A Life-Cycle Model with Unemployment Traps*

Fabio C. Bagliano   Carolina Fugazza
Giovanna Nicodano

Università di Torino and CeRP (Collegio Carlo Alberto)

This draft: September 1^{th} 2016

Abstract

The Great Recession has highlighted that long-term unemployment may become a trap with loss of human capital. In this paper we extend the life-cycle model of saving and portfolio choice to allow for long-term unemployment spells that have permanent effects on labor income. The risk of future human capital erosion dampens the incentive to invest in risky assets, the younger is the worker. The resulting optimal portfolio share invested in stocks becomes relatively flat in age, more in line with the available evidence and contrary to the predictions of traditional life-cycle models. The driver of such flattening in the life-cycle profile is the resolution of uncertainty, as the worker ages.

Keywords: Life-cycle portfolio choice, unemployment risk, human capital depreciation, age rule.

JEL classification: D91, E21, G11

*We thank Margherita Borella, Marie Brière, Frank DeJong, Jordi Gali, Jerome Glachant, Tullio Jappelli, Christian Julliard, Elisa Luciano, Sidney Ludvigson, Marco Pagano and Bas Werker for very useful comments and suggestions. We thank participants to the NETSPAR International Workshop on Pensions, 2016 and to the RiskForum2016 (Institut Louis Bachelier). We are grateful to CINTIA-Italy for funding.

1 Introduction

Unemployment leads to large and persistent earnings losses that increase in its duration due to skill deterioration. The magnitude of this effect varies over time and across industries and demographic groups (Rhum, 1991; Jacobson, Lalond and Sullivan, 1993; Davis and vonWacther 2011) as well as countries (Machin and Manning, 1999). Recently, the average duration of unemployment spells in developed economies has increased. In the US the share of unemployed workers who are jobless for more than one year doubled over the recent Great Recession episode, reaching 24% of total unemployment in 2014. Krueger, Cramer and Cho (2014) and Kroft, Lange, Notowidigdo and Katz (2016) show that the re-employability of the long-term unemployed progressively declines over time, so that they are more likely to exit labor force. More job openings do not lead to more employment among those who are jobless for more than six months, a pattern holding across all ages, industries and education levels (Ghayad and Dickens 2012). On the whole, these findings indicate that long-term unemployment may become a trap, often not supported by supplementary income provisions, given that unemployment benefits decline rapidly with unemployment duration.

In this paper, we embed in a life-cycle model of consumption and portfolio choice the possibility of entering long-term unemployment with its permanent consequences on human capital. We model working life careers as a three-state Markov chain driving the transitions between employment, short-term and long-term unemployment states, as in Bremus and Kuzin (2014), calibrated to broadly match observed US labor market features. Importantly, we allow for human capital erosion during unemployment. When unemployed, individuals receive benefits but simultaneously experience a cut, proportional to unemployment duration, in the permanent component of labor income which captures diminished future income prospects. This represents the observed permanent earning losses (Arulampalam et al., 2000; Arulampalam, 2001; Schmieder, von Wachter and Bender, 2013) due to skill loss (Neal, 1995; Keane and Wolpin, 1997; Edin and Gustavsson, 2008) during long-term unemployment.

Our results show that the risk of permanently losing labor income severely reduces the level of human capital at any age prior to retirement. This occurs even if the implied unconditional probabilities of being short-run unemployed (3.84%) and long-run unemployed (0.16%) are rather conservative, and the unemployment benefit replacement rate is set at the average level observed for the US. As for human capital erosion following unemployment, it is respectively equal to 20%

and 90% in case of short- term and long-term unemployment spells in a calibration that captures the relatively slow re-employment experienced in the US.

Such potential loss in human capital considerably lowers the optimal portfolio share invested in stocks with respect to the case of no unemployment risk. This is not surprising because of the reduced ability to hedge capital income uncertainty with labour income, albeit with low probability. In other words, our model draws the attention to a scenario opposite to the one depicted by Gomes, Kotlikoff and Viceira (2008), where the employed worker is able to increase labor supply as an additional buffer against future income uncertainty.

Importantly, optimal stock investment is no longer decreasing with age but remains remarkably flat over the whole working life. These findings are broadly consistent with the joint empirical evidence about investment decisions and average unemployment duration across education groups.[1] The traditional life-cycle model implies that households should reduce investments in risky stocks as they approach retirement (Bodie, Merton and Samuelson, 1992; Viceira 2001; Cocco, Gomes and Maenhout 2005). The reason is that human capital provides a hedge against shocks to stock returns, making financial risk bearing more attractive. Investment in stocks should therefore be relatively high at the beginning of working careers, when human capital is relatively large relative to accumulated financial wealth. Then, human capital typically decreases relative to financial wealth over the life cycle, leading to a gradual reduction in stock investment till retirement. This model implication is embodied in the popular financial advice of a stock exposure gradually decreasing with age. In our model with unemployment traps, such effect is instead moderated by the resolution of uncertainty concerning labour and pension income, as the worker safely approaches retirement age. When potential human capital erosion is considerable, the resolution of uncertainty effect compensates the hedge effect and the optimal investment in stocks is relatively flat over the life-cycle. This threat also shrinks the heterogeneity of optimal portfolio choices across agents.

Previous life-cycle models with unemployment leave instead the observed age pattern of stock holding during working life largely unexplained. Some versions of the life-cycle model account for the risk of being unemployed by introducing a (small) positive probability of zero labor income: in these models unemployment

[1] In the US, stock investment appears to be positively correlated with the level of education which is inversely related to the average probability of being unemployed. In particular, according to Current Population Survey data from the US Bureau of Labor Statistics, the average unemployment rate among college graduates was 2% in 2014, while it was 6% and 9% for high school and less than high school educated workers respectively.

risk affects income only during the unemployment spell with no consequences on subsequent earnings ability (Cocco, Gomes and Maenhout, 2005). Bremus and Kuzin (2014) model unemployment persistence allowing for both short-term and long-term unemployment. Given that there is no permanent consequence on subsequent earnings ability, the stock holding is still counterfactually decreasing in age till retirement although, on average, lower than what obtained without unemployment risk. Thus, it is the possibility of unemployment traps - rather than unemployment per se - that restrains risk taking by the young and middle-aged workers.

Prior research already emphasizes that the resolution of uncertainty over the life-cycle may flatten the age profile of stock investment (Bagliano, Fugazza and Nicodano, 2014). In their model such flattening depends on both the presence of another risky asset besides equities and a positive correlation between stock returns ans permanent labour income shocks. Moreover, it only appears when risk aversion or the variance of labour income shocks are higher than in the baseline calibration of Cocco et al. (2005). Our contribution adds, to an otherwise standard setting, the possibility of large human capital losses.

The rest of the paper is organized as follows. Section 2 presents the benchmark life-cycle model and briefly outlines the numerical solution procedure adopted. We detail the model calibration in Section 3, discuss results in Section 4 and perform robustness checks in Section 5. Section 6 concludes the paper.

2 The life-cycle model

We model an investor who maximizes the expected discounted utility of consumption over her entire life and wishes to leave a bequest as well. The effective length of her life, which lasts at most T periods, is governed by age-dependent life expectancy. At each date t, the survival probability of being alive at date $t+1$ is p_t, the conditional survival probability at t. The investor starts working at age t_0 and retires with certainty at age t_0+K. Investor's i preferences at date t are described by a time-separable power utility function:

$$\frac{C_{t_0}^{1-\gamma}}{1-\gamma}+E_{t_0}\left[\sum_{j=1}^{T}\beta^j\left(\prod_{k=0}^{j-2}p_{t_0+k}\right)\left(p_{t_0+j}\frac{C_{t_0+j}^{1-\gamma}}{1-\gamma}+\left(1-p_{t_0+j}\right)b\frac{\left(X_{t_0+j}/b\right)^{1-\gamma}}{1-\gamma}\right)\right] \quad (1)$$

where C_t is the level of consumption at time t, X_t is the amount of wealth the investor leaves as a bequest to her heirs in case of death, $b \geq 0$ is a parameter

capturing the strength of the bequest motive, $\beta < 1$ is a utility discount factor, and γ is the constant relative risk aversion parameter. Following Cocco, Gomes and Maenhout (2005), we do not model labour supply decisions, whereby ignoring the insurance property of flexible work effort allowing investors to compensate for bad financial returns with higher labour income, as in Gomes, Kotlikoff and Viceira (2008).

2.1 Labor and retirement income

During working life individuals receive exogenous stochastic earnings as compensation for labor supplied inelastically. Working life careers are modelled as a three-state Markov chain considering employment (e), short-term (u_1) and long-term unemployment (u_2). Unemployment may be short-term and last only one year or it may become long term and last two years. Individual labor market dynamics are driven by to the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} \end{pmatrix} \tag{2}$$

where $\pi_{ij} = \text{Prob}\left(s_{t+1} = j | s_t = i\right)$ with $i, j = e, u_1, u_2$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability π_{eu_1}. Since to become long-term unemployed she must first experience short-term unemployment, we set the probability for the employed to enter long-term unemployment at zero, $\pi_{eu_2} = 0$). If the worker is short-term unemployed at t ($s_t = u_1$), then she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or she becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2}$; consequently we set $\pi_{u_1u_1} = 0$. Finally, if she's long-term unemployed at t ($s_t = u_2$), since long-term unemployment lasts only two years, she is re-employed ($s_{t+1} = e$) with certainty, thus $\pi_{u_2e} = 1$ and $\pi_{u_2u_1} = \pi_{u_2u_2} = 0$.

