their entry decisions.\textsuperscript{61}  

\textsuperscript{61}See\ In re Auction Houses antitrust litigation (2001)\ for the full litigation text. Ashenfelter and Graddy (2005) provide a more detailed description of the case.
with those resulting from a five percentage point increase in seller commission.

The empirical results in Table 7 show that the five percent increase in buyer premium decreases the sale price and probability by about 4 percent, reduces the share of listings with a zero reserve price, and reduces the number of bidders per listing. Expected winning bidder surplus decreases by about 7 percent. I also estimate that sellers are significantly worse off: their expected surplus decreases by 17 percent. A finding that underlines how important it is to incorporate the interconnectedness of users in platform markets is that sellers are better off if their seller commission goes up by 5 percentage points than when the buyer commission increases by the same amount. In fact, this feature of the platform marketplace would be missed if users’ endogenous entry decisions would be ignored. The columns labelled “Without entry” in Table 7 provide welfare impacts when bidder and seller participation is kept constant at baseline levels and only their bidding and reserve pricing strategies respond to the increases in commissions. Results from such a model suggest that sellers would instead prefer if the five percent commission increase is targeted to buyers.

The estimates also depart from a third model that underlies the argumentation in previous studies looking at welfare impacts of commissions in auctions in relation to the SC case. Ashenfelter and Graddy (2005) conclude: though buyers received the bulk of the damages, a straightforward application of the economic theory of auctions shows that it is unlikely that successful buyers as a group were injured. This conclusion relies on the idea that buyers reduce their bid by the amount of buyer premium and sellers accept any price so that the economic incidence of any commission or premium fully falls on sellers because they are the price-inelastic party.62

The simulated commission increases show that in the BW data, increasing the commission to one side of the market by 5 percent decreases expected surplus to that side more than proportionally. On top of that, the other side is affected as well. Not addressing entry in user welfare evaluations significantly underestimates expected loss in surplus. The 5 percent increase in buyer premium that likely was the basis of the SC settlement negotiations decreases total user surplus by about 12 percent, so more than double the premium increase.63

Besides the amount of total damages estimated, also awarding five-sixths of the damages to winning bidders was plausibly flawed.64 I show that in the BW data, the economic incidence of commissions falls predominantly on sellers despite endogenous entry and also when accounting for optimal reserve prices. The loss in user surplus resulting from a 5 percent buyer premium increase is estimated to fall for 71 percent on sellers:

That the majority of the welfare loss from increasing commissions falls on sellers remains a valid conclusion also when holding bidder and seller participation fixed at baseline levels. My estimates furthermore reveal that sellers shoulder 71-89 percent of the loss in user surplus regardless of which

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62 This argumentation also underlies evaluation of the role of commissions in (wine) auctions by Marks (2009) and McAfee (1993). I have referred to this as the “one-sided market perspective”.

63 Relating this statistic to the numbers in Table 7; a 5 percent buyer premium increase reduces total user surplus by 11.87 percent: 100 * ((1 − 0.07169) + 7.719 + (1 − 0.17243) * 6.757) − (7.719 + 6.757)) / (7.719 + 6.757) = −11.87.

64 Ashenfelter and Graddy (2005) and Marks (2009) hypothesized this before based on the one-sided market perspective with inelastic sellers in which case winning bidders logically avoid a reduction in surplus by reducing their bid proportionally to any buyer premium increase.
commission increases.

7.4 Larger effects in high-value auctions

Results in the previous section are estimated with the main sample that excludes auctions exceeding 200 pounds. This section exploits the remaining data to examine whether results are different in auctions that are of higher “quality”.

The first three columns in Table 8 report estimated platform revenue, user surplus and other relevant platform descriptors at \((c_S = 0.1, c_B = 0)\) and the last three columns show these statistics for the reverse fee structure \((c_S = 0, c_B = 0.1)\). The first take-away from the Table supports the importance of high value listings. When charging \(c_S = 0.1\) for instance, while less than one fifth of listings and bidders is high value, those interactions generate about half of both platform revenue and volume. Columns 3 and 6 correspond to auctions with transaction prices below 200 pounds, e.g. those in the main sample central to the previous experiment. Platform revenue, total seller surplus and total surplus for winning bidders is all higher when charging sellers rather than buyers, reinforcing earlier findings to that effect. But while these differences are relatively modest, in high value auctions all parties are significantly better off when charging a 10 percent commission to sellers rather than buyers.

The different commission elasticities may be part of the explanation for the observed variation in fee structures in various wine auction platforms.\(^{65}\) Another suggestion offered in Table 8 is that a platform targeting only high-value users (those offering and bidding for high-value lots) can generate similar revenues as a platform attracting lower-priced lots with only one fifth of listings.

8 Conclusions

In this paper I examine the welfare impacts of fees charged in auction platforms using a new dataset of wine auctions web-scraped from an online wine auction platform. Despite the platform marketplace, empirical patterns suggest an absence of dependencies between listings. This is inconsistent with theoretical predictions from previous auction platform models tailored to explain strategic behaviour in auction platforms for more homogeneous goods but can be explained by costly inspection of the idiosyncratic goods for sale. The structural auction platform model that also includes endogenous entry of bidders and sellers and seller/listing selection plausibly captures

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\(^{65}\)Many wine auction platforms take a comparable portion of transaction prices in total commissions but allocate commissions very differently to the two user groups. For example, the US-based higher-end Acker, Merrall & Condit charges 21 percent buyer- and no seller commission, the Chicago Wine Company has the reverse structure charging nothing to buyers and 28 to sellers and Winebid.com charges both sides evenly at respectively 15 and 18 percent.
Table 7: Welfare impacts increases in buyer and seller commissions

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Increase buyer premium 5%</th>
<th>Increase seller commission 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Two-sided</td>
<td>Without entry</td>
</tr>
<tr>
<td>Buyer premium, $c_B$</td>
<td>0</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Seller commission, $c_S$</td>
<td>0.10</td>
<td>0.102</td>
<td>0.10</td>
</tr>
<tr>
<td>Platform revenue (x10.000)</td>
<td>2.715</td>
<td>25.774</td>
<td>31.734</td>
</tr>
<tr>
<td>Volume of sales (x10.000)</td>
<td>20.769</td>
<td>-10.597</td>
<td>-5.613</td>
</tr>
<tr>
<td>Expected surplus winning bidders (x10.000)</td>
<td>7.719</td>
<td>-7.169</td>
<td>-2.428</td>
</tr>
<tr>
<td>Expected surplus per winning bidder</td>
<td>28.695</td>
<td>-5.303</td>
<td>-2.428</td>
</tr>
<tr>
<td>Seller entry probability, $F_{v_0}</td>
<td>Z(v_0^*)$</td>
<td>0.855</td>
<td>-1.903</td>
</tr>
<tr>
<td>Mean number bidders per listing, $\lambda_r&gt;0$</td>
<td>4.759</td>
<td>-5.177</td>
<td>0</td>
</tr>
<tr>
<td>Share listings with no reserve, $p_r0$</td>
<td>0.374</td>
<td>-8.881</td>
<td>0</td>
</tr>
<tr>
<td>Sale probability</td>
<td>0.869</td>
<td>-3.834</td>
<td>-2.228</td>
</tr>
<tr>
<td>Sale price</td>
<td>91.189</td>
<td>-4.175</td>
<td>-2.286</td>
</tr>
</tbody>
</table>

Percentage change w.r.t. Baseline:

Table 8: Opposing fee structures and their differential impact on high-value lots

|                                | Combined | High-value | Regular | Combined | High-value | Regular |
|                                | Charge seller commission | Charge buyer premium |
| buyer premium, $c_B$           | 0        | 0          | 0       | 0.1      | 0          | 0       |
| Seller commission, $c_S$       | 0.1      | 0.1        | 0.1     | 0        | 0          | 0       |
| Platform revenue (x10.000)     | 5.133    | 2.468      | 2.665   | 4.064    | 1.662      | 2.402   |
| Volume of sales (x10.000)      | 44.120   | 23.317     | 20.803  | 33.079   | 15.153     | 17.926  |
| Share revenue from high-value  | 0.481    | 1          | 0       | 0.409    | 1          | 0       |
| Share volume from high-value   | 0.528    | 1          | 0       | 0.458    | 1          | 0       |
| Share of listings high-value   | 0.179    | 1          | 0       | 0.185    | 1          | 0       |
| Share of bidders high-value    | 0.158    | 1          | 0       | 0.122    | 1          | 0       |
| Expected surplus sellers (x10.000) | 8.542 | 1.761      | 6.781   | 7.51     | 1.063      | 6.447   |
| Expected surplus winning bidders (x10.000) | 12.267 | 4.534     | 7.733   | 10.336   | 3.047      | 7.289   |
| Expected surplus per seller    | 27.565   | 29.948     | 25.182  | 20.544   | 17.308     | 23.78   |
| Expected surplus per winning bidder | 15.396 | 17.56     | 5.631   | 11.31    | 16.983     | 5.637   |
| Seller entry probability, $F_{v_0}|Z(v_0^*)$ | 0.881 | 0.996      | 0.855   | 0.862    | 0.863      | 0.861   |
| Mean number bidders per listing, $\lambda_r>0$ | 4.559 | 3.745      | 4.748   | 4.023    | 2.543      | 4.455   |
| Share listings with no reserve, $p_r0$ | 0.354 | 0.243      | 0.378   | 0.299    | 0.142      | 0.335   |
| Sale probability               | 0.828    | 0.646      | 0.868   | 0.757    | 0.45       | 0.826   |
| Sale price                     | 188.21   | 633.052    | 91.072  | 176.159  | 584.581    | 83.657  |
the main features of other auction platforms as well, such as those for freelance jobs and second hand cars.

Empirical findings suggest that it would be a mistake for antitrust authorities or the legal system not to consider interconnectedness of bidders and sellers on an auction platform. Increasing fees to one side of the market affects both types of users, and network effects from entry are such that sellers are better off when the seller commission increases than when the buyer premium increases by the same amount. Results are placed in the context of a prominent commission-fixing case involving auction giants Sotheby’s and Christie’s. I show that relative to the benchmark rule that does not consider the economic incidence of the alleged overcharge or endogenous entry, total estimated damages are more than twice as high. Also, while sellers in the civic case received only one sixth of total damages, the economic incidence of increasing commissions by 5 percentage points falls to 71-89 percent on sellers. More conceptually, these counterfactual fee experiments highlight that it is feasible to estimate exact welfare impacts of platform fees from publicly available data, despite strategic interactions rendering these effects ex-ante ambiguous and despite the auction mechanism causing transaction prices to be endogenous to the fees.

Results also show that the platform can increase revenues from fees without reducing the volume of sales by (further) subsidizing bidders. Platform profitability can increase up to 80 percent when introducing a negative buyer premium, e.g. providing winning bidders a discount on the transaction price, paid for by a higher seller commission. When implemented right, this policy could even increase expected surplus for sellers and winning bidders on the platform. Although no auction platform currently charges negative fees, this recommendation is in line with the practice in other two-sided markets such as payment cards. I demonstrate that this fee policy could be implemented in a way that also increases expected surplus for potential bidders and sellers on the platform.

