Demand-Driven Labor Market Polarization

Diego Comin (Dartmouth)

joint work with

Ana Danieli (Northwestern)
Martí Mestieri (Northwestern)

FRAME Final Conference
US Labor Market Outcomes Have Polarized since 1980

- Since 80s, wage bill share of middle-skill workers declined.
US Labor Market Outcomes Have Polarized since 1980

- Since 80s, wage bill share of middle-skill workers declined.
- Employment shares and wages have also polarized.
US Labor Market Outcomes Have Polarized since 1980

- Since 80s, wage bill share of middle-skill workers declined.
- Employment shares and wages have also polarized.
- What drives the increase in inequality and polarization?
  - Skilled biased technical change.
  - Computerization and digitization of the economic activity.
  - Offshoring and international trade.
  - ...
Propose a New Mechanism: Demand-Driven Polarization

Contributions of the Paper

1. Establish **new empirical findings**:
   - High-income elastic sectors are intensive in high- and low-skill occupations relative to middle-skill.
   - Wage bill (and emp. shares) of high- and low-skill occupations concentrated in high-income elastic sectors.

2. Novel demand-driven mechanism based on nonhomotheticity of demand (Engel's Law):
   - Income grows → demand shifts to high-income-elastic sectors → relative demand of high- and low-skilled workers increases → polarization.

3. Quantify the effect of the mechanism using GE model:
   - Model incorporates skill-biased tech. change and offshoring.
   - Demand-driven mechanism accounts for 2/3 of observed polarization of the wage bill at the bottom and 1/2 the top from 1980 to 2016.
Propose a New Mechanism: Demand-Driven Polarization

Contributions of the Paper

1. Establish **new empirical findings**:
   - High-income elastic sectors are intensive in high- and low-skill occupations relative to middle-skill.
   - Wage bill (and emp. shares) of high- and low-skill occupations concentrated in high-income elastic sectors.

2. **Novel demand-driven mechanism** based on nonhomotheticity of demand (Engel’s Law):
   - Income grows $\rightarrow$ demand shifts to high-income-elastic sectors $\rightarrow$ relative demand of high- and low- skilled workers increases $\rightarrow$ polarization.

3. Quantify the effect of the mechanism using GE model:
   - Model incorporates skill-biased tech. change and offshoring.
   - Demand-driven mechanism accounts for 2/3 of observed polarization of the wage bill at the bottom and 1/2 the top from 1980 to 2016.
Propose a New Mechanism: Demand-Driven Polarization

Contributions of the Paper

1. Establish **new empirical findings**:
   - High-income elastic sectors are intensive in high- and low-skill occupations relative to middle-skill.
   - Wage bill (and emp. shares) of high- and low-skill occupations concentrated in high-income elastic sectors.

2. **Novel demand-driven mechanism** based on nonhomotheticity of demand (Engel’s Law):
   - Income grows → demand shifts to high-income-elastic sectors → relative demand of high- and low- skilled workers increases → polarization.

3. **Quantify** the effect of the mechanism using GE model:
   - Model incorporates skill-biased tech. change and offshoring.
   - Demand-driven mechanism accounts for 2/3 of observed polarization of the wage bill at the bottom and 1/2 the top from 1980 to 2016.
Related Literature

- Traditional mechanisms to explain polarization:
  - Routinization hypothesis: Autor, Levy and Murnane (2003),...

- Employment Shifts Between Sectors:

- Structural change and wage structure:
  - Barany and Siegel (18), Lee and Shin (18), Buera et al. (15).