As in Cocco, Gomes and Maenhout (2005), the employed individual receives a stochastic labor income given by the following process:

$$Y_t = H_t N_t \qquad\qquad t_0 \leq t \leq t_0 + K \tag{3}$$

where $H_t = F\left(t, \mathbf{Z}_t\right) P_t$ represents the permanent income component. In particular, $F\left(t, \mathbf{Z}_t\right) \equiv F_t$ denotes the deterministic trend component that depends on age

(t) and a vector of individual characteristics ($\mathbf{Z}_t$) such as gender, marital status, household composition and education. Consistent with the available empirical evidence, the logarithm of the stochastic permanent component is assumed to follow a random walk process:

$$\log P_t = \log P_{t-1} + \omega_t \tag{4}$$

where ω_t is distributed as $N(0, \sigma_\omega^2)$. N_t denotes the transitory stochastic component and $\log(N_t)$ is distributed as $N(0, \sigma_\varepsilon^2)$ and uncorrelated with ω_t.

In our set-up, differently from Bremus and Kuzin (2014), labor income received by the employed individual at time t depends on her past working history. In particular, we allow unemployment and its duration to affect the permanent component of labor income H. Since the empirical evidence suggests that the longer the unemployment spell the larger the worker's human capital depreciation (Schmieder, von Wachter and Bender, 2013), we let human capital erosion increase with unemployment duration. Thus, for the worker who at t is re-employed after one-year unemployment the permanent component H_t is equal to H_{t-1} eroded by a fraction Ψ_1, and the worker who is re-employed at t after a two-year unemployment spell experiences a reduction of the permanent income earned when entering unemployment, H_{t-2}, by a fraction Ψ_2, with $\Psi_2 > \Psi_1$. Then, the permanent component of labor income H_t evolves according to:

$$H_t = \begin{cases} H_t & \text{if } s_t = e \text{ and } s_{t-1} = e \\ (1-\Psi_1)H_{t-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\ (1-\Psi_2)H_{t-2} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 \end{cases} \qquad t = t_0, ..., t_0 + K \tag{5}$$

In the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income $H_{t-1} = F_{t-1}P_{t-1}$, whereas in the long-term unemployment state ($s_t = u_2$) no benefits are available: $\xi_2 = 0$. Thus, income received during unemployment is:

$$Y_t = \begin{cases} \xi_1 H_{t-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\ 0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e \end{cases} \qquad t = t_0, ..., t_0 + K \tag{6}$$

Finally, during retirement, income is certain and equal to a fixed proportion λ of the permanent component of labor income in the last working year:

$$Y_t = \lambda F\left(t, \mathbf{Z}_{t_{0+l}}\right) P_{t_{0+l}} \qquad t_0 + K < t \leq T \tag{7}$$

where retirement age is $t_0 + K$, $t_0 + l$ is the last working period before retirement and λ is level of the replacement rate.

2.2 Investment opportunities

We allow savings to be invested in a short-term riskless asset, yielding each period a constant gross real return R^f, and one risky asset, characterized as "stocks" yielding stochastic gross real returns R_t^s. The excess returns of stocks over the riskless asset follows

$$R_t^s - R^f = \mu^s + \nu_t^s \tag{8}$$

where μ^s is the expected stock premium and ν_t^s is a normally distributed innovation, with mean zero and variance σ_s^2. We do not allow for excess return predictability and other forms of changing investment opportunities over time, as in Michaelides and Zhang (2005).

At the beginning of each period, financial resources available for consumption and saving are given by the sum of accumulated financial wealth W_t and current labor income Y_t, that we call *cash on hand* $X_t = W_t + Y_t$. Given the chosen level of current consumption, C_t, next period cash on hand is given by:

$$X_{t+1} = (X_t - C_t)R_t^P + Y_{t+1} \tag{9}$$

where R_t^P is the portfolio return:

$$R_t^P = \alpha_t^s R_t^s + (1 - \alpha_t^s) R^f \tag{10}$$

with α_t^s and $(1 - \alpha_t^s)$ denoting the shares of the investor's portfolio invested in stocks and in the riskless asset respectively. We do not allow for short sales and assume that the investor is liquidity constrained, so that the nominal amount invested in each of then two financial assets are $B_t \geq 0$, $S_t \geq 0$, respectively for the riskless asset and stocks, are non negative in each period. All simulation results presented below are derived under the assumption that the investor's asset menu is the same during working life and retirement.

2.3 Solving the life-cycle problem

In this standard intertemporal optimization framework, the investor maximizes the expected discounted utility over life time, by choosing the consumption and the portfolio rules given uncertain labor income and asset returns. Formally, the

optimization problem is written as:

$$\max_{\{C_t\}_{t_0}^{T-1},\{\alpha_t^s\}_{t_0}^{T-1}} \left(\frac{C_{t_0}^{1-\gamma}}{1-\gamma} + E_{t_0} \left[\sum_{j=1}^{T} \beta^j \left(\prod_{k=0}^{j-2} p_{t_0+k} \right) \left(p_{t_0+j} \frac{C_{t_0+j}^{1-\gamma}}{1-\gamma} + \right. \right. \right.$$
$$\left. \left. \left. + \left(1 - p_{t_0+j}\right) b \frac{\left(X_{t_0+j}/b\right)^{1-\gamma}}{1-\gamma} \right) \right] \right) \tag{11}$$

$$s.t. \quad X_{t+1} = (X_t - C_t) \left(\alpha_t^s R_t^s + (1 - \alpha_t^s) R^f \right) + Y_{t+1} \tag{12}$$

with the labor income and retirement processes specified above and the no-short-sales and borrowing constraints imposed. Given its intertemporal nature, the problem can be restated in a recursive form, rewriting the value of the optimization problem at the beginning of period t as a function of the maximized current utility and of the value of the problem at $t+1$ (Bellman equation):

$$V_t \left(X_t, P_t, s_t\right) = \max_{\{C_t\}_{t_0}^{T-1},\{\alpha_t^s\}_{t_0}^{T-1}} \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \beta E_t \left[p_t V_{t+1} \left(X_{t_{+1}}, P_{t+1}, s_{t+1} \right) \right. \right.$$
$$\left. \left. + \left(1 - p_t\right) b \frac{\left(X_{t+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{13}$$

At each time t the value function V_t describes the maximized value of the problem as a function of three state variables: cash on hand at the beginning of time t (X_t), the stochastic permanent component of income at beginning of t (P_t), and the labor market state $s_t (= e,\ u_1,\ u_2)$. The Bellman equation can be written by making the expectation over the employment state at $t+1$ explicit, as:

$$V_t \left(X_t, P_t, s_t\right) = \max_{\{C_t\}_{t_0}^{T-1},\{\alpha_t^s\}_{t_0}^{T-1}} \left(\frac{C_t^{1-\gamma}}{1-\gamma} \right.$$
$$+ \beta \left[p_t \sum_{s_{it+1}=e,\mathrm{u}_1,u_2} \pi \left(s_{t+1} | s_t\right) \widetilde{E_t V}_{t+1} \left(X_{t_{+1}}, P_{t+1}, s_{t+1} \right) \right.$$
$$\left. \left. + \left(1 - p_t\right) b \sum_{s_{t+1}=e,\mathrm{u}_1,u_2} \pi \left(s_{t+1} | s_t\right) \frac{\left(X_{t+1}/b\right)^{1-\gamma}}{1-\gamma} \right] \right) \tag{14}$$

where $\widetilde{E_t V}_{t+1}$ denotes the expectation operator taken with respect to the stochastic variables ω_{t+1}, ε_{t+1}, and υ_{t+1}. The history dependence that we introduce in our set-up by making unemployment to affect subsequent labor income prospects prevents us to rely on the standard normalization of the problem with respect to the level of P_t. To highlight how the evolution of the permanent component of labor income depends on previous individual labor market dynamics we write the value function at t in each possible labor market state (dropping the term involving the bequest motive) as:

$$V(X_t, P_t, e) = u(C_t) + \beta p_t \begin{cases} \begin{cases} V(X_{t+1}, P_{t+1}, e) & \text{with prob. } \pi_{e,e} \\ \text{with } P_{t+1} = P_t e^{\omega_{it+1}} & \text{and} \\ X_{t+1} = (X_t - C_t)R_t^p + F_{t+1}P_{t+1}e^{\varepsilon_{t+1}} \end{cases} \\ \\ \begin{cases} V(X_{t+1}, P_{t+1}, u_1) & \text{with prob. } \pi_{e,u_1} \\ \text{with } P_{t+1} = (1-\Psi_1)P_t & \text{and} \\ X_{t+1} = (X_t - C_t)R_t^p + \xi_1 F_t P_t \end{cases} \end{cases}$$