Building on the model and results in this paper, a logical next step is to consider the platform fee setting problem more explicitly. The resulting structure could be used to predict the impact of mergers between auction platforms, thereby contributing to an empirical literature studying (price-)impacts of consolidation in two-sided markets (including e.g. Chandra and Collard-Wexler (2009), Song (2013) and Fan (2013)).

Another direction for further research is to test predictions from this paper using experimental variation in fees. This would be in line with e.g. Ostrovsky and Schwarz (2016) who conduct a field experiment to show that revenues from Yahoo! ad auctions increase significantly when adopting the optimal Myerson (1981) and Riley and Samuelson (1981) reserve prices.

References


A Omitted tables and figures

Figure 8: Ullage classification and interpretation

Source: Christie’s (2013). Numbers refer to auction house Christie’s interpretation of the fill levels, which are for Bordeaux-style bottles: 1) Into Neck: level of young wines. Exceptionally good in wines over 10 years old. 2) Bottom Neck: perfectly good for any age of wine. Outstandingly good for a wine of 20 years in bottle, or longer. 3) Very Top-Shoulder. 4) Top-Shoulder. Normal for any claret 15 years or older. 5) Upper-Shoulder: slight natural reduction through the easing of the cork and evaporation through the cork and capsule. Usually no problem. Acceptable for any wine over 20 years old. Exceptional for pre-1950 wines. 6) Mid-Shoulder: probably some weakening of the cork and some risk. Not abnormal for wines 30/40 years of age. 7) Mid-Low-Shoulder: some risk. 8) Low-Shoulder: risky and usually only accepted for sale if wine or label exceptionally rare or interesting. For Burgundy-style bottles where the slope of the shoulder is impractical to describe such levels, whenever appropriate [due to the age of the wine] the level is measured in centimetres. The condition and drinkability of Burgundy is less affected by ullage than Bordeaux. For example, a 5 to 7 cm. ullage in a 30 year old Burgundy can be considered normal or good for its age.
Table 9: First stage estimation results, main sample

<table>
<thead>
<tr>
<th>Dep. var: log transaction price per bottle with 2 or more bidders</th>
<th>Estimate</th>
<th>Std. error</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>4.042</td>
<td>0.165</td>
<td>0</td>
</tr>
<tr>
<td>Case of 6 bottles</td>
<td>0.276</td>
<td>0.096</td>
<td>0.004</td>
</tr>
<tr>
<td>Case of 12 bottles</td>
<td>1.473</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>Stored in warehouse</td>
<td>0.506</td>
<td>0.211</td>
<td>0.016</td>
</tr>
<tr>
<td>Special format (not 75cl)</td>
<td>0.229</td>
<td>0.068</td>
<td>0.004</td>
</tr>
<tr>
<td>Number bottles in lot</td>
<td>-0.215</td>
<td>0.017</td>
<td>0</td>
</tr>
<tr>
<td>Fill level: Base of Neck (BN)</td>
<td>-0.215</td>
<td>0.068</td>
<td>0.002</td>
</tr>
<tr>
<td>Fill level: High Shoulder (HS)</td>
<td>-0.394</td>
<td>0.148</td>
<td>0.008</td>
</tr>
<tr>
<td>Fill level: Low Shoulder (LS) or worse</td>
<td>-0.364</td>
<td>0.19</td>
<td>0.056</td>
</tr>
<tr>
<td>Fill level: Missing</td>
<td>0.012</td>
<td>0.055</td>
<td>0.83</td>
</tr>
<tr>
<td>Fill level: Mid Shoulder (HS)</td>
<td>-0.427</td>
<td>0.159</td>
<td>0.007</td>
</tr>
<tr>
<td>Fill level: Top Shoulder (TS)</td>
<td>-0.401</td>
<td>0.136</td>
<td>0.003</td>
</tr>
<tr>
<td>Fill level: Very Top Shoulder (VTS)</td>
<td>-0.139</td>
<td>0.116</td>
<td>0.25</td>
</tr>
<tr>
<td>Duty estimate</td>
<td>-0.026</td>
<td>0.009</td>
<td>0.004</td>
</tr>
<tr>
<td>VAT estimate</td>
<td>0.008</td>
<td>0.006</td>
<td>0.173</td>
</tr>
<tr>
<td>Shipping quote</td>
<td>0.007</td>
<td>0.004</td>
<td>0.091</td>
</tr>
<tr>
<td>Percentage range provided shipping quotes</td>
<td>-0.923</td>
<td>0.094</td>
<td>0</td>
</tr>
<tr>
<td>Percentage range provided shipping quotes, squared</td>
<td>0.361</td>
<td>0.048</td>
<td>0</td>
</tr>
<tr>
<td>Delivers to UK</td>
<td>0.066</td>
<td>0.044</td>
<td>0.136</td>
</tr>
<tr>
<td>Seller accepts returns</td>
<td>-0.14</td>
<td>0.151</td>
<td>0.355</td>
</tr>
<tr>
<td>Bottles can be collected from seller</td>
<td>0.027</td>
<td>0.044</td>
<td>0.535</td>
</tr>
<tr>
<td>Buyer can only collect bottles from seller</td>
<td>-0.237</td>
<td>0.11</td>
<td>0.032</td>
</tr>
<tr>
<td>Shipping with Royal Mail</td>
<td>0.025</td>
<td>0.065</td>
<td>0.22</td>
</tr>
<tr>
<td>Shipping with ParcelForce</td>
<td>-0.155</td>
<td>0.046</td>
<td>0.001</td>
</tr>
<tr>
<td>Mentions fast shipping</td>
<td>0.444</td>
<td>0.066</td>
<td>0</td>
</tr>
<tr>
<td>Insurance for loss or breakage is included in shipping cost</td>
<td>0.098</td>
<td>0.041</td>
<td>0.016</td>
</tr>
<tr>
<td>Can pay by bank</td>
<td>0.336</td>
<td>0.089</td>
<td>0</td>
</tr>
<tr>
<td>Can pay by Paypall</td>
<td>-0.033</td>
<td>0.045</td>
<td>0.461</td>
</tr>
<tr>
<td>Can pay by cheque</td>
<td>-0.139</td>
<td>0.047</td>
<td>0.003</td>
</tr>
<tr>
<td>Can pay in cash</td>
<td>-0.088</td>
<td>0.113</td>
<td>0.436</td>
</tr>
<tr>
<td>Wine Type: Assorted</td>
<td>0.089</td>
<td>0.068</td>
<td>0.199</td>
</tr>
<tr>
<td>Wine Type: White</td>
<td>-0.247</td>
<td>0.079</td>
<td>0.062</td>
</tr>
<tr>
<td>Wine Type: Sparkling</td>
<td>0.176</td>
<td>0.105</td>
<td>0.094</td>
</tr>
<tr>
<td>Wine Type: Fortified</td>
<td>-0.161</td>
<td>0.125</td>
<td>0.198</td>
</tr>
<tr>
<td>Wine Type: Rose</td>
<td>-0.186</td>
<td>0.324</td>
<td>0.566</td>
</tr>
<tr>
<td>Region of origin: Tuscany</td>
<td>-0.375</td>
<td>0.106</td>
<td>0</td>
</tr>
<tr>
<td>Region of origin: Bordeaux</td>
<td>-0.199</td>
<td>0.069</td>
<td>0.004</td>
</tr>
<tr>
<td>Region of origin: Australia</td>
<td>-0.377</td>
<td>0.112</td>
<td>0.001</td>
</tr>
<tr>
<td>Region of origin: Burgundy</td>
<td>-0.154</td>
<td>0.095</td>
<td>0.106</td>
</tr>
<tr>
<td>Region of origin: Rhone</td>
<td>-0.061</td>
<td>0.094</td>
<td>0.518</td>
</tr>
<tr>
<td>Region of origin: Champagne</td>
<td>0.161</td>
<td>0.124</td>
<td>0.195</td>
</tr>
<tr>
<td>Region of origin: Provence</td>
<td>-0.43</td>
<td>0.242</td>
<td>0.076</td>
</tr>
<tr>
<td>Region of origin: Veneto</td>
<td>-0.358</td>
<td>0.172</td>
<td>0.038</td>
</tr>
<tr>
<td>Region of origin: Rioja</td>
<td>-0.337</td>
<td>0.176</td>
<td>0.056</td>
</tr>
<tr>
<td>Region of origin: Alsace</td>
<td>-0.191</td>
<td>0.195</td>
<td>0.328</td>
</tr>
<tr>
<td>Region of origin: Piedmont/Lombardy</td>
<td>-0.397</td>
<td>0.133</td>
<td>0.003</td>
</tr>
<tr>
<td>Region of origin: South Australia</td>
<td>-0.348</td>
<td>0.113</td>
<td>0.002</td>
</tr>
<tr>
<td>Region of origin: Douro</td>
<td>-0.349</td>
<td>0.195</td>
<td>0.074</td>
</tr>
<tr>
<td>Region of origin: Mendoza</td>
<td>-0.438</td>
<td>0.347</td>
<td>0.206</td>
</tr>
<tr>
<td>Region of origin: Scotland</td>
<td>0.228</td>
<td>0.26</td>
<td>0.381</td>
</tr>
<tr>
<td>Region of origin: Oporto</td>
<td>-0.217</td>
<td>0.197</td>
<td>0.272</td>
</tr>
<tr>
<td>Region of origin: Assorted</td>
<td>0.098</td>
<td>0.084</td>
<td>0.244</td>
</tr>
<tr>
<td>Region of origin: Bekan Valley</td>
<td>0.103</td>
<td>0.366</td>
<td>0.78</td>
</tr>
<tr>
<td>Region of origin: United States</td>
<td>-0.103</td>
<td>0.197</td>
<td>0.6</td>
</tr>
<tr>
<td>Region of origin: California</td>
<td>0.002</td>
<td>0.159</td>
<td>0.992</td>
</tr>
<tr>
<td>Region of origin: Spain</td>
<td>-0.418</td>
<td>0.265</td>
<td>0.115</td>
</tr>
<tr>
<td>Region of origin: Cognac</td>
<td>0.672</td>
<td>0.301</td>
<td>0.026</td>
</tr>
<tr>
<td>Region of origin: Portugal</td>
<td>0.111</td>
<td>0.195</td>
<td>0.571</td>
</tr>
<tr>
<td>Region of origin: Loire</td>
<td>-0.759</td>
<td>0.268</td>
<td>0.005</td>
</tr>
<tr>
<td>Region of origin: Cuba</td>
<td>-0.115</td>
<td>0.346</td>
<td>0.74</td>
</tr>
<tr>
<td>Region of origin: Italy</td>
<td>-0.328</td>
<td>0.145</td>
<td>0.024</td>
</tr>
<tr>
<td>Region of origin: Oregon</td>
<td>-0.704</td>
<td>0.266</td>
<td>0.008</td>
</tr>
<tr>
<td>Region of origin: Ribera del Duero</td>
<td>-0.368</td>
<td>0.334</td>
<td>0.271</td>
</tr>
<tr>
<td>Region of origin: South Africa</td>
<td>-0.586</td>
<td>0.343</td>
<td>0.088</td>
</tr>
<tr>
<td>Region of origin: Islay</td>
<td>0.681</td>
<td>0.473</td>
<td>0.15</td>
</tr>
<tr>
<td>Region of origin: South West France</td>
<td>0.166</td>
<td>0.481</td>
<td>0.73</td>
</tr>
<tr>
<td>Region of origin: Other</td>
<td>-0.155</td>
<td>0.105</td>
<td>0.14</td>
</tr>
</tbody>
</table>

