Houses across time and across place*

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Abstract

This paper develops a model of the evolution of housing and of housing costs over time and across locations. It aims to understand how housing wealth and the cost of housing have moved over the past in different countries and how they might evolve into the future. We use a framework that combines features of a Ramsey two-sector growth model with a model of the geography of residential development that tracks the change in location of the population over time. We use the model to cast light on several issues: Can we expect housing costs to continue rising relative to the price of other goods? Are there conditions where housing costs can be expected to persistently rise faster than incomes? What accounts for very different recent histories of ratios of housing costs to incomes across countries? We find that taking account of the fixity of land supply, rising populations and the changing technology of transport are central to the different paths of housing costs and patterns of residential development across developed economies. We also find that the future path of housing costs is extremely sensitive to two parameters - elasticities of substitution between land and structure in creating housing services and substitutability between housing and consumption goods in utility. The interactions between factors that affect the geographic pattern of housing development and macroeconomic outcomes are explored and we draw out implications for policy. We find that in many countries it is plausible that house prices could now persistently rise faster than incomes.

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1 Introduction

Substantial fluctuations in the value of housing have major economic consequences. Financial regulators and central banks now have a range of policy tools (including limits on loan to value ratios on mortgages, capital weights on mortgage debt, limits on the value of home loans relative to household incomes) that they are increasingly willing to use to try to head off the sort of fluctuations in house values that have been associated with financial instability. (See Jordà, Schularick, and Taylor (2015) for the links between housing market conditions and financial instability). But to assess whether movements in housing prices are a sign of likely sharp corrections down the road requires some idea of the evolution of prices that are consistent with a sustainable path. This paper explores what such paths might look like and how they are particularly sensitive to parameters reflecting preferences and technology, and to the unchanging geographic features of countries and their evolving transport infrastructure. We find that even small variation in a handful of parameters around plausible central estimates can generate paths for house and land values that are dramatically different. This is sobering given the weight that is being put upon the use of cyclically varying macro-prudential tools to help preserve financial stability.

Looking forward we ask how likely changes in technology, and perhaps in preferences, can affect house and land values. Looking back, we are able to show how a simple model where these key parameters play a central role can account for the very different paths of prices seen at different stages of the recent economic histories of many developed countries.

Over the past seventy years or so it appears that house prices in many countries have risen very substantially faster than the price of (other) goods. Knoll, Schularick, and Steger (2017) present carefully constructed measures of average national house prices for 14 advanced economies since 1870. Their measure of national, real house prices (that is relative to consumer goods) averaged across all countries rises by about 300% in the period since 1945 (see their Figure 2). The problems of measuring house prices over long periods are well known, but such is the scale of this increase, which is large for every one of the countries studied, then it is most unlikely to be due to mis-measurement, especially as the authors go to some lengths to adjust for quality and other differences over time. In some countries national house prices have, on average and measured over many decades, risen faster than incomes, and not just faster than consumer goods prices. In the UK house prices relative to average household incomes by 2015 were, on many measures, around double the level from the late 1970's. In the US average national house prices seem not to have risen faster than incomes over the period since the end of the second world war, though as in nearly all advanced countries they have risen substantially faster than aggregate consumer prices. Albouy, Ehrlich, and Liu (2016) present evidence that housing's relative price, share of expenditure and "unaffordability" have all risen in the US since 1970. Rognlie (2016) presents evidence that the average share across G7 countries of private domestic added value that is accounted for by returns on housing has risen steadily since 1950; essentially all of the rise in the net capital share reflects a rising share of housing.

This rise in the relative price of housing across most developed countries in the period since the second world war has come as the proportion of the population living in big cities has risen in most developed countries[1]. It has also coincided with a period where transport costs have been flat or (more recently) often rising; that is markedly different from the period between the middle of the nineteenth century and the second world war when transport costs fell dramatically. These phenomena - rising relative price of housing, an end to falls in transport costs, greater urbanization in population - are plausibly linked. One of the aims of this paper is to explore the nature of that link and to develop a model which accounts for the patterns seen in the last 100 years or so. We use a framework that combines features of a Ramsey two-sector growth model with a model of the evolving geography of residential development that tracks the change in location of the population over time. We are not aware of much work which combines these features. We find that there are significant interactions between the geographical spread of housing across an economy with fixed land mass and macroeconomics aggregates (the capital stock, wealth and capital accumulation).

We use the model to cast light on several issues about the evolution of house and land values over the next several decades: Can we expect housing costs to continue rising relative to the price of other goods? Are there conditions where house prices can be expected to

[1] By 1950, only 30% of the world's population resided in cities. That share increased to 54% by 2015 and is now expected to increase to 66% by 2050. (Source: Atlas of Urban Expansion, 2016, volume 1). For developed countries these ratios are higher but have also followed an upward trajectory. Duranton and Puga (2013) note that the growth of population in large cities in developed countries has exceeded national population growth by substantial margins. Growth in population has shown up in ever larger big cities rather than in growth in the number of cities.

persistently rise faster than incomes so that the ratio of house prices to household labor incomes rises steadily? What accounts for the tendency of housing costs in some countries to rise in real terms but at a rate slower than the rise in incomes while in other countries housing cost to income ratios have been on an upwards trend for decades? Is there a natural limit to how expensive houses become in terms of consumer goods or incomes? As average incomes and populations change (and typically grow) could we expect the regional patterns of house prices and of housing developments to vary systematically? What determines whether regional differences in prices and the density of development rise or fall? Why did real house prices across countries seem to rise rather little in the 75 years before 1945 but then treble in the next 75 years? Can we expect technological progress, both in the way in which structures are combined with land to create housing and in the ease of travel, to help stop land and house prices rising faster than most other goods?

There is a large literature on the properties and the determinants of house prices, both across time and across regions within countries. (For a good review of the literature on this and many other aspects of housing economics see the survey paper by Piazzesi and Schneider (Piazzesi and Schneider (2016)) and the many references therein). The focus of much of this literature on national house prices is less on the very long term drivers of housing markets and more on business cycle variability in values. Much of the literature on regional differences in housing conditions does not focus on the macroeconomic backdrop so takes aggregate incomes, interest rates and population as given (and often constant). A substantial literature looks at how housing fits in to household decisions on portfolio allocation, borrowing and saving (see, for example, Campbell and Cocco (2003) and Campbell and Cocco (2007)). In much of this literature the changing way in which housing is supplied is not the focus of attention; supply is often assumed fixed or at least exogenous.Our focus is on the long run. One paper in the spirit of our own is Deaton and Laroque (2001); see also Kiyotaki, Michaelides, and Nikolov (2011), Grossmann and Steger (2016) and Favilukis, Ludvigson, and Van Nieuwerburgh (2017). Those papers, like this one, embed the housing sector within a model of the overall economy that endogenises growth, saving and asset prices. Any long run analysis has to model the changing supply of housing taking into account the fixity of land mass and the way in which endogenous shifts in the cost of land relative to structures changes the way in which houses are constructed. Land is obviously not homogenous and the impact of the most important way in which it differs (that is by location) varies over time as technological change means that distance may have a varying effect on value. One obvious way in which this happens is if transport costs change.

We introduce these features into a model where aggregate saving, production and the level of interest rates are simultaneously determined alongside the stock of housing and where house values differ across the economy because land is not homogenous. We explore the questions listed above focusing on the evolution over long horizons in the value of properties relative to incomes and other goods and how differences across regions vary as populations and average incomes change. We pay particular attention to how the technology for producing houses - specifically the substitutability between land and buildings - impacts long run outcomes.

We find that there is great sensitivity over time in the pattern of development, the types of houses built and the values of structures to even small changes in two key parameters: the degree of substitutability of land and structure in creating houses (a parameter Muth wrote extensively about Muth (1971)); the degree to which households substitute between housing and non-housing goods. We find that it is not difficult to find sets of parameters that are plausible, in the light of existing evidence, and which imply that house prices can rise faster than incomes for periods spanning generations. But at parameter values that are close (relative to the uncertainty about empirical estimates of those parameters) house prices follow radically different trajectories. Setting key elasticities to unity - as is often done in papers assuming log utility or Cobb Douglas production of housing (see for example Kiyotaki et al. (2011), Grossmann and Steger (2016), Favilukis et al. (2017)) turns out to be a massively important assumption. We also find that initial differences in the ratio between land area and population across countries create very different paths for housing costs over the next several decades. Differences in population density also mean that the incentives to invest in productive capital can diverge.

To focus on the essentials of the mechanisms we abstract from uncertainty and so our paths for prices and the pattern of development over time are perfect foresight equilibria. But one of the implications of the great sensitivity of such paths to small variations in the two key elasticities is that in a world where perfect foresight is obviously impossible - and where variability around underlying equilibrium paths might be substantial and excess exuberance might arise - it could be very hard to spot divergences in house values from sustainable levels. If a sustainable path for house prices at an elasticity of substitution between land and structures of 0.55 rises consistently faster than incomes while the path at an elasticity of 0.65 consistently falls relative to incomes then a financial regulator or central bank will find it difficult to figure out whether prices are diverging from a sustainable trajectory. We find that such dramatically different trajectories are indeed likely at values slightly above and slightly below what many studies find as a central estimate of key elasticities.

While the preference substitutability may be relatively constant over time, the elasticity of substitution between land and structures is likely to be affected by technological progress. We consider the very significant implications of this. We also consider how technological advances in transport influence outcomes; we find that the impact of such advances is likely to be different in the future than it was in the past.

To understand how housing markets develop over time we use a model with the following features:

Houses are constructed by profit maximizing firms combining land at different locations with structures using a technology which allows substitutability between the two types of input. Households make decisions on location, and consumption of goods and housing, which generate regional differences in the types of house built and the relative prices of houses to consumer goods. We assume that there are advantages of being close to central urban areas - wages may be higher there; amenities better; availability of goods greater. We do not analyze why this might be true and take it as a given. (We note however the extensive evidence consistent with this which has made it a standard assumption in a large literature following Krugman (1991).For a survey of the evidence see Combes and Gobillon (2014)). Density of population and of development across regions varies endogenously over time. The total supply of land is fixed but the extent to which land is used varies endogenously as the value of houses determines whether marginal land can be commercially developed. Aggregate production of goods can be used for immediate consumption or for investment in productive capital and structures, which depreciate at different rates. Interest rates clear the market for saving and investment and vary over time.

The model is described in detail in Section 2. In section 3 we describe how the model is solved and calibrated. Section 4 analyses equilibrium paths and their sensitivity to key parameters. Section 5 concludes.

2 The Model

2.1 The Dynasties

Our economy consists of a *continuum of dynasties* on the unit interval. Though the number of dynasties remains constant over time, the number of people in each dynasty grows at rate m which is therefore also the rate of population growth. If we normalize the size of each dynasty to be 1 at time $t = 0$, then the total population at time t, $n(t)$, is equal to e^{mt}. Each dynasty has command over the same initial endowment of resources, in the form of labor, L, capital, K, and land, R; the dynasties only differ in where they choose to live. Labor is supplied inelastically and in proportion to dynastic size at each period, and so the labor forces grows at rate m too.

Each dynasty derives utility from the consumption of goods, denoted C, and of housing services, S. Preferences over these goods at a given time t is described by a constant elasticity of substitution (CES) utility function

$$Q_{it} = \left[aC_{it}^{1-1/\rho} + (1-a)S_{it}^{1-1/\rho} \right]^{1/(1-1/\rho)} \tag{1}$$

where ρ is the elasticity of substitution between housing and consumption goods and a is a share parameter. The indices i and t index the quantity to dynasty i at time t. Dynastic welfare is the discounted power function of instantaneous utility

$$W_{i0} = \int_0^\infty \frac{1}{1-\gamma} Q_{it}^{(1-\gamma)} e^{-\theta t} dt \tag{2}$$

where γ reflects the degree of inter-temporal substitutability and θ is the discount factor. The consumption good is the numeraire, the real interest rate is r_t and the price of housing services at location l is p_{lt}. We shall shortly describe the model's geography.