$$V(X_t, P_t, u_1) = u(C_t) + \beta p_t \begin{cases} \begin{cases} V(X_{t+1}, P_{t+1}, e) & \text{with prob. } \pi_{u_1,e} \\ \text{with } P_{it+1} = (1-\Psi_1)P_{t-1}\, e^{\omega_{t+1}} & \text{and} \\ X_{t+1} = (X_t - C_t)R_t^p + F_{t-1}(1-\Psi_1)P_{t-1}e^{\varepsilon_{t+1}} \end{cases} \\ \\ \begin{cases} V(X_{t+1}, P_{t+1}, u_2) & \text{with prob. } \pi_{u_1,u_2} \\ \text{with } P_{t+1} = (1-\Psi_2)(1-\Psi_1)P_{t-1} & \text{and} \\ X_{t+1} = (X_t - C_t)R_t^p \end{cases} \end{cases}$$

$$V(X_t, P_t, u_2) = u(X_t) + \beta p_t \begin{cases} V(X_{t+1}, P_{t+1}, e) & \text{with prob. } \pi_{u_2,e} \\ \text{with } P_{t+1} = (1-\Psi_2)(1-\Psi_1)P_{t-1}e^{\omega_{t+1}} & \text{and} \\ X_{t+1} = (X_t - C_t)R_t^p + F_{t-2}(1-\Psi_2)(1-\Psi_1)P_{t-2}e^{\omega_{t+1}}e^{\varepsilon_{t+1}} \end{cases} \tag{15}$$

This problem has no closed form solution: hence the optimal values for consumption and portfolio shares depending on the values of each state variable at each point in time are obtained by means of numerical techniques. To this aim, we apply the standard backward induction procedure starting form the last possible period of life T. The optimal consumption and portfolio share policy rules are obtained for each possible value of the continuous state variables (X_t and P_t) using the stan-

dard grid search method.[2] Going backwards, for every period $t = T-1, T-2, ..., t_0$, the Bellman equation (14) is used to obtain the optimal rules for consumption and portfolio shares.

3 Calibration

Parameter calibration concerns investor's preferences, the features of the labor income process during working life and retirement, and the moments of the risky asset returns. For reference, we solve the model also abstracting from the unemployment risk as in Cocco, Gomes and Maenhout (2005); this scenario is referred to as *Calibration 1.*

The investor begins her working life at the age of 20 and works for (a maximum of) 45 periods (K) before retiring at the age of 65. After retirement, she can live for a maximum of 35 periods until the age of 100. In each period, we take the conditional probability of being alive in the next period p_t from the life expectancy tables of the US *National Center for Health Statistics.* As regards to preferences, we set the utility discount factor $\beta = 0.96$, and the parameter capturing the strength of the bequest motive $b = 2.5$ (which bears the interpretation of the number of years of her descendants' consumption that the investor intends to save for). Finally, the benchmark value for the coefficient of relative risk aversion is $\gamma = 5$. The latter choice is relatively standard in the literature (Gomes and Michaelides 2005; Gomes, Kotlikoff and Viceira 2008), capturing an intermediate degree of risk aversion, though Cocco, Gomes and Maenhout (2005) and Bremus and Kuzin (2014) choose a value as high as 10 in their benchmark setting. The riskless (constant) interest rate is set at 0.02, with expected stock premium μ^s fixed at 0.04 . The standard deviation of the returns innovations is set at $\sigma_s = 0.157$. Finally, we impose a zero correlation between stock return innovations and aggregate permanent labour income disturbances ($\rho_{sY} = 0$). Table 1 summarizes the benchmark values of relevant parameters as well as their changes considered through our analysis and robustness checks.

3.1 Labor income and unemployment risk

[2] The problem is solved over a grid of values covering the space of the state variables and the controls in order to ensure that the obtained solution is a global optimum.

The labor income process is calibrated using the estimated parameters for US households with high-school education (but not a college degree) in Cocco, Gomes and Maenhout (2005). After retirement, income is a constant proportion λ of the final (permanent) labour income, with $\lambda = 0.65$. In the benchmark case, the variances of the permanent and transitory shocks (ω_{it} and ε_{it} respectively) are $\sigma_\omega^2 = 0.0106$ and $\sigma_\varepsilon^2 = 0.0738$. The solution of the model with unemployment risk maintains the value of all parameters assumed above. The chosen transition probabilities between the three labor market states broadly reflect the transition rates between employment and the two unemployment states observed on average among US workers:[3]

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.96 & 0.04 & 0 \\ 0.95 & 0 & 0.05 \\ 1 & 0 & 0 \end{pmatrix} \tag{16}$$

The assumed transition matrix yields rather conservative values for the unconditional probabilities of being short-run unemployed (3.84%) and long-run unemployed (0.16%). We set the unemployment benefit replacement rate ξ_1 at the average level observed for the US. In particular, considering that the replacement rate with respect to last labor income is on average low and state benefits are paid for a maximum of 26 weeks,[4] we set $\xi_1 = 0.3$ in case of short-term unemployment spells and set a value of $\xi_2 = 0$ for the long-term unemployed.

To study long-run effects of unemployment on optimal asset allocation, we first consider a baseline calibration of the model with unemployment risk (*Calibration 2*), in which the permanent proportion of human capital erosion Ψ following unemployment is equal to 0% at all unemployment durations. This *Calibration 2* corresponds to the set up studied by Bremus and Kuzin (2014) who focus only on temporary effects of unemployment and disregards possible its possible permanent consequences.

A well established empirical literature on job displacement shows that job losses affect earnings far beyond the unemployment spell, though the range of the estimated effects varies considerably. For example, the estimates for immediate losses following displacement, may range from 30% (Couch and Placzek, 2010) to 40%

[3]The transition matrix reports the annual transition probabilities obtained by annualizing U.S. average quarterly transitions.

[4]No additional weeks of federal benefits are available in any state: the temporary Emergency Unemployment Compensation (EUC) program expired at the end of 2013, and no state currently qualifies to offer more weeks under the permanent Extended Benefits (EB) program.

(Jacobson, Lalond and Sullivan,1993). These earnings losses are shown to be persistent in a range of about 25% (Jacobson, Lalond and Sullivan, 1993a) and 15% (Couch and Placzek, 2010) of the pre-displacement earnings. These estimates abstract from the effect of the duration of unemployment following job losses, while Cooper (2013) finds, instead, that earning losses are larger the longer the unemployment duration. In addition, various studies indicate that workers who have been out of the labor force for a long time are less likely to find a job (Krueger, Cramer and Cho, 2014), which implies a more substantial human capital erosion. Based on administrative data, Jacobson et al. (1993a, 1993b and 2005) estimate that earning losses for displaced workers amount, on average, to 50% of the pre-displacement wage level. This evidence, combined with a 40% probability of finding a job after being 24 months unemployed and a 88% of transitioning from unemployment to out of the labor force each month (Kroft, Lange, Notowidigdo and Katz, 2016), leads us to calibrate an expected drop in human capital of about 90% following a long term unemployment spell (*Calibration 3,* our benchmark scenario). This captures the long lasting effects of long term unemployment on job careers in our simplified transition matrix. In order to contain the dimensionality of the transition matrix, we have to set the probability of finding a job after two years equal to 1, well above the estimate of Kroft, Lange, Notowidigdo and Katz (2016). In section 5. we relax this assumption and allow for more gradual human capital erosion along longer unemployment durations (up to five years) calibrated tightly to U.S. transition data.

4 Results

4.1 Optimal policies

In Figure 1 optimal stock shares are shown for the model without unemployment risk and the standard life-cycle result obtains. In particular, the figure plots the optimal stock share as a function of cash on hand for an average level of the permanent labor income component. For relatively large levels of the permanent component, labor income acts as an implicit asset and affects the optimal portfolio composition depending on investor's age and wealth. Under the considered standard calibration, labor income, though uncertain, is treated like a risk-free asset. At age 20, the sizable implicit holding of the risk free asset (through human capital) makes it optimal for the less wealthy investors to tilt their portfolio towards the risky financial assets. Indeed, for a wide range of levels of wealth, optimal

stock investment is 100%. The optimal stock holding decreases in financial wealth to offset the relatively lower implicit investment in (less risky) human capital.

Figures 2 and 3 display policy functions obtained from our model extended to account for unemployment risk. In this paper, the focus is on the consequences of unemployment in terms of labor income prospects after experiencing job loss. Job losses imply a cut in income during the unemployment spell when the individual receives only a relatively small benefit. We model the most common unemployment insurance scheme in the US, where the average unemployment benefit is 30% of the last wage during the first year of unemployment and zero afterwards. Our results are derived assuming replacement rates $\xi_1 = 0.3$ for short-term and $\xi_2 = 0$ for long-term unemployed.

In addition, our model accounts for the fact that unemployment may have severe effects in terms of individual skill erosion and thus on labor income prospects at re-employment. In the present set-up unemployment induces human capital erosion implying proportional cuts in individual's permanent labor income, with the reduction following a two-year unemployment spell ($\Psi_2 = 0.9$) being higher than that occurring in case of one-year unemployment ($\Psi_1 = 0.2$).