... Continued on next page
| Grape: Sangiovese | 0.348 | 0.107 | 0.001 |
| Grape: Corvina | -0.027 | 0.238 | 0.91 |
| Grape: Rhone Blend | 0.034 | 0.123 | 0.783 |
| Grape: Syrah | 0.213 | 0.151 | 0.157 |
| Grape: Bordeaux Blend | 0.153 | 0.066 | 0.019 |
| Grape: Other | -0.088 | 0.089 | 0.37 |
| Grape: Riesling | -0.008 | 0.198 | 0.968 |
| Grape: Nebbiolo | 0.253 | 0.145 | 0.082 |
| Grape: Cabernet Sauvignon | 0.163 | 0.167 | 0.328 |
| Grape: Chardonnay | 0.207 | 0.117 | 0.076 |
| Grape: Tempranillo | 0.334 | 0.209 | 0.111 |
| Grape: Malbec | -0.19 | 0.381 | 0.619 |
| Grape: Pinot Noir | -0.01 | 0.107 | 0.926 |
| Grape: Syrah/Shiraz | 0.246 | 0.107 | 0.022 |
| Grape: Port Blend | 0.744 | 0.243 | 0.002 |
| Grape: Semillon-Sauvignon Blanc Blend | 0.631 | 0.258 | 0.015 |
| Grape: Merlot | -0.337 | 0.193 | 0.08 |
| Grape: Champagne Blend | 0.673 | 0.228 | 0.003 |
| Grape: Barbera | -0.367 | 0.389 | 0.234 |

Excluded from table: popular vintage and market month fixed effects. Estimated coefficients = 141.

Table 10: First stage estimation results, high-value sample

<table>
<thead>
<tr>
<th>Dep. var: log transaction price per bottle with 2 or more bidders</th>
<th>Estimate</th>
<th>Std. error</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>6.242</td>
<td>0.319</td>
<td>0</td>
</tr>
<tr>
<td>Case of 6 bottles</td>
<td>-0.881</td>
<td>0.105</td>
<td>0</td>
</tr>
<tr>
<td>Case of 12 bottles</td>
<td>-0.514</td>
<td>0.131</td>
<td>0</td>
</tr>
<tr>
<td>Stored in warehouse</td>
<td>-0.705</td>
<td>0.248</td>
<td>0.005</td>
</tr>
<tr>
<td>Special format (not 75cl)</td>
<td>0.393</td>
<td>0.145</td>
<td>0.007</td>
</tr>
<tr>
<td>Number bottles in lot</td>
<td>-0.124</td>
<td>0.011</td>
<td>0</td>
</tr>
<tr>
<td>Fill level: Missing</td>
<td>-0.082</td>
<td>0.09</td>
<td>0.361</td>
</tr>
<tr>
<td>Fill level: Base of Neck (BN)</td>
<td>0.03</td>
<td>0.14</td>
<td>0.83</td>
</tr>
<tr>
<td>Fill level: Mid Shoulder (HS)</td>
<td>0.1</td>
<td>0.229</td>
<td>0.661</td>
</tr>
<tr>
<td>Fill level: Top Shoulder (TS)</td>
<td>0.265</td>
<td>0.355</td>
<td>0.456</td>
</tr>
<tr>
<td>Fill level: Very Top Shoulder (VTS)</td>
<td>-0.039</td>
<td>0.164</td>
<td>0.812</td>
</tr>
<tr>
<td>Fill level: High Shoulder (HS)</td>
<td>0.177</td>
<td>0.253</td>
<td>0.485</td>
</tr>
<tr>
<td>Fill level: Low Shoulder (LS) or worse</td>
<td>-0.13</td>
<td>0.334</td>
<td>0.696</td>
</tr>
<tr>
<td>Duty estimate</td>
<td>0.023</td>
<td>0.01</td>
<td>0.021</td>
</tr>
<tr>
<td>VAT estimate</td>
<td>-0.003</td>
<td>0.004</td>
<td>0.459</td>
</tr>
<tr>
<td>Shipping quote</td>
<td>0.005</td>
<td>0.004</td>
<td>0.16</td>
</tr>
<tr>
<td>Percentage range provided shipping quotes</td>
<td>-0.192</td>
<td>0.321</td>
<td>0.549</td>
</tr>
<tr>
<td>Percentage range provided shipping quotes, squared</td>
<td>0.064</td>
<td>0.15</td>
<td>0.672</td>
</tr>
<tr>
<td>Delivers to UK</td>
<td>-0.293</td>
<td>0.096</td>
<td>0.002</td>
</tr>
<tr>
<td>Seller accepts returns</td>
<td>0.002</td>
<td>0.164</td>
<td>0.992</td>
</tr>
<tr>
<td>Bottles can be collected from seller</td>
<td>0.036</td>
<td>0.086</td>
<td>0.676</td>
</tr>
<tr>
<td>Buyer can only collect bottles from seller</td>
<td>-0.546</td>
<td>0.196</td>
<td>0.006</td>
</tr>
<tr>
<td>Shipping with Royal Mail</td>
<td>-0.052</td>
<td>0.099</td>
<td>0.605</td>
</tr>
<tr>
<td>Shipping with ParcelForce</td>
<td>-0.218</td>
<td>0.12</td>
<td>0.071</td>
</tr>
<tr>
<td>Mentions fast shipping</td>
<td>0.124</td>
<td>0.129</td>
<td>0.336</td>
</tr>
<tr>
<td>Insurance is included in shipping cost</td>
<td>-0.133</td>
<td>0.076</td>
<td>0.081</td>
</tr>
<tr>
<td>Can pay by bank</td>
<td>-0.146</td>
<td>0.211</td>
<td>0.491</td>
</tr>
<tr>
<td>Can pay by Paypall</td>
<td>-0.092</td>
<td>0.083</td>
<td>0.27</td>
</tr>
<tr>
<td>Can pay by cheque</td>
<td>-0.091</td>
<td>0.078</td>
<td>0.244</td>
</tr>
<tr>
<td>Can pay in cash</td>
<td>0.015</td>
<td>0.225</td>
<td>0.948</td>
</tr>
</tbody>
</table>

Excluded from table: popular vintage, wine type, region, grape and market month fixed effects. Estimated coefficients = 133.
Nuits St George Les Boudots Domaine Leroy

Sold by wawemuse (13 ratings, 75% positive, 0% neutral.)

- Email the seller
- Show my bids on this auction
- Add this auction to my watch list

BID NOW

(Your bid is for 1 bottle of 750 ml.)

Your maximum bid: £
(At least £52.00)

Place Bid Now

<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>£50.00</th>
<th>2d 19h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid placed</td>
<td>No. of Bidders</td>
<td>Current price</td>
<td>Remaining time</td>
</tr>
</tbody>
</table>

Lot size: 1 bottle of 750 ml each  
Wine type: Red, 1985 vintage  
Tax status: Duty Paid  
Origin: Burgundy, France  
Fill level: Inoc Nock (IN)  
Grape variety: 

An incredibly rare bottle of the sublime Nuits St George Les Boudots from Domaine Leroy from the exceptional 1985 vintage. In great order, this legend of a wine has lain in the same Berlin cellar or decades. The last time this was on WineSearcher - 2016 - it was listed at £2,200, the reserve on this is a fraction of that.

PayPal preferred but will charge 4% for fees.

Other details

Aux Boudots' thin soils consist of gravel, crumbly limestone marl and a small amount of clay. This fragmented soil, along with the natural slope of the vineyard, gives good drainage, making sure that vines do not receive excessive water. Instead, vines have to grow deep into the ground in search of hydration, a process which lessens vigor and reduces grape yields. This ultimately leads to the production of small, concentrated berries which make excellent wines.

Payment methods: PayPal  
Returns policy: No returns  
Shipping Method: Courier delivery.  
Shipping paid by: buyer  
Cost of delivery: Will quote  
Delivers to UK and Singapore  
Other countries delivered to: Worldwide  
Insurance options: TBC

Figure 9: Listing page example: Nuits St George Les Boudouts, Domaine Leroy

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Figure 10: The survival probability of product listings

The survival probability is defined as the empirical probability that the same product is listed in consecutive months. For example, a survival probability of 0.2 at 5 months indicates that 20 percent of all products that were offered in the first month are also offered in months 2-6. The solid black line (bottom) pertains to the median product, of which only one is listed in a typical month, and the dash-dotted blue line (top) pertains to products in the 90th percentile of availability in a typical month. See Table 3 for deciles of the distribution of the number of comparable products per month. A key take-away from this plot is that about $\frac{7}{8}$th of typical listings doesn’t survive one month, i.e. doesn’t get listed in the next period.

Figure 11: Testing equality of reserve price distribution and approximation
B Testing the reserve price approximation

I approximate the reserve price as the average between the highest standing price for which the reserve price is not met and the lowest for which it is met. If all bids would be recorded in real time, this approximation would be accurate up to half a bidding increment due to the proxy bidding system. But to relieve traffic pressure on the site I only track bids on 30-minute intervals. The reserve price approximation could be more than half a bidding increment off if the bids are not placed at regular intervals. As a compromise with constant high website traffic a separate dataset is collected that accesses open listings at 30-second intervals for the duration of two weeks, to test the reserve price approximation in the main sample.

