Countercyclical Labor Income Risk and Portfolio Choices over the Life-Cycle

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Abstract

I structurally estimate a life-cycle model of portfolio choices that incorporates the relationship between stock market returns and the cross-sectional skewness of idiosyncratic labor income shocks. The cyclicality of this skewness can explain *(i)* why young households with modest financial wealth do not participate in the stock market and *(ii)* why, conditional on participation, the share of financial wealth invested in equity slightly increases with age until retirement. With an estimated relative risk aversion of 5 and yearly participation cost of $290, the model matches the evolution of wealth, of stock market participation and of the equity share of participants over the life-cycle. Without cyclical skewness, the same model requires a risk aversion above 10 or a participation cost above $1,000 to match the average equity share and cannot explain its decline over the life-cycle. Nonetheless, cyclical skewness reduces the aggregate demand for equity by at most 15%.

Keywords: Household finance, Labor income risk, Portfolio choices, Human capital, Life-cycle model, Simulated method of moments

JEL codes: G11, G12, D14, D91, J24, H06

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1 Introduction

Following Viceira (2001) and Cocco et al. (2005), a large literature studies how human capital risk affects the optimal demand for stocks over the life-cycle. Because the correlation between stock market returns and labor income shocks is close to zero, most papers conclude that human capital is a substitute for bonds and should increase the demand for equity. This conclusion creates two problems. First, as the share of human capital in total wealth declines over the life-cycle, so should the share of financial wealth invested in equity (Jagannathan and R. Kocherlakota (1996)).[1] This prediction is not supported by the data. On the contrary, the stock market participation rate of US households rises with age until retirement. Moreover, conditional on participation, the share of financial wealth they invest in equity does not decrease before retirement (Bertaut and Starr-McCluer (2002), Ameriks and Zeldes (2004)). Second, when human capital increases the demand for stocks, life-cycle models need unrealistically high levels of risk aversion to match the average equity share of working households.

These discrepancies are evidence that households make significant investment mistakes or that economists do not understand the nature of their background risk. In particular, life-cycle models ignore recent findings by Guvenen et al. (2014) that the cross-sectional skewness of labor income shocks is cyclical. Cyclical skewness implies that the worst human capital shocks happen during recessions, which themselves tend to coincide with financial crashes. As a consequence, workers investing in the stock market take the risk of losing their job and a large fraction of their savings at the same time. To illustrate this intuition, Figure 1 plots the evolution of the cross-sectional skewness of earnings growth among the US male population and that of S&P500 returns between 1978 and 2010 and shows that these two series move together.

[Insert Figure 1 about here]

In this paper, I show that introducing cyclical skewness in a standard life-cycle model of portfolio choices helps reconcile its predictions with the data and leads to more plausible estimates of risk aversion. To do so, I follow the dominant approach in the literature by building a life-cycle model in which a worker with constant relative risk aversion (CRRA) invests his wealth in a riskfree

[1] Beside these authors, Viceira (2001), Cocco et al. (2005), Campbell et al. (2001), Cocco (2005), Gomes and Michaelides (2005), Gomes et al. (2008), and Chai et al. (2011) reach the same conclusion.

asset or the stock market portfolio. The main innovation of the model is that it incorporates the relationship between stock market returns and the cross-sectional skewness of idiosyncratic labor income shocks. The model is estimated in two steps using the Simulated Method of Moments (SMM). In the first step, I estimate the joint dynamics of labor income and stock market returns using US Social Security panel data from 1978 and 2010. In a second step, I estimate the coefficient of relative risk aversion, the discount factor and a yearly stock market participation cost. These parameters are identified by the evolution of wealth, participation and equity holdings over the life-cycle in the US Survey of Consumer Finances (SCF).

My main findings are as follows. First, my model matches the data well with a relative risk aversion of $\gamma = 5$ and a yearly participation cost of only $290. Without cyclical model, the same model requires a level of relative risk aversion above 10 or a participation cost above $1,000 to match the average equity share and cannot explain its decline over the life-cycle. Second, comparative statics show that cyclical skewness has a strong effect on households with modest financial wealth. In particular, cyclical skewness causes the optimal equity share of young workers to practically drop from roughly 100% to close to 0%. Third, cyclical skewness has little consequences for households with high financial-wealth-to-wage ratios. As a consequence, cyclical skewness reduces the aggregate demand for equity by only 15% and increases the equity premium by at most half a percentage point, which contradicts previous findings by Constantinides and Ghosh (2017).

The first step of my quantitative exercise is to estimate the dynamics of labor income and yearly stock market returns. Idiosyncratic income shocks are modeled and estimated as in Guvenen et al. (2014) with one key difference. Instead of taking two different values in recessions and expansions, skewness is a continuous function of the average labor income shock, which itself correlates with stock market returns. The labor income process is estimated by targeting the cross-sectional mean, variance and skewness of idiosyncratic earnings growth in US Social Security data for each year from 1978 to 2010. Importantly, each of these moments are targeted for earnings growth over 1, 3 and 5 year periods. This allows me to disentangle the dynamics of transitory and persistent shocks and to estimate the persistence of the latter. In line with Guvenen et al. (2014), I find tail shocks to be quite persistent, and therefore more likely to have important portfolio implications.

In a second step, I incorporate this labor income process into a standard life-cycle model of portfolio choices. The model is similar to Cocco et al. (2005) but offers a more realistic computation

of Social Security pension benefits and takes into account the social safety net and stock market crashes.

I start the structural estimation by assuming no participation cost. The agent's discount factor and relative risk aversion are identified by the evolutions of wealth and unconditional equity shares over the life-cycle. The model with cyclical skewness closely matches these moments with a discount factor of $\beta = 0.95$ and a relative risk aversion of $\gamma = 7$. By contrast, the model without cyclical skewness cannot match the data and generates a very high estimate of γ (10.8) and a very low estimate of β (0.79), in line with previous results by Fagereng et al. (2017).[2]

In a second set of estimations, I introduce a fixed yearly participation cost which I identify using the evolution of stock market participation rates. For the model with cyclical skewness, I find an estimated risk aversion of $\gamma = 5.5$ and a yearly participation cost of \$290. This cost is enough to prevent the participation of young workers with limited financial wealth because their optimal equity share would be low in the absence of participation costs. By contrast, without cyclical skewness, young households would have very high optimal equity shares. For this reason, matching their low participation rate requires a much higher participation cost of \$1,010. Though it can match the evolution of participation rates, the model without cyclical skewness keeps generating a negative relationship between age and the equity share of participants.

Using this set of parameter estimates, I run comparative statics to show that cyclical skewness strongly reduces workers' optimal equity share. For households with modest financial-to-human wealth ratios, cyclical skewness reduces the optimal equity share from roughly 100% to close to 0%. Even absent any participation cost, most households with less than a year of salaries in wealth barely hold any stocks. Moreover, for relative risk aversions as low as 5, cyclical skewness reverses the predicted relationship between age and the equity share. I also find that, while the fear of stock market crashes only reduces the equity share of young households by a few percentage points when labor income shocks are normally distributed, its effect is much stronger in the presence of cyclical skewness. Tail events on the stock market and in individuals' careers matter more when they tend to coincide.

To assess the model's validity, I also evaluate its prediction regarding the relationship between

[2] Fagereng et al. (2017) estimate a similar model on Norwegian data and find $\gamma = 11$ and $\beta = 0.77$ with a participation cost of only \$69.

the equity share and the wealth-to-wage ratio within age groups. I find this relationship to be relatively flat in the data and in the model with cyclical skewness but to be strongly negative in the model without cyclical skewness. Moreover, holding age and human capital fixed, the optimal equity share in the presence of cyclical skewness is increasing and concave in financial wealth, a pattern documented in Calvet and Sodini (2014)'s study of Swedish twins and which these authors interpret as evidence of decreasing relative risk aversion.

Finally, I run counterfactual experiments to quantify the effect of cyclical skewness on the aggregate demand for stocks and the equity premium. First, I start by removing cyclical skewness from the model and shows that this increases the aggregate demand for equity by 9 to 15%. Then, I solve for the change in the equity premium that exactly offsets this increase in demand and find that a drop of 0.5 percentage points in the equity premium is enough to get the aggregate demand for stocks back to its initial level. The modest magnitude of my results contrasts with findings by Constantinides and Ghosh (2017) who argue that the cyclical skewness in *consumption* shocks can explain the equity premium. These authors assume that all households face the same consumption risk. I argue that this assumption is incorrect. Indeed, I show that if all households face the same labor income risk, the negative skewness of consumption shocks during recessions is driven by households with low financial wealth experiencing large negative labor income shocks. On the other hand, workers with substantial financial wealth do not need to reduce their consumption drastically when they receive the same labor income shocks. As a consequence, the cyclical skewness of consumption risk documented by Constantinides and Ghosh (2017) is unlikely to be representative of the risk faced by the average dollar-weighted investor.

Related literature My paper contributes to the literature on portfolio choices over the life-cycle. We know from early studies that absent any background risk and labor income, the optimal equity share of a CRRA investor does not vary with age (Merton (1969), Samuelson (1969)) and that in the presence of risk-free human capital, households should move away from stocks as they get closer to retirement (Merton (1971)). Even if human capital is risky, Viceira (2001) shows that it increases the demand for equity as long as labor income risk is uncorrelated with stock market returns. Cocco et al. (2005) document that the correlation between labor income shocks and returns is close to zero and find that, in a calibrated life-cycle model, young households

with very high risk aversion invest all their wealth in stocks. Fagereng et al. (2017) structurally estimate a similar model with stock market disasters using Norwegian data and find a relative risk aversion between 11 and 15 and a discount factor between 0.75 and 0.83. Relative to these papers, my contribution is to improve the model's ability to match the data and generate more realistic estimates of preference parameters, despite much more optimistic assumptions regarding the distribution of stock market returns.[3]

To do so, my paper follows the intuition of Storesletten et al. (2007) and Lynch and Tan (2011) that labor income risk is countercyclical. Relying on findings by Storesletten et al. (2004), these authors assume that the variance of labor income shocks is higher during recessions and show that this can generate a positive relationship between the optimal equity share and age. However, an important issue with these papers is that US administrative data show no evidence that variance is countercyclical (Guvenen et al. (2014)). Another closely related study is Benzoni et al. (2007) who document that aggregate wages and dividends are cointegrated and show that this cointegration may also generate a positive relationship between age and the optimal equity share when the volatility of dividend growth is set to match that of stock market returns. However, de Jong (2012) shows that this finding is not robust when stock returns are affected by other factors than dividend growth, such as time-varying discount rates. Overall, my paper confirms that labor income risk can generate a positive relationship between age and the optimal equity share, but, by contrast to previous studies, relies on well documented assumptions.

My paper also re-examines Mankiw (1986)'s idea that the concentration of aggregate shocks among a few individuals matters for asset pricing. Storesletten et al. (2007) and Constantinides and Ghosh (2017) respectively study the effects of countercyclical variance in labor income risk and cyclical skewness in consumption risk. My contribution to this literature is to show that, unless wealthy households face greater countercyclical labor income risk, the latter can only explain a limited fraction of the equity premium.

My paper also contributes to the literature on stock market participation. Vissing-Jorgensen (2002) argues that a yearly participation cost of $260 can explain the decision of 75% of non

[3]The equity premiums in Fagereng et al. (2017) and my paper are respectively 0.03 and 0.05, while the standard deviations of returns are 0.24 and 0.16. These numbers imply that, in Merton's model, I would need a relative risk aversion 2.5 times larger to match the same equity share.

participants. However, Andersen and Nielsen (2011) and Briggs et al. (2015) show that windfall wealth has limited effects on participation, which is not consistent with limited participation being caused by fixed costs. In my model, matching the rise of participation over the life-cycle requires a much higher cost when cyclical skewness is ignored. When it is taken into account, estimated costs are relatively low and cannot fully explain participation levels, leaving more space for alternative theories, which is more consistent with Andersen and Nielsen (2011) and Briggs et al. (2015).

2 Model

This section describes a discrete time life-cycle model of consumption and portfolio choices. The main novelty of this model is that the distribution of idiosyncratic labor income shocks tends to have negative (positive) skewness in years of bad (good) stock market returns.

2.1 Macroeconomic environment

Stock market returns The log return of the stock market index on year t is

$$s_t = s_{1,t} + s_{2,t} \tag{1}$$

where s_1 denotes the component of stock returns that covaries with labor market conditions. To take into account stock market crashes, I assume that s_1 follows a normal mixture distribution:

$$s_{1,t} = \begin{cases} s^-_{1,t} \sim \mathcal{N}(\mu^-_s, \sigma^2_{s_1}) & \text{with probability} \quad p_s \\ s^+_{1,t} \sim \mathcal{N}(\mu^+_s, \sigma^2_{s_1}) & \text{with probability} \quad 1 - p_s \end{cases} \tag{2}$$

whereas $s_{2,t}$ is normally distributed with variance $\sigma^2_{s_2}$. Without loss of generality, I impose $\mu^-_s < \mu^+_s$ and interpret μ^-_s as the expected log return in years of stock market crashes, and p_s the frequency of such events. On the other hand, μ^+_s is the expected log return during normal years.

