Technology Diffusion and Adoption

STEG Course on Macro Development
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Outline

- Background: Nonrivalry and a motivating fact
- Explore some theories of technology diffusion
  - Nelson-Phelps diffusion models
    - Will use Benhabib-Perla-Tonetti (JEG 2014) as reference
  - Idea diffusion models
    - See Buera-Lucas (ARE 2018) survey article
    - Will quickly use Perla-Tonetti (JPE 2014) as discrete-time reference
    - Will use Perla-Tonetti-Waugh (AER 2021) as continuous-time reference
- Briefly discuss models of innovation and idea diffusion
  - Benhabib-Perla-Tonetti (ECMA 2021)
  - Buera-Oberfield (ECMA 2020)
- Skipping creative-destruction models, networks, and learning
  - Skipping due to time constraints and my lack of knowledge
  - For creative destruction see Akcigit-Ates-Impullitti (2020) and Hsieh-Klenow-Nath (2020)

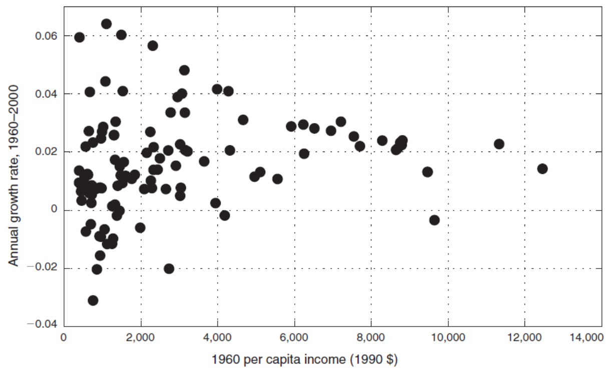
Goal of Class

1. Obtain broad overview of theories of technology diffusion
2. Understand two types of models well enough to use them in your research
    - Nelson-Phelps style models
    - Lucas-style idea diffusion models

Background: Nonrivalry

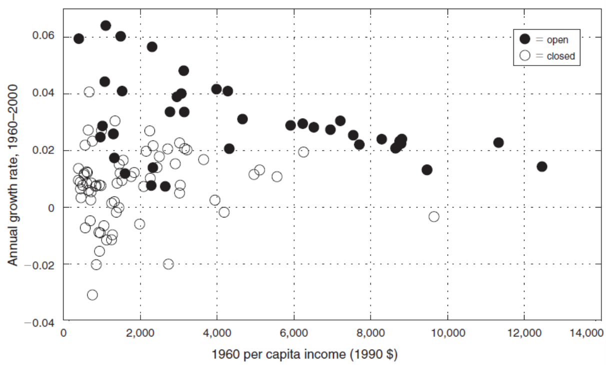
- Think of a technology as an idea (i.e., a blueprint or recipe)
- Nonrivalry is essential to understanding technology diffusion and growth
- Ideas are nonrival
  - A specific blueprint can be used both in a factory in India and in Brazil at a given time
  - A specific machine can be used either in a factory in India or Brazil at a given time
- This motivates the need for distinct models of the distribution and spread of ideas/technology
- The economics of nonrival goods is very different from classical "Adam Smith" economics

Income and Growth Rates: Lucas (AEJ: Macro 2009)



- Rich countries grow at ≈ 2%
- Poor countries have highly variable growth rates

Income, Growth Rates, and Openess: Lucas (AEJ: Macro 2009)



- Open and market-based economies form upper edge of growth triangle
- Upper edge is "growth potential" for open economies
- A demonstration of the "equalizing forces operating in the set of market economies"

Nelson-Phelps Diffusion Model

Stylized Overview of Nelson-Phelps (AER 1966)

- For simplicity, unit of observation is a country with constant population
- There is a frontier country that has the highest level of technological advancement
- There are follower countries, with varying distances to the frontier
- The rate of diffusion from the leader to a follower is a function of the distance from the follower to the leader
- Researcher specifies an exogenous differential equation for the rate of technology diffusion
- Let the frontier grow at an exogenous rate (a model of diffusion, not innovation)
- Comment: Simple to analyze because of pairwise dependence

Nelson-Phelps Diffusion Model

- Output in country i, with population $\bar{L}_i$ is $Y_i(t) := z_i(t)\ \bar{L}_i$
- Distribution of productivity z across countries. Productivity frontier $F(t) := \max\{z\}$
- The law of motion for productivity is:

$$\frac{\dot{z}}{z} = \tilde{D}\left(t, z; m\right) := \frac{1}{m}\left(1 - \left(\frac{z(t)}{F(t)}\right)^{m}\right)$$

- m is the key parameter of the diffusion equation

Nelson-Phelps Diffusion Model: Parameterization

- The law of motion for productivity is:

$$\frac{\dot{z}}{z} = \frac{1}{m}\left(1 - \left(\frac{z(t)}{F(t)}\right)^{m}\right)$$

- Nelson-Phelps (1966): confined exponential diffusion $m = -1$
  - The further $F(t)$ is ahead of $z(t)$, the faster the follower country grows
  - Used by Lucas (AEJ: Macro 2009) for open and market-based economies
- Logistic Diffusion if $m = 1$
  - Benhabib-Spiegel (Handbook of Economic Growth 2005) estimate $m > 1$
  - if distance to frontier is too large, "the follower will not be able to keep up, growth rates will diverge, and the income ratio of the follower to the leader will go to zero"