Dynasties maximize their welfare (2) given their endowment and prices. We use a dynasty as our decision making unit throughout. We could equally have done the analysis in per capita terms. If we assume that the flow of dynastic utility at time t is the sum of utilities of identical dynastic members then alive - whose number is proportional to $n(t)$ - then because the utility function is constant returns to scale (CRS) and population growth is constant the welfare function in (2) can be re-written as

$$W_{i0} = \int_0^\infty \frac{1}{1-\gamma} \left[a \left(\frac{C_{it}}{n(t)} \right)^{1-1/\rho} + (1-a) \left(\frac{S_{it}}{n(t)} \right)^{1-1/\rho} \right]^{\frac{(1-\gamma)}{(1-1/\rho)}} n(t) e^{-\widetilde{\theta} t} dt \tag{3}$$

where now the discount rate $\widetilde{\theta} = \theta + \gamma m$. Thus the dynastic welfare function (2) is equivalent to a welfare function that is the sum over members of the dynasty of their individual utilities but with a shifted discount factor.

We use the same notation to denote aggregate quantities but without the index i. These aggregate quantities are just the sum over dynasties of the respective quantities; thus the aggregate consumption of housing services at time t, S_t , is

$$S_t = \int_0^1 S_{it} di \tag{4}$$

2.2 Goods Production Sector

The production side of the economy consists of 2 sectors; a goods production sector and a housing production sector. The goods production sector uses Cobb-Douglas technology, F, to manufacture the single good. This good can be consumed, C, or invested in productive capital, I_K, or in residential buildings, I_B. We assume a constant rate of labor augmenting technical progress, g.Thus production in the goods sector is

$$C_t + I_t^K + I_t^B = F\left(K_t, L_t e^{gt}\right) = A K_t^{\alpha} \left(L_t e^{gt}\right)^{1-\alpha} \tag{5}$$

where α is the capital share of output. The stock of capital, K, and residential buildings, B, evolve over time as

$$\dot{K}_t = I_t^K - \delta^K K_t \tag{6}$$

$$\dot{B}_t = I_t^B - \delta^B B_t \tag{7}$$

where δ^K and δ^B are the respective constant depreciation rates of productive capital and residential buildings.

For the production sector to be in equilibrium, the net return to capital must equal the real interest rate, that is

$$r_t = \frac{\partial F}{\partial K_t} - \delta^K = \alpha A \left(\frac{L_t e^{gt}}{K_t} \right)^{1-\alpha} - \delta^K \tag{8}$$

2.3 Housing Sector

Land is not homogenous. House prices vary dramatically across different locations within a country so that the same mix of land and structure can be worth vastly more in some locations than others. Variability in the availability of land relative to population also varies enormously across countries that in other respects (e.g. income levels, productive capital per worker) are economically similar. Since the availability of land and the way it is combined with structures is central to housing supply this variability within and across countries cannot be ignored. There is a huge literature on the variability in land and house prices, and in the density of development and populations, across regions. The pioneering works are Alonso (1960), Alonso (1964), Mills (1967), Mills (1972) and Muth (1969); a resurgence in the literature was triggered by Krugman (1991) and Lucas (2001). We introduce variability in housing, location and land values in a tractable way that captures the essence of the idea that there are desirable locations and that being further from them creates costs. Land and house prices adjust to reflect that.

We assume a circular economy with a physical area given by a circle of radius $l_{\max}$, with the central business district (CBD) located, unsurprisingly, at the center. We adopt this monocentric assumption, though recognize that in actual economies there may be more than one central location. Despite the fact that the monocentric model has some obvious shortcomings - for example that land values do not decline monotonically in all directions form an urban center - it captures an important aspect of variation in land rents. Ahlfeldt (2011) finds that once allowance is made for various other geographic factors the monocentric model does a good job in explaining house and land price variability across space. The key assumption for us is not so much that there is only one such central location in each economy but rather that new centers will not emerge endogenously. It is striking that in many developed economies there have been few big new cities that have emerged even over long periods like the last 200 years (Glaeser and Kohlhase (2004)).

We assume a simple cost of distance function levied on consumption. Thus, we follow a large literature (e.g. Krugman (1980)) in using Samuelson's 'Iceberg' model of transport costs; that a fraction of any good shipped simply 'melts away' in transit. Formally, at distance l from the CBD, $1 + \lambda_t l$ of consumption good must be purchased to consume 1 unit of the good. We think of $\lambda_t l$ as the tax on location with λ_t as the impact at time t of distance on that tax rate. There is no tax on housing services in this formulation. However, introducing a tax on housing services makes little quantitative difference to the model. In an economy where all agents are identical, a common distance tax on all consumption is isomorphic to an economy with a distance tax on labor, (see Atkinson and Stiglitz (2015)). Of these three slightly different formulations - a distance tax on consumption of goods only, a tax on all consumption (including housing) or a labor tax - we stick with the traditional 'iceberg' formulation. However the results are almost identical across the three formulations.

There are many aspects of the cost of location and several interpretations of $\lambda_t l$. The most obvious is travel costs - you need to spend time and money on getting nearer to the center where you may work and where you can most easily buy goods and consume them. This idea goes back at least to von Thunen Von Thünen and Hall (1966). Many goods need to be brought to location l at greater cost than being brought to places nearer the center and that to go to the center and buy them cost you time and travel expenses; this is in the spirit of Samuelson's iceberg costs of moving things and the net effect is that the costs of such goods is raised by $\lambda_t l$. (There is evidence that in the US good are less expensive on average in cities than in other parts of the country - see Handbury and Weinstein (2014)).

It is also consistent with Krugman's model of commuting costs, where all dynasties have a fixed supply of labor but lose a proportion of this supply in commuting to the CBD for work. One might also view $\lambda_t l$ as the cost of being less productive away from the center where economies of scale, positive externalities or network effects means that wages and productivity are higher; there is extensive evidence for such agglomeration economies (Combes and Gobillon (2014)).

The cost of distance on all these interpretations is linked to transport costs. There is no reason to think that $\lambda_t l$ is constant over time - it reflects technology (most obliviously travel technology) which has changed a lot. We come back to that later.

Dynasties have the same preferences and endowments but differ only in where they choose to live. As dynasties are otherwise identical and markets competitive, prices adjust so that dynasties are indifferent to where they locate - the cost of distance for each dynasty being offset exactly by lower housing costs further from the center.

The housing of the dynasty at location l at time t is provided by combining buildings, B_{lt}, and land, R_{lt}, at location l[2]. The same CES technology is used at all locations: .

$$S_{lt} = H(B_{lt}, R_{lt}) = A_s \left[bB_{lt}^{1-1/\varepsilon} + (1-b)R_{lt}^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{9}$$

where ε is the elasticity of substitution between land and structure and b the share parameter in the housing market. A_s is a constant determining the units. B_{lt}, R_{lt} are the use of structures and land for the dynasty located at distance l at time t. As land is cheaper further away from the CBD, the mix of land to buildings will increases the further away from the center the house is located. To calculate how this mix changes with distance, we first calculate the price of housing services so that dynasties are indifferent to where they locate.

Let p_{lt} be the price of housing services at location l at time t. This is the user cost of housing - the rent that would have to be paid at time t for a unit of housing services at location l; it is also the opportunity cost of having wealth in the form of such housing. If the dynasty is to be indifferent between living at the center relative to any other location,

[2] This is a slight abuse of notation. By B_{lt}, R_{lt} we mean the amount of land and structure used in the house of the dynasty whose location at time t is centered at distance l.

the price of housing services must move so that for the same level of expenditure (including transport costs), the dynasty achieves the same level of utility wherever they are located. Hence, if we denote the level of expenditure of each dynasty at time t as E_t and the indirect utility at location l as Q^*_{lt} then

$$Q^*_{lt} = \sup_{C_{lt}, S_{lt}} \left(aC_{lt}^{1-1/\rho} + (1-a)S_{lt}^{1-1/\rho} \right)^{1/(1-1/\rho)} \tag{10}$$

given that

$$(1+\lambda_t l)C_{lt} + p_{lt}S_{lt} \leq E_t.$$

For equilibrium we require $Q^*_{lt} = Q^*_{0t}$ for all l. The indirect utility function, (10), is therefore

$$Q^*_{lt} = \frac{a^{1/(1-1/\rho)}E_t}{(1+\lambda_t l)} \left(1 + \left(\frac{p_{lt}}{(1+\lambda_t l)} \right)^{1-\rho} \left(\frac{(1-a)}{a} \right)^{\rho} \right)^{1/\rho/(1-1/\rho)} \tag{11}$$

which is achieved at location l when

$$C_{lt} = \frac{E_t}{(1+\lambda_t l)\left[1 + \left(\frac{p_{lt}}{(1+\lambda_t l)}\right)^{1-\rho}\left(\frac{(1-a)}{a}\right)^{\rho}\right]}, \qquad S_{lt} = \frac{E_t}{(1+\lambda_t l)\left[\left(\frac{a}{(1-a)}\right)^{\rho} + \left(\frac{p_{lt}}{(1+\lambda_t l)}\right)\right]}. \tag{12}$$

If the market is to be in equilibrium, then costs of housing must be such that at the given expenditure E_t the indirect utility is equal at each location. For that to hold we require that the price, p_{lt}, be the following function of the price at the center, p_{0t},

$$p_{lt} = \left(p_{0t}^{1-\rho} - \left(\frac{a}{1-a} \right)^{\rho} \left((1+\lambda_t l)^{1-\rho} - 1 \right) \right)^{1/(1-\rho)} \tag{13}$$

Price must also be non-negative. When $\rho < 1$, housing will only be built at location l if

$$l \leq \frac{1}{\lambda_t} \left(\left(p_{0t}^{1-\rho} \left(\frac{(1-a)}{a} \right)^{\rho} + 1 \right)^{1/(1-\rho)} - 1 \right) \tag{14}$$

We can now substitute for p_{lt} in the indirect utility function, equation (11), using equation

(13). Thus indirect utility function, Q^*_{lt}, at each location is therefore

$$Q^*_{lt} = \frac{E_t}{\left((1-a)^\rho p_{0t}^{1-\rho} + a^\rho\right)^{1/(1-\rho)}}.$$

which is only a function of the cost of housing at the center, p_{0t}.

Given this price function for p_{lt}, all dynasties derive the same utility for the same expenditure wherever they choose to locate. They all have the same labor endowment and face the same interest rate and wage, so starting with the same endowment they will all chose the same expenditure path over time. We can therefore solve for the optimal path for a dynasty living at the center, and know that every other dynasty will choose the same expenditure path though they choose to live at different locations.

The equilibrium condition in the housing sector is that all capital investment in residential buildings must earn the same real rate of return, r_t. This condition sets the mix of residential buildings to land at each location l. Given the rental rates p_{lt} at each location, the real return to buildings at location l is

$$r_t = \left(p_{lt}\frac{\partial H\left(B_{lt}, R_{lt}\right)}{\partial B_{lt}} - \delta_B\right) \tag{15}$$

Given the housing technology (9), then this condition combined with the demand for housing services in (12) gives the demand for residential buildings at location l

$$B_{lt} = \left(\frac{p_{lt}A_s^{1-1/\varepsilon}b}{r_t+\delta_B}\right)^\varepsilon S_{lt} \tag{16}$$

The condition (15) also implies the mix of buildings to land must satisfy

$$\left(\frac{r+\delta_B}{p_{lt}bA_s}\right)^\varepsilon = \left(b + (1-b)\left(\frac{R_{lt}}{B_{lt}}\right)^{1-1/\varepsilon}\right)^{1/(1-1/\varepsilon)}. \tag{17}$$

Given the demand for buildings in (16) this implies the size of the residential plot, R_{lt}. Thus from the interest rate and optimality condition (12), we can back out the implied demand for both buildings and land at each location.