- Other related mechanisms:
  - Trade, skill premium, structural change: Cravino Sotelo (18).
  - Consumption Spillovers: Manning (04), Mazzolari and Ragusa (13), Clemens et al. (16).
1. Documenting Two Facts.
2. Bare-bones model and quantification of wage bill polarization.
   ▶ Production: CRS technology with only labor.
   ▶ Representative household.
   ▶ Allow for skill-biased tech. change and offshoring.
3. Full model: prices and quantities.
   ▶ Production: Introduce capital, DRS in labor.
   ▶ Occupational choice: wages and employment shares.
4. Extensions:
   ▶ Trade.
   ▶ Looking back and ahead, from 1950 to 2036.
5. Conclusion
Documenting Two Correlations Supporting our Mechanism

1. US sectoral output growth is fastest in income-elastic sectors.

2. Wage bill share of high- and low-skill occupations is concentrated in income-elastic sectors.
Documenting Two Correlations Supporting our Mechanism

1. US sectoral output growth is fastest in income-elastic sectors.
   - Income grows $\rightarrow$ Demand shifts to high-income-elastic sectors.
   - Look at 8 broad sectors US economy (BEA, VA measures).
   - Measure income elasticity parameters.

2. Wage bill share of high- and low-skill occupations is concentrated in income-elastic sectors.
Documenting Two Correlations Supporting our Mechanism

1. US sectoral output growth is fastest in income-elastic sectors.
   ▶ Income grows $\rightarrow$ Demand shifts to high-income-elastic sectors.
   ▶ Look at 8 broad sectors US economy (BEA, VA measures).
   ▶ Measure income elasticity parameters.

2. Wage bill share of high- and low-skill occupations is concentrated in income-elastic sectors.
   ▶ Demand shifts to high-income-elastic sectors $\rightarrow$ Relative demand of high- and low- skilled workers increases.
   ▶ Classify occupations according to skill-intensity.
Documenting Two Correlations Supporting our Mechanism

1. US sectoral output growth is fastest in income-elastic sectors.
   - Income grows $\rightarrow$ Demand shifts to high-income-elastic sectors.
     - Look at 8 broad sectors US economy (BEA, VA measures).
     - Measure income elasticity parameters.

2. Wage bill share of high- and low-skill occupations is concentrated in income-elastic sectors.
   - Demand shifts to high-income-elastic sectors $\rightarrow$ Relative demand of high- and low- skilled workers increases.
   - Classify occupations according to skill-intensity.

**Key:** Estimate income elasticity parameters.
We Estimate Income Elasticities using HH Survey Data

- Study urban HH with age of head between 25 and 64.
  - Keep if responses in 4 rounds, not incomplete, 5th-95th income, positive total and food expenditure, . . .
We Estimate Income Elasticities using HH Survey Data

• Use household expenditure data: CEX Survey, 2000-2002.
• Study urban HH with age of head between 25 and 64.
  ▶ Keep if responses in 4 rounds, not incomplete, 5th-95th income, positive total and food expenditure, . . .
• Convert final good expenditures reported in the CEX into value added using the BEA’s 2000 input-output tables.
We Estimate Income Elasticities using HH Survey Data

- Study urban HH with age of head between 25 and 64.
  - Keep if responses in 4 rounds, not incomplete, 5th-95th income, positive total and food expenditure, . . .
- Convert final good expenditures reported in the CEX into value added using the BEA’s 2000 input-output tables.
- Obtain total expenditure $E_{ht}$ and expenditure shares $x_{hst}$ of HH $h$ in sector $s$ during quarter $t$.
- Use as HH controls $Z_h$ dummies for:
  - Age (25-37, 38-50, 51-64), number of earners ($\leq 2$, $2+$), household size ($\leq 2$, 3-4, $5+$), region of residence.
- Merge with BLS urban sectoral price series.
We Estimate a Nonhomothetic CES Demand System

- Each sector $s$ has a demand income elasticity parameter, $\epsilon_s$.
  - Normalized to 1 for one sector $\bar{s}$, $\epsilon_{\bar{s}} = 1$.
  - Expenditure elasticity proportional to $\epsilon_s$. 

- There is a common price elasticity $\sigma$ across sectors.

- Allow for heterogeneity in tastes:
  $$\zeta_{hst} \equiv \alpha_s + \Gamma_s X_h + \delta_r + \delta_t.$$ 

- Estimate system of equations for all sectors $s$ ($\neq \bar{s}$).

  $$\ln x_{hst} = \zeta_{hst} + (1 - \sigma) \ln \left( \frac{p_{hst}}{p_{h\bar{s}}} \right) + (1 - \sigma)(\epsilon_s - 1) \ln \left( \frac{E_{ht}}{p_{h\bar{s}}} \right) + \epsilon_s \ln x_{h\bar{s}} + \nu_{hst}.$$ 

- If $\epsilon_{\bar{s}} = 1 \rightarrow$ Homothetic CES.

