Inflating Away the Public Debt? An Empirical Assessment*

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Abstract

While it is well understood theoretically that higher inflation will lower the real value of outstanding government debt, empirically there is neither a method nor plausible estimates of how large this effect will be. We propose a method that takes an ex ante perspective of the government budget constraint, and relies on having detailed information on debt held by the public at different maturities, risk-neutral densities for future inflation at different horizons, and a set of plausible counterfactuals. Applying it to the United States in 2012, we estimate that the effects of higher inflation on the fiscal burden are modest. A more promising route to inflate away the public debt is to use financial repression, and we estimate that a decade of repression combined with inflation could wipe out almost half of the debt.

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1 Introduction

A higher inflation target has some benefits, and one of its most celebrated is to erode the real value of outstanding debt. Across centuries and countries, a common way sovereigns pay for high public debt is by having higher, and sometimes even hyper, inflation. At the same time, higher inflation is often not the way out and rarely does it come without some fiscal consolidation (Reinhart and Rogoff, 2009). It is an open empirical question how effective is higher inflation at alleviating the fiscal burden of a country. A more specific application is: with U.S. total public debt at its highest ratio of GDP since 1947, would higher inflation be an effective way to pay for it?

Providing an answer requires tackling two separate issues. The first is to calculate by how much would 1% unanticipated and permanent higher inflation lower the debt burden. If all of the U.S. public debt outstanding in 2012 (101% of GDP) had one year maturity and was held by the public then the answer is 1%. Noting that the the average maturity of the federal debt reported by the Office of Management and Budget is 5 years, another quick back-of-the-envelope answer would instead be 5%.[1] Unfortunately, as we will show, the approximations behind these calculations give misleading estimates. The debt number above is incorrect, because a large part of the debt is held by other branches of the government, and another large share has payments indexed to inflation. The maturity number is inaccurate because of a large composition bias in trying to simplify the rich distribution of debt held by the public at different maturities with ill-fitting single-parameter approximations.

The second issue is that assuming a sudden and permanent increase in inflation by an arbitrary amount is implausible and not helpful. After all, if the price level could suddenly

[1] This calculation is justified as follows: assuming the government owes X to be all paid in m periods, then its current market value is $V = Xe^{-(r+\pi)m}$ with continuous-time discounting and where r is the real interest rate and π is the rate of inflation, assumed to be constant. Then $\partial V/\partial\pi = -mV$. An alternative justification assumes that the government owes an amount $Xe^{-t/m}$ at every future date t, so the distribution of outstanding debt is exponential. Its market value today then is: $X/(r + \pi + 1/m)$. Since the nominal interest is approximately zero, we get approximately the same answer.

jump to infinity, the entire debt burden would be trivially eliminated. It is important first to recognize that inflation is stochastic, and that investors will take this risk into account when choosing to hold and pricing government debt. Moreover, there are many options to achieve higher inflation, either doing so gradually or suddenly, permanently or transitorily, in an expected or unexpected way, and we would like to know how they vary in effectiveness. Finally, because the central bank does not perfectly control inflation, we would like to discipline the counterfactual experiment by considering only changes in inflation that economic agents believe are plausible and achievable.

This paper addresses both issues, providing a method for estimating the effect of inflation on the fiscal burden. Section 2 lays out our approach, which takes a forward-looking approach to the intertemporal government budget constraint to arrive at a simple formula for the fiscal burden of outstanding debt. It is equal to a weighted average of the payments due at different horizon, with the weights given by the expected inverse of compounded inflation under a risk-neutral measure. Higher future inflation can lower the fiscal burden by affecting these weights.

Section 3 collects data on the payments due by the federal government to private entities associated with Treasury securities, using investor data on bond holdings of different maturities. We further separate private holdings between domestic and foreign, and we single out the Federal Reserve in the public holdings. Finally, using data on the real yield curve, we measure the market value of the debt held at different horizons by these different agents. The calculations that we put together should be useful to other researchers that need estimates of the financial commitments of the U.S. government.

Section 4 introduces data on inflation contracts, in the form of caps and floor derivatives that depend on the realizations of CPI inflation. Exploring the variety of contracts that are traded at a date with different strike prices, we can extract the implied stochastic discount factors for inflation used by market participants at different horizons. We build on, and go beyond, the calculations in contemporaneous work by Kitsul and Wright (2013) and Fleckenstein, Longstaff and Lustig (2013), providing risk-neutral density functions for both cumulative inflation and year-on-year inflation.

Section 5 sets up our counterfactuals. Aside from giving us the right stochastic discount factors, the data on options also constrain the inflation counterfactuals to not be too grossly out of line with what the market expects inflation to be. However, this requires knowing the joint risk-neutral distribution of inflation at different horizons, while the options data only has the marginal densities. We propose a new method to estimate this distribution, which has the intriguing feature of not using any time-series data on the realizations of inflation. It is based entirely on the information provided by the different options contracts at a given date, and relies on the theory of copulas.

Finally, section 6 performs our calculations using U.S. data for the end of 2012. Our main finding is that realistically higher inflation would have a relatively modest effect on the fiscal burden. This is driven by two complementary factors. First, the maturity of U.S government debt held by the public is quite low. By comparison, we show that with the outstanding debt composition of 2000 the effects would be much larger, and that the Federal Reserve, which in 2012 held significant long-term debt, would suffer a significant loss with higher inflation. Second and related, over only a few years, market participants put a very low probability on U.S. inflation being significantly high. In the near horizons, there is much debt but little extra inflation, and in the far horizons, there can be significant inflation but little debt. The total resulting effect is small. An interesting observation is that much of the effect would fall on foreign holders of the government debt, which tend to hold the longer maturities.

Section 7 explores two alternatives. First, we consider the possibility that higher inflation leads to lower real interest rates. Theoretically, we show that his has opposing effects on the fiscal burden and the overall effect is likely low. Second, we consider an alternative that is often mentioned for developing countries: financial repression. While, ultimately, the government could reap large gains from taxing all financial activities (as well as induce larger social costs) we consider an alternative akin to monetizing all of the outstanding debt. It consists of forcing bondholders to swap their existing claims for new zero-interest debt. We calculate both the debt relief that this would provide, as well as the impact that our inflation counterfactuals would now have. The effects are much larger.

Section 8 concludes with suggestions for further research.

Turning to our contributions to the literature, we cannot do full justice to the many studies on the link between fiscal policy and inflation. More recently, Cochrane (2011*b*) and Davig, Leeper and Walker (2011) have argued that high levels of U.S. debt may lead to higher inflation through the fiscal theory of the price level, while Aizenman and Marion (2011) argue that policymakers have a strong incentive to inflate this debt. Our goal is more applied as we try to provide a first attempt to quantify by how much inflation can actually lower the public debt burden. We contribute to this literature by providing estimates that might be useful to calibrate models in the future.

Closest to our question, Hall and Sargent (2011) provide an accounting decomposition of the evolution of public debt applied to U.S. historical data, while Reinhart and Sbrancia (2011) emphasize that inflation coupled with financial repression helped developed countries to pay their debts after World War II. Our methods are instead forward looking, which allow us to consider different counterfactual scenarios. We derive a quite general but simple formula to measure the impact of future inflation on the current debt burden, which the literature going forward might find useful.

Aizenman and Marion (2011) and Bohn (2011) also ask counterfactuals about the future, but they make rough approximations of the maturity of debt held by the public and treat inflation as deterministic. We tackle these two issues directly and as precisely as possible. Our data on the distribution of debt of different maturities held by different agents, available to other researchers, makes the approximations no longer necessary.

Krause and Moyen (2013) use a DSGE model, making many structural and behavioral assumptions and investigating many links through inflation may affect debt, fiscal surpluses and seignorage. We only assume no arbitrage in the government debt market, and focus exclusively on the debasement of debt. At the same time, they make many approximations in treating the data, whereas we go into more detail. Faraglia et al. (2013) and Leeper and Zhou (2013) also write DSGE models to study how optimal inflation depends on the maturity of government debt partly through its effect on the real value of debt. In the other direction, Arellano et al. (2013) develop a model of the maturity of debt as a function of the government's credibility to keep inflation low. Our goal is positive, not normative, and again our estimates should allow researchers to calibrate their models.

Berndt, Lustig and Yeltekin (2012) and Chung and Leeper (2007) use vector autoregressions to estimate the impact of fiscal spending shocks on different terms in the intertemporal budget constraint. We focus on inflation shocks instead, and we directly measure the impact on future discount rates using inflation options data. Giannitsarou and Scott (2008) show that fiscal imbalances do not help to forecast future inflation in six advanced economies. We instead use options market data to make forecasts, and we ask not whether in the past government have used inflation to pay for debts, but rather what would be the impact of doing it today. Our goal is to understand what are the limits to using the option to inflate, rather than to ask whether or not that option has been chosen in the past.