My estimation method requires that the estimated distribution of reserve prices is consistent for its population counterpart. Equality of the distribution of approximated reserve prices in the main sample and the distribution of (approximated) reserve prices in the smaller high frequency sample is tested with a two sample nonparametric Kolmogorov-Smirnov test. To account for different listing compositions the empirical reserve price distributions are right-truncated at the 90th percentile of the high frequency reserve price sample. The null hypothesis is that the two right truncated reserve price distributions are the same. In particular, letting $F_R^H$ and $F_R^M$ respectively denote the empirical distribution of right truncated approximated reserve prices in the high frequency (H) and main (M) sample, the Kolmogorov-Smirnov test statistic is defined as:

$$D_{h,m} = \sup_x |F_R^H(x) - F_R^M(x)|,$$

with $\sup_x$ the supremum function over $x$ values and $h$ and $m$ respectively denoting the relevant number of observations in the high frequency and main samples, which are 330 and 596 (only for sold lots). With $D_{h,m} = 0.059$, the null cannot be rejected at the 5 percent level ($D_{h,m} > 1.36\sqrt{\frac{h+m}{hm}}$, the p-value = 0.4406). The two empirical distributions are plotted in Figure 11.
C Independent listings: additional results

This section reports descriptive statistics that point to listings being independent of each other conditional on entry and the matching of bidders to listings.

a) Number bids per bidder

b) Reserve price

c) Transaction price

Figure 12: Evidence stylized fact: independence listings. Patterns suggesting that bidders do not cross-bid, thin out, and sellers do not compete

The box edges indicate the 25th to 75th percentile, the black line indicates the median, and whiskers indicate the 25th (75th) percentile minus (plus) 1.5 times the interquartile range or the sample extremes if those are less extreme. The residual reserve and transaction price in plot b) and c) are obtained from a linear regression of these outcomes $Y_{pm}$ on product dummies $\alpha_p$ in:

$Y_{pm} = \alpha_p + \epsilon_{pm}$. Plots display the relation between $\epsilon_{pm}$ and the number of listings of product $p$ in market $m$, $T_{pm}$.
Table 11: Evidence stylized facts: independent listings

<table>
<thead>
<tr>
<th>Competing listings: any wine</th>
<th>Dependent variable:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>- Ending within 30 days (of each other ... )</td>
<td>Nr bidders listing</td>
<td>0.0001</td>
<td>0.00002</td>
<td>-0.0001</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td>Bids per bidder</td>
<td>-0.001</td>
<td>-0.0002</td>
<td>0.008</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td>Transaction price</td>
<td>-0.003*</td>
<td>0.0003</td>
<td>0.062</td>
</tr>
<tr>
<td>Competing listings: same wine type (e.g. red)</td>
<td>Reserve price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ending within 30 days</td>
<td></td>
<td>-0.001</td>
<td>0.00004</td>
<td>0.012</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td></td>
<td>-0.0003</td>
<td>-0.0002</td>
<td>0.084</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td></td>
<td>-0.003</td>
<td>0.001</td>
<td>0.167</td>
</tr>
<tr>
<td>Competing listings: same wine region (e.g. Bordeaux)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ending within 30 days</td>
<td></td>
<td>-0.004</td>
<td>0.001</td>
<td>0.155</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td></td>
<td>-0.001</td>
<td>0.002</td>
<td>0.376</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td></td>
<td>-0.020**</td>
<td>0.006**</td>
<td>0.171</td>
</tr>
<tr>
<td>Competing listings: region x wine type (e.g. red Bordeaux)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ending within 30 days</td>
<td></td>
<td>-0.003</td>
<td>0.001</td>
<td>0.228</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td></td>
<td>0.011</td>
<td>0.004</td>
<td>1.134**</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td></td>
<td>-0.019</td>
<td>0.007*</td>
<td>0.905</td>
</tr>
<tr>
<td>Competing listings: region x wine type x vintage decade (e.g. red Bordeaux from 1980s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ending within 30 days</td>
<td></td>
<td>-0.012</td>
<td>0.001</td>
<td>-0.561</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td></td>
<td>-0.006</td>
<td>0.005</td>
<td>-0.465</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td></td>
<td>-0.061</td>
<td>0.004</td>
<td>-0.938</td>
</tr>
<tr>
<td>Competing listings: subregion x wine type x vintage decade (e.g. red Margaux from 1980s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ending within 30 days</td>
<td></td>
<td>-0.009</td>
<td>0.001</td>
<td>0.433</td>
</tr>
<tr>
<td>- Ending within 7 days</td>
<td></td>
<td>0.003</td>
<td>0.003</td>
<td>1.914**</td>
</tr>
<tr>
<td>- Ending within 2 days</td>
<td></td>
<td>-0.034</td>
<td>0.007</td>
<td>0.775</td>
</tr>
<tr>
<td>Product fixed effects:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample:</td>
<td>1,150</td>
<td>2,898</td>
<td>2,230</td>
<td>2,337</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.147</td>
<td>0.016</td>
<td>0.102</td>
<td>0.056</td>
</tr>
</tbody>
</table>

Significance levels: *p<0.1; **p<0.05; ***p<0.01. All reported coefficients are from a separate regression of the dependent variable on the number of competing listings, each row of the table corresponding to a different definition, including product fixed effects. The product fixed effect relates to the product aspect of the competing listing specification (so types of wine, or regions, etc.). The adjusted R^2 is from the regression with (subregion x type x vintage decade fixed effects).
D Additional details entry equilibrium

In this supplementary material, provides further intuition behind the entry equilibrium derived in the main text, for auctions with positive reserve prices. To economize on notation in the main text, I also specify listing-level surplus for bidders and sellers in this appendix. I consider the bidder entry equilibrium if the number of listings \( T > 0 \) would be known. This highlights that the equilibrium distribution of the number of bidders per listing is independent of the number of listings. It is then straightforward to see that also in expectation, for the simultaneous entry equilibrium presented in the main text, the equilibrium distribution \( f_N > 0; f^* \) is independent of the number of listings conditional on selection of sellers.

In what follows, \( \tilde{r} \) denotes the optimal reserve price increased with buyer premium, \( \tilde{r} = (1 + c_B) r^*(v_0, f) \). Before knowing their valuation, the expected bidder surplus in a listing with \( n \) bidders equals:

\[
\pi_b(n, f, r) \equiv \frac{1}{n} E[V_{(n:n)} - \max(V_{(n-1:n)}, \tilde{r})|V_{(n:n)} \geq \tilde{r}][1 - F_{V_{(n:n)}}(\tilde{r})],
\]

(36)

with the last term denoting the sale probability and the \( \max() \) term the transaction price including buyer premium. Expected surplus for a seller with valuation \( v_0 \) in a listing with \( n \) bidders:

\[
\pi_s(n, f, v_0) \equiv (E[\max(V_{n-1:n}, \tilde{r})|V_{n:n} \geq \tilde{r}](1 - c_S) - v_0) [1 - F_{V_{(n:n)}}(\tilde{r})]
\]

(37)

The following properties are also useful for solving the entry stage.

Lemma 3A. The listing-level impacts of number of bidders, commissions, and the seller’s valuation are:

a) Expected listing-level bidder surplus is decreasing in \( n, c_B, c_S \) and (in auctions with a positive reserve price) \( v_0 \)

b) Expected listing-level seller surplus is increasing in \( n \) and decreasing in \( c_B, c_S \) and \( v_0 \)

Proof is provided in Appendix G.

Bidder entry in auctions with a positive reserve price, if the number of listings would be known

Consider a different model in which the number of listings \( T > 0 \) would be known to potential bidders. Let \( \bar{v}_0 \) denote a candidate seller entry threshold and \( \Pi_{b,r>0}^T(f, \bar{v}_0; p) \) potential bidders’ expected surplus from entering the platform as a function of their entry probability \( p \), again if they

---

59Two observations are useful to make here. The first is also made in Ginsburgh et al. (2010): in a model without entry, the allocation of \((c_B, c_S)\) does not affect outcomes as long as a commission index \( \frac{c_B + c_S}{1} \) remains constant. I mention this here because this setting is previously used to study the impact of \( c_B \) and \( c_S \) on welfare in Ashenfelter and Graddy (2003, 2005) and Marks (2009). I refer to this as a “one-sided market perspective” (on the impacts of auction fees), by the definition in Rochet and Tirole (2006) of a market in which only the level and not the allocation of fees matters. I provide a numerical example of this case in the Appendix on page 66. The platform market with entry decisions involving sunk entry cost constitutes a two-sided market (Rochet and Tirole (2006)). The second is that a seller maximizing \( \pi_s(n, f, v_0) \) by his choice of reserve sets a reserve price that is too high from the platform’s perspective. To see why, note that the seller trades off the expected transaction price with \( v_0 \) while the platform revenue only involves a share of the transaction price. This observation does not seem lost in practice: eBay charges a reserve price fee equal to 4 percent of the reserve price (unconditional on selling) and BW platform has flat reserve price fees.
In this equilibrium the number of (competing) bidders per listing follows a Binomial distribution

\[ \Pi_{b,r>0}^{T,r>0}(f, \bar{v}_0; p) = \sum_{n=0}^{N^{B,r>0}-1} \mathbb{E}[\pi_b(n+1, f, v_0)|V_0 \in [v_{0,r=0}, \bar{v}_0]] f_{N^{r>0},r>0}^{T,r>0}(n; p) - e_{B,r>0}^o, \]  

(38)

It takes the expectation of \( \pi_b(n, f, v_0) \) (equation 36 with optimal \( r \)) over: i) possible seller values given sellers’ entry threshold and ii) the number of competing bidders given their entry probability. \( T^{r>0} \) superscripts in \( \Pi_{b,r>0}^{T,r>0}(f, \bar{v}_0; p) \) and \( f_{N^{r>0},r>0}^{T,r>0}(n; p) \) (equation 40) emphasize that they relate to the thought exercise in which \( T_{r>0} \) is known, while the true game’s simultaneous entry requires taking the expectation over \( T_{r>0} \) given candidate entry threshold \( \bar{v}_0 \) and \( N^S \). This is done to show more clearly that, in equilibrium, \( f_{N^{r>0},r>0}^{T,r>0} \) is independent of the realization of \( T_{r>0} \) which implies that it must also be independent of the expectation over \( T_{r>0} \). Bidding in one listing at a time, the entry problem for potential bidders is then equivalent to one in which they consider entry into a listing (also given that opportunity cost (listing inspection) \( e_{B,r>0}^o \) are associated with each listing). Components of equation (38) are then:

\[ \mathbb{E}[\pi_b(n+1, f, v_0)|V_0 \in [v_{0,r=0}, \bar{v}_0]] = \int_{v_{0,r=0}}^{\bar{v}_0} \pi_b(n+1, f, v_0) f_{V_0|v_{0,r=0}, \bar{v}_0}(v_0)dv_0 \]  

(39)

\[ f_{N^{r>0},r>0}^{T,r>0}(n; p) = \left( \frac{N^{B,r>0} - 1}{n} \right) \left( \frac{p}{T} \right)^n (1 - \frac{p}{T})^{N^{B,r>0}-1-n} \]  

(40)

where \( f_{N^{r>0},r>0}^{T,r>0}(n; p) \) denotes the Binomial probability that \( n \) out of \( N^{B,r>0} - 1 \) competing potential bidders arrive in the same listing as the potential bidder who considers entering the platform. \( \pi_b(n+1, f, v_0) \) is strictly decreasing in \( n \) due to the increasing failure rate assumption on \( F_{V|Z} \), which delivers a decreasing spacings property (Li (2005), discussed in more detail in the proof of Lemma 3A). Hence, the bidder entry problem is equivalent to the Levin and Smith (1994) entry equilibrium (which starts from expected bidder surplus decreasing in \( n \)). Given \( T_{r>0} \), the equilibrium bidder entry probability \( p^{*T,r>0} \) solves zero profit condition:\footnote{\( p^{*T,r>0} \) is used to distinguish the entry probability from the central one pertaining to the central case where the number of listings is not known. A no-trade entry equilibrium at \( p = 0 \) that trivially solves (6) always exists and excluding it requires the profit-maximizing platform to set fees such that entry is profitable for players on both sides and for players not to believe that the other side enters with zero probability.}

\[ p^{*T,r>0}(T_{r>0}, f, \bar{v}_0) = \arg_{p \in (0,1)} \Pi_{b}^{T,r>0}(f, \bar{v}_0; p) = 0 \]  

