

# False Advertising and Consumer Protection Policy

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## Abstract

There is widespread evidence that some firms use false advertising to overstate the value of their products. Using a model in which a regulator is able to punish false claims, we characterize a natural equilibrium in which false advertising occurs probabilistically and actively influences rational consumers. We solve for the optimal level of regulatory punishment under different welfare objectives and establish a set of demand and parameter conditions where optimal policy permits a positive level of false advertising. Further analysis considers wider issues, including the implications for industry self-regulation, product investment, and optimal policy across multiple heterogeneous markets.

**Keywords:** Misleading Advertising; Persuasion; Self-Regulation; Pass-through

**JEL codes:** D83; L15; L51; M37

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# 1 Introduction

Many adverts make explicit claims about product quality attributes such as effectiveness, durability, origin, and so forth. Therefore, in most countries, a consumer protection authority regulates the use of incorrect claims or ‘false advertising’. However, despite the potential for such sanctions, there is abundant evidence that some firms still engage in false advertising. Aside from a plethora of successful prosecutions involving firms such as Reebok, Skechers, L’Oreal, Kellogg’s and Lexus<sup>1</sup>, i) Dannon recently paid \$21 million to 39 US states after exaggerating the health benefits of its products, and ii) manufacturers, such as Nestle and Findus, are still awaiting charges for mislabeling their beef products in the European horse-meat scandal.<sup>2</sup> Further careful evidence for the existence of false advertising is provided by academic studies, which also document the ability of false advertising to actively increase consumer demand.<sup>3</sup> However, the traditional theoretical literature has had little to say about such false advertising. This is surprising because false advertising offers many important questions: How can it influence rational consumers? Under what circumstances can it harm consumers? Why are regulators not tougher in practice? Which markets should authorities prioritize when regulating adverts? Can industry self-regulation ever be socially optimal?

To answer these and other questions, this paper aims to better understand the fundamental equilibrium effects of advertising policy on false advertising, product pricing and a variety of welfare measures. We show how these effects can be analyzed by using familiar tools, with rational consumers and a general form of consumer demand. In particular, we consider the pricing and advertising behavior of a monopolist that is privately informed about whether its product is of ‘low’ or ‘high’ quality. The firm chooses a price and makes a claim about its product quality. Consumers then observe the claim and price, update their beliefs, and make their purchase decisions, before an authority potentially punishes any exaggerated claims. We think this set-up closely approximates many important markets where consumers are unable to verify claims, or can only do so after using the product for a long time, and where authorities play an active role in regulating false advertising. Indeed, in many jurisdictions, such as the US and the EU, the relevant authorities either monitor adverts directly or respond to consumer complaints, before then instigating potential punishments, often in the form of monetary fines or administration costs.<sup>4</sup>

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<sup>1</sup>See <http://www.ftc.gov/news-events/press-releases/2011/09/reebok-pay-25-million-customer-refunds-settle-ftc-charges>, <http://www.ftc.gov/news-events/press-releases/2012/05/skechers-will-pay-40-million-settle-ftc-charges-it-deceived>, and <http://www.asa.org.uk/Rulings.aspx> for a range of other cases. Accessed 04/15/15.

<sup>2</sup>See <http://www.ftc.gov/news-events/press-releases/2010/12/dannon-agrees-drop-exaggerated-health-claims-activia-yogurt>, and <http://www.bbc.co.uk/news/uk-21335872>. Accessed 04/15/15.

<sup>3</sup>E.g. Zinman and Zitzewitz (2014) and Cawley *et al* (2013). See also Mayzlin *et al* (2014) for false advertising in the form of fake user reviews.

<sup>4</sup>In the US, most federal-level regulation is conducted by the FTC which actively monitors markets and punishes any offenses with measures including orders to conduct corrective advertising, and monetary

In Sections 2 and 3 the paper first characterizes a natural equilibrium where the high type advertises truthfully but where the low type may engage in false advertising. When the punishment is sufficiently ‘strong’, there is no false advertising and so consumers infer product quality and each type responds with its optimal associated price. However, for ‘moderate’ punishments, the low type engages in false advertising with positive interior probability by mixing between i) pooling with the high type by using a false advert and a relatively high price, and ii) advertising truthfully and setting a relatively low price. Given the firm’s strategy, it is rational for consumers to believe that any high quality claim is correct with a non-zero probability. Therefore, false advertising can influence rational consumers and stimulate demand in equilibrium as consistent with empirical evidence (e.g. Zinman and Zitzewitz 2014, Cawley *et al* 2013). However, such false advertising never systematically deceives the consumers because their beliefs are correct on average. Nor does it raise the low type’s profits because any increase in profits is offset in equilibrium by the expected punishment. In the extreme, for lower ‘weak’ punishments, the low type always conducts false advertising within a full pooling equilibrium and so advertising becomes ineffective in changing consumers’ priors. Hence, our equilibrium provides an attractive, smooth unification of some familiar ideas within the information literature: when punishments are strong, advertising is equivalent to fully verifiable disclosure, and there is full separation; when punishments are weak, advertising is (almost) cheap talk and there is full pooling; however, when punishments are moderate, our equilibrium provides a novel case where adverts are partially verifiable and where the low type engages in false advertising with an interior probability.

Section 4 explores how changes in the level of regulation affect a variety of welfare measures. We first consider consumer surplus. Here, a reduction in the punishment can increase the probability of false advertising and generate two opposing effects. The first ‘persuasion’ effect harms consumers directly by prompting them to overestimate a low type’s product quality and so ‘overpurchase’ by buying too many units and/or buying at too high a price. This is akin to a formalization of Dixit and Norman’s (1978) classic effect of persuasive advertising. However, our effect derives from a change in consumer beliefs, rather than preferences. The second effect derives from the underlying impact of false advertising in damaging the credibility of claims and lowering consumers’ resulting quality expectations. This intuitive effect goes back to at least Nelson (1974) and is well-documented empirically (e.g. Darke and Richie 2007). However, rather than taking the traditional view that this effect is detrimental, we stress its benefits under a novel ‘price’ effect by showing how it

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penalties in the form of civil fines and/or consumer compensation. In Europe, most countries employ varying levels of industry self-regulation alongside statutory authorities, as coordinated by the European Advertising Standards Alliance. For instance, in the UK, most regulation is conducted by the industry-led Advertising Standards Authority (ASA) which is endorsed by various governmental bodies. The ASA often uses consumer complaints to guide its investigations, but it also monitors adverts directly. After a persistent offense and a subsequent referral to the governmental authorities, firms can be fined, and employees can even face imprisonment.

can counteract market power and prompt any type making a high quality claim to set lower prices. By then adapting some recent results on cost pass-through or ‘quality pass-through’ (e.g. Weyl and Fabinger 2013), we provide some demand and parameter conditions to compare the two effects and explicitly characterize the optimal punishment. In many cases, the persuasion effect dominates such that consumer surplus is maximized with strong punishments that eliminate all false advertising. However, we also formalize a range of other market conditions where the price effect dominates, such as the case where the high quality product is sufficiently desirable. Here, the regulator should implement some moderate or even zero punishment to permit a positive level of false advertising and generate a level of consumer surplus that exceeds that under full information (where there is no false advertising).

Next, we turn to the effects on profits. The low type always prefers weaker punishments, and the high type always prefers stronger punishments. However, from an ex ante perspective, the monopolist always weakly prefers strong punishments that eradicate false advertising. Therefore, in some circumstances, the firm’s preferences for punishments coincide with that of consumers, while in others, the monopolist prefers relatively *stronger* punishments. Hence, if the monopolist could commit to a punishment, as may be consistent with some forms of industry self-regulation in Europe, the monopolist would commit to a punishment that is weakly stronger than that desired by consumers.

Under a total welfare objective, we show that the regulator should implement a punishment that coincides exactly with the level preferred by either consumers or the monopolist. In some case, this involves the regulator permitting a positive level of false advertising. It also demonstrates how the optimal use of advertising regulation can be superior to a ban on low quality products through minimum quality standards.

Aside from false advertising, our results can also apply to situations where firms falsify evidence to qualify for some quality certificate or standard. In response to such false certification, the Organic Retail and Consumer Alliance has recently been established to attack the ‘rampant’ labeling fraud within the organic and natural food sectors<sup>5</sup>. By re-interpreting our model, we demonstrate cases where any certifier that places a positive weight on consumer surplus will optimally induce a positive level of false certification activity.

Within Sections 4 and 5, we show how our results are robust to a range of important issues. First, in contrast to a fixed punishment, we allow for more ornate punishments that can vary with the profit gained or the harm caused. With some stronger assumptions on consumer beliefs, the resulting equilibrium and policy results remain qualitatively similar to those in the main model. Second, we let the firm types vary in marginal production costs so that a high type may signal its quality through both its price as well as its high claim. Provided the difference in costs is not large, our equilibrium and results remain

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<sup>5</sup><https://www.organicconsumers.org/essays/orca-attack-%E2%80%98natural%E2%80%99-products-labeling-fraud>. Accessed 04/15/15.

robust. Third, we allow claims to be costly even when truthful. This increases the ability of the high type to separate, but our main results remain robust provided that such costs are not too large. Interestingly, if such costs derive from an advertising tax, then we suggest that advertising taxes can have the same effect on reducing the level of false advertising as punishments. However, in contrast to false advertising regulation, taxes also impose an additional burden on the high type.

Finally, before Section 7 concludes, Section 6 provides some substantial extensions. First, we extend the regulator’s problem into a multiple market context where a set of limited regulatory resources must be allocated over a range of heterogeneous markets that vary in consumer demand, quality levels, and quality priors. Under some mild assumptions, a regulator with a consumer surplus objective will regulate most markets at either the level previously prescribed in the main model or not at all. *Ceteris paribus*, the regulator should prioritize markets with weak levels of low quality and small probabilities of high quality goods. Second, we examine regulation in a competitive context where an established incumbent competes against a horizontally differentiated entrant with private product quality. We demonstrate the existence of an equilibrium with false advertising and derive a set of policy implications that are qualitatively similar to those under monopoly. To begin to understand how optimal regulation varies with the level of competition, we then examine the effects of an increase in the level of horizontal product differentiation. A reduction in competition induces weakly *stronger* regulation: it enhances the persuasion effect by inflating the price paid for a falsely-labeled product, while reducing a version of the price effect by making prices less sensitive to the firms’ product qualities. Lastly, we return to monopoly but allow for endogenous product quality investment. This creates a new mechanism that encourages weakly stronger policy. Intuitively, while an increase in punishment can harm consumers via a negative price effect, it can also prompt investment by restoring advertising credibility and enhancing the returns from a high quality product. However, cases remain where optimal policy still permits a positive level of false advertising.

*Related Literature:* Traditionally, economics has had relatively little to say about consumer protection policy. While some recent work has turned to this topic under a variety of other contexts<sup>6</sup>, we focus on false advertising. Aside from some early works such as Nelson (1974), this topic has also been largely ignored within the advertising literature (see the reviews by Bagwell 2007 and Renault 2014). However, false advertising has been considered in a few other recent papers that differ from our own in their focus and approach. One important issue in this area concerns how false advertising can be credible. Some papers bypass this by assuming that consumers naively believe all claims (e.g. Glaeser and Ujhelyi

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<sup>6</sup>Some examples include the effects of high-pressure sales tactics (Armstrong and Zhou 2014), the misuse of commissions for advice-giving intermediaries (Inderst and Ottaviani 2009) and the regulation of cancellation and refund rights (Inderst and Ottaviani 2013).

2010, Hattori and Higashida 2012). Here, false advertising simply expands demand and so the socially optimal level of false advertising can be positive because the associated increase in consumption can offset the distortion from imperfect competition. Other papers maintain consumer rationality and resolve the credibility issue endogenously by introducing legal penalties in ways more related to our paper. Closest is Piccolo *et al* (2014) who examine a duopoly model with homogeneous consumers and unit demand where one firm has a good product and the other has a bad product. Unlike us, they focus only on fully pooling and separating equilibria. The authors find that a zero punishment maximizes consumer surplus due to the pro-competitive effect of false advertising in making the firms appear closer substitutes. In contrast, within our richer framework, we characterize demand and parameter conditions where optimal policy can involve a zero, weak or strong punishment. Moreover, we document a different mechanism through which false advertising can be beneficial via its ability to erode monopoly power by reducing consumers' confidence in quality claims. In other closely related work, Corts (2013, 2014a, 2014b) only studies fully pooling and separating equilibria in a monopoly setting. However, he takes a different focus by assuming that the firm must choose whether or not to become informed of its own product quality. The findings outline some welfare results and suggest that finite penalties may be optimal because they induce the firm to make socially-valuable unsubstantiated claims.<sup>7</sup>

As a last note, our model also relates to a number of papers in the communication literature that study equilibrium lying and persuasion under full rationality (e.g. Kartik 2009, Kamenica and Gentzkow 2011). Of most relevance is Kartik (2009) which offers a general treatment of lying costs in a standard cheap-talk setting. Our paper allows for policy-related lying costs within a specific advertising context with optimal pricing.