Finally, while there are many ways to extract objective and subjective probability forecasts for inflation, including financial prices, surveys, and economic and statistical models, these methods tend to forecast the mean while being silent on higher moments. Crucially, they are not appropriate for pricing. Our goal is to measure the market value of different policies, so we need the risk-neutral probabilities that are relevant for pricing the government debt. Together with Kitsul and Wright (2013) and Fleckenstein, Longstaff and Lustig

(2013), we are one of the first papers to use data on inflation option contracts to extract the risk-neutral density for inflation and use it to ask macroeconomic questions. Kitsul and Wright (2013) look at the response of the density around monetary policy announcements, while Fleckenstein, Longstaff and Lustig (2013) assess the risk of deflation in the United States. We ask a different question. Our new method of moments using copulas can be used with other options data to estimate joint distributions for other series.

2 Theory: the debt burden and risk-neutral densities

Our goal is to measure the fall in the debt burden with higher inflation. This requires coming up with a workable definition of the debt burden, seeing the effect of inflation on it, and deriving a formula to estimate its size. We take each step on turn.

2.1 The public debt

Letting W_t denote the real market value of government debt at date t:

$$W_t = \sum_{j=0}^{\infty} \frac{H_t^j B_t^j}{P_t} + \sum_{j=0}^{\infty} Q_t^j K_t^j. \tag{1}$$

Going over each of the terms on the right-hand-side: B_t^j is the par value of zero-coupon nominal debt held at date t that has a maturity of j years, so that at date t the government expects to pay B_t^j dollars at date $t+j$. K_t^j is the par value of real debt held at date t that has a maturity of j years, referring mostly to Treasury indexed protected securities (TIPS). H_t^j is the market price (or inverse-yield) at which nominal debt with a maturity of j years trades at date t. Likewise, Q_t^j is the price (or inverse-yield) of TIPS with q maturity of j years at date t. Finally, P_t is the price level, and we will use the notation $\pi_{t,t+j} = P_{t+j}/P_t$ to denote gross cumulative inflation between two dates. The following normalizations apply:

$H_t^0 = Q_t^0 = 1$ and $P_0 = 1$.

Modeling the government debt this way involves some simplifications. First, the government often has a wide variety of non-market outstanding debt. The implicit assumption above is that their price is the same as that of marketable debt, which should be the case through the forces of arbitrage between these different securities. Second, we assume that coupon-paying bonds can be priced as portfolios of zero-coupon bonds. In this way, we limit the huge variety of debt instruments issued by the government and simply consider promised payments (either principal or coupon payments) at each point in time. Again, arbitrage should imply that this assumption is reasonable. Third, unfunded liabilities of the government like Social Security could be included in B_t^j, and the real assets of the government could be included in K_t^j, so that theoretically they pose no problem. In practice, measuring either of these precisely is a challenge that goes beyond this paper, so we will leave them out.

If all debt was short-term, then the expression in equation (1) would reduce to $B_t^0/P_t + K_t^0$. The first rule of thumb that we discussed in the introduction would be exact: an increase in P_t would lower the debt burden by an amount exactly proportionately to the nominal debt held by the public. However, with multiple maturities, higher inflation would surely affect the prices of the different government liabilities, so this equation is not enough.

2.2 The law of motion for debt

To pay for the debt, the government must either collect a real fiscal primary surplus of s_t, or borrow more from the private sector:

$$W_t = s_t + \sum_{j=0}^{\infty} \frac{H_t^{j+1} B_{t+1}^j}{P_t} + \sum_{j=0}^{\infty} Q_t^{j+1} K_{t+1}^j. \tag{2}$$

Combining the previous two equations provides a law of motion for debt. Looking forward from date 0 for t periods, we can write it as:

$$\begin{aligned} W_0 &= W_{t+1} \sum_{i=0}^{t} Q_i^1 + \sum_{i=0}^{t} Q_i^1 s_t \\ &+ \sum_{i=0}^{t} Q_i^1 \sum_{j=0}^{\infty} \left(H_i^{j+1} - H_i^1 H_{i+1}^j \right) \frac{B_{i+1}^j}{P_i} + \sum_{i=0}^{t} Q_i^1 \sum_{j=0}^{\infty} \left(Q_i^{j+1} - Q_i^1 Q_{i+1}^j \right) K_{i+1}^j \\ &+ \sum_{i=0}^{t} Q_i^1 \left(\frac{H_i^1 P_{i+1}}{P_i} - Q_i^1 \right) \sum_{j=0}^{\infty} \frac{H_i^{j+1} B_{i+1}^j}{P_{i+1}}. \end{aligned} \tag{3}$$

This equation makes apparent why it is difficult to answer our question. Inflation can affect almost every term on the right-hand side without a clear way to decompose them. Worse, in order to judge how a particular path for inflation $\{\pi_{0,i}\}_{i=0}^t$ affects the fiscal burden, we would need to know how inflation will change the slope of the yield curve at every maturity (the $H_i^{j+1} - H_i^1 H_{i+1}^j$ term) or the maturity composition that future governments will choose (the B_{i+1}^j term). Likewise, we would need to know the link between inflation the real yield curve (the $Q_i^{j+1} - Q_i^1 Q_{i+1}^j$ term) as well as the ex post differences between nominal and real returns (the $H_i^1 P_{i+1}/P_i - Q_i^1$ term). Finally, recall that this expression holds for every possible path of inflation as well as for realization of uncertainty in the economy. Therefore, there is an unwieldy large number of possible measures of how much the fiscal burden will change in the future.

Hall and Sargent (2011), and many that preceded them, partially overcome these problems by using a version of this equation to look backwards, instead of forward, in time. Therefore they have historical data on most of the terms above. Still, their decomposition of the factors affecting the evolution of the debt is only able to isolate the effect of inflation while keeping fixed every other interest rate, fiscal surplus, and outstanding bonds. Moreover, our question requires us to look forward to figure out how debt depends on future, not past, inflation.

2.3 Looking forward: the intertemporal budget constraint

Our approach replies on one assumption: that there is a unique stochastic discount factor to price all of these government liabilities. It is well understood that this is a strong assumption, requiring the absence of arbitrage, and there is plentiful evidence against it (Cochrane, 2011*a*). At the same time, government bond markets are among the most liquid in the United States, have fewer restrictions on short-selling, and serve as the fundamental asset for many traded derivatives. Therefore, in terms of their relative returns across maturities, assuming the efficiency of government bond markets is not a terrible approximation.

The stochastic discount factor at date t for a real payoff at date $t+j$ is denoted by $m_{t,t+j}$, and the conventional pricing equation for a j-period bond is:

$$1 = \mathbb{E}\left(\frac{m_{t,t+j}}{Q_t^j}\right) = \mathbb{E}\left(\frac{m_{t,t+j}P_t}{H_t^j P_{t+j}}\right). \tag{4}$$

Intuitively, a nominal bond costs $Q_{t,t+j}/P_t$ in real units at date t, and pays off $1/P_{t+j}$ real units in j periods; this return times the stochastic discount factor has to have an expectation of 1. The absence of arbitrage over time implies that the stochastic discount factors across any two maturities, k and j are linked by: $m_{t,t+j} = m_{t,t+k}m_{t+k,t+j}$ for $1 \leq k \leq j$.

Multiplying by stochastic discount factors at different dates, and taking expectations of equation (3), while taking the limit as time goes to infinity and imposing that the government cannot run a Ponzi-scheme, we get the following result.

$$W_0 = \mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t}\left(\frac{B_0^t}{P_t}\right)\right] + \mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t}K_0^t\right] = \mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t}s_t\right] \tag{5}$$

The first equality provides a workable measure of the debt burden. It does not depend on the prices of bonds, and it makes clear how not just the current but all future price levels matter. Moreover, as in the case with only short-term bonds, it shows that we can focus on only the nominal debt as long as we assume that changes in inflation do not affect the stochastic discount factor. We will maintain that assumption for our first calculations, and consider the relation between inflation and real interest rates in section 7.

The second equality shows that we can interpret our measures as saying how much fewer taxes the government can collect by lowering the debt burden. Note that higher inflation will potentially not only lower the real payments on the outstanding nominal debt, but also change primary fiscal surplus. In companion work (Hilscher, Raviv and Reis, 2014), we measure one of these main effects, through the seignorage revenues that higher inflation generates. Here, we focus solely on inflating away the outstanding public debt.

