# Jobs and technology in general equilibrium:

# A three-elasticities approach

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*Paper to be presented at*
*European Research Workshop in International Trade (ERWIT)*
*October 2020*

**Abstract:**

The impact of technological progress on jobs and wages has been subject to a good deal of empirical and some theoretical work. However, most of this literature has not addressed the general equilibrium interplay between the sectors in which change occurs, the productive factors that are affected, and the consequent changes in the structure of employment and returns to factors of production. This paper draws on tools from general equilibrium trade theory to provide an integrated approach to these issues. The analysis centres around three key elasticities linking technological change to jobs – the jobs-displacing substitution effect, the job-creating demand effect, and the general-equilibrium effects, through which factors are reallocated between sectors. The results highlight the role of relative factor intensities and the importance of openness in determining the effects of technology on jobs and wages. Furthermore, employment implications of including a non-traded sector are sketched.

Keywords: Technology and wages, technology and employment, three-elasticity approach

JEL classification: F11, F16, J30, O33

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## 1. Introduction.

Technological progress has been a driver of aggregate prosperity but also a driver of labour market disruption in the advanced economies. Many adverse labour outcomes have been linked to technical change ranging from stagnant wages and the polarisation of employment patterns to destruction of middleclass jobs and rising inequality – especially in the US. Given this track record, the rapid advances in digital technology, machine learning, and advanced robotics have raised concerns about the future of work (Brynjolfsson and McAfee 2014, Ford 2015).The rich and varied research on these issues to date have naturally been dominated by empirical work. Theoretical research on the economic mechanisms underpinning the technology-labour link has been somewhat less rich and varied. Much of the theory has focused on the nature of the technological change and its interaction with different types of workers. The baseline settings assumed tend to be partial equilibrium, or general equilibrium setups with limited feedback possibilities.

This paper seeks to broaden the range of economic mechanisms considered in technology-labour research by embedding the investigation of technology's impact in a simple general equilibrium model that allows a full range of feedback mechanisms. This yields new insights, and suggests that empirical work should at the very least consider the interaction between the nature of the technological shock and the nature of the sector in which it occurs.

Investigation of the effects of technical progress on employment patterns and wages requires analysis at several different levels. The first is between different factors and the technology within the sector or firm. These are the first-order, direct effects. The second is cross-sector effects arising through product market interaction; technical progress will change costs and prices and this will cause switching of expenditure by final consumers and other users of affected products. The third is also across sectors, but through interactions in factor markets rather than the goods markets. The employment and wage effects of technical change depend as much on the characteristics of sectors that factors move to (or are drawn from) as they do on the characteristics of sectors where technical change occurs. All three levels interact in determining outcomes.

The extensive literature on the effects of technical change has devoted less attention to the third of these mechanisms than to the first two, and the contribution of this paper is to provide a relatively rich modelling of the general equilibrium of the entire economy, and thereby extend (and in some cases reverse) results that come from models with a simpler general equilibrium structure.

The first level – modelling the direct impact – is most simply done by assuming that technical change is factor augmenting, i.e. simply raises the efficiency of a particular factor or factors in some activities. Effects depend on the elasticity of substitution, $\sigma$, between those inputs in production that directly benefit from technical change and those that do not. Recent work has looked at richer modelling of technical change, for example developing models of task production (Acemoglu and Restrepo 2018), and task production together with the use of more efficient 'robots' (Graetz and Michaels 2018). The present paper works with factor augmentation and we discuss its relationship to these alternative approaches in section 2 of the paper.

The second level – demand switching – brings a further elasticity into play, and this is the price elasticity of demand, $\varepsilon$, for the output of affected activities. This effect is absent in single sector models such as the 'canonical model' of Acemoglu and Autor (2011). The interaction between these two levels – between the elasticity of substitution and elasticity of demand – is shown with particular clarity in Graetz and Michaels (2018) where technical progress in a sector creates or destroys jobs in the innovating sector according to the sign of the difference between the two elasticities, $\varepsilon - \sigma$, creating jobs where $\varepsilon > \sigma$. [1]

The third level is also across sectors, and through factor market interaction. If the economy has factors that move between sectors, then the impact of technical change on employment and on factor prices depends on the patterns of factor use in each sector. For example, suppose that demand is extremely elastic, so technical change tends to expand output and employment. If the technical change affects one factor (in all sectors) then the relative price of this factor increases, and we show that the movement of factors between sectors depends on sectors' relative factor intensities. If the technical change is in one sector (all factors), then employment of factors in the sector increases, but the change in relative factor prices depends on the relative factor-intensities of sectors where employment is expanding or contracting.

These factor market interactions are well-analysed in Heckscher-Ohlin trade theory where they are represented by the dual Rybczynski and Stolper-Samuelson relationships. In the present context, these relationships underpin the 'third elasticity' essential to understanding the factor market implications of technical change. The Rybczynski theorem of trade theory says that (under well-defined circumstances) an increase in the supply of one factor tends to expand the output of the sector that is intensive in its use. Moreover, the expansion is more than proportional, while other sectors will contract. In the present context the basic insight is that technical progress has similarities with changes in factor endowments.[2]

The primary contribution of this paper is to provide a systematic analysis of the effect of factor augmenting technical change on employment and relative wages when all three of these effects are present. We look at contexts in which one or both factors may be augmented in one or both sectors of a two-factor, two-sector economy, and derive explicit expressions for price, wage, and factor reallocation effects. Whereas most of the trade literature's analysis of the Heckscher-Ohlin model is undertaken with fixed product prices (the small open economy assumption) we generalise this to include the product prices changes which are caused by technical change.[3] We also discuss the importance of the trade regime – what goods are traded, what are not traded, and the pattern of trade – for understanding the impact of technical change.[4]

The next section of the paper, Section 2, sets up a simple benchmark model which we use both to introduce elements of the full model, and to link with existing literature on modelling technical change. Section 3 sets up the general equilibrium and its comparative statics, focusing on the role of

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[1] In a more general setting demand shifts would also include income effects and derived demands through intermediates and the input-output matrix

[2] The observation dates back to Jones (1965) and is sometimes referred to as Jones' equivalence.

[3] Xu (2000) signs many of the relative wage effects, but does not look at factor reallocation or provide precise statements of conditions under which sign conditions hold. Dixit and Norman (1980) show how technical change can be analysed in their dual general equilibrium approach, focussing on trade and welfare effects. Jones (2000), building on Jones (1965), uses a model similar to ours; however, he models technical change differently, and does not focus on employment effects. Haskel and Slaughter (2002) focus on the effects of skill biased technical change (SBTC) on skill premia (relative wages) in a Heckscher-Ohlin model similar to ours, but they do not provide a full analysis of the interaction between effects.

[4] This connects with the earlier debate about the impacts of trade vs technical change on wages, see Krugman (2000).

the three elasticities, substitution, demand, and Rybczynski. In section 4 we look at the effects of technical change that is sector specific (augmenting both factors in one sector), factor specific (a single factor in both sectors), and both factor and sector specific. We draw out implications for the structure of employment and factor prices, pointing to the key role of the third elasticity, and to the very wide range of qualitative and quantitative outcomes that are possible. In section 5 we discuss various open-economy issues, while section 6 concludes.

**2. Model preview and literature context.**

We start with the simplest model in order to introduce building blocks of our approach, derive some benchmark results, and connect with existing literature. Technical progress occurs in a sector (which we call sector 1) which uses two factors of production, A and B, to produce an output, $X_1$. The production function has constant returns to scale and takes the form $X_1 = F(\alpha L_{A1}, L_{B1})$ where $L_{A1}, L_{B1}$ are quantities of each factor. Technical progress is, for the moment, assumed to just augment factor-A, with α giving the efficiency of $L_{A1}$. The corresponding unit cost is, in equilibrium, equal to price, so $p_1 = c(w_A/\alpha, w_B)$, where $w_A$ and $w_B$ are wages of each factor. Totally differentiating the production and cost functions gives (with proportionate changes denoted ^),

$$\hat{X}_1 = \omega_1\big(\hat{L}_{A1} + \hat{\alpha}\big) + (1 - \omega_1)\hat{L}_{B1}, \tag{1}$$

$$\hat{p}_1 = \omega_1(\hat{w}_A - \hat{\alpha}) + (1 - \omega_1)\hat{w}_B, \tag{2}$$

where $\omega_1$ is the cost share of factor-A in this sector. The definition of the elasticity of substitution between factors, $\sigma > 0$, is

$$\sigma \equiv -\left[\frac{\hat{L}_{A1} + \hat{\alpha} - \hat{L}_{B1}}{\hat{w}_A - \hat{\alpha} - \hat{w}_B}\right], \quad \text{so} \quad \left(1 - \frac{1}{\sigma}\right)\hat{\alpha} = \hat{w}_A - \hat{w}_B + \big(\hat{L}_{A1} - \hat{L}_{B1}\big)/\sigma\,. \tag{3}$$

What are the wage and employment effects of this technical progress?

In the simplest case, the canonical model of Acemoglu and Autor (2011), there are two factors and a single sector (sector 1) which fully employs the economy's fixed endowment of each labour type. Thus $\hat{L}_{A1} = \hat{L}_{B1} = 0$ and it follows immediately, from (3), that the "sign test" for the relative wage change depends on a single elasticity, $sign\ (\hat{w}_A - \hat{w}_B) = sign\ (\sigma - 1)$. In words, factor augmentation brings a direct increase in the wage of the factor experiencing the progress (factor-A), and also a change in relative wages as the increased factor-A supply (in efficiency units) is combined with the fixed stock of factor-B. If the two factors are gross substitutes ($\sigma > 1$) this change in relative wage is small, so the net effect on $\hat{w}_A - \hat{w}_B$ is positive. If they are gross complements ($\sigma < 1$) the effect is larger, reducing $w_A/w_B$.[5] The individual wage effects can be seen from (2) and (3) to be $\hat{w}_A = (\sigma + \omega_1 - 1)/\sigma)\hat{\alpha}$ and $\hat{w}_B = (\omega_1/\sigma)\hat{\alpha}$, holding $p_1$ constant. Thus B-type labour always gains from augmentation of A-type labour, and A-type labour gains as long as $\sigma + \omega_1 > 1$.

Graetz and Michaels (2018) show that a second elasticity matters when demand considerations are introduced. This is accomplished in our model by adding a second sector which uses factor-B (sector 2, the rest of the economy, assumed to be the numeraire). The sectors are linked in two ways, which

[5] This also illustrates the difference between factor-saving and factor-using technical progress. A factor-A augmenting technical change ($\hat{\alpha} > 0$ in this case) is *factor-A saving* if it at unchanged factor prices reduces the relative use of factor A in the sector, while it is *factor-A using* if it increases the relative use of factor A. From (3) it follows that $\hat{\alpha}$ is factor-A saving for $\sigma < 1$, and factor-A using for $\sigma > 1$.

we model in a simple (and, for the moment) ad hoc way. One is substitution in demand which, in its simplest form (quasi-linear preferences) with price elasticity $\varepsilon$, is

$$\hat{X}_1 = -\varepsilon \hat{p}_1. \tag{4}$$

The other is that both sectors use factor-B, and we suppose that this generates an elasticity of factor-B supply to sector 1 which we denote $\zeta$, so

$$\hat{L}_{B1} = \zeta \hat{w}_B. \tag{5}$$

If we furthermore take factor-A to be specific to sector 1, we have $\hat{L}_{A1} = 0$. Equations (1) – (5) can be solved to give explicit expressions for the impact of factor-A augmentation on wages and sector 1 employment,

$$\hat{w}_A = \frac{(\sigma-1)(\varepsilon+\zeta) + \omega_1(\varepsilon-\sigma)(1+\zeta)}{\sigma\varepsilon + \zeta[(1-\omega_1)\sigma + \omega_1\varepsilon]}\hat{\alpha}, \qquad \hat{w}_B = \frac{\omega_1(\varepsilon-\sigma)\hat{\alpha}}{\sigma\varepsilon + \zeta[(1-\omega_1)\sigma + \omega_1\varepsilon]}, \tag{6}$$

$$\hat{L}_{A1} = 0, \qquad \hat{L}_{B1} = \frac{\zeta\omega_1(\varepsilon-\sigma)\hat{\alpha}}{\sigma\varepsilon + \zeta[(1-\omega_1)\sigma + \omega_1\varepsilon]} \tag{7}$$

These expressions depend on the share of factor-A in sector 1, $\omega_1$, and on three elasticities: substitution between factors within a sector, $\sigma$, substitution in demand between sectors, $\varepsilon$, and supply of factor-B to sector 1, $\zeta$.

Focussing on the first two of these elasticities, we set $\zeta = 0$, so intersectoral interactions operate entirely via demand substitution. By inspection of (6), with $\zeta = 0$, the relative wage effects are as before, $sign\ (\hat{w}_A - \hat{w}_B) = sign\ (\sigma - 1)$, but the individual wage effects now depend on the interplay of two elasticities. In Graetz and Michaels (2018), the factor experiencing the progress are 'robots' (our factor A), and the other factor are workers (our factor B).[6] With $\zeta = 0$, (6) implies $\hat{w}_B = \omega_1(\varepsilon - \sigma)\,\hat{\alpha}/\varepsilon\sigma$, so workers gain when the demand responsiveness, $\varepsilon$, exceeds the intra-sectoral factor substitution, $sign\ (\hat{w}_B) = sign\ (\varepsilon - \sigma)$. The change in the robots' reward is $\hat{w}_A = [\varepsilon(\sigma - 1) + \omega_1(\varepsilon - \sigma)]\,\hat{\alpha}/\varepsilon\sigma$, so the technical progress raises their reward if $\varepsilon > \sigma > 1$.

Graetz and Michaels (2018) assume perfectly mobile labour between sectors, $\zeta = \infty$ in our notation, and hence $\hat{w}_B = 0$, and focus on the employment effect. In this case we see from (7) that $\hat{L}_{B1} = \omega_1(\varepsilon - \sigma)\hat{\alpha}/[(1-\omega_1)\sigma + \omega_1\varepsilon]$, thus confirming the two-elasticity result of Graetz and Michaels.[7]

The general 'three elasticity' case arises when $\zeta \neq 0$ and $\zeta \neq \infty$. Continuing to view factor-A as robots, the impact on workers' wages involves all three elasticities; however, the sign test still gives

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[6] Graetz and Michaels (2018) develop a model with a continuum of industries each of which produces using multiple tasks. Some industries are robot-using, in the sense that a fraction of tasks can be undertaken by robots, the remainder by labour. Technical progress can take the form of a reduction in the price of robots, and they derive the result that employment in robot-using industries increases if $\varepsilon - \sigma > 0$, where $\sigma$ is the elasticity of substitution between tasks (and hence between robots and labour).

[7] Autor and Dorn (2013) model technical progress as a continuous fall in the price of computers, and show how that will affect the demand for low-skilled labour to routine tasks in goods production versus manual work in the services sector. In the simplest version of their model, with only one type of labour, they reach a similar elasticity condition for whether labour will move to or from the sector in which more efficient computers can substitute for labour.

$sign\ (\hat{w}_B) = sign\ (\varepsilon - \sigma)$, as long as $\zeta \geq 0$; $\zeta$ only affects the magnitude of the change in this case. For the employment effect, on the other hand, we have $sign\ (\hat{L}_B) = sign(\zeta(\varepsilon - \sigma))$. In simple general equilibrium models $\zeta$ is always positive, but in richer models (section 3) there is a critical interaction between the nature of the technological progress and the nature of the sector in which it occurs, so the analogue of $\zeta$ can be positive or negative. In such cases, all three elasticities matter in determining the sign as well as the magnitude of the impact of technological progress.

