Estimating Production Functions of Multiproduct Firms*

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Abstract

I estimate production functions of multiproduct firms when technologies are product-specific but inputs are observable only at the firm-level. I provide an estimation strategy that solves for the unobservable inputs while correcting for the well-known simultaneity, collinearity and omitted price problems in production function estimation. The key insights of the estimation strategy are, first, using output demand estimates in identifying the product-level input allocations and production functions, and second, using an inverse of the production function to control for endogeneity. Multiproduct firms constitute a considerable share of firms, and even a greater share of production. Estimates of production functions and the implied productivity distributions serve as input for numerous economic studies.

Keywords: Multiproduct firm, production function, productivity

JEL codes: D24, L11, L25

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1 Introduction

A substantial share of firms is multiproduct firms[1], and even a greater share of goods is provided by these multiproduct producers. For example, in the US manufacturing sector in 1987 to 1997, 39% of the firms manufactured more than one product title, while these multiproduct firms accounted for 87% of the sector's output (Bernard, Redding and Schott, 2010). In a large sample of Finnish manufacturing plants on years 2004 to 2011[2], more than 60% of the plants produce at least two product titles. The product scopes range up to 82 titles, and the average product scope of multiproduct firms is 4.3 titles. In international trade multiproduct firms are even more widely present: they accounted for more than 99% of the US exports in 2000 (Bernard, Jensen, Redding and Schott, 2007). Moreover, the product assortments and their output shares vary both across firms,[3] and across time (Bernard, Redding and Scott, 2010).

Despite the empirical fact that multiproduct firms are prevalent, and hence many firms are likely to use several production technologies[4], the standard practice in production function estimation is to assume that all firms are singleproduct firms with a single production technology. Most often the output variable is the sum of sales revenue from the various products, and hence the production functions are estimated at the firm-level. The reason for this is pragmatic: to the best of my knowledge, there is no dataset that reports input allocation at the product-firm level for a cross-section of firms.

Unfortunately, the standard practice of ignoring product-specific production technologies, and assuming firm-level production functions instead, is likely to have severe implications on production function estimates. Using simulations, Valmari (2014) finds that the biases in the estimated firm-level parameters are substantial even when the true product-specific technologies are very similar. The directions and the magnitudes of the biases are determined by intricate functions of the true product-specific technologies and the product scopes of the firms in the industry. The estimated productivity levels have a relatively low correlation with the true firm-level productivity levels when the firms' product scopes are heterogenous, as they usually are.

In this paper I estimate product-specific production functions of firms that are mostly

[1] Multiproduct firms exist due to economies of scope. See, for example, Panzar (1989) for how production technology affects firm and industry structure.

[2] For the description of this data used in this paper, see section 5.1

[3] This is an observation on the data used in this paper.

[4] Hence I may also adopt the term *multitechnology firm* in this paper, but as *multiproduct firm* is already an established term in the literature, and also refers to the fact that these firms sell their goods in various product markets, I stick to the term *multiproduct firm*.

multiproduct producers. I provide a simple structural estimation strategy for product-level production functions when factors of production are observed only at the firm- or establishment-level, which is typical of most micro-level datasets. The challenges consist of solving for the unobservable product-level inputs and, as always in production function estimation, controlling for endogeneity problems, i.e., the endogeneity of inputs to the unobservable productivity. The first key insight underlying my estimation strategy is that by inverting the production function, the very definition of productivity can be used to control for the unobservable productivity level. The second insight that is that, once one can control for the unobservable productivity level, the demand for the final goods can be used to identify the unobservable input allocation as well as the production functions.

I demonstrate the method by using Finnish manufacturing data with output quantities and prices observed at the product-plant-level, and input quantities and prices at the plant-level. I estimate the product-level production functions used in two industries: "Sawmilling and planing of wood" (PRODCOM 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM 162). The empirical findings suggest that production functions should be estimated at the product- instead of the firm-level, and that multiproduct firms use multiple production technologies.

Production function estimates and the implied productivity distributions serve as input for various economic studies. Effects of a new technology or how a change in the level competition affects firms' productivity, market outcomes, and total welfare are typical examples. Productivity distributions speak to the question of how efficiently resources are allocated within industries. One stylized fact of the production function literature is that even within narrowly defined industries, productivity differentials between firms are substantial and persistent (Doms and Bartelsman, 2000; Syverson, 2011). Syverson (2004) finds that in four-digit SIC industries of the US manufacturing sector, on average, the plant at the 90th percentile of the productivity distribution produces almost twich as much as the plant at the 10th percentile with the same measured inputs. Hsieh and Klenow (2009) report even higher productivity differentials for China and India where, on average, the plant at the 90th percentile is more than five times as productive as the plant at the 10th percentile. Another stylized fact is that competition within the industry is correlated with productivity, and that competition narrows the productivity distribution.[5] Competition can have a causal effect on firms' productivity, i.e., shift the produc-

[5] See Berger and Hannan (1998), Dunne, Klimek and Schmitz (2010), Schmitz (2005), and Syverson (2004).

tivity distribution to the right. Alternatively, it can drive the least productive firms out of the industry, which truncates the productivity distribution from the left tail. Nevertheless, lack of competition has not been identified as the cause of the wide productivity distributions reported. This makes the first stylized fact of wide productivity distributions even more surprising. My estimation strategy may be used to examine whether some of the surprisingly large productivity differentials may be an outcome of incorrectly assuming industry- instead of product-specific production function parameters.

Accounting for product-specificity in production enables economists to study also new economic questions. For example, we don't yet fully understand what economic factors determine firms' productivity evolution and the productivity differentials observed between firms. As many key strategic decisions are made at the product-level, understanding production and profit maximization at the product-level is essential. Due to the practice of estimating productivity at the firm-level, the product-level factors are still largely unexplored. Furthermore, endogeneous product choices by firms, and how these endogeneities can be taken into account in, for example, demand estimation, entry models and policy simulations, have become a subject of interest in the recent industrial organization literature.[6] So far, however, the role of product-specific technology on product choice has not been studied.

In the next chapter I review shortly the literature on identification of production functions and production by multiproduct firms. The model and the estimation strategy are presented in chapters 3 and 4. In chapter 5, I introduce the dataset and provide further details of the estimation procedure. Empirical results are presented in chapter 6. Chapter 7 provides a discussion on how the identifying assumptions of my estimation strategy relate to the current production function literature, and chapter 8 concludes.

2 Literature

This paper relates to two bodies of literature. The first is about identification and estimation of production functions. The second is about production by multiproduct firms.

[6] See Ackerberg, Crawford and Hahn (2011), Draganska, Mazzeo and Seim (2009), and Seim (2006).

2.1 Identification of production functions

The current literature recognizes several identification issues that challenge the estimation of production functions. Marschak and Andrews (1944) first pointed out that inputs are not independent variables because firms set them with the aim of maximizing profit. More precisely, inputs are endogeneous to the productivity level that is unobservable to the econometrician. This endogeneity bias, often referred to as the simultaneity or transmission bias, is the identification problem most carefully considered in the literature. Traditional solutions are using instrumental variables or estimating a fixed effects model (Mundlak, 1961). In practice, however, these solutions have not performed well. Data sets usually fall short of appropriate instruments for the endogenous variables. Furthermore, the fixed effects model relies on an unrealistic assumption of firm productivity being constant over time. Failure to correct for the simultaneity bias leads to overestimated production function parameters for the flexible inputs such as materials and possibly also labor.

Another endogeneity problem is the selection bias. As first discussed by Wedervang (1965), econometricians do not observe a random sample of firms. A firm's decision to be active in the market depends on its productivity level as well as its fixed input stocks. Firms with a large capital stock may find it profitable to stay active in the market even if they face a negative productivity shock, while the same holds for firms with a small capital stock that face a positive productivity shock. Hence the fixed input stocks and the unobservable productivity levels of the firms observed are negatively correlated. If firm selection is not accounted for, the production function parameters for the fixed inputs, such as capital, are overestimated.

Olley and Pakes (1996, henceforth OP) were the first to correct for the selection bias, while also controlling for the simultaneity of inputs with a novel structural method. To take account of selection OP estimate survival probabilities for the observed firms. The insight that allows them to correct the simultaneity problem is that a firm chooses its investment level as a function of the firm's productivity. Hence the firm's demand for investment, which OP write as a nonparametric function, can be used to back out the unobservable productivity. The key assumptions that enable this identification strategy are (1) strict monotonicity of investment in productivity, (2) productivity as the only unobservable in investment demand, and (3) the timing of investment (labor) choices before (after) the productivity shock. To relax the rather strict assumption of a monotonic investment function, Levinsohn and Petrin (2003, henceforth

LP) propose using demand for intermediate inputs, rather than investment, in inverting out productivity. Wooldridge (2009) shows how the two-step estimators of OP and LP can be implemented in one step to improve efficiency.