2.4 A formula for the debt burden as a weighted average

The only uncertainty on how much the nominal debts that are outstanding and mature in t periods will pay is on the realization of the price level. Therefore, even though the stochastic discount factor depends, in principle, on all sources of uncertainty in the economy, only its marginal distribution with respect to inflation will lead to non-zero terms in the expression for the debt burden. Therefore:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t}\left(\frac{B_0^t}{P_t}\right)\right] = \sum_{t=0}^{\infty} B_0^t\,\mathbb{E}\left(\frac{m(\pi_{0,t})}{\pi_{0,t}}\right) = \sum_{t=0}^{\infty} R_t^{-1} B_0^t \int \left(\frac{f(\pi_{0,t})}{\pi_{0,t}}\right) d\pi_{0,t}. \tag{6}$$

The second equality uses the standard definition of a risk-neutral density $f(.)$, where R_t is the real risk-free return between 0 and t, which by definition does not depend on inflation.

Combining all the results gives our formula for the debt burden as a function of inflation.

Proposition 1. *The debt burden is a weighted average of the nominal payments that the government must make at all present and future dates:*

$$\sum_{t=0}^{\infty} \omega_t B_0^t \tag{7}$$

with weights given by:

$$\omega_t = R_t^{-1} \int \left(\frac{f(\pi_{0,t})}{\pi_{0,t}} \right) d\pi_{0,t} \tag{8}$$

This formula makes clear how future inflation affects the debt burden today. It takes account of inflation being stochastic and not perfectly controlled by evaluating an integral. It is forward-looking, and it delivers a single number in spite of all the possible future histories. It depends on inflation only, as all of its relevant effects on prices are captured in the inflation densities. Finally, it allows for a discussion of counterfactuals that is somewhat disciplined, in terms of either different realizations from these densities or shifts in the densities themselves. Therefore, it satisfies all of the requirements that we laid out to answer our question.

Using this formula requires two key inputs, the payments due to the public, and the risk-neutral densities, at each future maturity. The next two sections explain how we measure them for the United States.

3 Data: U.S. Treasuries held by the public by maturity

The total U.S. public debt at the end of 2012 reported by the Bureau of the Public Debt were \$16.4 trillion, or 101% of GDP. While this number may serve as a starting point for our calculations, it needs much further work. For one, it includes both nominal and indexed bonds. Proposition 1 makes clear that we want to measure only the nominal payments due. Moreover, it includes debt held by different branches of the U.S. government. However, when the real value of the Treasury securities held by the social security trust fund falls because of inflation, the net liabilities of the federal government are unchanged, as the Treasury will sooner or later have to cover these losses. Finally, we need to know the maturity structure of when payments are due, not just their aggregate marker value.

3.1 Data on private holdings

We use data from the Center for Research on Security Prices (CRSP) on private holdings of marketable government notes and bonds at the end of 2012. We measure monthly total nominal payments on nominal marketable debt, using both face value and coupons at each maturity. This includes bills, notes and bonds, all of which promise a stream of dollar payments into the future.

We exclude outstanding TIPS, which amounted to $0.9 trillion in 2012. We also exclude non-marketable debt, which amounted to a considerable $5.4 trillion. Fortunately, almost all of these non-market instruments—approximately $4.8 trillion—are held by Social security in intergovernmental accounts. We are left with $11.0 trillion of public debt to account for.

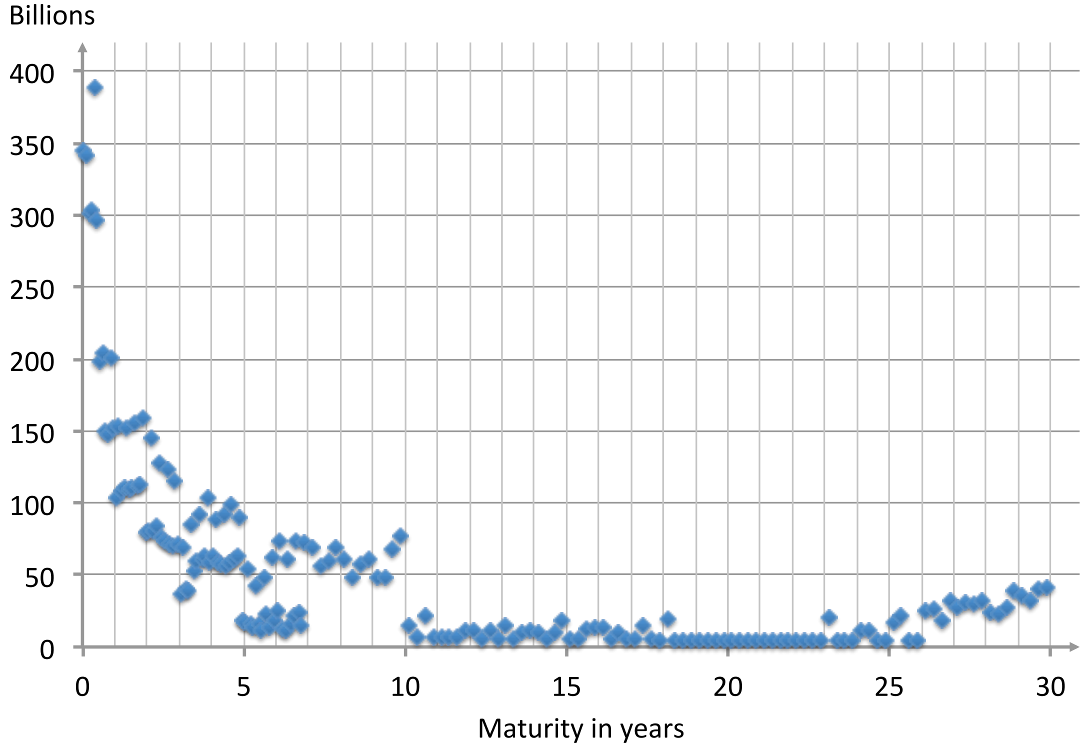
While this is sometimes misleadingly called “debt held by the public”, it includes the holdings by the Federal Reserve System. Any losses on the portfolio of the central bank will map directly into smaller seignorage payments to the Treasury (Hall and Reis, 2013), so the same argument that excludes Social Security holdings should apply to Federal Reserve holdings. The Federal Reserve Bank of New York keeps the information on each bond held by the Federal Reserve in its SOMA account, and we use this information, bond by bond, to also obtain detailed holdings for the Federal Reserve at each maturity. We exclude these $1.9 trillion from our baseline, but will consider them separately in the analysis.

Finally, we also exclude state and local branches of the government using data from the Treasury. Non-federal authorities issue a negligible share of the total debt ($0.2 trillion) but they hold approximately $0.7 trillion, especially in state and local pension funds.

CRSP does not have data on Treasury bills. We use the issues of the Treasury bulletin to obtain information on bills and follow the same steps as we did above for notes and bonds.

At the end of these manipulations, using the formula in equation (1), we calculate the market value of nominal debt using the nominal and real yield curve. The total market value of publicly-held marketable Treasury securities in 2012 was $8.5 trillion, or 53% of GDP. A

Figure 1: Government payments due to the private sector by maturity (B_0^t)



check on our estimates of the risk-neutral distributions is that we can also calculate the market value of debt according to equation (7). The discrepancy between the two measures is negligible, at about \$12 billion. We also have detailed data by maturity on how much is held by foreigners. The total is \$5.3 trillion, or 33% of GDP, of which 6.9% of GDP is held by Chinese. For the results that follow, it is important to keep in mind that inflation at most can improve the fiscal burden by 53% of GDP, but that a large share of possible losses might be borne by foreigners.

3.2 The maturity distribution of holdings

Figure 1 shows B_0^t as a function of t using monthly data. A noticeable feature of the distribution is that three quarters of the payments are due in less than 4.5 years. The average maturity of the U.S. government debt held by the public is 3.7 years according to our calculations, well below the 5.4 years reported by the Treasury.[2] Because most of the holdings of government debt by the Federal Reserve are today at very long maturities, the debt held by the public is of lower maturity.

Moreover, it is salient from figure 1 that simple approximations, like a single bond with a maturity equal to the average or an exponential profile, give crude approximations. A fitted exponential decay of Treasury bills would be much higher than what would be inferred from the Treasury bonds between 1 and 5 years. Moreover, between 5 and 10 years, the maturity distribution is not at all close to an exponential, oscillating around $50 billion. Finally, between 10 to 25 years, there is very little mass, which picks up between 25 and 30 years, but overall, there is little debt held by the public of maturity above 10 years. This is partly driven by the quantitative easing policies pursued by the Federal Reserve, which took many longer maturities from private hands, and partly by the lack of issuance of 30-year bonds between 2001 and 2006. To get reliable estimates of the fiscal burden, one must take the entire vector of observations displayed in the figure.