In Figure 2, we assume *Calibration 2* which implies no human capital depreciation due to job loss, with $\Psi_1 = \Psi_2 = 0$. Given that there are no permanent effects of job loss, optimal asset allocation are not remarkably different from the case that ignores unemployment risk.

The recent Great Recession spurred attention on the long -term unemployed and on their subsequent labor market prospects. In particular, Krueger, Cramer and Cho (2014) document that long-term unemployed experience a progressively declining re-employability over time and are more likely to exit the labor force. In *Calibration* 3 we account for such an extreme consequence of long term unemployment assuming a strong human capital depreciation following a two year unemployment spell. In particular, Ψ_1 is kept at 0.2 and Ψ_2 is increased up to 0.9, implying a 90% erosion of the individual permanent labor income component after the second unemployment year. In Figure 3 the resulting policy functions are shifted abruptly leftward. The optimal stock share is still decreasing in financial wealth but 100% stock investment is optimal only at very low levels of wealth. In this case, long-term unemployment implies the loss of a substantial portion of future labor income which severely reduces the level of human capital and increases its risk at any age. Thus, for almost all levels of financial wealth, stock investment is considerably lower than in the benchmark case of no unemployment risk.

4.2 Life Cycle Profiles

On the basis of optimal policy functions, we simulate the whole life-cycle consumption and investment decisions for 10,000 agents. Figure 4, in panel a), shows the average optimal stock shares plotted against age derived when unemployment risk is ignored (dotted line) and for the case in which it is accounted for (dashed and solid lines).

In case of no unemployment risk, the well known result on the age profile of optimal stock portfolio shares obtains. Over the life cycle the proportion of overall wealth implicitly invested in the riskless asset through human capital declines with age. Consequently, at early stages of the life cycle, optimal stock investment is about 100% and declines with age till retirement.

In case of unemployment risk, with no human capital erosion (*Calibration* 2) the optimal share of stocks in the portfolio still declines with age, though being lower at all ages, with a 100% optimal stock share only for very young investors. However, when long-term unemployment implies large skill erosion, as in *Calibration 3*, the optimal stock investment is almost flat and reduced at any age. As seen in the previous section, the risk of permanently losing a substantial portion of future labor income prospects reduces the level of human capital at all ages and increases its riskiness inducing a relatively lower optimal stock investment conditional on financial wealth at all ages. Moreover, the large amount of background risk increases precautionary savings and thus wealth accumulation over time implying less need to tilt the asset allocation towards stocks at any age (see Panel (b) of Figure 4). Consequently, the age profiles remains remarkably flat overall the working life and during retirement[5]. These results are at odds with previous studies that model unemployment risk. Among them, Bremus and Kuzin (2014) study the effects of unemployment persistence on asset allocation. In their model, the unemployment length affects income only during job loss. Thus, their results on optimal stock investing derived under a standard calibration, namely with moderate risk aversion equal to 5, are indistinguishable from those obtained in a standard life cycle model that disregards unemployment risk at all. Our results show instead that allowing for possible long run consequences of unemployment may significantly dampen the optimal incentive to invest in stocks, even under standard calibrations.

Panel (b) of Figure 4 reports the average financial wealth accumulated over the

[5] The relatively low investment in stocks during retirement is due to the presence of positive bequest motive, common to all parametrizations considered in the paper.

life cycle for the three cases considered. In front of the higher human capital risk in case of rare but possible strong human capital depreciation, individuals accumulates substantially more financial wealth during working life to buffer possible disastrous labor market outcomes.

4.2.1 Heterogeneity

The above results imply that the optimal stock investment is flat in age, even for a moderately risk averse worker. In the face of a very rare but large human capital depreciation, workers on average invest in stocks about 55% of the accumlated financial wealth.

The average patterns may hide considerable differences across agents. This section investigates the distribution across agents of both conditional optimal stock share and accumulated wealth.

The distributions of optimal stock shares and accumulated financial wealth for the case of no unemployment risk are displayed in Figure 5, panels (a) and (b). The distributions of optimal stock shares and financial wealth are highly heterogeneous for both workers and retirees, with the exception of young workers who tilt their entire portfolio towards stocks given the relatively riskless nature of their human capital. Heterogeneity of portfolio shares depends on the shape and movements through age of the policy functions, relating optimal stock shares to the amount of available cash on hand and on the level of cash on hand itself. Relatively steep policy functions imply that even small differences in the level of accumulated wealth result in remarkably different asset allocation choices.

At the early stage of the life cycle, when accumulated financial wealth is modest, it is optimal for everybody to invest almost 100% of it in stocks. As investors grow older, different realizations of background risk induce large differences in savings and wealth accumulation pushing investors on the steeper portion of their policy functions, determining a gradual increase in heterogeneity of optimal portfolio shares of shocks their working life. After retirement, investors decumulate their financial wealth relatively slowly, due to the bequest motive, and still move along the steeper portion of their relevant policy functions and the dispersion of optimal shares tends to persist.

In Figure 6, panels (a) and (b), we report the life-cycle distribution of stock share and financial wealth for the case with unemployment risk and human capital erosion. Compared to the case with no unemployment risk, the distribution of optimal stock shares is much less heterogeneous over all the life-cycle for both

workers and retirees. In particular, the heterogeneity shrinks during working life even for young workers given that the riskier nature of human capital induces lower investment in stocks and higher savings at all ages. In case of unemployment risk, policy functions are relatively flat (see Figure 3) implying that even large differences in the level of accumulated wealth result in homogenous asset allocation choices. Then as in the previous case, the shape of heterogeneity of stock shares and accumulated financial wealth over the life-cycle is due to different realization of background risk.

5 Robustness

5.1 Calibrating U.S. transition probabilities

Section 2 provides the essence of our argument imposing that unemployment lasts at most two years. This shortcut allows us to use a representation with just two states, but forces us to assume that after the second state the unemployed counterfactually gets a job with probability 1, albeit with a very high human capital erosion. In this section we extend the model to allow for a maximum unemployment duration of five years and for more gradual human capital erosion. Unemployment may be short-term and last only one year or it may become long term and last from two to five years. In Appendix 1 we report the model's analytic, while here we just outline the main differences with respect to the benchmark model introduced in Section 2.

To allow for very long-term, though rare, unemployment spells, we model working life careers as a six-state Markov chain considering employment (e), short-term (u_1) and long-term unemployment from two up to five years ($u_2, ...u_5$). Individual labor market dynamics are driven by the following transition matrix:

$$
\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} & \pi_{eu_3} & \pi_{eu_4} & \pi_{eu_5} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} & \pi_{u_1u_3} & \pi_{u_1u_4} & \pi_{u_1u_5} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} & \pi_{u_2u_3} & \pi_{u_2u4} & \pi_{u_2u_5} \\ \pi_{u_3e} & \pi_{u_3u_1} & \pi_{u_3u_2} & \pi_{u_3u_3} & \pi_{u_3u_4} & \pi_{u_3u_5} \\ \pi_{u_4e} & \pi_{u_4u_1} & \pi_{u_4u_2} & \pi_{u_4u_3} & \pi_{u_4u_4} & \pi_{u_3u_5} \\ \pi_{u_5e} & \pi_{u_5u_1} & \pi_{u_5u_2} & \pi_{u_5u_3} & \pi_{u_5u_4} & \pi_{u_5u_5} \end{pmatrix} = \tag{17}
$$

$$
= \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & 0 & 0 & 0 & 0 \\ \pi_{u_1e} & 0 & (1-\pi_{u_1e}) & 0 & 0 & 0 \\ \pi_{u_2e} & 0 & 0 & (1-\pi_{u_2e}) & 0 & 0 \\ \pi_{u_3e} & 0 & 0 & 0 & (1-\pi_{u_3e}) & 0 \\ \pi_{u_4e} & 0 & 0 & 0 & 0 & (1-\pi_{u_4e}) \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

The annual transition probabilities between labor market states are chosen to match the average annual unemployment rate at all durations in the U.S. Over $1990-2015$, the average annual unemployment rate was 6.1 percent, the average rate of workers unemployed for more than one year was about 0.91% (the rate for those unemployed for less than one year was 5.2%).[6] The long term unemployment rose dramatically during the $2007-2009$ recession and remained high thereafter. In particular, data from the Current Population Survey show that in 2013 the average rate of unemployed for more than one year and two years is about 1.4% and 0.8%, respectively (the corresponding values for year 2011 were 2.3% and 1.3%, respectively).

We use CPS data to calibrate the transition probabilities from employment to unemployment to reflect the risk of entering unemployment beside the observed average unemployment rates at different durations. According to the evidence based on CPS reported in Kroft et al. (2013), the annual transition probability from employment to unemployment is 4%. The annual outflow rate from unemployment to employment is steadily declining during the first year of unemployment and is quite high, thus we set at 90% the probability to leave unemployment after the first year (Kroft et al., 2016). However, after the first year the chance of exiting from unemployment is substantially reduced to about 50% per year. Consequently, we set at 50% the chance of exiting unemployment for a worker unemployed for less than 4 years. Given the duration dependence (Kroft et al. 2016), we set the tran-

[6] Source: OECD Data

sition probability out of unemployment from the fourth to the fifth year at 10%. Finally, as the maximum length of unemployment spells is five years, at the end of the fifth year, the worker is re-employed with certainty. Overall, our calibration is quite conservative, since the chance of being employed 15 months later for those who had been unemployed 27 weeks or more is only 36% (see the evidence on CPS data in Krueger et al., 2014).