Economic Interpretation of the Diffusion Function

- "Benefits to backwardness"
  - See Klenow and Rodriguez-Clare Handbook of Economic Growth (2005)
  - It is easier to imitate than innovate
    - So, grow faster while adopting existing technologies than when inventing new technologies
  - Can possibly skip some costly steps along the way
    - E.g., wireless cell phones skipping costly investment in landline infrastructure

- Barriers to adoption
  - Without sufficient human capital, it might not be possible to adopt frontier technologies
    - "Any sufficiently advanced technology is indistinguishable from magic." -Clarke's third law
  - Production complimentarity. See Hirschman 1958; Kremmer (QJE 1993); Jones (AEJ: Macro 2011)
  - Efficiency of given technology could fundamentally vary across countries
  - Institutional barriers. See Parente-Prescott (AER 1999); Acemoglu-Robinson

Nelson-Phelps Diffusion with Investment

- Benhabib-Perla-Tonetti (JEG 2014) add investment in innovation and adoption to standard NP Model
- The productivity of innovation is: σ
- The productivity of imitation is: $\tilde{D}\left(t, z; m\right)$

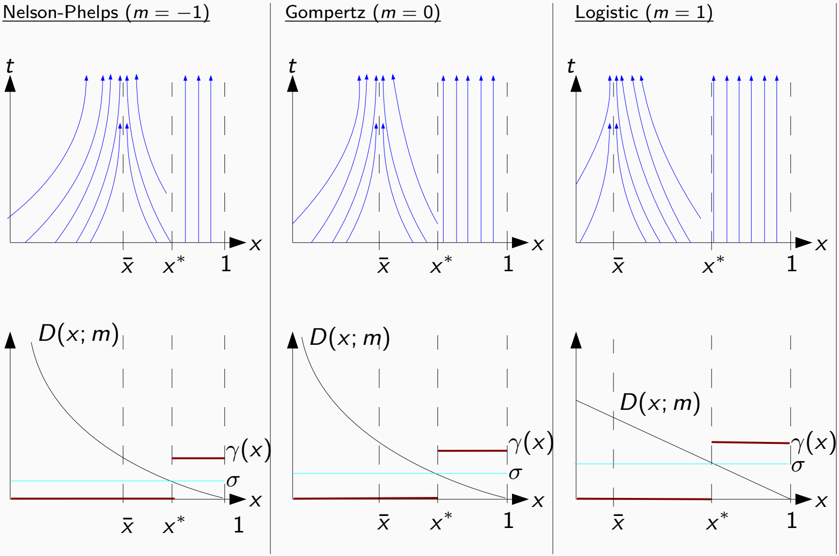
$$\tilde{D}\left(t, z; m\right) := \tfrac{c}{m}\left(1 - \left(\tfrac{z(t)}{F(t)}\right)^{m}\right)$$

- Countries choose to invest in innovation (γ) and imitation (s)
- The law of motion for productivity is:

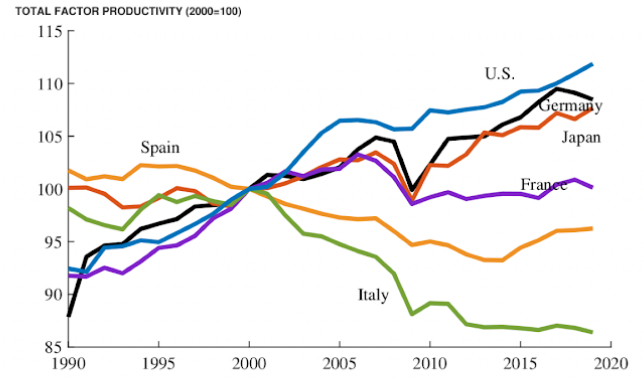
$$\frac{\dot{z}}{z} = \sigma\gamma + \tilde{D}(t, z; m)s$$

- Define relative productivity $x(t) := \frac{z(t)}{F(t)}$
- Note: BPT (JEG 2014) is a good paper to learn/refresh Hamiltonian math

Growth Dynamics in BPT (JEG 2014)



Could Falling Behind be Optimal Free Riding?



(Source: Chad Jones + PWT; Grumpy Economist Blog)

- Countries might choose to fall-back in relative terms
  - Definitely not obviously optimal to grow slowly
  - But should consider advantage to backwardness in the discussion
- With investment choice, NP model + logistic no longer permits countries to be left behind
  - Informs which barriers to adoption are present?