Aggregate labor income shocks Stock market returns are correlated with the growth of the log national wage index l. Specifically, the dynamics of l is

$$l_t - l_{t-1} = \mu_l + \lambda_{ls} s_{1,t} + \varepsilon_t \tag{3}$$

where ε_t follows $\mathcal{N}(0, \sigma_l^2)$, μ_l is the average growth rate of wages and λ_{ls} captures the relationship between stock market returns and the growth rate of the wage index.

2.2 Idiosyncratic labor income shocks

The agent's log annual salary is the sum of the log national wage index l_t and an idiosyncratic component y_{it}. The latter is split into a deterministic life-cycle component f_{it}, a persistent component z_{it} and a transitory shock η_{it}. The specific form of f_{it} is discussed latter.

$$y_{it} = f_{it} + z_{it} + \eta_{it} \tag{4}$$

2.2.1 Persistent income shocks

The behavior of the persistent component differs from the traditional AR(1) process to the extent that innovations follow a normal mixture. Specifically, the dynamics of z_i is

$$z_{it} = \rho_z z_{it-1} + \zeta_{it} \tag{5}$$

where:

$$\zeta_{it} = \begin{cases} \zeta_{it}^- \sim \mathcal{N}\left(\mu_{z,t}^-, \sigma_z^{-2}\right) & \text{with probability} \quad p_z \\ \zeta_{it}^+ \sim \mathcal{N}\left(\mu_{z,t}^+, \sigma_z^{+2}\right) & \text{with probability} \quad 1 - p_z \end{cases} \tag{6}$$

The values of p_z, $\mu_{z,t}^-$ and $\mu_{z,t}^+$ control the degree of asymmetry in the distribution of income shocks. To replicate the cyclicality of skewness, I define $\mu_{z,t}^-$ as an affine function of the log growth rate of the wage index:

$$\mu_{z,t}^- = \overline{\mu_z^-} + \lambda_{zl}(l_t - l_{t-1}) \tag{7}$$

Since idiosyncratic shocks have an expectation of zero, I impose

$$p_z \mu_{z,t}^- + (1 - p_z)\mu_{z,t}^+ = 0 \tag{8}$$

and, without loss of generality, $p_z \leq 0.5$. In this case, and if σ_z^- is large, p_z can be interpreted as the frequency of tail events, which tend to be promotions during expansions and layoffs in recessions.

2.2.2 Transitory income shocks

The transitory component of income is also modeled as a mixture of normals whose first and second components always coincide with the first and second components of the normal mixture governing the innovations to z_i.

$$\eta_{it} = \begin{cases} \eta_{it}^- \sim \mathcal{N}(0, {\sigma_\eta^-}^2) & \text{if} \quad \zeta_{it} = \zeta_{it}^- \\ \eta_{it}^+ \sim \mathcal{N}(0, {\sigma_\eta^+}^2) & \text{if} \quad \zeta_{it} = \zeta_{it}^+ \end{cases} \tag{9}$$

2.3 Pensions and safety net

2.3.1 Social Security

In the US, wages are subject to a Social Security payroll tax of $\tau = 12.4\%$ up to a limit known as the maximum taxable earnings and close to 2.5 times the national wage index[4]. Pensioners receive a percentage of the value of the national wage index when they retired. This percentage depends on the agent's historical taxable earnings, adjusted for the growth in the national wage index. Specifically, the initial pension P_i of agent i is defined by:

[4] In 2014, the value of the Social Security wage index was \$46,481 and the maximum amount of taxable earnings was \$117,000

$$\frac{P_i}{L_R} = \begin{cases} 0.9 \times S_{iR} & \text{if } S_{iR} < 0.2 \\ 0.116 + 0.32 \times S_{iR} & \text{if } 0.2 \leq S_{iR} < 1 \\ 0.286 + 0.15 \times S_{iR} & \text{if } 1 \leq S_{iR}. \end{cases} \tag{10}$$

where $L_R = e^{l_R}$ is the value of the wage in the first retirement year and S_{iR} is the agent's average historical taxable earnings. Specifically,

$$S_{it} = \sum_{k=t_0}^{t} \frac{\max\{Y_{ik}, 2.5\}}{t - t_0 + 1} \tag{11}$$

where $Y_{ik} = e^{y_{ik}}$. Because Social Security uses a bend point system, returns on investment are higher for individuals with low earnings records.

To avoid keeping track of historical earnings, many papers in the life-cycle literature model Social Security benefits as a percentage of the agent's last persistent/permanent income, a methodology that overestimates Social Security risk. In my model, a large negative shock close to retirement would have dramatic consequences on the value of Social Security entitlements, which is not the case in reality. To solve this problem, I keep track of the agent's average historical earnings.

2.3.2 Social safety net

Welfare programs limit the impact of human capital disasters on consumption, and thus may also alleviate the portfolio consequences of cyclical skewness. To take this into account, I incorporate into the model the Supplemental Nutrition Assistance Program (also known as the food stamp program) and, for individuals above 65 years, the Supplemental Security Income (SSI) program. Eligibility to these programs is limited to individuals with very low financial wealth (roughly \$2,000), which I model as less than 5% of the average national wage. After 65 years old, eligible individuals receive supplemental income such that their total income reaches at least 20% of the national average wage. Before 65 years old, eligible individuals with earnings below 20% of the national average wage receive benefits equal to that threshold minus 30% of their earnings. Hence, welfare benefits are defined by:

$$\frac{B_{it}}{L_{it}} = \begin{cases} \max\left\{0.2 - \frac{P_i}{L_{it}}, 0\right\} & \text{if } \frac{W_{it}}{L_{it}} < 0.05 \text{ and } t \geq R \\ \max\{0.2 - 0.3Y_i, 0\} & \text{if } \frac{W_{it}}{L_{it}} < 0.05,\ Y_i < 0.2 \text{ and } t < R \end{cases} \tag{12}$$

2.4 Household

I incorporate the stochastic model in the life-cycle problem of an agent controlling his consumption C and equity share π.

Preferences The agent has CRRA preferences and maximizes his expected utility, given by

$$V_{t_0} = \mathrm{E} \sum_{t=t_0}^{T} \beta^{t-1} \left(\prod_{k=0}^{t-1} (1 - m_k) \right) \frac{C_{it}^{1-\gamma}}{1-\gamma} \tag{13}$$

where γ is his coefficient of relative risk aversion, m_k the mortality rate at age k, β the discount factor and T the maximum lifespan.

Wealth dynamics Each year, he receives an income I_{it} that includes his net wage, pension and welfare benefits, that is:

$$I_{it} = (Y_{it} - \tau \max\{Y_{it}, 2.5\}) L_t + B_{it} + P_{it} \tag{14}$$

He decides how much to consume, and then invests his savings in bonds or in the stock market index. Short-selling and borrowing are not allowed. Holding equity incurs a fixed participation cost $c_{\pi,1}$ and a variable management fee $c_{\pi,2}$. I assume that $c_{\pi,1}$ and $c_{\pi,2}$ are respectively percentages of the wage index and assets under management. Noting π the equity share and r the risk free rate, the agent's wealth dynamics is

$$W_{it+1} = [W_{it} + I_{it} - C_{it}] \left[\pi_{it} e^{s_t - c_{\pi,2}} + (1 - \pi_{it}) e^{r}\right] - \mathbf{1}_{\pi_{it}>0} c_{\pi,1} L_t \tag{15}$$

Dynamic optimization The problem is solved by dynamic programming. Besides age, the state variables of the problem are the components of labor income l, z_i and η_i, Social Security record S_i and financial wealth W_i. Dropping indexes to simplify the notation, the Bellman equation

is

$$V_t(w,l,z,\eta,S) = \max_{[C_t,\pi]} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} + (1-m_t)\,\beta \mathrm{E} V_{t+1} \right\} \tag{16}$$

and the terminal value is the period utility derived from entirely consuming pension, benefits and financial wealth.

$$V_T(w,l,S) = \frac{(W+P+B)^{1-\gamma}}{1-\gamma} \tag{17}$$

Homothety of the value function Since the agent has CRRA, and because all benefits and participation costs are proportional to L_t, the dimensionality of the problem can be reduced by scaling financial wealth, wages and pension benefits by the national wage index. Noting $x = w - l$ the log of the financial wealth to wage index ratio, we have

$$V_t(w,l,z,\eta,S) = e^{(1-\gamma)l} V_t(x,0,z,\eta,S). \tag{18}$$

3 Estimation

The model is estimated in two steps. In the first step, I use moments derived from Social Security panel data and S&P500 data to estimate the parameters controlling the dynamics of wages and stock market returns. In a second step, I use data on the portfolio of US households to estimate their preferences and a fixed yearly stock market participation cost.

3.1 Data

The estimation uses two US datasets: the Social Security Master Earnings File (MEF) and the Survey of Consumer Finances (SCF).

Labor income risk Lacking access to the MEF, I rely on Guvenen et al. (2014) who report cross-sectional moments (mean, standard deviation, skewness) of the distribution of log labor income growth, net of life-cycle effects, for a representative sample of the US male population

between 25 and 60 years old, for each year from 1978 to 2010. I also rely on Guvenen et al. (2015) who report the evolution of within-cohort wage inequalities for the same sample.

Households' portfolios Moments relative to the wealth and portfolio choices of households are computed using the triennial Survey of Consumer Finances (SCF). I use publicly available data from nine waves between 1989 and 2013. I use survey weights, adjusted to give equal importance to all surveys. I restrict my sample to households between 23 and 82 years old, who are not business owners (*bus*=0) and whose net worth (*networth*) exceeds -$10,000. Table 4 reports key statistics for the final sample. Households constitute the unit of analysis.

[Insert Table 4 about here]

I define wealth as net worth normalized by the national average wage. The average wage in each survey is computed by taking the mean of wage incomes (*wageinc*) for households whose head is between 22 and 65 and part of the labor force (*lf*=1). The equity share is defined as equity holdings (*equity*) divided by financial wealth (*fin*), for households with positive financial wealth.

3.2 Stock market returns and labor income risk

The first step of the structural estimation is dedicated to the stochastic processes controlling macroeconomic and idiosyncratic shocks, which are estimated independently.

3.2.1 Macroeconomic shocks

Eight parameters control the dynamics of stock returns and aggregate labor income shocks: p_s, μ_s^-, μ_s^+, σ_{s_1}, σ_{s_2}, μ_l, λ_{ls}, σ_l. I estimate these parameters by SMM by targeting moments from the time series of the average wage growth and yearly S&P500 returns. The first 8 moments are the mean, standard deviation, and the third (skew) and fourth (kurtosis) standardized moments of each time-series. The last moment is the correlation coefficient of the two time series. Stock market moments are computed using yearly data between 1900 and 2015. The time series of average wage growth goes from 1978 to 2010.

[Insert Table 1 about here]

Table 1 reports the results of the estimation and shows that, despite being slightly overidentified, the model matches all moments very well. Note that the model is conservative regarding the frequency and severity of financial crashes. In the model, log returns below $-.35$ ($\approx -30\%$) occur three times per century. In the data, this actually happened five times since 1900 (1917, 1931, 1937, 1974 and 2008). For a log return of $-.35$, workers should expect their wage to drop by approximately 5%.[5] For the 2008 crisis, the model predicts an expected log wage shock of $-.062$ versus $-.064$ in the data.

3.2.2 Idiosyncratic shocks

Eight parameters control the distribution of idiosyncratic labor income shocks: p_z, ρ_z, $\overline{\mu_z^-}$, λ_{zl}, σ_z^-, σ_z^+, σ_η^-, σ_η^+. To estimate these parameters, I simulate an economy receiving the same aggregate shocks as the US economy between 1944 and 2010.[6] For each year between 1978 and 2010, I target the historical values of Kelly's skewness at the one, three and five-year horizons, as well as the standard deviation at the one and five-year horizons.[7] This constitutes a first set of 155 empirical moments targeted in the SMM.

The cyclicality of skewness is identified by targeting the historical values of these moments for each of the 33 years and using the true time series of aggregate labor income shocks as an input. The decomposition of the variance between transitory (σ_η^-, σ_η^+) and persistent shocks (σ_z^-, σ_z^+) and the persistence of the latter (ρ_z) are identified by targeting these moments at different horizons. One concern is that the model could overestimate the persistence of labor income shocks if workers face different life-cycle income profiles ex-ante (Guvenen (2009)). To address this issue, I assume that the deterministic component of y_i is of the form

$$f_i(t) = \overline{f}(t) + \varphi_i t + \alpha_i \tag{19}$$

where $\overline{f}(t)$ is a function of experience common to all workers while the two other terms are individual specific and normally distributed with standard deviations σ_α and σ_φ. Note that the

[5] $0.008 - 0.161 * 0.35 = 0.048$

[6] I use the NIPA tables to estimate aggregate income shocks before 1978.

[7] Guvenen et al. (2014) do not report the standard deviation at the three-year horizon.

main role of these parameters is to get a conservative estimate of ρ_z.[8] To avoid the multiplication of state variables, I ignore σ_φ in the life-cycle model and uses σ_α to initialize the distribution of z_i.

To identify σ_α and σ_φ, I also target the evolution of within-cohort inequalities between 25 and 60 years. Hence, I use 191 moments to estimate 10 parameters. To maintain a constant age structure within the economy, I start the simulation in 1944 and replace each cohort reaching 60 with young workers. More details on the estimation procedure are provided in Appendix A.1.