Following sections of this paper set out and analyse such a model where both sectors are fully specified. Factor supply elasticities come from the Rybczynski elasticity of trade models, which itself depends on shares of each factor in each sector. We fully characterise the responses of employment and factor prices to each combination of factor and sector specific technical changes.

Before moving to this, we discuss two alternative approaches to modelling technical progress. Haskel and Slaughter (2002) study skill-biased technical change (SBTC) in a two-sector open economy model similar to ours. Their focus is on whether the sector-bias or factor-bias of technical change is more important for skill premia (relative wage effects). However, rather than to model technical progress as factor augmentation, they model SBTC as an exogenous change in the share of skilled labour in a CES production function in one or both sectors. While general equilibrium mechanisms are similar to the model we present in section 3, below, our focus is different, as we study relative employment as well as relative wage effects, and derive specific results focussing on three key elasticities in determining these effects.

A task-based approach is adopted by Acemoglu and Restrepo (2018a, 2018b, 2019) in which technical progress takes two main forms. One is that tasks that were performed by labour become automated, principally requiring capital. The other is that new labour-using tasks emerge. The net effect on wages depends on the balance between these forces. The base-line modelling of this involves shifting weights on automated (capital using) and non-automated (labour using) tasks in a CES production function[8]. This is similar to a model with augmentation of the two factors, where augmentation terms are weights on the factor inputs, and therefore provides an attractive interpretation of factor augmentation. Their approach involves a single sector, so does not address the cross-sectoral interactions that are the focus of this paper.

These examples illustrate various ways in which the modelling of technical progress can be enriched to accommodate automation and the effects of more efficient robots. However, our main focus will be on general-equilibrium goods and factor market interactions, and to highlight that, we work with factor-augmenting technical progress in the sections below.

## 3. General equilibrium: the three elasticities

We now develop this approach into a fully specified 2x2 general equilibrium model, familiar from Heckscher-Ohlin (HO) trade theory, and derive the effects of technical progress on employment and wages. Results depend on three elasticities, including the Rybczynski elasticity, giving the relationship between the supply of goods and the endowment of factors of production, and its dual, the Stolper-Samuelson elasticity giving the relationship between goods prices and factor prices. The setting allows technical progress to augment efficiency in different combinations of factors and

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[8] Hence, the approach could be seen as providing a micro foundation for the the modelling of SBTC in Haskel and Slaughter (2002).

sectors. We start by setting out the analytics for a general case, and then move to specific combinations in order to establish our main results.

There are two sectors, $s$ =1, 2, and two factors, $f$ = A, B, which can be thought of as different types of labour. Production has constant returns to scale and is described by unit cost functions $c_s = C_s( w_A/\alpha_{As}, w_B/\alpha_{Bs})$ and production functions $X_s = F_s(\alpha_{As}L_{As}, \alpha_{Bs}L_{Bs})$, these containing factor and sector specific efficiency levels $\alpha_{fs}$. The economy's endowment of factor $f$ is fixed at level $L_f$ and full employment of factors means that factor inputs in each sector, $L_{fs}$, satisfy $L_{f1} + L_{f2} = L_f$ , $f = A, B$. Demand for goods is derived from the homothetic utility function of a representative consumer, so relative demand is a function of relative prices, $X_1/X_2 = (p_1/p_2)^{-\varepsilon}$, with elasticity $\varepsilon$. With perfect competition and constant returns, price equals average cost, so $p_s = c_s$.

We are interested in the impact of exogenous technical changes, the $\hat{\alpha}_{fs}$'s, on labour market outcomes (wage and employment effects). Using standard 'exact hat algebra' techniques on the three equilibrium conditions – the pricing, employment, and market-clearing conditions – and the production functions, we get four sets of 'equations of change' linking the technology shocks to changes in the endogenous variables.

$$\hat{p}_s = \omega_s( \hat{w}_A - \hat{\alpha}_{As}) + (1-\omega_s)(\hat{w}_B - \hat{\alpha}_{Bs}), \qquad s = 1,2, \tag{8}$$

$$\hat{X}_s = \omega_s\big( \hat{L}_{As} + \hat{\alpha}_{As}\big) + (1-\omega_s)\big(\hat{L}_{Bs} + \hat{\alpha}_{Bs}\big), \qquad s = 1,2, \tag{9}$$

$$\upsilon_f \hat{L}_{f1} + \big(1-\upsilon_f\big)\hat{L}_{f2} = \hat{L}_f = 0, \qquad f = A, B, \tag{10}$$

$$\hat{X}_1 - \hat{X}_2 = -\varepsilon(\hat{p}_1 - \hat{p}_2)\,, \tag{11}$$

where $\omega_s \equiv w_A L_{As}/p_s X_s$ is the cost-share of factor-A in industry $s$, and $\upsilon_f \equiv L_{f1}/L_f$ is the employment-share of factor $f$ in sector-1. Sectoral differences in factor-usage intensities are critical to the third elasticity, so it is important to note that differences in the cost-shares of factor-A in sector 1 versus 2, $\omega_1 - \omega_2$, and difference in the employment-shares of factor-A and B in sector 1, $\upsilon_A - \upsilon_B$, are two sides of the same coin. They always have the same sign and are equivalent ways of representing factor-intensity differences.[9]

The final equations are the definition of the elasticity of substitution between factors, $\sigma_s$, that relates relative demand for factors (measure in effective units) to the effective relative wage

$$\sigma_s \equiv -\left[\frac{\big( \hat{L}_{As} + \hat{\alpha}_{As}\big) - \big(\hat{L}_{Bs} + \hat{\alpha}_{Bs}\big)}{( \hat{w}_A - \hat{\alpha}_{As}) - (\hat{w}_B - \hat{\alpha}_{Bs})}\right], \qquad s = 1,2. \tag{12}$$

These five sets of linear equations, (8) to (12), together with a numeraire, determine the ten changes in the endogenous variables $\hat{p}_s, \hat{X}_s, \hat{L}_{fs},$ and $\hat{w}_f$. Explicit solutions can be derived (Appendix 1), and in what follows we draw out the central results. To simplify expressions, we assume that the elasticity of substitution is the same in both sectors, $\sigma_1 = \sigma_2 = \sigma$, and we focus on relative effects.

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[9] The shares $\upsilon_f$ and $\omega_s$ are linked by the relative sizes of the sectors and the factor intensity of each. The relationship takes the form $\upsilon_A = s\omega_1/\bar{\omega}$, $\upsilon_B = s(1-\omega_1)/(1-\bar{\omega})$ where $s$ is the share of sector 1 in GDP, and $\bar{\omega} \equiv s\omega_1 + (1-s)\omega_2$ is the average cost share of factor-A in the economy.

The wage impacts can be illustrated by looking at the relative wage changes induced by the technical shock. From the sectoral price changes in (8), we get the proportional change in $w_A/w_B$ as

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} \tag{13}$$

where $\Delta\hat{w} \equiv \hat{w}_A - \hat{w}_B$ and $\Delta\hat{p} \equiv \hat{p}_1 - \hat{p}_2$, The first key feature of (13) concerns $\beta_{SS}$, which is the familiar Stolper-Samuelson elasticity from HO theory and is given by $\beta_{SS} \equiv 1/(\omega_1 - \omega_2)$. That is, $\beta_{SS}$ is the proportional changes in relative wages generated by an exogenous proportional change in relative prices. Note that $|\beta_{SS}| > 1$ since both $\omega_s$'s are fractions, but the sign depends on sectoral factor intensities. If sector 1 is A-labour intensive, then $\beta_{SS} > 1$, but if it is B-labour intensive, $\beta_{SS} < -1$.

The second key feature of (13) is the term $\Delta\hat{\chi}$ which is the cost-saving bias of the technical shock and is given by $\Delta\hat{\chi} \equiv \hat{\chi}_1 - \hat{\chi}_2$ where $\hat{\chi}_s = \omega_s\hat{\alpha}_{As} + (1-\omega_s)\hat{\alpha}_{Bs}$. Expression (8) shows that $\hat{\chi}_s$ is the direct cost-share weighted reduction of costs in sector $s$, i.e. when wages are held constant. Thus $\Delta\hat{\chi}$ is the sector bias in the cost-savings. For example, if technical progress is greater in sector 1, say $\hat{\alpha}_{A1} > \hat{\alpha}_{A2}$ and $\hat{\alpha}_{B1} > \hat{\alpha}_{B2}$, then cost-savings are biased to sector 1, $\Delta\hat{\chi} > 0$. If the progress is biased towards sector 2, then $\Delta\hat{\chi} < 0$. Taken together, the first and second features imply that the wage impact will depend upon the interplay of the sectoral factor-intensities and the technology's sectoral bias. Finally, any endogenous change in relative goods prices, $\Delta\hat{p}$, will also have a familiar Stolper-Samuelson effect on relative factor prices. These price effects are endogenous, and we come back to them below.

The employment effects depend upon the general equilibrium shift in the production mix. Working though the comparative statics (see Appendix 1), the change in relative supply is

$$\Delta\hat{X} = \beta_{RY}\Delta V + \eta\Delta\hat{p}; \qquad \Delta \mathrm{V} \equiv (1-\sigma)\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}. \tag{14}$$

In this expression $\Delta\hat{X} \equiv \hat{X}_1 - \hat{X}_2$, $\beta_{RY} \equiv 1/(\upsilon_A - \upsilon_B)$, $\eta \equiv \sigma(\beta_{RY}\beta_{SS} - 1) \geq 0$, and $\Delta\hat{\lambda} \equiv \hat{\lambda}_A - \hat{\lambda}_B$, where $\hat{\lambda}_f \equiv \upsilon_f\hat{\alpha}_{f1} + (1-\upsilon_f)\hat{\alpha}_{f2}$ .

There are three salient features of (14). The first is $\beta_{RY}$ which is the familiar Rybczynski elasticity from HO theory.[10] It gives the responsiveness of relative output to changes in the relative factor endowment in a model without technical change. As noted, the signs of $\beta_{RY}$ and $\beta_{SS}$ are always identical and $|\beta_{RY}| > 1$. We also introduce elasticity $\eta$ which is the relative supply elasticity, i.e. the responsiveness of relative production, $\Delta\hat{X}$, to relative prices, $\Delta\hat{p}$, holding technology constant (as can be seen in eq. (14) taking $\Delta\hat{V} = 0$). This elasticity is not an independent parameter, as it takes the form $\eta \equiv \sigma(\beta_{RY}\beta_{SS} - 1) \geq 0$. However, it is sometimes useful in interpreting and simplifying expressions, and we note that as the sectoral factor intensities become more similar so $\eta$ goes to infinity.[11]

[10] The two elasticities $\beta_{SS}$ and $\beta_{RY}$ differ only because they are expressed as elasticities rather than derivatives (and expressed in terms of relative changes). Stolper-Samuelson derivatives give the changes in factor prices with respect to goods prices, and are equal to Rybczynski derivatives which give changes in outputs with respect to endowments. They are cross-partial derivatives of the GNP function, and thus equal by Young's theorem. The relationship between the two elasticities is $\beta_{RY} = \beta_{SS}\bar{\omega}(1-\bar{\omega})/s(1-s)$.

[11] If factor intensities in each sector are very similar (i.e. $|\omega_1 - \omega_2|$ and $|\upsilon_A - \upsilon_B|$ are very small) then $|\beta_{RY}|, |\beta_{SS}| \to \infty$ so, providing $\sigma > 0$, the elasticity of supply goes to infinity, i.e. the production possibility frontier (ppf) tends to linearity. If factor intensities are very different (tending to factor-sector specificity) then

The second feature is $\Delta\hat{\lambda}$. Note that $\hat{\lambda}_f$ is the employment-weighted increases if effective units of labour type *f*. For A-labour, for example, $\hat{\lambda}_A$ depends on the share of A employed in sector 1, $\upsilon_A$, and sector 2, $1-\upsilon_A$, as well as the size of the augmentations, $\hat{\alpha}_{A1}$ and $\hat{\alpha}_{A2}$; specifically, $\hat{\lambda}_A = \upsilon_A\hat{\alpha}_{A1} + (1-\upsilon_A)\hat{\alpha}_{A2}$. Thus $\Delta\hat{\lambda}$ is the factor-bias of the technical change. For instance, if the progress only augments A-labour, then $\Delta\hat{\lambda} > 0$, while if it only augments B-labour, $\Delta\hat{\lambda} < 0$. Observe that $\Delta\hat{\chi}$ also enters the expression, so it is important to recall that $\Delta\hat{\lambda}$ and $\Delta\hat{\chi}$ are not independent; they are two portrayals of the same technology shock. $\Delta\hat{\lambda}$ picks up the impact of technology on the relative effective supply of factors, holding all else equal, while $\Delta\hat{\chi}$ reflects the relative cost-saving aspects of the technology holding all else equal.

The third key feature of (14) is $\Delta V$. This is a collection of parameters and technology shocks that has a direct and intuitive interpretation. It is the 'relative-factor-endowment representation' of the technical change. To see this, note that if we considered a relative endowment change with static technology, the analogue to (14) would be $\Delta\hat{X} = \beta_{RY}\Delta\hat{L} + \eta\Delta\hat{p}$, where $\Delta\hat{L} \equiv \hat{L}_A - \hat{L}_B$, In other words, as far as (14) is concerned, we can find a change in relative endowments with static technology that is equivalent to any given technology shock with static endowments; $\Delta V$ gives the equivalence.

There are three elements of $\Delta V$ when writing it out as $\Delta V = \Delta\hat{\lambda} - \sigma\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}$. The first reflects the direct impact of the technology shock on the relative supply of factors (measured in effective units). The second and third terms capture factor substitution in the sectors due to the impact of technical change on the effective relative wages, $(w_A/\alpha_{As})/(w_B/\alpha_{Bs})$. The elasticity of substitution determines the strength of these effects. [12]

The general equilibrium employment responses are given by the labour demand functions and the adding-up condition (10). The labour demand changes are, as usual, linked to the technical changes, output changes, and own-product price changes (Appendix 1). Differencing the labour demands in sector 1 and 2, and focussing on proportional changes, $\hat{L}_{fs}$, we have

$$\Delta\hat{L}_f = \Delta\hat{X} + \sigma\Delta\hat{p} - (1-\sigma)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) \quad (15)$$

where $\Delta\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2}$. Since total employment of each factor is constant, the signs of these relative changes also give us the signs of absolute changes, i.e. $sign(\Delta\hat{L}_f) = sign(\hat{L}_{f1}) = -sign(\hat{L}_{f2}),\ f = A, B$.

To close the model, we solve the relative supply equation, (14), and the relative demand equation, (11), for $\Delta\hat{p}$. This yields

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$|\beta_{RY}|, |\beta_{SS}| \to 1$ and the relative elasticity of supply, $\eta$, goes to zero.