Idea Diffusion Models

Idea Diffusion Models

- Want: a microfounded process of diffusion to replace exogenous diffusion equation
- Want: adoption to be costly and the result of deliberate choice
- Want: focus on growth of the many low productivity firms, not just few frontier firms
  - Think about large gains of allocating resources to firms to using best practices. E.g., Bloom et. al. management literature
- Want: the distribution of existing technologies to influence adoption choice
  - Drops pairwise analysis of NP (not everyone adopts frontier technology)
  - Adds stochastic growth (order switching)
  - Generates a non-degenerate distribution of productivity in long-run
- Typical application to firms or workers allows connection to panel microdata

Background Knowledge

- Kortum (ECMA 1997)
  - Model of innovation via search for new ideas
  - Draw new ideas from exogenous fixed idea distribution
  - Use new idea if better than current idea
  - Extreme values - harder over time to get better ideas
  - Population growth $\rightarrow$ more draws
  - Power law distribution of potential ideas + exponential population growth $\rightarrow$ BGP

- Jovanovic and Rob (RESTUD 1989)
  - Model of diffusion of knowledge via interaction
  - Distribution of knowledge across people $G(z)$
  - Random meetings between person 1 and 2 with productivity $\{z_t^1, z_t^2\}$
  - $\{z_{t+1}^1, z_{t+1}^2\} = f(z_t^1, z_t^2)$
  - e.g., $z_{t+1}^1 = z_{t+1}^2 = \max\{z_t^1, z_t^2\}$

- Side note: Close connection between extreme values and differential equations

Perla-Tonetti 2014: Key Mechanism

- Heterogeneous firms: produce or search to adopt a new productivity
- Searchers randomly meet and copy the technology used by an existing producing firm
  - Like Kortum (1997) but search from distribution of known ideas
  - Peer-effect model like Jovanovic-Robb (1989)
- Selective search endogenously evolves distribution, shifting weight to higher productivity
- Aggregate state = productivity distribution, F_t, where $\min \mathrm{support}\{F_t\} = M_t$
- Note: Similar economics to Lucas-Moll (JPE 2014)

Environment and Technology

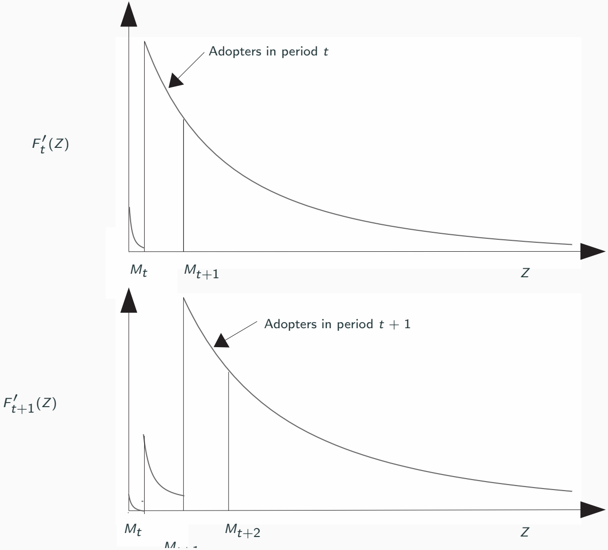
- Discrete time
- Fixed measure 1 of firms who discount at rate r_t (from consumer IMRS in GE)
- Linear production with capacity constraint: $Y_t^i = Z_t^i$
- Either produce with existing technology and keep it, or take a period to adopt a new technology
- Source distribution for new technology Φ_t

$$V_t(z) = \max\left\{z + \tfrac{1}{1+r_t}V_{t+1}(z), \tfrac{1}{1+r_t}\int V_{t+1}(z')d\Phi_t(z')\right\}$$

- Solution is reservation productivity each period: M_{t+1}. Why?
- Key idea to make this a model of adoption/diffusion: $\Phi_t(z)$ is related to $F_t(z)$

Evolution of the Productivity Distribution

Let $F_t(z) = \Phi_t(z)$ and consider time period t



Environment and Technology

Simplification for discrete time: assume adopters only meet non-adopters

$$V_t(z) = \max\left\{z + \frac{1}{1+r_t} V_{t+1}(z), \frac{1}{1+r_t} \int V_{t+1}(z') dF_t(z'|z' \geq \hat{z}_t)\right\}$$

- Solution is reservation productivity each period: M_{t+1}
- Firms uses forecast of aggregate $\hat{z}_t$ to calculate value
- In RE equilibrium, $\hat{z}_t = M_{t+1}$ (just like big K little k)

Evolution of F is a Truncation

F_{t+1} is F_t truncated at m_{t+1}:

$$f_{t+1}(z) = f_t(z) + f_t(z \,|\, z \geq m_{t+1}) F_t(m_{t+1}) = \frac{f_t(z)}{1 - F_t(m_{t+1})}$$

Given an initial condition F_0, $m_0 \equiv \min \operatorname{support}\{F_0\}$, and a sequence $\{m_{t+1}\}$:

$$f_t(z) = \frac{f_0(z)}{1 - F_0(m_t)} \tag{1}$$

- For BGP, in addition to aggregate output growing at constant rate, restrict normalized distribution to be constant over time (all quantiles grow at same rate)

A BGP Equilibrium

Proposition

Given a Pareto initial condition and parameter restrictions (i.e. $F_0(z) = 1 - \left(\frac{m_0}{z}\right)^{\alpha}$*)*

An equilibrium exists with the following properties (see paper for eqm. definition)

1. *The growth rate is:* $g = \left(\beta \frac{\alpha}{\alpha - 1}\right)^{\frac{1}{\gamma - 1 + \alpha}}$
2. *Minimum of support:* $m_t = m_0 g^t$
3. *Production:* $Y_t = \frac{\alpha}{\alpha - 1} g^{1-\alpha} m_t$
4. *Measure of adopters:* $S_t = 1 - g^{-\alpha}$
5. *The value function is piecewise-linear, with kinks at* $\{m_{t+1}\}$*. That is,* $\forall\, s \in \mathbb{N}$

$$V_t(z) = \frac{1+r}{r}\left(1 - \left(\frac{1}{1+r}\right)^{s}\right) z + \left(\frac{1}{1+r}\right)^{s} \bar{W} g^{t+s},\ z \in [m_0 g^{t+s}, m_0 g^{t+s+1}]$$