Ackerberg, Caves and Frazer (2006, henceforth ACF) observe that the identification strategies of OP, and especially of LP, suffer from collinearity problems. ACF point out that in both estimation strategies the static labor input is collinear with the nonparametric input demand function that is inverted for the unobservable productivity. ACF provide an alternative identification strategy that uses the insights of OP and LP but with slightly modified timing assumptions avoids the aforementioned collinearity problem. However, they also acknowledge that if a gross output production function with more than one flexible input is estimated, there is one identification problem remaining. As shown by Bond and Söderbom (2005), in the absence of inter-firm variation in the input prices, flexible inputs are collinear with each other and with any fixed inputs.

Some studies attempt to control for the collinearity problem by estimating a value added production function that has only one flexible input. However, Gandhi, Navarro and Rivers (2013) show that the value added specification is not a resolution to the collinearity problem, but induces a so-called value added bias instead. In excluding flexible inputs, which are collinear with productivity and other inputs, the degree of productivity heterogeneity is overstated and the elasticity estimates for the fixed inputs are biased. Gandhi et al. show that if the value added bias is not corrected, the estimated inter-firm productivity differences are orders of magnitude larger, and even of opposite sign, than the productivity differences obtained when correcting for the bias. They provide a strategy to correct for the collinearity and simultaneity problems for both gross output and value added specifications. Gandhi et al. make the same assumptions regarding timing of input choices and evolution of productivity as ACF, but identification is based on a transformation of the firm's short-run first order conditions.

Also the so-called monotonicity assumption of the aforementioned proxy estimators has been contested. Ornaghi and Van Beveren (2011) compare the performance of the proxy method proposed by OP, and modifications to it by LP, ACF, and Wooldridge. The methods differ in the proxy variables, assumptions on the timing of input decisions and when investments translate into productive capital, and moment conditions. However all the estimators are based on the so-called monotonicity assumption that the proxy variable monotonically increases in the unobservable productivity term. As noted by Ornaghi et al., if the monotonicity assumption is

violated, the estimators yield inconsistent estimates. They propose a diagnostic tool for testing whether the monotonicity assumption holds for the estimators. Ornaghi et al. find that the assumption fails to hold in the majority of cases. The assumption holds in all three industries examined in at least 90% of the cases only for three estimators: OP/LP with non-linear least squares, OP/LP with GMM, and Wooldridge's one-step estimator with the assumptions of OP. Furthermore, there is a large degree of heterogeneity in the results, which indicates that the timing assumptions and the choice of the estimator affect the estimates.

Another type of identification problem is the omitted price bias, which occurs whenever the production function is estimated using sales revenue and/or input expenditure data, and output and/or input prices are not equal across firms. Harrison (1994) discusses the bias with input prices, and Klette and Griliches (1996) with output prices. Despite the considerable biases these inter-firm price differentials can induce, they have been ignored to a large extent in the empirical literature. The explanation is largely practical: output and input are often measured in sales revenue and expenditures only.

The most recently remarked identification problem concerns firms' endogeneous product selection. Bernard, Redding and Schott (2009) note that most firms make production decisions at a more disaggregated level than what is observed in the data and therefore studied in the productivity literature. They consider single-product firms that choose one out of two heterogeneous goods based on the productivity of the firm, as well as the production technologies and demand for the goods. Bernard et al. derive the productivity bias that arises in revenue production function estimation when endogeneous product selection is not accounted for. The so-called product bias is determined, not surprisingly, by the same factors that influence product selection. The empirical implications of ignoring product endogeneity have not been considered.

Also the functional form assumptions have been challenged. When estimating the Cobb-Douglas production function the vast majority of firm-level studies assume that productivity is Hicks neutral, i.e. that a change in productivity does not change the input shares used. Using data on U.S. manufacturing plants Raval (2012) shows that a CES production function with labor augmenting productivity differences better accounts for the characteristics of the firms observed, as compared to the Hicks neutral Cobb-Douglas technology.

2.2 Multiproduct firms

A large share of the recent literature on multiproduct firms is written in the context of international trade, perhaps because international trade flows are dominated by multiproduct firms. In 2000, firms that exported more than one product title, as defined at the ten-digit level, accounted for more than 99% of the US export value (Bernard, Jensen, Redding and Schott, 2007). A number of studies centers on how reductions in barriers to international trade affect firms' productivity and product scope. Nearly every study finds that as reductions in trade barriers lead to increased competition, the firms that remain active become more productive. Theoretical findings on the product scope, which is a potential channel for productivity effects to take place, are mixed. As a consequence to reductions in trade barriers, product scopes are found to decrease[7], increase[8], or both[9]. Empirical evidence indicates that increased competition drives firms to concentrate on the goods they are most competent in and drop the least productive products from the selection of exported goods,[10] unless industrial regulations hinders firms from doing so (Goldberg, Khandelwal, Pavcnik andTopalova, 2010). In other words, empirical evidence suggests that firms' productivity across goods vary.

Multiproduct firms are widely present also within national markets. As in the global markets, firms' production decisions are not restricted to entry and exit decisions at the extensive margin and production scale adjustments at the intensive margin. In fact, changes in product scope, i.e. in the intra-firm extensive margin, are substantially more frequent than changes in the extensive margin (Bernard, Redding and Schott, 2010; Broda and Weinstein, 2010). Dropping old goods and starting production of new ones are central decisions in firms' production and competition strategy. Bernard, Redding and Schott (2010) find that changes in product scope lead to productivity gains for US manufacturing firms. Product choices are key variables also in strategic actions between firms, with implications on market structure[11], competition[12], and incentives to invest in product quality[13].

An assumption that frequently underlies theoretical studies as well as interpretations of

[7] See Bernard, Redding and Schott (2011), Eckel and Neary (2010), Mayer, Melitz and Ottaviano (2014), and Nocke and Yeaple (2013).

[8] See Feenstra and Ma (2007), and Ma (2009).

[9] See Allanson and Montagna (2005).

[10] See Bernard, Redding and Schott (2011), Mayer, Melitz and Ottaviano (2013), and Baldwin, Caves, Gu (2005).

[11] See Eaton and Schmitt (1994).

[12] See Ju (2003), Johnson and Myatt (2003, 2006), and Roson (2012).

[13] See Eckel, Iacovone, Javorcik and Neary (2011).

empirical findings is that multiproduct firms conduct flexible manufacturing. Flexible manufacturing means that producers can add new goods to their product assortment without making considerable investments in production technology, albeit the good-specific marginal costs increase as the product scope grows (e.g. Eckel and Neary, 2010). Flexible manufacturing is closely related to the concept of core competency, which means that a multiproduct firm can produce one or a few of its goods more efficiently than the rest of its goods (e.g. Bernard, Redding and Schott, 2011). Production function estimation does not typically accommodate the concepts of flexible manufacturing or core competency, however, apart from a few exceptions discussed below.

Virtually all estimates of production functions are implicitly based on the assumption that all of the firm's output is produced with a firm-level technology.[14] The first set of papers that make an exception evaluate cost minimization with a nonparametric methodology. Cherchye, De Rock and Vermeulen (2008) allow for product-specific technologies as well as economies of scope that result from joint input use and input externalities. Their methodology does not require observable input allocation. Cherchye, De Rock, Dierynck, Roodhooft and Sabbe (2011) build on Cherchye, De Rock and Vermeulen (2008) using a methodology based on data envelopment analysis. In contrast to Cherchye, De Rock and Vermeulen (2008), they use information on output-specific inputs and joint inputs. As a result the discriminatory power of the efficiency measurement is higher, and the efficiency value of the decision making unit can be decomposed into output-specific efficiency values. However, the methodology is not suited for any typical firm- or plant-level dataset due to the requirement on observable input allocation. Cherchye, Demuynck, De Rock and De Witte (2011) distinguish between two assumptions: cooperative cost minimization at the firm level, and uncooperative minimization at the level of output department. The advantage of these nonparametric methodologies is that they do not require functional form assumptions. On the other hand, the typical endogeneity biases are not treated.