## 2 Model

A monopolist sells one product to a unit mass of consumers. The monopolist is privately informed about its product quality  $q$ . Specifically, the product is of low quality  $L$  with probability  $x \in (0, 1)$ , and of high quality  $H$  with probability  $1 - x$ . Average ex ante quality can then be defined as  $\bar{q} = xL + (1 - x)H$ . For our main analysis we assume that marginal costs are independent of product quality, and normalized to zero.<sup>8</sup> Each consumer has a unit demand and values a given product of quality  $q$  at  $q + \varepsilon$ , where  $\varepsilon$  is a consumer's privately known match with the product. This match is drawn independently across consumers using a distribution function  $G(\varepsilon)$  with support  $[a, b]$  where  $-\infty \leq a < b \leq \infty$ . The associated density  $g(\varepsilon)$  is strictly positive, continuously differentiable, and has an increasing hazard

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<sup>7</sup>In some broader work, Barigozzi *et al* (2009) study false comparative advertising rather than false quality advertising.

<sup>8</sup>Our assumption of cost symmetry simplifies the analysis, and is reasonable if, for example, quality investments are sunk. We allow for asymmetric costs in Section 5.

rate.

The monopolist sends a publicly observable advertisement or ‘report’  $r \in \{L, H\}$  at no cost. A report  $r = z$  is equivalent to a claim “My product is of quality  $z$ ”. Note that the binary report space is without loss because there are only two firm types and reports are costless. False advertising is defined as the use of a high quality report  $r = H$  when the product has a low quality,  $q = L$ . A regulator is able to verify any advertised claim, and impose a penalty  $\phi$  if it is false. For our main analysis, we assume that the regulator can costlessly choose any level of punishment,  $\phi \geq 0$ , in order to maximize one of three possible objectives: consumer surplus, total welfare, or total profit. Any punishments that involve the use of a fine go straight to the regulator, and are not used to compensate consumers.<sup>9</sup>

The timing of the game is as follows. At stage 1 the regulator publicly commits to a penalty  $\phi$  for false advertising. At stage 2 the monopolist privately learns its product quality. It then announces a price  $p$  and makes an advertising claim  $r \in \{L, H\}$ . At stage 3 consumers decide whether or not to buy the product, taking into account  $\phi$  as well as the firm’s price and claim. Finally at stage 4 the regulator verifies the advertised claim, and fines the firm  $\phi$  if it was false. The solution concept is Perfect Bayesian Equilibrium (PBE). All omitted proofs are included in Appendix A, unless stated otherwise.

### 3 Benchmark with Known Quality

As a preliminary step towards solving the model, we first consider a benchmark case in which the firm is known to have quality  $q$ . Quality claims are then redundant because it will be weakly optimal for the firm to use truthful advertising,  $r(q) = q$ . Therefore, the firm’s only task is to choose a price  $p$ . An individual consumer buys the product if and only if  $\varepsilon \geq p - q$ , such that demand equals  $D(p - q) = 1 - G(p - q)$ . The firm chooses its price to maximize  $p[1 - G(p - q)]$ . It is then immediate that:

**Lemma 1.** *Suppose the firm is known to have quality  $q$ . The firm’s optimal price,  $p^*(q)$ , is increasing in  $q$  and satisfies:*

$$p^*(q) = \begin{cases} 0 & \text{if } q \leq \underline{q} \\ \frac{1-G(p^*(q)-q)}{g(p^*(q)-q)} & \text{if } q \in (\underline{q}, \tilde{q}) \\ a + q & \text{if } q \geq \tilde{q} \end{cases} \quad (1)$$

where  $\underline{q} = -b$  and  $\tilde{q} = -a + 1/g(a)$ .

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<sup>9</sup>Allowing fines to be directed back to consumers is equivalent to the approach taken within the product liability literature. This generates a range of different issues, which are not the focus of our paper. See Daughety and Reinganum (2013) for more.

The interpretation is straightforward. When  $q \leq \underline{q}$ , quality is so low that the firm would make zero sales even if it priced at marginal cost. The market is therefore inactive, and we normalize the firm's price to zero without loss. When instead  $q \in (\underline{q}, \tilde{q})$ , the firm optimally sells to some but not all consumers, such that  $p^*(q)$  satisfies the usual monopoly first order condition. After differentiating this first order condition, one finds

$$\frac{dp^*(q)}{dq} = \frac{1 - \sigma}{2 - \sigma} \quad (2)$$

where  $dp^*(q)/dq$  is the level of ‘quality pass-through’, and where  $\sigma(D) = D \cdot D'' / [D']^2$  is the curvature of the inverse demand function (see Aguirre *et al* 2010, and Weyl and Fabinger 2013). It then follows that  $dp^*(q)/dq \in [0, 1)$  because  $D(p - q)$  is logconcave in price such that  $\sigma \leq 1$ . Intuitively, an increase in quality  $q$  produces a parallel rightward shift in the inverse demand curve. The firm optimally responds to this by both charging a higher price, and by selling to strictly more consumers. Finally if  $q \geq \tilde{q}$ , quality is so high that the firm prefers to sell to the entire market. Price then equals the willingness-to-pay of the marginal consumer  $a + q$ , and hence increases one-for-one with quality.<sup>10</sup> Henceforth to avoid some uninteresting cases, we assume that  $\bar{q} > \underline{q}$  (or  $\bar{q} + b > 0$ ) such that a product of average quality makes strictly positive profit.

The profit earned by a firm of known quality  $q$  can then be written as

$$\pi^*(q) = p^*(q) \cdot [1 - G(p^*(q) - q)] \quad (3)$$

It is straightforward to show that  $\pi^*(q)$  is increasing and convex in  $q$ , given our assumption that  $1 - G(\cdot)$  is logconcave. Finally consumer surplus can be expressed as

$$v^*(q) = \int_{p^*(q)}^{b+q} [1 - G(z - q)] dz \quad (4)$$

When  $q \leq \underline{q}$  no consumer buys the product and so  $v^*(q) = 0$ . When  $q \geq \tilde{q}$  all consumers buy and  $p^*(\tilde{q}) = a + q$ , such that consumer surplus is independent of quality with  $v^*(q) = v^*(\tilde{q}) = \int_a^b [1 - G(z)] dz$ . However when  $q \in (\underline{q}, \tilde{q})$  the market is partially covered, and it is easily verified that  $v^*(q)$  is both positive and strictly increasing in  $q$ . Further, by making use of equation (2), one can also note that consumer surplus is convex in quality if and only if

$$\frac{d\sigma(D)}{dD} > -\frac{2 - \sigma(D)}{D} \quad (\text{Condition 1})$$

Thus, if Condition 1 holds, consumer surplus behaves as in Figure 1. Condition 1 requires price to be ‘not too convex’ in quality and is equivalent to an assumption used within a recent

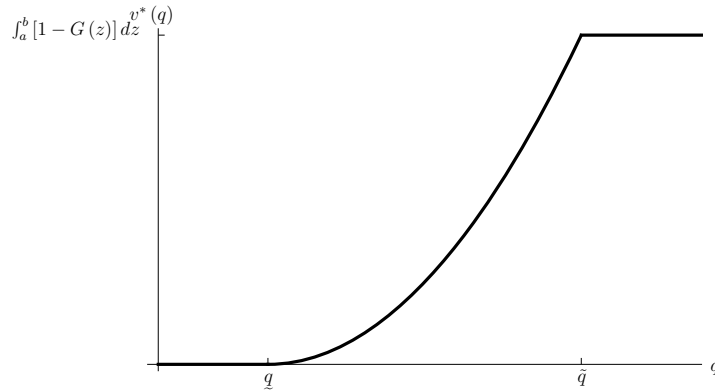
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<sup>10</sup>The threshold  $\tilde{q}$  is finite if  $a > -\infty$  such that consumers’ idiosyncratic matches are bounded from below.



literature on third-degree price discrimination with reference to marginal cost (see Cowan 2012, and Chen and Schwartz 2013). The condition is satisfied by many common demand functions, including all those with decreasing or constant pass-through. For example Bulow and Pfleiderer (1983) characterize a rich class of constant-pass-through demand functions, which includes linear, exponential, and constant elasticity. Results in Fabinger and Weyl (2012) also imply that the condition can be satisfied for other examples, such as the AIDS demand function which has decreasing pass-through. Certain parametrizations of increasing pass-through demand functions can also satisfy the condition, including normal and logit demand. Further details are provided in Appendix B, including a proof that Condition 1 is preserved under arbitrary truncations of the match distribution  $G(\cdot)$ .

Figure 1: Equilibrium Consumer Surplus,  $v^*(q)$



## 4 Privately-Known Quality

Henceforth we assume that the firm is privately informed about its quality. A high quality firm may then try to signal its type. As is typical in signaling games, there exists a large number of Perfect Bayesian Equilibria (PBE) because consumers can attribute any off-equilibrium claim (or price) to the low type.

We approach the equilibrium selection issue in the following way. Firstly, we restrict attention to equilibria in which a high type always makes a truthful claim  $r(H) = H$ . This allows us to focus on the incentives of a low type to engage in false advertising. Secondly, we restrict consumer beliefs to depend only on the firm's claim, and thus to be independent of the firm's price. The rationale for doing this is as follows. Notice that conditional on making a high claim and charging a price  $p$ , the payoffs of the low- and high type differ only by the expected punishment  $\phi$  because the types have the same marginal cost. In other words, the preferences of the two types are perfectly aligned with respect to the price they charge. It

therefore seems unnatural that consumers should infer anything from the firm's price.<sup>11</sup>

To explore this further, let  $q_H^e \equiv E(q|r = H)$  denote consumers' belief about quality following a high claim. Given our second restriction, the firm optimally charges  $p^*(q_H^e)$  when it makes a high claim (irrespective of its actual type). Interestingly this is also the unique price selected by Mailath *et al*'s (1993) Undefeated Equilibrium refinement.<sup>12</sup> Specifically, notice that conditional on its claim, a firm's pricing decision can be viewed as a special type of signaling 'game' where forward induction refinements like D1 and the Intuitive Criterion have no bite because the two seller types have identical preferences over price. However, the 'game' does have a unique (pure strategy) Undefeated PBE, in which both types pool on the price  $p^*(q_H^e)$ .

After applying our equilibrium restrictions, we derive the following result.

**Proposition 1.** *Suppose a high type always sends a truthful claim, and that consumer beliefs depend only on the firm's claim. There then exists a unique PBE equilibrium (up to off-path beliefs<sup>13</sup>), in which:*

- i) A high type firm claims  $r = H$  and charges  $p^*(q_H^e)$*
- ii) A low type firm randomizes. With probability  $y^*$  it claims  $r = H$  and charges  $p^*(q_H^e)$ .*

*With probability  $1 - y^*$  it claims  $r = L$  and charges  $p^*(L)$ .*

- iii) The probability that the low type uses false advertising,  $y^*$ , is determined as follows:*

- *When  $\phi \leq \phi_1 \equiv \pi^*(\bar{q}) - \pi^*(L)$ ,  $y^* = 1$*
- *When  $\phi \geq \phi_0 \equiv \pi^*(H) - \pi^*(L)$ ,  $y^* = 0$*
- *When  $\phi \in (\phi_1, \phi_0)$ ,  $y^* \in (0, 1)$  and uniquely solves*

$$\pi^*(q_H^e) - \phi = \pi^*(L) \quad (5)$$

- iv). Consumer beliefs about the firm's type are*

$$\Pr(q = H|r = L) = 0 \quad \text{and} \quad \Pr(q = H|r = H) = \frac{1 - x}{1 - x + xy^*}$$

*Expected quality when the firm makes a high claim is*

$$q_H^e(y^*) = \frac{xy^*}{1 - x + xy^*}L + \frac{1 - x}{1 - x + xy^*}H \quad (6)$$

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<sup>11</sup>In Section 5 we extend our analysis to allow for asymmetric costs and price-dependent fines, such that the types' preferences are no longer perfectly aligned.

<sup>12</sup>See Mezzetti and Tsoulouhas (2000) for a formal definition and development of this refinement. For other recent applications and uses of the refinement, see Gill and Sgroi (2012), Perez-Richet and Prady (2012), Miklos-Thal and Zhang (2013) and Lauermann and Wolinsky (2015).

<sup>13</sup>Note that when  $\phi < \phi_1$  the claim  $r = L$  is off-path, and a range of beliefs  $\Pr(q = L|r = L)$  lead to the same equilibrium play.

Proposition 1 characterizes a natural equilibrium where false advertising can arise. A low quality firm has the following tradeoff when choosing its advertised claim. Firstly if the firm reports truthfully with  $r = L$ , consumers correctly infer that its product is of low quality. Consumers then demand  $1 - G(p - L)$  units from the firm, such that it optimally charges a low price  $p^*(L)$  and earns a low profit  $\pi^*(L)$ . Secondly, if a low quality firm engages in false advertising and pools with the high type to claim  $r = H$ , it attracts an expected punishment  $\phi$ , but also prompts the rational consumers to Bayesian update and raise their expectations of the quality of the product to

$$q_H^e(y^*) = \frac{xy^*L + (1-x)H}{1-x+xy^*} \geq \bar{q} > L \quad (7)$$

where  $y^*$  is the (equilibrium) probability that a low type makes a false claim, and where we henceforth simplify notation by writing  $q_H^e(y^*)$  as  $q_H^e$ . Therefore by making a false claim, a low quality firm persuades consumers to overestimate its product quality. This shifts out the demand curve to  $1 - G(p - q_H^e)$  and allows the the firm to charge a higher price  $p^*(q_H^e)$ , and earn a higher profit (excluding any punishment)  $\pi^*(q_H^e)$ . However, such false advertising never systematically deceives consumers because their beliefs are correct on average due to the additional possibility that the high report comes from a high type.