Table 2 reports the results of the SMM and Figure 2 plots targeted and simulated moments. The model replicates correctly the cyclicality of skewness at different horizons as well as the stability of the standard deviation and, unlike Guvenen et al. (2014), its high value.[9]

[Insert Table 2 about here]

[Insert Figure 2 about here]

A standard deviation of labor income shock above 0.5 may seem counter-intuitive but can be explained by the high peakness of the distribution. Since $p_z = .136$, in any given year, 86.4% of workers receive shocks from normal distributions with relatively low standard deviations: respectively 0.037 and 0.089 for the persistent and transitory components. This means that most of the variance comes from the remaining 13.6% who receive shocks with very high standard deviations: respectively 0.562 and 0.895. A plausible economic interpretation is that a fraction of the population switches job or become unemployed while the vast majority does not experience any noticeable shock.

Another important result is the very high persistence of innovations to z. By increasing the volatility of the human capital stock, an overestimation of ρ_z would create spurious portfolio effects, in particular among young households. Four things can reduce this concern. First, my

[8]The literature on labor income risk offers two alternative views on the rise of within-cohort inequalities over the life-cycle. In models with "restricted income profiles" (RIP), workers face similar life-cycle income profiles and inequalities result from large and highly persistent shocks. By contrast, models with "heterogeneous income profiles" (HIP) assume individual specific deterministic drifts and require lower shock persistence to explain the data (see Guvenen (2009)). I adopt the HIP specification, which is the less likely to overstate the role of income risk in portfolio choices by overestimating ρ_z.

[9]In Guvenen et al. (2014), transitory shocks are normally distributed and the model cannot match the standard deviation. I find that this problem can be solved by modeling transitory shocks using the normal mixture described in equation (9)

estimate of ρ_z is below that of the baseline specification of Guvenen et al. (2014). Second, the model matches the ratio of standard deviations at the five and one year horizons, a simple measure of persistence. Third, the model does not exaggerate the evolution of within-cohort inequalities over the life-cycle. Finally, the high persistence must be interpreted in conjunction with the high value of σ_η^-, which indicates that a large fraction of the most extreme shocks is transitory and recovered within one year. Therefore, only shocks that do not quickly dissipate turn out to be very persistent. This is consistent with findings by Guvenen et al. (2015) that workers who experience a large drop in their income recover one-third of that loss within one year, but have to wait 10 years to recover more than two-thirds.

3.3 Preferences and participation cost

In the second step of the structural estimation, I use SCF data to estimate the coefficient of relative risk aversion, the discount factor and a fixed yearly participation cost. Technical details on the numerical resolution of the model and its estimation are provided in Appendix B.

3.3.1 Preset parameters

Table 3 reports preset parameters. I set the real interest rate to 2% and variable management fees to 1%. From the agent's point of view, this calibration implies an equity premium slightly above 5%, in the upper range of values used in previous papers.

[Insert Table 3 about here]

I approximate $\overline{f}(t)$ with a quadratic polynomial that matches the average log wage by age observed in Social Security data. Households start working at 23 and retire at 65. Death probabilities are taken from Social Security actuarial life tables.

In the data, households start with different levels of wealth. I compute the centiles of the distribution of wealth between 23 and 24 years old in the SCF and use them to generate 100 groups with different initial wealth in my simulation. Having all households start with the same level of wealth would lead to an initial participation rate close to one or zero.

3.3.2 Identification

I run several sets of estimations and target the evolution over the life-cycle of the following variables: financial wealth, conditional or unconditional equity share, and stock market participation rate.

Empirically, the relationship between these variables and age is estimated as follows. First, I build 3-years age and cohort groups as in Ameriks and Zeldes (2004). Since the SCF is triennial, each cohort group moves exactly from one age group to the next between two surveys. Second, I disentangle age effects from year and cohort effects using the methodology of Deaton and Paxson (1994), as in Fagereng et al. (2017). Specifically, for each variable of interest, I run OLS regressions on a set of age, year and cohort group dummies and solve the collinearity problem by assuming that year dummies sum to zero and are orthogonal to a time trend. As a result, any time trend is captured by cohort effects. Finally, I compute the predicted values of each variable, for each age group between [23; 25] and [62; 64], putting equal weights on all cohorts born after 1946.

This constitutes a vector $\mathbf{m}$ of $3 \times 14 = 56$ empirical moments. The SMM procedure seeks the parameters γ, φ and $c_{\pi,1}$ that minimize

$$\left(\mathbf{m} - \widehat{\mathbf{m}}\left(\gamma, \varphi, \mathbf{c}_{\pi,\mathbf{1}}\right)\right)' \mathbf{W} \left(\mathbf{m} - \widehat{\mathbf{m}}\left(\gamma, \varphi, \mathbf{c}_{\pi,\mathbf{1}}\right)\right) \tag{20}$$

where $\widehat{\mathbf{m}}\left(\gamma, \varphi, \mathbf{c}_{\pi,\mathbf{1}}\right)$ is the vector of simulated moments generated by the model, and $\mathbf{W}$ is the inverse of the covariance matrix of the empirical moments, which is estimated by bootstrapping the true data. As described in the following section, some moments are not targeted across all specifications.

The identification is straightforward and well understood. A higher discount factor (β) reduces the accumulation of financial wealth. A higher risk aversion (γ) reduces the optimal equity share. Finally, a higher fixed participation cost prevents the participation of households whose optimal equity share represents a small dollar amount.

3.3.3 Results

Models without participation cost I start by estimating the coefficient of relative risk aversion and the discount factor by targeting the evolutions of wealth and that of the unconditional

equity share. Estimation results for the models with and without cyclical skewness are respectively reported in columns (1) and (4) of Table 5. Simulated moments and their empirical counterparts are reported in Panel A of Figure 3.

Without cyclical skewness, the SMM fails to match the data and returns unlikely estimates of the discount factor ($\beta = 0.79$) and relative risk aversion ($\gamma = 10.81$). The bond-like properties of human capital imply a positive relationship between the human-to-financial wealth ratio and the optimal risky share, and therefore a decline of the equity share over the life-cycle. A low β mitigates this problem by slowing down the decline of the human-to-financial wealth ratio, but this prevents the model from matching the level of wealth at 65. My estimates are very close to Fagereng et al. (2017), who find $\gamma = 11$ and $\beta = 0.77$ in a similar model with stock market disasters and a small participation cost of $69.

When cyclical skewness is taken into account, human capital is no longer bond-like and the model is able to match the data with more reasonable estimates of the discount factor ($\beta = 0.95$) and relative risk aversion ($\gamma = 6.97$).

Models with participation cost In a second step, I introduce a fixed yearly participation cost and estimate it, alongside β and γ. To do so, I distinguish the intensive and extensive margins by targeting the evolution of stock market participation rates and that of the equity share among participants. Columns (2) and (5) of Table 5 reports the results and Panel B of Figure 3 the model fitness.

Without cyclical skewness, participation costs improve the model's ability to fit the data and to produce more plausible estimates of β and γ. Estimated participation costs represent 2.1% of the average wage, that is $1,010 in 2015. When human capital is bond-like, matching low participation rates among young workers requires high fixed-costs because workers with low financial wealth want to invest their entire wealth in the stock market. By preventing their participation, the fixed cost also reduces the average conditional equity share, which, in turn, affects the estimated value of γ, which falls from 10.79 to 6.46. The discount factor rises from 0.79 to 0.95 because wealth accumulation drives the rise in participation. However, the model still fails to match the positive relationship between age and the conditional equity share.

With cyclical skewness, estimated participation costs are much lower, representing only 0.6% of

the average wage ($290). Because workers with low financial wealth would not want to invest much in the stock market anyway, a lower fixed cost is required to match low participation rates among young households. The model still generates a conditional equity share that slightly increases with age but overestimates participation among older households.

4 Discussion

4.1 Policy function

Portfolio choice models generally predict that human capital increases (decreases) the demand for stocks when its exposure to stock market risk exceeds (is below) the agent's optimal risky share. This generally implies that, absent participation costs, the optimal equity share should be a monotonous function of the wage-to-financial wealth ratio. This intuition does not hold in the presence of cyclical skewness. This is apparent in Panel A of Figure 4, which plots the optimal equity share of a 40 years old agent as a function of his persistent income (z) and financial wealth (w). Panel B displays the same policy function assuming that labor income shocks are normally distributed.

[Insert Figure 4 about here]

In Panel A, for a given level of human capital (z), the optimal equity share first increases with financial wealth, reaches a maximum, and then decreases. By contrast, financial wealth always reduces the optimal equity share in Panel B. In this case, human capital unambiguously generates a positive demand for stocks as in Viceira (2001), because the covariance between labor income shocks and stock market returns is negligible.

The difference between the two surfaces represents the effect of cyclical skewness, which is the strongest for households with high wage-to-financial wealth ratios ($z - w$). From the agent's point of view, what matters is the risk of severe shocks to her lifetime consumption. His fear of losing his job during a recession is therefore much more serious when most of his future consumption depends on labor income. A simple way to hedge against this risk would be to short-sell the stock market.

As the agent starts accumulating financial wealth, disastrous labor income shocks have milder implications in terms of consumption. Because left-tail consumption risk becomes less of a concern, the agent gives more attention to the low covariance between labor income shock and stock returns and the optimal equity becomes a positive function of z and negative function of w. At the limit, when $w - z$ gets very large, the agent follows portfolio rules close to Merton's solution.[10]

4.2 Empirical policy function

To evaluate my two models, I now turn to their ability to replicate empirical policy functions, that is relationships between the current state of the household and its portfolio choice. Empirical policy functions offer a natural starting point for model evaluation (Bazdresch et al. (2018)).

Beside age, the main state variable of the model that we observe in the data is the wage-to-wealth ratio. Hence, I start by estimating the empirical relationship between the conditional equity share and twenty quantiles of wage-to-wealth ratio using OLS with age and cohort fixed-effects. Then, I run the same OLS regression in simulated data from my models with and without cyclical skewness using estimated parameters reported in columns (2) and (5) of Table 5. The OLS coefficients for all quantile dummies are reported in Figure 5.

[Insert Figure 5 about here]

Clearly, the model with cyclical skewness fits the empirical policy function much better. In the SCF data, the conditional equity share and the wage-to-wealth ratios are not correlated. This is also true in simulated data from the model with cyclical skewness. On the contrary, simulated data from the model without cyclical skewness show a strong positive relationship between the wage-to-wealth ratio and the conditional equity share. Given the low correlation between labor income shocks and stock market returns, this relationship is consistent with theoretical predictions from Viceira (2001). Hence, only the model with cyclical skewness matches how the equity share of stock market participants varies within age-groups.

Note that the relationship between age and the equity share used to estimate the model is also an empirical policy function. But this relationship is mostly driven by the evolution of the human

[10]Assuming normally distributed log-returns, the solution to Merton's portfolio problem is $\frac{\mu - r}{\gamma \sigma^2} = \frac{.063 - .02}{5.6 \times .189^2} \approx .21$

capital-to-financial wealth ratio. Indeed, without labor income, the optimal equity share would be independent of age and wealth (Samuelson (1969), Merton (1969)). One can therefore argue that the relationship between the equity share and the wage-to-wealth ratio provides a more direct benchmark to evaluate the model.

We also know from Calvet and Sodini (2014)'s study of Swedish twins that (i), controlling for human capital, the elasticity of the risky share to financial wealth is positive, and (ii) that this elasticity is three times larger in the bottom quartile of financial wealth than in the top quartile. Figure 4 shows that these two facts can only be reconcile with the model when it incorporates cyclical skewness.

4.3 Decomposition of effects

In this section, I show that cyclical skewness has a strong effect on the optimal equity share and that, without cyclical skewness, stock market disasters would not matter nearly as much. I also find that taking into account Social Security is important to match the data.

To do so, I start by simulating the model with normally distributed income shocks and returns, using parameter estimates reported in column (2) of Table 5. Then, I introduce or remove specific elements of the model to see how the simulated data change. Figure 6 reports the evolution of the equity share over the life-cycle depending on the presence of cyclical skewness and stock market disasters.

[Insert Figure 6 about here]

Normally distributed shocks Scenario (a) represents the evolution of the equity share when all shocks are normally distributed and shocks to human capital are only correlated with stock returns at the national level. This calibration of the model is similar to the situation considered by Cocco et al. (2005) and Viceira (2001), but with a lower level of risk aversion and higher variance of labor income shocks. In the absence of participation costs, the result is quantitatively close to Cocco et al. (2005).

Stock market disasters Scenario (b) takes into account stock market crashes. In my model, taking into account the left-tail risk of the stock market reduces the optimal equity share by only

a few percentage points. In Fagereng et al. (2017), the introduction of left-tail risk in stock returns has an effect three times larger on the optimal equity share, but the difference may arise from the much higher levels of risk aversion ($\gamma \geq 10$) considered by these authors.

Cyclical skewness In Scenario (c), households face cyclical skewness, but stock returns are log-normally distributed. Introducing cyclical skewness has a dramatic effect on the level of the equity share and reverses the sign of its relationship with age. For young households, absent participation costs, the optimal equity share drops from close to 100% to close to 0%. The effect is much smaller for households getting closer to retirement because most of their future consumption comes from financial wealth ans Social Security entitlements.