The calibrated transition matrix for all labor market states is reported in the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} 0.95 & 0.05 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0.5 & & & \\ 0.5 & & & 0.5 & & \\ 0.1 & & & & 0.9 & \\ 1 & & & & & 0 \end{pmatrix} \tag{18}$$

The maintained transition probabilities yield rather conservative values for the unconditional unemployment rates at different durations, as reported in the following table:

	unemployment durations (years)				
	u_1	u_2	u_3	u_4	u_5
Unconditional unemployment probabilities	4.72%	0.47%	0.24%	0.12%	0.11%

In particular, the calibrated transitions imply an average annual rate for those unemployed for less than or equal to one year of 4.72% . In line with what observed in the data, the overall average rate for those who experience unemployment for more than one year is 0.93%: 0.47% is the rate for unemployment with duration lower than two years, and so on, up to 0.11% which is the rate of unemployment with duration up to 5 years.

For the worker who at t is unemployed after a $i-$year unemployment spell the human capital is eroded by a fraction Ψ_i, with $i = 1, ...5$. In this version of the model, unemployment induces more gradual human capital erosion, being 20% of the existing human capital the fraction eroded at all unemployment durations shorter than 5 years ($\Psi_i = 0.2$ for $i = 1, ...4$) and assuming a huge human capital drop of 90% only in case of the very rare event of an unemployment spell lasting 5 years ($\Psi_5 = 0.9$).

In Figure 7 we report the life-cycle mean simulated stock profiles (panel a) and financial wealth profiles (panel b), for both the extended and the baseline model with human capital erosion. Our results confirm that a very high human capital erosion associated to very long lasting unemployment, though very rare, induces to self -insure through higher wealth accumulation and lowers optimal investing in stocks.

5.2 Correlation between labor income and stock returns

In this section, we consider the case with unemployment risk and human capital erosion and let the stock return innovations be positively correlated with the innovations in permanent labour income ($\rho_{sY} > 0$). According to the empirical evidence the estimates of this correlation for the U.S. differ widely. Cocco, Gomes and Maenhout (2005) report estimated values not significantly different from zero across various education groups, whereas Campbell, Cocco, Gomes and Maenhout (2001) and Campbell and Viceira (2002) find higher values, ranging from 0.33 for households with no high-school education to 0.52 for college graduates. However, Cocco, Gomes, and Maenhout (2005) provide estimates between -0.01 and 0.02, while Heaton and Lucas (2000) between -0.07 and 0.14. We adopt an intermediate positive value of $\rho_{sY} = 0.3$.

In Figure 8 we report the optimal portfolio shares of stocks (panel a) and of financial wealth (panel b) for both no correlation (dotted line) and positive correlation (solid line) between labor income shocks and stock returns. While the shape of life cycle profiles are relatively unaffected, the average stock share is lower at all ages. In case of positive correlation, labour income is closer to an implicit holding of stocks, reducing the incentive to invest in stocks at all ages. More specifically, investors are relatively more exposed to stock market risk and will find it optimal to offset such risk by holding a lower fraction of their financial portfolio in stocks if compared with the case of no correlation (see Bagliano et al. 2014). The stock share reaches a bottom level of about 40% for the average investor at around the age of 25, and remains substantially flat until retirement. At the age of 65 the human capital becomes riskless, since pension income is certain and therefore uncorrelated with stock return innovations. Thus investors rebalance their portfolio towards stocks: during retirement, the level and time profile of the stock share are very close to the case with no correlation. The relative increase in human capital risk due to positive correlation is reflected in higher financial wealth accumulation for precautionary motives (see panel b).

6 Conclusions

According to the Congressional Budget Office (2012), long term unemployment may produce a self-perpetuating cycle. With this background motivation, this paper uncovers the effects of unemployment traps on life-cycle savings and investment. Our analysis shows that even a small probability of experiencing human capital erosion due to long-term unemployment is able to generate optimal conditional stock shares more in line with those observed in the data. Due to the possibility of human capital loss, young workers face higher uncertainty concerning future income and social security pension levels than older ones. At the same time, young workers with continuous careers have larger human capital than older ones. When potential human capital erosion is considerable, conditional on a highly-unlikely long unemplyment spell, the first effect offsets the second one and the optimal investment in stocks is relatively flat over the life-cycle. This result departs from the implications of previous life cycle models that do not allow for unemployment traps.

Our calibrations suggest an alternative, and more balanced, design for target date investment funds. Of course, this paper only considers savings and asset allocation, together with unemployment benefits, as a partial hedge against shocks to human capital. There are instead other sources of human capital erosion, such as bad health (Hugonnier, Pelgrin and St-Amour, 2013) that may correlate with unemployment, as well other hedges (Low and Pistaferri, 2015). Future work may assess whether the flat asset allocation over the life cycle is robust to such additions.

References

[1] Arulampalam W, Booth AL and Taylor MP (2000) Unemployment persistence. Oxford Economic Papers 52(1): 24–50.

[2] Arulampalam W (2001) Is unemployment really scarring? Effects of unemployment experiences on wages. Economic Journal 111(475): F585–606.

[3] Bagliano F. C., C. Fugazza and G. Nicodano (2014), Optimal life-cycle portfolios for heterogeneous workers, Review of Finance, 18, 6, 2283-2323.

[4] Bodie Z., R.C. Merton and W. Samuelson (1992), Labour Supply Flexibility and Portfolio Choice in a Life Cycle Model, *Journal of Economic Dynamics and Control*, 16, 427-449.

[5] Bremus, F. M. and V. Kuzin (2014), Unemployment and portfolio choice: Does persistence matter?, *Journal of Macroeconomics*, vol. 40(C), pages 99-113.

[6] Campbell J. Y., J. Cocco, F. Gomes and P. Maenhout (2001), Investing Retirement Wealth: a Life-Cycle Model, in Campbell J.Y. and M. Feldstein (eds.), *Risk Aspects of Investment-Based Social Security Reform*, University of Chicago Press.

[7] Campbell J. Y. and L. Viceira (2002) *Strategic Asset Allocation: Portfolio Choice for Long-Tem Investors*, Oxford University Press

[8] Cocco, J. F., Gomes and P. Maenhout (2005), Consumption and Portfolio Choice over the Life Cycle, *Review of Financial Studies*, 18, 491-533.

[9] Congressional Budget Office (2012), Understanding and Responding to Persistently High Unemployment. http://www.cbo.gov/publication/42989.

[10] Cooper, E. (2013), The Effect of Unemployment Duration on Future Earnings and Other-Outcomes, Federal Reserve Bank of Boston Working Paper No. 13-8.

[11] Couch, K. A. and D. W. Placzek. 2010. "Earnings Losses of Displaced Workers Revisited." *American Economic Review*, 100(1): 572-89.

[12] Davis, S. J., and T. von Wachter (2011), Recessions and the Costs of Job Loss, *Brookings Papers on Economic Activity* 43, no. 2: 1–72.

[13] Edin, P. A., and M. Gustavsson. (2008), Time Out of Work and Skill Depreciation, *Industrial Labor Relations Review*, 61(2): 163-180.

[14] Gayad, R. and W. Dickens (2012),What Can We Learn by Disaggregating the Unemployment-Vacancy Relationship?, Federak Reserve Bank of Boston, 12-3.

[15] Gomes, F.J., L.J. Kotlikoff and L.M. Viceira (2008), Optimal Life-Cycle Investing with Flexible labour Supply: A Welfare Analysis of Life-Cycle Funds, *American Economic Review*, 98, 2, 297-303.

[16] Gomes, F.J. and A. Michaelides (2005), Optimal Life-Cycle Asset Allocation:Understanding the Empirical Evidence, *Journal of Finance*, 60, 869-904.

[17] Jacobson, L., R. LaLonde, and D. Sullivan (1993a), Earnings Losses of Displaced Workers, *American Economic Review*, 83(4): 685-709.

[18] Jacobson, Louis, Robert LaLonde, and Daniel Sullivan (1993b), The Costs of Worker Dislocation. Kalamazoo, MI: W. E. Upjohn Institute for Employment Research.

[19] Jacobson, Louis, Robert LaLonde, and Daniel Sullivan (2005a), Estimating the Returns to Community College Schooling for Displaced Workers. Journal of Econometrics, 125(1-2): 271–304.

[20] Heaton, J. and D. Lucas (2000), Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk. *Journal of Finance*, 55(3), 1163-1198.

[21] Hugonnier, J., F. Pelgrin and P. St-Amour. (2013) Health and (Other) Asset Holdings, *Review of Economic Studies,* 80(2), 663-710.

[22] Keane, M. P., and K. I. Wolpin (1997),The Career Decisions of Young Men, *Journal of Political Economy*, 105(3): 473–522.