As rental rates, p_{lt}, fall the further we are from the center, equation (17) implies that the ratio of land to buildings rises. Substituting out for the rental rate using equation (13), one can alternatively express this ratio as a function of the rental rates at the center and location l. If $\varepsilon < 1$(and there is a great deal of empirical evidence to suggest it is, see below) then for a large enough country there comes a distance from the center when it is no longer possible to earn a return r_t on residential building - even when combined with an infinite amount of land. If this distance is less than the radius of the economy, then the condition that $(r + \delta_B) > 0$ gives the edge of residential development at time t. Thus the edge of the inhabited region of the economy at time t ($l_{t,\mathrm{Edge}}$) is given by a distance from the center of

$$l_{t,\mathrm{Edge}} = \min\left(l_{\max}, \frac{1}{\lambda_t}\left(\left(\left(1 + \left(p_{0t}^{1-\rho} - \left(\frac{(r_t + \delta_B)}{A_s b^{1/(1-1/\varepsilon)}}\right)^{1-\rho}\right)\left(\frac{(1-a)}{a}\right)^{\rho}\right)^{1/(1-\rho)} - 1\right)\right) \tag{18}$$

For $\varepsilon < 1$, this condition is tighter than the condition for the positive rental price in equation (14). For $\varepsilon > 1$ the limits to the urban sprawl are determined by (14) and therefore for these values of ε the inhabited region stretches a distance from the center of

$$l_{t,\mathrm{Edge}} = \min\left(l_{\max}, \frac{1}{\lambda_t}\left(\left(p_{0t}^{1-\rho}\left(\frac{(1-a)}{a}\right)^{\rho} + 1\right)^{1/(1-\rho)} - 1\right)\right). \tag{19}$$

To complete the description of the housing sector, we consider the price at time t of land at a distance l from the center (p_{lt}^{Land}). At all points land needs to generate a return equal to the real interest rate. This implies that the rent on the land, $p_{l\tau}\frac{\partial H(B_{l\tau},R_{l\tau})}{\partial R_{l\tau}}$, plus capital gains generates a return that equals the interest rate,

$$p_{l\tau}\frac{\partial H\left(B_{l\tau}, R_{l\tau}\right)}{\partial R_{l\tau}} + \dot{p}_{lt}^{Land} = r_t p_{lt}^{Land}. \tag{20}$$

Integrating this relationship forward implies that the price of land is the discounted value of all future land rents, that is

$$p_{lt}^{Land} = \int_t^{\infty} e^{-\int_t^{\tau} r_v dv} p_{l\tau}\frac{\partial H\left(B_{l\tau}, R_{l\tau}\right)}{\partial R_{l\tau}} d\tau \tag{21}$$

$$= \int_t^{\infty} e^{-\int_t^{\tau} r_v dv} p_{l\tau}(1-b)A_s\left(b\left(\frac{B_{lt}}{R_{lt}}\right)^{1-\frac{1}{\varepsilon}} + (1-b)\right)^{\frac{1/\varepsilon}{(1-1/\varepsilon)}} d\tau \tag{22}$$

Equations (15) and (20), along with the CES production function for housing services, also allow us to write the user cost of housing:

$$\begin{aligned} p_{lt} &= p_{lt}\frac{\partial H\left(B_{lt},R_{lt}\right)}{\partial B_{lt}}\left(\frac{B_{lt}}{S_{lt}}\right)+p_{lt}\frac{\partial H\left(B_{lt},R_{lt}\right)}{\partial R_{lt}}\left(\frac{R_{lt}}{S_{lt}}\right) \\ &= \left(r_t+\delta_B\right)\left(\frac{B_{lt}}{S_{lt}}\right)+\left(r_t p_{lt}^{Land}-\dot{p}_{lt}^{Land}\right)\left(\frac{R_{lt}}{S_{lt}}\right) \end{aligned} \tag{23}$$

This says that the rent on housing must generate on net real returns to residential buildings and land that are equal to the interest rate, r_t.

To evaluate aggregate quantities in the housing sector, we need to be able to integrate across all locations rather than over dynasties. We therefore define a mapping, $i_t(l)$, that identifies the dynasty living at location l at time t. We let $i_t(0)=0$ at the center. At radius l, the area of residential land in an annulus of width dl is $(2\pi l)\,dl$. The number of dynasties living in this annulus is equal to this area divided by the size of the residential land plot at this location, R_{lt}. As dynasties are identical, the ordering of dynasties is unimportant. We shall therefore assume the ordering implicit in the following differential relationship is satisfied

$$di_t(l)=\left(\frac{2\pi l}{R_{lt}}\right)dl. \tag{24}$$

Using this relationship, we can calculate the total demand for consumption goods, housing services and residential buildings (residential capital) by integrating over the inhabited area of the economy. Thus aggregate consumption of housing services given in equation (4) is calculated as

$$S_t=\int_0^1 S_{it}di=\int_0^{l_{t,\mathrm{Edge}}} S_{lt}\left(\frac{2\pi l}{R_{lt}}\right)dl \tag{25}$$

and similarly for the aggregate residential building stock

$$B_t=\int_0^1 B_{it}di=\int_0^{l_{t,\mathrm{Edge}}} B_{lt}\left(\frac{2\pi l}{R_{lt}}\right)dl. \tag{26}$$

The aggregate value of land wealth (which we denote by LW) is given by

$$LW_t=\int_0^{l_{\max}} p_{lt}^{Land}\left(2\pi l\right)dl. \tag{27}$$

Aggregate consumption of goods must include the sum of all dynastic consumption which includes the distance tax or consumption 'melt'. Hence aggregate consumption is equal

$$C_t = \int_0^{l_{t,\mathrm{Edge}}} (1+\lambda_t l) C_{lt} \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{28}$$

And finally the number of housed dynasties is

$$D_t = \int_0^{l_{t,\mathrm{Edge}}} \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{29}$$

(It will be an equilibrium condition that $D_t = 1$ for all t).

We have described the equilibrium in the housing sector as a function of 3 variables; the rental price at the center, p_{0t}, the real interest rate r_t and expenditure by each dynasty on goods, E_t. Given a time path for these variables, equilibrium in the housing sector describes a path for the aggregate demand for consumption goods, equation (28), the aggregate demand for residential buildings, equation (26), and number of housed dynasties, equation (29). These aggregate variables need to be consistent with the production sector for the economy to be in equilibrium.

3 Equilibrium Conditions

We are now in a position to define the equilibrium path of the economy. Given an initial total capital stock, $K_0 + B_0$, an equilibrium exists if for all t there exists positive finite prices r_t, w_t and p_{0t} (the real interest rate, wage rate and rental price of housing at the center) that supports a competitive equilibrium. We first state conditions for equilibrium on the demand side.

3.1 Demand Side Equilibrium

Given these prices we first describe the path of consumption for dynasty 0, which is assumed to live at the center $l = 0$. This dynasty, like all others, has an initial wealth (excluding human capital) equal to aggregate physical capital divided by the number of dynasties. Aggregate capital is $K_0 + B_0$ while the value of land is LW_0 ; the dynasties have unit mass so each dynasty's initial wealth is equal to $K_0 + B_0 + LW_0$

1. Dynasty 0 has consumption and housing demands that satisfy the intertemporal budget constraint

$$K_0 + B_0 + LW_0 + \int_0^\infty e^{-\int_0^t r_\tau d\tau} w_t e^{mt} dt = \int_0^\infty e^{-\int_0^t r_\tau d\tau} \left(C_{0t} + p_{0t} S_{0t}\right) dt, \qquad (30)$$

2. the intertemporal efficiency condition holds

$$\frac{\frac{d}{dt}\left(Q_{0t}^{-\gamma} \frac{\partial Q_{0t}}{\partial C_{0t}}\right)}{Q_{0t}^{-\gamma} \frac{\partial Q_{0t}}{\partial C_{0t}}} = \theta - r_t \qquad (31)$$

3. and the intratemporal efficiency condition holds

$$p_{0t} = \frac{\partial Q_{0t}}{\partial S_{0t}} / \frac{\partial Q_{0t}}{\partial C_{0t}} = \frac{(1-a)}{a} \left(\frac{C_{0t}}{S_{0t}}\right)^{1/\rho}$$

   for all t.

Given the convexity of the utility function, prices r_t, w_t and p_{0t} generate a unique path C_{0t} and S_{0t} that satisfy these efficiency conditions and budget constraint. The expenditure path of dynasty zero is therefore $E_t = (C_{0t} + p_{0t} S_{0t})$. The price of housing at all other locations must be such that the cost of reaching the same level of utility as the dynasty at the center is also $C_{0t} + p_{0t} S_{0t}$. So all other dynasties will have the same expenditure path as dynasty 0 and so also satisfy the intertemporal efficiency condition. However their intratemporal efficiency condition will be expressed in terms of their living at location l

$$\frac{p_{lt}}{(1+\lambda_t l)} = \frac{\partial Q_{lt}}{\partial S_{lt}} / \frac{\partial Q_{lt}}{\partial C_{lt}} = \frac{(1-a)}{a} \left(\frac{C_{lt}}{S_{lt}}\right)^{1/\rho} \qquad (32)$$

where rental prices at location l are given by equation (13)

$$p_{lt} = \left(p_{0t}^{1-\rho} - \left(\frac{a}{1-a}\right)^{\rho} \left((1+\lambda_t l)^{1-\rho} - 1\right)\right)^{1/(1-\rho)}.$$

Aggregating over all housed dynasties gives paths for aggregate consumption, equation (28), aggregate demand for residential buildings, equation (26), and number of housed dynasties, equation (29). These aggregate quantities need to match what is generated by the supply side of the economy.

3.2 Supply Side Equilibrium

Given the initial total capital stock, K_0+B_0, and the aggregate path for residential buildings, B_t, consumption goods, C_t and labor, L_t, then the first order differential equation implied by production goods technology equation (5),

$$\dot{K}_t + \dot{B}_t = AK_t^{\alpha} \left(L_t e^{gt}\right)^{1-\alpha} - \delta_K K_t - C_t - \delta_K B_t \tag{33}$$

implies a unique path for the capital stock K_t. This path of these aggregate supply variables must be supported by the equilibrium prices r_t, w_t and p_{0t}. The supply side conditions for a competitive equilibrium are that

1. Wages satisfy the profit maximizing condition of firms and can be expressed in terms of the interest rate

$$w_t = (1-\alpha)\, A^{1/(1-\alpha)} \left(\frac{\alpha}{r_t + \delta_K}\right)^{\alpha/(1-\alpha)} \tag{34}$$

2. net returns to capital and to residential buildings are equal to the interest rate

$$r_t = \frac{\partial F}{\partial K_t} - \delta^K = \alpha A \left(\frac{L_t e^{gt}}{K_t}\right)^{1-\alpha} - \delta^K \tag{35}$$

$$r_t = \left(p_{lt} \frac{\partial H\left(B_{lt}, R_{lt}\right)}{\partial B_{lt}} - \delta_B\right) \tag{36}$$

3. the production of housing services (the number of housed dynasties) is equal to the demand for housing services (the number of dynasties to be housed)

$$1 = D_t = \int_0^{l_{t,\mathrm{Edge}}} \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{37}$$

are satisfied for all t.As condition (36) is satisfied by the construction of an equilibrium in the housing sector, the three conditions (34), (35) and (37) imply a unique solution for the three prices r_t, w_t and p_{0t}.

3.3 Balanced Growth Path

Effective labor supply grows at the sum of the rate of productivity plus population growth, $g+m$. We now show if the location tax, λ_t (which is predominantly a proxy for the costs of travel) falls at half this rate, $(g+m)/2$, then, as long as the urban expansion does not approach the edge of the country, $l_{t,\mathrm{Edge}} < l_{\max}$, the economy will tend toward a balanced growth path where all economic aggregate quantities grow at the rate $g+m$ too. Our model, therefore, admits a balanced growth path, even though one of the factors is land and is in fixed supply. This is because it is not efficient to use all available land - the costs of travel make in uneconomic to use the land further than $l_{t,\mathrm{Edge}}$ from the centre - but as the costs of travel fall it becomes economically viable to use more land, effectively increasing its supply. If improvements in travel technology imply the cost of travel falls at exactly the rate $(g+m)/2$, then the usable area of land increases at rate $(g+m)$ as does the size of the economy.

We shall now demonstrate that the economy converges to a steady-state balanced growth path if the location tax declines at a rate of $(g+m)/2$. We do so by showing that in a country of infinite size, $(l_{\max}=\infty)$, there exists a balanced steady state if the location tax falls at $(g+m)/2$. We then appeal to a Turnpike Theorem, to argue that when a country is of finite size, the economy will hug this optimal path until the urban expansion approaches the edge of the country.

To demonstrate the existence of a steady state growth path, we have to show there exists an equilibrium growth path when aggregate quantities growing at rate $(g+m)$ and supporting prices are constant. The argument is complicated slightly by the nature of the spatial economy. We proceed by assuming a steady state and then verify that this satisfies the equilibrium conditions.