Cyclical skewness & Stock market disasters Scenario (d) combines the effects of scenarios (b) and (c). The difference between (c) and (d) is greater than between (a) and (b) which suggests an interaction effect between cyclical skewness and stock market crashes. For young agents with high wage-to-financial wealth ratios, cyclical skewness strongly amplifies the importance of stock market tail risk. When income shocks are normally distributed, stock market tail risk reduces the optimal equity share by roughly 6 percentage points over the entire life-cycle. By contrast, assuming no participation cost, when cyclical skewness is taken into account, stock market tail risk reduces π by 16 percentage points at 25 years old, 11 points at 35, 8 points at 45 and 6 points at 55.

Social Security To a large extent, Social Security acts as a mandatory investment in bonds. If entitlements perfect bond properties, and under the assumptions that *(i)* the mandatory yearly investment is below the optimal saving rate of the agent, *(ii)* that the total investment in Social Security entitlements is less than what the agent wants to invest in bonds, then one dollar of investment in Social Security should reduce financial wealth and bond holdings by one dollar, but equity holdings should be unaffected.

[Insert Figure 7 about here]

Panels A and B of Figure 7 appear to be in line with these predictions at retirement age. However, equity holdings are slightly reduced by Social Security among younger households, which

is consistent with the facts that entitlements are somewhat risky as their value evolves with the national wage index. The evolution of the equity share, reported in Panel C, is quite different with Social Security. In the second half of the agent's career, the risky share is higher because entitlements reduce financial wealth, and therefore the denominator. In the first half of the agent's career, the risky share is lower. One possible explanation is that payroll taxes delay the accumulation of precautionary savings. Unlike financial wealth, Social Security entitlements cannot be used to smooth consumption in the event of a large negative income shock. This lack of liquidity could deter workers from investing in stocks.

At age 65, equity holdings are identical in the two scenarios. At that point, Social Security entitlements are risk-free and crowd out bonds. We know from Merton (1971) that optimal equity holdings represent the same fraction of *total* wealth, including the present value of entitlements. Hence, identical equity holdings in the two scenarios indicate identical total wealth and perfect substitution between Social Security and private savings.

Social safety net I also run a counterfactual experiment in which I remove the SNAP and SSI welfare programs. I find that the effect of these programs on the equity share does not exceed a few percentage points, and only at the beginning of the life cycle. In the model, households quickly accumulate enough wealth to become ineligible to these programs, which, of course, is not true empirically.

4.4 Aggregate effect on the demand for equity

How does the cyclical skewness of labor income risk affect the optimal demand for equity and the equity premium? To answer this question, I run a counterfactual experiment in which I assume that labor income shocks are normally distributed, but have otherwise the same variance, persistence and covariance with stock market returns. Holding the equity premium constant, I then compute the change in the aggregate demand for equity in simulated data, including from retired households. This change represents the effect of cyclical skewness on the demand for equity. In a second step, I solve for the change in the equity premium that offsets the effect of cyclical skewness and raises the aggregate demand for stocks back to its initial value. This change

corresponds to the effect of cyclical skewness on the equity premium assuming that the supply of stocks in inelastic, and therefore represents an upper bound on the effect of cyclical skewness.

As reported in columns (1) and (2) of Table 6, I find that cyclical skewness reduces aggregate demand for stocks by only 13% to 15%, a reduction that could be offset by increasing the equity premium by half a percentage point. Columns (1) and (2) corresponds to the specifications with and without fixed participation costs. The effect of cyclical skewness on the demand for equity can be decomposed into two components. First, cyclical skewness reduces the share of aggregate financial wealth invested in equity by 17% to 21%. But cyclical skewness also increases aggregate financial wealth by 4% to 5% by stimulating precautionary savings, which attenuates the first effect.

[Insert Table 6 about here]

These findings contrast with the conclusions of Constantinides and Ghosh (2017), who argue that cyclical skewness in consumption risk can explain a variety of asset pricing puzzles, including the equity premium. One key difference between their methodology and mine, is that Constantinides and Ghosh (2017) assume all households to face the same distribution of consumption shocks and estimate their model by targeting the cross-sectional skewness of consumption growth. By contrast, I assume all households to face the same distribution of labor income shocks. Under my assumption, the distribution of consumptions shocks is very different across the wealth distribution. Households with very high wage-to-wealth ratios reduce their consumption drastically when they receive a dramatic labor income shocks, but households with high financial wealth do not. Hence, the negative skewness in the cross-sectional distribution of household consumption growth documented by Constantinides and Ghosh (2017) is probably not representative of the risk faced by the average dollar-weighted investor.

To further illustrate this intuition, Figure 8 plots the contribution of different deciles of financial wealth and age to the cyclical skewness of consumption risk in simulated as well as their share of aggregate financial wealth. Here, I measure cyclical skewness using cokurtosis, defined as:

$$\kappa = \frac{\mathrm{E}\left[(\delta_c - \mathrm{E}(\delta_c))^3 \, (s - \mathrm{E}(s))\right]}{\sigma_{\delta_c}^3 \sigma_s} \tag{21}$$

where $\delta_{c,it} = \log\left(\frac{C_{it+1}}{C_{it}}\right)$ is the growth rate of consumption of household i and σ_{δ_c} its cross-sectional standard deviation. A large κ indicates that the distribution of δ_c is left (right) skewed in years of low (high) stock returns. Because κ is a sum over households, computing the contribution of each subgroup to κ is straightforward. As shown in Figure 8, individuals with low financial wealth contribute disproportionately to κ. By contrast, the top decile concentrates close to 50% of financial wealth and barely contributes to κ. Hence, the countercyclical consumption risk faced by the average household cannot explain the behavior of the average dollar-weighted investor.

[Insert Figure 8 about here]

5 Robustness

5.1 Alternative theories of stock market participation

My estimation relies on the assumption that a fixed monetary cost explains low stock market participation rates. This assumption is difficult to reconcile with reduced-form evidence that windfall wealth has limited effects on participation (Andersen and Nielsen (2011), Briggs et al. (2015)).[11] Moreover, a number of alternative solutions to the participation puzzle has been proposed: lack of trust (Guiso et al. (2008)), disappointment aversion (Ang et al. (2005)), narrow framing (Barberis et al. (2006)) and ambiguity aversion (Campanale (2011), Peijnenburg (2016)) among others.

This literature raises two questions. Are my findings robust when alternative theories of non-participation are taken into account? And to which extend can the model accommodate these alternative explanations?

To answer these questions in a simple way, I reestimate the model assuming that a fraction of the population never participates. For these households, the model is solved as if the participation cost was infinite.

Columns (3) and (6) of Table 5 reports the results for the models with and without cyclical skewness. Panel C of Figure 3 their fitness. Estimated parameters are similar to columns (2)

[11] Andersen and Nielsen (2011) show that receiving 134,000 euros after an unexpected inheritance raises the probability of Danish inheritors entering the stock market by only 21 percentage points. They also observe that the majority of households inheriting stocks actively exit the equity market. Briggs et al. (2015) find that among Swedish lottery players, a $150,000 windfall gain raises the probability of stock ownership from non-participants by only 12 percentage points.

and (5). Without cyclical skewness, the estimated share of never participants is below 2%. This number rises to 23% in the presence of cyclical skewness. I also report the effect of cyclical skewness on the aggregate demand for equity in column (3) of Table 6 and find previous conclusions to be robust.

The identification now works as follows. The fixed participation cost determines the speed at which the participation rate rises with age while the fraction of never participants explains why some households do not participate when their financial wealth peaks, that is when they retire. In the model without cyclical skewness, matching the trend in participation age requires a very large fixed cost because young households are willing to invest in stocks. The fixed cost is large enough to explains why some households close to retirement do not participate and therefore leaves little room for alternative theories of non-participation. By contrast, the model with cyclical skewness requires a much lower fixed cost to explain the trend, because the latter is also explained by the unwillingness of young households to hold stocks. In that case, the fixed cost is too low to explain why some households do not participate when they get close to retirement, which in turn leads to a much higher estimate of the fraction of never-participating households.

Overall, I find that, in the model without cyclical skewness, 96% of non participants below retirement age would buy stocks if they were wealthier. Only 56% would do the same in the model with cyclical skewness, which is more consistent with empirical evidence on the causal effects of wealth on participation.

5.2 Relative risk aversion

Figure 9 plots the life-cycle profile of the equity share for different levels of relative risk aversion, with cyclical skewness and stock martket crashes ("all effects"), and when all shocks are normally distributed ("no effect"). Cyclical skewness reduces the equity share significantly for levels of γ of at least 5. Below this level, too many households hit the upper constraint at $\pi = 1$ for the two scenarios to be clearly distinguishable. When $\gamma \geq 6$, young households with very little financial wealth avoid the stock market, even in the absence of participation costs. Adding fixed costs delays participation by a few years.

[Insert Figure 9 about here]

While coefficients of relative risk aversion around 10 are common in the household finance and asset pricing literatures, laboratory experiments (Holt and Laury (2002), Harrison and Rutström (2008), Andersen et al. (2008)), life-cycle consumption models (Gourinchas and Parker (2002)) and the elasticity of labor supply (Chetty (2006)) generally suggest a relative risk aversion below 2. Though my paper pushes down the estimate of γ implied by households' equity holdings, it falls short of finding sizable results for low values of γ.

One possible explanation is that my model leaves aside many sources of background risk or that CRRA utility underestimates households' preference for skewness. Perhaps more interestingly, the model fails to match the large fraction of US households who reach retirement with very little wealth. Hence, the model largely underestimates the fraction of households for which cyclical skewness have very large effects. The model also neglects that a large share of household's net worth is real-estate and may be difficult to mobilize for consumption in case of large income shocks.

6 Conclusion

In this paper, I study whether the cyclical skewness of idiosyncratic labor income shocks can reconcile life-cycle models of portfolio choices with the US data. I find that cyclical skewness can explain both the limited stock market participation among households with modest financial-to-human wealth ratios, and why the conditional equity share rises with age. Moreover, I find that omitting cyclical skewness leads to a three-fold overestimation of stock market participation costs.

Overall, the model with cyclical skewness can fit the data with plausible parameters: a relative risk aversion of 5, a discount rate of 8% and a yearly participation cost below $300. By contrast, in the absence of cyclical skewness, the life-cycle model generates a negative relationship between age and the equity share of participants which is not observed in the data.

Contrary to prior research, I find that countercyclical income risk has limited effects on the aggregate demand for equity because it does not significantly affect the portfolios of wealthy households.

References

Ameriks, John and Stephen P Zeldes, "How Do Household Portfolio Shares Vary with Age?," 2004.

Andersen, Steffen and Kasper Meisner Nielsen, "Participation Constraints in the Stock Market: Evidence from Unexpected Inheritance Due to Sudden Death," *Review of Financial Studies*, 2011, *24* (5), 1667–1697.

_ , Glenn W. Harrison, Morten I. Lau, and E. Elisabet Rutström, "Eliciting Risk and Time Preferences," *Econometrica*, 2008, *76* (3), 583–618.

Ang, Andrew, Geert Bekaert, and Jun Liu, "Why stocks may disappoint," *Journal of Financial Economics*, June 2005, *76* (3), 471–508.

Barberis, Nicholas, Ming Huang, and Richard H. Thaler, "Individual Preferences, Monetary Gambles, and Stock Market Participation: A Case for Narrow Framing," *American Economic Review*, September 2006, *96* (4), 1069–1090.

Bazdresch, Santiago, R. Jay Kahn, and Toni M. Whited, "Estimating and Testing Dynamic Corporate Finance Models," *The Review of Financial Studies*, 2018, *31* (1), 322–361.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Godlstein, "Portfolio Choice over the Life-Cycle when the Stock and Labor Markets Are Cointegrated," *Journal of Finance*, October 2007, *62* (5), 2123–2167.

Bertaut, Carol C. and Martha Starr-McCluer, "Household portfolios in the United States," 2002. Literaturangaben.

Briggs, Joseph S., David Cesarini, Erik Lindqvist, and Robert Östling, "Windfall Gains and Stock Market Participation," October 2015, (21673).

Calvet, Laurent E. and Paolo Sodini, "Twin Picks: Disentangling the Determinants of Risk-Taking in Household Portfolios," *Journal of Finance*, 04 2014, *69* (2), 867–906.

Campanale, Claudio, "Learning, Ambiguity and Life-Cycle Portfolio Allocation," *Review of Economic Dynamics*, 2011, *14* (2), 339–367.

Campbell, John Y., João F. Cocco, Francisco J. Gomes, and Pascal J. Maenhout, "Investing Retirement Wealth: A Life-Cycle Model," in "Risk Aspects of Investment-Based Social Security Reform" NBER Chapters, National Bureau of Economic Research, Inc, March 2001, pp. 439–482.

Chai, Jingjing, Wolfram Horneff, Raimond Maurer, and Olivia S. Mitchell, "Optimal Portfolio Choice over the Life Cycle with Flexible Work, Endogenous Retirement, and Lifetime Payouts," *Review of Finance*, 2011, *15* (4), 875–907.

Chetty, Raj, "A New Method of Estimating Risk Aversion," *American Economic Review*, December 2006, *96* (5), 1821–1834.