The precise equilibrium characterization depends upon how strong the level of policy is, as measured by the size of the punishment,  $\phi$ . If  $\phi \leq \pi^*(\bar{q}) - \pi^*(L)$ , policy is ‘weak’. The low type always uses false advertising,  $y^* = 1$ , because the punishment from doing so is so small. The equilibrium has full pooling, with both types claiming  $r = H$  and charging a price  $p^*(\bar{q})$ . Advertising is then completely uninformative, and so when faced with a high claim, consumers just maintain their prior,  $q_H^e = \bar{q}$ .

On the contrary if  $\phi \geq \pi^*(H) - \pi^*(L)$ , policy is ‘strong’. The low type never uses false advertising,  $y^* = 0$ , because if it did, the regulator would punish it very severely. Therefore policy enables the two types to perfectly separate: the low type claims  $r = L$  and charges  $p^*(L)$ , whilst the high type claims  $r = H$  and charges  $p^*(H)$ . Advertising is then perfectly informative, and so consumers fully believe any high claim i.e.  $q_H^e = H$ .

Finally and perhaps most interestingly, if  $\phi \in (\phi_1, \phi_0)$ , policy is ‘moderate’. In equilibrium the low type makes a false claim with probability  $y^* \in (0, 1)$ , as defined by (5). This ensures that the low type is indifferent between lying and telling the truth, and is therefore willing to randomize. The equilibrium now has partial separation: sometimes the low type separates by claiming  $r = L$  and charging  $p^*(L)$ , but other times it pools with the high type, by using false advertising to claim  $r = H$  and charge  $p^*(q_H^e)$ . Consequently advertising claims are only partially informative. Note that randomization is an essential feature of the equilibrium. For example there does not exist an equilibrium with full separation: given the associated consumer beliefs  $q_H^e = H$ , a low type would do better to deviate and use false

advertising to get  $\pi^*(H) - \phi$ , rather than tell the truth and get  $\pi^*(L)$ . Similar reasoning shows that there does not exist an equilibrium with full pooling.

**Lemma 2.** *A firm is less likely to engage in false advertising when policy is stronger. That is,  $y^*$  is weakly decreasing in  $\phi$ .*

When policy is either ‘strong’ or ‘weak’ the low type has a strict preference for truth-telling or lying respectively, such that small changes in  $\phi$  have no effect on its behavior. However when policy is ‘moderate’,  $y^*$  satisfies the indifference condition (5) and is strictly decreasing in  $\phi$ . Intuitively, in order to maintain indifference of the low type as  $\phi$  increases, consumers must become more confident about the credibility of high reports. Since consumers use Bayesian updating, this is only possible if  $y^*$  is strictly lower. One implication of Lemma 2 which we will repeatedly exploit in the rest of the paper, is that the regulator can implement any  $y^* \in [0, 1]$  through its judicious choice of the punishment  $\phi$ .

## 4.1 The Effects of Policy on Consumer Surplus

We now consider the effects of policy on a variety of welfare measures, starting with consumer surplus. In light of Proposition 1 we can write expected consumer surplus as

$$E(v) = x(1 - y^*)v^*(L) + (xy^* + 1 - x)v^*(q_H^e) \quad (8)$$

In words, with probability  $x(1 - y^*)$  the firm sends a low report, consumers correctly infer quality to be low, face the associated price,  $p^*(L)$ , and so receive a surplus  $v^*(L)$ . With complementary probability  $1 - x + xy^*$  the firm sends a high report, consumers correctly infer an updated expected quality of  $q_H^e$ , face the associated price,  $p^*(q_H^e)$ , and so receive  $v^*(q_H^e)$ . Hence,  $E(v)$  is a simply convex combination of  $v^*(L)$  and  $v^*(q_H^e)$ .

Before providing a more intuitive explanation below, we first note some immediate effects from a marginal increase in the probability of false advertising,  $y^*$ . As  $y^*$  increases, i) consumers are less likely to receive a low claim, via a reduction in  $x(1 - y^*)$ , and ii) consumers correctly lower their resulting expectations of quality for any product with a high claim,  $q_H^e$ . Equivalently, a small increase in  $y^*$  induces a mean-preserving contraction in consumers’ posterior belief about the firm’s quality. Under the assumption that Condition 1 holds such that  $v^*(q)$  is convex for intermediate qualities, Figure 1 suggests that there are three distinct cases of interest. Firstly when  $q_H^e < \tilde{q}$ , Jensen’s inequality implies that consumers benefit from having a more dispersed posterior, such that a small increase in  $y^*$  lowers  $E(v)$ . Secondly though, when  $L < \tilde{q} \leq q_H^e$  it is easy to see that  $E(v)$  is increasing in  $y^*$  as consumers are actually made better off by a small increase in lying. Thirdly when  $\tilde{q} \leq L$ ,  $E(v) = v^*(\tilde{q})$ ,

such that regulation has no effect on consumer surplus. In light of this logic, it is then straightforward to prove:

**Proposition 2.** *Suppose the regulator seeks to maximize consumer surplus, and that Condition 1 holds.*

- a) *When  $H \leq \tilde{q}$ , the regulator uses a strong policy  $\phi^* \geq \phi_0$  to induce  $y^* = 0$ .*
- b) *When  $\bar{q} < \tilde{q} < H$ , the regulator uses a moderate policy  $\phi^* = \pi^*(\tilde{q}) - \pi^*(L)$  to induce  $y^* = \frac{(H-\tilde{q})(1-x)}{(H-\bar{q})(1-x)+\bar{q}-\tilde{q}} \in (0, 1)$ , such that  $q_H^e = \tilde{q}$ .*
- c) *When  $L < \tilde{q} \leq \bar{q}$ , the regulator uses a weak policy  $\phi^* \leq \phi_1$  to induce  $y^* = 1$ .*
- d) *When  $\tilde{q} \leq L$ , the regulator is indifferent over all  $\phi$ .*

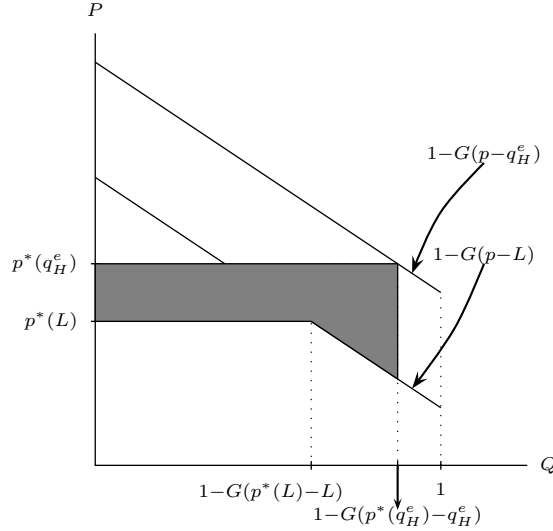
Proposition 2 provides a range of demand and parameter conditions where a consumer-oriented regulator may prefer to *refrain* from completely eradicating false advertising with the use of a strong penalty. To understand its insights, observe that an increase in the level of false advertising,  $y^*$ , produces two effects. On the one hand, consumers are more likely to receive a false advert and so be persuaded to buy a potentially low quality product at an inflated price. On the other hand, the increase in lying damages the credibility of advertising and reduces consumers' quality expectations for a product with a high claim, which then induces any such product to have a lower price. In more detail, one can write

$$\frac{\partial E(v)}{\partial y^*} = -x [v^*(L) - v^*(q_H^e|L)] - (1 - x + xy^*) D(p^*(q_H^e) - q_H^e) \frac{\partial p^*(q_H^e)}{\partial q_H^e} \frac{\partial q_H^e}{\partial y^*} \quad (9)$$

The first term is a 'persuasion' effect. Conditional on the firm having a low quality product (which happens with probability  $x$ ), a marginal increase in lying replaces the surplus that the consumer would have received if the firm had told the truth,  $v^*(L)$ , with the surplus associated with false advertising,  $v^*(q_H^e|L) = v^*(q_H^e) - (q_H^e - L) D(p^*(q_H^e) - q_H^e)$ . To explain this latter surplus, note that after observing such a false advert with price  $p^*(q_H^e)$ , the consumers expect a quality of  $q_H^e$  and so purchase  $D(p^*(q_H^e) - q_H^e)$  units. However, as quality is actually low, they receive  $q_H^e - L$  utils less than they anticipated on each unit purchased. This effect harms consumers by prompting them to pay too much and to potentially buy too many units of a low quality product, as represented by the shaded area in Figure 2. The effect is equivalent to a formalization of the loss in consumer surplus caused by persuasive advertising, as identified in the seminal paper by Dixit and Norman (1978). However, as noted in the introduction, our false advertising 'persuasion' effect arises from a change in consumers' beliefs rather than their preferences.

The second term in (9) is a 'price' effect. An increase in  $y^*$  lowers the probability that a high-claim is true. This lowers the credibility of advertising, reduces consumers'

Figure 2: The Persuasion Effect of False Advertising



confidence in high reports, and lowers their rational expectation of the relevant product quality,  $\partial q_H^e / \partial y^* < 0$ . While this effect on credibility is typically thought to be detrimental, little attention has been paid to its benefits in inducing price reductions. In particular, with the probability that the firm uses a high claim,  $1 - x + xy^*$ , the reduction in credibility lowers market power and prompts the firm to reduce its price.

Proposition 2 can then be understood in terms of our two effects. Suppose  $L < \tilde{q}$ . It can then be shown that the persuasion effect dominates if and only if the market remains uncovered following a high claim, with  $\tilde{q} > q_H^e$ . Hence, a consumer-oriented regulator should reduce false advertising but only to the point where the high-claim market just becomes covered. Therefore, when  $H < \tilde{q}$  the regulator should eradicate false advertising with strong policy. However, when  $H > \tilde{q}$ , our results state that it is optimal to use a weaker form of policy to induce a positive level of lying  $y^* \in (0, 1]$  in order to exploit the beneficial price effect of false advertising.<sup>1415</sup>

In the final part of this subsection, we now consider how some other parameters affect

<sup>14</sup>In the remaining case, where  $L \geq \tilde{q}$ , the two effects exactly cancel so the regulator is indifferent over  $\phi$ . Intuitively all consumers buy irrespective of the claim, paying either  $a + L$  following a low claim, or  $a + q_H^e$  following a high claim. The average price paid is  $a + \tilde{q}$ , which is unaffected by policy. For completeness, we also note a scenario with homogeneous consumers, with  $a = b = 0$ . There,  $p^*(q) = q$  and  $\underline{q} = \tilde{q} = 0$  such that i)  $v^*(q) = 0$  and ii)  $p_q^*(q) = 1$  for all  $q \geq 0$ . Again, the two effects exactly cancel. (However, the results with  $a = b$  under a profit or total welfare objective do not differ from our later findings.)

<sup>15</sup>When Condition 1 does not hold, while more complex, we can show that optimal policy permits a (weakly) higher level of false advertising, such that the main insight of Proposition 2 is strengthened. Firstly, when  $q_H^e \geq \tilde{q}$ , a small increase in  $y^*$  still benefits consumers. Secondly though, when  $q_H^e < \tilde{q}$ , it is no longer necessarily true that a small increase in  $y^*$  harms consumers. Thus, optimal policy is weakly more lenient towards false advertising. For example, when  $L > \underline{q}$ , expected consumer surplus is *always* maximized at  $y^* = 1$  irrespective of whether  $H \gtrless \tilde{q}$ .

optimal policy under a consumer surplus objective. Specifically, we consider how the values of the other market variables,  $x$ ,  $H$  and  $L$  affect optimal policy. When  $L \geq \tilde{q}$ , the authority is indifferent. However:

**Corollary 1.** *When  $L < \tilde{q}$ , a consumer-oriented regulator is more tolerant of false advertising when products are better ( $L$  and  $H$  are higher) and when the probability of a low type,  $x$ , is smaller.*

The regulator should allow the low type to engage in a higher level of false advertising with the use of weaker policy when product quality levels are higher, and/or when the probability of a low type is smaller. Intuitively, under these conditions, the high-claim market is closer to being covered and so the price effect becomes relatively more powerful.

## 4.2 The Effects of Policy on Profits

To begin, we consider the effects of policy on the profits earned by each individual firm type. high type always tells the truth and consequently earns  $E(\pi_H) = \pi(q_H^e)$ . A low type tells the truth with probability  $1 - y^*$  and earns  $\pi^*(L)$ , but also lies with probability  $y^*$ , to earn a profit  $\pi^*(q_H^e)$  with expected punishment  $\phi$ . This gives an overall expected payoff for a low type of  $E(\pi_L) = y^*[\pi^*(q_H^e) - \phi] + (1 - y^*)\pi^*(L)$ . One can then show the following.

**Lemma 3.** *A high type's profit is increasing in  $\phi$  and reaches its maximum at  $\phi \geq \phi_0$ . A low type's profit is decreasing in  $\phi$  and reaches its maximum at  $\phi = 0$ .*

This result is very intuitive. Stronger regulation that eradicates false advertising benefits a high type because it strengthens the credibility of advertising and allows consumers to update more optimistically upon seeing a high claim. However, stronger regulation hurts a low type, because it (weakly) reduces the profit that can be earned by lying and mimicking a high type.