[12] The second reflects the substitution towards the augmented factor due to the reduced effective relative price at given factor prices. For instance, an increased relative productivity of factor A in sector 1, $\hat{\alpha}_{A1}$, implies that the sector becomes more A-intensive in its production techniques, with the size of the shift intermediated by the degree of factor substitution, $\sigma$. The third term reflects the indirect impact that technical change has on the relative wage (holding goods prices constant), and thus on the relative intensity of factor usage in both sectors. For instance, if $\Delta\hat{\chi} > 0$, so the technology shock is biased towards sector 1, and sector 1 is A-labour intensive, so $\beta_{SS} > 1$, then the effective relative wage of A will tend rise and this will tend to make both sectors less A-intensive.

$$\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon + \eta}\Delta V. \tag{16}$$

Note that the sign of the relative price change depends upon the nature of the shock, $\Delta V$, and the nature of the sectors, $\beta_{Ry}$. For instance, if the technology acts like a relative increase in the supply of A labour, and sector 1 is relatively A-labour intensive, then the relative price of good 1 will fall. The magnitude of this general equilibrium effect is mitigated by the general equilibrium demand and supply elasticities $\varepsilon + \eta$.

**3.1 The one, two, and three elasticity approaches.**

The expressions (13) and (15), where $\Delta\hat{p}$ is given by (16), reveal the impact of technical change on relative wages and employment patterns. The closed-form responses of wages and employment patterns are

$$\Delta\hat{w} = \frac{-\beta_{SS}\beta_{Ry}}{\varepsilon + \eta}\Delta V + \beta_{SS}\Delta\hat{\chi}, \qquad \Delta\hat{L}_f = \frac{(\varepsilon - \sigma)\beta_{Ry}}{\varepsilon + \eta}\Delta V + (\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) \tag{17}$$

The connections are complex since there are direct effects and general equilibrium feedback effects. The direct, or 'on impact' effects, of factor augmenting progress are twofold: an increase in the supply of effective units of labour, and a change the wage per effective unit of labour. Both of these produce first-order responses in output, prices, and wages – as well as secondary, general equilibrium effects. The general equilibrium effects operate via i) factor substitution within sectors, ii) labour reallocation across sectors, iii) cost-driven price changes (and hence wage changes), and iv) product substitution in consumption.

To clarify mechanisms, we abstract momentarily from some of the secondary effects by assuming away factor substitution within sectors (by taking $\sigma = 0$), and price-related general equilibrium feedback by taking $\Delta\hat{p} = 0$.[13] The labour market effects are then simply

$$\Delta\hat{w} = \beta_{SS}\Delta\hat{\chi}, \qquad \Delta\hat{L}_f = \beta_{RY}\Delta\hat{\lambda} - \left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) \tag{18}$$

The expression for $\Delta\hat{w}$ shows that in this case, the relative wage effect depends only upon the nature of the shock and the third elasticity, $\beta_{SS}$, relating to factor reallocation across sectors. This clearly illustrates the interplay between the nature of the technology shock and the nature of the sector in which it occurs. The interplay is strongly reminiscent of Stolper-Samuelson theorem in that the relative wage will rise for the factor that is used intensively in the sector which is experiencing the greatest technology-induced cost savings. For example, if the cost savings is biased towards sector 1, so $\Delta\hat{\chi} > 0$, then the factor used intensively in sector 1 gains relative to the other factor. However, unlike the Stolper-Samuelson theorem, there is no need for one factor to lose; some forms of technical change can raise both wages.

The expression for $\Delta\hat{L}_f$, when $\sigma = \Delta\hat{p} = 0$, has two terms, which reflect two very different mechanisms. This is most easily seen if we write (18) as $\left(\hat{L}_{f1} + \hat{\alpha}_{f1}\right) - \left(\hat{L}_{f2} + \hat{\alpha}_{f2}\right) = \beta_{RY}\Delta\hat{\lambda}$. Hence, the first term in (18) reflects the impact of shifting relative output on the pattern of employment

[13] We can think of this as a ceteris paribus thought-experiment, or the small open economy case with Leontief technology.

measured in efficiency terms, since when $\sigma = 0$, $\Delta\hat{X} = \beta_{RY}\Delta\hat{\lambda}$. Given the lack of factor substitution, an expansion of the relative output of sector 1 will tend to shift workers of both types from sector 2 to sector 1. This is a "pure" Rybczynski effect. The second term captures the direct impact of factor augmentation on the difference between actual employment and employment measured in efficiency units. If, for example the progress is only in A-labour and it is faster in sector 1, i.e. $\hat{\alpha}_{A1} > \hat{\alpha}_{A2}$, then even if $\hat{L}_{A1} + \hat{\alpha}_{A1} > \hat{L}_{A2} + \hat{\alpha}_{A2}$, we may have $\hat{L}_{A1} < \hat{L}_{A2}$, i.e. A-workers moving out of sector 1 and into sector 2. As the two terms in the employment equation in (18) may have opposite signs, illustrating the relationship is best done in the context of specific shocks (Section 4).

Allowing price changes ($\Delta\hat{p} \neq 0$) but continuing to suspend factor substitution ($\sigma = 0$), changes nothing for the employment shift, $\Delta\hat{L}_f$ in (18). It does, by contrast, add a new influence on the wage change. Specifically

$$\Delta\hat{w} = \frac{-\beta_{RY}\beta_{SS}}{\varepsilon}\Delta\hat{\lambda} + \beta_{SS}\Delta\hat{\chi} \qquad (19)$$

The new term (the first term on the right-hand side) reflects the general equilibrium influence of the factor-bias aspect of the technology change on wages. For example, if the factor augmentation is biased towards A relative to B, so $\Delta\hat{\lambda} > 0$, then relative output will rise according to $\Delta\hat{X} = \beta_{RY}\Delta\hat{\lambda}$, which will depress the relative price of good 1 according to $\Delta\hat{p} = -\Delta\hat{X}/\varepsilon$. This, again, will tend to depress the relative wage according to the Stolper-Samuelson link, $\Delta\hat{w} = \beta_{SS}\Delta\hat{p}$. The first term is the combination of these mechanisms. Its sign is unambiguous since a relative increase in A labour will always result in a relative decline in A's wage, when all else is held constant. Once again, the two terms may have opposite signs, and illustrating the overall effect is best done in the context of specific shocks (Section 4).

Finally, allowing intra-sector factor substitution considerations back into the model ($\sigma \neq 0$) opens the door to two additional general equilibrium feedback mechanisms. The first concerns the impact of the effective wage on the relative factor intensity in both sectors. This works through $\Delta$V, thus affecting the 'relative factor-endowment representation' of the technical change. For instance if the effective wage of A-labour falls – either because the relative wage of A falls, $\Delta\hat{w} < 0$, or because the technological progress is biased towards A-labour, $\Delta\hat{\lambda} > 0$ – both sectors will find it optimal to employ a higher A-to-B labour ratio, giving a negative impact on $\Delta$V. In general, the sign of the impact of $\sigma$ on $\Delta$V is given by $sign\ (\beta_{SS}\Delta\hat{\chi} - \Delta\hat{\lambda})$. This is a general equilibrium effect involving characteristics of the technical change as well as of the sectors; the next section will show examples of how we can sign this for specific cases. The second effect of $\sigma$ is the standard labour-saving or labour-using effect that turns on whether $\sigma < 1$ or $\sigma > 1$. If the elasticity of substitution is above unity, then the demand for effective units of labour will expand more than the factor augmentation and, in this case, additional workers will be demanded, not just more units of effective labour.

The key takeaway is that that labour market impact of technological progress depend upon three elasticities (not one or two as in the previous literature), and that the sign of the effects depends upon interactions between the nature of the shock and the nature of the sector and factor in which the shock is focused. These interactions bear a strong resemblance to the Stolper-Samuelson and Rybczynski theorems of trade theory.

To further illustrate develop the intuition for the general equilibrium mechanisms, we turn to illustrating the relationship in the context of specific shocks.

## 4. Labour market effects of technical progress.

The expressions derived in the preceding section enable us to establish the effects of different types of technical change on prices, wages, output and employment. We look first at two combinations of change. Sector-specific augmentation, where both factors experience the same augmentation in one sector, while their efficiency in the other sector is unchanged. And factor-specific augmentation[14], where one factor experiences the same augmentation in both sectors, the other factor's efficiency held constant. We then turn to looking at the effects of a single sector and factor-specific change, $\hat{\alpha}_{fs}$. Unless otherwise stated, we will label sectors such that sector 1 is A-intensive, so $\beta_{RY}$ and $\beta_{SS} > 1$.

### 4.1 Sector-1 specific progress: $\hat{\boldsymbol{\alpha}}_{A1} = \hat{\boldsymbol{\alpha}}_{B1} = \hat{\boldsymbol{\alpha}}_{1} > \mathbf{0}$: $\hat{\boldsymbol{\alpha}}_{A2} = \hat{\boldsymbol{\alpha}}_{B2} = \mathbf{0}$.

Sector 1 specificity is given by $\hat{\alpha}_{A1} = \hat{\alpha}_{B1} = \hat{\alpha}_1 > 0$ and $\hat{\alpha}_{A2} = \hat{\alpha}_{B2} = 0$. This is sector 1 relative cost-saving, $\Delta\hat{\chi} = \hat{\alpha}_1$ , while its factor-A saving effect depends on the relative factor intensity of the sectors, $\Delta\hat{\lambda} = (\upsilon_A - \upsilon_B)\hat{\alpha}_1 = \hat{\alpha}_1/\beta_{Ry}$. Hence, in this case we have $\Delta V = (1+\eta)\hat{\alpha}_1/\beta_{RY} > 0$, and changes in endogenous variables are

$$\Delta\hat{p} = -\frac{1+\eta}{\varepsilon+\eta}\hat{\alpha}_1 \leq 0, \tag{16'}$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \beta_{SS}\left\{\frac{\varepsilon-1}{\varepsilon+\eta}\right\}\hat{\alpha}_1, \tag{13'}$$

$$\Delta\hat{L}_A = \Delta\hat{L}_B = (\varepsilon-\sigma)(-\Delta\hat{p}) + (\sigma-1)\hat{\alpha}_1 = \frac{\sigma+\eta}{\varepsilon+\eta}(\varepsilon-1)\hat{\alpha}_1. \tag{15'}$$

The sign patterns from these equations are clear. The price effect is (weakly) negative. The wage effect depends on two elasticities; if $\varepsilon > 1$, then it is positive if $\beta_{SS} > 0$, i.e. it raises the price of the factor used intensively in the sector experiencing the technical progress. While the sign of the wage effect depends on factor intensities, the sign of the employment effect does not, with employment of both factors in the affected sector increasing if $\varepsilon > 1$, i.e. there is a powerful demand effect.

For magnitudes of effects, it is apparent that a higher demand elasticity, $\varepsilon$, reduces the size of the price fall (16'), so more of the effect of the technical change is passed through to wages (13'). More labour (of both types) is reallocated to the sector with the technical change, the more elastic is the demand curve (15').

Factor market linkages operate through $\beta_{RY}$ and $\beta_{SS}$ each of which has larger absolute value the more similar are factor intensities in the two sectors. Recalling that $\eta = \sigma(\beta_{RY}\beta_{SS} - 1)$, extreme similarity implies a high price elasticity of supply. In this case, the relative price falls by the full amount of the factor augmentation (16', with $\eta \to \infty$), relative wages do not change (13') and relative employment changes by $(\varepsilon - 1)\hat{\alpha}_1$. Generally, the magnitude of the relative employment changes are larger the more similar are the sectors if $\sigma < \varepsilon$ and vice versa if $\sigma > \varepsilon$. Hence, for high $\varepsilon$, for example for a small open economy, larger employment changes occur the more similar are the sectors' factor intensities.

[14] This is related to analysis of skill-biased technical change (SBTC) in the literature.

### 4.2 Factor-A specific technological progress: $\hat{\alpha}_{A1} = \hat{\alpha}_{A2} = \hat{\alpha}_A > 0$, $\hat{\alpha}_{B1} = \hat{\alpha}_{B2} = 0$ .

Factor-A specific augmentation in both sectors takes the form $\hat{\alpha}_{A1} = \hat{\alpha}_{A2} = \hat{\alpha}_A > 0$, leaving factor-B efficiency unchanged, $\hat{\alpha}_{B1} = \hat{\alpha}_{B2} = 0$. Cost- and factor-saving measures are then $\Delta\hat{\chi} = (\omega_1 - \omega_2)\hat{\alpha}_A = \hat{\alpha}_A/\beta_{SS}$ and $\Delta\hat{\lambda} = \hat{\alpha}_A$, so $\Delta V = \hat{\alpha}_A > 0$, i.e. as if the relative endowment of factor A increases by $\hat{\alpha}_A$.[15] Price, wage, and employment changes are

$$\Delta\hat{p} = \frac{-\beta_{RY}}{\varepsilon + \eta}\hat{\alpha}_A, \tag{16''}$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \left[-\frac{\beta_{RY}\beta_{SS}}{\varepsilon + \eta} + 1\right]\hat{\alpha}_A = \left[\frac{\varepsilon - 1 + (\sigma - 1)\,\eta/\sigma}{\varepsilon + \eta}\right]\hat{\alpha}_A, \tag{13''}$$

$$\Delta\hat{L}_A = \Delta\hat{L}_B = (\varepsilon - \sigma)(-\Delta\hat{p}) = \frac{(\varepsilon - \sigma)\beta_{RY}}{\varepsilon + \eta}\hat{\alpha}_A. \tag{15''}$$

Does factor augmentation in a sector increase of decrease employment of the augmented factor in that sector? Since both sectors face the same wages and, for factor-specific technical change, the same change in factor efficiency, it must be the case that the change in relative employment across sectors is the same for both factors, $\Delta\hat{L}_A = \Delta\hat{L}_B$. Sector 1 expands and 2 contracts if $(\varepsilon - \sigma)\beta_{RY} > 0$, i.e. the direction of change depends on all three elasticities.

Technical progress in this case works as if it was an increase in the relative endowment of factor-A. There are two ways to restore general equilibrium: either by increasing output and employment in the A-intensive sector, or by increasing the A-intensity of one or both of the sectors. The first of these dominates if $\varepsilon$ is large. From the Rybczynski theorem of trade theory we know that if goods prices are given (as in a small open economy case, or more generally with very high demand elasticity $\varepsilon$), full employment is maintained by increased output of the A-intensive sector and contraction of other sectors; employment of both factors increases in the A-intensive sector[16].

However, for a large (or closed) economy such changes in output affect goods prices. There is a fall in the relative price of the A-intensive good which decreases the price of factor-A (the Stolper-Samuelson theorem), hence increasing A-intensity in both sectors. To accommodate this change in factor-intensity, the A-intensive sector has to contract, losing employment of both factors to the B-intensive sector.[17] The dividing line between cases is $\sigma = \varepsilon$, as is clear from eqn. (15''). The third elasticity tells us which sector is A-intensive, with $\beta_{Ry} > 0$, if this is sector 1.

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[15] Note that in this case the two factor substitution elements of $\Delta V$, $\beta_{SS}\Delta\hat{\chi}$ and $\Delta\hat{\lambda}$, cancel each other, leaving $\Delta V = \Delta\hat{\lambda} = \hat{\alpha}_A$.

[16] As the Rybzynski theorem applies to changes in factor endowments, while we here study factor-specific technical progress, it should be noticed that for a small, open economy, the factor prices and factor intensities will remain unchanged *in efficiency units*, so so if $\Delta\hat{p} = 0$ we have $\Delta\hat{w} - \hat{\alpha}_A = 0$, $\hat{L}_{As} + \hat{\alpha}_A - \hat{L}_{Bs} = 0$ in this case. To restore full employment of both factors, we need $\Delta\hat{X} = \beta_{RY}\hat{\alpha}_A$, and $\Delta\hat{L}_A = \Delta\hat{L}_B = \Delta\hat{X}$.