(41)

In this equilibrium the number of (competing) bidders per listing follows a Binomial distribution with mean \( (N^{B,r>0} - 1)p^{*T,r>0}_{T_{r>0}} \) and variance \( (N^{B,r>0} - 1)p^{*T,r>0}_{T_{r>0}}(1 - p^{*T,r>0}_{T_{r>0}}) \).\footnote{For completeness this is derived in Appendix G. The variance of \( N_{r>0} \) would be larger when also taking the expectation over \( T_{r>0} \) given \( N^S \) and \( \bar{v}_0 \).}

A key property is that \( p^{*T,r>0}_{T_{r>0}} \) is independent of \( T_{r>0} \): bidders only derive positive surplus from the listing that they are matched to (e.g. \( T_{r>0} \) does not affect \( \mathbb{E}[\pi_b(n+1, f, v_0)|V_0 \in [v_{0,r=0}, \bar{v}_0]] \)) so the zero profit condition guarantees that in equilibrium a change in \( T_{r>0} \) causes \( p^{*T,r>0}_{T_{r>0}} \) to
adjust to keep $\int_{N>0}^{T,r>0} f$ constant. The same reasoning applies when $T_{r>0}$ is the stochastic outcome of the simultaneous seller entry process: the seller entry threshold only affects the equilibrium mean number of bidders per listing through $E[\pi_b(n + 1, f, v_0)|V_0 \in [v_0_{r=0}, \bar{v}_0]]$ and not through its effect on the distribution of $T_{r>0}$. This is defined more formally in the main text that describes the simultaneous-move entry equilibrium.

E  Monte Carlo simulations: a recursive algorithm

This Section discusses the Monte Carlo simulations and provides details about numerical approximation of the entry equilibrium. Simulated auctions are structured according to the idiosyncratic-good auction platform model presented before with:

<table>
<thead>
<tr>
<th>Input parameters:</th>
<th>Equilibrium values:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(Z) = 0.5Z, Z \sim N(0, 2)$</td>
<td>$v_0^* = 5.999, P_{V</td>
</tr>
<tr>
<td>$(U_0, U) \sim N(5, 0.5)$</td>
<td>$\lambda_{r=0}^* = 4.499, \lambda_{r&gt;0}^* = 6.112$</td>
</tr>
<tr>
<td>$e_B^0 = e^0_{B,r=0} = 10 = e^0_{B,r&gt;0}, e_S^0 = 5$</td>
<td></td>
</tr>
<tr>
<td>$f = {e_S = 5, e_B = 0, c_B = 0, c_S = 0.1}$</td>
<td></td>
</tr>
<tr>
<td>$p_{r=0} = 0.10, N^S = 4000$</td>
<td></td>
</tr>
</tbody>
</table>

Hence with the chosen valuation distributions, opportunity cost and platform fees, the seller entry threshold equals 5.599 so that in equilibrium 88.47 percent of the 4000 potential sellers enter the platform. The marginal seller, or any seller who sets a positive reserve price receives on average 4.499 bidders in his listing. Furthermore, the mean number of bidders in zero reserve auctions is calculated to be 6.112 and 10 percent of listings have no reserve price (exogenously determined here).

A recursive algorithm.

Recall the multi-step estimation method outlined on page 24. I will now provide details on steps 4 and 5:

4) solving for the entry equilibrium given estimated parameters

5) re-estimating seller parameters at the updated entry equilibrium.

When iterating on these estimation steps until convergence, they describe the iterative nested pseudo likelihood (NPL) in Aguirregabiria and Mira (2002, 2007). Roberts and Sweeting (2010) are the first to apply this algorithm to the auction literature to study auctions with selective bidder entry. Studies by Pesendorfer and Schmidt-Dengler (2010), Kasahara and Shimotsu (2012) and Egesdal et al. (2015) provide conditions under which NPL does (not) converge to the true equilibrium. A best-response stable equilibrium is a sufficient condition for the NPL algorithm to converge to it and this is certainly guaranteed (Proposition 1) by the game reducing to a single agent discrete choice problem with unique equilibrium. Aguirregabiria and Mira (2002) find that in single-agent games asymptotic efficiency is independent of the number of iterations.

Monte Carlo (MC) simulations evaluate the proposed estimation method in Steps (1)-(3) and
the updating according to steps (4)-(5), either once or iterating until the maximum difference between updated and previous seller parameters is less than $10^{-3}$. This choice of convergence objective is motivated by the centrality of $\theta_s$ as structural parameters in contrast to the seller entry probability or threshold. Furthermore, calculation of $v_0^*$ depends on opportunity cost to the marginal seller $e_o^S$, which, following its identification argument, is a function of both seller parameters and the entry threshold. If the initial estimate $v_T$ is an overestimate of the truth that will be reflected in low opportunity cost that reinforce a high $v_0^*$ in the recursion. To address this, an exponentially vanishing trimming parameter $\tau^j$ is subtracted from the seller entry probability obtained in iteration $j$ to enforce that $v_T > v_0^*$. Using $\tau^j = 0.05 \exp(-j)$ the trimming parameter at $j = 1$ equals 0.05 and vanishes quickly to virtually zero in than 5 iterations.

Four sets of 500 MC experiments are conducted. The first implements the algorithm with the true value of $e_o^S$, the second imposes a fixed but wrong value of twice $e_o^S$, and the third recursively solves for $e_o^S$ given $\theta_s^{j-1}$. Columns 1-3 of Table 12 show that all three ways to deal with $e_o^S$ (the true value, a wrong value, solving recursively) deliver virtually identical estimates of $\theta_s$. This confirms that equating seller opportunity cost to some fixed value (usually 0, for instance in Seim (2006) for a discrete choice entry game with firm heterogeneity) is truly a normalisation for the estimation of seller parameters in our setting. Another take-away is that when the first stage is estimated precisely and the resulting initial estimate $v_T$ is close as well, the recursive method delivers no benefits only computational cost. Intuitively, this is because seller parameters are obtained using the first order condition of optimal reserve prices that is independent of the threshold for all observations not censored by it.

Column 4 of Table 12 shows the benefit of updating at least once in the presence of small sample estimation bias. In this MC experiment $Z$ is measured with noise, implemented by adding draws from $Unif(-1,1)$ to it after simulating values and bids. The initial estimate of the seller entry probability now overestimates the truth (0.978 compared to 0.885). Especially standard deviation $\sigma_s$ is affected. One iteration effectively addresses this problem, and further iterations remain consistent but do not deliver benefits or improve precision.

Based on MC simulation results, estimation in the remainder of this paper is done with one update of $\theta_s$ based on solving the equilibrium and normalising $e_o^S = e_o^B$. The gray colored rows in Table 12 correspond to the single recursion solution.

**F Numerical approximation of the entry equilibrium**

Computation of the surplus functions is based on Monte Carlo integration with importance sampling. For details on numerical methods see Judd (1998); I will summarize only their application. The following is implemented on homogenized values and with estimated parameters but for brevity I omit the conditioning on $Z$ and $(\hat{\theta}_b, \hat{\theta}_s)$. A grid of 150 points $v_0^m$ is drawn from $F_{\hat{v}_0}$. For all values on that grid, and for $n = 0, 1, \ldots, 15$ the value of $\pi_s(n, f, v_0^m)$ in (37) is calculated for two scenario’s: when setting $r = r^*(v_0^m)$ and when setting $r = 0$. Also the value of $\pi_b(n + 1, f, v_0^m)$ is calculated
Table 12: Monte Carlo simulations

<table>
<thead>
<tr>
<th>Bidder side</th>
<th>Iteration $j$</th>
<th>Truth</th>
<th>Given true $e_{oS}$</th>
<th>Given wrong $e_{oS}$ (double)</th>
<th>Estimating $e_{oS}$</th>
<th>Estim. $e_{oS}$ noisy first stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Med. AD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>$j$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>0</td>
<td>5</td>
<td>5</td>
<td>0.021</td>
<td>0.015</td>
<td>5</td>
<td>0.02</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td>0.499</td>
<td>0.02</td>
<td>0.013</td>
<td>0.499</td>
<td>0.02</td>
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<tr>
<td>$e_{oB}$</td>
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<td>9.942</td>
<td>0.689</td>
<td>0.481</td>
<td>9.943</td>
<td>0.681</td>
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<td></td>
<td>4.499</td>
<td>4.507</td>
<td>0.159</td>
<td>0.108</td>
<td>4.506</td>
<td>0.158</td>
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<tr>
<td></td>
<td>6.112</td>
<td>6.105</td>
<td>0.137</td>
<td>0.093</td>
<td>6.105</td>
<td>0.137</td>
</tr>
<tr>
<td>$\lambda_{r&gt;0}$</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_{r=0}$</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Seller side, include solving entry game (if $j &gt; 0$)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>$j$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>4.998</td>
<td>0.014</td>
<td>0.01</td>
<td>4.998</td>
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</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4.994</td>
<td>0.027</td>
<td>0.013</td>
<td>4.998</td>
<td>0.034</td>
</tr>
<tr>
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<td>5</td>
<td>4.994</td>
<td>0.027</td>
<td>0.013</td>
<td>4.998</td>
<td>0.032</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4.994</td>
<td>0.029</td>
<td>0.013</td>
<td>4.998</td>
<td>0.034</td>
</tr>
<tr>
<td>$J$</td>
<td>5</td>
<td>4.994</td>
<td>0.029</td>
<td>0.013</td>
<td>4.998</td>
<td>0.034</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>$j$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.023</td>
<td>0.019</td>
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<td>0.023</td>
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<td>0.034</td>
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<td>0.02</td>
<td>0.493</td>
<td>0.032</td>
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<td>0.039</td>
<td>0.023</td>
<td>0.496</td>
<td>0.035</td>
</tr>
<tr>
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<td>0.48</td>
<td>0.038</td>
<td>0.021</td>
<td>0.493</td>
<td>0.031</td>
</tr>
<tr>
<td>$e_{oS}$</td>
<td>$j$</td>
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<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
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<td>2</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>10</td>
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</tr>
<tr>
<td>$J$</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>$F_{0}\mid Z(e_{0}^*)$</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>0</td>
<td>0.885</td>
<td>0.894</td>
<td>0.015</td>
<td>0.012</td>
<td>0.894</td>
<td>0.014</td>
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<td>1</td>
<td>0.885</td>
<td>0.878</td>
<td>0.064</td>
<td>0.015</td>
<td>0.842</td>
<td>0.016</td>
</tr>
<tr>
<td>2</td>
<td>0.885</td>
<td>0.88</td>
<td>0.074</td>
<td>0.02</td>
<td>0.837</td>
<td>0.029</td>
</tr>
<tr>
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<td>0.885</td>
<td>0.881</td>
<td>0.077</td>
<td>0.021</td>
<td>0.838</td>
<td>0.031</td>
</tr>
<tr>
<td>$J$</td>
<td>0.885</td>
<td>0.881</td>
<td>0.077</td>
<td>0.021</td>
<td>0.837</td>
<td>0.028</td>
</tr>
</tbody>
</table>

| Number iterations ($J$) | 6.862 | 2.989 | 7.412 | 3.003 | 8.266 | 2.189 | 8.41 | 2.055 |