4 The marginal densities for inflation

The weights in equation (8) require knowing the term structure of risk-free real rates and the risk-neutral density of inflation at different maturities. For the former, we use standard estimates of the real yield curve from Gurkaynak, Sack and Wright (2010). Since, by assumption, they do not depend on inflation, this has little effect on the calculations. For the densities, we use new data on option contracts.

[2]This is the Fisher-Weil measure; the Macaulay duration is 4.1 years. The Treasury estimate comes from the Quarterly Refunding Documents and refer to the total public debt.

4.1 The data

The market for over-the-counter USD inflation options emerged in 2002. By 2011, trading in the inter-dealer market was close to 22 billion, while today approximately 100 billion are traded. Kitsul and Wright (2013) and Fleckenstein, Longstaff and Lustig (2013) use data from these markets as well, and argue that since 2009 the market has been liquid enough to reliably reflect market expectations of inflation. We use daily data of caps and floors on CPI inflation, which is available through Bloomberg.[3] For our baseline estimates, we use the prices on December 31 of 2012. The appendix compares these with the average prices in the month before and after to ensure there was nothing special about this particular date.

Like Kitsul and Wright (2013) and Fleckenstein, Longstaff and Lustig (2013), we use data on zero-coupon caps and floors that pay off if average inflation between the start of the contract and its maturity lies above or below the strike price.[4] The strike price ranges are -2% to 3% (floors) and 1% to 6% (caps), in 0.5% increments, so 22 separate prices.[5] We have data for maturities between 1 and 15 years, except for maturities of 11 years, and the data for the 2 and 9 year maturities are of generally lower quality.

Unlike Kitsul and Wright (2013) and Fleckenstein, Longstaff and Lustig (2013), we also use data on year-on-year (yoy) inflation caps and floors. These contracts are portfolios of caplets and floorlets that pay off at the end of each year if inflation during that year is above or below the strike price. The maturity of these goes from from 1 to 10 years for yoy options, although the 9-year maturity is again of generally lower quality.

[3] Kitsul and Wright (2013) use data provided by an interdealer organization, whereas we use the raw reported numbers. These require a considerable amount of cleaning, but allow us to do an extensive amount of cross checking, described in the appendix.

[4] Specifically, cumulative inflation is compared to the annually compounded strike price.

[5] For the overlapping range of strike prices we use both option prices to reduce measurement error.

4.2 Estimating the risk-neutral density for inflation

As the classic work of Breeden and Litzenberger (1978) noted, given a rich enough set of option contracts with observable prices, it is possible to recover non-parametrically the distribution of inflation without even making any specific distributional assumption about inflation or its link to other asset prices.

Note that through this procedure, the data do not reveal point expectations of future inflation but rather the risk-neutral distributions for the future. That is, the strike prices reflect the likelihood of different values of inflation, the risk associated with them, and the market price of this risk. This distinguishes our measures from many of the common measures of inflation expectations. Unlike opinion surveys, we are extracting risk-neutral rather than subjective expectations, and we do so from observing profit-making behavior. Unlike the break-even rate of inflation, from comparing real and nominal yields, we have a whole distribution for inflation instead of a single expected number. Moreover, we do not need to worry about the liquidity differences between nominal bond and TIPS markets, or the price of the embedded floor which ensures that TIPS always pay back at least par value. Finally, unlike models of the term structure that use the yield curve to extract market-based inflation expatiations, our measure does not rely on the associated (often strong) identifying assumptions in these models.

The price X_0 of a simple European call option with maturity t with a strike price S on inflation $\pi_{0,t}$ with a risk-neutral density $f(\pi_{0,t})$ and the risk-free rate R_t is:

$$X_0 = R_t^{-1} \int_S^{\infty} (\pi_{0,t} - S) f(\pi_{0,t}) d\pi_{0,t}. \tag{9}$$

As Breeden and Litzenberger (1978) show, it follows from this formula that:

$$f(\pi_{0,t}) = R_t \left(\frac{\partial^2 X_0}{\partial S^2} \right). \tag{10}$$

Therefore, we can extract the risk-neutral density by observing how the price of the option varies with changes in the strike price.[6] Intuitively, if the price of a call declines quickly with the strike price, then the outcome at that point is more likely. Because our data give us many option prices for different strike prices and at different intervals for inflation, we can estimate this partial derivative by using the differences in these prices.

In practice, because there is considerable noise in our data and because call-pricing functions are not always well behaved, we need to do considerable work in treating the data. In particular, first we drop option prices from the data if they contain simple arbitrage opportunities: (i) if the call (put) premium does not monotonically decrease (increase) in the strike price, (ii) if the option premium does not increase monotonically with maturity, and (iii) if butterfly spreads do not have positive prices (more on this in the appendix). Next, we calculate the implied volatilities from the option prices, smooth them with a spline, and convert back to option prices. This allows us to smooth the data before we take several differences. We then use equation (10) and the delta method to calculate two finite differences that approximate the second partial derivative. The appendix provides more details.

The appendix also outlines a second approach to extract the risk-neutral density, also proposed by Breeden and Litzenberger (1978), but which does not use equation (10). Instead, one can use our very rich menu of traded options to construct a portfolio (a butterfly spread) that approximate the price of an Arrow-Debreu security. From these, the risk-neutral density follows immediately. The results are similar and we discuss them in the appendix.

4.3 The estimates

Using our data on zero-coupon floors and caps, we extract the risk-neutral density $f(\pi_{t,t+k})$, with $k = 1, .., 15$. This gives a term structure of the cumulative inflation distributions.

[6] To see the result, take derivative of the pricing equation with respect to S to obtain the cumulative density function $F(S) = R_{0,t}\frac{\partial X_0}{\partial S} + 1$, and a second derivative to obtain the result.

In turn, using data on year-on-year inflation swaps, we construct forward distributions for year-on-year inflation $f(\pi_{t+k-1,t+k})$ with $k = 1, .., 10$. Because we have 22 strike prices, these distributions come in $N = 21$ equally-spaced bins.

Figures 2 and 3 plot the distributions at the end of 2012. Noticeably, the mode of inflation in 2013 was only 1%. Beyond 5 years, all of the distributions are bell-shaped and with similar median and mode, between 2% and 3%. Moreover, all the distributions have fat tails.Kitsul and Wright (2013) interpret the tails as saying that investors perceive both very high and very low inflation as the costly states of the world.

Another interesting feature of the distributions is that, as the horizon increases, the variance increases. The distribution becomes more spread out, either because extreme events far in the future are perceived as more costly, or because there is more uncertainty about inflation. Importantly for our calculations, note that, for cumulative inflation, even the 90th percentile is never above 5% per year at any horizon. Sustained high risk-adjusted inflation is perceived as being a very remote possibility.

5 The counterfactuals and the joint distribution

To evaluate the weights in our formula, all that we need are the marginal distributions for cumulative inflation in the previous section. However, for our experiments, we would like to know the joint distribution of inflation across multiple years to consider different types of shifts in this distribution and to be able to draw sequences for inflation. First we describe how to obtain this distribution, and after we present those experiments.

5.1 A method of moments copula-based estimator

Understanding how the realizations of a random variable are related over time is, of course, the classic problem in time-series modeling. We could in principle use one of the dozens of