[17] It is worth noting that $\sigma$ enters the condition due to a general equilibrium mechanism through which relative factor prices adjust to changes in relative goods prices, thus inducing changes in factor intensities and a need for structural change. This differs from the $\sigma$ effect in partial equilibrium models, where it captures the direct substitution between, for example, robots and labour, if robots become cheaper or more efficient.

The change in relative wages is ambiguous. The direct cost-saving effect of augmentation raises the wage of factor-A, although the relative wage per efficiency unit, $\Delta\widehat{w} - \hat{\alpha}_A$, goes down as the price of good using factor A intensively falls. It is clear from (13'') that for high values of $\varepsilon$ the relative wage of A-labour will increase, the ultimate case being a small, open economy, for which we would have $\Delta\widehat{w} = \hat{\alpha}_A$. For a large economy with low $\varepsilon$, the price effect may dominate, giving $\Delta\widehat{w} < 0$. A sufficient condition for $\Delta\widehat{w} > 0$ is $\varepsilon > 1$ and $\sigma > 1$; while if both elasticities are less than unity, we have $\Delta\widehat{w} < 0$. In other cases, the condition for $\Delta\widehat{w} > 0$ is $\varepsilon - 1 > (1-\sigma)\,\eta/\sigma$.Recalling that $\eta/\sigma = \beta_{RY}\beta_{SS} - 1 > 0$, the condition can be written $\varepsilon > \beta_{RY}\beta_{SS} - \sigma(\beta_{RY}\beta_{SS} - 1)$ Hence, there is a trade-off between high demand elasticity and high elasticity of substitution, since both contribute to a positive impact on the use of A-labour at unchanged factor prices. The balance between these forces is sensitive to factor-intensity differences between sectors. If factor intensities are very similar, then $\beta_{RY}\beta_{SS} \to \infty$ and $\Delta\widehat{w} \to (1 - 1/\sigma)\hat{\alpha}_A$ . If they are very different (tending to factor-sector specificity), then $\beta_{RY}\beta_{SS} \to 1$ and $\Delta\widehat{w} \to (1 - 1/\varepsilon)\hat{\alpha}_A$ .

Finally, it is worth emphasising that these effects are qualitatively different from what a partial equilibrium approach would give. Since both sectors experience technical progress, a partial equilibrium model would give employment effects depending on $\varepsilon - \sigma$ in each sector, so if they have the same elasticities, they would both face similar employment effects. In the general equilibrium model, on the other hand, there is always increased employment of both factors in one of the sectors, and reduced employment of both factors in the other, which being determined by the sign of $\beta_{RY}$.

Results for this and the previous case are tabulated in Table 1. The table highlights the role of the third elasticity $(\beta_{Ry}, \beta_{SS})$; when technical progress is sector-specific, the third elasticity determines the sign of the relative wage effect, while the relative employment effects are given from the sector specificity. When technical progress is factor-specific, on the other hand, the third elasticity determines the sign of the relative employment effects, while the relative wage effect is given by the factor specificity. The table also illustrates the importance of the demand elasticity in determining the signs of the relative wage and relative employment effects. Finally, it reveals that the elasticity of substitution in production only matters when technical progress changes the relative productivity of the two factors (in one or both sectors).

**Table 1*: Relative employment and relative wage response to technical change***

| | **Sector-1 augmenting** | | **Factor-A augmenting** | | |
|---|---|---|---|---|---|
| Relative factor intensity | $\omega_1 > \omega_2$<br>$\beta_{Ry}, \beta_{SS} > 0$ | $\omega_1 < \omega_2$<br>$\beta_{Ry}, \beta_{SS} < 0$ | | $\omega_1 > \omega_2$<br>$\beta_{Ry}, \beta_{SS} > 0$ | $\omega_1 < \omega_2$<br>$\beta_{Ry}, \beta_{SS} < 0$ |
| $\varepsilon < 1$ | $\Delta\hat{L}_A = \Delta\hat{L}_B < 0$ | | $\varepsilon < \sigma$ | $\Delta\hat{L}_A = \Delta\hat{L}_B < 0$ | $\Delta\hat{L}_A = \Delta\hat{L}_B > 0$ |
| $\varepsilon < 1$ | $\Delta\widehat{w} < 0$ | $\Delta\widehat{w} > 0$ | $\varepsilon < \beta_{RY}\beta_{ss}$<br>$-\sigma(\beta_{RY}\beta_{ss} - 1)$ | $\Delta\widehat{w} < 0$ | |
| $\varepsilon > 1$ | $\Delta\hat{L}_A = \Delta\hat{L}_B > 0$ | | $\varepsilon > \sigma$ | $\Delta\hat{L}_A = \Delta\hat{L}_B > 0$ | $\Delta\hat{L}_A = \Delta\hat{L}_B < 0$ |
| $\varepsilon > 1$ | $\Delta\widehat{w} > 0$ | $\Delta\widehat{w} < 0$ | $\varepsilon > \beta_{RY}\beta_{ss}$<br>$-\sigma(\beta_{RY}\beta_{ss} - 1)$ | $\Delta\widehat{w} > 0$ | |

**4.3 Factor-sector specific augmentation.**

There is no reason to think that augmentation in a sector is the same for both factors, or that augmentation of a factor is the same in both sectors. Each of the four factor-sector efficiencies may change in different ways, in which case the interaction between the three elasticities becomes more complex. The general formulae of section 3 apply, and here we illustrate adjustment for augmentation that takes place in sector 1, which we take to be the A-intensive sector; we look first at the case of factor-A augmentation, ($\hat{\alpha}_{A1}$), and then at B-augmentation, ($\hat{\alpha}_{B1}$).

Augmentation of factor-A is $\hat{\alpha}_{A1} > 0$, with all other $\hat{\alpha}_{fs} = 0$. This gives $\Delta\hat{\chi} = \omega_1\hat{\alpha}_{A1}$, $\Delta\hat{\lambda} = \upsilon_A\hat{\alpha}_{A1}$, and $\Delta V = \{(1-\sigma)\upsilon_A + \sigma\beta_{SS}\omega_1\}\hat{\alpha}_{A1}$. It should be noted that $\Delta V > 0$ as long as sector 1 is A-intensive, since in this case $\beta_{SS}\omega_1 > 1 > \upsilon_A$. This is a good example of a case where the direct factor-saving effect, $(1-\sigma)\upsilon_A$, could be negative, while the overall factor-endowment equivalence of the technical change is still positive, due to the changes in relative factor prices. Equations (16), (13) and (15) are

$$\Delta\hat{p} = -\frac{\beta_{RY}\,\Delta V}{(\varepsilon+\eta)} = -\frac{\beta_{RY}\{(1-\sigma)\upsilon_A + \sigma\beta_{SS}\omega_1\}}{(\varepsilon+\eta)}\hat{\alpha}_{A1}$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \omega_1\hat{\alpha}_{A1}\} = \frac{\beta_{SS}\{\omega_1(\varepsilon-1) + (\sigma-1)(\beta_{RY}\upsilon_A - \omega_1)\}}{(\varepsilon+\eta)} \qquad (20)$$

$$\Delta\hat{L}_A = (\varepsilon-\sigma)(-\Delta\hat{p}) + (\sigma-1)\hat{\alpha}_{A1}, \quad \Delta\hat{L}_B = (\varepsilon-\sigma)(-\Delta\hat{p}).$$

The relative price of the A-intensive good (good 1) falls, as $\Delta V$ and $\beta_{RY}$ are both positive in this case. The wage change is ambiguous; positive for any combination of $\varepsilon > 1$ and $\sigma > 1$, negative for any combination of $\varepsilon < 1$ and $\sigma < 1$, and always more likely to be positive the more elastic demand is, since $-\Delta\hat{p}$ is falling in $\varepsilon$.

Employment effects (as well as relative wage effects) are illustrated in Figure 1a, which has parameters $\sigma$ and $\varepsilon$ on the vertical and horizontal axes respectively.[18] Sector 1 employment of factor-B increases if $\varepsilon > \sigma$, as is clear from (20), i.e. if the demand effect following from the price change is greater than the (general equilibrium) substitution effect. The sector's employment of factor-A combines direct effects with general equilibrium effects. Employment increases for sufficiently large $\varepsilon$, as expected. At lower values of $\varepsilon$ outcomes depend on $\sigma$, but in a non-monotonic way, with employment falling at intermediate values of $\sigma$. The intuition is seen from a closer inspection of the $\Delta\hat{L}_A$ equation in (20): at $\varepsilon = \sigma = 1$ we have $\Delta\hat{L}_A = 0$. A small reduction in $\sigma$ from that point, keeping $\varepsilon = 1$, yields $\Delta\hat{L}_A < 0$, while as $\sigma \to 0$ we get $\Delta\hat{L}_A > 0$. Hence, as $\sigma$ falls below 1, to restore $\Delta\hat{L}_A = 0$ first requires an increase and then a reduction in the demand elasticity. Similarly, keeping $\sigma = 1$, yields $\Delta\hat{L}_A < 0$ for $\varepsilon < 1$ and $\Delta\hat{L}_A > 0$ for $\varepsilon > 1$.

Finally, it should be observed that for any combination of $\varepsilon > 1$ and $\sigma > 1$, we have $\Delta\hat{L}_A > 0$ and $\Delta\hat{w} > 0$. Hence, biased technical progress for the factor used intensively in a sector yields increased employment of the factor in that sector and increased relative wage for the factor, as long as both

[18] Computed with the two sectors symmetric, s = ½, $\omega_1 = 1 - \omega_2$. If $\sigma \neq 1$ then $\omega_1, \omega_2$ are endogenous. The figure uses equations from section 3 for local variations around equilibrium points at which $\omega_1 = 1 - \omega_2 = 0.6$.

elasticities exceed 1. For $\varepsilon > 1$ and $\sigma < 1$ it is interesting to see that there are areas in which the sector's relative employment of the factor increases, yet the relative factor price falls.[19]

If technical change augments factor-B in sector 1, ($\hat{\alpha}_{B1} > 0$, augmentation of the factor un-intensive in the sector, all other $\hat{\alpha}_{fs} = 0$), then $\Delta V = \{-(1-\sigma)\upsilon_B + \sigma\beta_{SS}(1-\omega_1)\}\hat{\alpha}_{B1}$, which can be positive or negative, as factor-saving (for factor B) and the cost-saving (for sector 1) impacts draw in different directions. For low values of $\sigma$ the factor-saving impact dominates, and $\Delta V < 0$; for higher values of $\sigma$ the cost-saving impact dominates, and $\Delta V > 0$. For the endogenous variables we have

$$\Delta\hat{p} = -\frac{\beta_{RY}\,\Delta V}{(\varepsilon+\eta)} = \frac{\beta_{RY}\{(1-\sigma)\,\upsilon_B - \sigma\beta_{SS}(1-\omega_1)\}}{(\varepsilon+\eta)}\hat{\alpha}_{B1}$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + (1-\omega_1)\hat{\alpha}_{B1}\}, \tag{21}$$

$$\Delta\hat{L}_A = (\varepsilon-\sigma)(-\Delta\hat{p}), \qquad \Delta\hat{L}_B = (\varepsilon-\sigma)(-\Delta\hat{p}) + (\sigma-1)\hat{\alpha}_{B1}.$$

Relative employment and wage changes are illustrated in Figure 1b.

In this case, the price may go either way, reflecting the sign of $(-\Delta V)$. Hence for very low values of $\sigma$ (less than 0.167 in the example of Figure 1b), the relative price of good 1 increases, and the production of good 1 falls. For higher values of $\sigma$ the price falls and, as usual, high $\varepsilon$ tends to raise employment of both factors in the sector. For factor-A (the now non-augmenting factor), the sign is determined from $\varepsilon - \sigma$ alone, while for factor-B the negative factor-saving effect implies that factor-B may be reallocated from sector 1 to sector 2 even for higher values of $\varepsilon$ when $\sigma < 1$.

When demand is inelastic ($\varepsilon$ < 1), it should be observed that there is a range of values of $\sigma$ for which there is reduced employment of both factors in the sector that experiences technical progress. This is true whether the factor augmentation is for the intensive or the non-intensive factor.

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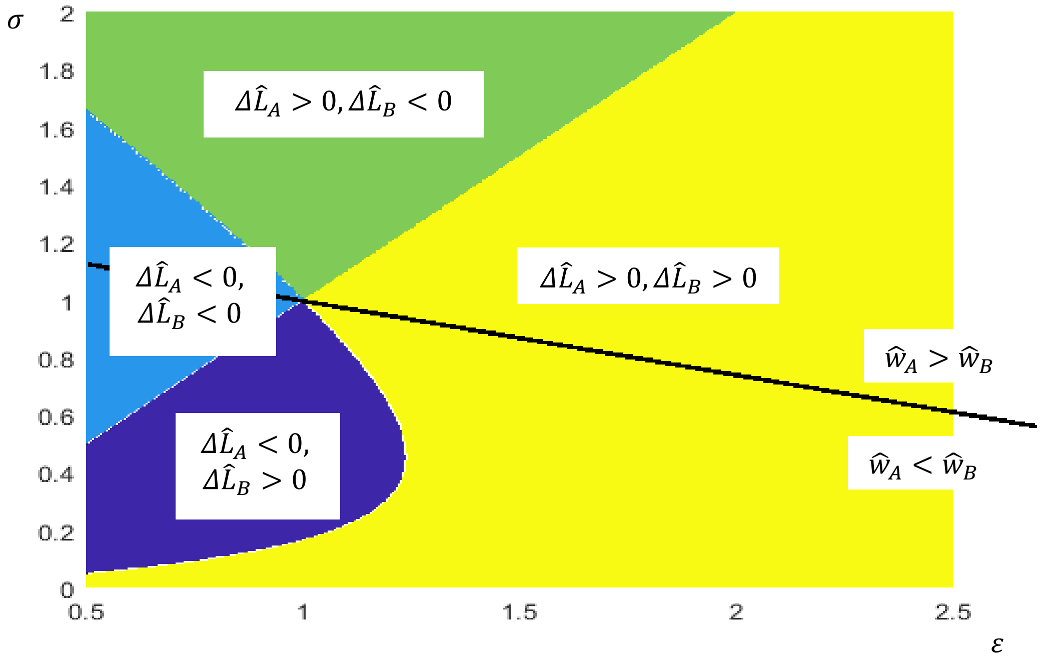
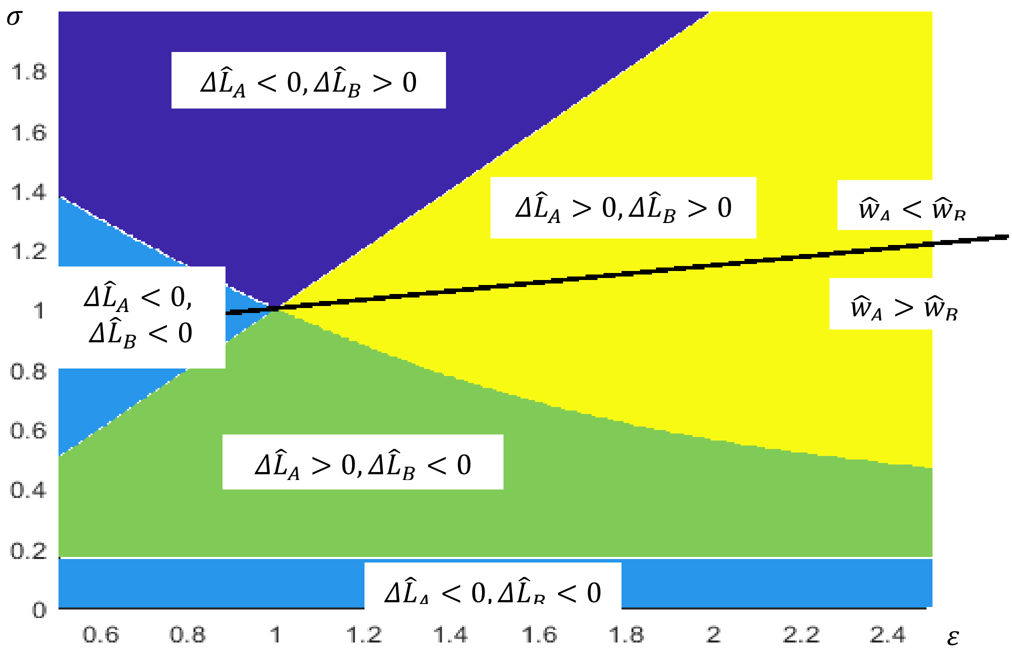
[19] Similarly, for $\varepsilon < 1$ and $\sigma > 1$ there are areas with $\Delta\hat{L}_A < 0$ and $\Delta\hat{w} > 0$. This illustrates clearly the fact that there is not a one-to-one correspondence between relative wage effects and relative employment effects in the sector experiencing technical progress.