[23] Kroft K., F. Lange, M.J. Notowidigdo and L. Katz (2016), Long Term Unemployment and the Great Recession: the Role of Composition, Duration Dependence, and Non Participation, Journal of Labour Economics, 34(S1), S7-S54.

[24] Krueger, A. B., J. Cramer and D. Cho. (2014), Are the Long-Term Unemployed on the Margins of the Labor Market?" BPEA, forthcoming.

[25] Low, H., and L. Pistaferri. (2015), Disability Insurance and the Dynamics of the Incentive Insurace Trade-Off, *American Economic Review,* 105(10), 2986-3029

[26] Machin, S. and A.Manning (1999), The Causes and Consequences of Long-Term Unemployment in Europe, in Handbook of Labour Economics, 3, Part C, 3085–3139; O. Ashenfelter & D. Card (ed.).

[27] Michaelides, A. and y. Zhang (2005), Stock Market Mean Reversion and Portfolio Choice over the Life Cycle, *Journal of Financial Quantitative Analysis*, forthcoming.

[28] Neal, D. (1995), Industry-Specific Human Capital: Evidence from Displaced Workers, *Journal of Labor Economics*, 13(4): 653-677.

[29] Ruhm, C. J. (1991), Are Workers Permanently Scarred by Job Displacements?, *American Economic Review*, 81(1): 319–24.

[30] Schmieder, J. F., T. von Wachter, and S. Bender (2013), The Causal Effect of Unemployment Duration on Wages: Evidence from Unemployment Insurance Extensions, Working Paper.

[31] Viceira, L.M. (2009), Life-Cycle Funds, in A. Lusardi ed., *Overcoming the Saving Slump: How to Increase the Effectiveness of Financial Education and Saving Programs*, University of Chicago Press.

7 Appendix 1

In the appendix, we extend the model to allow for possible maxiumum unemployment duration of 5 years. In particular, working life careers are modelled as a six-state Markov chain considering employment (e), short-term (u_1) and long-term unemployment ($u_2, ...u_5$). Unemployment may be short-term and last only one year or it may become long term and last from two to five years. Individual

labor market dynamics are driven by the following transition matrix:

$$\Pi_{s_t,s_{t+1}} = \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & \pi_{eu_2} & \pi_{eu_3} & \pi_{eu_4} & \pi_{eu_5} \\ \pi_{u_1e} & \pi_{u_1u_1} & \pi_{u_1u_2} & \pi_{u_1u_3} & \pi_{u_1u_4} & \pi_{u_1u_5} \\ \pi_{u_2e} & \pi_{u_2u_1} & \pi_{u_2u_2} & \pi_{u_2u_3} & \pi_{u_2u4} & \pi_{u_2u_5} \\ \pi_{u_3e} & \pi_{u_3u_1} & \pi_{u_3u_2} & \pi_{u_3u_3} & \pi_{u_3u_4} & \pi_{u_3u_6} \\ \pi_{u_4e} & \pi_{u_4u_1} & \pi_{u_4u_2} & \pi_{u_4u_3} & \pi_{u_4u_4} & \pi_{u_4u_6} \\ \pi_{u_5e} & \pi_{5u_1} & \pi_{u_5u_2} & \pi_{u_5u_3} & \pi_{u_5u_4} & \pi_{u_5u_6} \end{pmatrix} = \tag{19}$$

$$= \begin{pmatrix} \pi_{ee} & \pi_{eu_1} & 0 & 0 & 0 & 0 \\ \pi_{u_1e} & 0 & (1-\pi_{u_1e}) & 0 & 0 & 0 \\ \pi_{u_2e} & 0 & 0 & (1-\pi_{u_2e}) & 0 & 0 \\ \pi_{u_3e} & 0 & 0 & 0 & (1-\pi_{u_3e}) & 0 \\ \pi_{u_4e} & 0 & 0 & 0 & 0 & (1-\pi_{u_4e}) \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where $\pi_{ij} = \text{Prob}\,(s_{t+1} = j|s_t = i)$ with $i, j = e, u_1, ..., u_5$. If the worker is employed at t ($s_t = e$), she continues the employment spell at $t+1$ ($s_{t+1} = e$) with probability π_{ee}, otherwise she enters short-term unemployment ($s_{t+1} = u_1$) with probability π_{eu_1}. Since to become long-term unemployed she must first experience short-term unemployment, we set the probability for the employed to enter long-term unemployment at zero, $\pi_{eu_2} = ... = \pi_{eu_5} = 0$). If the worker is short-term unemployed at t ($s_t = u_1$), then she exits unemployment ($s_{t+1} = e$) with probability π_{u_1e} or she becomes long-term unemployed ($s_{t+1} = u_2$) with probability $\pi_{u_1u_2}$; consequently we set $\pi_{u_1u_1} = 0$. Similarly, for all other possible unemployment durations, π_{u_ie} is the probability of being re-emplyed and $\pi_{u_iu_j}$ is the probability of becoming unemployed for the j^{th} year, conditional on being a i year unemployed, with $\pi_{u_iu_j} = 0$ if $j \neq i+1$. Finally, since long-term unemployment lasts five years, she is re-employed ($s_{t+1} = e$) with certainty, thus $\pi_{u_2e} = 1$.

Labor income follows the process as in Section 2.

As in Section 2, for the worker who at t is re-employed after a $i-$year unemployment spell the permanent component H_t is equal to H_{t-1} eroded by a fraction Ψ_i, and in the short-term unemployment state ($s_t = u_1$) individuals receive an unemployment benefit as a fixed proportion ξ_1 of the previous year permanent income, whereas in the long-term unemployment states ($s_t = u_i$, with $i > 1$) no benefits are available: $\xi_i = 0$.

Then the model is as presented in Section 2. Here we just report the value function in each possible labor market states.

$$V(X_t,P_t,e)=u(C_t)+\beta p_t\begin{cases}\begin{cases}V(X_{t+1},P_{t+1},e) & \text{with prob. } \pi_{e,e}\\ \text{with } P_{t+1}=P_te^{\omega_{it+1}} & \text{and}\\ X_{t+1}=(X_t-C_t)R_t^p+F_{t+1}P_{t+1}e^{\varepsilon_{t+1}}\end{cases}\\ \\ \begin{cases}V(X_{t+1},P_{t+1},u_1) & \text{with prob. } \pi_{e,u_1}\\ \text{with } P_{t+1}=(1-\Psi_1)P_t & \text{and}\\ X_{t+1}=(X_t-C_t)R_t^p+\xi_1F_tP_t\end{cases}\end{cases}$$

$$V(X_t,P_t,u_i)=u(C_t)+\beta p_t\begin{cases}\begin{cases}V(X_{t+1},P_{t+1},e) & \text{with prob. } \pi_{u_i,e}\\ \text{with } P_{t+1}=(1-\Psi_i)P_{t-i}\,e^{\omega_{t+1}} & \text{and}\\ X_{t+1}=(X_t-C_t)R_t^p+F_{t-i}P_{t+1}e^{\varepsilon_{t+1}}\end{cases}\\ \\ \begin{cases}V(X_{t+1},P_{t+1},u_j) & \text{with prob. } \pi_{u_i,u_j}\\ \text{with } P_{t+1}=\prod_{l=1}^{i}(1-\Psi_l)P_{t-i} & \text{and}\\ X_{t+1}=(X_t-C_t)R_t^p\end{cases} \quad \text{with } i,j=1,...,5 \text{ and } j>i\end{cases}$$

$$V(X_t,P_t,u_5)=u(X_t)+\beta p_t\begin{cases}V(X_{t+1},P_{t+1},e) & \text{with prob. } \pi_{u_7,e}\\ \text{with } P_{t+1}=\prod_{l=1}^{5}(1-\Psi_l)P_{t-5}e^{\omega_{t+1}} & \text{and}\\ X_{t+1}=(X_t-C_t)R_t^p+F_{t-5}P_{t+1}e^{\varepsilon_{t+1}}\end{cases} \tag{20}$$

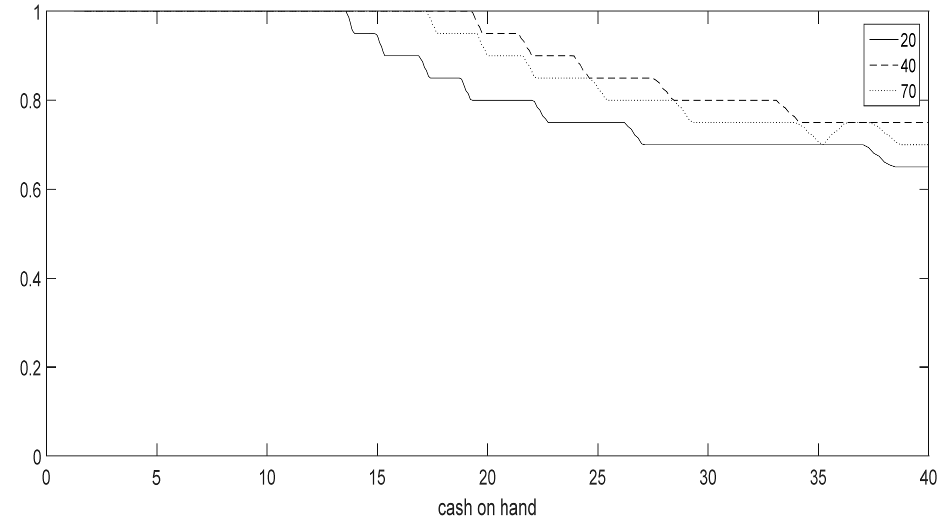
The model is solved using standard numerical techniques as outlined in Section 2.