Gray rows correspond to the algorithm that solves for the entry equilibrium once given initial parameter estimates, as per the adopted estimation method.
for $n = 0, 1, ..., 15$ and for both scenario’s, which involves the expectation over values of $V_{n:n}$ and also an inner integral over realizations of $V_{n-1:n}$ conditional on $V_{n:n} = v_n$:

$$\pi_b(n, f, r) = \int_\mathbb{R} v_n - \max((1 + c_B)r, \int_\mathbb{R} v_n dF_{V_{n-1:n}}|_{V_{n:n} = v_n}(v_{n-1})) dF_{V_{n:n}}(v_n) \tag{42}$$

$$F_{V_{n:n}}(v_n) = \int_\mathbb{R}^{v_n} nF_V(x)^{n-1} f_V(x) dx$$

$$F_{V_{n-1:n}}|_{V_{n:n} = v_n}(v_{n-1}) = \int_\mathbb{R}^{v_n} \frac{(n-1)F_V(y)^{n-2} f_V(y)}{F_V(v_n)^{n-1}} dy$$

Samples from both distributions are obtained with globally adaptive quadrature (on support $[0, \bar{v}]$ with $\bar{v}$ the $1.0000e^{-9}$th percentile of $F_V$) setting the inverse of the distribution equal to 250 equally spaced probabilities on $[0, 1]$, and then evaluated on 2500 points with linear interpolation, separately for each $n$. Then for each $r \in \{r^*(v_0^n)\}_{n=1}^{250}$, $\pi_b(n+1, f, r)$ is calculated by drawing 250 values from the simulated highest and second-highest valuation samples conditional on $v_n \geq \tilde{r}$ and taking averages. For the positive reserve price case, $\pi_s(n, f, v_0^n)$ and $\pi_b(n+1, f, v_0^n)$ are linearly interpolated on finer grids with 500 points for each $n$. These four matrices are pre-calculated once for each set of parameter draws so essentially for each iteration (and fee combination, in the counterfactual experiment). Let $S_{r=0}$ and $B_{r=0}$ of dimension [16x1] denote the listing-level surplus matrices for sellers and bidders when the seller sets no reserve price and $S_{r>0}$ and $B_{r>0}$ the matrices of dimension [16x150] in the positive reserve price case.

The entry equilibrium solves equation (??), where $\Pi_b(f, \bar{v}_0, \lambda)$ includes: $E[\pi_b(n_1, f, v_0)|V_0 \leq \bar{v}_0]$. In the optimal positive reserve price case it is approximated, for each $n$, by the elementwise product of the $n + 1$th row of $B_{r>0}$ with vector $[f_{v_0^0}(v_0^0), ..., f_{v_0^n}(v_0^n)]$ such that $f_{v_0^0}(v_0^m) = 0$ for $v_0^m > \bar{v}_0$, using local linear interpolation on the margin and with the resulting non-zero elements of the row summing to $1$. Also subtracting bidder entry cost delivers the expected bidder surplus for each number of $n$ competing bidders, and for each $\bar{v}_0$ the equilibrium Poisson parameter $\lambda_{r>0}^*$ is approximated as the value that sets $\Pi_b(f, \bar{v}_0, \lambda) = 0$ to within a $1e^{-6}$ tolerance of either the function value or the parameter.

The calculation of $\lambda_{r>0}^*$ is nested in the seller entry equilibrium. Every candidate $\bar{v}_0$ maps to a $\lambda_{r>0}^*$. For the two columns in $S_{r>0}$ corresponding to the values nearest to $\bar{v}_0$ the expected surplus is calculated by placing weight on row $k$ corresponding to $f_N(k - 1; \lambda_{r>0}^*(\bar{v}_0))$, linear interpolation to get expected surplus at $\bar{v}_0$ for each $n = k - 1$, and summing over all $k$. The equilibrium $v_0^*$ is approximated within a $1e^{-6}$ tolerance level.

Section 5.4 explains that, unless all sellers find it optimal to set a zero reserve price, the marginal seller will set a positive reserve price and there is also a “screening value” of $V_0$ below which sellers will set no reserve price. Given that in the data only about one third of sellers sets a reserve, the entry problem is about the marginal seller who will set a positive reserve price and expects corresponding bidder entry. To economize on notation I disregard the screening value here but it is incorporated in the computation of $\lambda_{r>0}^*$ and hence in $v_0^*$.
G  Omitted proofs

Optimal reserve price.

Proof Lemma 2. For brevity I omit conditioning on characteristics $Z$, and define hat and check notation as: $\hat{x} = x(1 + c_B)$ and $\check{x} = \frac{x}{1 + c_B}$. Let $R$ denote expected revenue for a seller with valuation $v_0$ when setting reserve price $r$ in an auction with $n$ bidders:

$$R = v_0 F_V(\check{r})^n + (1 - c_S)rnF_V(\check{r})^{n-1}[1 - F_V(\check{r})] + (1 - c_S)\int_{\check{r}}^{0} \check{x}n(n - 1)F_V(x)^{n-2}[1 - F_V(x)]f_V(x)dx$$

The three terms in the above equation for $R$ cover three cases: i) no sale takes place, ii) a sale takes place but the second-highest bid is less than the reserve price and iii) the sale takes place and the second-highest bid exceeds the reserve. Maximizing $R$ with respect to $r$:

$$\frac{\partial R}{\partial r} = v_0 n F_V(\check{r})^{n-1}f_V(\check{r})(1 + c_B) + (1 - c_S)n F_V(\check{r})^{n-1}[1 - F_V(\check{r})]$$

$$+ (1 - c_S)rn(n - 1)F_V(\check{r})^{n-2}f_V(\check{r})(1 + c_B)[1 - F_V(\check{r})]$$

$$- (1 - c_S)rn F_V(\check{r})^{n-1}f_V(\check{r})(1 + c_B)$$

$$- (1 - c_S)rn(n - 1)F_V(\check{r})^{n-2}[1 - F_V(\check{r})]f_V(\check{r})(1 + c_B)$$

$$r\{ (1 - c_S)n F_V(\check{r})^{n-1}f_V(\check{r})(1 + c_B) \} = v_0 n F_V(\check{r})^{n-1}f_V(\check{r})(1 + c_B)$$

$$+ (1 - c_S)n F_V(\check{r})^{n-1}[1 - F_V(\check{r})]$$

Re-arranging, this delivers the optimal reserve price $r^*(v_0, f)$ which solves:

$$r^*(v_0, f) \equiv r = \frac{v_0}{1 - c_S} + \frac{1 - F_V(r(1 + c_B))}{(1 + c_B) f_V(r(1 + c_B))}$$

The optimal reserve price has the familiar Riley and Samuelson (1981) reserve price form. A unique $r^*(v_0, f)$ solves this equation $\forall (v_0, f)$ given that $F_V$ is assumed to satisfy the increasing failure rate (IFR) property. It has the following properties:

$$i) \frac{\partial r^*(v_0, f)}{\partial v_0} > 0$$

$$ii) \frac{\partial r^*(v_0, f)}{\partial c_B} < 0$$

$$iii) \frac{\partial r^*(v_0, f)}{\partial c_S} > 0$$

Property $ii$ also relies on the IFR assumption and requires $(r, c_B)$ to be small enough s.t. $F_V(r(1 + c_B))) < 1$.\(^\text{70}\)

\(^{70}\)My calculations differ slightly from Ginsburgh et al. (2010) who I believe omit that a successful sale requires the highest value to exceed the reserve price increased by buyer premium, $r(1 + c_B)$.
Non-neutrality of fees.
The fee structure is non-neutral if not just their total amount but also their allocation between different sides of the platform affect platform profitability (Rochet and Tirole (2006)). Non-neutrality is a prerequisite for the central questions in this paper to be of relevance. The presence of transaction cost, in the case of the auction platform stemming from costly listing inspection on the bidder side, entry cost, listing fees and opportunity cost for sellers, is known to generate non-neutrality (Rochet and Tirole (2006)). When taking entry as given, the fee structure is neutral in the auction stage when the non-linear commission index \( \frac{c_B + c_S}{1 + c_B} \) remains constant. I illustrate this point with a simple numerical example with optimal reserve prices.

Example 1. Bidder valuations are distributed according to \( U[0,1], n = 2 \). Sellers have a valuation of \( v_0 \) and set the optimal reserve price: 
\[
 r = \frac{v_0}{2(1-c_S)} + \frac{1}{2(1+c_B)} 
\]
(solving the optimal reserve price formula with uniformly distributed valuations). The sale probability equals \( 1 - F(r(1 + c_B))^2 = 1 - \left[ \frac{v_0(1 + c_B)}{2(1-c_S)} + \frac{1}{2} \right]^2 \). Let \( X \) denote: \( \max(\mathbb{E}[V_{n-1}], \frac{r}{1+c_B}) \). Because bidders scale their bid down by \( c_B \) and sellers pay a commission on the transaction price the expected seller profit equals:
\[
 \pi_s = (X \frac{1-c_S}{1+c_B} - v_0)\{1 - \left[ \frac{v_0(1+c_B)}{2(1-c_S)} + \frac{1}{2} \right]^2 \} 
\]
It is independent of the allocation of commissions to buyers and sellers as long as the fraction \( \frac{1-c_S}{1+c_B} \) remains constant. For example, their expected surplus is the same when respectively i) \( c_S = 0.1, c_B = 0 \) and ii) \( c_S = 0.01, c_B = 0.1 \), both cases result in \( \frac{1-c_S}{1+c_B} = 0.9 \):
\[
i) (0.9X - v_0)\{1 - \left[ \frac{v_0}{2 \cdot 0.9} - \frac{1}{2} \right]^2 \} 
\]
\[
ii) (0.99X - v_0)\{1 - \left[ \frac{v_0 \cdot 1.1}{2 \cdot 0.99} - \frac{1}{2} \right]^2 \} 
\]
In particular, the optimal reserve price adjusts to keep the sale probability constant and therefore also the expected transaction price will be the same. However, neutrality does not refer to the seller’s profit but to that of the platform who designs the fee structure. Expected platform revenue is a slightly modified function accounting for the fact they retain the share \( c_S + c_B \) of a transaction. The below shows that the platform fee structure is neutral when holding constant the fraction \( \frac{1-c_S}{1+c_B} \) (coming from the seller maximization problem, or \( \frac{c_B + c_S}{1+c_B} \) as in Ginsburgh et al. (2010)). Expected platform revenue is the same for fee structures i and ii:
\[
i) 0.1X\{1 - \left[ \frac{v_0}{2 \cdot 0.9} - \frac{1}{2} \right]^2 \} 
\]
\[
ii) 0.11X\{1 - \left[ \frac{v_0 \cdot 1.1}{2 \cdot 0.99} - \frac{1}{2} \right]^2 \} 
\]
On take-away from this example is that in the auction stage sellers should be slightly better off with a say 10 percent seller commission and no buyer premium than with a 10 percent buyer premium and no seller commission. They are equally well off with a 10 percent buyer premium and a 1 percent seller commission.
Proof Lemma 3A. Expected bidder surplus decreases in number bidders.