Figure 2: Risk-neutral densities for cumulative annualized inflation

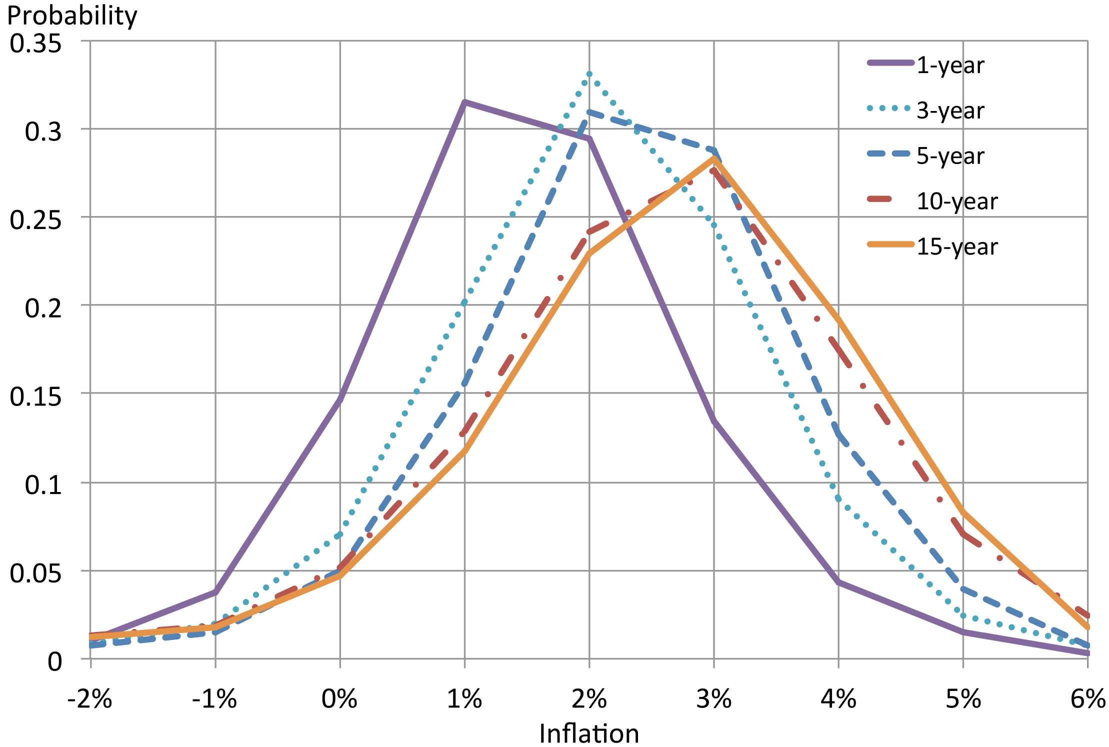
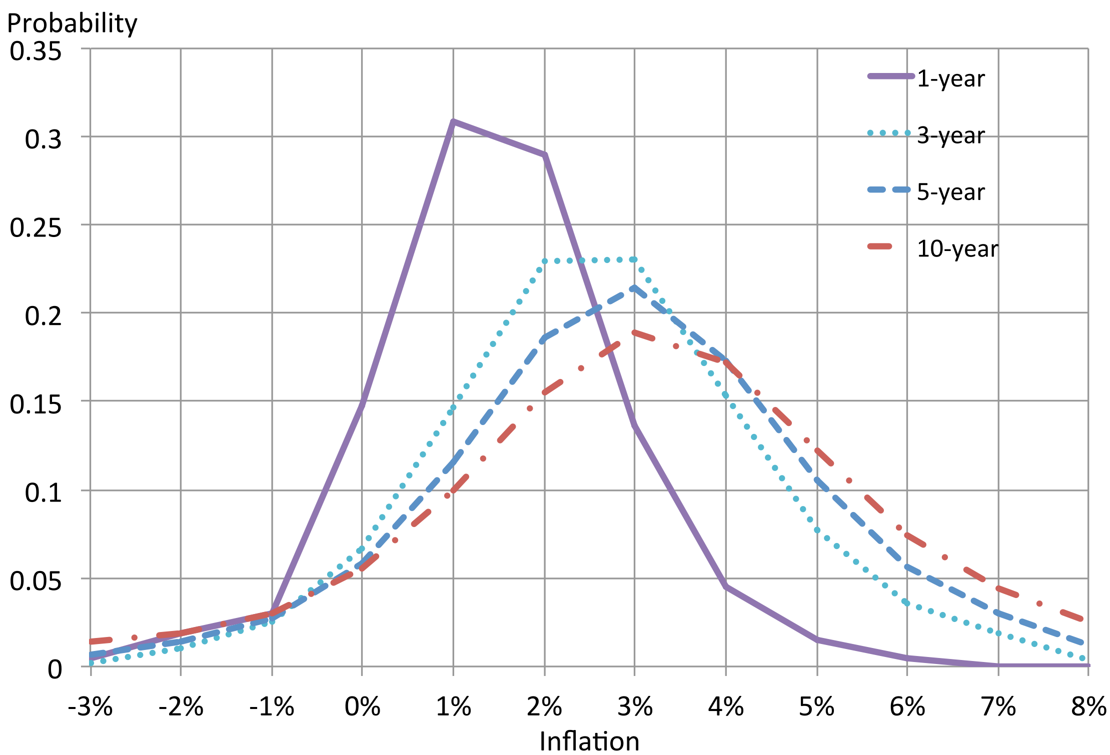


Figure 3: Risk-neutral densities for year-on-year inflation



dynamic models for inflation that have been estimated. However, our particular data on inflation contracts provides a novel way to approach this problem.

Consider the problem of obtaining the joint density between annual inflation over the next two years: $f(\ln \pi_{t,t+1}, \ln \pi_{t+1,t+2})$. Sklar (1959) shows that there exists a copula function $C(.) : [0,1]^2 \to [0,1]$ such that:

$$f(\ln \pi_{t,t+1}, \ln \pi_{t+1,t+2}) = C\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2})\right). \quad (11)$$

This function captures the co-dependence between the two random variables, so that we can obtain the joint density given information on the marginals.

We use a parametric version of the copula function $\hat{C}\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2}), \rho\right)$ that depends on a vector of parameter ρ of dimension M. The typical approach in the literature that estimates copulas would be to use the time series for past inflation to estimate both the marginal densities and the parameters in ρ. Our unusual data allows us to approach the problem quite differently.

To start, we already have estimates of the marginal densities for yoy inflation. Moreover, from the zero-coupon options, we also have another marginal distribution: $f(\ln \pi_{t,t+2}) = f(\ln \pi_{t,t+1} + \ln \pi_{t+1,t+2})$. From the definition of the distribution:

$$f(\ln \pi_{t,t+2}) = \int_{\ln \pi_{t,t+1} + \ln \pi_{t+1,t+2} = \ln \pi_{t,t+2}} \hat{C}\left(f(\ln \pi_{t,t+1}), f(\ln \pi_{t+1,t+2}), \rho\right) d\pi_{t,t+1} d\pi_{t+1,t+2}. \quad (12)$$

Since we have N bins on the marginal distributions, this expression gives N moment conditions. In principle, we can use these to estimate the M unknown parameters in ρ.

The appendix extends this logic to show that:

Proposition 2. *Given data for the marginal distributions $f(\pi_{t,t+k})$ and $f(\pi_{t+k-1,t+k})$ for k=1...K, we can obtain the joint distribution $f(\pi_{t,t+1}, \pi_{t+1,t+2}, ...\pi_{t+K-1,t+K})$ by solving for* *the M parameters in the ρ vector given the N conditions as long as $N \geq M$:*

$$f(\ln \pi_{t,t+n}) = \int_{\Pi} \hat{C}\left(f(\ln \pi_{t,t+1}), ...f(\ln \pi_{t+K-1,t+K}), \rho\right) d\pi_{t,t+1}...d\pi_{t+K-1,t+K}, \tag{13}$$

where the integration is over the set Π such that: $\ln \pi_{t,t+1} + ... \ln \pi_{t+n-1,t+n} = \ln \pi_{t,t+n}$*, for* $n = 1, ...N$.

These can be used to estimate ρ as a GMM procedure, even though these are not moments of the distribution of the random variable, as is usual, but rather the distributions themselves.

5.2 The joint distribution for inflation

We used one-parameter families to model co-dependence between any two variables in the copula, so that $M = K(K-1)/2$. Since we have 10 maturities ($K = 10$) and 21 bins ($N = 21$) the condition $N \geq M$ in the proposition was easily met. Our baseline results use a multivariate Gaussian copula, while the appendix shows results with a couple of alternative copulas. This does not assume normality for inflation, as we just take the marginal distributions delivered by the options, which are distinctively non-normal. Rather, it simply assumes that the joint dependence of inflation over time resembles a normal distribution in the sense that if the marginals were normal, then the multivariate would be too.

We picked ρ by minimizing the equally-weighted squared sum of differences from the moment conditions above. The appendix shows plots of the actual and predicted estimation, confirming that the fit is quite good. Table 1 shows the implied correlation parameters across maturities.[7]

Annual risk-adjusted inflation is not very persistent according to the market's percep-

[7] Our data on options contracts only goes to 10 years, but we need weights all the way to 30 years because that is the highest maturity of debt. For inflation beyond 10 years, we extrapolate by assuming that the joint distribution is a stationary Markov process of order 9 with parameters given by the distribution from 1 to 10 years. The appendix discusses the details.

Table 1: Estimated correlation coefficients of yoy inflation in the joint distribution

Maturity	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	1.000	0.257	0.269	0.301	0.290	0.242	0.248	0.311	0.245	0.284
2		1.000	0.231	0.248	0.250	0.238	0.263	0.287	0.285	0.233
3			1.000	0.305	0.291	0.271	0.270	0.263	0.238	0.276
4				1.000	0.301	0.256	0.237	0.262	0.285	0.261
5					1.000	0.312	0.271	0.236	0.298	0.289
6						1.000	0.294	0.263	0.303	0.273
7							1.000	0.280	0.282	0.287
8								1.000	0.279	0.284
9									1.000	0.283
10										1.000

Notes: Estimated correlation coefficients for yoy inflation between date 2012+j and 2012+l, in column j, row l.

tions, with correlation coefficients between 0.2 and 0.3. This is not out of line with the historical experience on actual CPI inflation, as its serial correlation between 1992 and 2012 is 0.01, but it was significantly higher in the past. More surprising is that the autocorrelation does not seem to fall with maturity. This suggests that the simple autoregressive processes of low order typically estimated on past data do not match the forward-looking expectations of market participants.