We now turn to the effects of policy on ‘*ex ante*’ expected equilibrium profits,  $E(\Pi) = xE(\pi_L) + (1 - x)E(\pi_H)$ . After some simple manipulations,  $E(\Pi)$  can be shown to be piecewise linear:

$$E(\Pi) = \begin{cases} \pi^*(\bar{q}) - x\phi & \text{if } \phi < \phi_1 \\ \pi^*(L) + (1 - x)\phi & \text{if } \phi \in [\phi_1, \phi_0] \\ x\pi^*(L) + (1 - x)\pi^*(H) & \text{if } \phi > \phi_0 \end{cases} \quad (10)$$

Intuitively, when  $\phi < \phi_1$  the low type lies with probability one such that  $q_H^e = \bar{q}$ . Both types then earn  $\pi^*(\bar{q})$ , but the low type also incurs an expected penalty  $\phi$ . When instead  $\phi > \phi_0$ ,

the types fully separate, with the high type earning  $\pi^*(H)$ , the low type earning  $\pi^*(L)$ , and no punishments being incurred. Finally, when  $\phi \in [\phi_1, \phi_0]$ , the low type is indifferent between lying and telling the truth,  $\pi^*(q_H^e) - \phi = \pi^*(L)$ , such that  $E(\pi_L) = \pi^*(L)$  and  $E(\pi_H) = \pi^*(L) + \phi$ . It then follows that:

**Proposition 3.** *Suppose the regulator seeks to maximize ex ante expected profits.*

- a) *When  $L < \tilde{q}$ , the regulator uses a strong policy  $\phi^* \geq \phi_0$  to induce  $y^* = 0$ .*
- b) *When  $\tilde{q} \leq L$ , the regulator is indifferent between a strong policy with  $\phi^* \geq \phi_0$  to induce  $y^* = 0$ , and a very weak policy with  $\phi^* = 0$  to induce  $y^* = 1$ .*

This can be understood from equation (10) which implies that in order to maximize ex ante expected profits, the firm should never pay the penalty in equilibrium - a firm-oriented regulator would either set  $\phi = 0$  and allow full pooling, or set  $\phi > \phi_0$  and induce full separation. Given that  $\pi^*(q)$  is convex in  $q$ , it is straightforward to see that full separation is (weakly) dominant.

Interestingly, Proposition 3 implies that from an ex ante perspective the monopolist itself always (weakly) prefers higher punishments. Hence, if the monopolist could credibly commit to effective self-regulation in some way, then it would weakly prefer a commitment to not using false advertising. Such self-regulation would be acceptable to consumers in many circumstances, as the monopolist's preferred level of punishment (weakly) coincides with that of consumers when  $L \geq \tilde{q}$  or  $H < \tilde{q}$ . This offers some initial support for the industry-led regulation that is popular in Europe. However, in the remaining circumstances, where  $L < \tilde{q} \leq H$ , such self-regulation would go against consumers' preferences because the monopolist prefers a level of punishment that is strictly *higher* than that preferred by consumers. This (mis-)alignment between the firm's and consumers' preferences is further explored in the next subsection.

### 4.3 The Effects of Policy on Total Welfare

We now examine total welfare. To begin, suppose the punishment,  $\phi$ , is in the form of a fine which is as valuable to the government as to the firm. Any punishment then acts as a simple welfare transfer such that expected total welfare can be written as

$$E(TW) = x(1 - y^*)[v^*(L) + \pi^*(L)] + (1 - x + xy^*)[v^*(q_H^e) + \pi^*(q_H^e)] \quad (11)$$

If we now denote  $y_{E(v)}^*$  and  $y_{E(\pi)}^*$  as the optimal level of false advertising for a regulator with a consumer surplus or profit objective, respectively, (as solved for in Propositions 2 and 3), then we can state:



**Proposition 4.** *Suppose the regulator seeks to maximize total welfare, and that Condition 1 holds.*

- a) *When  $H \leq \tilde{q}$ , the regulator induces  $y^* = y_{E(v)}^* = y_{E(\pi)}^* = 0$ .*
- b) *When  $L < \tilde{q} < H$ , there exists an  $\hat{L} \in q \in (\tilde{q}, \tilde{q})$  such that the regulator induces*

$$y^* = \begin{cases} y_{E(\pi)}^* = 0 & \text{if } L < \hat{L} \\ y_{E(v)}^* \in (0, 1] & \text{if } L > \hat{L} \end{cases}$$

- c) *When  $L \geq \tilde{q}$ , total welfare is the same for all  $y^* \in [0, 1]$ .*

Surprisingly, Proposition 4 shows that under certain conditions a welfare-maximizing regulator permits false advertising. In particular, it strictly prefers a weaker form of punishment,  $\phi < \phi_0$ , whenever the product qualities  $L$  and  $H$  are relatively large. Intuitively, in cases outside market coverage, a monopolist uses its market power to restrict output below the socially efficient level. False advertising then changes this output distortion in two ways. Firstly it raises consumers' expectation of a lying low type's product quality, enabling it to *expand* its output. Secondly, it lowers consumers' expectation of a high type's product quality, and thus causing it to *reduce* its output. The net effect of these two output changes depends crucially on the level of market coverage. When  $q_H^e \leq \tilde{q}$ , the market is not fully covered. A small reduction in false advertising is then beneficial for welfare,  $\partial E(TW)/\partial y^* < 0$ , because a unit of output is more socially valuable when it is produced by the high type. However when  $q_H^e > \tilde{q} > L$ , the firm sells to all consumers when it makes a high claim. Consequently a small change in  $y^*$  has no effect on the output produced by a high type, nor on its contribution to total surplus. Instead, an increase in  $y^*$  i) increases the probability of a low type claiming  $r = H$  and generating surplus  $v^*(q_H^e) + \pi^*(q_H^e) - (q_H^e - L)^{16}$ , and ii) decreases the probability that it claims  $r = L$  and generates surplus  $v(L) + \pi(L)$ . Let  $\Delta(L)$  be the difference between these two surpluses:

$$\Delta(L) = v^*(q_H^e) + \pi^*(q_H^e) - v^*(L) - \pi^*(L) - (q_H^e - L) \quad (12)$$

There exists a unique  $\hat{L}$  such that  $\Delta(L) > 0$  when  $L > \hat{L}$ , and  $\Delta(L) < 0$  when  $L < \hat{L}$ . It is then straightforward to prove that  $dE(TW)/dy^*$  is strictly positive (negative) when  $L$  is above (below)  $\hat{L}$ . In other words, the output expansion induced by false advertising is socially beneficial if and only if  $L$  is relatively large. Intuitively when  $L$  is large, the socially optimal output of a low type is also large and so false advertising is beneficial since it brings the firm's actual output closer to the social optimum. However when  $L$  is small, the optimal level of output for a low type is also small and so false advertising is detrimental since it

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<sup>16</sup>On average a high report generates surplus  $v^*(q_H^e) + \pi^*(q_H^e)$ . However when the firm's quality is actually low, each unit of output is worth  $q_H^e - L$  less to consumers than the average.

increases the firm's output by so much that most of the additional units are valued at less than marginal cost.

Now return to Proposition 4. When  $H \leq \tilde{q}$ , the regulator chooses to fully eliminate false advertising. Recalling our earlier results, the regulator thus implements the outcome preferred by both consumers,  $y_{E(v)}^*$ , and firm,  $y_{E(\pi)}^*$ . Next suppose that  $\bar{q} < \tilde{q} < H$ . At  $y^* = 1$  the market is not fully covered and so, from above, the regulator reduces  $y^*$  until  $q_H^e$  just hits  $\tilde{q}$ ; after this, the regulator either stops intervening if  $L > \hat{L}$ , or entirely eliminates false advertising if  $L < \hat{L}$ . Finally suppose that  $L < \tilde{q} \leq \bar{q}$ , such that a firm reporting  $r = H$  always sells to the entire market. Given the above discussion, the regulator chooses  $y^* = 0$  if  $L < \hat{L}$ , and chooses  $y^* = 1$  if  $L > \hat{L}$ .<sup>17</sup> In these last two cases, the preferred level of lying for the firm,  $y_{E(\pi)}^*$ , is mis-aligned with the consumers' preferred level,  $y_{E(v)}^*$ . Depending upon parameters, the regulator implements either the firm-optimum or the consumer-optimum. Thus policy is (weakly) too lenient for the firm and (weakly) too strong for consumers.

Finally, consider the case where some of the punishment,  $\phi$ , is 'lost' and does not contribute to total welfare. Here, the analysis is messier but qualitatively similar to that in Proposition 4. In particular it is clear that:

*Remark 1.* Suppose that a fraction  $\tau > 0$  of the punishment is deadweight loss. There still exists parameters such that a welfare-maximizing regulator strictly prefers to induce a positive level of false advertising.

To illustrate, consider the case where  $L < \tilde{q} \leq \bar{q}$ . When there is no deadweight loss,  $\tau = 0$ , Proposition 4 showed that the optimal policy induces  $y^* = 1$ , with  $\phi = 0$ . However, when  $\tau > 0$ , it becomes even more attractive for a welfare-maximizing authority to set  $\phi = 0$ .

## 4.4 Further Comments

Before moving on to the robustness analysis and extensions in Sections 5 and 6, we briefly make some further comments.

### 4.4.1 Minimum Quality Standards

An alternative consumer protection policy to false advertising regulation involves an outright ban on low quality products or a minimum quality standard whereby firms are punished for attempting to sell a product with a sufficiently low quality (e.g. Leland 1979). We now show how optimal advertising regulation is weakly superior to a minimum quality standard in the context of our model. First, it is immediate that a ban is inferior to advertising regulation if  $\pi^*(L) > 0$  or  $v^*(L) > 0$ . Hence, from this point forward we suppose that  $L$  is sufficiently low

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<sup>17</sup>When  $L \geq \tilde{q}$  the firm sells to every consumer regardless of its report. False advertising thus has no effect on the firm's output, or the surplus that it generates.

such that  $\pi^*(L) = v^*(L) = 0$ . Second, consider an authority with a profit objective. Here, we know  $y^* = 0$  is always weakly optimal such that expected profits are equivalent to those under full information,  $E(\pi) = (1 - x)\pi^*(H)$ . Consequently, a ban on low quality products would generate the same level of expected profits and the two policies are equivalent. Third, now consider an authority with a consumer surplus or total welfare objective. For some cases,  $y^* = 0$  is optimal and so the two policies are equivalent for the same reasoning as above. However, for other cases, we know that the authority strictly prefers to allow the low market to be active with  $y^* > 0$ . Consequently, in these cases, a ban on low quality products is strictly inferior to the optimal use of false advertising regulation.

#### 4.4.2 Costly Reports and Advertising Taxes

In contrast to our initial assumption, firms may find it costly to issue reports even when they are truthful due to the existence of advertising costs or taxes. Suppose that the publication of a report  $r \in \{L, H\}$  now costs  $A > 0$ , but that the firm also has a costless option of not sending a report,  $r = \Theta$ . Note that if  $A > \pi^*(H) - \pi^*(L)$ , neither type is willing to send a report, and only a pooling equilibrium with  $r = \Theta$  exists. Thus we will assume  $A \leq \pi^*(H) - \pi^*(L)$ , and again restrict attention to equilibria in which  $r(H) = H$  and consumer beliefs are independent of the firm's price. It then follows that the report  $r = L$  is never issued in such an equilibrium: if it were, the sender would be inferred to have low quality, and so would strictly benefit by deviating and sending no report. Consequently the low type will choose between sending no report and charging  $p^*(L)$ , or sending  $r = H$  and charging  $p^*(q_H^e)$ . The equilibrium is then qualitatively the same as before: i)  $y^* = 1$  if  $\phi \leq \phi_1 \equiv \pi^*(\bar{q}) - \pi^*(L) - A$ , ii)  $y^* = 0$  if  $\phi \geq \phi_0 \equiv \pi^*(H) - \pi^*(L) - A$ , and iii)  $y^* \in (0, 1)$  and uniquely solves  $\pi^*(q_H^e) - \phi - A = \pi^*(L)$  when  $\phi \in (\phi_1, \phi_0)$ .<sup>18</sup> One interesting observation is that  $\partial y^* / \partial A = \partial y^* / \partial \phi$  such that report costs and punishments have the same effect on the incentive to engage in false advertising. Hence, an advertising tax can be an effective way to increase the truthfulness of advertising. However, in contrast to false advertising regulation, it imposes an additional burden on the high type.