Comparative Statics of Growth Rate

Proposition

The following properties hold for a solution to the BGP:

1. $\frac{\partial g}{\partial \beta} > 0$ *and* $\frac{\partial g}{\partial \gamma} < 0$
2. g *is independent of* $\min \mathrm{support}\{F_0\}$
3. $\frac{\partial g}{\partial \alpha} < 0$
    - $\downarrow \alpha$ *is* $\uparrow$ *inequality in Pareto*
    - *Fatter tail generates higher growth*
    - *Adoption is a real-option; option value is increasing in variance*

Planner's Problem

- The planner makes the search vs. produce decision
- Describe recursively, with $f(\cdot)$ the state with $\min \operatorname{support}\{f\} = m(f)$
- Chooses the growth rate $g(f) \geq 1$ such that $m' = g(f)m(f)$, where $m(f) \equiv \min \operatorname{support}\{f\}$
- Maximizes the consumer's utility

$$U(f) = \max_{g \geq 1} \left\{ \frac{\left(\int_{gm(f)}^{\infty} z\, f(z) \mathrm{d}z \right)^{1-\gamma}}{1-\gamma} + \beta U(f') \right\}$$

$$s.t.\ f'(z) = \frac{f(z)}{1 - F(g\, m(f))}$$

Planner vs. Competitive Equilibrium

Comparing first-best to competitive equilibrium:

$$g_{fb} = \left(\beta \frac{\alpha}{\alpha - 1}\right)^{\frac{1}{\gamma - 1}}, \quad g_{ce} = \left(\beta \frac{\alpha}{\alpha - 1}\right)^{\frac{1}{\gamma - 1 + \alpha}}$$

- $g_{fb} > g_{ce}$
- Signs of $\frac{\partial g}{\partial \beta}$, $\frac{\partial g}{\partial \gamma}$ and $\frac{\partial g}{\partial \alpha}$ same as the CE
- The wedge increases with higher inequality: $\frac{d(g_{fb}/g_{ce})}{d\alpha} < 0$
- Economic mechanism: External spillovers and the free-rider problem

Next Steps

- Embed this model of adoption in a richer macro/trade model
  - Use continuous time to increase tractability and simplify analysis
- What are firms adopting in the long-run?
  - Models of innovation and adoption
  - The role of the initial distribution

Perla-Tonetti-Waugh (AER 2021): Trade and Growth

- Technical Innovations in PTW relative to PT
  - Continuous time
  - Monopolistic competition (compare to z = profits = output)
  - Labor as factor of production (GE in r and w)
  - Entry/exit and endogenous measure of varieties
  - Same endogenous adoption + exogenous innovation (GBM) to match firm dynamics
  - Transition dynamics (not just BGPs)
  - Serious calibration (caveat: symmetric countries)
- I will mostly skip the economics of the paper to focus on teaching methods

Model: Time, Countries, and Consumers

Continuous time, infinite horizon economy.

N symmetric countries

Consumers with period utility:

$$U_i(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \log C_i(\tau) d\tau$$

$$C_i(t) = \left(\sum_{j=1}^{N} \int_{\Omega_{ij}(t)} Q(v,t)^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}} .$$

- $\rho =$ discount factor.
- $\Omega_{ij}(t) =$ varieties consumed.
- $\sigma =$ elasticity of substitution across varieties.

Consumers inelastically supply L units of labor.

Model: Firm's Technology

Large pool of monopolistically competitive firms in each country.

Firms are. . .

- Heterogeneous over productivity, Z.
- Sole producers of variety, v.
- Have linear production technologies using labor, ℓ,

$$Q(Z) = Z\ell.$$

- Face fixed cost and iceberg trade costs to export.
- Have the option to pay a cost and receive a new productivity draw.

Overview of Firm's Optimization Problems

Incumbent firms' decisions can be divided into static and dynamic optimization problems and there is entry and exit

Static Problem: Produce and Export...

Given Z, choose price and labor to maximize profits Π_{ji}, for each market j

- Fixed costs (of hiring labor) to export to foreign market, affected by parameter $\kappa \geq 0$
- Iceberg trade costs to ship goods abroad, $d \geq 1$

Very standard. I won't go through this today.

Overview of Firm's Optimization Problems

Incumbent firms' decisions can be divided into static and dynamic optimization problems and there is entry and exit

Dynamic Problem...

1. Non-adopting firms' productivity evolves exogenously according to geometric Brownian motion:

$$\mathrm{d}Z_t/Z_t = (\mu + \upsilon^2/2)\mathrm{d}t + \upsilon \mathrm{d}W_t$$

- μ is the drift parameter
- υ is the volatility parameter
- and W_t is standard Brownian motion

Overview of Firm's Optimization Problems

Incumbent firms' decisions can be divided into static and dynamic optimization problems and there is entry and exit

Dynamic Problem...