\[ i) \frac{\partial \pi_b(n, f, v_0)}{\partial n} \leq 0 \]

It holds strictly in the uninteresting case when \( v_0 \) s.t. \( F_V(r^*(v_0, f)(1 + c_B)) = 1 \). Otherwise, expected bidder surplus decreases in the number of bidders in a listing because \( F_V \) satisfies the increasing failure rate (IFR) property. Li (2005) prove that IFR implies decreasing spacings so that \( \mathbb{E}[V_{(n+1,n+1)} - V_{(n,n+1)}] - \mathbb{E}[V_{(n,n)} - V_{(n-1,n)}] \leq 0 \). This holds without a reserve price or fees and since both are independent of \( n \) this proves the statement.

\[ ii) \frac{\partial \pi_b(n, f, v_0)}{\partial c_B} \leq 0 \]

While expected bidder surplus conditional on a sale is independent of \( c_B \), the probability of selling is decreasing in \( c_B \). The optimal reserve price decreases in \( c_B \), but not enough to keep the sale probability constant given that sellers trade this off against the sale price. Formally, denote \( r = r^*(v_0, f) \) and using hat notation \( \hat{r} = r^*(v_0, f)(1 + c_B) \):

The sale probability and expected bidder surplus decreases in \( c_B \) and \( c_S \).

\[ \frac{\partial (1 - F_{V_{(n,n)}}(\hat{r}))}{\partial c_B} = -f_{V_{(n,n)}}(\hat{r})(1 + c_B) \]

which is negative because \( f_{V_{(n,n)}} \) is a PDF and therefore \( \in [0, 1] \). The same result follows from the derivative of the sale probability with respect to \( c_S \). As \( c_S \) does not affect bidder surplus in other ways it also follows that:

\[ iii) \frac{\partial \pi_b(n, f, v_0)}{\partial c_S} \leq 0 \]

Expected bidder surplus in positive reserve price auctions decreases in the seller valuation.

\[ iv) \frac{\partial \pi_b(n, f, v_0)}{\partial v_0} \leq 0 \]

This intuitively follows from the reserve price strictly increasing in \( v_0 \) and expected bidder surplus decreasing in the reserve price. The latter is necessary for sellers to have a unique optimal \( r^* \) (less than \( \bar{v}_0 \)), so it relies on the IFR assumption. Formally, let \( \hat{r} = r^*(v_0, f)(1 + c_B) \) and \( F_{n-1}(x|v_n) = \)
\[ P[V_{n-1:n} \leq x|V_{n:n} = v_n] \] with similar notation for conditional densities, and \( F_n = F_{V_{n:n}} \). For \( n > 0 \):

\[
\pi_b(n, f, v_0) = \mathbb{E}[V_{n:n} - \max(V_{n-1:n}|V_{n:n} \geq \tilde{r}, \tilde{r})|V_{n:n} \geq r] [1 - F_n(\tilde{r})] \\
\frac{\partial \pi_b(n, f, v_0)}{\partial v_0} = \int_{\tilde{r}}^{\bar{v}} [v_n - F_{n-1}(\tilde{r}|v_n)\tilde{r} - \int_{\tilde{r}}^{v_n} x f_{n-1}(x|v_n)] f_n(v_n)dv_n \\= \tilde{r} f_n(\tilde{r})(1 + c_B) - H(\bar{r}, \tilde{r}) f_n(\tilde{r})(1 + c_B) + \int_{\tilde{r}}^{\infty} \frac{\partial H(v_n, \bar{r}) f_n(v_n)}{\partial \bar{r}} dv_n \\
= -\tilde{r} f_n(\tilde{r})(1 + c_B) + F_{n-1}(\tilde{r}|v_n)\tilde{r} f_n(\tilde{r})(1 + c_B) + \\
\int_{\tilde{r}}^{\bar{v}} f_n(v_n)[-(1 + c_B)F_{n-1}(\tilde{r}|v_n) - f_{n-1}(\tilde{r}|v_n)(1 + c_B)\tilde{r} \\
+ f_{n-1}(\tilde{r}|v_n)\tilde{r}(1 + c_B)]dv_n \\
= \int_{\tilde{r}}^{\bar{v}} -(1 + c_B)F_{n-1}(\tilde{r}|v_n)f_n(v_n)dv_n
\]

The third line follows from applying the Leibnitz rule. The last line follows from the fact that in the fourth line \( F_{n-1}(\tilde{r}|v_n) = 1 \) so that the two remaining terms on the first line cancel out and also the last two terms in square brackets on the fifth line cancelling. Given that \( f_n \) and \( F_{n-1}(|v_n) \) are both always \( \geq 0 \), \( \frac{\partial \pi_b(n, f, v_0)}{\partial \bar{r}} \leq 0 \). The derivative equals 0 if: i) \( \bar{r} \geq \tilde{v} \), or ii) \( n = 0 \) which results in \( F_{n-1}(.|v_n) = 0 \) and otherwise the derivative is strictly negative.

**Expected seller surplus increases in the number of bidders.**

\[
\mathbb{E}[V_{n-1:n}] \text{ increases in } n \text{ without restrictions on } F_V \text{ (unlike for the bidder side in i) and the sale probability increases in } n \text{ as } r^*(v_0, f) \text{ is independent of } n \text{ and } F_{V_{n:n}} \text{ is stochastically increasing in } n. \text{ No other aspects of } \partial \pi_s(n, f, v_0) \text{ relate to } n \text{ so this delivers the result.}
\]

**Expected seller surplus decreases in commissions.**

\[
\frac{\partial \pi_s(n, f, v_0)}{\partial c_B} \leq 0 \\
\frac{\partial \pi_s(n, f, v_0)}{\partial c_S} \leq 0
\]

In addition to the sale probability decreasing in \( c_B \) and \( c_S \) (in parts iii and iv above), the share of the transaction price received by the seller in the event of a sale when the reserve price is set at 0, \( \frac{\mathbb{E}[V_{n-1:n}]}{1+c_B}(1-c_S) \) is decreasing in both \( c_B \) and \( c_S \). The reserve price is furthermore decreasing in \( c_B \) so that completes the proof for vi. The reserve price is increasing in \( c_S \) but vii holds because if the reserve price is binding the increased \( c_S \) in \( r^*(v_0, f) \) exactly cancels out with the higher share of the transaction price to pay to the platform. Commissions have no impact (vi and vii hold with
equality) only in the uninteresting case when $F_{V(n,n)(r^*(v_0,f))} = 1$.

Expected seller surplus decreases in $v_0$.

$$iv) \frac{\partial \pi_s(n, f, v_0)}{\partial v_0} \leq 0$$

Intuitively, a higher seller valuation reduces gains from trade. The decreased sale probability is not offset by a benefit from a higher $v_0$. Formally, relating the result to $iv$ above: $H(v_n, r)$ equals expected payment to the seller if the highest valuation equals $v_n$ and the reserve is set at $r$. It follows directly that $\frac{\partial \pi_s(n, f, v_0)}{\partial v_0} < 0$ because the derivative adds to the derived effect of $v_0$ on $\pi_b(n, f, v_0)$ the negative terms: i) $-rf_n(r)(1 + c_B)$ and ii) the contribution from the loss of higher value when the seller sells the item:

$$\pi_s(n, f, v_0) = \int_{r}^{v_0} H(v_n, r)f_n(v_n)dv_n - v_0(1 - F_n(r))$$

$$\frac{\partial \pi_b(n, f, v_0)}{\partial v_0} = \frac{\partial v_0}{\partial r} \int_{r}^{v_0} -(1 + c_B)F_{n-1}(r|v_n)f_n(v_n)dv_n$$

$$+ (v_0 - r)f_n(r)(1 + c_B)$$

This is negative due to $\frac{\partial r}{\partial v_0} > 0$ and $v_0 < r$.

The equilibrium bidder entry probability in auctions with positive reserve prices decreases in the seller entry threshold.

Proof Lemma 4. The zero profit condition follows from bidders being indifferent in equilibrium between staying out and entering the platform. Uniqueness follows from their expected surplus strictly decreasing in $p$ (as listing-level surplus is decreasing in $n$, Lemma 3A). The entry probability is decreasing in $v_0$ because $\pi_b(n+1, f, v_0)$ is decreasing in $v_0$ (Lemma 3A) so that the expectation over $v_0$, $E[\pi_b(n+1, f, v_0)|V_0 \in [v_0, r=0, \bar{v}_0]$ is decreasing in $\bar{v}_0$ (shown below). $\pi_b(n+1, f, v_0)$ is furthermore decreasing in $(e_B, c_S)$ (Lemma 3A), so the zero profit condition dictates that higher commissions must result in a lower entry probability, and the same holds for a higher sunk entry cost $e_B$ or opportunity cost $e_B^o$. Seller entry cost $e_S$ do not affect expected bidder surplus so conditional on $\bar{v}_0$ they do not affect $p^*_r(f, \bar{v}_0)$. Given that $N^{B,r>0}$ and $N^{S}$ do not affect $E[\pi_b(n+1, f, v_0)|V_0 \in [v_0, r=0, \bar{v}_0]$ the zero profit condition dictates that $p^*_r(f, \bar{v}_0)$ is such the equilibrium distribution $f_{N,r>0}(n; p^*_r, \bar{v}_0)$ is invariant to $N^{B,r>0}$ and $N^{S}$.