De Loecker, Goldberg, Khandelwal and Pavcnik (2012) estimate production functions to examine how trade liberalization affects product-specific marginal costs and price markups. They use data on singleproduct firms and the estimation strategy of Ackerberg et al. (2006) to estimate good-specific production function parameters, which are assumed to be the same for

[14] There is an early literature on estimating cost functions of multiproduct firms. See, for example, Brown, Caves and Christensen (1979) and Caves, Christensen and Tretheway (1980). The early multiproduct cost functions allow for the fact that production technologies across goods vary, but they do not correct the typical endogeneity problems such as the simultaneity or selection bias.

single- and multiproduct firms. In estimating the product-level input allocations De Loecker et al. assume that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e. independent of the input type. They show that cost efficiency as well as profitability vary across the various products firms produce. They also find a positive correlation between productivity and the size of the product scope, and suggest that firms may use reductions in marginal costs to finance the development of new products. The method adopted by De Loecker et al. is perhaps closest to the empirical strategy presented in this paper, and the assumptions underlying their estimation method are discussed in section 6.1.

Dhyne, Petrin and Warzynski (2013) study price, markup, productivity and quality dynamics of Belgian manufacturing firms. They modify the proxy approach of Wooldridge (2009) to estimate a product-level production function where the output of a given good is related to the firm-level inputs, the output quantities of the other goods the firm produces, and an unobservable firm-level productivity term. Estimating the production function does not require solving for the unobservable input allocations. However, the output elasticities of the inputs as well as the productivity levels are assumed constant across goods. Dhyne et al. also estimate a variable cost function for multiple goods, which takes into account the productivity shocks that are implied by the production function estimates.

3 Model

The model consists of good-specific production and demand functions, and assumptions on the timing of production decisions. Production functions are typically estimated without considering demand for the goods, but in this study output demand is they key for identifying good-specific input allocations and production functions. When firms have market power in the output market, the production decisions are functions of the downward-sloping output demand curves. Functional forms and also most of the other assumptions are familiar from empirical microeconomic literature. The only exception is that the production function is specified at the product-level instead of the firm-level. The key assumptions in identifying the empirical model are discussed in more detail in chapters 4 and 7.

3.1 Production

Firm j produces n_{jt} goods at time t. Production technology i is a good-specific Cobb-Douglas production function with three inputs, materials M_{ijt}, labor L_{ijt}, and capital K_{ijt}:

$$Q_{ijt} = \exp(\beta_{0i}) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp(\omega_{ijt}). \tag{1}$$

Parameters β_{Mi}, β_{Li}, and β_{Ki} denote the marginal products of materials, labor, and capital for good i, and β_{0i} is a constant. All the production function parameters are good-specific. The productivity term ω_{ijt} varies across goods, firms, and time. It can be divided into expected productivity, $E[\omega_{ijt}|\omega_{ijt-1}]$, and a mean zero productivity shock, ξ_{ijt}:

$$\omega_{ijt} = E[\omega_{ijt}|\omega_{ijt-1}] + \xi_{ijt}. \tag{2}$$

Productivity ω_{ijt} comprises all factors other than M_{ijt}, L_{ijt}, and K_{ijt} that affect the firm's production volume in a given product line and time period. Examples of such factors are management and organization of production and down-time due to, for example, maintenance work and defect rates in the manufacturing process (Ackerberg, Caves and Frazer, 2006). Productivity $\exp(\omega_{ijt})$ follows a first-order Markov process. The firm's decision maker forms an expectation of period t's productivity, $E[\omega_{ijt}]$, as a function of the previous period's productivity ω_{ijt-1}. The productivity shock ξ_{ijt} represents a deviation from the expected productivity that takes place or becomes observable at the beginning of period t. The shocks ξ_{ijt} may or may not be correlated across the product lines of the firm. For example, managerial changes may have a similar effect on all the product lines, but they may also have different impacts. Similarly, productivity ω_{ijt} may or may not be correlated across the product lines. The firm may have achieved heterogeneous productivity levels due to, for example, different paths of learning and experience. Also physical economies of scope are captured in the total factor productivity term $\exp(\omega_{ijt})$.

Labor L and capital K are substitutable across the product lines of the firm. All the factors of production are continuously divisible and exclusive across product lines. This means that they can be flexibly allocated across the different product lines, and that any given share of a firm-level input stock is used in only one product line at a time. Furthermore, none of the production functions utilizes other inputs than M_{ijt}, L_{ijt}, and K_{ijt}. This rules out utilization

of by-products as factors of production.

3.2 Demand

The firm faces a downward sloping and isoelastic demand curve for each of its goods:

$$Q_{ijt} = \exp\left(\alpha_{ij}\right) P_{ijt}^{\eta_i} \exp(\varepsilon_{ijt}). \tag{3}$$

Price elasticity of demand, η_i, is good-specific and assumed to be lower than -1. Price elastic demand is required to rule out cases where firms produce marginally small output quantities of various goods. The level of demand, denoted by α_{ij}, depends on unobservable factors such as the quality of the good. These factors vary across goods and firms, but they are constant over time. Any shocks to the good- and firm-specific demand level are captured by ε_{ijt}. The shocks can be caused by changes in buyers' preferences or income, prices of substitutes or complementary goods, or the number of buyers in the market, for example.

3.3 Timing of production decisions

The three types of inputs, M_{ijt}, L_{ijt}, and K_{ijt}, differ in how they are determined. The product-level materials M_{ijt} is a flexible input, set or adjusted at the time of production. It is also a static input, meaning that it doesn't have dynamic implications such as adjustment costs. The firm-level human resources[15] L_{jt} and capital stock K_{jt}, on the other hand, are fixed at the time of production, and they are formed in a dynamic process. L_{jt} is chosen in the previous period $t-1$, while the related costs are paid in the period of production. K_{jt} is determined as a function of the previous period's capital stock and investment, $K_{jt} = f\left(K_{jt-1}, I_{jt-1}\right)$. However, the product-level inputs L_{ijt} and K_{ijt} are allocated in the period of production, subject to the the firm-level constraints $\sum_i L_{ijt} \leq L_{jt}$ and $\sum_i K_{ijt} \leq K_{jt}$.

The outline of the production decisions is as follows. At time $t-1$, the firm observes its current level of human resources L_{jt-1} and capital stock K_{jt-1}, the expected productivity in product lines i at time t, $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$, as well as any other observable factors that affect its future profits. The firm then chooses whether to remain active in the market in period t, and if

[15] L_{jt} is typically a flexible input in structural production function models. I assume fixed L_{jt} to be fixed because it is more realistic of the Finnish labor market, as discussed in section 7. However the model can be estimated under either assumption: flexible or fixed labor input.

so, what product titles i to produce. Then, the firm decides on the next period's level of human resources L_{jt} and, by setting the level of capital investment I_{jt-1}, capital stock K_{jt}.

At time t the productivity shocks ξ_{ijt} and the demand shocks ε_{ijt} realize and become observable to the firm. The firm observes also the price of materials, P_{Mjt}. P_{Mjt} is an exogenous variable, which may reflect the level of bargaining power the firm possesses in the input markets, for example. P_{Mjt} is not a function of the input quantities purchased, however, which implies that there are no cost economies of scope or scale in the form of lower input prices. The firm then chooses the quantities of product-level materials M_{ijt}. At the same time the firm decides how to allocate its human resources L_{jt} and the capital stock K_{jt} among the different product lines the firm is active in, i.e., it sets L_{ijt} and K_{ijt}.

The timing assumptions of this model are similar to the assumptions previously made in the production function literature. These assumptions are compared to those in the previous literature in section 7.

3.4 Firm's optimization problem

The firm maximizes the present discounted value of future profits by making three decisions. First, it chooses which goods i to produce in the next period $t+1$, denoted by $D_{ijt+1} = 1$ if it produces good i at $t+1$, and $D_{ijt+1} = 0$ if otherwise. Second, the firm decides how much human resources L_{jt+1} to employ in the next period. Third, the firm invests I_{jt} to determine the next period's capital stock K_{jt+1}. These decisions are made given the expected demand and productivity for the goods in the next period, as well the expected future material price.

The Bellman equation for the firm's firm-level dynamic optimization problem is:

$$V\left(S_{jt}\right) = \max_{D_{ijt+1}, L_{jt+1}, I_{jt}} \sum_{i} \Pi_{ijt}\left(S_{jt}\right) - C\left(I_{jt}\right) + \frac{1}{1+\rho} E[V\left(S_{jt+1}\right) | S_{jt}, D_{ijt}, L_{jt+1}, I_{jt}] \tag{4}$$

where $\Pi\left(S_{jt}\right)$ is the static profit earned in period t, $S_{jt} = \left(\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt}\right)$ is the vector of state variables, $C\left(I_{jt}\right)$ is the cost of investment, and ρ is the discount rate. The dynamic optimization problem gives rise to policy functions $D\left(S_{jt}\right)$, $L\left(S_{jt}\right)$ and $I\left(S_{jt}\right)$.