Cocco, Joao F., "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies*, 2005, *18* (2), 491–533.

_ , Francisco J. Gomes, and Pascal J. Maenhout, "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies*, 2005, *18* (2), 491–533.

Constantinides, George M. and Anisha Ghosh, "Asset Pricing with Countercyclical Household Consumption Risk," *The Journal of Finance*, 2017, *72* (1), 415–460.

de Jong, Frank, "Portfolio Implications of Cointegration Between Labor Income and Dividends," *De Economist*, December 2012, *160* (4), 397–412.

Deaton, Angus S. and Christina Paxson, "Saving, Growth, and Aging in Taiwan," in "Studies in the Economics of Aging" NBER Chapters, National Bureau of Economic Research, Inc, March 1994, pp. 331–362.

Fagereng, Andreas, Charles Gottlieb, and Luigi Guiso, "Asset Market Participation and Portfolio Choice over the Life-Cycle," *The Journal of Finance*, 2017.

Gomes, Francisco and Alexander Michaelides, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, 04 2005, *60* (2), 869–904.

Gomes, Francisco J., Laurence J. Kotlikoff, and Luis M. Viceira, "Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds," *American Economic Review*, May 2008, *98* (2), 297–303.

Gourinchas, Pierre-Olivier and Jonathan A. Parker, "Consumption over the Life-Cycle," *Econometrica*, 2002, *70* (1), 47–89.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales, "Trusting the Stock Market," *Journal of Finance*, December 2008, *63* (6), 2557–2600.

Guvenen, Fatih, "An Empirical Investigation of Labor Income Processes," *Review of Economic Dynamics*, January 2009, *12* (1), 58–79.

_ **, Fatih Karahan, Serdar Ozkan, and Jae Song**, "What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Risk?," *NBER Working Papers*, January 2015, *20913.*

_ **, Serdar Ozkan, and Jae Song**, "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, 2014, *122* (3), 621 – 660.

Harrison, Glenn and Elisabet E. Rutström, "Risk Aversion in the Laboratory," 06 2008, *12*, 41–196.

Holt, Charles A. and Susan K. Laury, "Risk Aversion and Incentive Effects," *American Economic Review*, December 2002, *92* (5), 1644–1655.

Jagannathan, Ravi and Narayana R. Kocherlakota, "Why Should Older People Invest Less in Stocks Than Younger People?," 02 1996, *20*, 11–23.

Lynch, Anthony W. and Sinan Tan, "Labor income dynamics at business-cycle frequencies: Implications for portfolio choice," *Journal of Financial Economics*, 2011, *101* (2), 333–359.

Mankiw, N. Gregory, "The equity premium and the concentration of aggregate shocks," *Journal of Financial Economics*, September 1986, *17* (1), 211–219.

Merton, Robert C., "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, August 1969, *51* (3), 247–57.

— , "Optimum consumption and portfolio rules in a continuous-time model," *Journal of Economic Theory*, December 1971, *3* (4), 373–413.

Peijnenburg, Kim, "Life-Cycle Asset Allocation with Ambiguity Aversion and Learning," *Journal of Financial and Quantitative Analysis*, 2016, *forthcoming.*

Samuelson, Paul A, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *The Review of Economics and Statistics*, August 1969, *51* (3), 239–46.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, June 2004, *112* (3), 695–717.

— , Chris Telmer, and Amir Yaron, "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," *Review of Economic Dynamics*, October 2007, *10* (4), 519–548.

Tauchen, George, "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

Viceira, Luis M., "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income," *Journal of Finance*, 04 2001, *56* (2), 433–470.

Vissing-Jorgensen, Annette, "Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures," April 2002, (8884).

7 Tables and figures

Table 1: Estimated parameters: stock market returns and aggregate labor income shocks

Note: This table reports parameter estimates for the dynamics of log returns and aggregate labor income shocks. The S&P500 log return in year t is $s_t = s_{1,t} + s_{2,t}$, where $s_{2,t} \sim \mathcal{N}(0, \sigma_{s_2}^2)$. With probability p_s, t is a year of stock market crash and $s_{1,t} = s_{1,t}^- \sim \mathcal{N}(\mu_s^-, \sigma_{s_1}^2)$. Otherwise, $s_{1,t} = s_{1,t}^+ \sim \mathcal{N}(\mu_s^+, \sigma_{s_1}^2)$. The average log change in wages is $l_{t+1} - l_t = \mu_l + \lambda_{ls} s_{1,t} + \varepsilon_t$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma_l^2)$. The SMM targets the first, second, third (skew) and fourth (kurt) standardized moments of each time series as well as their correlation coefficient. In the specification "without crashes", I assume log returns and aggregate labor income shocks to be normally distributed by imposing $p_s = \mu_s^- = \sigma_l = 0$ and only target the means, standard deviations and correlation coefficient.

Panel A: Estimated parameters

	Log returns					Aggregate income shocks		
Model	p_s	μ_s^-	μ_s^+	σ_{s_1}	σ_{s_2}	μ_l	λ_{ls}	σ_l
With stock market crashes	.146	-.245	.115	.077	.114	.008	.161	.017
Without crashes			.063	.118	.141	.003	.250	

Panel B: Moments

	Log returns				Aggregate income shocks				
	Mean	Std	Skew	Kurt	Mean	Std	Skew	Kurt	$\text{Corr}_{s,\delta l}$
Data	.063	.185	-.623	3.323	.018	.029	-.663	3.368	.638
Model									
– with stock market crashes	**.063**	**.189**	**-.596**	**3.368**	**.019**	**.030**	**-.663**	**3.433**	**.638**
– without crashes	**.063**	**.185**	.000	3.000	**.018**	**.029**	.000	3.000	**.638**

Table 2: Estimated parameters: idiosyncratic labor income shocks

Note: This table reports parameter estimates for the dynamics of idiosyncratic income shocks. The idiosyncratic component of log wages has a transitory component η_{it} and a persistent component z_{it} with persistence ρ_z. With probability $p_z \leq 0.5$, the agent receives a transitory shock $\eta_{it} = \eta_{it}^- \sim \mathcal{N}(0, {\sigma_\eta^-}^2)$, and a persistent shock $\zeta_{it} = \zeta_{it}^- \sim \mathcal{N}(\mu_{z,t}^-, {\sigma_z^-}^2)$, where $\mu_{z,t}^- = \overline{\mu_z^-} + \lambda_{zl}(l_{t+1} - l_t)$ depends on the aggregate labor income shock $l_{t+1} - l_t$, which is an exogenous input. Otherwise, the transitory and persistent shocks are respectively $\zeta_{it} = \zeta_{it}^+ \sim \mathcal{N}(\mu_{z,t}^+, {\sigma_z^+}^2)$ and $\eta_{it} = \eta_{it}^+ \sim \mathcal{N}(0, {\sigma_\eta^+}^2)$. Since expected idiosyncratic shocks are equal to zero, I impose $p_z\mu_{z,t}^- + (1-p_z)\mu_{z,t}^+ = 0$. σ_α and σ_φ represent the standard deviation of fixed effects in levels and trends. The model "with cyclical skewness" is estimated by targeting moments from US Social Security panel data presented in figure 2. In the model "without cyclical skewness", income shocks are lornormally distributed and other parameters calibrated to keep persistence and variance constant.

		Persistent shocks					Transitory shocks		Ex-ante heterogeneity	
Model	p_z	ρ_z	$\overline{\mu_z^-}$	λ_{zl}	σ_z^-	σ_z^+	σ_η^-	σ_η^+	σ_α	σ_φ
With cyclical skewness	.136	.967	-.086	4.291	.562	.037	.895	.089	.280	.004
Without cyclical skewness		.967				.216		.341	.280	.004

Table 3: Benchmark calibration

Parameter		Value
Financial markets		
r	risk-free log rate	.02
$c_{\pi,2}$	proportional management fees	.01
Life-cycle income profile		
θ_1	effect of age on log wage	.1237
θ_2	effect of $age^2/10$ on log wage	-.0125
θ_0	constant	-3.015
Preferences		
t_0	age of first employment	23
R	age of retirement	65
T	maximum life span	100

Table 4: Summary statistics of SCF sample

Note: This table reports summary statistics for the SCF 1989-2013 sample. The sample is restricted to households whose head is between 23 and 81 years old, with net worth above -$10,000. The wage statistics is computed for households with a wage earner between 23 and 65 years. The equity share is expressed as a fraction of financial wealth, and only reported for households with positive financial wealth.

	Mean	Std Deviation	Observations
Year	2000.83	7.710	121,194
Age	48.84	16.18	121,194
Net worth	290,114	1,104,176	121,194
Financial wealth	154,382	841,440	121,194
Wage	71,109	62,972	94,476
Equity share	0.234	0.307	113,609
Participation	.484	.500	113,609
Conditional equity share	.451	.289	64,851

Table 5: Estimated parameters: preferences and participation cost

Note: This table reports results of my structural estimation. Panel A shows the estimated parameters and their standard errors while Panel B indicates which sets of life-cycle moments from the SCF data are targeted. Columns (1) and (2) assume no fixed participation cost and target the evolution of wealth and of the unconditional equity share. Columns (3) and (4) include a per period participation cost which is a fraction of the national average wage and, beside wealth, target the life-cycle profiles of the participation rate and the conditional equity share. In columns (5) and (6), I introduce a fraction of the population which never participates in the stock market. In columns (1), (3) and (5), the model takes into account the cyclical skewness of idiosyncratic income shocks, whereas in columns (2), (4) and (6) labor income shocks are log normally distributed with identical variance, persistence and covariance with stock returns as in columns (1), (3) and (5). All models include stock market crashes.

	With cyclical skewness			Without cyclical skewness		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Estimated parameters						
Relative risk aversion	6.974	5.543	5.274	10.810	6.464	6.501
	(.004)	(.004)	(.003)	(.007)	(.002)	(.004)
Discount factor	0.946	0.890	0.921	0.798	0.955	0.954
	(.004)	(.006)	(.005)	(.012)	(.004)	(.008)
Fixed participation cost		0.006	0.007		0.021	0.020
		(.000)	(.000)		(.000)	(.000)
Fraction of never participants			0.233			0.015
			(.001)			(.001)
Panel B: Targeted life-cycle moments						
Wealth	✓	✓	✓	✓	✓	✓
Equity share	✓			✓		
Participation rate		✓	✓		✓	✓
Conditional equity share		✓	✓		✓	✓

Table 6: Aggregate effect of cyclical skewness on the demand for equity

Note: This table reports the effects of cyclical skewness on the aggregate demand for equity. Using models (1), (2) and (3) of Table 5, I simulate the life of 10^6 individuals and sum their equity holdings over all ages, including retirement. Holding everything else equal, I then remove cyclical skewness from the model and repeat the computation. The first line of the table reports the log difference in aggregate demand for equity between the model without and the model with cyclical skewness. The log difference is explained by the change in aggregate wealth and the change in the aggregate equity share. I also compute the change in the equity premium (in both μ_s^- and μ_s^+) that exactly offsets the effect of removing cyclical skewness. Specifically, I use Newton's method to find the equity premium for which the aggregate demand in the model without cyclical skewness equals the demand in the model with cyclical skewness and the true equity premium.

	Model		
	(1)	(2)	(3)
Aggregate effect of cyclical skewness on:			
– ln(equity share)	-.172	-.205	-.140
– ln(wealth)	.039	.054	.048
– ln(demand for equity)	-.133	-.151	-.092
Equivalent change in equity premium	-.004	-.005	-.004

Figure 1: Skewness of income shocks and stock returns in the US

Note: This graph plots the evolution of the cross-sectional skewness of log wage yearly changes (line) in the US from 1978 to 2010 and yearly S&P500 log returns (bars). Here, skewness is defined as $\frac{(d9-d5)-(d5-d1)}{d9-d1}$, where di denotes the i^{th} decile of the log change in wage. Those deciles are computed by Guvenen et al. (2014) using Social Security panel data. Their sample is restricted to male workers between 25 and 60 years old and log wage changes are adjusted for life-cycle effects. S&P500 data are taken from Robert Shiller's website.