Assume for all time $t \geq t_0$, supporting prices are constant; $r_t = r_{t_0}$, $w_t = w_{t_0}$ and $p_{0t} = p_{0t_0}$ and aggregate quantities all grow at rate $(g+m)$. If this is to be consistent with the equilibrium condition (31), then the interest rate r_t satisfies the balanced growth path (or Ramsey) condition

$$r_t = r_{t_0} = \theta + \gamma\,(g+m)\,.$$

To describe the spatial economy along the balanced growth path, introduce the scaled location variable, $\widetilde{l}\,(l,t) = le^{-(g+m)(t-t_0)/2}$ which we also denote as $\widetilde{l}_t$ for short. Along the balanced growth path, this scaling maps the growing urban area back onto the urban area at t_0. This enables us to describe all variables in the spatial economy for $t \geq t_0$ in terms of their values at t_0. For example, as prices of housing at the centre are constant, $p_{0t} = p_{0t_0}$, and $\lambda_t = \lambda_{t_0} e^{-(g+m)(t-t_0)/2}$, then equation (13) implies that $p_{lt} = p_{\widetilde{l}_t t_0}$; that is the price of housing at time t and location l is equal to the price of housing at time t_0 and location $\widetilde{l}_t$. Similarly for the edge of the urban area, equations (18) or (19) imply that $l_{t_0,\text{Edge}} = \widetilde{l}\,(l_{t,\text{Edge}}, t)$; thus the urban area at time t maps back onto the urban area at time t_0. As distances are scaled back by $e^{-(g+m)t_\Delta/2}$, areas will be scaled back by the square of this; hence we can write $R_{\widetilde{l}_t t_0} = R_{lt} e^{-(g+m)(t-t_0)}$. Under these definitions we show that the equilibrium conditions are satisfied at t if they are satisfied at t_0.

Firstly look at the equilibrium condition (37) that demand for land equals the supply of land. Assume this is satisfied at t_0. Then at time t we can map all quantities onto their respective values at t_0 to show that

$$\int_0^{l_{t,\text{Edge}}} \left(\frac{2\pi l}{R_{lt}}\right) dl = \int_0^{l_{t_0,\text{Edge}}} \left(\frac{2\pi \widetilde{l}_t}{R_{\widetilde{l}_t t_0}}\right) d\widetilde{l}_t = 1$$

and so (37) is satisfied at time t. Similarly let $C_{lt} = C_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$, $S_{lt} = S_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$ and $B_{lt} = B_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$ and substitute into equations (28), (26) and (25) respectively to show that this implies $C_t = C_{t_0} e^{(g+m)(t-t_0)}$, $S_t = S_{t_0} e^{(g+m)(t-t_0)}$ and $B_t = B_{t_0} e^{(g+m)(t-t_0)}$.

Given the spatial economy is consistent with a balanced growth path, it is trivial to show the rest of the economy is too. As both aggregate consumption and housing grow at the rate $(g + m)$ then equations (31) and (36) are satisfied at t as they are satisfied at t_0. Finally as aggregate capital K is also growing at rate $g + m$, then the production constraint (33) as well as the budget constraint (30) are all satisfied at t as they are satisfied at t_0.

Thus in a country of infinite size, there exists a steady state balanced growth path. For a country of finite size, it will be optimal to converge towards this path as close as possible for as long as possible before the constraint of the fixed factor (land) forces the economy to significantly diverge from this path. Our numerical simulations have exactly this property for the special case when λ_t falls at the rate $(g+m)/2$.

4 Solution technique

At each point in time we must solve for an equilibrium in the housing sector. We have described this equilibrium in Section 2.3 in terms of 3 variables, the triplet $(p_{0t},\ E_t,\ r_t)$ which are the price of shelter at the centre, the total expenditure of each dynasty on shelter and the consumption good, and the real interest rate respectively. For an equilibrium in this sector we require that the sum of dynastic consumption, equation (28), is equal to the aggregate production of consumption goods; that the dynastic demand for residential buildings, equation (26), is equal to the aggregate supply of residential buildings; and that the housed dynastic population is equal to the aggregate population, equation (37). Thus we have 3 variables and 3 constraints. For a given aggregate consumption, C_t, aggregate residential stock of buildings, B_t and dynastic population (normalized to 1) we can solve for the unique value of the triplet $(p_{0t}, E_t,\ r_t)$ that satisfies (28), (26) and (37).

The solution of the two-sector growth model amounts to solving for the path of the stock variables, that is the stock of housing B_t and the stock of productive capital K_t for $0 \leq t \leq T$. We solve a discrete time approximation to the continuous time model described in this paper using the relaxation approach first described in Laffargue (1990) and Boucekkine (1995). We let a period be a year, so that $t\ = 0, 1, 2... T$ and solve for B_t and K_t at these points.To demonstrate that the path for these state variables describes the complete growth path for of economy, note that given such a path one can calculate investment I_t^K and I_t^B from discrete time equivalents of equations (6) and (7) respectively and C_t from the production equation (5) and then finally solve for the housing sector as described in the previous paragraph.

However, there is one further consideration. To be able to solve for the economy in this way, we need a terminal condition for these state variables at $t = T+1$ in order to calculate I_t^K and I_t^B at $t = T$. To choose these terminal values, we appeal to the Turnpike theorem of McKenzie (1976). We can assume any resonable values for the state variables at $T+1$; for the optimal growth path between our initial condition and this terminal value will 'hug' the optimal growth path of the infinite horizon problem as closely as possible for as long as possible before deviating off to the given terminal value. We therefore assume that $K_{T+1} = B_{T+1} = 100$ for a large T and only report the growth path for $t \ll T$[3].

We are solving for $2(T+1)$ unknowns; B_t and K_t for $t = 0, 1, 2... T$ and therefore need to $2(T+1)$ constraints. The first constraint is the initial condition, that $B_t + K_t$ equals initial stock of capital. A further T constraints stem from the dynamic efficiency condition, equation (31), at $t = 1, 2... T$. The other $(T+1)$ constraints are that the marginal product of residential buildings equals the marginal product of reproductive capital, equation (15) and (8) at $t = 0, 1, 2... T$. Hence, along the optimal path, the model variables as described by that path of B_t and K_t must solve these $2(T+1)$ constraints. We write these conditions as a set of non-linear equations denoted $f(B_t, K_t) = 0$ where f is a $2(T+1)$-vector.

The relaxation approach starts from an initial guess for the path of the state variables[4]. It then uses a standard Newton-Raphson iterative procedure to solve the $2(T+1)$ non-linear equations. We set the convergence condition to a change of less than 10^{-6} between iterations. To acheive this level of accuracy took no longer than a couple of minutes of a Intel Core i5 2.7GHz processor .

4.1 Numerical simulations:

We outline how we set key parameters. Since the distance cost parameter is central to the model, and the facts on its evolution are less clear cut than for other parameters, we consider the empirical evidence on transport costs in some detail. First we consider parameters where there is considerable evidence on plausible values.

$\delta_K = 0.06$

[3] In practise we solve the model for 450 annual periods, and report answers only for the first 250. We checked that the path over these first 250 periods was different by less than 10^{-5} if we solved the model instead over 600 periods.

[4] This guess is not critical. However our initial guess was constructed by assuming $K_t/B_t = 3$ for all t. We then set K_t equal to

$$K_t = \left(K_0 - \left(K_0 - \frac{K_{T+1}}{e^{g(T+1)}} \right) \frac{t}{T+1} \right) e^{gt}$$

where K_0 was chosen so that the initial condition was satisified and K_{T+1} was equal to our chosen terminal value for K_t. This guess was good enough to ensure quick convergence.

Davis and Heathcote (2005) use a quarterly value for depreciation of business capital of 0.0136 (annual of around 0.054). Kiyotaki et al. (2011) use 10%.

$\delta_B = 0.02$

Fraumeni (1997) reports that in the US structures depreciate at a rate between 1.5% and 3% a year. Van Nieuwerburgh and Weill (2010) use 1.6% in their simulations. Davis and Heathcote (2005) use a quarterly value for depreciation of the housing stock of 0.0035 (annual of around 0.014). Hornstein (2009) suggests a figure of 1.5%.

$m = 0.05$.

In the 100 years up to 2011 average annual growth in population in the UK, Germany, France and Italy was 0.4%, 0.2%, 0.4% and 0.5 respectively. In Japan it was 0.9, but more recently population stopped rising. US population growth has been higher; it rose at an annual rate of around 1.2% in the 100 years from 1911 to 2011. It has grown slightly more slowly in the 50 years to 2011 at rate of around 1.1% a year.

$g = 0.02$

We use a figure based on historical long run growth in productivity in many developed economies of around 2%.

$\gamma = 0.8$

There is much evidence that the degree of inter-temporal substitutability is less than 1. Hall (1988) estimated it was close to zero. Subsequent work suggests a significantly higher value, but still less than unity (see Ogaki and Reinhart (1998) and Vissing-Jørgensen (2002)).

$\theta = 0.034$

We set the rate of time preference to 3.4%. This is consistent with a pure rate of time preference for individuals $(\widetilde{\theta})$ of 3.8%, a rate of growth of population of 0.5% a year and a inter-temporal substitutability of 0.8. ($\widetilde{\theta} = \theta + \gamma m$)

$\alpha = 0.3$

This is about the share of capital in private domestic value added in developed economies in recent years (Rognlie (2016)).

$\varepsilon = [0.5 - 1.0]$

Muth (1971) estimates the elasticity of substitution between land and structures in producing housing at 0.5; later work finds a slightly higher level, but well under 1. Thorsnes (1997) puts estimates in range 0.5 to 1. Ahlfeldt and McMillen (2014) suggest it might be a bit under 1. Kiyotaki et al. (2011) constrain it to 1 in their calibrated model. But the weight of evidence is for a number under 1. Combes, Duranton, and Gobillon (2016) use a rich data set on French housing and estimate a figure of around 0.8 for the substitutability between land and structures (they quote a central estimate of 0.795 when they use instrumental variables estimation). For our base case we use a value of 0.5.

$\rho = [0.5 - 1]$

There are many estimates from the empirical literature on housing of the elasticity of substitution between housing and consumption in utility. Ermisch, Findlay, and Gibb (1996) summarizes that literature and puts the absolute value at between 0.5 and 0.8; Rognlie (2016) uses a range of 0.4 to 0.8. Kiyotaki et al. (2011) constrain it to 1 in their calibrated model. Van Nieuwerburgh and Weill (2010) use 0.5 for the price elasticity of demand for housing, basing their choice on micro studies. Albouy, Ehrlich, and Liu (2014) and Albouy et al. (2016) find strong US evidence for a value of 2/3. For our base case we use a value of 0.6.

4.1.1 The cost of distance:

λ reflects the cost of living further from the (economic) center of the country - it is essentially a tax on distance. The most natural measure of that tax is the cost of travelling to the center. Some of that cost is the price of a rail, bus or plane ticket or of fuel to drive a car. Much of it is in the form of time taken to get to work or to get to shops, restaurants, or theaters. It is clear that this cost, and its rate of change, vary substantially over time. The development of railways in the second part of the nineteenth century dramatically brought down the time cost of commuting. Increased use of cars in the twentieth century brought cost down further. There have been great increases over the twentieth century in the average distance people travel to work, a phenomenon linked to urban sprawl and made possible by improvements in transport infrastructure and technology. But the rate of improvement in travel speeds has slowed in the past fifty years - on some measures it has stopped completely. It is also likely that the cost of moving goods has fallen by more than the cost of moving people.

There is an extensive literature on the evolution of passenger travel over time focusing on average distances travelled in a day, average time spent travelling and the implied average speed at which people travel. There is a good deal of evidence that average time spent traveling is surprisingly constant over time and across countries at about 1 hour a day. Ausubel, Marchetti, and Meyer (1998), Schafer (2000), Schafer and Victor (2000), Levinson and Kumar (1994) all present evidence that, a least up to the 1990s, there has been near constancy of travel times, supporting a hypothesis originally put forward by Zahavi (Zahavi and Talvitie (1980)). Figure 1 of Schafer and Victor (2000) is particularly striking revealing a clustering of average daily travel times at around 1 hour for countries of very different standards of living and across different periods up to the early 1990s.

It is the inverse of average travel speeds that we take as our measure of the cost of distance. Ausubel et al. (1998) show estimates of the average daily distance travelled by US citizens over the period 1880 to 1998. If time spent traveling is roughly constant this is a measure of the evolution of average travel speed. The average rise over the whole period is 2.7%. But the data (see Figure 3 of Ausubel) show a clear falling off in the growth of travel distances from about 1970. The growth in distance is at its fastest between around 1900 and 1930.