Instead of solving for the dynamic optimization problem[16], I follow the examples of Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg, Caves and Frazer (2006), and

[16] Because the dynamic optimization problem is not solved, further specification of the determinants of the dynamic variables is not needed.

solve only the static profit maximization problem, which is sufficient for identifying the production function parameters. The static profit maximization problem consists of allocating the firm-level human resources L_{jt} and capital stock K_{jt} among the various product lines i, and setting the product-specific materials M_{ijt} for each product line:

$$\max_{M_{ijt},L_{ijt},K_{ijt}} \Pi_{ijt} = \sum_i P_{ijt}Q_{ijt} - P_{Mjt}M_{ijt} \text{ s.t. } \sum_i L_{ijt} \leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \tag{5}$$

Substituting in the inverse demand, $P_{ijt} = \left(Q_{ijt}\left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-1}\right)^{\frac{1}{\eta_{ijt}}}$, as well as the production functions, the static profit maximization problem becomes:

$$\begin{aligned} \max_{M_{ijt},L_{ijt},K_{ijt}} \Pi_{ijt} &= \sum_i \left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} - P_{Mjt}M_{ijt} \\ \text{s.t.} \sum_i L_{ijt} &\leq L_{jt} \text{ and} \sum_i K_{ijt} \leq K_{jt}. \end{aligned} \tag{6}$$

The optimization problem yields a Lagrangian equation with two constraints. The constraints account for not exceeding the firm-level human resources L_{jt} and capital stock K_{jt} when the firm makes input allocations to the product lines. More precisely, given that the firm maximizes profit, L_{jt} and K_{jt} are always fully utilized and the constraints are binding as $\sum_i L_{ijt} = L_{jt}$ and $\sum_i K_{ijt} = K_{jt}$. The Lagrangian is:

$$\begin{aligned} Lagr &= \sum_i \left(\exp(\alpha_{ijt}+\varepsilon_{ijt})\right)^{-\frac{1}{\eta_i}} \left(\exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}} L_{ijt}^{\beta_{Li}} K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1} \\ &\quad - P_{Mjt}M_{ijt} + \lambda_{Ljt}\left(L_{jt} - \sum_i L_{ijt}\right) + \lambda_{Kjt}\left(K_{jt} - \sum_i K_{ijt}\right). \end{aligned} \tag{7}$$

The first-order conditions for static profit maximization are (JT is the number of firm-time

-observations):

$$\begin{aligned}
\frac{\partial Lagr}{\partial M_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)(\exp(\alpha_{ij}+\varepsilon_{ijt}))^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Mi}}{M_{ijt}}-P_{Mjt}=0 \\
\forall\ i &= [1,n_{jt}] \qquad (8) \\
\frac{\partial Lagr}{\partial L_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)(\exp(\alpha_{ij}+\varepsilon_{ijt}))^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Li}}{L_{ijt}}-\lambda_{Ljt}=0 \\
\forall\ i &= [1,n_{jt}] \qquad (9) \\
\frac{\partial Lagr}{\partial K_{ijt}} &= \left(\frac{1}{\eta_i}+1\right)(\exp(\alpha_{ij}+\varepsilon_{ijt}))^{-\frac{1}{\eta_i}}\left(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}\exp\left(\omega_{ijt}\right)\right)^{\frac{1}{\eta_i}+1}\frac{\beta_{Ki}}{K_{ijt}}-\lambda_{Kjt}=0 \\
\forall\ i &= [1,n_{jt}] \qquad (10) \\
\frac{\partial Lagr}{\partial \lambda_{Ljt}} &= L_{jt}-\sum_i L_{ijt}=0\ \forall\ jt=[1,JT] \qquad (11) \\
\frac{\partial Lagr}{\partial \lambda_{Kjt}} &= K_{jt}-\sum_i K_{ijt}=0\ \forall\ jt=[1,JT]\,. \qquad (12)
\end{aligned}$$

Although the production functions are product-specific, production of the goods is interdependent because the firm-level human resources and capital stock are fixed at the time of production, and hence the firm has to allocate these inputs across the product lines. The allocation is done as a function of the various demand conditions, production technologies, and the price of materials. Interdependency in production may arise also due to physical economies of scope, which take place when the firm produces several goods and therefore reaches higher productivity levels than when producing only one good.

3.5 Measurement error

The observed variables are product-level Q_{ijt} and P_{ijt}, and firm-level M_{jt}, L_{jt}, K_{jt} and P_{Mjt}. The firm-level materials, M_{jt}, is measured with multiplicative measurement error:

$$\epsilon_{Mjt}=\frac{M_{jt}}{\sum_{i=1}^{n_{jt}}M_{ijt}}-1. \qquad (13)$$

The other observed variables are measured with zero measurement error.

4 Identification and Estimation Strategy

Firm-level Cobb-Douglas production functions have been estimated in numerous studies. With respect to estimation, the product-specific functions of this paper differ from the firm-level

functions in one important aspect: the product-specific inputs are unobservable to the econometrician. This implies that all the elements in the production function are unobservable: input quantities, the marginal outputs of the inputs, and total factor productivity. In other words, not only are the the inputs endogenous to the unobservable productivity, which is a standard problem in production function estimation, but they are also unobservable. Clearly, these two problems are closely related.

My identification strategy is based on two insights: one for controlling the endogeneity of inputs to the unobservable productivity level, and another for identifying the unobservable input allocations. The first insight is that, by definition, output is a function of the firm's productivity: the more productive the firm is, the greater its output for any given level of inputs. The unobservable productivity level can be written as a function of the input allocations and the marginal outputs of the three inputs, β_{Mi}, β_{Li}, and β_{Ki}. I will use this definition of productivity in solving the product-level inputs.

The second insight is that firms make their production decisions as a function of supply-side factors, such as productivity, fixed inputs, and prices of the flexible inputs, but also as a function of the demand for the goods. Intuitively, the higher the demand for a given good, the more inputs the firm is willing to allocate to the product line. Shocks in output demand provide a source of variation for identifying the optimal input allocations. Furthermore, as an overidentifying assumption I can use the notion that the product-level inputs estimated sum up to the observable firm-level inputs.

The optimal input choices are solved analytically from the firm's static profit maximization problem, as a function of the productivity term ω_{ijt} and up to the production function parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} (recall that the state variables $S_{jt} = \left(\alpha_{ijt}, \eta_{ijt}, \varepsilon_{ijt}, L_{jt}, K_{jt}, \omega_{ijt}, P_{Mjt}\right)$):

$$
\begin{aligned}
M_{ijt} &= f_M\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) && (14)\\
L_{ijt} &= f_L\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) && (15)\\
K_{ijt} &= f_K\left(S_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right). && (16)
\end{aligned}
$$

As explained above, the first key of the estimation strategy is using the definition of the productivity term ω_{ijt} in controlling for the endogeneity of inputs. Inverting the production function

for ω_{ijt}, I get:

$$\omega_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\beta_{0i})M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}}\right). \tag{17}$$

By substituting this definition of ω_{ijt} in the analytical input functions $M_{ijt}, L_{ijt}, K_{ijt}$, I obtain:

$$\begin{aligned} M'_{ijt} &= g_M\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) & (18)\\ L'_{ijt} &= g_L\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right) & (19)\\ K'_{ijt} &= g_K\left(S'_{ijt}, Q_{ijt}, \beta_{0i}, \beta_{Mi}, \beta_{Li}, \beta_{Ki}\right), & (20)\end{aligned}$$

where S'_{ijt} denotes the state variables without ω_{ij}. By imposing $M'_{ijt} = M_{ijt}$, $L'_{ijt} = L_{ijt}$, and $K'_{ijt} = K_{ijt}$, and substituting $M'_{ijt}, L'_{ijt}, K'_{ijt}$ and the definition of ω_{ijt} in the production function, I take account of the unobservable productivity level. The production function for good i can then be written as:

$$Q_{ijt} = \exp\left(\beta_{0i}\right) M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}} \exp\left(\omega_{ijt}\right), \tag{21}$$

where β_{0i}, β_{Mi}, β_{Li} and β_{Ki} are the only unobservables. But when written in this form, an infinite number of parameters β_{0i}, β_{Mi}, β_{Li} and β_{Ki} solve the empirical production function. This is because ω_{ijt} is inverted from the production function itself. However, the production function can be identified using the structure of the productivity process, which is a function of the expectation of productivity $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$, and the productivity shock ξ_{ijt}.