2. Incumbent firms choose **when** to adopt a new technology, Z

- Draw new productivity Z, related to **equilibrium** distribution $\Phi(Z, t)$
- $X(t)$ is the cost of hiring ζ units of labor to draw a new productivity

Overview of Firm's Optimization Problems

Incumbent firms' decisions can be divided into static and dynamic optimization problems and there is entry and exit

Entry and Exit...

- Entrants receive initial productivity from $\Phi(Z, t)$ at cost $\frac{X(t)}{\chi}$, where $0 < \chi < 1$
- Exit at exogenous rate δ

Detour: Stopping Times and Free Boundary Problems

- Choosing a time at which to adopt is an optimal stopping time problem
- Tight link between optimal stopping times and free boundary differential equations
    - See, books by Stokey (2008) and Dixit-Pindyck (1994) and Ben Moll's excellent lecture notes
- The idea is to recursively choose productivity threshold for when to stop
- Math: the value function and optimal threshold from the stopping problem satisfy a
    - Continuation value equation: describes the value of keeping current technology
    - Value matching equation: at the adoption threshold productivity, the value of adopting equals the value of continuing
    - Smooth pasting equation: ensures no gain from adopting with a little smaller or larger productivity

Detour: Recursive Continuation Value Heuristic

An asset that pays dividend Z each period has value equal to the dividend plus capital gains:

$$V(t, Z) = Z\Delta + \frac{1}{(1 + r\Delta)} V(t + \Delta, Z)$$

Multiply by $(1 + r\Delta)$, subtract V(t,Z), and divide by Δ

$$rV(t, Z) = Z + \frac{V(t+\Delta, Z) - V(t, Z)}{\Delta}$$

Take the limit

$$rV(t, Z) = Z + \frac{\partial V(t, Z)}{\partial t}$$

Summary of a Firm's Dynamic Problem

1. The value function in the continuation region

$$r(t)V(Z,t) = \Pi(Z,t) + \frac{\partial V(Z,t)}{\partial t}, \quad \text{if } \mu = \upsilon = 0$$

2. Value matching condition with $M(t)$ adoption threshold

$$V(M(t),t) = \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t) - X(t)$$

3. Smooth pasting condition

$$\frac{\partial V(M(t),t)}{\partial Z} = 0$$

4. Free Entry Condition

$$X(t)/\chi \geq \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t)$$

Summary of a Firm's Dynamic Problem

1. The value function in the continuation region

$$r(t)V(Z,t) = \Pi(Z,t) + \left(\mu + \frac{\upsilon^2}{2}\right) Z \frac{\partial V(Z,t)}{\partial Z} + \frac{\upsilon^2}{2} Z^2 \frac{\partial^2 V(Z,t)}{\partial Z^2} + \frac{\partial V(Z,t)}{\partial t}$$

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4. Free Entry Condition

$$X(t)/\chi \geq \int_{M(t)}^{\infty} V(Z,t)\mathrm{d}\Phi(Z,t)$$

Law of Motion for the Productivity Distribution

The productivity distribution (with CDF $\Phi(Z,t)$) evolves according to the following Kolmogorov Forward Equation (KFE):

$$\frac{\partial \Phi(Z,t)}{\partial t} = \underbrace{\Phi(Z,t)\left(\underbrace{S(t)+E(t)}_{\text{adopt or enter}}\right)}_{\text{distributed below } Z} - \underbrace{S(t)}_{\text{adopt at } M(t)} - \underbrace{\delta\Phi(Z,t)}_{\text{Death}}\ldots$$

$$-\underbrace{\left(\mu-\frac{\upsilon^2}{2}\right)Z\,\frac{\partial \Phi(Z,t)}{\partial Z}}_{\text{deterministic drift}} + \underbrace{\frac{\upsilon^2}{2}Z^2\,\frac{\partial^2 \Phi(Z,t)}{\partial Z^2}}_{\text{Brownian motion}}.$$

A solution to this is a truncation

$$\phi(Z,t) = \frac{\phi(Z,0)}{1-\Phi(M(t),0)}$$

The probability density function at date t is a truncation of the initial distribution at the reservation adoption productivity.

The Initial Productivity Distribution

Assumption

The initial distributions of productivity are Pareto,

$$\Phi(Z,0) = 1 - \left(\frac{M(0)}{Z}\right)^{\theta} \quad \textit{with density} \;\; \phi(Z,0) = \theta M(0)^{\theta} Z^{-1-\theta}.$$

Lemma

Pareto assumption and the solution to the KFE implies

$$\phi(Z,t) = \theta M(t)^{\theta} Z^{-1-\theta}.$$

If the initial density is Pareto with shape θ, it remains Pareto with shape θ.

Outline for the rest of PTW (AER 2021)

In the no GBM, no exit model ask some qualitative questions about the balanced growth path...

1. How do changes in variable trade costs affect growth?
2. What is the role of reallocation vs. market size effects?
3. How do changes in variable trade costs affect welfare?

Ask some quantitative questions in the general setup of model...

4. Calibrate to aggregate and firm dynamics data.
5. Study a local decomposition to identify the sources of the gains from trade and how they differ from those in an efficient economy.
6. Study a large decrease in trade costs inclusive of the transition path.