Table 1 Calibration parameters

Working life (max)	20 -65
Retirement (max)	65 -100
Discount factor (β)	0.96
Risk aversion (γ)	5
Replacement ratio (λ)	0.68
Variance of permanent shocks to labour income (σ^2_ε)	0.0106
Variance of transitory shocks to labour income (σ^2_n)	0.0738
Maximum unemployment duration (years)	2 (5)
Unemployment benefit	
Frist year unemployed (ξ_1)	0.3
Second year unemployed (ξ_2)	0
Human capital erosion	
Frist year unemployed (Ψ_1)	0 (0.2)
Second year unemployed (Ψ_2)	0 (0.9)
Riskless rate	2%
Excess returns on stocks (μ^s)	4%
Variance of stock returns innovations (σ^2_s)	0.025
Stock ret./permanent lab. Income shock correlation (ρ_{sY})	0 (0.3)

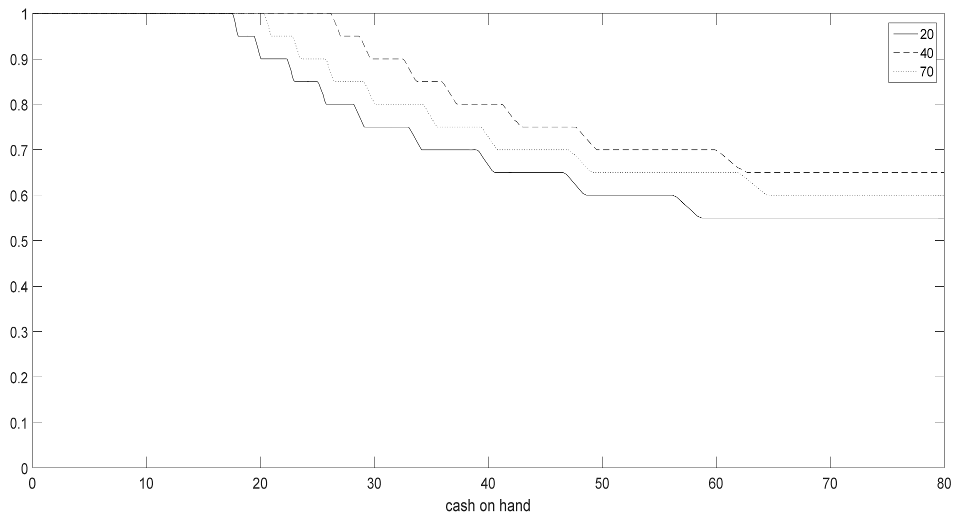
The table reports the benchmark values of relevant parameters and, in parenthesis, alternative values we experiment with in the paper.

Figure 1 Policy functions: baseline case without unemployment risk



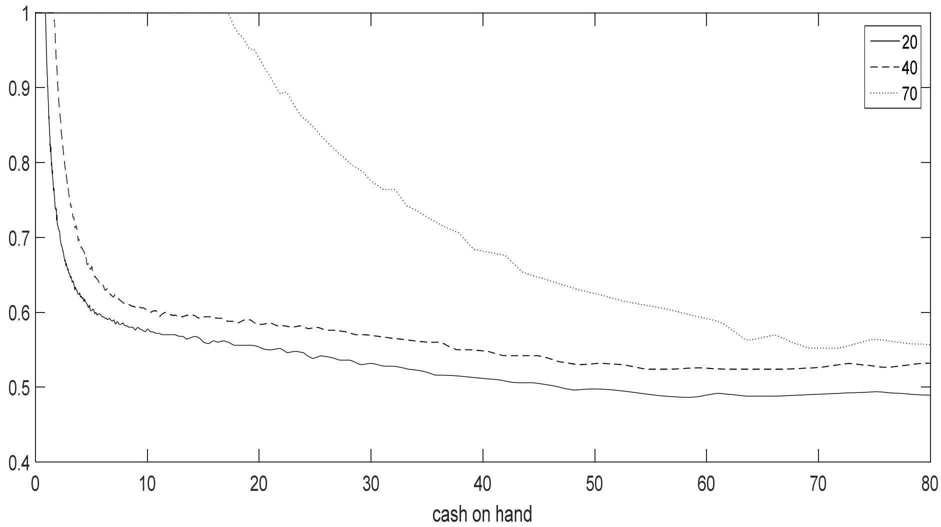
The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in the baseline case, without unemployment risk. The policies are plotted for selected ages 20, 40, and 70.

Figure 2 Policy function: with unemployment risk, no human capital depreciation



The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in case of unemployment risk. Risk aversion γ=5, social security replacement ratio λ=0.68. The parameters governing the human capital erosion during short-term and long-term unemployment spells are equal to 0 (Ψ_1, Ψ_2). The policies are plotted for selected ages: 20, 40, and 70.

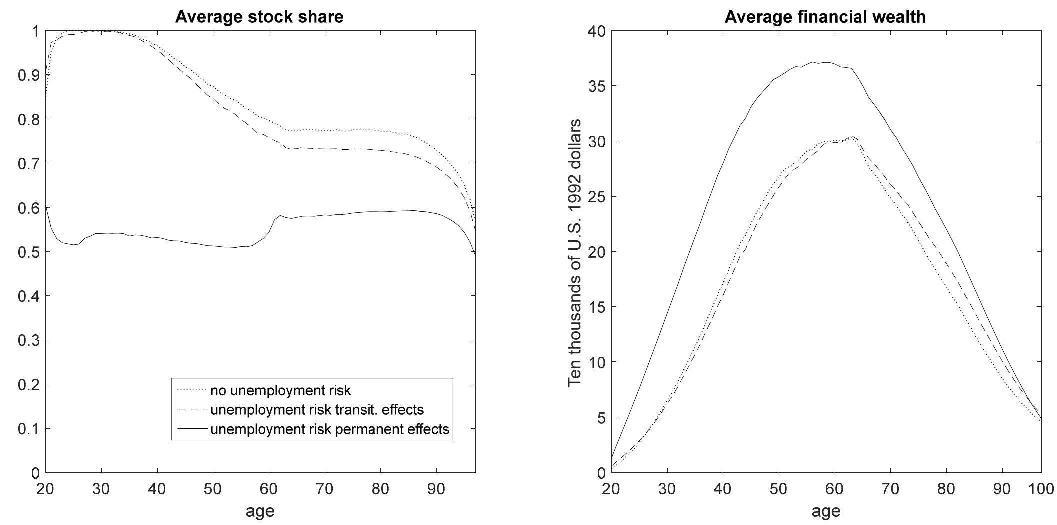
Figure 3 Policy functions: with unemployment risk, human capital depreciation



The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in case of unemployment risk. Risk aversion γ=5, social security replacement ratio λ=0.68. The parameters governing the human capital erosion during short-term and long-term unemployment spells are equal to 0.2 (Ψ_1) and 0.9 (Ψ_2), respectively. The policies are plotted for selected ages: 20, 40, and 70.

Figure 4 Life cycle average profiles

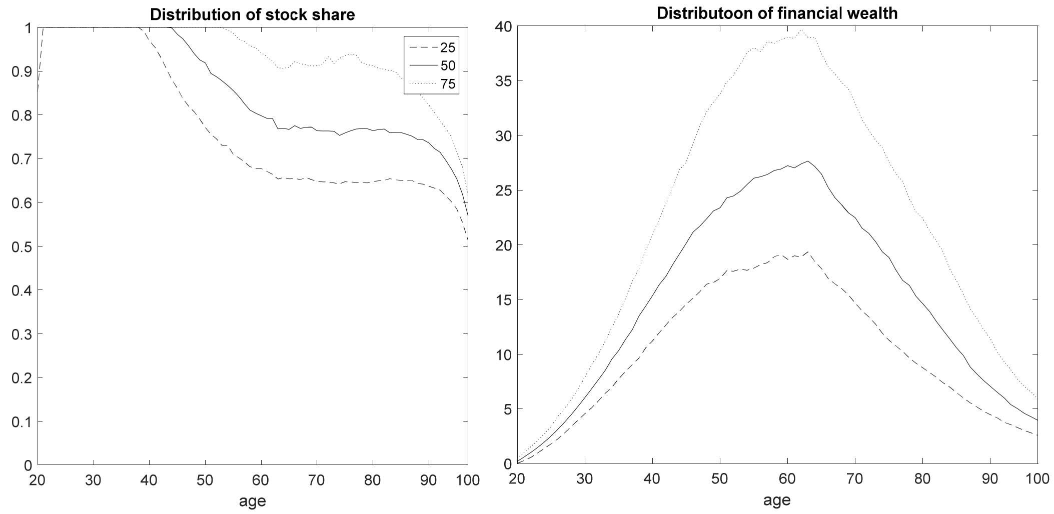
(a) **(b)**



The figure displays the mean simulated stock profiles (a) and accumulated financial wealth (b) for individuals of age 20 to 100. Risk aversion γ=5, social security replacement ratio λ=0.68. Three cases are considered*:* the model without unemployment risk; the model with unemployment risk and no human capital erosion, (Ψ_1=Ψ_2=0); the model with unemployment risk and human capital erosion, (Ψ_1=0.2) and (Ψ_2=0.9).