Glaeser and Kohlhase (2004) estimate that goods are effectively costless to transport now but that the cost to people of moving themselves is high. They say:

"....according to the 2001 Consumer Expenditure Survey, 18% of total expenditures for the average household is spent on vehicle purchases, gasoline and other vehicular expenses (e.g., insurance). This cash cost fails to include the far more important time costs of moving people, and these time costs are not withering away with technological progress. Instead, as wages continue to rise, these time costs should rise roughly in proportion to wages."

They present evidence of increasing travel times to work for all major US cities between 1980 and 2000. They conclude "...that people-moving costs are not declining within US cities." This is in marked contrast to what had happened in the first half of the twentieth century and, especially, in the second half of the nineteenth century.

The contrast between very rapid rises in travel speeds in the late nineteenth and early twentieth century and a more recent stagnation in speeds is stark and may be one reason why changes in real house prices seemed very much smaller in most developed countries between 1850 and the second world war than in the 70 odd years since then. In an earlier version of Knoll et al. (2017) they consider this a significant factor behind the much smaller rise in land prices between 1850 and the middle of the twentieth century than in the period since then. Rapid falls in passenger transport costs (reflecting rises in average speed) substantially increased the available quantity of land within a given travel time of large urban centers in the period 1850-1940; such increases have been far smaller in the period since. We aim to calibrate that change in λ and see if, along with population changes, it can plausibly account for the changing pattern of real house prices noted at the outset.

Detailed data on travel speeds is rather patchy, especially for the late nineteenth and early twentieth century. We rely on surveys from various industrial economies to piece together a plausible path for the overall evolution of travel speeds.

Survey evidence suggests that in the UK in the late nineteenth century around 60% of journeys people undertook were made on foot (Pooley and Turnbull (1999), Table 5). Fifty years earlier - and before the spread of railways - the proportion walking would have been much higher. By 1920 a substantial proportion (around 50%) used trains or buses for journeys and the proportion walking had fallen to under 30%. For those travelling to London trains and buses accounted for close to 80% of journeys over 1920-29 (Pooley and Turnbull (1999), table 6). They report average journey speeds for Britain in the period 1920-1929 of 13.9 kilometers per hour (Table 3). In 1850 it is likely that this average speed was not much more than 4-5 kilometers per hour. Something that is plausibly close to a tripling of average travel speeds took place in the seventy years between 1850 and the mid 1920's. Leunig (2006) reports an enormous (16 fold) increase in distances travelled in the UK between 1865 and 1912 and very large improvements in average travel speeds. In the next 70 years, according to the data reported in Pooley and Turnbull (1999), travel speeds rose by about 80%; over this period car use rises greatly but the improvement in average travel speeds is very substantially smaller than when railways and buses displaced walking in the earlier period. For those travelling into London, which is clearly the economic center of Britain, improvements in average travel speeds between 1925 and 2000 was markedly slower than the average for all journeys in the UK.

Based on this UK historical data the annual improvement in speeds over the period 1850-1925 was around 1.5% a year while over the period 1925-2000 it dropped to about 0.8% a year.

This substantial slowdown in the improvement in travel speeds is borne out by data from many developed countries. The evidence from Anusbel et al shows something similar for the US where walking and travel by horse account for a sharply falling share of travel over time replaced first by trains and later by cars. The common pattern of the spread of railways is one factor behind a general and sharp rise in speeds in the later part of the nineteenth century. Railroad density (kilometers of railways per 1000 people) show enormous expansion in Germany, Britain and the US between 1850 and 1920, when railroad density peaked (Hugill (1995)). More recently, and now that car journeys have become the most used means of travel in developed countries, improvements in travel speeds may have fallen. Evidence of flat or even rising travel times into large cities exists for many countries: Glaeser and Kohlhase (2004) present evidence of rising congestion and of travel times into major US cities since 1990; van Wee, Rietveld, and Meurs (2006) present evidence of increasing travel times in Holland. In considering the future evolution of house prices it may be plausible to consider that λ is now constant.

For the base case we assume λ falls by 1.5% a year between 1870 and 1945, then falls by 0.8% a year until 1995 and is then constant.

4.1.2 Other parameters

Finally are a group of parameters where there is less evidence on plausible values (or where the value is in some sense arbitrary) and where we will search for parameters that, in conjunction with other parameters, match key features of the data.

Parameters that fall into this category are:

- constant in goods production function A
- constant in housing production function A_s

- weight on consumer goods (relative to housing consumption) in utility a
- weight of buildings in production of housing b

We chose parameter values that generate simulation results in the base case that imply values for key macroeconomic ratios that broadly match data for industrialised economies in recent decades. The key ratios (or prices) are:

1. relative shares of productive capital and residential structures in the total capital stock:

Rognlie (2016) shows estimates on the contribution of the return on housing and all other capital to private domestic value added for several developed countries. In the 2000s housing, on average across developed countries, contributes about 8% of private domestic value added. That has been on a clear upward trend (see his figure 3). Non-housing capital contributes just over 20% on average. The overall capital share is about, or slightly above, 30% for most countries. If the rates of return on capital, land and structures were the same this would seem to imply a low ratio of structures to productive capital (no more than 8/30). But Kiyotaki et al. (2011) report the ratio of housing to total tangible assets for the US as 0.41 (the 1952-2005 average).

2. share of overall household spending on housing:

For the US Piazzesi and Schneider (2016) quote a figure of 17.8% for the average ratio of consumption on housing (actual and imputed rent) to all consumption (including durables) over the period 1960 to 2015. The ratio is remarkably stable (see Fig 1 of their paper). Jaccard (2012) uses a share parameter based on US data that means that housing consumption represents about seventeen percent of total consumption. Albouy et al. (2016) quote a figure of 18.1% for 2014. This figure is close to that for the UK, France and Germany.

3. total housing wealth and the relative value of land and structures in overall value of housing:

Piazzesi and Schneider (2016) show data for the US over 1960-2010. In recent years the housing stock has been worth about 1.7 of annual GDP (having peaked at over 2 just before financial crisis). Structures are worth about 1 x annual US GDP, land was worth about the same at the peak but more recently has fallen to about 0.8 annual GDP. Davis and Heathcote (2007) estimate the average share of land in US residential tangible assets over

1930-2000 to be about 25%. That aggregate US ratio is rising. It is considerably higher in the UK, and much higher in many US cities (around 50%). In the UK the total residential housing stock in 2015 was approximately 2.8 times annual GDP; the value of residential land is likely to have been around half of that total value so around 1.4 of annual GDP. In France, Germany and Italy the value of total housing wealth to GDP in recent years appears, like in the UK, to be larger than in the US. Piketty and Zucman (2014) have a figure of 2.9 as the 2000-2010 average of the housing wealth to national income ratio for France. For Germany over 2000-2011 the ratio averaged 2.2 and for Italy it averaged 3.25. For Japan the ratio of private housing wealth to national income averages 2.2 over this period; the ratio of total housing wealth to national income would be higher (see online appendix at http://gabriel-zucman.eu/capitalisback/).

4. real interest rate

Piazzesi and Schneider (2016) show data for the ratio of house price to rent that averages about 13 over 1960-2015. The implied yield of about 8% exceeds the real return by depreciation of structures and other property expenses, but real capital gains need to be added. This probably brings the return to under 8% and this figure also includes a risk premium. The risk adjusted return is likely to be nearer 5%. Yields on inflation indexed government bonds give a clearer measure of a safe real rate. Such yields have been falling across the world (King and Low (2014)) and average well under 5% over past 30 years. An alternative, and probably more relevant, guide is the net return on corporate capital, a number which is certainly closer to 5% than to the recent near zero real yield on government bonds.

Table 1 below (column 2) brings together the diverse evidence on some of these key ratios from various industrial economies in recent decades. It shows an estimate for ratios that the model should roughly replicate. The table shows 7 ratios we are trying to match; since we have only four free parameters (A, A_s, a, b) we can only hope to broadly match the evidence.

	Model	Observed
Real return on capital:	5.6%	≈5%
Share of housing in total consumption :	20.9%	≈18%
Share of housing value added in GDP :	16.0%	8-12%
Share of the returns to capital in GDP :	35%	≈33%
Value of capital stock to GDP :	164%	150-200%
Value of residential buildings to GDP :	104%	100-150%
Land rent capitalized relative to GDP :	145%	80-150%

Table 1: Comparison of Model generated and observed Target Ratios.

5 Results

We focus on what the model implies will happen over long periods. For the simulations we focus on a period of 250 years. We think of this period as running from towards the end of the nineteenth century to 100 years ahead - from around 1870 to 2120. We chose the first 150 years of this period to roughly correspond to the period over which Knoll et al. (2017) measure house and land prices. So we set population growth rates and the trajectory of travel costs over the first 150 years to reflect our assessment of their broad shape over the period since 1870. We noted above that the estimates from Knoll et al. (2017) averaged across all of the (now rich) countries they analyze show the real price of housing roughly flat for almost 100 years and then almost tripling since about 1950. But there is diversity across countries in the path of real house prices since the end of the second world war - real price increases have tended to be significantly higher in Europe and in Japan than in the US.

We begin by exploring what a version of our model which is calibrated to roughly match average characteristics of the developed countries in their sample (in terms of population growth, capital stock to GDP, labor share, real return on capital and transport cost changes) generates for the shape of housing markets over the past and into the future. For this base case we set ϵ=0.5 and ρ =0.6.

Table 1 shows what the model generates for key ratios for period 150 - which we think of as around now (2020) - and compares them with recent typical ratios for large developed economies.

The calibration generates figures for the capital share of GDP and for the ratio of the (non-residential) capital stock to GDP that closely match the US figures and are in line for most other developed economies. The rate of return on capital is around 5.5%. The share of housing value added in GDP also looks high for the US but less so for European countries (real estate activities - largely rent and imputed income on dwellings - made up around 11.5% of European Union GDP in 2014).The capitalised value of land to annual GDP looks high for the US though the value of residential structures to GDP looks about right - overall the ratios of total wealth in residential property (calitalised value of land plus structure) look closer to what is typical in Europe and for Japan than for the US. That overall value of housing (land and structures) to GDP is around 2.50 in the simulation which is high relative to recent US data (around 1.7) but more typical for many European countries (the UK ratio is 2.8 for 2015; over 2000-2010 it averaged 2.8 in France and over that period in Germany and in Japan it averaged around 2.2).

The overall value of residential land and structures matches what has been seen in the crowded old world (Europe, Japan), but does less well for the land-rich, new world (the US). But this is based on valuing land by capitalizing its current rent - or annuitising the current rental stream. This is common practice for the commercial valuation of land. But a different, and much higher, value is implied by our results if we take the present value of the future realized stream of rents. This is because the simulation using the base parameters generates a steeply rising path for land rents into the future, the present value of which is around three times as great as the current land rent divided by the real interest rate. But as we shall see below, parameter values slightly different from those used in the base generate paths for future housing costs and land values that are markedly different from the base case. Indeed that is one reason why in the commercial valuation of land people often use two observables - today's actual rental values and today's long term interest rate - to value that most durable asset, land. Since we are here assessing the ability of the model to broadly match actual valuations it makes sense to calculate values generated by the model (today's rents on land and the current interest rate) and see if those imply capitalized land values that match actual valuations. If people today did believe future house prices would follow the trajectory shown in the base case simulations then residential land would probably be worth considerably more. Although the capitalized value of current land rents and the perfect foresight value of future land rents are different the trajectories of house