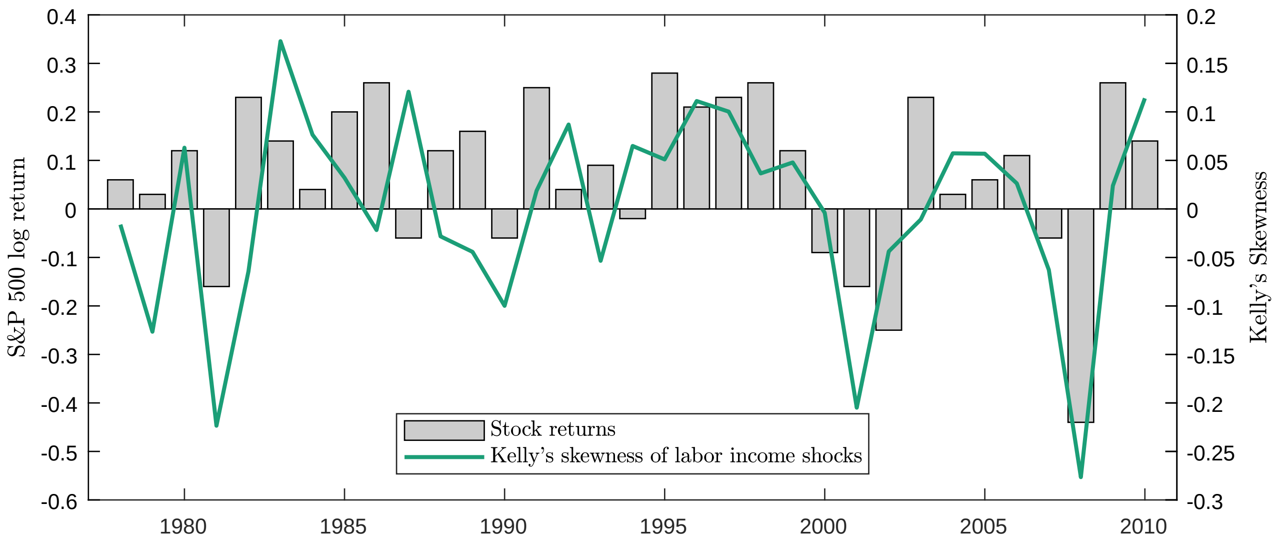


Figure 2: Fitness of idiosyncratic labor income risk model

Note: This figure reports the empirical and simulated values of moments targeted in the estimation of the dynamics of idiosyncratic labor income shocks. Panels A to E report, for each year, cross-sectional moments from the distribution of labor income log growth, net of life-cycle effects, at the one, three and five year horizons among US male salaried workers between 25 and 60 years old. Here, skewness is defined as $\frac{(d9-d5)-(d5-d1)}{d9-d1}$. Panel F reports the average historical within-cohort standard deviation of labor income at each age. All these moments are computed by Guvenen et al. (2014) and Guvenen et al. (2015) using Social Security panel data. I simulate 67 cohorts of 7,500 individuals, with the first cohort entering the labor market in 1944. Historical aggregate income shocks between 1943 and 2010 are exogenous inputs in the simulation and come from Social Security data (1978-2010) and NIPA tables (1944-1977).

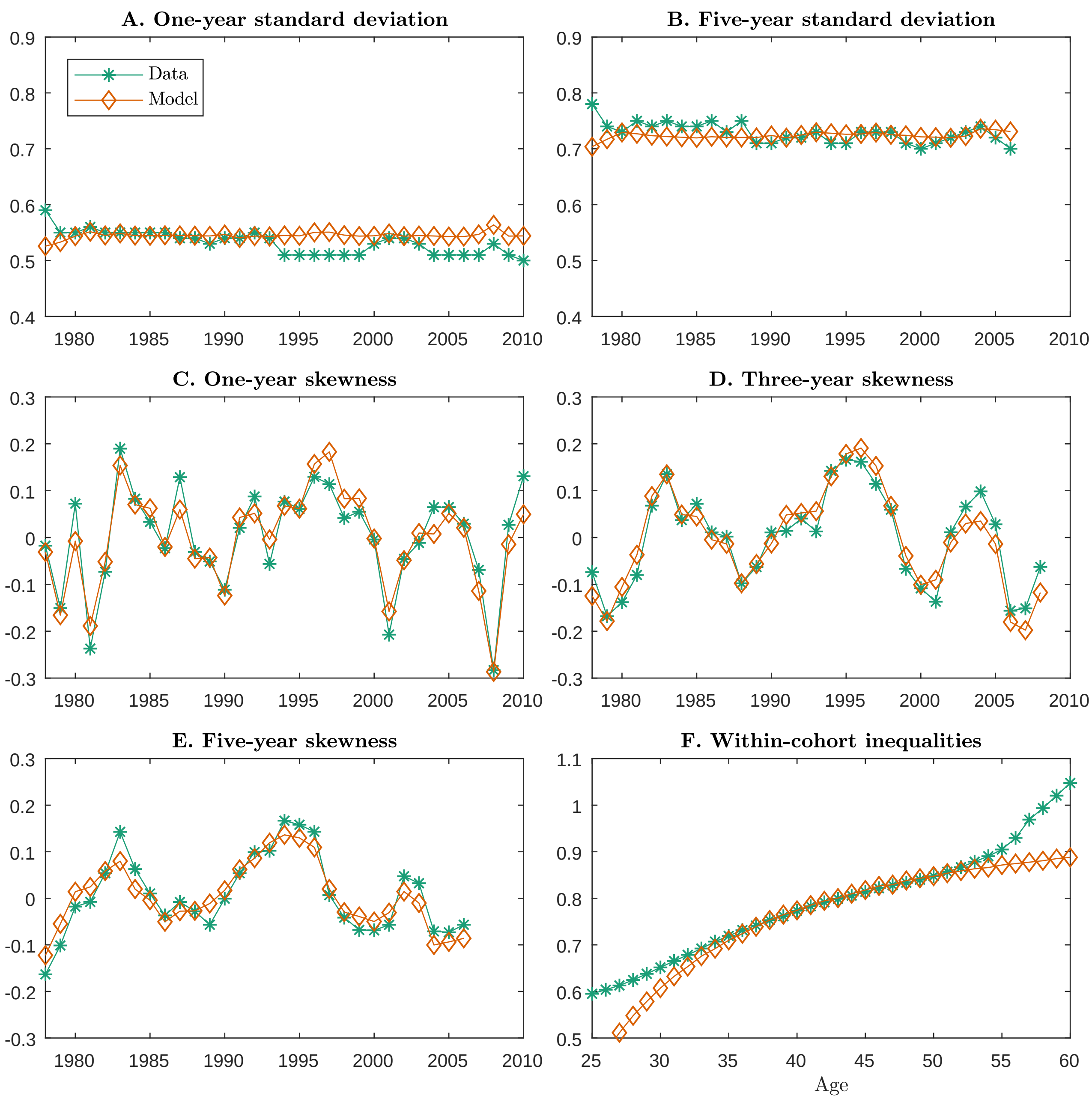


Figure 3: Fitness of estimated life-cycle models

Note: These graphs plot the value of moments targeted in the SMM procedures reported in Table 5. Empirical life-cycle patterns are estimated using the SCF (1989-2013) and the methodology of Deaton and Paxson (1994). 95% confidence intervals are reported in grey. In Panel A, I assume no participation cost and target the evolution of wealth and unconditional equity shares. The models "with cyclical skewness" and "without cyclical skewness" respectively refers to models (1) and (4). In Panel B, I introduce a per period participation cost and target the life-cycle patterns of participation and that of the conditional equity share. Simulated moments refers to models (2) and (5). Finally, in Panel C, I assume that a fraction of the population never holds any stock, which corresponds to models (3) and (6).

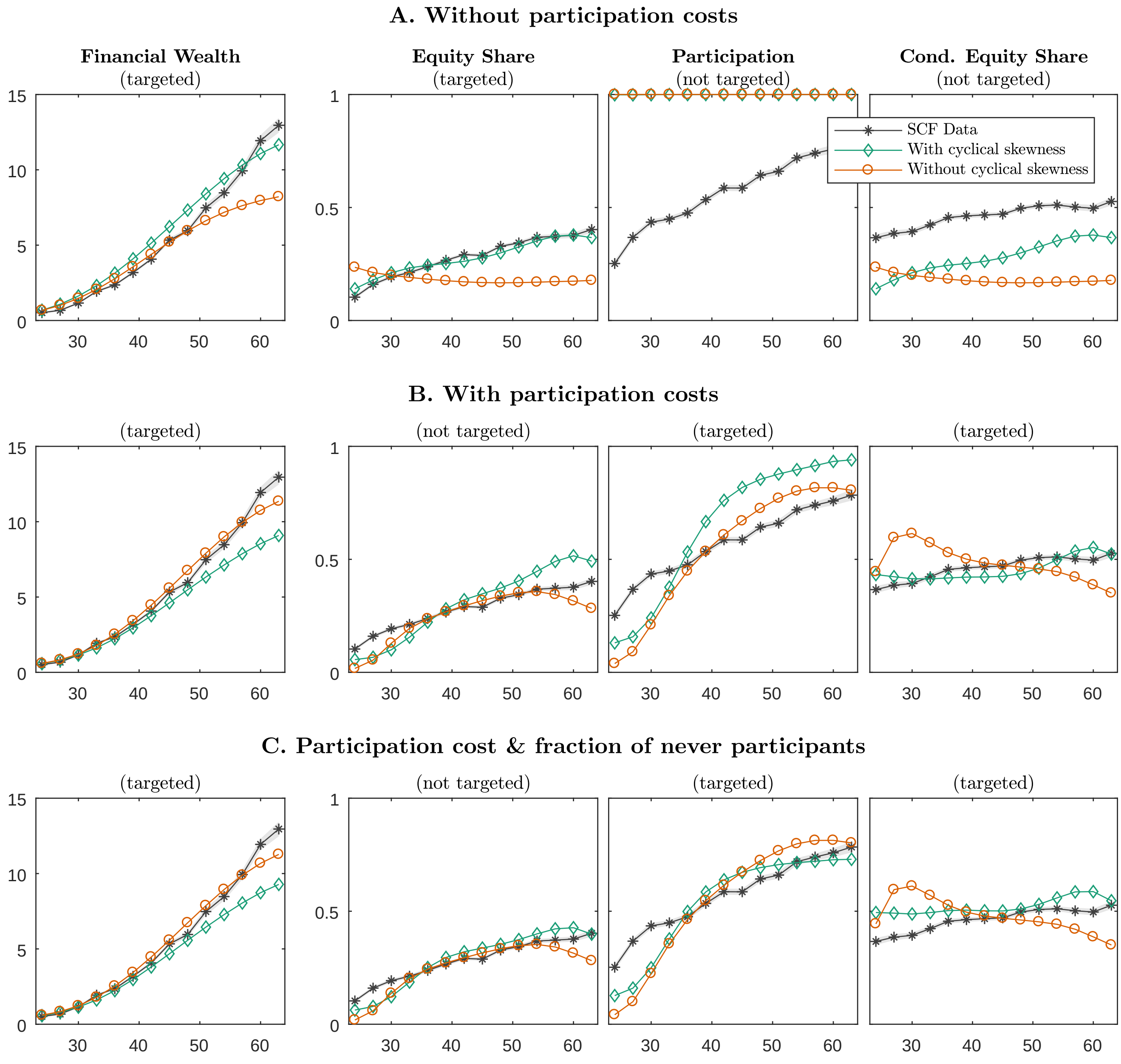
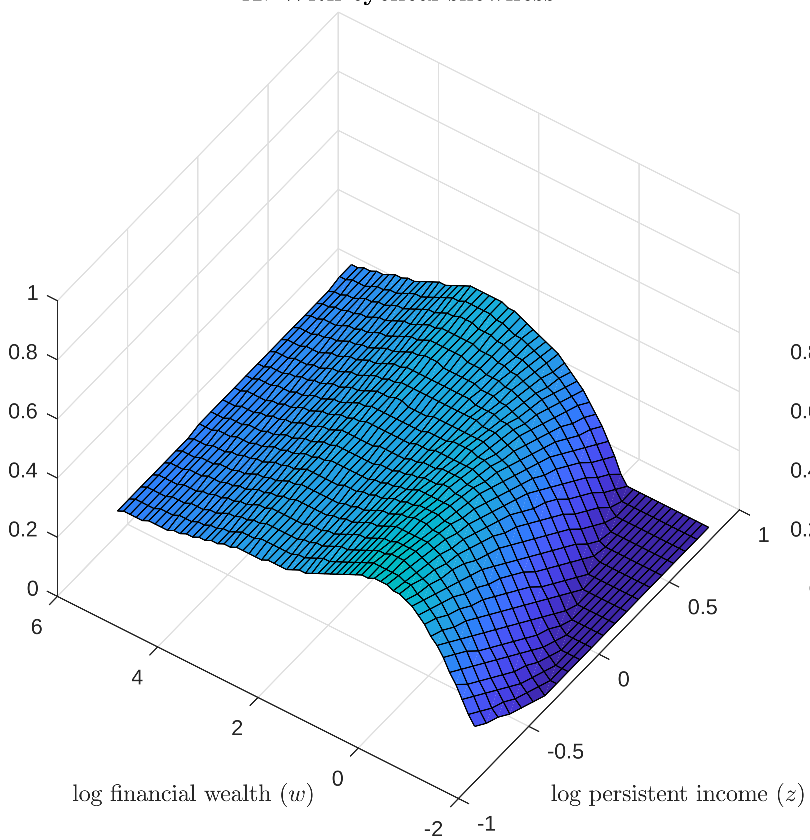


Figure 4: Optimal equity share at 40 without participation cost

Note: This figure represents the optimal equity share for a 40 years old worker, whose mean historical taxable wage has been $S = 1$ so far, as a function of two state variables: the logs of her financial wealth x and the log of her persistent income z. The transitory part of income is zero. Wealth and persistent income are scaled by the national wage index. The participation cost is set to zero. In panel A, all model parameters are set to their benchmark values, and replicates the skewness of stock returns and the cyclical skewness of idiosyncratic income shocks. In panel B, persistent income shocks are lognormally distributed with same persistence, variance and covariance with stock returns as in panel A. Stock market disasters occur in both specifications.