Using the productivity shock ξ_{ijt} in identification is a standard practice in structural production function models (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves and Frazer, 2006). Lagged static inputs, in this paper M_{ijt}, are correlated over time but uncorrelated with the productivity shock. Fixed inputs, in this case L_{ijt} and K_{ijt}, are chosen prior to observing ξ_{jt}. Hence, they are not correlated with the productivity shock. As the fixed inputs L_{ijt} and K_{ijt} are subject to different input costs, the two variables are not collinear.

Given the standard assumptions I make regarding the timing of input choices, and given that there are sufficiently many sources of identifying variation, the above moments can be modified to suit the production function specified in this paper. The productivity shocks only

have to be specified at the product-level:

$$
\begin{aligned}
E\left[\xi_{ijt}|M_{jt-1}\right] &= 0 \; \forall \; i=[1,N] & (22)\\
E\left[\xi_{ijt}|L_{jt}\right] &= 0 \; \forall \; i=[1,N] & (23)\\
E\left[\xi_{ijt}|K_{jt}\right] &= 0 \; \forall \; i=[1,N]\,. & (24)
\end{aligned}
$$

The firm-level M_{jt-1}, L_{jt}, and K_{jt} are correlated with the product-level M_{ijt}, L_{ijt}, and K_{ijt} because the firm-level variables are sums of the product-level inputs. An additional instrument is the price of the flexible input, correlated with M_{ijt} but uncorrelated with ξ_{ijt}:

$$
E\left[\xi_{ijt}|P_{Mjt}\right] = 0 \; \forall \; i=[1,N]\,. \tag{25}
$$

P_{Mjt} is a valid instrument even if measured with error because the measurement error is not correlated with the productivity shock.

Demand for good i would also be a valid instrument. Demand for good i correlates positively with the input choices M_{ijt}, L_{ijt} and K_{ijt}, while it is uncorrelated with the productivity shock ξ_{ijt}. Unfortunately, the demand is unobservable. However, the prices realized are informative about the underlying demand. Price for good i depends on the output quantity produced and the level of productivity at which it is produced, that is, P_{ijt} is correlated with the productivity shock and hence not a valid instrument. However, lagged price P_{ijt-1} is correlated with the demand for good i also at time t, and hence with the input choices M_{ijt}, L_{ijt} and K_{ijt}, because demand for good i is correlated over time as denoted by α_{ij}. At the same time, P_{ijt-1} is uncorrelated with the productivity shock:

$$
E\left[\xi_{ijt}|P_{ijt-1}\right] = 0 \; \forall \; i=[1,N]\,. \tag{26}
$$

I also use the fact that product-level inputs M_{ijt} add up to the firm-level input M_{jt}, which is observable but measured with measurement error. Any firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}, is expected to be zero. A valid instrument for identifying β_{Mi} is the product of output price and quantity, $P_{ijt}Q_{ijt}$, which is uncorrelated with the measurement error in materials ϵ_{Mjt}, but correlated with the use of materials M_{ijt}:

$$
E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right] = 0 \; \forall \; i=[1,N]\,. \tag{27}
$$

These moment conditions identify the production technologies.

Identification of the demand functions requires an instrument[17] for the endogeneous prices. The material price P_{Mjt}, human resources L_{jt}, and capital stock K_{jt} correlate with the product prices but they are uncorrelated with the product- and firm-specific demand shocks ε_{ijt}:

$$\begin{aligned} E\left[\varepsilon_{ijt}|P_{Mjt}\right] &= 0\ \forall\ i=[1,N] && (28)\\ E\left[\varepsilon_{ijt}|L_{jt}\right] &= 0\ \forall\ i=[1,N] && (29)\\ E\left[\varepsilon_{ijt}|K_{jt}\right] &= 0\ \forall\ i=[1,N]\,. && (30)\end{aligned}$$

The model is identified with these moments and estimated by GMM.

4.1 Solving for ξ_{ijt}, ε_{ijt}, and ϵ_{Mjt}

The productivity shock ξ_{ijt} is:

$$\xi_{ijt}=\log\left(\frac{Q_{ijt}}{\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}}}\right)-E\left[\omega_{ijt}|\omega_{ijt-1}\right] \tag{31}$$

where M_{ijt}, L_{ijt}, K_{ijt} and $E\left[\omega_{ijt}|\omega_{ijt-1}\right]$ are unknown. M_{ijt}, L_{ijt}, and K_{ijt} are solved from the first-order conditions for static profit maximization, the definition of productivity for the estimation equation, $\omega_{ijt}=\log\left(Q_{ijt}(\exp\left(\beta_{0i}\right)M_{ijt}^{\beta_{Mi}}L_{ijt}^{\beta_{Li}}K_{ijt}^{\beta_{Ki}})^{-1}\right)$, and the demand function inverted for price, $P_{ijt}=\exp\left(\alpha_{ij}+\varepsilon_{ijt}\right)^{-\frac{1}{\eta_i}}Q^{\frac{1}{\eta_i}}$. By substitution:

$$\begin{aligned} M_{ijt} &= \left(\frac{1}{\eta_{ijt}}+1\right)P_{ijt}Q_{ijt}\frac{\beta_{Mi}}{P_{Mjt}}\ \forall\ i=[1,n_{jt}] && (32)\\ L_{ijt} &= \frac{\left(\frac{1}{\eta_{ijt}}+1\right)P_{ijt}Q_{ijt}\beta_{Li}L_{jt}}{\sum_i\left(\frac{1}{\eta_{ijt}}+1\right)P_{ijt}Q_{ijt}\beta_{Li}}\ \forall\ i=[1,n_{jt}] && (33)\\ K_{ijt} &= \frac{\left(\frac{1}{\eta_{ijt}}+1\right)P_{ijt}Q_{ijt}\beta_{Ki}K_{jt}}{\sum_i\left(\frac{1}{\eta_{ijt}}+1\right)P_{ijt}Q_{ijt}\beta_{Ki}}\ \forall\ i=[1,n_{jt}]\,. && (34)\end{aligned}$$

Given M_{ijt}, L_{ijt}, K_{ijt}, and the implied ω_{ijt}, the productivity process is estimated with the following estimation equation:

$$\omega_{ijt}=g\left(\omega_{ijt-1}\right)+\xi_{ijt} \tag{35}$$

[17] For a discussion on instruments used in demand estimation, see, for example, Ackerberg, Benkard, Berry and Pakes (2007).

where $g(\omega_{ijt-1})$ is a second-order polynomial of the lagged productivity term $\omega_{ijt-1}(\beta_{Mi}, \beta_{Li}, \beta_{Ki})$, and ξ_{ijt} is the productivity shock.[18]

Given the solution for M_{ijt} (32), the multiplicative input measurement error ϵ_{Mjt} is computed as:

$$\epsilon_{Mjt} = \frac{M_{jt}}{\sum_{i=1}^{n_{jt}} M_{ijt}} - 1. \tag{36}$$

The demand shock ε_{ijt} is:

$$\varepsilon_{ijt} = \log\left(\frac{Q_{ijt}}{\exp(\alpha_{ij}) P_{ijt}^{\eta_i}}\right) \tag{37}$$

where the unobservable product-firm -specific demand level, $\alpha_{ij}$is (T_{ij}^{-1} is the number of time periods in which firm j has produced good i):

$$\alpha_{ij} = T_{ij}^{-1} \sum_{t=1}^{T_{ij}} \log\left(\frac{Q_{ijt}}{P_{ijt}^{\eta_i}}\right). \tag{38}$$

5 Data and Empirical Implementation

5.1 Data

I use the Longitudinal Database on Plants in Finnish Manufacturing (LDPM) and the Industrial output data of Statistics Finland on years 2004 - 2011. The two datasets include plants that belong to manufacturing firms with at least 20 employees, and a subset of plants of firms with less than 20 employees. The reporting units are mainly plants. The only exceptions are in the Industrial output data, where a few plants belonging to the same firm report jointly. For these reporting units I aggregate the observations in the LDPM accordingly.