The Balanced Growth Path

Proposition (Growth on the BGP)

Given parametric assumptions and parameter restrictions, there exists a unique Balanced Growth Path Equilibrium with growth rate

$$g = \frac{\rho(1-\chi)}{\chi\theta}\frac{\bar{\pi}}{\bar{\pi}_{\text{min}}} - \frac{\rho}{\chi\theta},$$

where

- $\bar{\pi} =$ *profits of the average firm.*
- $\bar{\pi}_{\text{min}} =$ *profits of the marginal, just adopting firm.*
- *And the profit ratio has the closed form expression*

$$\frac{\bar{\pi}}{\bar{\pi}_{\text{min}}} = \frac{\left(\theta + (N-1)(\sigma-1)d^{-\theta}\left(\kappa\frac{\chi}{\rho(1-\chi)}\right)^{1-\frac{\theta}{\sigma-1}}\right)}{(\theta-\sigma+1)}.$$

The Balanced Growth Path

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where

- $\bar{\pi}$ = *profits of the average firm.*
- $\bar{\pi}_{\text{min}}$ = *profits of the marginal, just adopting firm.*

Key feature: Growth encodes the trade-off that firms' face in a simple way:

Expected benefits (average profits) vs. the opportunity cost (forgone profits).

Trade, Profits, and Growth

Proposition (Comparative Statics: Trade, Profits, and Growth)

A decrease in variable trade costs...

1. *Decreases a country's home trade share*

$$\varepsilon_{\lambda_{ii},d} \;=\; \theta(1-\lambda_{ii}) \;>\; 0.$$

2. *Increases the spread in profits between the average and marginal firm*

$$\varepsilon_{\bar{\pi}_{rat},d} \;=\; \left(\frac{-(\sigma-1)}{1+\lambda_{ii}(\theta-1)}\right)\varepsilon_{\lambda_{ii},d} \;<\; 0.$$

3. *Increases economic growth*

$$\varepsilon_{g,d} \;=\; \left(\frac{\chi(1+\theta-\sigma)}{(\sigma-1)(1-\chi)}\lambda_{ii}-1\right)^{-1}\varepsilon_{\lambda_{ii},d} \;<\; 0.$$

Reallocation or Market Size Effects?

Proposition (Growth with No Selection into Exporting)

In the model with $\kappa = 0$, in which all firms sell internationally, the growth rate is

$$g = \frac{\rho(1-\chi)}{\chi\theta} \frac{\bar{\pi}}{\bar{\pi}_{\text{min}}} - \frac{\rho}{\chi\theta},$$

where the ratio of average profits to minimum profits is

$$\frac{\bar{\pi}}{\bar{\pi}_{\text{min}}} = \frac{\theta}{1+\theta-\sigma}.$$

Without reallocation effects, trade has no impact on growth.

Consumption Effects

Proposition (Variety, Labor, and Consumption)

A decrease in variable trade costs...

1. *Reduces domestic varieties.*

$$\varepsilon_{\Omega,d} = \left(1 - \frac{1+\theta-\sigma}{\theta\sigma(1-\chi)}\lambda_{ii}\right)^{-1} \varepsilon_{\lambda_{ii},d} \; > \; 0.$$

2. *Reduces the share of workers in goods production.*

$$\varepsilon_{\tilde{L},d} = \left(\frac{\theta\sigma(1-\chi)}{1+\theta-\sigma}\lambda_{ii}^{-1} - 1\right)^{-1} \varepsilon_{\lambda_{ii},d} \; > \; 0.$$

3. *Reduces the initial level of consumption.*

$$\varepsilon_{c,d} = \varepsilon_{\tilde{L},d} + \frac{\varepsilon_{\Omega,d} - \varepsilon_{\lambda_{ii},d}}{\sigma - 1} < 0.$$

Welfare

Steady-state utility is a function of the level of consumption and its growth rate

$$\bar{U} = \frac{\rho \log(c) + g}{\rho^2}.$$

Proposition (Welfare Effects)

The change in utility from a change in trade costs is

$$\varepsilon_{\bar{U},d} = \frac{\rho^2}{\bar{U}} \left(\rho \ \varepsilon_{c,d} + g \ \varepsilon_{g,d}\right).$$

Welfare depends on competing forces...

- Loss in consumption level from less varieties, more "investment" in technology adoption.
- Faster economic growth.

Global Analysis: The Gains from Trade

How does the economy react to a larger reduction in trade costs **inclusive of transition dynamics**?

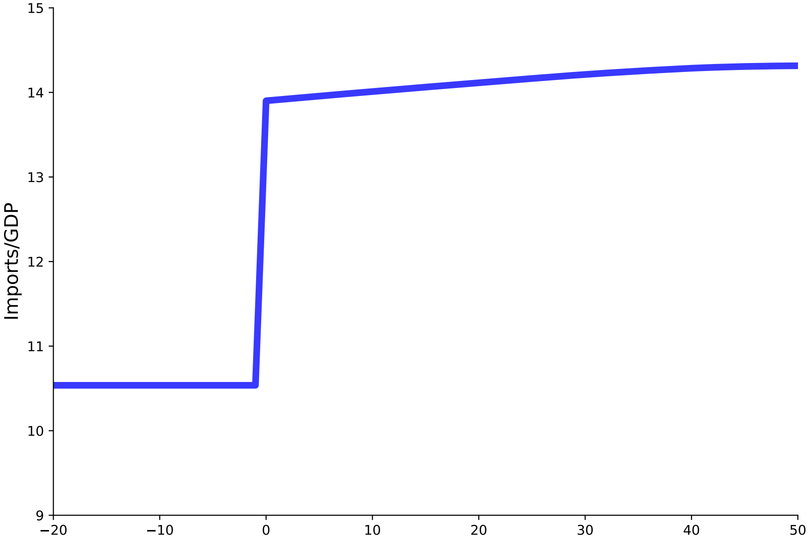
The quantitative experiment:

- Start from the economy on calibrated BGP,
- Shock the economy with an unanticipated ten percent permanent reduction in trade costs,
- Study how the economy transits to the new low-trade-cost BGP equilibrium.