#### 4.4.3 An Alternative Interpretation: False Certification

In many markets, governmental or non-governmental bodies act to publish product quality information in the form of quality certificates, labels or standards (see the review by Dranove and Jin 2010). Examples include the labeling of Organic, Fair Trade, or Eco-Friendly products, or hotel star classifications. However, evidence suggests that low quality firms sometimes exploit these certification processes. To show how our model can be re-interpreted

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<sup>18</sup>However, note that full pooling is impossible if  $A > \pi^*(\bar{q}) - \pi^*(L)$ . Even with  $\phi = 0$ , the low type must send  $r = \Theta$  with positive probability in order to make the high type be willing to send  $r = H$ .

to help understand such false certification, consider a ‘certifier’ that publicly issues quality certificates,  $r \in \{L, H\}$ . For simplicity, suppose that a firm of type  $q$  can obtain a certificate to verify its actual quality,  $r(q) = q$ , at zero cost. However, instead, a low type can falsely obtain a high quality certificate,  $r(L) = H$ , by incurring a cost,  $\phi$ , which we now interpret as the cost of falsifying any evidence required by the certifier. Then consider the following game. In Stage 0, the certifier publicly announces its certification requirements. The ‘toughness’ of these requirements, then determines the implied level of  $\phi$ . In Stage 1, the firm then learns its quality type, and selects which certificate to apply for,  $r \in \{L, H\}$ . In Stage 2, the certifier then publicly issues the certificate and the firm chooses its price, before consumers then make their purchase decisions in Stage 3. Even though the costs of lying are incurred before any market transactions in the form of false certification costs, rather than after any market transactions in the form of false advertising costs, one can verify that the game is isomorphic to our main model, such that low types engage in false certification with probability  $y^*$ . Crucially, the low type still faces an equivalent set of payoffs from lying,  $\pi^*(q_H^e) - \phi$ , relative to truth-telling,  $\pi^*(L)$ . As such, all our prior welfare and policy results remain. In particular, i) a certifier with a consumer surplus or total welfare objective may deliberately induce false certification,  $y^* > 0$  with a weak set of requirements,  $\phi < \pi^*(H) - \pi^*(L)$ , and ii) a certifier with a profit objective, such as an industry body, may use requirements that are tougher than those preferred under a consumer surplus or total welfare objective.

## 5 Robustness

The main model assumed that both types had the same marginal cost, and that any regulatory punishment was independent of the firm’s price. Jointly, these assumptions imply that both types have the same preference over optimal prices when making a high report. This played an important role when selecting amongst equilibria. Before moving on to our model extensions in Section 6, this section now relaxes both of these assumptions, and shows how our existing results can be generalized.

### 5.1 More Complex Punishments

In practice, and in contrast to our initial assumption of a fixed punishment for all false advertising,  $\phi$ , one might expect the punishment to depend on how much the firm gains from its misleading claim or on how much harm is caused to consumers. To capture these and other possibilities in a parsimonious way, now consider a general false advertising punishment function  $\phi(p, q^e)$ , where  $p$  is the firm’s price and  $q^e$  is the consumers’ belief about the firm’s product quality. In the first stage, the regulator now commits to a function  $\phi(p, q^e)$ , specifying a punishment for every price  $p \in \mathfrak{R}$  and every expected quality  $q^e \in [L, H]$ . In

order to rule out some uninteresting cases, we assume that the punishment is always strictly positive, with  $\phi(p, q^e) > 0$  everywhere. The game and move order are otherwise exactly as before. For simplicity, we also assume that conditional on its report, each type's price is chosen according to a pure strategy.

As a preliminary step, notice that if both types send the same report with positive probability, they must charge the same price when sending that report. If types did not pool in this way, consumers would be able to infer the firm's type based upon its price and one type would wish to deviate. Notice also that if the firm reports  $r = H$ , charges price  $p$  and induces a belief  $q^e$ , the high type earns  $\phi(p, q^e)$  more than the low type. Since  $\phi(p, q^e)$  depends on  $p$ , it is no longer necessarily true that the types have the same preference over what price to charge. Hence, we now require a different approach to equilibrium selection. Firstly, as before, we focus on equilibria in which the high type always sends a high report. Secondly, we restrict attention to equilibria in which the high type charges its sequentially optimal price. In other words, if the expected quality of a firm with  $r = H$  is  $q_H^e$ , then both types charge  $p^*(q_H^e)$  when sending a high report.<sup>19</sup> The rationale for this second restriction is that since the high type is the one being mimicked, it should have 'leadership' in deciding upon the pooling price. Since the high type has no need to engage in false advertising, the pooling price is independent of the regulator's choice of  $\phi(p, q^e)$ . We may then state the following:

**Lemma 4.** *Suppose that  $\pi^*(q) - \phi(p^*(q), q)$  is continuous and strictly increasing in  $q \in [L, H]$ . There is a unique equilibrium (up to off-path beliefs) satisfying the above two restrictions, in which:*

- i) *The high type claims  $r = H$  and charges  $p^*(q_H^e)$ .*
- ii) *The low type claims  $r = H$  and charges  $p^*(q_H^e)$  with probability  $y^*$ ; and claims  $r = L$  and charges  $p^*(L)$  with probability  $1 - y^*$ .*
- iii) *When  $\pi^*(L) \leq \pi^*(\bar{q}) - \phi(p^*(\bar{q}), \bar{q})$ ,  $y^* = 1$ . When  $\pi^*(L) \geq \pi^*(H) - \phi(p^*(H), H)$ ,  $y^* = 0$ . Otherwise  $y^* \in (0, 1)$  uniquely solves*

$$\pi^*(L) = \pi^*(q_H^e) - \phi(p^*(q_H^e), q_H^e)$$

- iv)  *$q_H^e = L + (H - L)\frac{1-x}{1-x+xy^*}$ , and consumer beliefs satisfy*

$$\Pr(q = H | \{r, p\} = \{H, p^*(q_H^e)\}) = \frac{1-x}{1-x+xy^*} \text{ and } \Pr(q = H | \{r, p\} \neq \{H, p^*(q_H^e)\}) = 0$$

- v) *If  $\phi(p, q^e)$  increases for all  $(p, q^e)$ ,  $y^*$  decreases.*

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<sup>19</sup>This is also the unique pooling price selected by Mezzetti and Tsoulouhas's (2000) Strongly Undeclared Equilibrium refinement, provided that  $\phi(p, q^e) \geq \phi(p^*(L), L)$  and that  $\phi(p, q^e)$  is not too sensitive to changes in  $p$  and  $q^e$ . A proof of this is available upon request.

Hence, even when the regulator uses a more ornate punishment scheme, the resulting equilibrium is qualitatively the same as in Proposition 1. The only difference is that pessimistic beliefs must be adopted off-path in order to ensure that the two types pool when sending a high report. Thus, we can again view the regulator as choosing a lying probability  $y^*$  to maximize its objective function. The desired  $y^*$  can then be implemented by using a fixed fine  $\phi$  as modeled earlier, or a more ornate fine  $\phi(p, q^e)$  as modeled here, with either approach producing the same final outcome.

## 5.2 Asymmetric Costs

We now consider an alternative scenario in which the types have different marginal costs (but return to the assumption of a simple fine,  $\phi(p, q^e) \equiv \phi$ ). Suppose that a firm of quality  $q$  has constant marginal cost  $c(q)$ , where  $c'(q) \geq 0$  and  $c''(q) \leq 0$ . Let  $\pi(p, q^e; i) \equiv (p - c(i)) [1 - G(p - q^e)]$  denote the profit earned by a firm with true quality  $i \in \{L, H\}$ , and price  $p$ , but which is believed by consumers to have quality  $q^e$ . It is convenient to further denote  $p^*(q^e; i) = \arg \max_p \pi(p, q^e; i)$ . Under cost asymmetry, it is well-known that price can be used to signal quality. In particular, suppose  $\phi = 0$  such that claims are redundant, and that  $c(H) - c(L)$  is not too large. Refinements such as D1 or the Intuitive Criterion then select a separating equilibrium, in which the low type charges  $p^*(L; L)$ , whilst the high type charges the  $p_H$  which solves

$$\Pi(p^*(L; L), L; L) = \Pi(p_H, H; L)$$

The high type distorts its price upwards above  $p^*(H; H)$ , such that the low type has no strict incentive to mimic. Separation is made possible precisely because  $c(H) > c(L)$ . Nevertheless an important drawback of such separating equilibria is that consumers necessarily hold correct beliefs about product quality at the point of purchase. This contrasts with the evidence given in the introduction, that suggests how false advertising is prevalent, and how it actively influences consumers' behavior. Consequently, we approach equilibrium selection as before by focusing on equilibria where  $r(H) = H$ , and where, conditional on sending a high report, the types pool on the high type's preferred price,  $p^*(q_H^e; H)$ . Provided  $c(H) - c(L)$  is not too large, we can then show that there is a unique (up to off-path beliefs) equilibrium which is qualitatively the same as that derived earlier in Proposition 1.<sup>20</sup> Policy comparative statics also remain unchanged. In particular we can define a threshold  $\tilde{q}(q) = c(q) - a + 1/g(a)$ , such that a firm with known quality  $q$  covers the market if and only if  $q \geq \tilde{q}(q)$ . When  $H < \tilde{q}(H)$ , all parties agree that stronger regulation is best. Relative to symmetric costs, there is now an additional reason to eradicate false advertising. With asymmetric costs, a low type which falsely claims  $r = H$  is required to distort its price upwards, by charging

<sup>20</sup>The proofs are lengthy and so omitted from the paper, but are available upon request.

$p^*(q_H^e; H)$  instead of its preferred (lower) price  $p^*(q_H^e; L)$ . This upward distortion provides a further loss to consumer surplus, total surplus, and ex ante profits. When instead  $L < \tilde{q}(L)$  and  $H \geq \tilde{q}(H)$ , such that only the high report market is covered, we again find that i) the firm prefers stronger regulation than consumers, and that ii) a welfare-maximizing authority sides perfectly with one party, depending upon quality level of the low quality product.<sup>21</sup>

## 6 Extensions

This section now analyzes some substantial extensions to the main model, including i) limited enforcement and multiple markets, ii) competition, and iii) endogenous quality investment.

### 6.1 Costly Enforcement and Multiple Markets

We now extend the main model to a realistic situation where the authority has jurisdiction to regulate false adverts across many (heterogeneous) markets, but has only limited resources to do so. This environment introduces many additional issues due the different potential costs and benefits of intervening within each market. Suppose there is now a unit mass of independent products indexed by  $i \in [0, 1]$ . Each product is supplied by a unit mass of geographically-isolated ‘local monopolists’. A supplier of product  $i$  has low quality  $L_i$  with probability  $x_i \in (0, 1)$ , and high quality  $H_i$  with complementary probability  $1 - x_i$ . A consumer enjoys utility  $q_i + \varepsilon_i$  from consuming one unit of product  $i$  when its quality is  $q_i$ ;  $\varepsilon_i$  is a consumer-specific idiosyncratic match term, distributed on  $[a_i, b_i]$  according to a density function  $g_i(\varepsilon_i)$  whose hazard rate is increasing. To allow for limited regulatory resources, we assume that adverts are now regulated by allocating inspectors to product markets and that the mass of inspectors,  $M$ , is finite. This is consistent with the idea that each inspector must be trained to inspect and evaluate claims about any specific product  $i$ . In particular, in Stage 1, the regulator publicly commits to allocating its inspectors across products, where  $m_i$  denotes the the mass of inspectors allocated to product  $i$  and where  $\int m_i di \leq M$ . In Stage 2, each firm learns its quality, and then announces a price and a report  $r_i \in \{L_i, H_i\}$  about its product quality. In Stage 3, consumers observe the report and price set by the firms for each product within their geographical market, and then make their purchase decisions. As the products are independent, each purchase decision is also independent. In Stage 4, each of the  $m_i$  inspectors allocated to product  $i$  randomly checks one firm that made a high quality claim for that product. If any high claim is inspected and found to be false, the

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<sup>21</sup>Given the assumption that  $c(H) - c(L)$  is not too large, there is one remaining case where the market is always covered,  $L \geq \tilde{q}(L)$ . As before policy has no effect on consumer surplus, total welfare, or ex ante profits.

authority administers a ‘large’ exogenous fine  $F^{22}$ . We assume that the fine is set at the national level in ways related to legal norms and constraints. Hence, the regulator is only able to influence punishments for each product by varying the monitoring probability.

We start by fixing the regulator’s resource allocation, and focusing on the implications for pricing and advertising. The expected punishment for false advertising on product  $i$  is now

$$\phi_i \equiv \frac{m_i}{1 - x_i + x_i y_i} \cdot F \quad (13)$$

because the probability of being monitored equals the number of inspectors divided by the total number of high claims for that product. Analogous to the main model, we can define two critical thresholds

$$m_i^1 = \frac{(\pi_i^*(\bar{q}_i) - \pi_i^*(L_i))}{F} \quad \text{and} \quad m_i^0 = \frac{(\pi_i^*(H_i) - \pi_i^*(L_i))(1 - x_i)}{F}$$

It is then straightforward to extend Proposition 1 to this richer setting. In particular, given the same restrictions that we imposed earlier, there is a unique equilibrium in which i) when  $m_i \leq m_i^1$  all suppliers of product  $i$  pool and claim to have a good product, ii) when  $m_i \geq m_i^0$  suppliers of product  $i$  separate and advertise truthfully, and iii) when  $m_i \in (m_i^1, m_i^0)$  low quality suppliers of product  $i$  randomize between truthful and false advertising.