I estimate the production functions of firms in Division 16, "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". The products are classified according to Eurostat's 8-digit PRODCOM (Production communautaire) codes that are supplemented by national 10-digit subclasses. Goods within the fairly narrowly defined titles are therefore comparable in physical quantities. The titles are

[18] The parameters in the polynomial $g(\omega_{ijt-1})$, denoted by γ_i, enter the moment conditions linearly. Hence they can be concentrated out from the estimation routine for the nonlinear parameters. The linear parameters γ_i are obtained by regressing the productivity level implied by a given set of parameter values $\omega_{ijt}(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki})$ on the second-order polynomial terms of the implied lagged productivity $\omega_{ijt-1}(\beta'_{Mi}, \beta'_{Li}, \beta'_{Ki})$.

provided in Table 1. For each product title a plant produces in a given year, I observe the output measured in a physical unit as well as the sales revenue. These two yield the average price of the good in the given year. Similarly for the intermediate products and materials I observe physical quantities and expenditures by the PRODCOM titles. The "price" of materials is computed as the Elteto-Koves-Szulc (EKS) multilateral price index. For firm a it can be expressed as follows:

$$P_{EKS}^{a} = \prod_{j=1}^{J} \left(\frac{P_F\left(q^j, q^a, p^j, p^a\right)}{P_F\left(q^j, q^b, p^j, p^b\right)} \right)^{\frac{1}{J}}, \tag{39}$$

where q^j and p^j are the quantity and price vectors of firm j, and $P_F\left(q^j, q^a, p^j, p^a\right)$ is the bilateral Fisher price index between firm a and firm j, $j = 1, ..., J$ (J is the number of firms), which is given by

$$P_F\left(q^j, q^a, p^j, p^a\right) = \left(\frac{q^j * p^a}{q^j * p^j} * \frac{q^a * p^a}{q^a * p^j} \right)^{\frac{1}{2}}, \tag{40}$$

where $q^j * p^j = \sum_{n=1}^{N} q_n^j p_n^j$ (N is the number of product titles). Similarly for $P_F\left(q^j, q^b, p^j, p^b\right)$, where b stands for the base firm chosen. The EKS multilateral index satisfies the circularity (transitivity) requirement, which implies that the same index is obtained irrespective of whether firms are compared with each other directly, or through their relationships with other firms (Hill, 2004; Neary, 2004). The EKS multilateral index is thus well-suited for my purpose of comparing firms when no representative firm exists, and bundles of goods differ between firms.

The labor input is measured in labor costs that comprise salary and social payments. The monetary value of the capital stock is estimated using the perpetual inventory method, $K_{jt} = \delta K_{jt-1} + I_{jt-1}$, where $\delta = 0.9$ and I_{jt} is investment.

The estimation methodology poses certain requirements on the observations. First, all product titles need to be observed in at least four pairs of observations, each pair being from two consecutive years in a given firm. This is because for each product title there are four non-linear parameters to be estimated, and because estimating the 1st order Markov process of productivity evolution requires sequences of at least two observations. Second, observations with missing variables cannot be used in estimation. Observations that do not fulfill the aforementioned criteria are dropped from the sample.

Note that measurement error in output is assumed zero. Unfortunately, there is no other output variable that could be used to verify the accuracy of the product-specific sales revenue variables. The only other output variable available is the plant-level gross output reported in

the LDPM. Gross output is defined as the sum of sales revenue, deliveries to other plants of the firm, changes in inventories, production for own use, and other business revenue, deducting capital gains and acquisition of merchandise. Not suprisingly, gross output is not equal to the sum of product-specific sales revenues from production in all of the plants. As the definition of gross output goes, there are several potential explanations for this. Plants may produce output that is not included in the sales revenue from production (deliveries to other plants of the firm, positive changes in inventories, production for own use), or the sales revenue data may include output produced in some previous year (negative changes in inventories). Moreover, because capital gains and acquisition of merchandise are deducted from gross output, it is not possible to make strong inferences about potential measurement error in output. Unfortunately, the various components of gross output are not reported in the LDPM, and hence I cannot identify why gross output may differ from sales revenue. However, to reduce the likelihood of using observations with major measurement error in output, I use only those observations for which the ratio of sum of sales revenue to gross output is at least 0.6 but not more than 1.4.

In the final sample there are 2053 good-plant-year -level observations and 904 plant-year -level observations, collected from 190 plants during 8 years. In total, 42 different product titles are produced. Plants' product assortments range from 1 up to 17 product titles. A plant produces on average 3.25 product titles.

5.2 Product line specification

Every product title i is related to four nonlinear parameters that need to be estimated: price elasticity η_i, and output elasticities β_{Mi}, β_{Li} and β_{Ki}. If I defined the parameters at the 8- or 10-digit level, I would need to estimate $42 \times 4 = 168$ nonlinear parameters. At least in my setting this is a too large a number of nonlinear parameters to be estimated. Instead, I define the parameters at the 3-digit level, which yields two product categories: "Sawmilling and planing of wood" (PRODCOM code 161), and "Manufacture of products of wood, cork, straw and plaiting materials" (162). This specification implies estimating $2 \times 4 = 8$ nonlinear parameters. The parameters governing the productivity process $g(\omega_{ijt-1})$ are also specified at the 3-digit level. The constants β_{0i} are specific to the goods as defined at the 8- or 10-digit level. Also the productivity levels ω_{ijt} and the productivity shocks ξ_{ijt} are specific to the 8- or 10-digit titles.

There are 15 titles in category 161, and 27 titles in category 162. A plant produces on average 2.17 product titles in category 161 and 1.08 titles in category 162. 56% of the plants in

the sample produce at least one good in category 161, and 61% of the plants produce at least one good in category 162.

5.3 Optimal instruments

To improve the estimator's efficiency, I replace some of the moment conditions discussed above by moments with optimal instruments. Amemiya (1974) derives optimal instruments for non-linear models, and Arellano (2003) provides an overview of optimal instruments in linear and nonlinear models. Reynaert and Verboven (2012) show that adopting Chamberlain's (1987) optimal instruments in estimating the randon coefficients logit demand model of Berry, Levinsohn, Pakes' (1995) reduces the small sample bias and increases the estimator's efficiency and stability.

The optimal instrument is the expected value of the derivative of the structural error term with respect to the parameter, computed at an initial estimate of the parameters:

$$z_{ijt} = E\left[\frac{\partial \xi_{ijt}(\theta)}{\partial \theta'} \mid X_{ijt}\right] \tag{41}$$

where θ contains the parameters to be estimated, $\theta = (\eta, \beta, \gamma)$, and X_{ijt} comprises the observables, $X_{ijt} = (Q_{ijt}, P_{ijt}, P_{Mjt}, L_{jt}, K_{jt})$. Because the optimal instruments are non-linear functions of the parameters to be estimated, they cannot be computed directly from the data. Instead the optimal instruments are updated after each stage of GMM. In the first stage I use starting values that are an educated guess of the parameters. For the subsequent rounds, the optimal instruments are recomputed using the parameter estimates from the previous stage of GMM.

I replace all the supply-side moments with productivity shocks ξ_{ijt} and standard instruments by moments with optimal instruments. As compared to the empirical model with standard instruments, the objective function appears smoother, and the estimates less responsive to the starting values. This is because the functional forms imposed are exploited to a fuller extent.

I do not adopt optimal instruments for the other moments, i.e. the moments that contain the measurement error ϵ_{Mjt} or demand shock ε_{ijt}. The reason is that writing optimal instruments when the structural error term is a function of endogenous observations is complicated (Arellano

2003). In summary, the moment conditions I use are:

$$\begin{array}{cc} \text{Moment} & \text{Parameter to be identified} \\ E\left[\xi_{ijt}|z_{Mijt}\right]=0\ \forall\ i=[1,N] & \beta_{Mi} \\ E\left[\xi_{ijt}|z_{Lijt}\right]=0\ \forall\ i=[1,N] & \beta_{Li} \\ E\left[\xi_{ijt}|z_{Kijt}\right]=0\ \forall\ i=[1,N] & \beta_{Ki} \\ E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right]=0\ \forall\ i=[1,N] & \beta_{Mi} \\ E\left[\varepsilon_{ijt}|P_{Mjt}\right]=0\ \forall\ i=[1,N] & \eta_i \\ E\left[\varepsilon_{ijt}|L_{jt}\right]=0\ \forall\ i=[1,N] & \eta_i \\ E\left[\varepsilon_{ijt}|K_{jt}\right]=0\ \forall\ i=[1,N] & \eta_i \end{array} \tag{42}$$

As four moment conditions are sufficient for exact identification of the model, there are three overidentifying restrictions in the above set of moments. Some of the 8- or 10-digit product titles have at least four but less than seven observation pairs. In these cases I cannot use all the seven moment conditions. Instead of dropping observations of the product title entirely, I drop some of the overidentifying moments for these products. For product i with only four observations pairs, I adopt moments $E\left[\xi_{ijt}|z_{Mijt}\right]=0$, $E\left[\xi_{ijt}|z_{Lijt}\right]=0$, $E\left[\xi_{ijt}|z_{Kijt}\right]=0$, and $E\left[\varepsilon_{ijt}|P_{Mjt}\right]=0$. Moment $E\left[\epsilon_{Mjt}|P_{ijt}Q_{ijt}\right]=0$ ($E\left[\varepsilon_{ijt}|L_{jt}\right]=0$) [$E\left[\varepsilon_{ijt}|K_{jt}\right]=0$] is used when there is at least five (six) [seven] observation pairs.