Our estimates also provide a more nuanced view of how well anchored are long-run expectations. On the one hand, the autocorrelation coefficients are all well below one, suggesting that agents expect inflation to revert to its mean, in risk-adjusted terms. On the other hand, even as far as maturity 10, the autocorrelation is still 0.28, so that even a one-year deviation of inflation from target affects market expectations of risk-adjusted inflation 10 years after.[8]

[8]This is consistent with the findings of Gurkaynak, Sack and Swanson (2005) who find that long-term forward rates respond to macroeconomic news by significantly more that a mean-reverting model of short term rates would suggest.

5.3 The counterfactuals

Infinite sudden inflation would debase all of the nominal debt. However, market expectations of inflation should reflect this possibility. If in the inflation contracts that we observe, investors are placing less than 10% probability on inflation being above 4%, scenarios where inflation is suddenly infinity are of little relevance.

We take two distinct approaches to construct scenarios where inflation is higher. The first approach measures the probability that debt would fall by different amount. It is similar to the measures of value-at-risk that are routinely used. Using our estimated joint distribution for yoy inflation, we draw a large number of histories and, ordering them by their impact on the real value of debt, we estimate probabilities that debt will fall by some threshold.

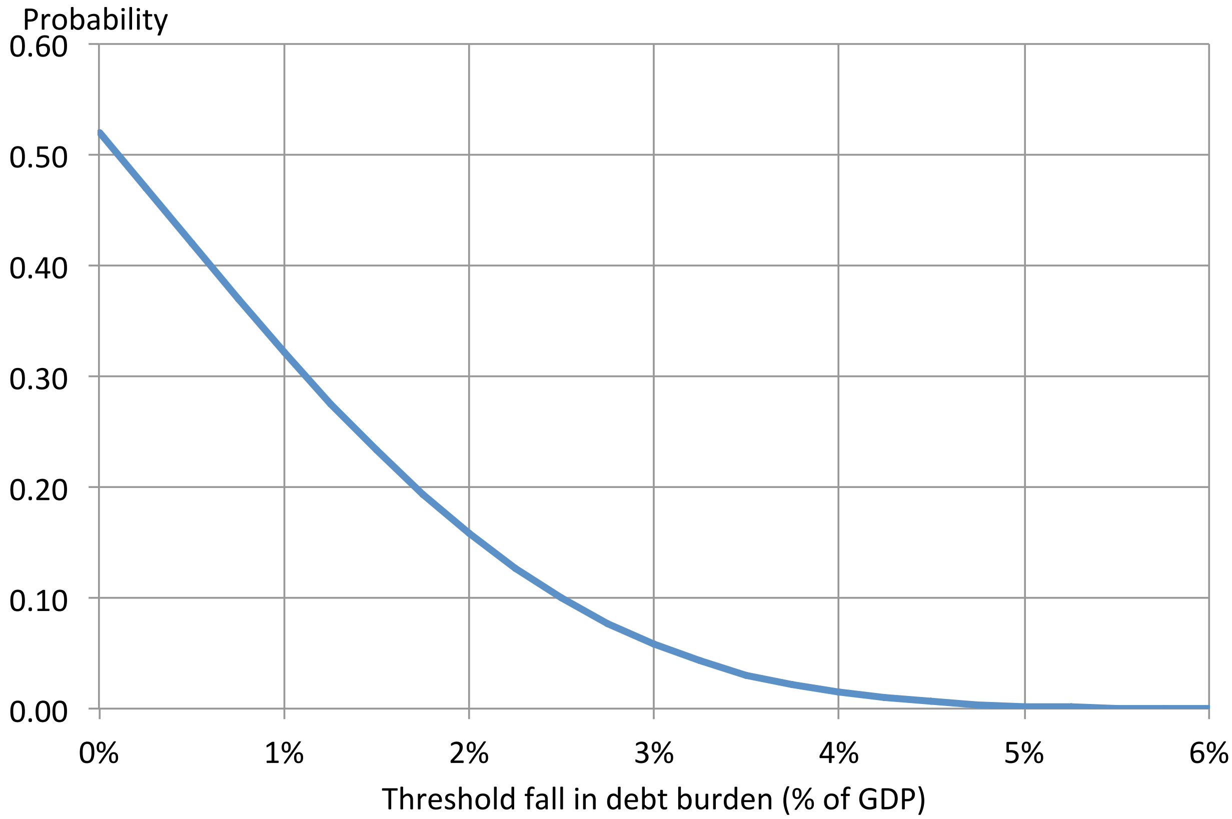
The second approach shifts the distribution for inflation to the right. In practice, we propose a new distribution $\hat{f}(.)$, recalculate the real value of the debt using equation (7), and subtract it from the market value of debt to obtain our estimate of the fall in real debt. This approach is the stochastic equivalent of asking what would happen if inflation was x% higher. The shift x is pinned down to be consistent with the plausible set of scenarios in our original distribution $f(.)$.

6 Estimates of debt debasement

Figure 4 shows the probability that the fiscal burden will fall by more than a few percentage points of GDP, according to the risk-neutral densities for inflation. Strikingly, the numbers are all quite small. The probability that debt falls by more than 5% of GDP is less than 0.2%. Having the real value of the debt fall by at least 1% of GDP due to inflation variation is likely, with a probability of 32% but anything more than even just 3% has the diminutive probability of 5.8%.

Table 2 presents conventional value-at-risk measures separately for each investor. Note

Figure 4: Probability that fall in value of debt exceeds threshold



that most of the loss is borne by foreigners. This happens not just because they hold more debt than domestics, but mostly because they hold longer maturity debt. Therefore, extreme situations where a succession of high realizations of yoy inflation lead to large cumulative inflation affect foreigners more than domestics. Chinese investors are not such a large fraction of foreign holdings, and they hold shorter maturities than other foreigners, so they lose relatively less. The last column in the table shows the effect on a non-private holder of debt, the central bank. The Federal Reserve would potentially suffer large losses as a share of its portfolio, since in 2012 it held mostly long-term bonds.

Table 2: Percentiles of the the distribution of losses for bondholders

	Holders of the debt				
Percentile	*Privately held (53%)*	*Domestic (20%)*	*Foreign (33%)*	*China (7%)*	*Central Bank (12%)*
90th	2.5%	0.9%	1.6%	0.4%	1.2%
95th	3.1%	1.1%	2.0%	0.5%	1.5%
99th	4.2%	1.5%	2.8%	0.7%	2.0%

Notes: Each cell shows the real losses of the debt held by the agent in the column that occur with the probability shown in the row.

6.1 The impact of higher inflation on the debt burden

To understand why significant debt debasement is so unlikely, we conduct several counterfactuals. Table 3 reports the results.

The first experiment shifts the marginal distributions for yoy inflation at every maturity so that the new median is at the old 90%. We think of this experiment as capturing an announcement that the inflation target of the Fed is now expected to be higher. It tries to approximate the effects of following the suggestion by Blanchard, Dell'Ariccia and Mauro (2010) of raising the inflation target in developed countries.[9]

The second experiment instead sets to 0 the mass in the density within the 90% percentile, and scales the density outside of this range proportionately. This corresponds to a commitment by the central bank to pursue much higher inflation for sure in the future. Only inflation realizations at the right tail of the current distribution become possible. At the same time, because there is no shift to the right as in the first case, very high levels of inflation are also not that likely.

[9]To be clear, we are shifting the risk-neutral distribution of inflation, not the actual inflation target of the central bank. The link between the two may be quite complicated, depending both on the effectiveness of central bank policy as well as on changes in private assessments of risk. In the extreme case where the central bank controls the distribution of inflation, and where inflation is "pure" in the sense of Reis and Watson (2010), so that changes in inflation are independent of changes in relative prices, then the two are the same.