A. With cyclical skewness



B. Without cyclical skewness

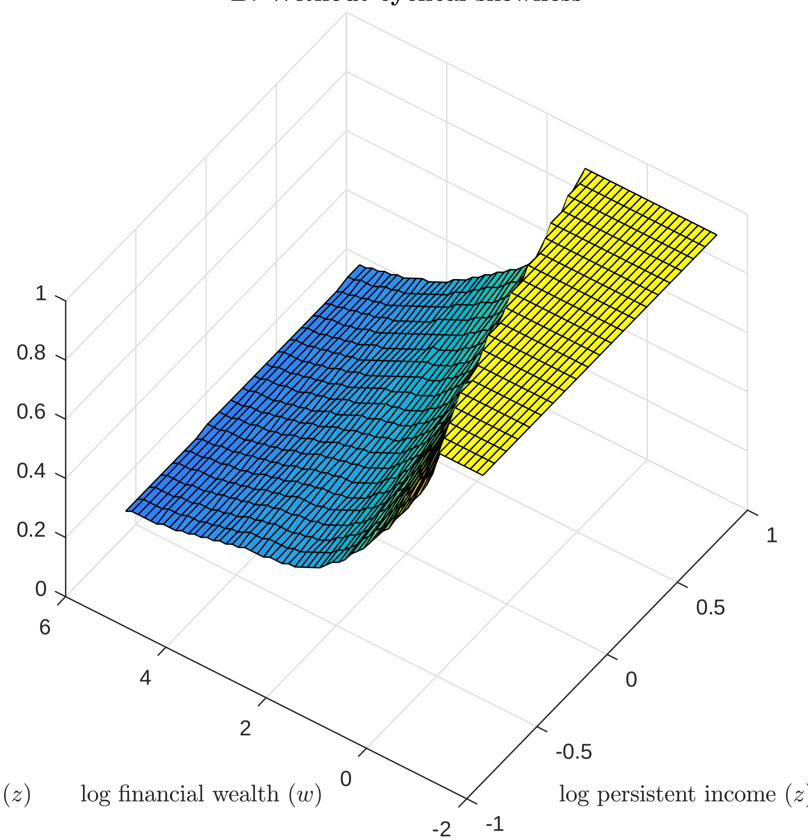


Figure 5: Conditional Equity Share and Wage-to-Wealth ratio

Note: This figure shows the relationship between the conditional equity share and the wage-to-wealth ratio, in the SCF (1989-2013) as well as in simulated data from models (2) and (5). Specifically, the conditional equity share is regressed on a set of 20 dummy variables representing quantiles of the wage-to-wealth ratio, as well as age and cohort dummies. The graph plots the value of the OLS coefficients associated to each quantile. Model (2) refers to my main specification, where labor income shocks display cyclical skewness. By contrast,labor income shocks are log-normally distributed in model (5). Stock market disasters occur in both specifications. Both models use estimated parameters reported in Table 5.

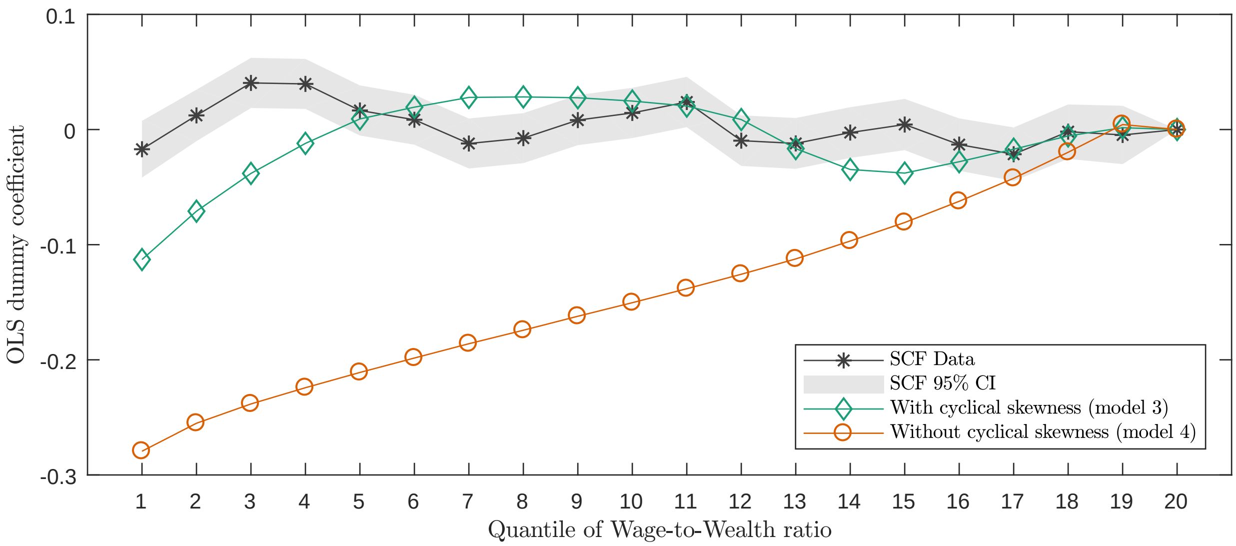


Figure 6: Decomposition of effects on the equity share

Note: These graphs plot the simulated life-cycle profile of the equity share under different scenarios. In Scenario (a), log returns and idiosyncratic income shocks are normally distributed. In scenario (b), I introduce stock market crashes. Scenario (c) incorporates the cyclical skewness of income shocks but assumes that log returns are normally distributed. Scenario (d) combines the effects of stock market crashes and cyclical skewness. Returns, transitory and persistent income shocks have identical means, variances and covariances in all scenarios.

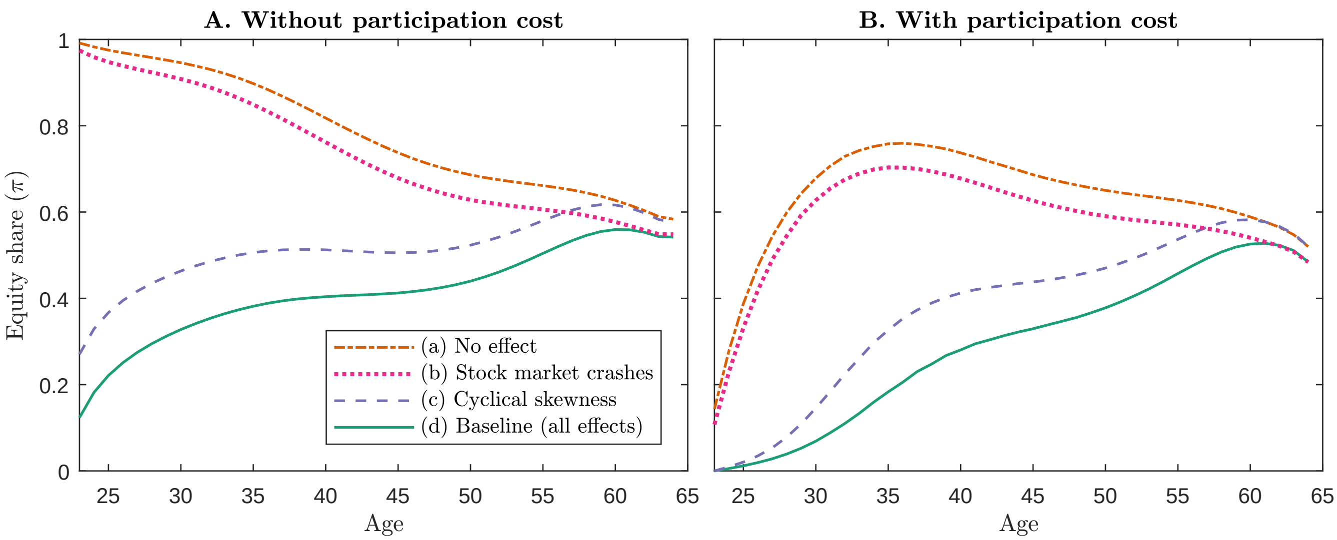


Figure 7: Effect of Social Security

Note: These graphs plot the life-cycle profiles of financial wealth, equity holdings and the unconditional equity share in the benchmark calibration, in the presence of participation costs, with and without Social Security. In the absence of Social Security, the agent does not receive any pension nor pay payroll taxes. Financial wealth and equity holdings are scaled by the national average wage.

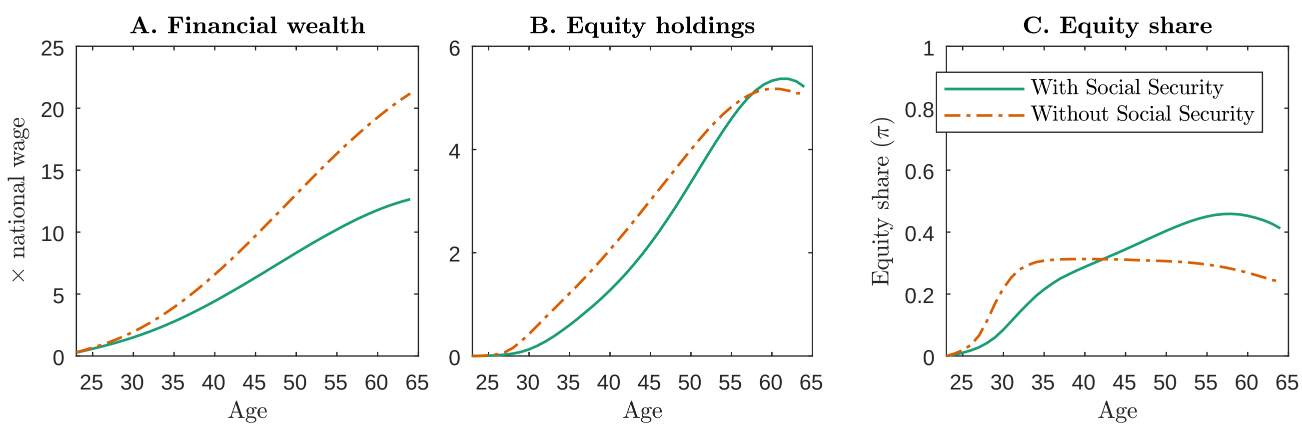


Figure 8: Decomposition of countercyclical consumption risk by deciles of wealth and age

Note: This graph plots the relative contribution of each decile of wealth and age to the cyclical skewness of consumption growth in the simulated data. Cyclicality is measured using cokurtosis, defined in equation (21). Using estimated parameters reported in column (2) of Table 5, I simulate the life-cycle of 10^6 individuals, including retirement. Simulated data are split by deciles of financial wealth (left panel) and deciles of age (right panel). The graph reports the relative contribution of each decile to the total cokurtosis and total financial wealth.

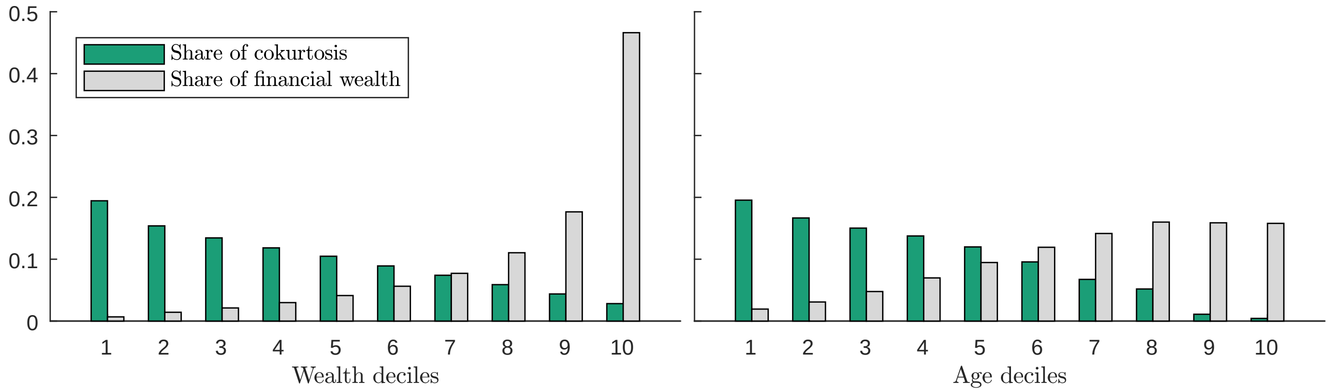
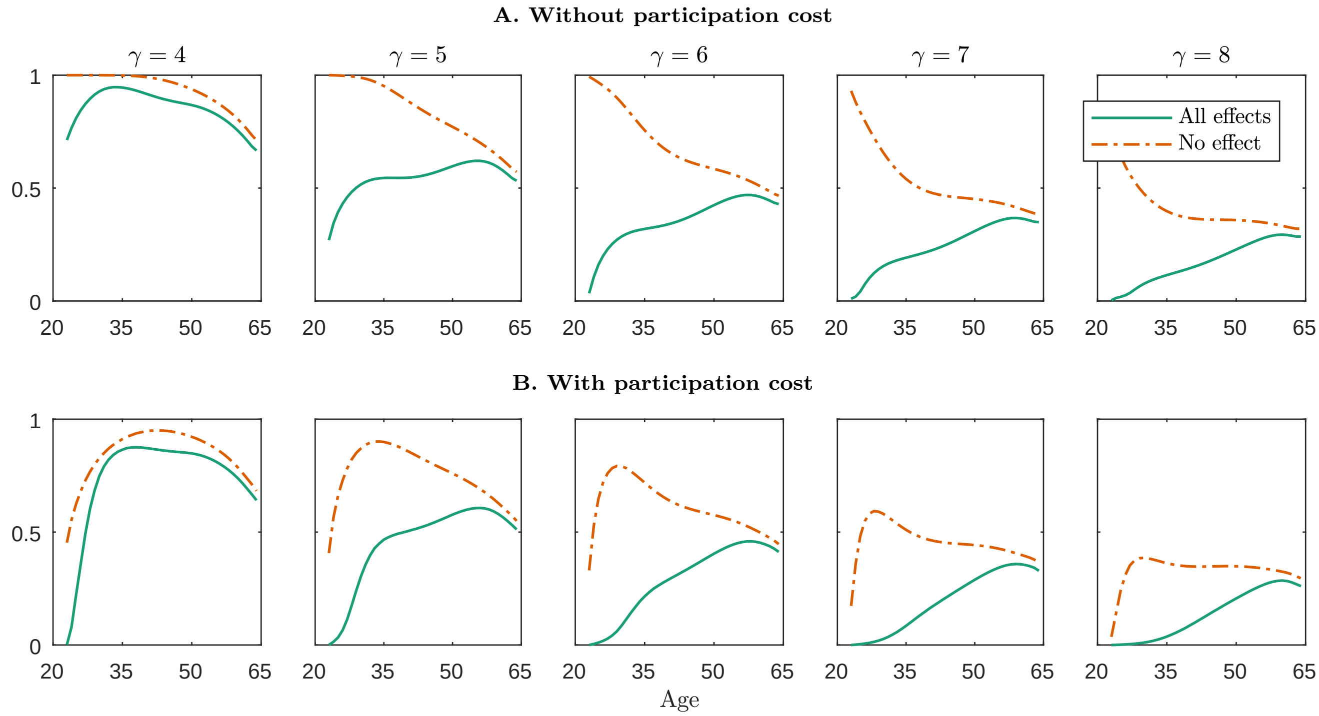