Figure 5 Life cycle percentile profiles – No unemployment risk

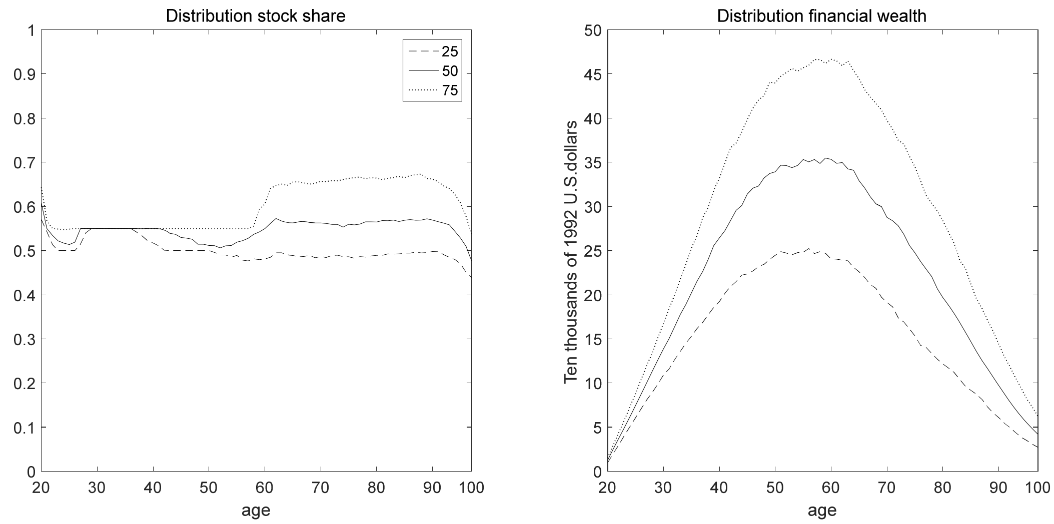
(a) **(b)**



The figure displays the distribution of simulated stock profiles (a) and accumulated financial wealth (b) for individuals of age 20 to 100. Risk aversion γ=5, social security replacement ratio λ=0.68.

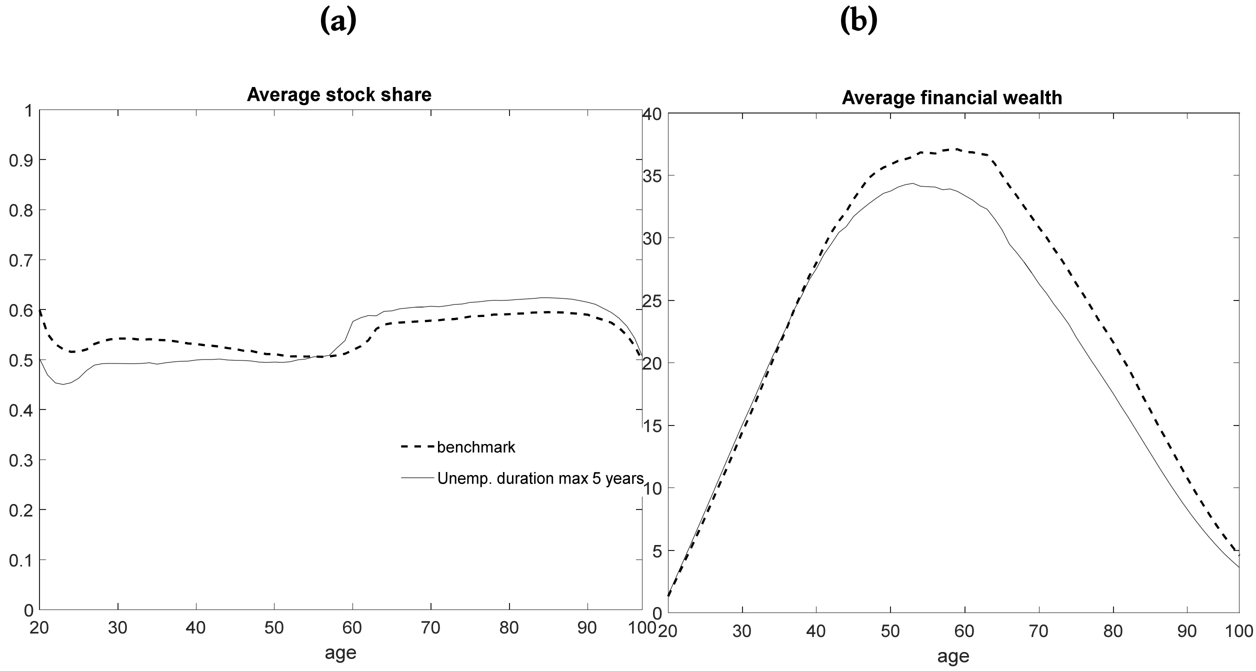
Figure 6 Life cycle percentile profiles – Unemployment risk with human capital erosion

(a) **(b)**



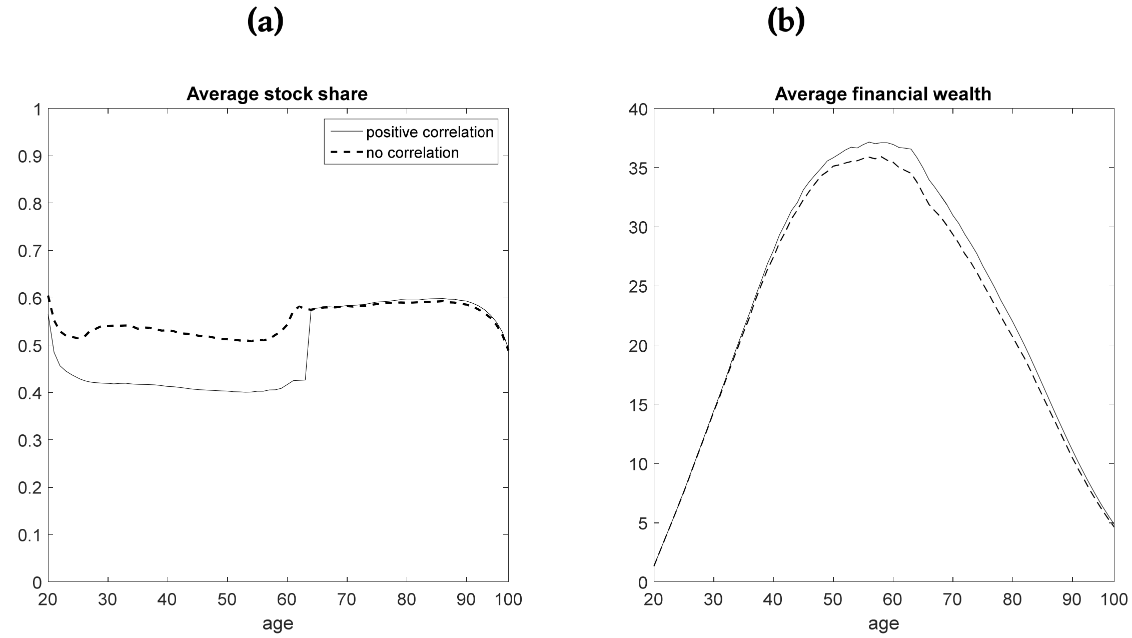
The figure displays the distribution of simulated stock profiles (a) and accumulated financial wealth (b) for individuals of age 20 to 100. Risk aversion γ=5, social security replacement ratio λ=0.68; human capital erosion: Ψ_1=0.2 and Ψ_2=0.9.

Figure 7 Life cycle average profiles – Calibrating U.S. transitions



The figure displays the simulated mean stock profiles (a) and mean accumulated financial wealth (b) for individuals of age 20 to 100. Risk aversion γ=5, social security replacement ratio λ=0.68. Maximum duration of unemployment is 5 years. Transition probabilities among labor market states are calibrated against U.S. data to match the annual unemployment rates at all durations. Human capital erosion is constant at all durations lower than five years (Ψ_i=0.2 with i=1,…,4) and is 90% in case of unemployment lasting 5 years (Ψ_5=0.9). *Benchmark* is the model with three labor market states and human capital erosion.

Figure 8 Life cycle profiles with unemployment risk: positive correlation between labor income and stock returns



The figure displays the distribution of simulated stock profiles (a) and accumulated financial wealth (b) for individuals of age 20 to 100. Risk aversion γ=5, social security replacement ratio λ=0.68; human capital erosion: Ψ_1=0.2 and Ψ_2=0.9. Positive correlation between labor income shocks and innovation to stock returns ρ_{sY}=0.3.