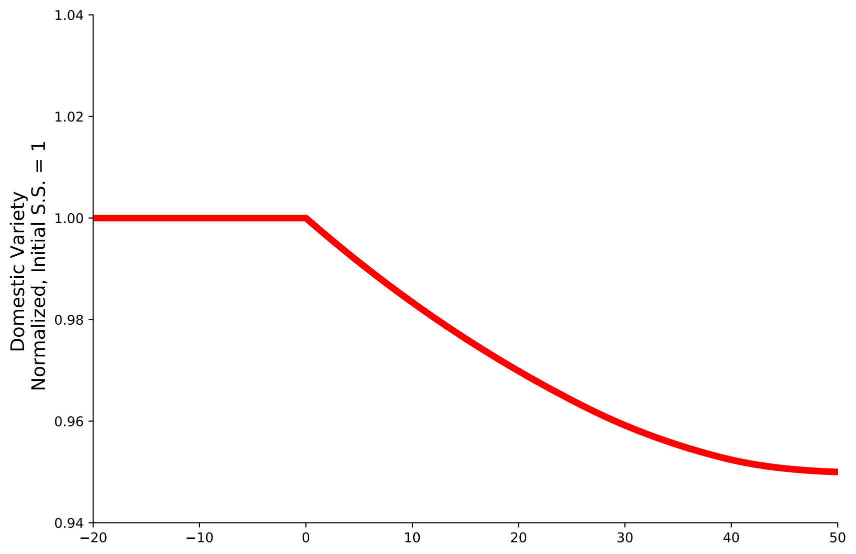
Technical details:

- Characterize the math problem as a differential-algebraic system of equations (DAEs)
- Use a professional DAEs solver to compute transitions
- Avoid home-brewing own PDE solver with finicky dependence on grids
- In background: upwind finite-difference methods (again, see Ben Moll's notes)
- See `https://github.com/jlperla/PerlaTonettiWaugh.jl` for replication code (Julia) and documentation, including a simple warm-up case

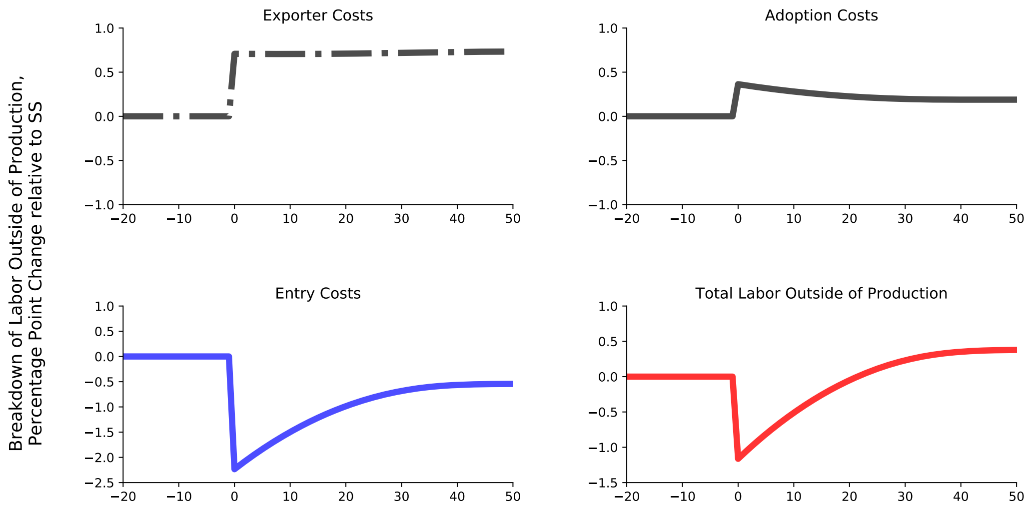
Trade: Near Instantaneous Jump



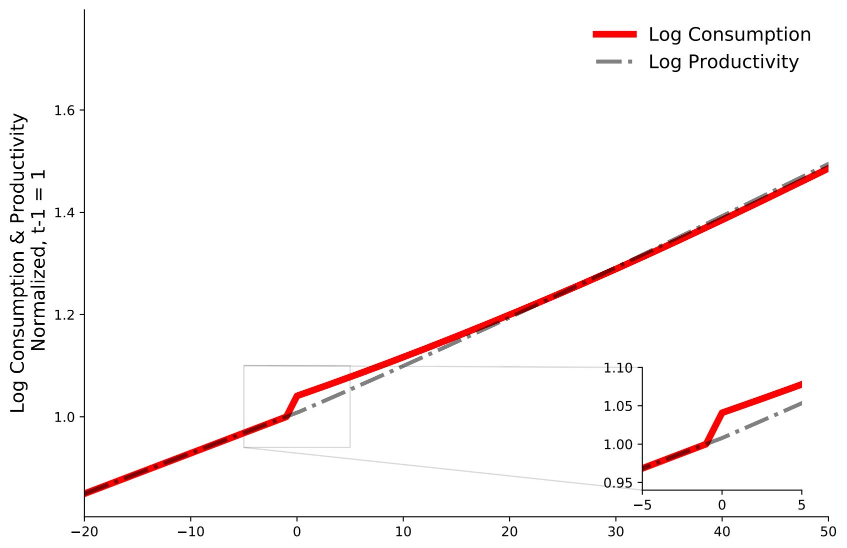
Domestic Variety $\Omega(t)$: Slow Adjustment as Firms Exit, Delayed Entry