Table 3: Counterfactual impact of higher inflation on value of debt

Inflation counterfactual	Holders of the debt: Privately held (53%)	Domestic (20%)	Foreign (33%)	China (7%)	Central Bank (12%)
1. Permanently higher	3.6%	1.3%	2.3%	0.5%	1.8%
2. Right tail only	4.1%	1.4%	2.7%	0.7%	1.9%
3. Higher and more variable	3.6%	1.3%	2.3%	0.6%	1.8%
4. Higher for sure	3.7%	1.3%	2.3%	0.6%	1.8%
5. Unexpected increase	1.6%	0.6%	1.0%	0.2%	0.7%
6. Gradual increase	2.2%	0.9%	1.3%	0.3%	1.3%
7. Temporary increase	1.2%	0.4%	0.8%	0.2%	0.4%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP.

The table shows that the first experiment lowers the debt burden by 2.7% while the second lowers it by 4.1%. Again, foreigners absorb a large share of the losses, because they both hold more debt and especially at longer maturities. Again, in spite of holding 40% less debt than domestic private bondholders, the Federal Reserve loses at least 30% more because of holding longer maturity debt.

6.2 The role of uncertainty

Inflation is uncertain, and because we are using its probability distribution, we can inspect separately what effect that has on the value of debt.

Our third experiment again shifts the marginal density so the new median is the old 90 percentile, but now this is accomplished by scaling the marginal densities of inflation at every maturity proportionately. It is often said that higher average inflation would come with more variable inflation, and this experiment tries to capture this possibility.

Still to understand the effect of volatility, we ask what would happen to the debt burden if inflation became deterministic. In the fourth counterfactual, we now assume that year-on-year inflation is exactly equal to the average inflation in our estimates.

Finally, the fifth counterfactual considers the effect of unexpected inflation. Now, we assume that after an initial unexpected jump of inflation upwards, the distribution of inflation looking forward is equal to the conditional expectation that we have estimated. Therefore, whereas in the previous experiment all of the changes at all maturities were unexpected, now only the change in the first year catches agents by surprise, but they adjust their expectations right after.

From table 3, we see that more uncertainty lowers the effectiveness of inflation at debasing the debt. Intuitively, because the real value of future nominal payments are convex in inflation, uncertainty raises their value and so lowers the benefits of raising inflation. Yet, increasing uncertainty or eliminating it, as in cases 3 and 4, has a quantitatively negligible effect on the estimates. Also as expected, in case 5, if agents adjust their expectations after one year of surpass inflation, the estimates are significantly smaller. In this case, in spite of the quite extreme shift in the distribution for inflation that we considered, the fall in the real value of debt is far from even a meager 2% of GDP.

6.3 The time path for inflation

The five experiments so far assumed that the inflation distribution would change immediately and permanently. The sixth case considers instead a temporary increase in inflation, with the distribution for yoy inflation shifting rightwards to the new mean at the 90th percentile for the next year, but only at the 80th percentile the year after, and so on, so that for maturities of 5 or more years there is no change. The seventh case considers a gradual increase, with the one-year inflation distribution unchanged, while the 2-year shifts horizontally so the new mean is at the old 60th percentile, and so on until the fifth year after which we have the same permanent shift as in the first case.

The last two rows in table 3 show that both of these reasonable deviations from the first

Table 4: Expected adjusted average annual inflation for different counterfactuals

	Horizon			
Distribution for inflation	*1-year*	*3-year*	*5-year*	*30-year*
Baseline	1.5%	2.2%	2.6%	3.2%
Counterfactuals				
1. Permanently higher	3.2%	4.2%	4.8%	5.9%
2. Right tail only	4.4%	5.0%	5.4%	5.1%
3. Higher and more variable	3.2%	4.3%	5.0%	6.1%
4. Higher for sure	3.2%	4.2%	4.8%	5.9%
5. Unexpected increase	3.3%	3.2%	3.5%	3.8%
6. Gradual increase	1.5%	2.6%	3.6%	5.7%
7. Temporary increase	3.2%	3.4%	3.4%	3.3%

Notes: Each cell reports $1/E(n/\pi_{0,n})$, the harmonic mean of inflation at horizon n.

counterfactual again cut significantly the effect of inflation on debt. If the central bank takes a few years to achieve the new target, then the benefit for the Treasury is only 1.6%.

6.4 Why such low numbers? The role of inflation

All of the estimates in tables 2 and 3 were surprisingly small. Even though we are describing unlikely and extreme scenarios, the debt never lost even 10% of its real value. Why is this the case?

Table 4 shows the harmonic of inflation for both the baseline and each of the counterfactuals at different maturities. We take the harmonic, instead of an arithmetic mean, since proposition 1 shows that it is the expectation of the inverse of inflation that matters for debt valuation.

The table shows that even when we shift the distributions to the 90th percentile, inflation between 2012 and 2013 would increase by at most 3% across experiments. From the perspective of actual market-based distributions, anything larger than this seems unreasonable. But from the perspective of debt valuation, these are not large numbers. It would take shifting the distribution of inflation so further to the right in experiment 1 so that the new median for annual inflation was close to 11% in order to raise the debt debasement effect to 10% of GDP.[10] But in that case, the new and old distributions for inflation would have close to zero overlap, making this scenario, literally, incredible.

6.5 Why such low numbers? The role of maturity

Even if inflation only increases by 2 or 3% more on average, after 30 years this builds up to more than half of a cut in terms of the real values of payments far into the future. If all the U.S. debt in private hand was of very long duration, we might expect that inflation could significantly reduce its real value. However, as we saw in figure 1 most, there is little debt of long maturities held in private hands.

Table 5 investigates the effect of maturity on our estimates by only considering the effect of the higher inflation on the debt with maturity below 1 year or 4.5 years, which includes 75% of the market value of debt. The numbers are significantly lower than when all the debt is included, and well below three quarters of them. This confirms that most of the benefits from higher inflation come from the longer maturity debt. There is just too little of this debt held by the public to have a large effect.[11]

The last column of the table confirms this conclusion. We built the distribution of debt held in private hands by maturity for the year 2000, using the same steps as we followed for our 2012 calculations. The average maturity in 2000 was 5.06 year, compared to the 3.70 years in 2012. Whereas in 2012 only 6% of the market value of debt was in maturities above

[10]Note that, across experiments, increasing average inflation does not lead to proportional increases in the extent of debt debasement. A proportional rule of thumb, like the ones discussed in the introduction, will deliver an estimate that is off by as much as one third of the real number.

[11]Another curious result is that the temporary increase now has a larger effect than the gradual, since the former has a greater impact on the short end of the maturity structure.

Table 5: Counterfactual impact of higher inflation with different maturity distributions

Inflation counterfactual	*Including only debt of maturity up to:* 1 year	*4.5 years*	*All*	With the maturity distribution of debt in 2000
1. Permanently higher	0.1%	1.1%	3.6%	7.1%
2. Right tail only	0.2%	1.6%	4.1%	7.6%
3. Higher and more variable	0.1%	1.1%	3.6%	7.1%
4. Higher for sure	0.1%	1.1%	3.7%	7.2%
5. Unexpected increase	0.1%	0.7%	1.6%	5.1%
6. Gradual increase	0.0%	0.3%	2.2%	6.0%
7. Temporary increase	0.1%	0.7%	1.2%	4.6%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP.

10 years, in 2000 these long-term bonds accounted for 17% of the total debt.

We then repeated our experiments using the 2000 distribution, scaled up proportionately so that the total market value of the debt is the same as in 2012. The question we are asking is whether inflation would be significantly more effective if the public debt held in the private sector was of as long duration as in 2000. The answer is a clear yes: the effects are approximately twice as large.[12]

7 Financial repression and the Fisher effect

When we performed our counterfactual experiments, we implicitly kept constant the safe real interest rate that agents apply to government bond, R_t. Yet, either because changes in inflation cause changes in real returns (the Fisher effect, or the Phillips curve), or because with inflation may come financial repression, it is important to investigate this assumption.

[12]Even though average maturity went from 3.7 to 5.1, the fall in the real value not debt increased by a factor of 2. Again, the simple approximations discussed in the introduction are grossly inaccurate by ignoring the actual distribution of debt by maturity.

7.1 The Fisher effect

The real neutrality of inflation is a hotly debated topic. At one extreme, when the distribution for inflation shifts $f(.)$ in our counterfactuals, perhaps the real interest rate R_t is unchanged as there is a classical dichotomy between nominal and real variables. At the other extreme, if nominal interest rates are pegged, perhaps because of the zero lower bound, higher inflation lowers the real interest rate one-to-one.

One might think that if inflation lowers real interest rates, then because the government needs to pay less to roll over its debt, it follows that the fiscal burden is smaller. This is not the case. If the interest rate is lower, this also means that investors discount the future debt by less. Because these two effects exactly offset, the effect on rolling over the debt is zero. By our assumption of no arbitrage opportunities, the real interest paid on the government bonds and the real interest that private agents use to discount the future are always the same, so inflation will have no effect.