We now consider the regulator’s optimal resource allocation across products. In order to make the problem interesting and tractable, we make three additional assumptions. First, we assume that  $H_i \leq \bar{q}_i$  for each market  $i$ . This implies that absent the resource constraint, the regulator would completely eradicate false advertising by setting  $m_i \geq m_i^0$ . Second, we assume that the regulator’s resource constraint binds, with  $\int m_i^0 di > M$  such that the optimal policy solution with unlimited resources is no longer feasible. Third, while we allow the shape of the demand curve to vary across products, we assume that all products exhibit constant pass-through, consistent for example with linear, exponential or constant elasticity demand. We focus on the case where the regulator has a consumer surplus objective:

**Proposition 5.** *Given the above assumptions, for almost every market the optimal resource allocation satisfies  $m_i^* \in \{0, m_i^0\}$ . The optimal policy allocates positive resources to markets where, ceteris paribus i)  $L_i$  is lower, ii)  $H_i$  is higher, and iii)  $x_i$  is higher.*

The optimal policy concentrates resources amongst a subset of products. Intuitively, when  $m_i \in [0, m_i^1]$  all suppliers of product  $i$  claim to have a good product and charge  $p_i^*(\bar{q}_i)$ ; only when  $m_i \in (m_i^1, m_i^0)$  do changes in policy have a meaningful impact on firm behavior and consumer surplus. Therefore  $m_i^1$  acts like a fixed cost of intervening in market  $i$ , and

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<sup>22</sup>Here, ‘large’ means that  $F > \pi_i(H_i) - \pi_i(L_i)$  for all  $i$ , such that any firm prefers to avoid false advertising if it expects to be monitored with certainty.



makes it optimal to concentrate regulatory resources. Moreover, given the assumption of constant pass-through, it turns out that the marginal effect of increasing  $m_i$  on expected consumer surplus from product  $i$  is constant. Therefore, conditional on allocating positive resources to product  $i$ , it is optimal to keep doing so until the unconstrained optimum  $m_i^0$  is reached. Finally, Proposition 5 also makes recommendations about which types of market are most suitable for regulatory intervention. Firstly the regulator should prioritize products where the difference between good and bad products  $H_i - L_i$  is largest. This is because the gain in consumer surplus induced by truthful advertising, is large relative to the regulatory cost  $m_i^0$ . Secondly the regulator should prioritize products where the fraction of bad firms is largest. This is because from equation (13), the amount of resources required to induce truth-telling by firms is particularly low.

## 6.2 Competition

In this subsection, we extend our results into a competitive context. The introduction of competition brings new issues. However, we demonstrate the existence of a related equilibrium with false advertising and detail some policy implications that are qualitatively similar to those found in the main monopoly model. Consider a setting where an established incumbent,  $I$ , with quality  $q_I > 0$ , competes against an entrant,  $E$ , with quality,  $q_E$ . Product differentiation is modeled using a Hotelling line, such that a consumer with location  $z \in [0, 1]$  can gain  $U_I(z) = q_I - p_I - tz$  or  $U_E(z) = q_E - p_E - t(1 - z)$  from trading with the respective firms. While the incumbent's product quality is known, the entrant's quality is private information. Specifically, the entrant's product quality  $q_E$  equals  $L$  with probability  $x \in (0, 1)$  and  $H > L$  with probability  $1 - x$ . In line with the main model, we define the entrant's ex ante average quality level as  $\bar{q}_E = xL + (1 - x)H$  and assume that the entrant is always able to make positive profits in equilibrium if consumers expect it to have such a quality level (which later requires  $\bar{q}_E > q_I - 3t$ ). The game proceeds with i) the regulator publicly selecting  $\phi$ , ii) the entrant learning its quality and issuing a report  $r \in \{L, H\}$ , iii) the entrant and incumbent simultaneously selecting their prices,  $p_E$  and  $p_I$ , iv) consumers making their purchase decisions, and v) the regulator administering any potential punishments. We assume that marginal costs are zero, and that consumers' outside option is sufficiently poor that in equilibrium they all buy the product.

First, consider the benchmark case where  $q_E$  is public information. After some computations, Nash equilibrium prices and profits can be shown to satisfy

$$p_i^*(q_i, q_j) = \begin{cases} 0 & \text{if } q_i \leq q_j \\ t + \frac{q_i - q_j}{3} & \text{if } q_i \in (q_j, \tilde{q}_i) \\ q_i - q_j - t & \text{if } q_i \geq \tilde{q}_i \end{cases} \quad (14)$$

$$\pi_i^*(q_i, q_j) = \begin{cases} 0 & \text{if } q_i \leq \underline{q}_i \\ \frac{t}{2} + \frac{q_i - q_j}{3} + \frac{(q_i - q_j)^2}{18t} & \text{if } q_i \in (\underline{q}_i, \tilde{q}_i) \\ q_i - q_j - t & \text{if } q_i \geq \tilde{q}_i \end{cases} \quad (15)$$

where  $\underline{q}_i = q_j - 3t$  and  $\tilde{q}_i = q_j + 3t$ . Therefore when  $q_E \leq \underline{q}_E$  the entrant is uncompetitive, and so ends up charging marginal cost and earning zero profit. When instead  $q_E \in (\underline{q}_E, \tilde{q}_E)$  both firms are active. An increase in  $q_E$  shifts out the demand curve of the entrant at the expense of the incumbent, causing the entrant to charge more (and earn more) and the incumbent to charge less (and also earn less). Finally when  $q_E \geq \tilde{q}_E$  the entrant's product is sufficiently strong that it monopolizes the entire market. After further computations, consumer surplus can be shown to be quasiconvex as in the main model, and to satisfy

$$v^*(q_I, q_E) = \begin{cases} q_E + \frac{t}{2} & \text{if } q_E \leq \underline{q}_E \\ -\frac{5t}{4} + \frac{q_I + q_E}{2} + \frac{(q_I - q_E)^2}{36t} & \text{if } q_E \in (\underline{q}_E, \tilde{q}_E) \\ q_I + \frac{t}{2} & \text{if } q_E \geq \tilde{q}_E \end{cases} \quad (16)$$

Second, when the entrant's product quality is private information, we can state:

**Proposition 6.** *In our competitive context, assuming that i) the high entrant type always issues a high report, and ii) consumer beliefs depend only on the entrant's claim, there exists an equilibrium related to the monopoly model where the low quality entrant type uses false advertising with probability  $y_c^*$ . Optimal policy remains qualitatively similar to that in the monopoly model.*

As the full derivation for Proposition 6 is lengthy, we only provide a sketch here and focus more on its intuition. However, full details are available on request. Given consumers' updated beliefs about  $q_E$ , our equilibrium restrictions imply that the firms will set the Nash equilibrium prices given by equation (14). Consequently a low quality entrant type can either i) issue a truthful report so that consumers infer its quality level, and thus earn  $\pi_E^*(q_I, L)$ , or ii) pool with the high quality entrant, be believed to have quality  $q_E^e = L + (H - L) \frac{1-x}{1-x+xy_c^*}$ , and thus earn  $\pi_E^*(q_I, q_E^e) - \phi$ . Similar to before, the resulting level of false advertising  $y_c^*$  is determined by the level of  $\phi$ : i) if  $\phi \leq \phi_1 \equiv \pi_E^*(q_I, \bar{q}_E) - \pi_E^*(q_I, L)$ , then  $y_c^* = 1$ , ii) if  $\phi \geq \phi_0 \equiv \pi_E^*(q_I, H) - \pi_E^*(q_I, L)$  then  $y_c^* = 0$ , and iii) if  $\phi \in (\phi_1, \phi_0)$ , then  $y_c^* \in (0, 1)$  uniquely solves  $\pi_E^*(q_I, q_E^e) - \phi = \pi_E^*(q_I, L)$ .

Under a consumer surplus objective, policy aims to maximize  $E(v^*) = x(1-y_c^*)v^*(q_I, L) + (1-x+xy_c^*)v^*(q_I, q_E^e)$ . Since  $v^*(q_I, q_E)$  is strictly convex in  $q_E \in (\underline{q}_E, \tilde{q}_E)$ , one can show that policy will a) always eradicate false advertising, if  $H < \tilde{q}_E$ , b) induce an interior level of false advertising,  $y_c^* \in (0, 1)$ , to prompt  $q_E^e = \tilde{q}_E$  if  $\bar{q}_E < \tilde{q}_E < H$ , and c) induce full false

advertising,  $y_c^* = 1$ , if  $\tilde{q}_E \leq \bar{q}$ . These results can be understood with a version of our previous effects by writing:

$$\begin{aligned} \frac{\partial E(v)}{\partial y_c^*} = & -x [v^*(q_I, L) - v^*(q_I, q_E^e) + (q_E^e - L)D_E^*(q_E^e, q_I)] \\ & - (1 - x + xy_c^*) \left[ D_I^*(q_I, q_E^e) \frac{\partial p_I^*(q_I, q_E^e)}{\partial y_c^*} + D_E^*(q_E^e, q_I) \frac{\partial p_E^*(q_E^e, q_I)}{\partial y_c^*} \right] \quad (17) \end{aligned}$$

where  $D_I^*(q_I, q_E^e)$  and  $D_E^*(q_E^e, q_I)$  are equilibrium demands for the incumbent and entrant respectively. The first term in (17) is a revised form of the ‘persuasion’ effect. Conditional on the entrant having low quality, a marginal increase in lying replaces the market level of consumer surplus that would have been expected had the entrant told the truth,  $v^*(q_I, L)$ , with the surplus associated with false advertising,  $v^*(q_I, q_E^e) - (q_E^e - L)D_E^*(q_E^e, q_I)$ . The second term in (17) is the revised ‘price’ effect. Conditional on the entrant using a high claim, an increase in lying lowers the credibility of the entrant’s advertising and reduces  $q_E^e$ . This forces the entrant to lower its price, but allows the incumbent to raise its price (when active). Once weighted by the demand at each firm, the net price effect may no longer be beneficial to consumers. Indeed, when  $q_E^e < q_I$ , the net price effect damages consumer welfare and provides a further incentive to strengthen regulation.<sup>23</sup> However, a beneficial price effect still dominates when  $q_E^e \geq \tilde{q}_E$ , while the persuasion effect (and possible negative price effect) dominates when  $q_E^e < \tilde{q}_E$ . Hence, a consumer-oriented regulator should reduce false advertising as much as possible, subject to the constraint that the entrant does not attract the whole market.

Optimal policy under an industry profit objective aims to maximize  $E(\Pi) = E(\pi_I^*) + E(\pi_E^*)$ . As  $E(\pi_I^*)$  and  $E(\pi_E^*)$  are both convex in  $q_E^e$ , one can show that both firms’ expected profits are weakly maximized with a zero level of false advertising. Therefore, as in the monopoly case, if the industry could credibly commit to effective self-regulation, it would weakly prefer to commit to not using false advertising. This preference is strict unless the low quality entrant type is so much better than the incumbent that it attracts the whole market in equilibrium, which happens when  $L \geq \tilde{q}_E$ .

Finally under a total welfare objective, the regulator selects the optimal regulation under either the consumer surplus or industry profits objective. In particular, policy will a) always eradicate false advertising if  $H < \tilde{q}_E$ , b) induce a potentially positive of false advertising  $y_c^* \in [0, 1]$  if  $\tilde{q}_E < H$ , and c) remain indifferent when  $\tilde{q}_E \leq L$ . Thus, industry self-regulation (under an industry profit objective) will again be weakly too strong from a total welfare or

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<sup>23</sup>A third potential effect, implicit in (17), involves a re-allocation effect where an increase in lying prompts some consumers to switch from the entrant to the incumbent. However, by definition, these consumers are indifferent between the two firms, and so this effect equals zero.

consumer perspective.

To deepen the intuition of these results, and to begin to understand the relationship between the level of competition and optimal advertising regulation in this context, we now consider the effects of an increase in the level of horizontal product differentiation,  $t$ .<sup>24</sup>

**Corollary 2.** *Following a decrease in the level of competition (via an increase in  $t$ ), optimal policy under a consumer surplus, industry profit or total welfare objective becomes weakly stronger by permitting a weakly lower level of lying.*

Our model suggests that a less competitive, more differentiated market should have stronger advertising regulation. An increase in  $t$  raises the quality threshold  $\tilde{q}_E = q_I + 3t$  needed for the entrant to attract the whole market. Therefore, under either a consumer surplus objective or a total welfare objective, optimal policy permits a lower level of false advertising when  $t$  is larger. The intuition for this can be understood in terms of our two effects: an increase in  $t$  i) enhances the persuasion effect by increasing the harm caused by paying an inflated price for a falsely-labeled product, and ii) reduces the price effect by making prices less sensitive to the firm's product qualities.<sup>25</sup>

### 6.3 Endogenous Quality Investment

In this final subsection, we return to monopoly but allow for endogenous product quality. We now assume that the firm is initially endowed with low quality  $L$ , but can upgrade to high quality  $H$  by paying an investment cost  $C$ . This cost is drawn privately from a distribution  $T(C)$  on  $[0, \infty)$ , with corresponding density  $t(C)$  which is strictly positive everywhere. The move order is then as follows. As before, at stage 1 the regulator commits to a punishment  $\phi$ . At stage 2 the firm learns its investment cost  $C$ , and privately chooses whether or not to upgrade. The game then proceeds as in the main model with the firm choosing its report and price, before consumers make their purchase decisions, and the regulator instigates any potential punishments. We maintain our equilibrium selection approach, and also let  $x^*(\phi)$  denote the endogenous probability that a firm chooses to have low quality.