**Figure 1:** ***Factor- and sector-specific technical progress.***

**Panel a**: Augmentation of factor-A in the A-intensive sector: $\hat{\alpha}_{A1} > 0$




**Panel b:** Augmentation of factor-B used in A-intensive sector: $\hat{\alpha}_{B1} > 0$



## 5. Open economy issues

We have so far worked with a closed economy, either a single country, or a group of countries closed to 'outside', although in some of the cases, small, open economy examples have been mentioned. Trade openness raises a number of issues.

### 5.1 Determination of prices

Trade openness changes the demand elasticities faced by products, since this now depends on world markets rather than domestic ones. The effect is typically to raise the price elasticity of demand faced by a single country (or group of countries), and thereby lead to smaller price changes, larger changes in the scale of production (and consequent movement of factors between sectors) and larger (more positive) wage effects to affected factors[20].

The key mechanism is the way in which openness effects demand elasticities. For a sector open to trade the demand elasticity depends on the elasticity of final demand in each country, and also on the share of world supply that this country produces. To clarify, let country $i$ have share $S_i$ of world supply and share $D_i$ of world demand, and let all countries have the same preferences (demand elasticity $\varepsilon$) and supply elasticities $\eta$. The countries experiencing the technical change are, collectively, country 1, and world product market clearing is

$$X_1 + \sum\nolimits_{i>1} S_i\, p^{\eta} = \sum\nolimits_{i} D_i\, p^{-\varepsilon}.$$

Differentiating, with shares summing to unity, $\sum_i D_i = 1$ and $S_1 + \sum_{i>1} S_i = 1$, the demand elasticity faced by country 1 is $\hat{X}_1/\hat{p} = -\{(1-S_1)\eta + \varepsilon\}/S_1$. Thus, if the price falls, final demand increases (in all countries), and supply falls in countries that produce the good, but have not experienced the technical change. Clearly, if $S_1 = 1$ then the elasticity is simply $\varepsilon$, and as $S_1$ goes to zero so the small open economy case of infinite demand elasticity is approached. For open economies, the analysis of preceding sections applies with a relatively high demand elasticity throughout. Thus, and unsurprisingly, employment is more likely to expand if countries experiencing the technical change have large export opportunities to the rest of the world (see Krugman 2000 for this argument). If, on the other hand, technical progress takes place in parallel in all countries, the relevant elasticity is $\varepsilon$ and the closed-economy framework applies.

### 5.2 Tradable and non-tradable sectors

What is the impact of having a non-tradable sector, and can it in any sense cushion the impact of technical change?

We note first that the two-sector model we have outlined can be interpreted as having one tradable and one non-tradable sector. Suppose that one of the goods produced in the country under consideration (home) is non-traded (sector 1), while sector 2 produces tradables. These can be thought of as a bundle of different goods, but all with fixed relative prices (i.e. produced with identical technology) and the same price elasticities of demand. They can be traded with the rest of the world for good 3, itself a composite of goods with fixed relative prices in the composite. If the price ratio $p_2/p_3$ is

[20] With low elasticity of demand, the fall in the relative goods price tend to dampen the factor price changes of the factor experiencing technical progress. With higher elasticity, the counteracting goods price effect is lower, and thus the factor price effect more positive.

constant – unchanged by technical change in home – then analysis is exactly as in preceding sections. The relative sizes of sectors 1 and 2 in the home economy are, from the demand side, determined by relative price $p_1/p_3(= p_1/p_3)$.[21]

A richer model adds a third production sector to our two-sector model, so goods 1 and 2 are tradable, and there is a new sector, $s$ = 0 producing non-tradable output. What then is the impact of having a non-tradable sector, and can it in any sense cushion the impact of technical change? Analysis is simplified by two facts. First, the size of the non-traded sector is determined by domestic demand, and the supply of factors available to tradable sectors is the economy's endowment minus factor use in the non-tradable sector (see Appendix 2). Second, if there are two factors of production and two traded goods, then factor prices are fixed by technology and the world prices of the two traded goods, as long as there is diversified production. If world prices are fixed, the only way that the presence of the non-traded sector changes outcomes is by changing the amount of the endowment available to traded goods production.

A full analysis of a (simple) model with a non-traded sector is given in Appendix 2. Some key insights can be illustrated without going into the details, by focussing on how adjustments in the non-traded sector affect factor supply to the traded goods sectors. Hence, we need to look at how demand for non-tradeable goods and services change as a consequence of technological progress in the traded-goods sectors, and what the implications of this are for factor use in non-tradeables. The demand effect depends on the change in income and the relative price of non-tradables following technical change; the factor market consequences depend on the factor intensity in non-traded versus traded sectors.

If we label production (and demand) of non-tradable $X_0$, its price $p_0$, national income $Y$, and income and demand elasticity for non-tradables $\mu_0$ and $\varepsilon_0$, respectively, we can in general write

$$\hat{X}_0 = \mu_0\hat{Y} - \varepsilon_0\hat{p}_0 \tag{22}$$

Income and price changes are related to factor prices in the following way

$$\hat{Y} = \overline{\omega}\widehat{w}_A + (1-\overline{\omega})\widehat{w}_B; \quad \hat{p}_0 = \omega_0\widehat{w}_A + (1-\omega_0)\widehat{w}_B$$

If we to simplify assume $\mu_0 = \varepsilon_0 = 1$[22], we can thus write

$$\hat{X}_0 = (\overline{\omega} - \omega_0)(\widehat{w}_A - \widehat{w}_B) \tag{22'}$$

If we, to simplify even further, focus on a small, open economy, then we know from (18) that $\Delta\widehat{w} = (\widehat{w}_A - \widehat{w}_B) = \beta_{SS}\Delta\hat{\chi}$. Hence, in this special case we have $\hat{X}_0 = (\overline{\omega} - \omega_0)\beta_{SS}\Delta\hat{\chi}$.

We can use this simple setup to illustrate how a non-traded sector may impact the production and employment effects of technical progress. To be specific, let us look at a case where $\Delta\hat{\chi} > 0$, i.e.

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[21] Price ratio $p_2/p_3$ will change – a terms of trade effect – if foreign demand for good 2 is less than perfectly elastic. Technically this price reduction is, in its supply side impact, like sector-2 specific technical regress. In the simplest case (where home consumers only consume goods 1 and 3), this could be modelled exactly as the combination of positive factor augmentation in home (the technical change) combined with negative or positive sector-2 specific augmentation (the sign depending on whether the initial technical change expanded or contracted home production in sector 2).

[22] This could, e.g., come from a two-level CES utility function with elasticity =1 at the top level between traded and non-traded goods, and $\varepsilon$ between the two traded goods.

technical progress favouring the A-intensive sector. This could be factor-specific progress for factor A or sector-specific progress in the A-intensive sector, as analysed in sector 4. A few observations follow directly:

If the factor intensity of the non-traded sector coincides with that of the overall economy ($\omega_0 = \overline{\omega}$), there are no added effects through the non-traded sector, as demand and production of non-traded goods remain unchanged.[23] If the factor intensities deviate, the production of non-traded goods adjusts and so does the factor supply to traded goods sectors. We will look briefly at the two possible cases.

First, assume $\omega_0 < \overline{\omega}$, so non-traded production is less A-intensive than the economy as a whole, and hence also less A-intensive than the two traded sectors combined. In this case, we have from (22') $\hat{X}_0 > 0$, and non-traded production attracts more of both factors. How will this affect production and employment in the two traded goods sectors? There are two effects: supply of both factors to the traded sectors decline, and the relative supply of factor A goes up, since non-traded production uses relatively more factor B. Using standard Rybczynski logics, this implies that the relative production of good 1 increases, and so does the relative employment of both factors in sector 1, i.e. $\Delta\hat{X} = \hat{X}_1 - \hat{X}_2 > 0$ and $\Delta\hat{L}_f = \hat{L}_{f1} - \hat{L}_{f2} > 0, f = A, B$. However, since the total factor supply to sector 1 and 2 goes down in this case, there is no direct correspondence between these relative changes and the changes in each traded sector. From the relative changes, we can conclude that $\hat{X}_2 < 0$ and $\hat{L}_{f2} < 0$, but we cannot say if production in sector 1 goes up or down.

If, on the other hand, $\omega_0 > \overline{\omega}$, then we have $\hat{X}_0 < 0$ and both factors are released from the non-traded to the traded sectors. Since the non-traded sector now is more A-intensive, the relative supply of factor A to traded sectors will again increase. Hence, as in the previous case, we have $\Delta\hat{X} = \hat{X}_1 - \hat{X}_2 > 0$ and $\Delta\hat{L}_f = \hat{L}_{f1} - \hat{L}_{f2} > 0$. However, since total factor supply to these sectors now increases, we can in this case conclude that production and employment in sector 1 will go up, while sector 2 may go up or down.

Even if this is a very simple example, it illustrates more general insights from adding a non-traded sector (see Appendix 2 for more details). First, relative factor intensities are decisive for both questions posed above – what happens to non-traded production and how does this impact relative production of traded goods. Second, the changes due to the non-traded sector tend to reinforce the initial changes in relative production and employment between the traded sectors.[24] And finally, there is no longer a direct correspondence between changes in relative and absolute employment in the traded sectors, as the total factor supply to these sectors will change.

These results extend beyond the restrictive assumptions made in this simple example. If we look at a large economy rather than a small, open one, we know that the relative factor prices are given by $\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\}$, so even if $\Delta\hat{\chi} > 0$, we may have $\Delta\hat{w} < 0$, as shown in section 3 and 4. However, qualitatively the impact through the non-traded sector is still given from (22') and the reasoning regarding relative factor intensities above still applies. If the factor intensity of the non-

[23] The income and price effects cancel each other in this case. If we look at a more general demand function, we can use (22) to see how this impact will be be modified. However, there will still be a condition based on relative factor intensities; albeit a more complicated one.

[24] In Appendix 2 it is shown that the adjustments through the non-traded sector work as if the relative supply elasticity, $\eta$, increases, i.e. the non-traded sector facilitates the relative changes between traded sectors.

traded sector is the same as for the whole economy, there is no added effect. If it deviates, the non-traded sector adjustments will tend to reinforce the initial relative production and employment effects in the traded sectors. Hence, the relative price effects tend to be smaller than in a model without the non-traded sector; in this sense, we could say that the non-traded sector tends to cushion the impact of technological change on relative prices. However, the actual modelling of the general equilibrium can be very complex in this case, as the non-traded adjustments affect relative supply of traded goods and hence relative goods and factor prices.

As a second example, if income or price elasticity of non-traded goods differ from unity, we must use (22) rather than (22') in the analysis; yet the logic of looking at changes in factor demand from non-traded production and how that impacts the production of traded goods is still valid. In general, if income goes up due to technical change, and demand for non-tradables is either more income elastic or less price elastic than in the example above, then there will be a tendency for increased demand and production of non-tradables, and hence reduced supply of both factors to the traded goods production. The relative changes will be as analysed above.

## 6. Concluding remarks

In this paper we have focussed on general equilibrium labour market effects of factor augmenting technical progress. While the literature has illustrated the importance of substitution versus demand effects, we emphasise that a third dimension – factor reallocation between sectors – must be taken into account to understand labour market implications of technical change. We show that the wage and sectoral employment effects of technical change depend on interactions between the nature of the technical change and the nature of the sector in which it occurs. These three dimensions – the factor substitution, the demand substitution, and the intersectoral reallocation of factors – are captured by three elasticities: the elasticity of substitution between factors in the production in each sector, the elasticity of substitution between the goods in demand (called the demand elasticity), and the Rybczynski elasticity of structural change between the sectors. Moreover, we show that when we allow for the full set of general equilibrium feedback effects, the close connection between wage effects and employment effects that is evident in partial equilibrium, or restricted general equilibrium analyses does not carry over in a richer setting.

In summary, we will highlight three main contributions of the paper relative to existing literature: i) general equilibrium matters; ii) the nature of the technological change as well as the nature of the sector(s) in which it occurs matter; and iii) there is not a one-to-one correspondence between relative wage effects and relative employment effects. We will discuss each of these briefly in the remainder of this section.

General equilibrium matters in many ways. Technically, it implies including adding-up constraints and accompanying general equilibrium goods- and factor price effects in the analysis. This gives rise to several insights, not normally emphasised in the literature on technical change and labour markets. One is that employment conditions in one sector depends not only on its own technology but also on technological change in other sectors. A second insight is that sectors with similar technological change may experience fundamentally different employment effects, depending on the characteristics of the sectors. And a third is that in our setting, the "standard" 2-elasticity condition $(\varepsilon - \sigma)$ has a new, general equilibrium explanation. $\varepsilon$ captures the demand effect of a cost-advantage, much like in a partial equilibrium setting. However, $\sigma$ occurs for a different reason. In a partial equilibrium model, the experiment is typically to look at the labour market effects of, say, cheaper or more efficient robots. Then the elasticity of substitution captures the substitution of robot for labour, and $\varepsilon - \sigma$ is the trade-off between these two effects. In the general equilibrium setting, the elasticity of substitution captures the structural changes needed to accommodate changes in relative factor prices and thus in factor intensities in both sectors.

A second contribution concerns how the interaction between the nature of technical change and the nature of the sector(s) in which it occurs determine the labour market outcomes. If technical change is sector-specific (or more generally, biased towards a sector) then the nature of that sector determines the relative factor price effects. If it is unskilled-labour intensive and the elasticity of demand is sufficiently high, the relative wage of unskilled workers will increase. If, on the other hand, the sector is skill-labour intensive, unskilled labour will experience a decline in its relative wage. Low $\varepsilon$ gives the opposite effects, but still depending on relative factor intensities. In any case, however, employment of both factors will increase in the sector experiencing technical progress for high $\varepsilon$, and decline if the elasticity is low.