For outstanding bonds, the formula in proposition 1 shows that if the interest rate is lower at any horizon, then the weight ω_t at that horizon is actually *larger*. Therefore, the debt burden actually rises with increases in inflation if there is a Fisher effect. The reason is that only the discounting effect is present, since their real return was already set in the past. In other words, if inflation induces private holders of outstanding debt whose par value is fixed to discount the future less, the market value of this debt becomes higher.[13]

[13] The real interest rate also appears on the right-hand side of equation (5) discounting future fiscal surpluses. In the horizon of the next few years, the United States is likely running a deficit, so the lower real interest rates that inflation may bring about would again make the fiscal burden worse. By the time the fiscal surpluses are expected to be positive, it becomes difficult to believe that the non-neutrality of inflation would remain, so real interest rate are probably unchanged. Therefore, the Fisher effect also worsens the fiscal burden through this term. The more likely effect of some gains is through s_t, which we are taking as exogenous, being higher with higher inflation, for instance because of higher seignorage revenues. That is not the question in this paper, which limits itself to the effects on the debt, and leaves for future work the effect on the entire fiscal position of the government.

7.2 Financial repression

In this context, financial repression is a way to drive a wedge between the interest rate that discounts the future and the interest rate paid by government bonds. This wedge works like a tax on the returns of government debt and as such provides a source of revenue that reduces the fiscal burden. The literature on financial repression, which dates back at least to McKinnon (1973), offers many examples of how this tax is collected and enforced, through channels like caps on interest rates, direct lending to the government by captive domestic savers, or financial regulation, among others. In theory, this would show up as a term $1 - \tau_t$ in each of the terms in our formula in proposition 1, but at this general level, we cannot say more empirically both in terms of the size of τ_t or how it varies with inflation.

To make progress, we focus on a particular policy that has been used by policymakers in the past, and which has a clear link to inflation. Reinhart and Sbrancia (2011) discuss how in the 1945-80 period, many developed countries used a combination of caps on interest rates on government bonds and inflation to liquidate the World War II debt. To formalize this, suppose that the government is somehow able to force the holders of outstanding debt to roll it over for "special" debt that sells for a higher price (or pays a lower return) than the market price for identical private securities. This can be achieved for instance by forcing banks to accept this special debt and hold it under the excuse of financial regulation and stability. That is, the government sells today for price $\tilde{H}_t$, bonds that promise to pay $\tilde{B}_{t+1}$ next period. The holders of maturing bonds B_0^t are forced to take these as payment so $B_0^t = \tilde{H}_t \tilde{B}_{t+1}$.

If the price of the new securities was infinity, or equivalently if the government paid zero back to the bond holders, then clearly it could completely eliminate the fiscal burden. As with infinite sudden inflation, we do not take this case to be realistically interesting for the United States. Instead, we assume that the new price is bounded above, $\tilde{H}_t \leq 1$ so that there is a zero lower bound on nominal interest rates. When the inequality binds, this special debt can be though of as required reserves. In that case, the government will be forcing the holders of maturing bonds to roll over their assets at a zero interest rate.

To simplify the algebra, we assume that all new debt has a maturity of solely one year and that there are no real bonds. The law of motion at any date after 0 for debt now becomes:

$$W_t = s_t + \left(\frac{\tilde{H}_t - H_t}{P_t} \right) \tilde{B}_{t+1} + W_{t+1} \frac{H_t P_{t+1}}{P_t}. \tag{14}$$

Because the price at which the government sells the bonds is above the market price, $\tilde{H}_t > H_t$, this expression makes clear that financial repression works like a source of tax revenue.

Iterating forward and following the same steps as before, the present value budget constraint is:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t} \left(\frac{B_0^t}{P_t} \right) \right] = \mathbb{E}\left[\sum_{t=0}^{\infty} m_{0,t} \left(s_t + \left(\frac{\tilde{H}_t - H_t}{P_t} \right) \tilde{B}_{t+1} \right) \right]. \tag{15}$$

The revenue from financial repression appears on the right-hand side. Recall that $B_0^t = \tilde{H}_t \tilde{B}_{t+1}$, so this revenue subtracts from the real value of outstanding debt on the left-hand side just like a tax on its holders would. Rearranging, similar algebra as the one that led to proposition 1 shows that the debt burden now is:

$$E_0 \sum_{t=0}^{\infty} m_{0,t} \left(\frac{H_t}{\tilde{H}_t} \right) \left(\frac{B_0^t}{P_t} \right) \leq \sum_{t=0}^{\infty} \frac{B_0^t}{P_0} E_0 \left(\frac{m_{0,t+1}}{\pi_{0,t+1}} \right). \tag{16}$$

The inequality binds in the case of extreme financial repression, where the government rolls over its past debt through zero-interest required reserves. This expression shows that the effect of financial repression is essentially equivalent to delaying all payments on the debt for one year at zero interest, or shifting the maturity structure for one year. Recalling our previous finding that it is the current short maturity structure that leads to modest benefits form inflation, we can guess that repression can significantly raise this effectiveness.

Generalizing the previous argument to have financial repression for N periods, we obtain

Table 6: The effect of financial repression and inflation

Duration of repression	*Repression*	*Higher inflation*	*Total*
1 year	0.8%	4.5%	5.3%
5 year	5.4%	7.9%	13.3%
10 year	13.1%	10.4%	23.5%

Notes: Each cell shows the fall in the real value of debt as a ratio of GDP as a result first of repression, and then inflation under experiment 1. The last column is the sum of the two previous.

a new version of proposition 1:

Proposition 3. *With financial repression for N periods, the debt burden is still equal to $\sum_{t=0}^{\infty} \omega_t B_0^t$ but now the weights are:*

$$\begin{aligned} \omega_t^r &= R_t^{-1} \int \left(\frac{f(\pi_{0,t})}{\pi_{0,t}} \right) \left(\frac{H_t^N}{\tilde{H}_t^N} \right) d\pi_{0,t} \\ &\leq R_{t+N}^{-1} \int \left(\frac{f(\pi_{0,t+N})}{\pi_{0,t+N}} \right) d\pi_{0,t} \end{aligned} \tag{17}$$

The inequality binds with extreme financial repression, when the outstanding debt is converted into required zero-interest paying reserves.

The final result shows that $\omega_t^r = \omega_{t+N}$ with extreme financial repression. The weights in the formula for the debt burden are therefore generally lower, as long as inflation and real interest rates are positive. The higher is inflation then the smaller is ω_t^r not just, as before, because the debt is debased, but also because now the zero-interest rate reserves earn a lower real return. The potential for inflation to lower the debt burden is higher.

Table 6 shows the effect of extreme financial repression. The first column of numbers shows that financial repressions alone, with no additional inflation, can significantly lower the real value of debt. The next column then conducts our experiment 1, where the inflation distribution shifts to the right. The effect of inflation is now much higher than before.

This confirms our conclusion of the previous section that a longer maturity of debt is the key ingredient that makes inflation effective at lowering the real value of debt. Financial repression is the tool by which an originally short maturity becomes longer.

8 Conclusion

At the start of this paper, we reported naive, but often used, calculations that 1% more inflation for 5 years would be enough to generate fall in the real value of the U.S. public debt of 5% of GDP. Our estimates are significantly lower. We estimated that the probability of the debt falling this much was actually 0.2%. Even though we considered experiments where inflation was 3 to 6% permanently higher, our largest estimate of the fall in the real value of debt was 4.1%. Many of our perhaps more realistic estimates, where the changes in the distribution of inflation were temporary or partially expected give estimates that are never close to even 2% of GDP.

To reach these estimates, we provided three contributions. We derived a simple new formula to calculate the effect of the inflation distribution on the real value of debt. We showed that to provide reliable estimates, one must carefully construct the actual distribution of public debt by maturity that is held by private agents. Finally, we showed how to construct the joint risk-neutral distribution for inflation over time using date from inflation options at a single date in time and a novel estimation method based on copulas and moments.

Our analysis was also able to indicate how inflation could be more effective at lowering the debt burden. One ingredient would be higher inflation, although we found that it would take double-digit inflation to reach significant fiscal gains, and this is perceived as impossible by markets over the next few years. More interesting, having debt outstanding of longer maturities would significantly increase the effectiveness of inflation. Even just going back to the maturity in 2000 would double the impact of inflation. Financial repression, which we showed is a way to force an ex-post extension of maturities, would likewise be effective, and if pursued for a decade should lower the real value of privately-held public debt by 45%.

If inflation will not pay for the public debt, then what will? Since market prices today put a low probability of the United States defaulting, the inescapable budget constraint implies that private agents must be expecting budget surpluses. How sensible are these expectations is a much wider debate then one modest paper can tackle.

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