The punishment  $\phi$  now affects firm behavior in a more complicated way. Firstly, when  $\phi = 0$  the firm earns the same payoff regardless of its quality, so there is no incentive to upgrade, and therefore  $x^*(0) = 1$ . Secondly, when  $\phi \geq \phi_0 = \pi^*(H) - \pi^*(L)$  false advertising is dominated, advertising is fully credible, and so a product of quality  $q$  earns  $\pi^*(q)$ . The firm upgrades if and only if  $C \leq \phi_0$ , such that  $x^*(\phi) = 1 - T(\phi_0) \in (0, 1)$ . Thirdly, when  $\phi \in (0, \phi_0)$  the level of false advertising is necessarily positive. This implies that the

<sup>24</sup>The proof is based on the details of Proposition 6, and so it is also available on request.

<sup>25</sup>Under an industry profit objective,  $y_c^* = 0$  is weakly optimal for all  $t$ . However in cases where initially  $L > \tilde{q}_E$  such that the regulator is indifferent, an increase in  $t$  can make  $y_c^* = 0$  strictly optimal.

difference in payoffs of a low and high quality firm is exactly  $\phi$ , such that  $x^*(\phi) = 1 - T(\phi)$ . Therefore, an increase in  $\phi$  makes investment more likely because it improves the credibility of advertising and so raises the net return from investing in high quality. However, the impact of  $\phi$  on false advertising is more ambiguous. Indeed, it is possible to generate examples where  $y^*$  increases in  $\phi$  over some range of punishments. Intuitively, when  $\phi$  increases, investment may cause  $x^*(\phi)$  to decrease so much that, ceteris paribus, the gains from false advertising,  $\pi^*(q_H^e) - \phi$ , rise, and prompt a higher  $y^*$ . Nevertheless, despite this complication, a higher  $\phi$  always translates into a larger  $q_H^e$  and therefore into more credible advertising.

Now consider the optimal punishment chosen by a regulator whose objective is to maximize consumer surplus. It is useful to rewrite (8) from earlier only in terms of  $x^*(\phi)$  and  $q_H^e$ :

$$E(v) = v^*(L) + (H - L)(1 - x^*(\phi)) \frac{v^*(q_H^e) - v^*(L)}{q_H^e - L} \quad (18)$$

We can then state<sup>26</sup>:

**Lemma 5.** *Fix  $L$  and  $H$  and suppose that Condition 1 holds. A consumer-oriented regulator chooses a weakly higher  $\phi$  when quality is endogenous. However a strictly positive level of false advertising is still optimal provided that  $L < \tilde{q}$  and  $t(\phi_0) \cdot (H - L) < T(\phi_0)$ .*

A stronger policy induces higher  $q_H^e$  and thus makes a claim of high quality more credible. On the one hand, as shown earlier, such an increase in  $\phi$  can harm consumers via a negative price effect. However, on the other hand, once quality is endogenous, the increase in advertising credibility can benefit consumers by making the firm more likely to invest in its product. As a result, optimal policy becomes weakly stronger relative to the case of exogenous quality. Nevertheless, the policymaker may still refrain from completely eliminating false advertising. This occurs if  $t(\phi_0) \cdot (H - L) < T(\phi_0)$  which loosely implies that a marginal decrease in  $\phi$  from  $\phi \approx \phi_0$  only causes a relatively small reduction in the probability that the firm upgrades.

## 7 Conclusions

This paper has analyzed the effects of consumer protection policy on false advertising. Despite its prevalence and importance, false advertising has previously remained under-studied as an equilibrium phenomenon. However, by using standard tools, we have shown how it can arise in equilibrium and how it can influence rational consumers. Moreover, the paper has provided conditions under which weak, rather than strong, regulation can be optimal for consumers and society due the positive effects of false advertising in counteracting firms' market power.

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<sup>26</sup>The proof is available on request

Our model offers some original predictions that we hope will be tested in future empirical work. First, we make specific predictions about how changes in policy should affect the credibility of advertising and the associated prices charged by firms. The otherwise-excellent existing empirical studies often lack price data (e.g. Cawley *et al* 2013, Zinman and Zitzewitz 2014). However, with the right data and an exogenous shift in policy, future research may be able to test such effects. Second, we predict that false advertising should be *probabilistic* for moderate levels of regulatory punishment. Indeed, at such levels of punishment, mixing is an inherent feature of equilibrium because fully separating and pooling equilibria cannot exist. This issue has yet to be addressed within existing empirical work.

Future theoretical work would also be valuable. To focus our analysis, we assumed that other credibility mechanisms involving firm reputations or firm-consumer contracts are not available, as consistent with cases where consumers can only assess a product's value with a sufficient delay, if at all. Future work in other settings where consumer protection policy can interact with reputational or contractual mechanisms would clearly be of value.

## Appendix A: Main proofs

**Proof of Lemma 1.** (i) If  $q \leq \underline{q}$  demand is zero at any strictly positive price, so profit is (weakly) maximized at  $p^* = 0$ . (ii) If  $q > \underline{q}$  the firm's price must satisfy  $p \geq a + q$  otherwise it could increase  $p$ , still sell to all consumers, and strictly increase its profit. Provided  $p \in [a + q, b + q]$  we can differentiate the profit function  $p[1 - G(p - q)]$  with respect to  $p$  and get a first order condition

$$1 - p \frac{g(p - q)}{1 - G(p - q)} = 0 \quad (19)$$

(a) When  $q \in (\underline{q}, \tilde{q})$  the left-hand side of (19) is strictly positive at  $p = a + q$ , strictly negative as  $p \rightarrow b$ , and strictly decreasing in  $p$  given our assumption that  $1 - G(\varepsilon)$  is logconcave. Hence there is a unique  $p$  that solves equation (19). Since the left-hand side of (19) is weakly increasing in  $q$  and decreasing in  $p - q$ , but strictly decreasing in  $p$ , it readily follows that  $\partial p^*(q)/\partial q \in [0, 1)$ . (b). When  $q \geq \tilde{q}$  the left-hand side of (19) is strictly negative at all  $p > a + q$  and hence  $p^* = a + q$ .  $\square$

**Proof of Proposition 1.** It is straightforward to show that this is a valid PBE and we therefore omit a formal proof. Instead, we begin by showing that this is the unique PBE (up to off-path beliefs) in which  $r(H) = H$  and consumer beliefs are price-independent. Define  $\beta_H^e = \Pr(q = H | r = H)$  and  $\beta_L^e = \Pr(q = H | r = L)$ .

(a) Beliefs must satisfy Bayes' rule where possible. Therefore fixing  $y^*$ ,  $\beta_H^e = \frac{1-x}{1-x+xy^*}$ . Moreover  $\beta_L^e = 0$  if  $y^* < 1$ . However Bayes' rule places no restrictions on  $\beta_L^e$  if  $y^* = 1$ .

(b) Prices must maximize profits given consumer beliefs. Therefore any firm claiming  $r = L$  must charge  $p^*(q_L^e)$ , and any firm claiming  $r = H$  must charge  $p^*(q_H^e)$ , where

$$q_H^e = (1 - \beta_H^e)L + \beta_H^e H \quad \text{and} \quad q_L^e = (1 - \beta_L^e)L + \beta_L^e H$$

(c) The high type should prefer to report  $r = H$ . This is clearly true because, from part (b), it will earn  $\pi^*(q_L^e)$  if it reports  $r = L$ , but earn  $\pi^*(q_H^e) > \pi^*(q_L^e)$  if its reports  $r = H$ .

(d) The low type's report should be optimal. (i) Using (b), there is an equilibrium with  $y^* = 0$  if and only if  $\phi \geq \pi^*(H) - \pi^*(L)$ . (ii) Using (b), there is an equilibrium with  $y^* = 1$  if  $\phi \leq \pi^*(\bar{q}) - \pi^*(q_L^e)$ . We know from (a) that Bayes rule doesn't restrict  $q_L^e$  in this instance, however we also know that  $q_L^e \geq L$ . So with the appropriate off-path beliefs, an equilibrium with  $y^* = 1$  exists for all  $\phi \leq \pi^*(\bar{q}) - \pi^*(L)$ . (iii) Using (b) an equilibrium with  $y^* \in (0, 1)$  requires that equation (5) hold. Notice that since  $q_H^e \in [\bar{q}, H]$ , equation (5) cannot hold for  $\phi \notin [\phi_1, \phi_0]$ . However (5) does have a unique solution for any  $\phi \in [\phi_1, \phi_0]$ , with  $y^* = 0$  when  $\phi = \phi_0$ , and  $y^* = 1$  when  $\phi = \phi_1$ .

(e) Summing up then, there is a unique PBE (up to off path beliefs).  $\square$

**Proof of Lemma 2.** This follows directly from Proposition 1. a)  $y^* = 1$  when  $\phi \leq \phi_1$ . b)  $y^* \in (0, 1)$  and satisfies equation (5) when  $\phi \in (\phi_1, \phi_0)$ . Recall that  $\partial \pi^*(q_H^e) / \partial q_H^e > 0$ , and note that equation (7) implies  $dq_H^e / dy^* < 0$ . Totally differentiating equation (5) then gives  $\partial y^* / \partial \phi < 0$ . c) Finally  $y^* = 0$  when  $\phi \geq \phi_0$ .  $\square$

**Proof of Proposition 2.** Given Lemma 2 we can first solve for the optimal  $y^*$ , and then use Proposition 1 to find the  $\phi^*$  needed to implement it. Using equations (7) and (8) we can write that

$$\frac{\partial E(v)}{\partial y^*} = x \left[ v^*(q_H^e) - v^*(L) - \frac{dv^*(q_H^e)}{dq} \times (q_H^e - L) \right] \quad (20)$$

As a preliminary step, consider the following three subcases. (i) If  $q_H^e < \tilde{q}$  then (20) is strictly negative, because  $v^*(q)$  is strictly convex for all  $q \in (\underline{q}, \tilde{q})$  via (Condition 1). (ii) If  $q_H^e \geq \tilde{q} > L$  then (20) is strictly positive, because  $dv^*(q)/dq$  is strictly positive for  $q < \tilde{q}$  but zero for  $q \geq \tilde{q}$ . (iii) For the same reason, it follows that (20) is zero if  $L \geq \tilde{q}$ . We are now able to prove the main result. (a) When  $H \leq \tilde{q}$ , case (i) applies for all  $y \in [0, 1]$ , such that  $y^* = 0$ . (b) Now consider  $\bar{q} < \tilde{q} < H$ , and define  $y' = \frac{(H-\tilde{q})(1-x)}{(H-\tilde{q})(1-x)+\tilde{q}-\bar{q}}$ . Case (i) applies for all  $y \in (y', 1]$ , and case (ii) applies for all  $y \in [0, y']$ , such that  $y^* = y'$ . (c) When  $L < \tilde{q} \leq \bar{q}$  case (ii) applies for all  $y \in [0, 1]$  such that  $y^* = 1$ . (d) When  $L \geq \tilde{q}$ , case (iii) applies such that  $E(v)$  is the same for any  $y \in [0, 1]$ . Finally for each of (a) through to (d), Proposition 1 can be used to find the associated  $\phi^*$ .  $\square$

**Proof of Corollary 1.** First consider comparative statics in  $H$ . Using Proposition 2 we have that  $y^* = 0$  when  $H \leq \tilde{q}$ ,  $y^* \in (0, 1)$  and strictly increasing in  $H$  when  $H \in (\tilde{q}, \frac{\tilde{q}-xL}{1-x})$ , and  $y^* = 1$  when  $H \geq \frac{\tilde{q}-xL}{1-x}$ . Second consider comparative statics in  $L$  and  $x$ . Observe from Proposition 2 that when  $H \leq \tilde{q}$ ,  $y^* = 0$  independent of  $L$  or  $x$ . Therefore henceforth assume that  $H > \tilde{q}$ . According to Proposition 2, we have  $y^* \in (0, 1)$  and strictly increasing in  $L$  when  $\bar{q} < \tilde{q} \iff L < \frac{\tilde{q}-(1-x)H}{x}$ , and  $y^* = 1$  when  $L \geq \frac{\tilde{q}-(1-x)H}{x}$ . Similarly we have  $y^* = 1$  when  $\bar{q} \geq \tilde{q} \iff x \leq \frac{H-\tilde{q}}{H-L}$ , but  $y^* \in (0, 1)$  and strictly decreasing in  $x$  when  $x > \frac{H-\tilde{q}}{H-L}$ .  $\square$

**Proof of Lemma 3.** The results follow immediately by using the discussion on page 15. More formally, we can verify the comparative statics by writing the expected profits for the two types in full as

$$E(\pi_L) = \begin{cases} \pi^*(\bar{q}) - \phi & \text{if } \phi < \phi_1 \\ \pi^*(L) & \text{if } \phi \geq \phi_1 \end{cases} \quad \text{and} \quad E(\pi_H) = \begin{cases} \pi^*(\bar{q}) & \text{if } \phi < \phi_1 \\ \pi^*(L) + \phi & \text{if } \phi \in [\phi_1, \phi_0] \\ \pi^*(H) & \text{if } \phi > \phi_0 \end{cases}$$