If the technical change is factor-specific (or more generally, biased towards a factor), then nature of the sectors determines the employment effects. If unskilled labour becomes more efficient, then employment of both factors in the unskilled-labour intensive sector will increase if $\varepsilon > \sigma$, and decrease if this condition is not satisfied. Employment of both factors in the other sector will move in the opposite direction. So even if both sectors experience technological progress for unskilled labour, employment in one of the sectors will always go down. With factor-specific technological progress, the relative wage effect is determined from the factor specificity, and independent of the factor intensities of the two sectors. However, it depends on the elasticities of demand and substitution, both of which has a positive impact on the relative wage of the augmented factor.

If we go back to our robot example from section 2, and now define factor-A as robots and factor-B as labour, the full analysis shows that as robots become more efficient, the demand for labour will go up in the robot-intensive sector and down in other sectors if $\varepsilon > \sigma$, while the opposite happens if $\varepsilon < \sigma$. So even if both sectors benefits directly from better (or cheaper) robots, and both sectors face the same elasticities $\varepsilon$ and $\sigma$, the employment effects will be different – one sector experiencing increased employment and the other a decline. Or, in the notation of section 2, the labour supply elasticity, $\zeta$, may be positive or negative, depending on the characteristics of the sectors.

Finally, since most previous studies have focussed on relative wages and not included results on structural changes and relative employment effects, it should be emphasised that relative wage and relative employment effects are not necessarily two sides of the same coin. Our results show clearly that technical progress for a factor in a sector, for instance unskilled labour augmentation in the unskilled-labour intensive sector, may well give a fall in the relative wage for unskilled workers, even if the demand for (both types of) labour increases in that sector. And vice versa, we may see an increase in the relative price of the factor, even if the demand for that factor in the sector declines. These examples highlight that it is necessary to study not only relative factor prices effects, but also the employment effects. Again, this illustrates that general equilibrium matters.

In addition to going through all relevant cases in some detail, the paper also sketches how the existence of a non-traded sector may dampen the price and reinforce the relative employment effects of technical change in the traded goods sectors.

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## Appendix 1. Complete model

In this appendix the complete 2-sector model is developed, assuming CES cost (and production) functions and exogenously given factor endowments.

Cost functions:

$$C_s(\, w_A, w_B) = \left[\omega_s\left(\frac{w_A}{\alpha_{As}}\right)^{1-\sigma} + (1-\omega_s)\left(\frac{w_B}{\alpha_{Bs}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = p_s, \quad s = 1{,}2$$

This yields:

$$\omega_s\widehat{w}_A + (1-\omega_s)\widehat{w}_B = \hat{p}_s + \omega_s\hat{\alpha}_{As} + (1-\omega_s)\hat{\alpha}_{Bs} = \hat{p}_s + \hat{\chi}_s, \;\; s = 1{,}2$$

Where $\hat{\chi}_s = \omega_s\hat{\alpha}_{As} + (1-\omega_s)\hat{\alpha}_{Bs}$ is the cost-saving impact of any combination of factor augmenting technical progress in sector *s*. We have

$$\widehat{w}_A - \widehat{w}_B = \frac{1}{\omega_1 - \omega_2}\{(\hat{p}_1 + \hat{\chi}_1) - (\hat{p}_2 + \hat{\chi}_2)\}$$

Using $\Delta\widehat{w} \equiv \widehat{w}_{\mathrm{A}} - \widehat{w}_{\mathrm{B}}, \Delta\hat{p} \equiv \hat{p}_1 - \hat{p}_2$, $\Delta\hat{\chi} \equiv \hat{\chi}_1 - \hat{\chi}_2$, and $\beta_{SS} \equiv 1/(\omega_1 - \omega_2)$ (where $\beta_{SS}$ captures the Stolper-Samuelson effect as an elasticity) this is

$$\Delta\widehat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} \qquad (A1.1)$$

From the cost function we get the labour demand

$$L_{\mathrm{fs}} = X_s\frac{\partial C_s}{\partial w_f} = \omega_i\frac{X_s}{\alpha_{fs}}\left(\frac{p_{\mathrm{s}}}{w_f/\alpha_{fs}}\right)^{\sigma} \quad s = 1{,}2, \qquad f = A, B$$

Differentiation these equations and using $L_{f1} + L_{f2} = L_f$ (where $L_f$ is the exogenously given endowment of factor *f*), $\upsilon_f \equiv \frac{L_{f1}}{L_f}$ (where $\upsilon_f$ is the share of factor *f* used in sector 1), and $\hat{\lambda}_f \equiv \upsilon_f\hat{\alpha}_{f1} + (1-\upsilon_f)\hat{\alpha}_{f2}$, as a summary measure of the factor-augmenting impact for factor *f*, we get

$$\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} = \frac{1}{\upsilon_A - \upsilon_B}\begin{bmatrix}(1-\upsilon_B) & -(1-\upsilon_A)\\ -\upsilon_B & \upsilon_A\end{bmatrix}\begin{bmatrix}(1-\sigma)\,\hat{\lambda}_A - \sigma[\upsilon_A\hat{p}_1 + (1-\upsilon_A)\hat{p}_2 - \widehat{w}_A]\\ (1-\sigma)\,\hat{\lambda}_B - \sigma[\upsilon_B\hat{p}_1 + (1-\upsilon_B)\hat{p}_2 - \widehat{w}_B]\end{bmatrix}$$

Using $\beta_{Ry} \equiv \frac{1}{\upsilon_A - \upsilon_B}$ (where $\beta_{Ry}$ captures the Rybczynski effect as an elasticity) we then have

$$\Delta\hat{X} \equiv \hat{X}_1 - \hat{X}_2 = \beta_{Ry}\{(1-\sigma)\left(\hat{\lambda}_A - \hat{\lambda}_B\right) + \sigma(\widehat{w}_A - \widehat{w}_B)\} - \sigma(\hat{p}_1 - \hat{p}_2)$$

Which, using (A1.1) and the change notation, can be written

$$\Delta\hat{X} = \beta_{Ry}\{(1-\sigma)\,\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}\} + \sigma\left(\beta_{Ry}\beta_{SS} - 1\right)\Delta\hat{p}$$

where $\Delta\hat{\lambda} \equiv \hat{\lambda}_A - \hat{\lambda}_B$. Define $\eta \equiv \sigma\left(\beta_{Ry}\beta_{SS} - 1\right)$ as the supply elasticity of relative production with regard to relative price, i.e. rotation along the production possibility frontier. Note that $\beta_{Ry}$ and $\beta_{SS}$ always have the same sign, and their magnitudes exceed 1. The sign depends on relative factor intensities; if sector 1 is relatively A-labour intensive, we have $\omega_1 > \omega_2$, $\upsilon_A > \upsilon_B$ and thus $\beta_{SS} > 1$ and $\beta_{Ry} > 1$.

Further, define $\Delta \text{V} \equiv (1-\sigma)\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}$; this is the "relative factor-endowment representation" of the technical change. See section 3 in the main text for a discussion of this. We can then write

$$\Delta\hat{X} = \beta_{Ry}\Delta \text{V} + \eta\Delta\hat{p} \qquad (A1.2)$$

Together with the CES demand, $\Delta\hat{X} = -\varepsilon\Delta\hat{p}$ , we have

$$\beta_{Ry}\Delta \text{V} + \eta\Delta\hat{p} = -\varepsilon\Delta\hat{p}$$

$$\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon+\eta}\Delta \text{V} \qquad (A1.3)$$

Finally, employment effects follow from the labour demand functions, and can be expressed as:

$$\Delta\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2} = \Delta\hat{X} + \sigma\Delta\hat{p} - (1-\sigma)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) = (\varepsilon-\sigma)(-\Delta\hat{p}) - (1-\sigma)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right) \, (A1.4)$$

Equations (A1.1)-(A1.4) give the comparative static effects of any combinations of technical shocks, $\hat{\alpha}_{fs}$, using

$$\Delta\hat{\lambda} = \hat{\lambda}_A - \hat{\lambda}_B = \upsilon_A\hat{\alpha}_{A1} + (1-\upsilon_A)\hat{\alpha}_{A2} - [\upsilon_B\hat{\alpha}_{B1} + (1-\upsilon_B)\hat{\alpha}_{B2}] \qquad (A1.5)$$

$$\Delta\hat{\chi} = \hat{\chi}_1 - \hat{\chi}_2 = \omega_1\hat{\alpha}_{A1} + (1-\omega_1)\hat{\alpha}_{B1} - [\omega_2\hat{\alpha}_{A2} + (1-\omega_2)\hat{\alpha}_{B2}]$$

Below we will go through various possible cases, using this system of equilibrium equations.

### A1.1 Sector-specific progress $\hat{\boldsymbol{\alpha}}_{A1} = \hat{\boldsymbol{\alpha}}_{B1} = \hat{\boldsymbol{\alpha}}_1$ and $\hat{\boldsymbol{\alpha}}_{A2} = \hat{\boldsymbol{\alpha}}_{B2} = \hat{\boldsymbol{\alpha}}_2$

This yields $\Delta\hat{\lambda} = \hat{\lambda}_A - \hat{\lambda}_B = (\upsilon_A - \upsilon_B)(\hat{\alpha}_1 - \hat{\alpha}_2) = \frac{(\hat{\alpha}_1 - \hat{\alpha}_2)}{\beta_{Ry}}$ and $\Delta\hat{\chi} = \hat{\chi}_1 - \hat{\chi}_2 = (\hat{\alpha}_1 - \hat{\alpha}_2)$.
Furthermore, $\Delta \text{V} = (1+\eta)\frac{(\hat{\alpha}_1 - \hat{\alpha}_2)}{\beta_{Ry}}$.

Note that $\hat{\alpha}_1 - \hat{\alpha}_2$ captures possible sector-specific shocks in both industries. Normally, we will focus on only one, so $\hat{\alpha}_1 > 0$ and $\hat{\alpha}_2 = 0$. In the general case, we get

$$\Delta\hat{p} = \frac{-1}{\varepsilon+\eta}\left\{(1-\sigma) + \sigma\beta_{Ry}\beta_{SS}\right\}(\hat{\alpha}_1 - \hat{\alpha}_2)$$

Remember that $\beta_{Ry} > 1$ and $\beta_{SS} > 1$ (assuming sector 1 to be A-intensive) so the curly bracket is always positive, independent of $\sigma$. Using $\sigma\beta_{Ry}\beta_{SS} = \eta + \sigma$, this can also be written:

$$\Delta\hat{p} = \frac{-(1+\eta)}{\varepsilon+\eta}(\hat{\alpha}_1 - \hat{\alpha}_2) \qquad (A1.3')$$

So any sector-specific progress that favours sector 1, $(\hat{\alpha}_1 - \hat{\alpha}_2) > 0$, yields increased relative production and reduced relative price of good 1.

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \beta_{SS}\frac{\varepsilon-1}{\varepsilon+\eta}(\hat{\alpha}_1 - \hat{\alpha}_2) \qquad (A1.1')$$

$$\Delta\hat{L}_f = (\varepsilon-\sigma)(-\Delta\hat{p}) - (1-\sigma)(\hat{\alpha}_1 - \hat{\alpha}_2) = \frac{(\sigma+\eta)}{\varepsilon+\eta}(\varepsilon-1)(\hat{\alpha}_1 - \hat{\alpha}_2) \qquad (A1.4')$$

So for $\hat{\alpha}_1 - \hat{\alpha}_2 > 0$ we have $\Delta\hat{p} < 0$ always, $\Delta\hat{L}_f > 0$ for $\varepsilon > 1$, and $\Delta\hat{w} > 0$ for $\varepsilon > 1$ and $\omega_1 > \omega_2$ i.e. sector 1 being A-labour intensive, as we will assume throughout, unless otherwise specified. However, it is important to notice that in this case it only matters for the wage effect.

**A1.2 Factor-specific progress: $\hat{\boldsymbol{\alpha}}_{A1} = \hat{\boldsymbol{\alpha}}_{A2} = \hat{\boldsymbol{\alpha}}_A$ and $\hat{\boldsymbol{\alpha}}_{B1} = \hat{\boldsymbol{\alpha}}_{B2} = \hat{\boldsymbol{\alpha}}_B$**

This yields $\Delta\hat{\lambda} = \hat{\lambda}_A - \hat{\lambda}_B = \hat{\alpha}_A - \hat{\alpha}_B$, $\Delta\hat{\chi} = \hat{\chi}_1 - \hat{\chi}_2 = \frac{(\hat{\alpha}_A - \hat{\alpha}_B)}{\beta_{SS}}$, and $\Delta V = \hat{\alpha}_A - \hat{\alpha}_B$

Note that $\hat{\alpha}_A - \hat{\alpha}_B$ captures possible factor-specific shocks for both factors. Normally, we will focus on only one, so $\hat{\alpha}_A > 0$ and $\hat{\alpha}_B = 0$. In the general case, we get

$$\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon + \eta} (\hat{\alpha}_A - \hat{\alpha}_B) \qquad (A1.3'')$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \left\{\frac{-\beta_{Ry}\beta_{SS}}{\varepsilon + \eta} + 1\right\}(\hat{\alpha}_A - \hat{\alpha}_B) \qquad (A1.1'')$$

$$\Delta\hat{L}_f = (\varepsilon - \sigma)(-\Delta\hat{p}) \qquad (A14'')$$

The relative price effect of $\hat{\alpha}_A - \hat{\alpha}_B > 0$ is negative as long as sector 1 is A-labour intensive (with the limiting case of a small, open economy giving $\Delta\hat{p} = 0$) . The sign of the relative wage effect depends on the sign of $\varepsilon + \eta - \beta_{Ry}\beta_{SS} = \varepsilon - \sigma + (\sigma - 1)\beta_{Ry}\beta_{SS}$. The condition can also be written $(\varepsilon - 1) + (\sigma - 1)\,\eta/\sigma$, which shows that $\Delta\hat{w} > 0$ for any combinations of $\varepsilon > 1$ and $\sigma > 1$, and $\Delta\hat{w} < 0$ for any combinations of $\varepsilon < 1$ and $\sigma < 1$. It should also be noted that for wages in efficiency terms, we have

$$\Delta\hat{w} - (\hat{\alpha}_A - \hat{\alpha}_B) = \frac{-\beta_{Ry}\beta_{SS}}{\varepsilon + \eta} (\hat{\alpha}_A - \hat{\alpha}_B) = \frac{\Delta\hat{p}}{\beta_{SS}} < 0$$

Which is always negative, as long as $\Delta\hat{p} \neq 0$. So $\hat{\alpha}_A - \hat{\alpha}_B > 0$ makes A-labour relatively cheaper in efficiency terms, thus inducing substitution towards using more labour in both industries.

Hence, the two-elasticity rule seemingly applies in this case; however, note that it is multiplied by the relative price effect, which depends on $\beta_{Ry}$.[25] Hence, $\hat{\alpha}_A - \hat{\alpha}_B > 0$ and $\varepsilon > \sigma$ yields increase employment of both factors in sector 1 if sector 1 is A-labour intensive. It gives reduced employment of both factors in sector 1 if it is B-labour intensive, even if technology improves in the sector.

**A1.3 Factor- and sector-specific technical progress – complete set of cases**

From the system of equations (A1.1), (A1.4) and (A1.4), using (A1.5), all cases of individual factor-augmenting changes are easily worked out.