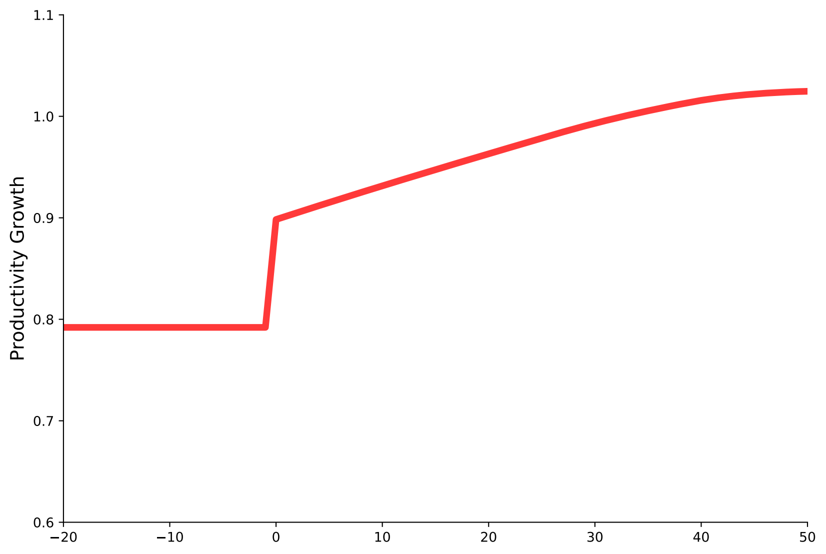
Reallocation Effects: Labor Reallocates to Expand Trade and Technology Adoption



Implication # 1: Consumption Level Overshoots



Implication # 2: Productivity Growth Slowly Rises



Innovation and Diffusion

Benhabib-Perla-Tonetti (ECMA 2021): Innovation and Adoption

- Perla-Tonetti and Perla-Tonetti-Waugh papers have models of adoption driven growth
- Imitation can't drive growth forever. Perla-Tonetti relied on power law initial distribution
  - See Luttmer (2020) for discussion of uniqueness and hysteresis
- Want: Model in which endogenous innovation pushes out finite productivity frontier
- Want: Model that emphasizes that low productivity firms improve without doing R&D/innovation in the same way that high productivity firms do
- Q: How do adoption and innovation interact to determine shape of productivity distribution (think how important θ is in trade models, where does it come from)
- Q: When and how does adoption affect long run growth

Benhabib-Perla-Tonetti (ECMA 2021): Innovation and Adoption

- Q: How do adoption and innovation interact to determine shape of productivity distribution
  - Initial distribution has finite support
  - Finite-state Markov process for innovation $\rightarrow$ finite support $\forall\ t$ (GBM?)
  - Innovation by top z firms pushes out frontier stretching distribution and fattening tail
  - Adoption helps low z firms keep up with the pack. Compresses distribution
  - Comparative statics on costs of adoption and innovation to see effect on endogenous tail index (dist shape)

- Q: When and how does adoption affect long run growth
  - With finite frontier, innovation rate of frontier firms is the aggregate growth rate
  - Adoption irrelevant? No.
    - Lower cost of adoption lowers incentives to innovate
      —option value/free rider problem
    - Partially excludability: Adopters pay licensing fee (nash bargain)
      More adopters, more fees, more incentive to innovate

Buera-Oberfield (ECMA 2020)

- What if there were one process that captured adoption and innovation
- Building on Kortum and Jovanovic-Rob:
- A new productivity q is the combination of an insight from another producer q' and some original idea z drawn from an exogenous distribution
  $q = zq'^{\beta}$
- z reminiscent of Kortum 1997 and q' reminiscent of idea diffusion models
- Can sustain long-run growth because new ideas are being created, not just copying existing ideas

Conclusion

- Technology diffusion, broadly defined, is everywhere:
  - Cross-country diffusion affects the distribution and dynamics of per capita GDP across countries
  - Diffusion across firms affects the distribution and dynamics of size and profits
  - Diffusion of knowledge across workers can affect the distribution and dynamics of wages (see Jarosch-Oberfield-Rossi-Hansberg ECMA 2021)
- Today I discussed some simple models that can be used to structure analysis of aggregate and micro productivity dynamics
- There is a rich literature and I omitted many great papers
- Much work remains to improve the models, in part by confronting them with more (macro and micro) data
  - See Brooks-Donovan-Johnson (2021) using RCT and survey data in Kenya
  - See König-Song-Storesletten-Zilibotti (2020) for R&D and productivity growth data in China