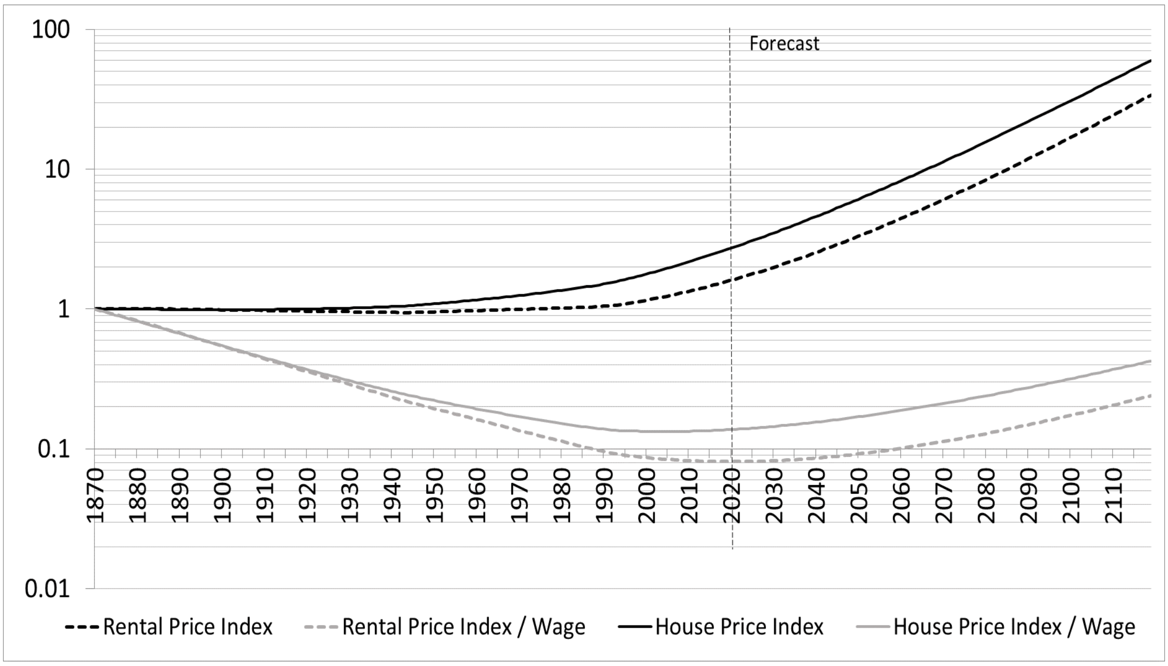


Figure 1: Path of the rental price and the rental price relative to wage.

prices using either method to value the land component are very similar. This is reassuring given our focus on how house values will evolve relative to incomes and relative to the price of other goods.

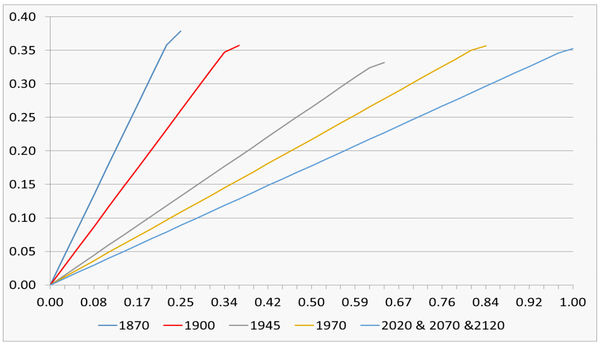
Figure 1 shows the path for the real cost of housing (the per period cost of using a unit of housing in terms of consumer goods) for this simulation. This is the rental equivalent and labelled "rental price index". The figure also shows an index of real house prices. The figures here are average costs of housing across locations at each point in time. There is little change in the real price of housing between 1870 and the middle of the twentieth century. The same is true for real house prices. Between around 1945 and 2020 the user cost of housing (the rental equivalent) rises by almost 70%. The average house price rises by considerably more - the level in 2020 is about 2.6 times as great as in 1945. Based on the findings of Knoll et al. (2017) this is somewhat lower than the average increase for developed countries on the whole - where average prices trebled - but much faster than in the US. But over the next 100 years, and assuming no further improvement in transport costs, both the user cost of housing and average house prices are forecast by the model to rise at a much stronger rate which is accelerating. Between 2020 and 2070 the user cost of housing (or rents) almsot quadruple and house prices rise by even more. Over the fifty years after that real housing costs go up almost six fold. The figure also shows what happens to the cost of housing relative to average wages.

Over the period since 1870 costs follow a trajectory that sees them fall steadily (though at a declining rate) relative to average wages. That stops from around now; over the next 50 years the cost of housing relative to wages rise by about 40%; over the fifty years after that housing costs double relative to wages. Over the next 100 years housing costs are predicted to rise at ever faster rates relative to average real wages so that by 2120 housing costs (for constant quality housing) are about three times greater than today in terms of average labor incomes.

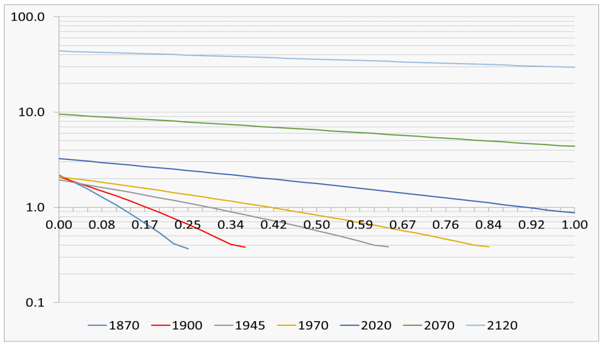
The key factor here is that with relatively low substitutability between land and structure (0.5), less than unit elasticity of substitution between housing and goods (0.6) and with no assumed further improvement in transport costs then housing costs rise at much faster rates than in the past. The scale of this effect, as we shall see, is highly sensitive to those factors.

The spatial pattern of development, the density of population across regions and the evolution of transport costs are shown in the panels of figure 2. Here we show the dispersion of outcomes across locations at intervals of several decades. The horizontal axis measures the distance from the center as a proportion of the total radius of the economy. As transport costs decline rapidly through the late nineteenth century and up until about the middle of the twentieth century people are able to commute greater distances and it becomes feasible to live further from the center. The model implies that the average distance that people live from the center (which in the stylized model one would expect is linked to the average commute distance) roughly doubles between 1870 and about 1920. The limit of commercially feasible residential development over this period also roughly doubles. The average distance people live from the center is not predicted to change much after 2000 which in the base case is some 5 years after when we assume transport improvements stop. Travel costs remain a stable proportion of total spending.

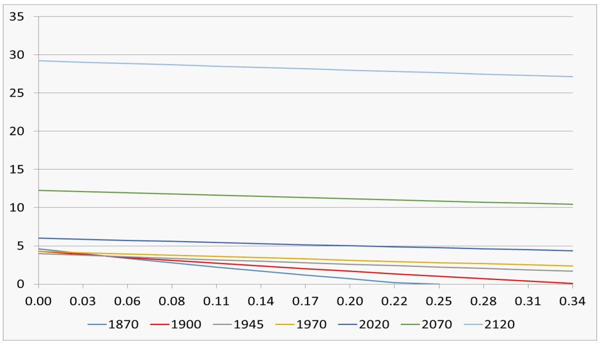
For the household that lives the average distance from the center at the end of the nineteenth century the cost of living at that location is the equivalent of facing prices for consumer goods higher than at the center by about 17%. So with consumption of goods just



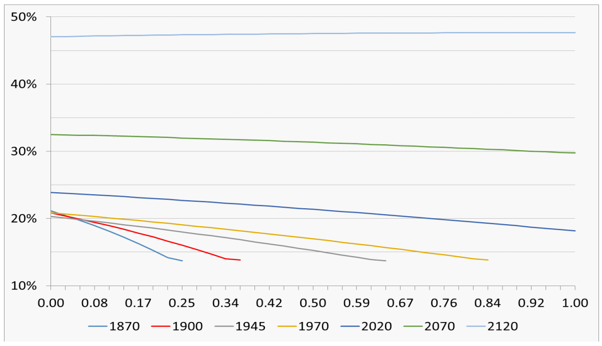
Transport Costs Over Time



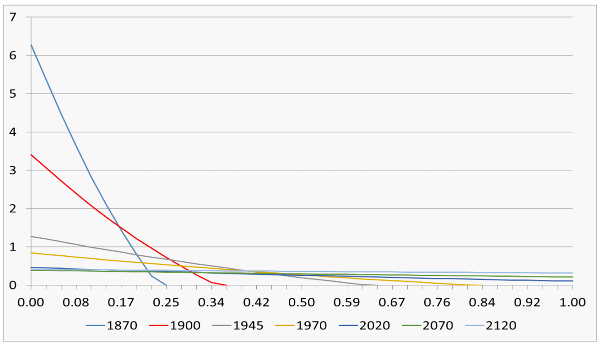
Rental Price of Housing



Structure to Land Ratio



% Expenditure on Housing



Population Density

Figure 2: The Distribution of Activity over time

under 80% of spending (the rest is spending on housing) this is roughly the equivalent of income being 12% lower. In terms of time that would roughly be worth 12% of the average work week. By 2020 the transport tax was a little higher - rising to the equivalent of close to 20% on the price of consumer goods. Consumption of goods was predicted to be around 80% of all spending so that the cost of distance could be seen as the time equivalent of about 13.3% of average hours worked. That would be a daily commute time of just over 1 hour for an 8 hour work day, or an average commute time per trip to work of slightly above 30 minutes - a plausible figure for average travel time to work in major cities. The typical travel cost varies rather little over time - with average distance travelled rising roughly in line with increasing travel speeds up until transport technology stops improving.

Prices are always highest at the center but one implication of improving transport over the past 150 years is that the ratio of housing costs further from the center to those at the center rises. The proportion of the country which is developed for housing also rises. In 1870 only land within a circle whose radius was about 1/4 of the distance to the edge of the country was developed for housing. By 1920 land half way from the center to the edge had houses on it; by about 1980 a combination of rising population, growing incomes and declining travel costs meant that almost all the country had some residential development. Density of population is always the greatest at the center, but population density is forecast to become much more equal over the next decades; indeed the density 100 years ahead is (rather implausibly) projected to be fairly similar across much of the country. This feature of the model is unlikely to be robust to a more realistic modelling of transport congestion which would allow for average travel speeds to decline if a rising proportion of the population chose to trave further each day. It is also sensitive to assumed rates of population growth.

How do these predictions on the geographical pattern of development, of population densities, on commuting and on the price-distance gradient match up to the historical record?

Two features of the simulations are clear and are obvious implications of the fall in transport costs over the past. These are that average commuting distances are predicted to have risen significantly and that price gradients (the fall in rent as distance from the center rises) have become less steep. Both features are strongly supported by the historical evidence. We noted above the strong upward trajectory of commuting distances over time. The decline in the house price-distance gradient over time has been noted many times in the literature and goes back at least to Mills (1972) who noted evidence of flattening in the gradient in the US which began in the nineteenth century. There is evidence of flattening in land price gradients for numerous cities, including: Chicago (McMillen (1996)); Berlin (Ahlfeldt and Wendland (2011)); Cleveland (Smith (2003)); New York (Atack and Margo (1998)); Sydney (Abelson (1997)).

Other predictions of the simulations are less obvious and not so clearly consistent with the historical evidence. The model predicts that the developed areas around cities should have consistently expanded (as has population) but that the density of population at the center will decline and density become more even across the country. This is precisely what has happened in just about all major world cities over the past 25 years. The population of cities in developed countries increased by a factor of 1.2 between 1990 and 2015; their urban extents increased by a factor of 1.8. In developed countries population densities in cities declined at 1.5% a year during this period. (Source: Atlas of Urban Expansion, 2016, vol 1 available at www.atlasofurbanexpansion.org). Table 2 gives details for 12 of the largest cities in the developed world. In every case the area of each city that is built up went up substantially between 1990 and 2014 - on average it close to doubled. In every case population also increased substantially. But in every case population grew by less than the built up area and population density fell. Across these 12 cities population density fell by 25%.

The model also predicts that commuting distances will tend to rise over time but that expenditure on transport will be roughly constant as a share of total spending.

The simulation results on the past and future trajectory of housing are based on a calibration of the model that does a reasonable job at matching economy characteristics for European countries and Japan. But it gets the housing market wrong for the US - the value of housing relative to GDP is too high for the US, the rate of growth of real housing costs over the past 70 years is also too high as are land values. One obvious reason why this might be true is that the population density of the US is far lower than in most European countries and very much lower than in Japan. Land area relative to population in the US

	1990		2014	
	Built up Urban Area Hectares	Population Density Person/Hectares	Built up Urban Area Hectares	Population Density Person/Hectares
Berlin	24,707	131.5	68,742	56.2
Chicago	340,561	21.5	510,971	17.4
Hong Kong	5,933	617.5	9,253	467.1
Houston	116,357	23.5	272,394	19.8
London	131,464	64.8	177,272	63.2
Los Angeles	353,940	34.9	459,046	33
Madrid	20,632	176.5	56,019	93.8
Milan	51,115	68.6	178,364	35.9
New York	509,235	31.9	747,852	24.6
Paris	127,790	72.5	198,625	56
Sydney	69,122	41.5	110,033	37.4
Tokyo	278,694	104.7	448,929	77.4

Table 2: Table recording both Area and Population Denisty in 1990 and 2014 for selected cities. Source: Atlas of Urban Expansion, 2016, volume 1.