**Pure A-progress in sector 1:** $\hat{\alpha}_{A1} > 0$, and $\hat{\alpha}_{fs} = 0$ for all other $\hat{\alpha}_{fs}$.

Price effects: $\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon+\eta}\{(1-\sigma)\,\upsilon_A + \sigma\beta_{SS}\omega_1\}\hat{\alpha}_{A1}$ and $\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \omega_1\hat{\alpha}_{A1}\}$

[25] Furthermore, $\sigma$ now captures general equilibrium changes, as discussed in section 3 of the main text.

Employment effects: $\Delta\hat{L}_A = (\varepsilon - \sigma)(-\Delta\hat{p}) - (1-\sigma)\hat{\alpha}_{A1}$ and $\Delta\hat{L}_B = (\varepsilon - \sigma)(-\Delta\hat{p})$

**Pure B-progress in sector 1:** $\hat{\alpha}_{B1} > 0,$ and $\hat{\alpha}_{fs} = 0$ for all other $\hat{\alpha}_{fs}$.

Price effects: $\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon+\eta}\{-(1-\sigma)\,\upsilon_B + \sigma\beta_{SS}(1-\omega_1)\}\hat{\alpha}_{B1}$ and $\Delta\widehat{w} = \beta_{SS}\{\Delta\hat{p} + (1-\omega_1)\hat{\alpha}_{B1}\}$

Employment effects: $\Delta\hat{L}_A = (\varepsilon - \sigma)(-\Delta\hat{p})$ and $\Delta\hat{L}_B = (\varepsilon - \sigma)(-\Delta\hat{p}) - (1-\sigma)\hat{\alpha}_{B1}$

**Pure A-progress in sector 2:** $\hat{\alpha}_{A2} > 0,$ and $\hat{\alpha}_{fs} = 0$ for all other $\hat{\alpha}_{fs}$.

Price effects: $\Delta\hat{p} = \frac{-\beta_{Ry}}{\varepsilon+\eta}\{(1-\sigma)\,(1-\upsilon_A) - \sigma\beta_{SS}\omega_2\}\hat{\alpha}_{A2}$ and $\Delta\widehat{w} = \beta_{SS}\{\Delta\hat{p} - \omega_2\hat{\alpha}_{A2}\}$

Employment effects: $\Delta\hat{L}_A = (\varepsilon - \sigma)(-\Delta\hat{p}) + (1-\sigma)\hat{\alpha}_{A2}$ and $\Delta\hat{L}_B = (\varepsilon - \sigma)(-\Delta\hat{p})$

**Pure B-progress in sector 2:** $\hat{\alpha}_{B2} > 0,$ and $\hat{\alpha}_{fs} = 0$ for all other $\hat{\alpha}_{fs}$.

Price effects: $\Delta\hat{p} = \frac{\beta_{Ry}}{\varepsilon+\eta}\{(1-\sigma)\,(1-\upsilon_B) + \sigma\beta_{SS}(1-\omega_2)\}\hat{\alpha}_{B2}$ and $\Delta\widehat{w} = \beta_{SS}\{\Delta\hat{p} - (1-\omega_2)\hat{\alpha}_{B2}\}$

Employment effects: $\Delta\hat{L}_A = (\varepsilon - \sigma)(-\Delta\hat{p})$ and $\Delta\hat{L}_B = (\varepsilon - \sigma)(-\Delta\hat{p}) + (1-\sigma)\hat{\alpha}_{B2}$.

### A1.4 Some useful observations

***Small open economy***

For a small, open economy, the relevant equations are (since $\Delta\hat{p} = 0$)

$\Delta\hat{X} = \beta_{Ry}\{(1-\sigma)\,\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}\} = \beta_{Ry}\Delta V;\quad \Delta\widehat{w} = \beta_{SS}\Delta\hat{\chi},$ and $\Delta\hat{L}_f = \Delta\hat{X} - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$.

***Factor augmenting vs factor saving impact***

The factor saving effect of a technological change $\hat{\alpha}_{A1}$ is defined as the change in the use of the relevant factor per unit of output at unaltered factor prices. Hence, in our CES-model the factor-saving effect of $\hat{\alpha}_{A1}$ is $(1-\sigma)\hat{\alpha}_{A1}$. For this to be positive, we need to assume $\sigma < 1$; otherwise $\hat{\alpha}_{A1} > 0$ leads to increased used of labour A per unit of output in sector 1, due to the substitution from labour B to labour A, and hence a negative factor-saving effect (or positive factor-using effect). However, it is important to notice that in our context, the decisive term is not the factor-saving effect defined in this way, but rather $\Delta V \equiv (1-\sigma)\,\Delta\hat{\lambda} + \sigma\beta_{SS}\Delta\hat{\chi}$. The difference being the factor substitution taking place due to a change in relative factor prices (at given goods prices) when technology changes.

***Relationship between various elasticities and variables***

From the definitions of shares variables, we can write

$$\upsilon_A - \upsilon_B = \frac{L_{A1}}{L_A} - \frac{L_{B1}}{L_B} = \frac{w_A L_{A1}}{w_A L_A} - \frac{w_B L_{B1}}{w_B L_B} = \frac{s\omega_1}{\overline{\omega}} - \frac{s(1-\omega_1)}{1-\overline{\omega}} = \frac{s(1-s)(\omega_1-\omega_2)}{\overline{\omega}(1-\overline{\omega})}$$

Where s is the share of sector 1 in GDP, and $\overline{\omega} = \frac{w_A L_A}{w_A L_A + w_B L_B} = s\omega_1 + (1-s)\omega_2$ is the average cost share of factor A in the economy. Since $\beta_{SS} \equiv \frac{1}{\omega_1 - \omega_2}$ and $\beta_{Ry} \equiv \frac{1}{\upsilon_A - \upsilon_B}$.

$$\beta_{Ry} \equiv \frac{1}{\upsilon_A - \upsilon_B} = \frac{\varpi(1-\varpi)}{s(1-s)(\omega_1 - \omega_2)} = \frac{\varpi(1-\varpi)}{s(1-s)}\beta_{SS}$$

In a ***symmetric economy***, with an initial situation with $w_A L_A = w_B L_B$ and $s = 0.5$, we have $\beta_{Ry} = \beta_{SS}$.

**Appendix 2. Model with non-traded sector**

In this appendix we will introduce a non-traded sector in the model in a simple way. Labelling non-traded sector as 0, the labour market equilibrium conditions are, in general, $\sum_{s=0}^{n} L_{fs} = L_f,\ f = A, B$. With three sectors, we can rewrite this as:

$$L_{f1} + L_{f2} = L_f - L_{f0} \equiv L_f^T, \quad f = A, B$$

Where $L_f^T$ is the "factor supply" to the two traded sectors. Hence, an increase in the production of non-traded goods will lower the factor supply to traded sectors, while reduced non-traded production releases more factors. The production and employment effects depend on relative factor intensities.

To stay as close to the previous analysis as possible, we will now define $\upsilon_f^T = \frac{L_{f1}}{L_f^T}$ as sector 1's share of traded sector employment of factor *j*, and let $\upsilon_f^0 = \frac{L_{f0}}{L_f^T}$ be the non-traded employment relative to the traded sector employment of factor *f*. For factor market equilibrium, the equivalence of (10) in section 3 becomes

$$\upsilon_f^T \hat{L}_{f1} + (1 - \upsilon_f^T)\hat{L}_{f2} = \left(\frac{dL_f - dL_{f0}}{L_f^T}\right) = -\upsilon_f^0 \hat{L}_{f0}, \qquad j = A, B, \qquad (10')$$

For non-traded production we will, to simplify, assume no technical progress and fixed coefficients ($\sigma_0 = 0$), so we have $\hat{L}_{j0} = \hat{X}_0$[26]. Given that the technical progress is assumed to take place only in the traded sectors, our summary measures of factor-saving progress become $\hat{\lambda}_f^T \equiv \upsilon_f^T \hat{\alpha}_{f1} + (1 - \upsilon_f^T)\hat{\alpha}_{f2}$ and $\Delta\hat{\lambda}^T \equiv \hat{\lambda}_A^T - \hat{\lambda}_B^T$. For the cost-saving measures there is no difference, since the cost shares in each sector are unchanged. The system of equations (see Appendix 1) then becomes

$$\begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = \frac{1}{\upsilon_A^T - \upsilon_B^T} \begin{bmatrix} (1 - \upsilon_B^T) & -(1 - \upsilon_A^T) \\ -\upsilon_B^T & \upsilon_A^T \end{bmatrix} \begin{bmatrix} (1-\sigma)\,\hat{\lambda}_A^T - \sigma[\upsilon_A^T \hat{p}_1 + (1 - \upsilon_A^T)\hat{p}_2 - \hat{w}_A] - \upsilon_A^0 \hat{X}_0 \\ (1-\sigma)\hat{\lambda}_B^T - \sigma[\upsilon_B^T\,\hat{p}_1 + (1 - \upsilon_B^T)\hat{p}_2 - \hat{w}_B] - \upsilon_B^0 \hat{X}_0 \end{bmatrix}$$

Solving this for differences in relative production changes we get

$$\Delta\hat{X} = \hat{X}_1 - \hat{X}_2 = \frac{1}{\upsilon_A^T - \upsilon_B^T}\left\{(1-\sigma)\Delta\hat{\lambda}^T - \left(\upsilon_A^0 - \upsilon_B^0\right)\hat{X}_0 + \sigma[\hat{w}_A - \hat{w}_B]\right\} - \sigma\Delta\hat{p}$$

Define $s_T \equiv s_1 + s_2 = 1 - s_0$ be the total share of GDP coming from the traded sectors, and $s_i^T = s_i / s_T$ to be the share of sector i in traded GDP. Let $\omega_T \equiv s_1^T \omega_1 + s_2^T \omega_2 = s_1^T \omega_1 + (1 - s_1^T)\omega_2$ be the

[26] We could allow for sector-specific technical progress in the non-traded sector by writing $\hat{L}_{j0} = \hat{X}_0 - \hat{\alpha}_0$.

average cost share of A-factor in traded goods production. Then we can write $\upsilon_A^0 = \frac{\omega_0 s_0}{\omega_T(1-s_0)}$, $\upsilon_B^0 = \frac{(1-\omega_0)s_0}{(1-\omega_T)(1-s_0)}$, and

$$\upsilon_A^0 - \upsilon_B^0 = \frac{(1-\omega_0)s_0}{(1-\omega_T)(1-s_0)}\left(\frac{\omega_0}{1-\omega_0}\Big/\frac{\omega_T}{1-\omega_T} - 1\right) = \frac{s_0}{(1-s_0)}\frac{(\omega_0-\omega_T)}{(1-\omega_T)\omega_T}$$

Using $\beta_{Ry}^T \equiv 1/(\upsilon_A^T - \upsilon_B^T)$ as the Rybczynski elasticity in this case, it has the standard property, but the magnitude may differ from the case without a non-traded sector. Then we have

$$\Delta\hat{X} = \beta_{Ry}^T\left\{(1-\sigma)\Delta\hat{\lambda}^T - \frac{s_0}{(1-s_0)}\frac{(\omega_0-\omega_T)}{(1-\omega_T)\omega_T}\hat{X}_0 + \sigma\Delta\hat{w}\right\} - \sigma\Delta\hat{p} \qquad (A2.1)$$

So far we have only used the assumption that there are fixed coefficients in the non-traded sector. To proceed, we need to add a demand side, since $\hat{X}_0$ is determined from the demand effect for non-traded goods and services. We will do this in the simplest possible way, by assuming that the demand for non-tradables is determined from Cobb-Douglas preferences, such that the income share going to non-traded goods is given.[27] Then we have $\hat{X}_0 = \hat{Y} - \hat{p}_0$.

For income and prices, the following applies:

Factor prices are determined from the $C_1(w_A, w_B) = p_1$ and $C_2(w_A, w_B) = p_2$ as long as there is positive production of both traded goods. Then (13), $\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\}$, still applies. For income and the price of non-traded goods, we have

$$\hat{Y} = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B; \quad \hat{p}_0 = \omega_0\hat{w}_A + (1-\omega_0)\hat{w}_B \qquad (A2.2)$$

Where $\bar{\omega}$ as before is the average cost share of factor A in the economy, and we can write $\bar{\omega} = (1-s_0)\omega_T + s_0\omega_0$. Then demand effects for non-traded goods can be written

$$\hat{X}_0 = \hat{Y} - \hat{p}_0 = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B - [\omega_0\hat{w}_A + (1-\omega_0)\hat{w}_B] = (\bar{\omega}-\omega_0)(\hat{w}_A - \hat{w}_B)$$

Which gives $\hat{X}_0 = (1-s_0)(\omega_T-\omega_0)\Delta\hat{w}$. Using all this in (A2.1) yields

$$\Delta\hat{X} = \beta_{Ry}^T\left\{(1-\sigma)\Delta\hat{\lambda}^T + \left[\frac{s_0(\omega_0-\omega_T)^2}{(1-\omega_T)\omega_T} + \sigma\right]\beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\}\right\} - \sigma\Delta\hat{p} \qquad (A2.1')$$

Finally, define the following elasticities

$$\eta_T \equiv \sigma\left(\beta_{Ry}^T\beta_{SS} - 1\right) \text{ and } \eta_0 \equiv \beta_{Ry}^T\beta_{SS}\frac{s_0(\omega_0-\omega_T)^2}{(1-\omega_T)\omega_T}$$

It is sometimes convenient to write $\eta_0 = \beta_{Ry}^T\beta_{SS}\,\varphi$ with $\varphi \equiv \frac{s_0(\omega_0-\omega_T)^2}{(1-\omega_T)\omega_T} > 0$ as long as $s_0 > 0$ and $\omega_0 \neq \omega_T$. We then get:

$$\Delta\hat{X} = \beta_{Ry}^T\{(1-\sigma)\Delta\hat{\lambda}^T + \beta_{SS}(\sigma+\varphi)\Delta\hat{\chi}\} + (\eta_T + \eta_0)\Delta\hat{p}$$

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[27] This could e.g. come from a two-level utility function where at the top-level there is Cobb-Douglas between non-traded and a nest of traded goods, and at the next level CES with elasticity $\varepsilon$ between the traded goods.

or, defining $\Delta V^T \equiv \{(1-\sigma)\Delta\hat{\lambda}^T + \beta_{SS}(\sigma+\varphi)\Delta\hat{\chi}\}$; this is the equivalent to $\Delta V$ in the 2-sector model, where we note that the substitution effect of $\Delta\hat{\chi}$ is amplified by the non-traded sector,

$$\Delta\hat{X} = \beta_{Ry}^T \Delta V^T + (\eta_T + \eta_0)\Delta\hat{p}$$

$$\Delta\hat{p} = \frac{-\beta_{Ry}^T}{\varepsilon + \eta_T + \eta_0}\Delta V^T$$

Comparing with the 2-sector model, the only difference is that the supply elasticity $\eta$ is replaced by $\eta_T + \eta_0$. $\eta_T$ captures structural changes through factor substitution between the two traded goods sectors; recall that it is = 0 if $\sigma = 0$ and positive for any $\sigma > 0$. $\eta_0$ captures structural changes appearing through changes in factor use in the non-traded sector. Note that we have $\eta_0 \geq 0$ and always positive as long as $s_0 > 0$ and $\omega_0 \neq \omega_T$ (the same applies for $\varphi$). $\eta_0$ is higher the larger the share of the non-traded sector is, and the more its factor intensity differs from the average traded sector intensity.