The production function parameters $\beta_{Mi}, \beta_{Li}, \beta_{Ki}$ and the price elasticities η_i are obtained by iterated GMM.

6 Results

As there are multiple parameters to be estimated that enter the GMM objective function non-linearly, finding the global minimum can be challenging. To make sure that the estimation routine reaches the global minimum of the GMM objective function, I experiment with various minimization algorithms, of which the Gauss-Newton algorithm turns out to perform best. I also run the estimation routine with a large set of alternative starting values. Several rather different starting values yield the same minimum, which I acknowledge as the global minimum of the objective function.

The estimation results are presented in Table 2. The two production functions and demand functions estimated are for two groups: "Sawmilling and planing of wood" (PRODCOM titles

161), and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM titles 162). All the non-linear parameter estimates are statistically significant.[19] Also, the estimates of the two groups are statistically different from each other. The output elasticity of materials is considerably higher in the technology for titles 162 than in the technology for 161 (β_M for 162 is 0.74 and β_M for 161 is 0.38). The output elasticity of labor, again, is considerably lower in the technology for titles 162 (β_L for 162 is 0.12 and β_L for 161 is 0.35). Both technologies have output elasticity of capital of the same magnitude (β_K for 161 is 0.19 and β_K for 162 is 0.18). The demand for titles 161 is more price elastic than the demand for titles 162, as η for titles 161 is -1.30 and η for 162 is -1.12. This is intuitive because products of wood, cork, straw and plaiting materials are likely to be more differentiated than the output of sawmilling and planing of wood. Hansen's J-test does not reject the null hypothesis of valid overidentification restrictions (Prob[Chi-sq.(264)>J] is 0.4632).

7 Discussion on Identification

The structural production function literature focuses on correcting for endogeneity biases. Several papers build on the insight of Olley and Pakes (1996) that because inputs are set as a function of the firm's productivity, input demand can be inverted for the unobservable productivity term. Subsequently this idea, referred to as the proxy method, has been used by Levinsohn and Petrin (2003), Ackerberg, Caves and Frazer (2006), Wooldridge (2009), and Doraszelski and Jaumandreu (2013). Gandhi, Navarro and Rivers (2013) use firms' short run first order conditions to control for the collinearity of inputs. Most of the assumptions underlying my identification strategy are familiar from this literature. I make also some novel assumptions, and relax some of the assumptions previously made.

All the moment conditions, in my and other structural production function estimation strategies, are based on assumptions about the timing of input choices with respect to productivity shocks. In addition, I specify the role of demand shocks in production choices. Materials M_{ijt} are chosen only after the demand and productivity shocks ε_{ijt} and ξ_{ijt} have been observed, while the firm-level labor L_{jt} and capital stock K_{jt} are determined before the shocks. These assumptions are standard in the literature, apart from taking account of the demand shocks in production decisions, and assuming L_{jt} to be a fixed variable. The reason for treating L_{jt} as

[19] The product-firm specific demand levels α_{ij}, the 42 constants β_{0i}, and the parameters governing the productivity process $g(\omega_{ijt-1})$ are not reported.

a fixed input in not technical, but this assumption is made to account for the environment in which the data has been generated: employment protection legislation plays a significant role in Finland. The OECD indicators of employment protection (OECD, 2013) measure the strictness of legislation on individual and collective dismissals and the strictness of hiring employees on temporary contracts. The measures are based on information about statutory and case laws, collective bargaining agreements, and advice by officials from OECD member countries and country experts. According to these indicators, the Finnish labor market was of the OECD average in the strictness of employment protection during the period of 2004 to 2011. Based on this measure, fixed labor input is a realistic assumption. In case the method of this paper is to be used for estimating production functions in an economy where flexible labor input is a more appropriate assumption, the empirical model can be adjusted accordingly. As in other structural production function models, one flexible input is required for inverting out the unobservable productivity ω_{ijt}. I also further specify that the product-level labor and capital allocations L_{jt} and K_{ijt} are set as endogeneous to ε_{ijt} and ξ_{ijt}. This assumption not only facilitates the estimation of L_{jt} and K_{ijt}, but also allows firms to reallocate human resources and capital as response to demand and productivity shocks.

An important difference in the timing assumptions of this and other structural estimation strategies is that I assume away any productivity shocks once the flexible inputs have been set, and measurement error in output Q_{ijt}. I make the assumption in order to solve for the unobservable input allocations, while controlling for the unobservable productivity ω_{ijt}. At the same time, and in contrast to the rest of the literature, I allow for measurement error in the flexible inputs M_{jt} observed at the firm-level. This provides me an additional moment condition for identifying β_{Mi}, as compared to the other production models: sales revenue from a given product correlates positively with the flexible input M_{ijt} allocated to the product line, but is uncorrelated with the firm-level measurement error in M_{jt}, denoted by ϵ_{Mjt}.

In addition to the timing assumptions, the proxy methods require two more key assumptions. First, input demand is assumed monotonic in productivity. In other words, cases where input demand may decrease due to improved efficiency are assumed away. However, this assumption may be unrealistic in settings where firms face downward sloping demand curves. I relax the monotonicity assumption by using the definition of productivity itself in controlling for endogeneity.

Second, the proxy methods require the assumption that productivity ω_{ijt} is the only scalar

unobservable that affects the input choices. Unobservable inter-firm variation in, say, input prices or output demand, as well as optimization and measurement error in the flexible inputs, are assumed away. I also need to make the scalar unobservability assumption for estimating product-level inputs. However, I do allow for measurement error in the flexible inputs. I also allow for inter-firm variation in input prices and output demand. In fact, I need input prices and estimates of output demand for estimating the input allocations. At the same time, variation in the input prices resolves the collinearity problem between the flexible input M_{ijt} and the other inputs. What the scalar unobservability assumption in my application implies is that the price a firm pays for its flexible input, P_{Mjt}, does not depend on the quantity purchased M_{ijt}. By modelling supply in the input market this assumption could be relaxed, however. As in other empirical strategies, I also assume that the input demand function is continuous. In other words, firms can purchase precisely the input quantity that maximizes their profit. This seems justified after eyeballing the firm-level input data.

The last set of supply-side assumptions that I make concerns the fixed inputs labor and capital. Units of the firm-level input stocks L_{jt} and K_{jt} are substitutable between product lines, and there are no adjustment costs in (re)allocating labor or capital to other product lines. In fact, these assumptions are not specific to this product-specific model, but they are made implicitly in all firm-level estimations when firms produce more than one type of good.

In contrast to the other structural methods, the one of this paper requires demand estimates for identifying the unobservable input allocations. Identification of the demand function is based on two assumptions. First, any unobservables that affect the demand for a given good of a given firm, e.g. product quality, are constant over time. This assumption may be realistic for some industries, and unrealistic for others. If unrealistic, the demand model can be replaced with a more flexible one. Second, changes in input prices and fixed input stocks shift the supply curve, while the demand curve, including the demand shock ε_{ijt}, is not affected. Using material prices and fixed input stocks as instruments is a standard practice. Also note that the estimated product-level inputs M_{ijt}, L_{ijt}, and K_{ijt} enter the production function as generated regressors. In order for the production function estimates to be consistent, all the instruments, generated and observed, need to be uncorrelated with the residuals (Wooldridge, 2002). In other words, if the moment conditions are valid, the parameter estimates are consistent.