	Model	Observed
Real return on capital:	5.5%	≈5%
Share of housing in total consumption :	19.7%	≈18%
Share of housing value added in GDP :	14.6%	8-12%
Share of the returns to capital in GDP :	34%	≈33%
Value of capital stock to GDP :	168%	150-200%
Value of residential buildings to GDP :	107%	≈100%
Land rent capitalized relative to GDP :	120%	≈100%

Table 3: Comparison of Model generated and observed Target Ratios for Radius 10 Economy.

at around 2015 was about 7 times as great as in Germany, just over 8 times as great as in the U.K. and around 10 times as great as in Japan. We ran the model with the same calibration as above except that the area of the economy relative to population is 10 times as great. The key ratios for 2020 are shown in Table 3.

The value of capitalized land rents (that is aggregate land rent divided by the real interest rate) relative to GDP falls from 145% to 120% and the value of total housing to GDP falls from just over 250% to around 225% - both figures are now closer to US than to average European ratios. The share of consumption of housing to total consumption and the value added from housing relative to GDP also fall and are closer to US data. The rise in real house prices over the past 75 years is also much lower: with more land per person the rise in real housing costs (rental costs) in the post second world war period is about

0.3% a year while in the calibration better suited to European population density it was close to 1% (see figure 3; both figures are a little higher for the grwoth in house price rises rather than rents). Knoll et al. (2017) (online Appendix, table 4) report that the average annual rate of real house price growth in the US in the post world war two period was 0.2% while the average growth for Germany, France, the UK and Japan were 1%, 4.5%, 6% and 2% respectively.

Using the calibration based on current relative US population density also gives a significantly lower growth of real housing costs over the next 100 years, but one that still reveals much faster rises than over the past. Over the next 50 years real housing costs (rents) are projected to rise by 40% (around 0.7% a year) and in the 50 years after that (between about 2070 and 2120) by a further 50%, or by about 0.8% a year. But unlike in the simulations based on population densities more typical for Europe and Japan, housing costs - both rents and house prices - are projected to consistently fall gradually relative to real wages (figure 3). The key factor here is that it takes far longer to start running out of land - where we define that as reaching a point at which some residential development is undertaken in all parts of the country. In the simulations were we set the overall land to population ratio at a level one tenth the level assumed for "the US" that point is reached at around 2020. For the simulations with land 10 times as plentiful relative to population ("the US") that point is only reached in 2120 - right at the end of the simulation horizon. As we shall see this difference does not only affect house values, the price-distance gradient and the distribution of population densities - it also affects the incentives to invest in productive capital, the capital to output ratio and the rate of return (the interest rate).

5.1 Sensitivity to key elasticities

We assess the sensitivity of projections of future housing costs to three factors: 1. substitutability between land and structure in creating housing; 2 substitutability of housing for consumption goods in utility; 3 variations in the path for transport costs (speed of commuting). In each case we focus on what changes mean for the cost of housing relative to consumer goods and the cost of housing relative to incomes.

1. $\varepsilon = [0.75]$

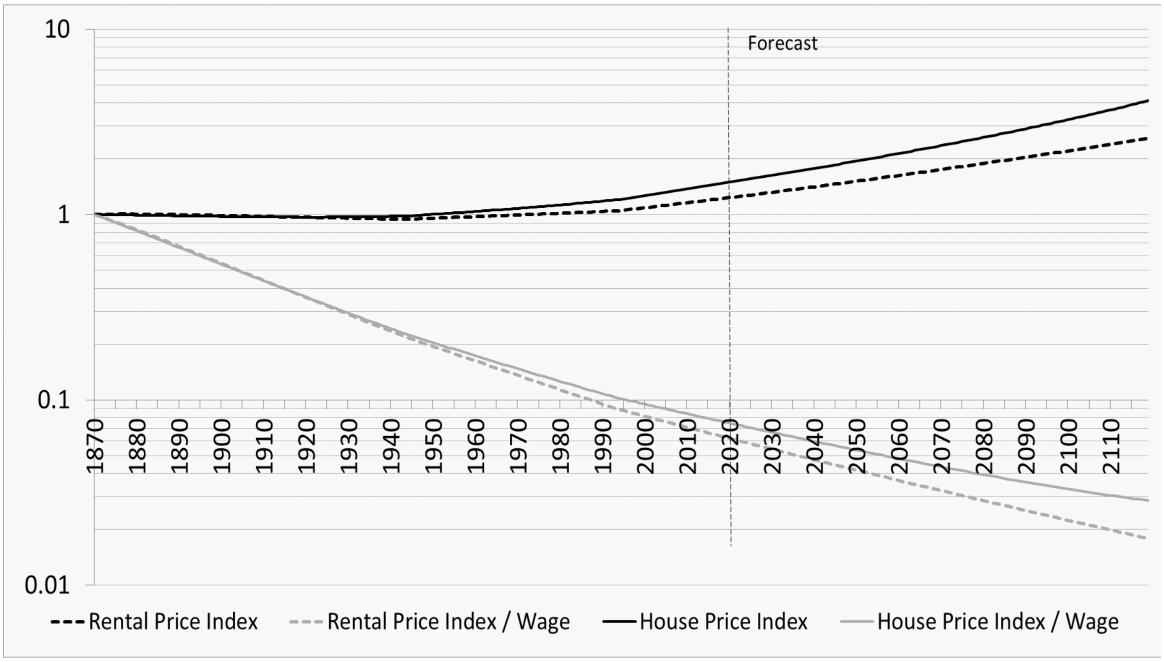


Figure 3: Path of rental price and rental price relative to wages.

When $\varepsilon = 0.75$ the path of housing costs over the future looks very different from the base case (figure 4). (We illustrate here using the population to land area ratio more typical for Europe and Japan; but using the ratio 10 time larger gives a similar sensitivity to variations in ε). There is some very gentle rise in real housing costs over the next 100 years, but at a rate significantly below the growth in average earnings. So instead of housing costs (rental equivalents) over the next 50 years rising nearly 40% relative to average labor incomes (when $\varepsilon = 0.5$) costs *fall* by about 40% relative to average earnings. Over that 50 year period house prices fall relative to incomes by about 30% when $\varepsilon = 0.75$ rather than rising strongly faster than incomes when $\varepsilon = 0.5$. If $\varepsilon = 0.99$ rental costs and house prices barely rise any faster than the cost of consumer goods and they decline markedly relative to earnings to less than half their 2020 value by 2070. Clearly there is enormous sensitivity in the path of housing costs and of house prices to even relatively small changes in the substitutability between land and structure in creating homes. That degree of substitutability is partly a matter of technology and partly a matter of preferences, reflecting the fact that ε effectively plays a dual role as summarizing production possibilities but also trade-offs in creating utility by consuming different combinations of structure and land (or building and space). This point was recognized by McDonald (1981). He noted that a model with utility a function of consumption of goods and housing, and with housing produced via a CES function combining land and structures, can be interpreted as a weakly separable utility function rather than a model of the production of housing. This implies that estimates of the elasticity of substitution, ε , can be thought of as estimates of a production parameter or as an estimate of a parameter of a weakly separable utility function. The production function of housing can as well be thought of as an aggregator reflecting tastes. Both tastes and production technology can change and so one should not think of ε as fixed. The degree to which one can substitute structure for land has probably changed significantly over time - new building techniques now make it possible to build 100 story apartment blocks with very small footprints. That creates a different trade-off between use of land and structure in creating units of housing than was available when the Empire State building was constructed - a building that had a footprint that was enormously larger relative to its height than the sort of pencil thin apartment blocks recently constructed in Manhattan. But whether having a living space a mile up in the air - potentially shrouded in cloud for some of the time and remote from life on the ground - is really a good substitute for a second floor apartment is not a question of technology. So while ε may rise with technological improvements - and put some limit to the rise in housing costs in the face of fixed land area - that scope is strictly limited by preferences which might be completely insensitive to changing housing costs.

2. $\rho = [0.80]$

Figure 5 shows that a slightly higher degree of substitutability between housing and consumer goods (0.80 versus 0.6 in the base case) has a very substantial effect on the future evolution of housing costs. Over a 100 year horizon the impact is as great as varying the elasticity of substitution between land and structures - it is the difference between housing costs rising steadily relative to wages (and ending up 3 times more expensive a century from now) or being flat relative to wages.

3. Transport costs:

In the base case we assume that improvements in travel speed stopped in 1995 and do not change over the next 125 years. If instead travel times from 2020 once again start to fall at 0.8% a year - the same rate as assumed between 1945 and 1995 - there is, not surprisingly,