Figure 9: Sensitivity of π to risk aversion

Note: These graphs plot the evolution of the mean equity share over the life-cycle for different levels of relative risk aversion. In the "no effect" scenario, log returns and income shocks are normally distributed with same variance, covariance and persistence as in the model with cyclical skewness and stock market crashes ("all effects").



INTERNET APPENDIX

A Stock market returns and labor income risk

A.1 Idiosyncratic shocks

This section details the parametric estimation of the idiosyncratic income risk process. I largely follow the methodology of Guvenen et al. (2014) and detailed in their own appendix. I target the following moments: the standard deviation of log income changes at the 1 and 5 year horizons, and Kelly's skewness of log income changes at the 1, 3 and 5 year horizons for each year between 1978 and 2010. There are two differences in targeted moments. First, I do not include the mean, which, in my paper, is generated by another process. Since I focus on idiosyncratic shocks, the mean is subtracted from the moments provided by Guvenen et al. (2014). Second, I target the evolution of within-cohorts inequalities (standard deviation of log-income) between 25 and 60 years old, in order to control for the existence of individual fixed-effects in income growth rates. This last set of moment is taken from Guvenen et al. (2015). In total, I target 191 moments.

For each moment m_t of the skewness and standard deviation times series, the distance between the empirical moment and the simulated one is normalized. Specifically, the distance d is computed as:

$$d_{m_t}(\Phi) = \frac{m_{t,data} - m_{t,simu}(\Phi)}{\overline{m_{data}}} \tag{22}$$

where Φ is a vector of parameters and $\overline{m_{data}}$ is the historical mean of the absolute value of moment m.

For life-cycle inequalities, I compute the standard deviation for each age/year, and take the average across years. The result is the historical standard deviation of log income for each age. The distance between the simulation and the target is simply

$$d_m(\Phi) = \frac{m_{data} - m_{simu}(\Phi)}{m_{data}} \tag{23}$$

The weighting matrix Ω is diagonal and weights are chosen so that 3/4 of the total is given to the skewness and standard deviation of income changes and the remaining to life-cycle inequalities. Within each of these two groups of moments, all targets receive equal weights. As in Guvenen et al. (2014), the simulation starts in 1944 to maintain a constant age structure and each individual reaching 60 is replaced by a young worker.Aggregate income shocks before 1978 are derived from NIPA tables[12]. Each cohort includes $1,000$ individuals, which means that at

[12] This time-series is borrowed from Emmanuel Saez' website: http://eml.berkeley.edu/~saez/TabFig2012prel.xls

any time the economy is occupied by 36,000 workers. The goal of the SMM is to minimize

$$\min_{\Phi} d(\Phi)'\Omega d(\Phi) \tag{24}$$

where $d(\Phi)$ is a vector composed of the values of $d_m(\Phi)$ for all moments m.

The optimization is done in two steps. In the first stage, I generate 10^5 quasi-random vectors of parameters from a Halton sequence and evaluate the value function for each of them. In the second step, I use the Nelder-Mead method to perform a local optimization using the best 2,000 results from the first stage as starting points. Like GOS, I simulate several (5) economies and averages moments accross these economies in order to smooth the surface of the objective function.

A.2 Macroeconomic shocks

The SMM used to estimate the macroeconomic risk model is straightforward. I minimize the sum of the distances, measured in percentage as in equation (23). All moments are nearly matched by using the identity matrix.

B Numerical resolution of the model

B.1 Grid design

The state and control variables of the problem are discretized as follows. Financial wealth X is measured in national wages and takes values between 0.01 and 200. The grid is uniformly distributed between the logs of these two values with a mesh size of $\Delta(x) = .025$. The persistent component of wage takes uniformly distributed values between -3 and 3, that is .05 and 20 the national wage index, with a mesh size of $\Delta(z) = .05$. Similarly, the transitory component can take 41 equally distributed values between minus two and two standard deviations. Social Security entitlements are measured in terms of b, the percentage of the national wage that the agent would get as benefit if he had 65 years old. The upper bound of the grid is the upper limit of this percentage given Social Security rules. The lower bound is 0.001 and the mesh size $\Delta(b(S)) = .03$. Consumption is defined as a position between two bounds. I define minimum and maximum consumption such that the agent stays within the grid and has positive consumption. She chooses consumption on a grid that spans over this interval. The grid is constructed using the log of consumption with of a mesh size $\Delta(x) = .01$. Finally, the equity share can take 101 equally spaced values between 0 and 1.

This discretization generates a grid of considerable size. I detail how the problem is solved in the following sub-sections.

– Table B1.

B.2 Discretization of stochastic processes

Three state variables of the problem have a stochastic dynamics: η, z, and x.

Transitory income shocks — The dynamics of η is discretized assuming that

$$P_d(\eta_t = \tilde{\eta}) = P\left(\eta_t = \tilde{\eta} \pm \frac{\Delta(\eta)}{2}\right) \tag{25}$$

where $\tilde{\eta}$ is a point on the grid of η and P and P_d are respectively the continuous and discretized probability.

Wealth and persistent income — The discretization of the joint dynamics of x and z requires a more specific methodology due to several difficulties.

First, a transition matrix must be computed for each possible value of π. Moreover, each of these 101 matrices would include $450 \times 121 \times 450 \times 121 \approx 3 \times 10^9$ elements. To alleviate this dimensionality problem, I use the fact that $P(x_{t+1} = x_1 + \delta_x | x_t = x_1, \pi) = P(x_{t+1} = x_2 + \delta_x | x_t = x_2, \pi)$ to compute the transition matrix for only one value of x_t. I also use the fact that the transition matrix of x_t to x_{t+1} is a band matrix (when extremely small probabilities are rounded to zero) to restrict variations in x to $\pm .75$, which corresponds to $\pm$ 25 grid points. As a result, to each value of π corresponds a transition matrix of $121 \times 51 \times 121 \approx 7.5 \times 10^5$ elements.

The second difficulty is the non-trivial relationship between x and z: shocks to x are correlated with $\delta_{l,t}$, which in turn determines parameters of the distribution of z. Finally, following equation (18), the value function must be adjusted for variations in the scaling state variable l. I circumvent the last two difficulties using the following method:

1. In a first step, I discretize δ_l using a grid of 101 points uniformly distributed between $-.15$ and $.15$. I then compute the transition matrix from x_t to (x_{t+1},$\delta_{l,t}$). Since these two random variables are correlated in a trivial way, the discretized probability of any pair $(\tilde{x}, \tilde{\delta}_l)$ on the grid can be approximated using the following rule:
$$P_d(x_{t+1} = \tilde{x} \cap \delta_{l,t} = \tilde{\delta}_l | x_t, \pi) = P\left(x_{t+1} = \tilde{x} \pm \frac{\Delta(x)}{2} \bigcap \delta_{l,t} = \tilde{\delta}_l \pm \frac{\Delta(\delta_l)}{2} \middle| x_t, \pi\right) \tag{26}$$
where $\Delta(\delta_l)$ the distance between two grid points.

2. In a second step, I compute the transition matrix of z for each value of the grid of δ_l. Each component of the mixture is discretized using the Tauchen method (Tauchen (1986)). The transition matrix of z is the weighted sum of the transition matrices associated with the two components of the mixture.

3. Finally, one can then use the law of total probability along the δ_l dimension to compute the joint transition matrix of x and z. Specifically, for any pair $(\tilde{x}, \tilde{z})$ on the grid, we have
$$\begin{aligned} P_d(x_{t+1} = \tilde{x} \cap z_{t+1} = \tilde{z} | x_t, z_t, \pi) = \textstyle\sum_{\tilde{\delta}_l} \quad & P_d(x_{t+1} = \tilde{x} \cap \delta_{l,t} = \tilde{\delta}_l | x_t, \pi) \\ & \times P_d(z_{t+1} = \tilde{z} | z_t, \tilde{\delta}_l) \end{aligned} \tag{27}$$

However, because I need to take into account how variations in l affect the value function of the agent, I

actually use an adjusted version of equation (27):

$$\tilde{P}_d(x_{t+1} = \tilde{x} \cap z_{t+1} = \tilde{z} | x_t, z_t, \pi) = \sum_{\tilde{\delta}_l} \begin{array}{l} P_d(x_{t+1} = \tilde{x} \cap \delta_{l,t} = \tilde{\delta}_l | x_t, \pi) \\ \times P_d(z_{t+1} = \tilde{z} | z_t, \tilde{\delta}_l) e^{(1-\gamma)\tilde{\delta}_l} \end{array} \tag{28}$$

As a result of the positive drift of l, this formula implies that the rows of the transition matrix sum to a number slightly below one.

B.3 Resolution of the Bellman equation

The model is solved by dynamic programming. Each year is split into two sub-periods. In the first sub-period, the agent receives her transitory income shock, receives her wage and decides how much to consume. In the second sub-period, she decides how to invest her remaining wealth, and then receives shocks to her wealth and her persistent income. Following the logic of backward induction, the two sub-periods are solved in reverse order.

Consumption sub-period — For a given consumption, the continuation value of the agent is determined by the new value of x and the new value of b, which depends on her current wage and income record.

Portfolio choice sub-period — In the second period, the agent chooses her equity share by maximizing her expected utility. In this period, I need to extrapolate the value function along the x dimension because equity returns can move the agent across the borders of the grid. This has little practical consequences since the wealth grid spans from 0.01 to 200 national wages. Transitory and persistent income shocks happen in different sub-periods, even though the same Bernoulli variable determines which component of their respective mixtures is received by the agent. To solve this problem, I keep track of this Bernoulli variable. Although this method multiplies the state space by two, it remains much less computationally intensive than using a joint transition matrix of z, x and η.

Extrapolation methodology — The negative of the value function is of the Cobb-Douglas form and therefore log-linear with respect to the log of consumption. To take advantage of this property, I use log-linear extrapolations of the negative of V, and then take the negative of the exponential of the result to retrieve the extrapolated value function.

Retirement period — Retirement years may be seen as degenerated working years, where state variables z and η become irrelevant. Yet, because benefits are no longer wage indexed, the value function is no longer scalable by the national wage. To avoid unnecessary complications, I simply assume a fixed wage index during retirement. The only economic shortcoming of this assumption is to freeze the dollar value of fixed participation costs for retirees.

B.4 Simulated Method of Moments

The SMM procedure follows the same global optimization routine as the one described in appendix section A.1. In the first (global) stage, I test 2,000 quasi-randoms vectors of parameters. In the second stage, I run 10 local

optimization around the best points of the first stage. I check that the local optimizations converge to the same point and take it as evidence of a global minimum.

To compute moments, I simulate a sample of 3×10^5 individuals, each receiving different idiosyncratic and macroeconomic shocks. As a results, there is no cohort and year effects in the simulated data. The predicted equity share for a given age group is simply the mean among all individuals in that age group. The same logic applies for wealth, participation and the conditional equity share.

The standard errors are estimated by computing the Jacobian matrix J of the moments with respect to estimated parameters. The standard errors are the square roots of the diagonal elements of matrix Q, which is defined as:

$$Q = \left[J^T \mathbf{W} J\right]^{-1} \tag{29}$$