$\square$

**Proof of Proposition 3.** It is immediate from (10) that i)  $\phi = 0$  strictly dominates any  $\phi \in (0, \phi_1]$ , and ii)  $\phi \geq \phi_0$  strictly dominates any  $\phi \in (\phi_1, \phi_0)$ . The increase in ex ante profits caused by moving from  $\phi = 0$  to  $\phi \geq \phi_0$  is

$$x\pi^*(L) + (1-x)\pi^*(H) - \pi^*(\bar{q}) \propto \frac{\pi^*(H) - \pi^*(\bar{q})}{H - \bar{q}} - \frac{\pi^*(\bar{q}) - \pi^*(L)}{\bar{q} - L} \quad (21)$$

where we have used the fact that  $\bar{q} = xL + (1-x)H$ . a) When  $L < \tilde{q}$  equation (21) is strictly positive, because  $\pi(q)$  is convex everywhere and strictly convex for  $q \in (\underline{q}, \tilde{q})$ . b) When  $\tilde{q} \leq L$  equation (21) is zero, because  $\pi(q) = a + q$  for all  $q \geq \tilde{q}$ .  $\square$

**Proof of Proposition 4.** Given Lemma 2 we need only solve for the optimal  $y^*$ . Using equations (7) and (11) we can write that

$$\frac{\partial E(TW)}{\partial y^*} = x \left[ v^*(q_H^e) - v^*(L) + \pi^*(q_H^e) - \pi^*(L) - \left[ \frac{dv^*(q_H^e)}{dq} + \frac{d\pi^*(q_H^e)}{dq} \right] \times (q_H^e - L) \right] \quad (22)$$

As a preliminary step, consider the following three subcases. (i) If  $q_H^e < \tilde{q}$  then (22) is strictly negative, because  $v^*(q) + \pi^*(q)$  is strictly convex for all  $q \in (\underline{q}, \tilde{q})$ . (ii) If  $q_H^e \geq \tilde{q} > L$  then



(22) is proportional to  $\Delta(L)$  (defined on page 17), because  $dv^*(q)/dq = 0$  and  $d\pi^*(q)/dq = 1$  for all  $q \geq \tilde{q}$ . In view of the latter, we can then write

$$\Delta(L) = v^*(\tilde{q}) - v^*(L) + a - \pi^*(L) + L \quad (23)$$

Notice that (23) is strictly concave in  $L$ , is strictly negative as  $L \rightarrow \underline{q}^{27}$ , and equals zero as  $L \rightarrow \tilde{q}$ . Consequently there exists a unique  $\hat{L}$  which solves  $\Delta(L) = 0$ , such that  $\Delta(L) > 0$  when  $L > \hat{L}$ , and  $\Delta(L) < 0$  when  $L < \hat{L}$ . (iii) Finally if  $L \geq \tilde{q}$  (22) is zero because  $v^*(L) = v^*(\tilde{q})$  and  $\pi^*(L) = a + L$ . We are now able to prove the main result. (a) When  $H \leq \tilde{q}$ , case (i) applies for all  $y \in [0, 1]$ , such that the optimum is  $y^* = 0$ . (b) Now consider  $\bar{q} < \tilde{q} < H$ , and define  $y' = \frac{(H-\bar{q})(1-x)}{(H-\bar{q})(1-x)+\bar{q}-\tilde{q}}$ . Case (i) applies for all  $y \in (y', 1]$ , and case (ii) applies for all  $y \in [0, y']$ . Therefore the optimum is  $y^* = 0$  if  $L < \hat{L}$ , and  $y^* = y'$  if  $L > \hat{L}$ . (c) When  $L < \tilde{q} \leq \bar{q}$  case (ii) applies for all  $y \in [0, 1]$ . Therefore the optimum is  $y^* = 0$  if  $L < \hat{L}$ , and  $y^* = 1$  if  $L > \hat{L}$ . (d) When  $L \geq \tilde{q}$ , case (iii) applies such that  $E(TW)$  is the same for any  $y \in [0, 1]$ .  $\square$

**Proof of Lemma 4.** Our first equilibrium restriction implies that in any equilibrium with  $y^* < 1$ , beliefs satisfy  $\Pr(q = L | r = L) = 1$ , such that a firm sending  $r = L$  optimally charges  $p^*(L)$  and earns  $\pi^*(L)$ . Our second equilibrium restriction implies that in any equilibrium, a firm sending  $r = H$  must charge price  $p^*(q_H^e)$ . Let Condition 2 refer to the assumption that  $\pi^*(q) - \phi(p^*(q), q)$  is strictly increasing in  $q \in [L, H]$ . Let  $y$  refer to the probability with which a low type claims  $r = H$ , and let  $q^e = L + (H - L)\frac{1-x}{1-x+xy}$ . To prove the result, it is then sufficient to show there is no PBE with  $y \neq y^*$ . Firstly suppose that  $\pi^*(L) \leq \pi^*(\bar{q}) - \phi(p^*(\bar{q}), \bar{q})$ . There is no equilibrium with  $y < 1$ . If a low type reports  $r = L$  it gets  $\pi^*(L)$ . If a low type reports  $r = H$  it gets  $\pi^*(q^e) - \phi(p^*(q^e), q^e)$ , which strictly exceeds  $\pi^*(\bar{q}) - \phi(p^*(\bar{q}), \bar{q})$  by Condition 2. Secondly suppose that  $\pi^*(L) \geq \pi^*(H) - \phi(p^*(H), H)$ . There is no equilibrium with  $y > 0$ . If a low type reports  $r = L$  it gets at least  $\pi^*(L)$ . (If  $y < 1$  it gets exactly  $\pi^*(L)$ , but if  $y = 1$  freedom in choosing off-path beliefs means that it might get more.) If a low type reports  $r = H$  it gets  $\pi^*(q^e) - \phi(p^*(q^e), q^e)$ , which strictly exceeds  $\pi^*(\bar{q}) - \phi(p^*(\bar{q}), \bar{q})$  by Condition 2. Thirdly suppose that  $\pi^*(L) = \pi^*(q_H^e) - \phi(p^*(q_H^e), q_H^e)$ . A similar argument to the first shows there is no equilibrium with  $y \in [0, y^*)$ , whilst a similar argument to the second shows there is no equilibrium with  $y \in (y^*, 1]$ . Finally we prove that if  $\phi(p, q^e)$  increases for all  $(p, q^e)$ , then  $y^*$  decreases. Let  $y_{pre}^*$  and  $y_{post}^*$  denote equilibrium  $y^*$  before and after the policy change. If  $y_{pre}^* = 0$  it is clear that  $y_{post}^* = 0$ . If  $y_{pre}^* = 1$  it is clear that  $y_{post}^* \leq 1 = y_{pre}^*$ . Finally if  $y_{pre}^* \in (0, 1)$  it is clear that  $\pi^*(L) > \pi^*(q) - \phi(p^*(q), q)$  for all  $q \leq q^e(y_{pre}^*)$ , such that necessarily  $y_{pre}^* > y_{post}^*$ .  $\square$

<sup>27</sup>To see this, note that  $v^*(\tilde{q}) = \int_a^b [1 - G(z)] dz < b - a$ , and that  $\underline{q} = -b$ .

**Proof of Proposition 5.** The regulator chooses  $\{m_i^*\}$  in to maximize  $\int E(v_i)$  subject to i)  $\int m_i^* \leq M$  and ii)  $m_i^* \geq 0$  for all  $i \in [0, 1]$ .

*Step 1.* The marginal benefit from increasing the number of inspectors in market  $i$  is

$$\frac{\partial E(v_i)}{\partial m_i} = \begin{cases} 0 & \text{if } m_i \leq m_i^1 \\ F \left( 1 - \frac{dp_i^*(q)}{dq} \right) & \text{if } m_i \in (m_i^1, m_i^0) \\ 0 & \text{if } m_i \geq m_i^0 \end{cases} \quad (24)$$

This is derived as follows. First, the marginal benefit is zero for  $m_i \leq m_i^1$  because  $y_i^* = 1$ , and also zero for  $m_i \geq m_i^0$  because  $y_i^* = 0$ . Second, using the definition of  $E(v_i)$  and the low quality firm's indifference condition  $\pi(q_i^e) - \pi(L_i) = \frac{m_i F}{1 - x_i + x_i y_i}$ , we can write that

$$\frac{\partial E(v_i)}{\partial m_i} = \frac{\partial E(v_i)}{\partial y_i} \frac{\partial y_i}{\partial m_i} = F \frac{v_i(q_i^e) - v_i(L_i) - v_i'(q_i^e)(q_i^e - L_i)}{\pi_i(q_i^e) - \pi_i(L_i) - \pi_i'(q_i^e)(q_i^e - L_i)} = F \left( 1 - \frac{dp_i^*(q)}{dq} \right) \quad (25)$$

where the final equality uses the fact that with constant pass-through,  $v_i'(q) = (1 - dp_i^*(q)/dq)\pi_i'(q)$ .

*Step 2.* At the optimum, all but a measure zero set of products have  $m_i^* \in \{0, m_i^0\}$ . First note that any allocation with  $m_i \in (0, m_i^1)$  for some  $i$ , is (weakly) dominated. Second, suppose there is a positive measure of products  $P$  with  $m_i \in (m_i^1, m_i^0)$ . Using (24) these products can be ranked according to  $\partial E(v_i)/\partial m_i$ . The regulator can strictly increase expected consumer surplus by setting  $m_i = 0$  for a subset of the  $P$  products with the lowest  $\partial E(v_i)/\partial m_i$ , and transferring them all to a subset of the  $P$  products with the highest  $\partial E(v_i)/\partial m_i$ , subject to never allocating more than  $m_i^0$  to any of the latter. This contradicts the optimality of the original resource allocation.

*Step 3.* An optimal allocation gives positive resources to products with the highest benefit-to-cost ratio which, using (24) can be written as

$$BC_i = F \left( 1 - \frac{dp_i^*(q)}{dq} \right) \left( \frac{m_i^0 - m_i^1}{m_i^1} \right)$$

It is straightforward (but lengthy) to show that  $BC_i$  is increasing in  $H_i$  and  $x_i$  but decreasing in  $L_i$ , whenever demand belongs to the constant pass-through class given by  $1 - G(p) = X \left( 1 - \frac{1-\sigma}{2-\sigma} \frac{p}{m} \right)^{\frac{1}{1-\sigma}}$ .  $\square$

## Appendix B: Further information on Condition 1

For ease of exposition, let us temporarily suppose that the monopolist has constant marginal cost  $c < b$ . Then let  $p^*(c, q)$  denote the monopolist's optimal price, and  $v^*(c, q)$  denote

realized consumer surplus, where  $p^*(0, q) \equiv p^*(q)$  and  $v^*(0, q) \equiv v^*(q)$ . It is straightforward to show that  $v_{qq}^*(c, q) = v_{cc}^*(c, q)$  and moreover that

$$v_{qq}^*(c, q) > 0 \iff g(\cdot) \left( \frac{dp}{dc} \right)^2 - (1 - G(\cdot)) \frac{d^2p}{dc^2} > 0 \quad (26)$$

Since  $dp/dq = 1 - (dp/dc)$  and  $d^2p/dq^2 = d^2p/dc^2$ , this is equivalent to Condition 1.. Firstly (26) holds when  $d^2p/dc^2 = 0$  i.e. pass-through is constant, as would be the case for example with linear, exponential, or constant elasticity demands. Secondly (26) holds when  $d^2p/dc^2 < 0$  i.e. pass-through is decreasing, as would be the case for example with the single-good AIDS demand function (Weyl and Fabinger 2013). Thirdly (26) holds for other demand functions too. For example the condition is satisfied for subsets of the support when  $G(\cdot)$  is a (and so demand is derived from) logit, normal, log-normal, or Weibull distribution (Cowan 2012, Chen and Schwartz 2013).

**Lemma 6.** *Condition 1 is preserved under truncation and affine transformations.*

*Proof.* Suppose the random variable is defined on  $[a, b]$ . After some algebra, Condition 1 holds on an interval  $[l, h] \subseteq [a, b]$  if and only if for all  $v \in [l, h]$ :

$$\frac{g''(v)}{g(v)} - 2 \left( \frac{g'(v)}{g(v)} \right)^2 + 2 \left( \frac{g(v)}{1 - G(v)} \right)^2 > 0 \quad (27)$$

i) Truncations. Consider a new random variable on interval  $[\hat{a}, \hat{b}] \subset [a, b]$  whose cumulative distribution function is  $\hat{G}(v) = [G(v) - G(\hat{a})] / [G(\hat{b}) - G(\hat{a})]$ . Suppose the intersection of  $[l, h]$  and  $[\hat{a}, \hat{b}]$  is non-empty. After some algebra, Condition 1 holds under  $\hat{G}(v)$  for some  $v$  if and only if

$$\frac{g''(v)}{g(v)} - 2 \left( \frac{g'(v)}{g(v)} \right)^2 + 2 \left( \frac{g(v)}{G(\hat{b}) - G(v)} \right)^2 > 0 \quad (28)$$

which is weaker than (27). Therefore Condition 1 is preserved under truncations.

ii) Affine transformations. Let  $t(v)$  be a linear transformation from  $[\hat{a}, \hat{b}]$  to  $[a, b]$ , and let  $\dot{G}(v) = G(t(v))$  be a cumulative distribution function defined on  $[\hat{a}, \hat{b}]$ . It is straightforward to check that (27) holds at  $v$  for distribution function  $\dot{G}(\cdot)$ , if and only if (27) holds at  $t(v)$  for distribution function  $G(\cdot)$ . Hence, Condition 1 is preserved under affine transformations.  $\square$

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