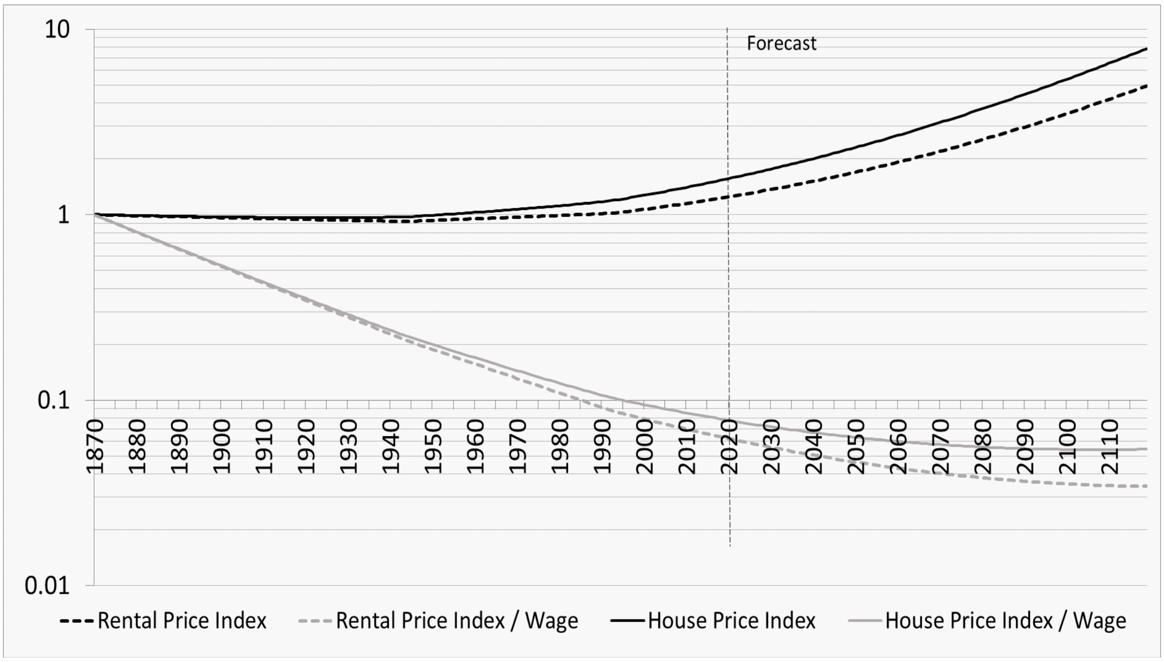


Figure 4: Future Cost of Housing, ε=0.75

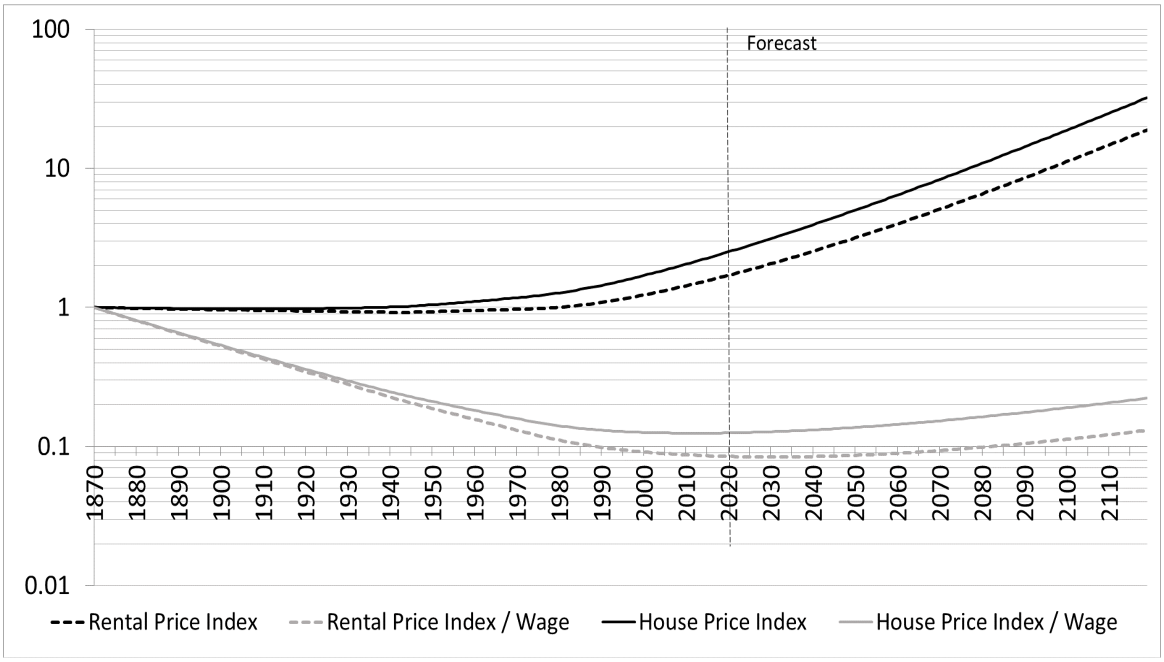


Figure 5: Future Cost of Housing, $\rho = 0.80$

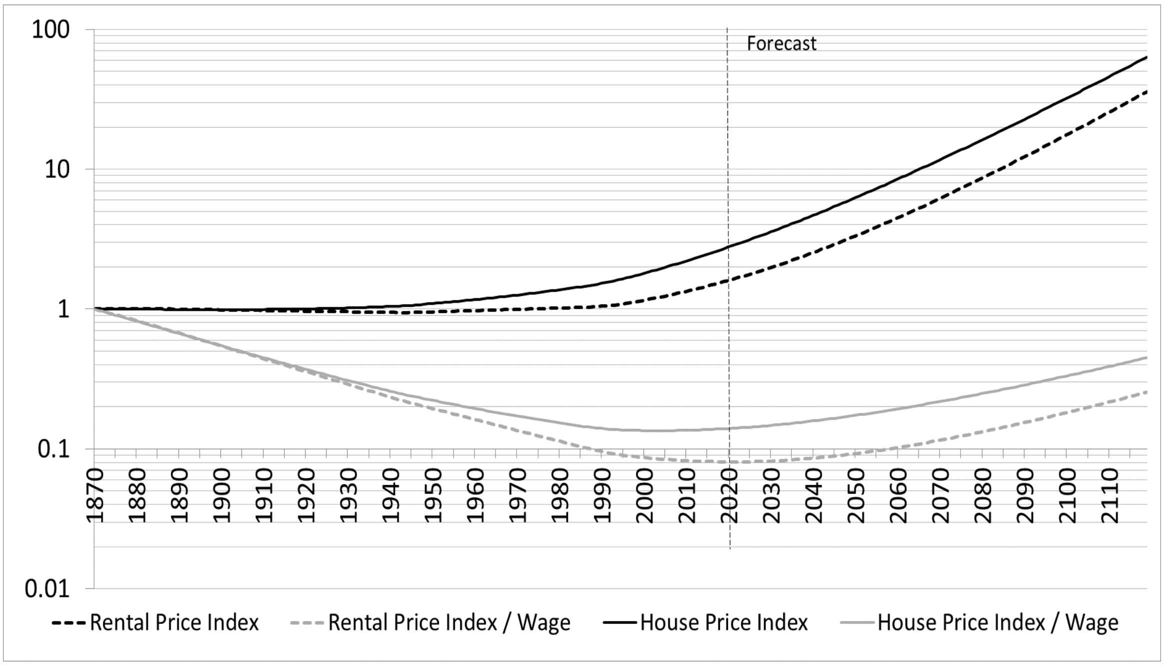


Figure 6: Future Costs of Housing, continuing improvement in transport speeds.

predicted to be a somewhat more equal density of population over the country. There is also greater investment in structures and overall consumption of housing is greater. The impact upon welfare is not trivial. Fifty years ahead average per period utility is just over 3% higher if transport speeds keep falling rather than level off; 100 years ahead utility is just over 4.5% higher. Although investment in housing, and therefore housing consumption, is higher if transport costs fall average housing costs between 2020 and 2120 are similar (figure 6 is fairly similar to figure 1). But the regional dispersion of house values looks different when travel costs continue falling; relative to a scenario of unchanging travel costs, prices at the center are over 6% lower while prices at the periphery are 10% higher.

The simulations illustrate that the impact of further falls in transport costs on the overall level of housing costs over the future is far smaller than its impact in the past. We ran a simulation assuming no transport improvements at all between 1870 and 1970; the level of real housing costs in 1970 was twice as high as in the base case which calibrated travel improvements to evidence on speed of moving passengers. Yet assuming continuing improvements in travel speeds from now on had a small effect on house prices 100 years ahead relative to the assumption of no further changes in passenger speeds. Why do transport cost improvements have a relatively small impact upon the trajectory of average housing costs in the future and yet have been very significant over the past? The main reason is that there are diminishing returns to travel improvements. In our model the travel improvements between 1870 and 1920 substantially increase the proportion of the economy where it is viable to live. By about 1980 (for the simulations using European-Japanese population to land ratios) almost all the country has some residential development, although density is still markedly lower near the periphery than at the center. Over this period there has effectively been a major expansion in the physical size of the country that is relevant for the day to day lives of the population. After that date further improvements have some impact on density differences between areas that have already been developed, but it does not increase the area that has some housing on it. Obviously this is a feature of a highly stylized model and it does not accord with one countervailing force strongly at work over the twentieth century which is the decline in the number of people working in agriculture, a phenomenon that has almost certainly increased the proportion of people living in cities.

Summary of results The key points to emerge from the simulation results are these:

There is very great sensitivity of the overall trajectory of average housing costs to two parameters: ε, the elasticity of substitution between land and structure in creating housing, and ρ the elasticity of substitution between housing and consumption goods in utility. Small changes in these elasticities have dramatic impacts upon the evolution of average housing costs over the next decades. Average housing costs begin to rise most rapidly once most areas within an economy have been developed after which further improvements in transport infrastructure and falls in travel times have much less of an impact than when they made feasible residential development in areas that had previously been unviable. This is a key factor behind the predicted far less rapid rise in average housing costs in the US than in most European countries - the model generates a path for future housing costs that is far steeper in economies where population relative to land area is already sufficiently high that there are few parts of the country not already developed, at least to some extent. This has implications for growth in output, for investment, labor productivity and welfare. We find that the incentive to invest in productive capital in the "crowded old world" declines steadily into the future. With base case parameters the capital to GDP ratio declines from about 164% to around 112% between 2020 and 2120. The return on productive capital rises (from 5.5% to 6.2%). In the economy where the land to population ratio is 10 times as great - "the US" - there is little change in the capital to GDP ratio or in the rate of return on productive capital over this period; the rate of return stays at around 5.5%. This result that the squeeze on available land, and the ramping up in housing costs that comes in the densely populated economies, erodes the incentives to invest in productive capital would not arise if there were a single, integrated global capital market.

5.2 Caveats and Limitations:

There are two important aspects of housing markets that play no role in our stylized model: the availability of mortgage credit to facilitate house purchase and the impact of planning (or zoning) restrictions on housing supply.

In assuming that dynasties are long-lived and are alike in all respects (except for where they chose to live) we give no useful role to mortgages. Essentially we have representative agents so there are not distinct groups alive at the same time some of whom want to save and some of whom want to borrow. Obviously such a model has no meaningful role for mortgages or indeed any debt. It is not that holding debt and financial assets could not be envisaged in the model - we could certainly allow households to have mortgages and save in financial assets that both pay the same interest rate. But in a closed-economy model with representative households the only equilibrium would be one where each household held an amount of interest bearing assets equal to its interest paying debt. Neither does the model make any meaningful distinction between owner occupation and renting; the cost of either form of tenure is the user cost of housing. So the model does not tie down the owner-occupation rate. We could interpret the outcomes as being ones where some households rent and simultaneously own shares in property owning companies that rent out property and pay the returns to shareholders. Ultimately the members of dynasties alive at any time own all the housing stock - and it does not matter whether it is held as owner occupied property or as claims on properties that are rented out. Members of those dynasties pass housing wealth down to later members of the same dynasty. This does reflect an important aspect of reality: people do pass on lots of wealth (both at death and in life) and housing is, for most families, the largest part of bequests. But we completely miss out on issues of inequality and the impact of credit restrictions by making assumptions that dynasties are long-lived and are all alike except for where they live.

We also ignore restrictions on housing supply stemming from regulations on new developments (e.g. zoning rules and other planning laws). There is a large literature suggesting these are significant (see for example Quigley and Raphael (2005), Glaeser, Gyourko, and Saks (2005b), Glaeser, Gyourko, and Saks (2005a), Jaccard (2011)). In the context of our model, relaxation of planning restrictions could be modelled as an increase in $l_{\max}$ - the measure of the physical size of the economy. But until development reaches the edge of the economy changes in $l_{\max}$ do not ease a restriction which, in reality, actually binds at points close to urban centers in many cities and not at the periphery of the country. In some ways one could interpret changes in λ (the cost of distance in our model) as reflecting zoning restrictions and not just travel costs. A fall in λ eases the pressure on space at close to the center of the economy. There is of course a lower limit on λ of 0 and so one cannot rely on it ever falling to offset other forces driving up in housing costs in the long run, and perhaps much the same can be said for easing restrictions on residential development.

6 Conclusions

This paper develops a model of the changes, over time and across locations, in housing and of housing costs. It aims to understand how housing wealth and the cost of housing have moved over the past and how they might evolve into the future. We use a framework that combines features of a Ramsey two-sector growth model with a model of the geography of residential development that tracks the change in location of the population over time.

We find that so long as improvements in travel technology proceed at a pace that is in a fixed proportion to the growth in productive potential (the sum of labor force growth and general productivity growth) there is a balanced growth path with no change in real house prices. But once travel improvements fall back we are no longer on a balanced growth path and real house prices and rents rise. How fast they then rise becomes dependent on a range of parameters that play little role when travel improvements were at a rate near half GDP growth. We then find that plausible parameter estimates plugged into this growth model can easily generate ever rising housing costs – relative to the price of other goods *and* to incomes. But there is great sensitivity of that to parameters that reflect both preferences (between different characteristics of houses) and technology. One key technology factor is how one combines structures and land to create housing. That has changed - the New York skyline shows that it is now possible to erect super-tall residential buildings on small plots of land and squeeze more residential space from a plot than was possible in the past. Whether that can drive our parameter ε to higher levels that fundamentally change the likely future cost of housing is an interesting question.

Price sensitivity of demand for housing - reflecting the substitutability of housing for other goods and services in creating satisfaction - is another factor with a very powerful effect on the longer term path of housing costs. Transport improvements have been very important in the past evolution of prices - improvements in commuting speeds have held down rises in the cost of housing. But we find that it is not likely that further improvements will be as powerful a force in keeping housing costs down.

Our results suggest that there is no reason to believe rising housing costs relative to incomes necessarily represent a risk to stability or are unsustainable. Rising housing costs (and rising house prices) relative to incomes need not be unsustainable and need not mean that financial stability risks are rising. For that reason there should be no automatic policy response to a rise in house prices relative to incomes. But the great sensitivity of the equilibrium (or fundamental) housing cost trajectory to small changes in two key elasticities means it is hard to know whether house prices relative to incomes rising to levels not seen before is the start of a bubble or just the natural path we should expect in an economy with a rising population, growing incomes and with a population density higher than in the past. Using a simple metric like the house price to income ratio relative to its history is unlikely to be a very reliable guide to whether prices are out of line with fundamentals. Having a system of rules (e.g. on acceptable loan to value ratios for mortgages) that would tend to bind ever tighter as house price to income ratios rise will be problematic if the fundamental equilibrium is that this ratio should be rising for many decades.

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