Qualitatively, all the price effects from the 2-sector model apply, as do the results for relative employment in the two traded sectors. However, for absolute employment effects, we need to take into account the employment changes in the non-traded sector as well.

For the general case, we have from (15)

$$\Delta\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2} = \Delta\hat{X}_1 + \sigma\Delta\hat{p} - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}) = (\varepsilon - \sigma)(-\Delta\hat{p}) - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$$

Using the general equilibrium conditions $\upsilon_f^T\hat{L}_{f1} + (1-\upsilon_f^T)\hat{L}_{f2} = -\upsilon_f^0\hat{L}_{f0}$ we get

$$\hat{L}_{f1} = (1-\upsilon_f^T)\Delta\hat{L}_f - \upsilon_f^0\hat{L}_{f0} \quad \text{and} \quad \hat{L}_{f2} = -[\upsilon_f^T\Delta\hat{L}_f + \upsilon_f^0\hat{L}_{f0}] \qquad (A2.3)$$

Recall that in the 2-sector model, with $\hat{L}_{f0} = 0$, the signs of $\hat{L}_{f1}$ and $\hat{L}_{f2}$ were always opposite and followed directly from $\Delta\hat{L}_f$. Now the total employment in the traded sectors can change ($\hat{L}_{f0} \neq 0$) and we may thus have cases in which employment in both traded sectors may go up or down simultaneously.

We will work out the details for the same examples as we have used before – a sector-specific and a factor-specific shock. It is convenient to start with a small, open economy case before we look at the general case.

### A2.1 Sector-specific shock in sector 1: $\Delta\hat{\lambda}^T = \hat{\alpha}_1/\beta_{Ry}^T$ and $\Delta\hat{\chi} = \hat{\alpha}_1$

In this case we have $\Delta V^T = (1 + \eta_T + \eta_0)\hat{\alpha}_1/\beta_{Ry}$.

***Small, open economy:***

$$\Delta\hat{X} = \beta_{Ry}^T\Delta V^T = (1 + \eta_T + \eta_0)\hat{\alpha}_1$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{\chi}\} = \beta_{SS}\hat{\alpha}_1$$

$$\hat{X}_0 = (1-s_0)(\omega_T - \omega_0)\Delta\hat{w} = (1-s_0)(\omega_T-\omega_0)\beta_{SS}\hat{\alpha}_1$$

$$\Delta\hat{L}_f = \Delta\hat{X}_1 - (1-\sigma)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}) = (\eta_T + \eta_0 + \sigma)\hat{\alpha}_1$$

$$\hat{L}_{f1} = \left(1 - \upsilon_f^T\right)\Delta\hat{L}_f - \upsilon_f^0\hat{X}_0 \; = \left\{\left(1 - \upsilon_f^T\right)(\eta_T + \eta_0 + \sigma) - \upsilon_f^0(1 - s_0)(\omega_T - \omega_0)\beta_{SS}\right\}\hat{\alpha}_1$$

If $\omega_0 > \omega_T$ this is clearly positive, as in this case, $\hat{X}_0 < 0$ and sector 1 attracts factors both from sector 2 and from the non-traded sector. If, on the other hand, $\omega_0 < \omega_T$ the non-traded sector expands, and the employment effect for sector 1 depends on the relative weight of the two effects.

$$\hat{L}_{f1} = \left\{\left(1 - \upsilon_f^T\right)(\eta_T + \eta_0 + \sigma) - \upsilon_f^0(1 - s_0)(\omega_T - \omega_0)\beta_{SS}\right\}\hat{\alpha}_1$$
$$= \left\{\left(1 - \upsilon_f^T\right)\beta_{Ry}^T\beta_{SS}(\sigma + \varphi) - \upsilon_f^0(1 - s_0)(\omega_T - \omega_0)\beta_{SS}\right\}\hat{\alpha}_1$$

Using $\upsilon_A^0 = \frac{\omega_0 s_0}{\omega_T(1-s_0)}$, $\upsilon_B^0 = \frac{(1-\omega_0)s_0}{(1-\omega_T)(1-s_0)}$, $\upsilon_A^T = \frac{\omega_1 s_1^T}{\omega_T} = \frac{\omega_1 s_1}{\omega_T(1-s_0)}$, $\upsilon_B^T = \frac{(1-\omega_1)s_1}{(1-\omega_T)(1-s_0)}$, $\varphi = \frac{s_0(\omega_0-\omega_T)^2}{(1-\omega_T)\omega_T}$ and $\beta_{Ry}^T = \frac{\omega_T(1-\omega_T)}{s_1^T(1-s_1^T)(\omega_1 - \omega_2)}$ we get

$$\hat{L}_{A1} = \beta_{Ry}^T\beta_{SS}(1 - \upsilon_A^T)\sigma\hat{\alpha}_1 + \beta_{SS}\frac{s_0(\omega_T - \omega_0)(\omega_2 - \omega_0)}{s_1^T(\omega_1 - \omega_2)}\hat{\alpha}_1$$

$$\hat{L}_{B1} = \beta_{SS}\beta_{Ry}^T(1 - \upsilon_B^T)\sigma\hat{\alpha}_1 + \beta_{SS}\frac{s_0(\omega_T - \omega_0)(\omega_2 - \omega_0)}{s_1^T(\omega_1 - \omega_2)}\hat{\alpha}_1$$

The first term in these two employment effects is the effect in an economy without a non-traded sector, and for a small, open economy we know that this is positive. This is because the increase in relative wage for factor A reduces the A-intensity in both sectors and shifts employment towards the A-intensive sector.

The second term is the added effects due to the non-traded sector. If $\omega_0 < \omega_2$ $(< \omega_T)$ the expansion of non-traded production will lead to increased employment of both factors in sector 1 and reduced employment of both factors in sector 2. The reason is that the non-traded sector requires so much factor B relative to factor A that sector 2 has to contract significantly. If $\omega_T > \omega_0 > \omega_2$ the expansion of non-traded production will tend to reduce employment of both factors in sector 1.

For sector 2 employment we have

$$\hat{L}_{f2} = -\left[\upsilon_f^T\Delta\hat{L}_f + \upsilon_f^0\hat{L}_{f0}\right] = -\left\{\upsilon_f^T(\eta_T + \eta_0 + \sigma)\hat{\alpha}_1 + \upsilon_f^0(1 - s_0)(\omega_T - \omega_0)\beta_{SS}\hat{\alpha}_1\right\}.$$

This is clearly negative for $\omega_T > \omega_0$ since both terms in the bracket are positive in that case (the second case discussed above). For $\omega_T < \omega_0$ we can write, similarly as for sector 1,

$$\hat{L}_{f2} = -\beta_{SS}\beta_{Ry}^T\upsilon_f^T\sigma\hat{\alpha}_1 - \beta_{SS}\frac{s_0(\omega_T - \omega_0)(\omega_1 - \omega_0)}{(1 - s_1^T)(\omega_1 - \omega_2)}\hat{\alpha}_1.$$

Hence, the substitution effect is negative, as expected. For the effect due to the non-traded sector it is positive for $\omega_1 > \omega_0 > \omega_T$. For $\omega_0 > \omega_1$ the increased factor supply to traded goods is so A-biased that it leads to increased employment in sector 1 and reduced employment in sector 2.

So, contrary to the 2-sector model, there are cases where employment in both traded sectors may increase, i.e. when $\omega_1 > \omega_0 > \omega_T$ , and where both decline, i.e. when $\omega_T > \omega_0 > \omega_2$. However, if the substitution effect is sufficiently strong, it may dominate over the non-traded effect, and thus give increased employment in sector 1 and reduced in sector 2. In this case, the existence of a non-traded sector will allow sector 1 to expand more than it otherwise would have been able to do.

***Large country case (sector specific progress):***

$$\Delta\hat{p} = \frac{-(1+\eta_T+\eta_0)}{\varepsilon+\eta_T+\eta_0}\hat{\alpha}_1$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p}+\Delta\hat{\chi}\} = \beta_{SS}\frac{\varepsilon-1}{\varepsilon+\eta_T+\eta_0}\hat{\alpha}_1$$

$$\hat{X}_0 = (1-s_0)(\omega_T-\omega_0)\Delta\hat{w} = (1-s_0)(\omega_T-\omega_0)\beta_{SS}\frac{\varepsilon-1}{\varepsilon+\eta_T+\eta_0}\hat{\alpha}_1$$

$$\Delta\hat{L}_f = (\varepsilon-\sigma)(-\Delta\hat{p}) - (1-\sigma)(\hat{\alpha}_{f1}-\hat{\alpha}_{f2}) = \frac{(\eta_T+\eta_0+\sigma)}{\varepsilon+\eta_T+\eta_0}(\varepsilon-1)\hat{\alpha}_1 = \frac{\beta_{Ry}^T\beta_{SS}(\sigma+\varphi)}{\varepsilon+\eta_T+\eta_0}(\varepsilon-1)\hat{\alpha}_1$$

As before, the relative employment in sector 1 increases if the demand effect dominates. For employment in sector 1 we now have

$$\hat{L}_{f1} = (1-v_f^T)\Delta\hat{L}_f - v_f^0\hat{X}_0 = \frac{\varepsilon-1}{\varepsilon+\eta_T+\eta_0}\{(1-v_f^T)(\eta_T+\eta_0+\sigma) - v_f^0(1-s_0)(\omega_T-\omega_0)\beta_{SS}\}\hat{\alpha}$$

The curly bracket is exactly the same as in the small, open economy case. By the same reasoning, we also get a similar expression for $\hat{L}_{f2}$. If we look at the case with no substitutability, $\sigma = 0$, we can summarize the results generated by the non-traded sector as follows:

The sign of $\hat{L}_{f1}$ is given by $(\varepsilon-1)(\omega_T-\omega_0)(\omega_2-\omega_0)$.

The sign of $\hat{L}_{f2}$ is given by $-(\varepsilon-1)(\omega_T-\omega_0)(\omega_1-\omega_0)$.

For $\varepsilon > 1$ the effects are qualitatively as in the small, open economy case. Hence, we will look at the case of $\varepsilon < 1$, where we know that $\hat{w}_A - \hat{w}_B < 0$ and $\hat{X}_0 < 0$ if $\omega_T > \omega_0$ (and vice versa). Then we have

- $\omega_T > \omega_0 > \omega_2$: $\hat{L}_{f0} < 0$; $\hat{L}_{f1} > 0$; $\hat{L}_{f2} > 0$.
- $\omega_T > \omega_2 > \omega_0$: $\hat{L}_{f0} < 0$; $\hat{L}_{f1} < 0$; $\hat{L}_{f2} > 0$.
- $\omega_1 > \omega_0 > \omega_T$: $\hat{L}_{f0} > 0$; $\hat{L}_{f1} < 0$; $\hat{L}_{f2} < 0$.
- $\omega_0 > \omega_1 > \omega_T$: $\hat{L}_{f0} > 0$; $\hat{L}_{f1} < 0$; $\hat{L}_{f2} > 0$.

Again, we have cases where employment in both traded sectors move in the same direction, contrary to in the 2-sector model. These are all Rybczynski-like effects, and follow from the need for structural changes to accommodate adjustments in "factor supply" (here caused by changes in factor demand from the non-traded sector). For $\sigma > 0$ there will in addition be factor substitution such that both traded sectors become more A-intensive (since the relative wage of A-workers go down). That will tend to lower employment in sector 1, increase employment in sector 2, and dampen the price effects.

**A2.2 Factor-specific shock for factor A: $\Delta\hat{\lambda}^T = \hat{\alpha}_A$ and $\Delta\hat{\chi} = \hat{\alpha}_A/\beta_{SS}^T$**

In this case we have: $\Delta V^T = (1+\varphi)\hat{\alpha}_A$. The equilibrium solution will be similar to what we have worked out before, and in the general (large country case) we get:

$$\Delta\hat{p} = \frac{-\beta_{Ry}^T\Delta V^T}{\varepsilon+\eta_T+\eta_0} = \frac{-\beta_{Ry}^T(1+\varphi)}{\varepsilon+\eta_T+\eta_0}\hat{\alpha}_A$$

$$\Delta\hat{w} = \beta_{SS}\{\Delta\hat{p} + \Delta\hat{\chi}\} = \Delta\hat{w} = \beta_{SS}\left\{1 - \frac{\beta_{SS}\beta_{Ry}^{T}(1+\varphi)}{\varepsilon + \eta_T + \eta_0}\right\}\hat{\alpha}_A$$

$$\hat{X}_0 = (1 - s_0)(\omega_T - \omega_0)\Delta\hat{w}$$

$$\Delta\hat{L}_f = (\varepsilon - \sigma)(-\Delta\hat{p}) = \frac{\beta_{Ry}^{T}(1+\varphi)}{\varepsilon + \eta_T + \eta_0}(\varepsilon - \sigma)\hat{\alpha}_A$$

These are all equal to the 2-sector economy, only replacing $\eta$ with $\eta_T + \eta_0$ (where $\eta_0 = \beta_{SS}\beta_{Ry}^{T}\varphi$ has been used to simplify notation and expressions). For employment effects it is convenient to look at a small, open economy first, where $\Delta\hat{L}_f = \Delta\hat{X} = \beta_{Ry}^{T}(1+\varphi)\hat{\alpha}_A$ and $\Delta\hat{w} = \hat{\alpha}_A$:

$$\hat{L}_{f1} = \left(1 - \upsilon_f^{T}\right)\Delta\hat{L}_f - \upsilon_f^{0}\hat{X}_0 = \left(1 - \upsilon_f^{T}\right)\beta_{Ry}^{T}(1+\varphi)\hat{\alpha}_A - \upsilon_f^{0}(1 - s_0)(\omega_T - \omega_0)\hat{\alpha}_A$$

For $\omega_0 > \omega_T$ this is clearly positive, since in this case $\hat{X}_0 < 0$ so the non-traded sector releases factors to the traded sectors, and reinforced the reallocation of both types of labour to sector 1. For $\omega_0 < \omega_T$ we need to look at the details of the trade-off:

$$\hat{L}_{A1} = (1 - \upsilon_A^{T})\beta_{Ry}^{T}(1+\varphi)\hat{\alpha}_A - \upsilon_A^{0}(1 - s_0)(\omega_T - \omega_0)\hat{\alpha}_A$$
$$= (1 - \upsilon_A^{T})\beta_{Ry}^{T}\hat{\alpha}_A + \frac{s_0(\omega_T - \omega_0)(\omega_2 - \omega_0)}{s_1^{T}(\omega_1 - \omega_2)}\hat{\alpha}_A$$

The first term is the effect without a non-traded sector, while the second terms is the additional adjustment due to the non-traded sector. This is almost identical to the similar adjustment in the sector-specific case; the only difference being that the term was multiplied by $\beta_{SS} = 1/(\omega_1 - \omega_2)$ and thus independent of the sign of $(\omega_1 - \omega_2)$. As long as we assume that sector 1 is the A-intensive one, the effects of the non-traded sector are qualitatively similar for the two cases.

The large country case can be developed in a similar way; however, it becomes slightly more complicated as the elasticity conditions for $\Delta\hat{L}_f$ and $\Delta\hat{w}$ are not the same. Without going into the details, it could be stated that for sufficiently high $\varepsilon$ relative to $\sigma$ the employment effect will be similar to in the small, open economy case. For lower $\varepsilon$ the signs will switch, as in the sector-specific case.