To sum up, recall that the estimation biases acknowledged in the literature are: selection, simultaneity, collinearity, omitted price, and product bias, as discussed in section 2. The esti-

mation strategy of this paper does not consider the selection bias[20]. Nevertheless, it is possible to extend the strategy to control for market entry and selection to various product lines by computing propensity scores for market entry, as in Olley and Pakes (1996). Furthermore, the selection bias may be less of a problem when product-level capital is a quasi-flexible variable, i.e. capital allocations to product lines are made in the period of production given a fixed firm-level capital stock. Recall that the selection bias arises due to a negative correlation between firms' capital stock and productivity level in the sample. But when capital allocations to product lines are set as a function of productivity and demand, as in the multiproduct case, it is not obvious whether the correlation between capital and productivity is positive or negative. Hence identifying β_{Ki} is now potentially subject to two opposing biases: selection bias (towards zero), and simultaneity bias (away from zero). The simultaneity bias of β_{Ki} is corrected as the biases of β_{Li} and β_{Mi}. The selection bias is not corrected for, but the problem is alleviated due to allocation of capital across product lines.

The other four of the five biases are accounted for. The simultaneity bias is corrected by writing input functions explicitly as a function of the unobservable productivity. Identifying variation in material prices and fixed inputs stocks resolves the collinearity problem. The omitted price bias doesn't occur because input and output prices are observed, and physical quantity measures are used instead of sales revenues and input expenditures. The so-called product bias is corrected by allowing for good-specific production technology, and by taking account of the role of output demand in production decisions.

The identification strategy accommodates also other functional forms than the Cobb-Douglas production function and the isoelastic demand function used in this paper. The requirement on the production model, as in most structural production models, is that there has to be at least one input that is chosen as a function of the unobservable productivity. The data is required to include observations of at least two consecutive periods, and report physical output and sales revenue by product title. Such data, fortunately, is provided by many national statistical offices in Europe, for example.

[20] In fact, the method of Olley and Pakes (1996) is the only one that corrects for the selection problem, while the other structural methods focus on accounting for the simultaneity problem.

7.1 Comparison with De Loecker et al.

There are a few recent papers that also accommocate for multiproduct firms and product-specific production technologies, as mentioned in the literature review. The method of De Loecker, Goldberg, Khandelwal and Pavcnik (2012, henceforth DLGKP) is perhaps closest to the method presented in this paper. DLGKP and I have rather similar datasets where input allocations within firms are unobservable. We also make many similar identifying assumptions that are standard in the structural production function literature, as DLGKP use the empirical model and estimation strategy of Ackerberg, Caves and Frazer (2006). Nevertheless, our key assumptions and empirical strategies that address the unobservable input allocations are quite different.

Both DLGKP and I assume that single- and multiproduct firms use similar product-specific technologies. DLGKP are able to utlize this assumption to a fuller extent because they observe sufficiently many singleproduct firms to estimate the technology parameters using data on those firms only. This enables DLGKP to estimate the parameters without simultaneously solving for the unobservable input allocations. The input allocations are computed using the parameter estimates and the observable variables. DLGKP assume that the share of a firm's materials, labor, and capital allocated to a given product line is constant, i.e. independent of the input type. This implies that a firm produces all of its goods with the same materials-labor-capital -ratio. However, a profit maximizing or cost minimizing firm would not allocate inputs to product lines with such constant ratios. Even when the technology parameters are correctly estimated, estimates of the unobservable productivity levels are affected by this assumption. On the other hand, DLGKP avoid making the assumption of zero productivity shocks after the flexible inputs have been set, which I need to make. Moreover, DLGKP do not require estimates of output demand.

8 Conclusion

This paper contributes to a large empirical literature on production function estimation, which underlies even a larger body of applied economic research. To take account of the empirical fact that a remarkable share of firms is multiproduct firms, I provide a method to estimate product-specific production functions when some or all firms produce multiple goods. The method does not require data on input allocation to various product lines. Instead, output demand

is estimated to identify input allocation to the product lines and the production functions. Endogeneity of the input allocation to the unobservable productivity level is controlled for by using the functional form of the production function. The method is demonstrated by estimating production functions for goods in the industry "Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials". I find that the technologies used in "Sawmilling and planing of wood" (PRODCOM 161) and "Manufacture of products of wood, cork, straw and plaiting materials" (PRODCOM 162) are statistically different from each other. The empirical findings suggest that production functions should be estimated at the product- instead the firm-level, and that multiproduct firms use multiple production technologies.

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9 Tables and Figures

Table 1

PRODCOM	Title
16.10.10.33	Coniferous wood; sawn or chipped lengthwise, sliced or peeled, of a thickness > 6 mm, e
16.10.10.33.10	*Kuusipuu, höylätty tai hiottu, päistään jatkettu, sahattu tai veistetty pituussuunnassa, ta*
16.10.10.33.20	*Mäntypuu, höylätty tai hiottu, päistään jatkettu, sahattu tai veistetty pituussuunnassa, ta*
16.10.10.35	Spruce wood (Picea abies Karst.), fir wood (Abies alba Mill.)
16.10.10.37	Pine wood (Pinus sylvestris L.)
16.10.10.50	Wood, sawn or chipped lengthwise, sliced or peeled, of a thickness > 6mm (excluding co
16.10.21.10	Coniferous wood continuously shaped (including strips and friezes for parquet flooring, n
16.10.23.03	Coniferous wood in chips or particles
16.10.23.05	Non-coniferous wood in chips or particles
16.10.41.00.10	*Sahanpuru*
16.10.41.00.20	*Polttohake*
16.10.41.00.40	*Rimat, syrjät, tasauspätkät yms.*
16.10.41.00.60	*Kuori*
16.10.41.00.80	*Muu puujäte (pois lukien sahanpuru, polttohake, kuori, rimat, syrjät, tasauspätkät, pelle*
16.21.11.00	Plywood, veneered panels and similar laminated wood, of bamboo
16.21.12.14	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceedi
16.21.12.17	Plywood consisting solely of sheets of wood (excluding of bamboo), each ply not exceedi
16.21.13.13	Particle board, of wood
16.21.21.18.30	*Viilut vanerointia, ristiinliimattua vaneria yms puuta varten, havupuuta, sahattu pituuss*
16.21.21.18.80	*Viilut vanerointia, ristiinliimattua vaneria yms puuta varten, lehtipuuta, sahattu pituuss*
16.21.22.00	Densified wood, in blocks, plates, strips or profile shapes
16.22.10.60	Parquet panels of wood (excluding those for mosaic floors)
16.23.11.10	Windows, French-windows and their frames, of wood
16.23.11.50	Doors and their frames and thresholds, of wood
16.23.19.00.12	*Rakennuspuusepän ja kirvesmiehen tuotteet seiniä varten, puuta*
16.23.19.00.16	*Rakennuspuusepän ja kirvesmiehen tuotteet portaita varten, puuta*
16.23.19.00.26	*Puuosat saunaa varten*
16.23.19.00.32	*Levyelementit (myös liimalevyt ja solulevyt), puuta*
16.23.19.00.36	*Kattoelementit, puuta*
16.23.19.00.42	*Liimapuupalkit ja -pilarit*
16.23.19.00.46	*Pysty- ja vaakapalkit (pois lukien liimapuupalkit ja -pilarit)*
16.23.19.00.52	*Hirsikehikot puutaloja varten*
16.23.19.00.90	*Muut rakennuspuusepän ja kirvesmiehen tuotteet, puuta (pois lukien ovet, ikkunat, tuotte*
16.23.20.00.20	*Puiset asuinrakennukset, vakituista asumista varten*
16.23.20.00.40	*Puiset asuinrakennukset, vapaa-ajan asumista varten*
16.23.20.00.60	*Puusaunat (ulkosaunat, valmiit tai kokoamattomina osina)*
16.23.20.00.90	*Puurakennukset (valmiit tai kokoamattomina osina) (pois lukien asuinrakennukset tai sa*
16.24.11.35	Box pallets and load boards of wood (excluding flat pallets)
16.24.13.20	Cases, boxes, crates, drums and similar packings of wood (excluding cable drums)
16.24.13.50	Cable-drums of wood
16.29.14.90	Other articles of wood (excluding pallet collars)

Table 2: Parameter estimates

PRODCOM 161: Sawmilling and planing of wood

PRODCOM 162: Manufacture of products of wood, cork, straw and plaiting materials

Parameter	Est.	Std. Err.
PRODCOM 161		
β_M	0.37	0.008
β_L	0.36	0.011
β_K	0.20	0.008
η	-1.29	0.020
PRODCOM 162		
β_M	0.73	0.002
β_L	0.13	0.003
β_K	0.18	0.003
η	-1.13	0.004
Prob[Chi-sq.(264)>J]	0.4632	
Number of observations	2053	