It remains to be shown formally that $\frac{\partial E[\pi_b(n, f, v_0)|V_0 \in [v_0, r=0, \bar{v}_0]}{\partial v_0} < 0$. Without loss of generality,
let $v_{0,r=0} = v_0 = 0$.

$$\frac{\partial E[\pi_b(n, f, v_0)|V_0 \leq \bar{v}_0]}{\partial \bar{v}_0} = \pi_b(c, \bar{v}_0, n)f_{\bar{v}_0|V_0 \leq \bar{v}_0}(\bar{v}_0) + \int_0^{\bar{v}_0} \frac{\partial \pi_b(n, f, v_0)f_{\bar{v}_0|V_0 \leq \bar{v}_0}(v_0)(\bar{v}_0)}{\partial \bar{v}_0} dv_0$$

$$= \frac{\pi_b(c, \bar{v}_0, n)f_{\bar{v}_0}(\bar{v}_0)}{F_{\bar{v}_0}(\bar{v}_0)} - \int_0^{\bar{v}_0} \frac{\pi_b(n, f, v_0)f_{\bar{v}_0}(v_0)}{(F_{\bar{v}_0}(v_0))^2} dv_0$$

$$= \frac{f_{\bar{v}_0}(\bar{v}_0)}{F_{\bar{v}_0}(\bar{v}_0)} \left[ \pi_b(c, \bar{v}_0, n) - \int_0^{\bar{v}_0} \frac{\pi_b(n, f, v_0)f_{\bar{v}_0}(v_0)}{F_{\bar{v}_0}(v_0)} dv_0 \right]$$

$$= \frac{f_{\bar{v}_0}(\bar{v}_0)}{F_{\bar{v}_0}(\bar{v}_0)} \left[ \int_0^{\bar{v}_0} \left( \pi_b(c, \bar{v}_0, n) - \pi_b(n, f, v_0) \right) f_{\bar{v}_0}(v_0)dv_0 \right]$$

The last line follows from: $\int_0^{\bar{v}_0} \frac{f_{\bar{v}_0}(v_0)}{F_{\bar{v}_0}(v_0)} dv_0 = 1$ and $\pi_b(c, \bar{v}_0, n) \perp v_0$. Finally given that $\frac{\partial \pi_b(n, f, v_0)}{\partial \bar{v}_0} \leq 0$ so $\forall v_0 < \bar{v}_0$ the derivative is negative. A qualifier is that the no trade equilibrium is excluded from consideration.

\[\square\]

**Seller entry equilibrium**

**Proof Lemma 4.** Since $\pi_s(n, f, v_0)$ is strictly decreasing in $v_0$ (Lemma 3A), for any $v_0$ for which a potential seller finds it profitable to enter he would also enter with a lower value. Hence their pure strategy entry decision is characterized by a threshold value that makes the marginal seller indifferent when competing sellers adopt the same entry threshold. $\Pi_s(f, \bar{v}_0; p_{r>0}^*(f, \bar{v}_0))$ denotes the expected seller surplus for the marginal seller with valuation equal to candidate threshold $\bar{v}_0$ when competing sellers follow that threshold strategy. An equilibrium exists because sellers reaction function is continuous in their own value and competing sellers threshold. Sellers have a unique best response for any competing seller entry threshold given that $\frac{\partial \Pi_s(f, \bar{v}_0; p_{r>0}^*(f, \bar{v}_0))}{\partial \bar{v}_0} < 0$. Given that $p_{r>0}^*(f, \bar{v}_0)$ is strictly decreasing in $\bar{v}_0$ (Lemma 3), and 2) entry of competing sellers does not affect $\Pi_s(f, \bar{v}_0; p_{r>0}^*(f, \bar{v}_0))$ in other ways, the best response function is strictly decreasing in competing sellers $\bar{v}_0$: $\frac{\partial \Pi_s(f, \bar{v}_0; p_{r>0}^*(f, \bar{v}_0))}{\partial \bar{v}_0} < 0$. Symmetry then delivers a unique $v_0^*(f)$ that is the fixed point in seller value space that solves equation 8.

The equilibrium threshold must be decreasing in $c_B$ and $c_{B,r>0}^2$ because both reduce $p_{r>0}^*(f, \bar{v}_0)$ for any candidate threshold (Lemma 3) and do not affect expected seller surplus otherwise. $\frac{\partial v_0^*(f)}{\partial c_B} < 0$ unless 1) the impact of a lower resulting entry threshold on $p_{r>0}^*(f, \bar{v}_0)$ outweighs the direct negative impact of $c_B$ on $p_{r>0}^*$ and 2) the resulting higher $p_{r>0}^*$ outweighs the negative direct impact of higher $c_B$ on $\pi_s(n, f, v_0)$. The same goes for $c_S$. A slightly modified argument holds for seller entry fees that do not directly impact $p_{r>0}^*(f, \bar{v}_0)$: $\frac{\partial v_0^*(f)}{\partial c_S} < 0$ unless the positive impact of a lower entry threshold on $p_{r>0}^*(f, \bar{v}_0)$ (increasing expected seller surplus) outweighs the direct negative impact of a higher listing fee.

\[\square\]

**Bidder entry equilibrium in auctions with zero reserve price**
Proof Lemma 5. The zero profit condition follows from bidders being indifferent in equilibrium between staying out and entering the platform. Uniqueness follows from \( \Pi_{b,r=0}(f;p) \) strictly decreasing in \( p \) for any fee structure that induces non-trivial entry (\( p \in (0,1) \)). As \( \Pi_{b,r=0} \) decreases in buyer and seller commission and also the bidder entry fee and opportunity cost undoubtedly reduce expected surplus from entering, \( f_{N,r=0} \) decreases in \((c_B,e_B,e_o^B)\). Population sizes and the seller entry threshold do not directly affect bidder surplus, so the zero profit condition dictates that in equilibrium \( f_{N,r=0}(n;p^*_r=0) \) remains constant.

Poisson decomposition property for number of bidders per listing

Proof Lemma 6. The proof concerns the statement that when \( N^B \) potential bidders enter a platform with \( T \) listings with probability \( p \), the distribution of the number of bidders per listing is approximately Poisson with mean \( \frac{N^B p T}{T} \). Let \( M \) denote the total number of bidders on the platform, distributed \( \text{Binomial}(N^B p,N^B p(1-p)) \). The limiting distribution of \( M \) when the population of potential bidders \( N^B \to \infty \) and associated \( p \to 0 \) s.t. \( N^B p \) remains constant is \( \text{Poisson}(\lambda = N^B p) \). Bidders on the platform get uniformly allocated over \( T \) listings, entering each listing with probability \( q = \frac{1}{T} \). Due to the stochastic number of bidders on the platform, the probability that \( m \) bidders get allocated in listing \( t \) and \( n \) enter into other listings also includes the probability that \( m+n \) bidders enter the platform.

\[
\begin{align*}
f_{N_t,N_{-t}}(m,n) &= \exp(-\lambda)\lambda^{m+n} \frac{(m+n)!}{m!n!} (1-q)^n \\
&= \exp(-\lambda)\lambda^{m+n} (1-q)^n \frac{m!}{n!} (43)
\end{align*}
\]

This joint distribution can be manipulated to reach the conclusion. The \( (m+n)! \) cancels out:

\[
\begin{align*}
f_{N_t,N_{-t}}(m,n) &= \exp(-\lambda)\lambda^{m+n} \frac{m!}{m!n!} (1-q)^n \\
&= \exp(-\lambda)\lambda^{m+n} (1-q)^n (44)
\end{align*}
\]

Using that \( x^{(a+b)} = x^a x^b \), and rewriting the multiplications:

\[
\begin{align*}
f_{N_t,N_{-t}}(m,n) &= \exp(-\lambda)\frac{\lambda^m (1-q)^n \lambda^n p^m}{m!} \\
&= \exp(-\lambda)\frac{(\lambda(1-q))^n (\lambda q)^m}{m!} (45)
\end{align*}
\]

Taking a convex combination of \( \lambda \):

\[
\begin{align*}
f_{N_t,N_{-t}}(m,n) &= \frac{\exp(-\lambda q)(\lambda q)^m}{m!} \frac{\exp(-\lambda(1-q))(\lambda(1-q))^n}{n!} \\
&= \exp(-\lambda q)(\lambda q)^m (46)
\end{align*}
\]

The marginal probability of having \( m \) bidders in listing \( t \) takes the expectation over possible values of bidders allocated to other listings, \( N_{-t} \):

\[
\begin{align*}
f_{N_t}(m) &= \sum_{n=0}^\infty \exp(-\lambda q)(\lambda q)^m \frac{\exp(-\lambda(1-q))(\lambda(1-q))^n}{n!} \frac{m!}{n!} \\
&= \exp(-\lambda q)(\lambda q)^m (47)
\end{align*}
\]
This shows that if the number of bidders on the platform follows a Poisson distribution with mean \( \lambda = N^B p \), and bidders enter \( T \) listings with equal probability \( q = \frac{1}{T} \), the number of bidders in each listing follows a Poisson distribution with mean \( \frac{N^B p}{T} \). This is referred to as the decomposition property of the Poisson distribution*, e.g. in Myerson (1998). Note that this result does not require the number of bidders to be independent of the number of listings. Hence the decomposition property also applies to the auction platform where the expected number of bidders is a function of the (expected) number of listings. The \( t \) subscript is dropped from \( f_{N_t} \) as the distribution is identical for all listings \( t = \{1, \ldots, T\} \).

*Binomial decomposition property

This result can also be derived from a decomposition property of the Binomial distribution. If \( N^B \) potential bidders enter the platform with probability \( p \) and get allocated over \( T \) listings with equal probability \( \frac{1}{T} \), the below shows that the number of bidders per listing follows a Binomial distribution, \( N \sim \text{Binom}(N^B \frac{p}{T}, N^B \frac{p}{T}(1 - \frac{p}{T})) \) by the law of iterated expectations and iterated variance:

\[
P[N = n] = \sum_{m=0}^{N^B} P[N = n|m]P[M = m] = \sum_{m=0}^{N^B} \binom{N^B}{m} p^m (1-p)^{N^B-m} \binom{m}{n} \frac{1}{T} \left( 1 - \frac{1}{T} \right)^{m-n}
\]

\[
\mathbb{E}[N] = \mathbb{E}_M[\mathbb{E}[N|m]] = \mathbb{E}_M[\frac{m}{T}] = \frac{\mathbb{E}[M]}{T} = \frac{N^B p}{T}
\]

The law of iterated variance states that \( \text{Var}(N) = \mathbb{E}_M[\text{Var}(N|M = m)] + \text{Var}(\mathbb{E}_M[N|M = m]) \):

\[
\mathbb{E}_M[\text{Var}(N|M = m)] = \mathbb{E}_M[m \frac{1}{T} (1 - \frac{1}{T})] = (N^B) p^1 \frac{1}{T} (1 - \frac{1}{T}) = N^B p \frac{1}{T} - (N^B) p (\frac{1}{T})^2
\]

\[
\text{Var}(\mathbb{E}_M[N|M = m]) = \text{Var}(\frac{M}{T}) = \frac{1}{T}^2 \text{Var}(M) = \left( \frac{1}{T} \right)^2 N^B p (1 - p) = -\left( \frac{1}{T} \right)^2 N^B p^2 + \left( \frac{1}{T} \right)^2 N^B p
\]

\[
\text{Var}(N) = \mathbb{E}_M[\text{Var}(N|M = m)] + \text{Var}(\mathbb{E}_M[N|M = m]) = \frac{N^B p}{T} (1 - \frac{p}{T})
\]

The last line follows from the last terms on the first and second line cancelling out and rearranging. The large population assumption combined with success probability of entering in listing \( t \) (for any \( t \in \{1, \ldots, T\} \)) equal to \( \frac{p}{T} \) also renders \( f_N \) Poisson with mean \( \frac{N^B p}{T} \). \( \square \)