- System of equations, estimate using GMM.
We Estimate a Nonhomothetic CES Demand System

- Each sector $s$ has a demand income elasticity parameter, $\epsilon_s$.
  - Normalized to 1 for one sector $\bar{s}$, $\epsilon_{\bar{s}} = 1$.
  - Expenditure elasticity proportional to $\epsilon_s$.
- There is a common price elasticity $\sigma$ across sectors.
- Allow for heterogeneity in tastes: $\zeta_{sht} \equiv \alpha_s + \Gamma_s X_h + \delta_r + \delta_t$. 
  
  If $\epsilon_s = 1 \rightarrow$ Homothetic CES.
  
  System of equations, estimate using GMM.
We Estimate a Nonhomothetic CES Demand System

- Each sector $s$ has a demand income elasticity parameter, $\epsilon_s$.
  - Normalized to 1 for one sector $\bar{s}$, $\epsilon_{\bar{s}} = 1$.
  - Expenditure elasticity proportional to $\epsilon_s$.
- There is a common price elasticity $\sigma$ across sectors.
- Allow for heterogeneity in tastes: $\zeta_{sht} \equiv \alpha_s + \Gamma_s X_h + \delta_r + \delta_t$.
- Estimate system of equations for all sectors $s(\neq \bar{s})$.

$$\ln x_{hst} = \zeta_{hst} + (1 - \sigma) \ln \left( \frac{p_{hst}}{p_{h\bar{s}t}} \right) + (1 - \sigma)(\epsilon_s - 1) \ln \left( \frac{E_{ht}}{p_{h\bar{s}t}} \right) + \epsilon_s \ln x_{h\bar{s}t} + \nu_{hst}.$$ 

- If $\epsilon_s = 1 \rightarrow$ Homothetic CES.
- System of equations, estimate using GMM.
We Instrument Prices and Expenditures

- Want to isolate relative price variation coming from shifts in the supply curve.
- Use average relative price in other regions controlling for time and region dummies.
We Instrument Prices and Expenditures

- Want to isolate relative price variation coming from shifts in the supply curve.
- Use average relative price in other regions controlling for time and region dummies.
- Household expenditures have measurement error.
- Use HH annual income and HH income quintile as instruments ($\sim$ NPV).
Estimation Results of Nonhomothetic CES

<table>
<thead>
<tr>
<th>Income Elasticity Parameters ${\epsilon_s}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Education and Health Care (6)</td>
<td>3.50 (0.18)</td>
</tr>
<tr>
<td>Arts, Entertainment, Recreation and Food Services (7)</td>
<td>2.04 (0.08)</td>
</tr>
<tr>
<td>Government (G)</td>
<td>1.00</td>
</tr>
<tr>
<td>Finance, Professional, Information, other services (excl. gov’t) (FIRE, PROF, 51, 81)</td>
<td>0.98 (0.04)</td>
</tr>
<tr>
<td>Manufacturing (31G)</td>
<td>0.57 (0.04)</td>
</tr>
<tr>
<td>Retail, Wholesale Trade and Transportation (42, 44RT, 48T)</td>
<td>0.37 (0.04)</td>
</tr>
<tr>
<td>Construction (23)</td>
<td>0.14 (0.06)</td>
</tr>
<tr>
<td>Agriculture, Mining and Utilities (11, 21, 22)</td>
<td>0.10 (0.04)</td>
</tr>
</tbody>
</table>

Price Elasticity $\sigma$                                                  0.63 (0.01)

Std. Err. Clustered at HH level in parenthesis. Number of HH is 20,843.
Correlation of Sectoral Growth with Income Elasticity > 0

Nominal VA Growth Rate 1980-2016

Income Elasticity Parameters

- Education and Health Services
- Financial and Professional Services
- Wholesale, Retail and Transportation
- Manufacturing
- Agriculture, Mining and Utilities

Sectors 15 Sectors
Documenting Two Correlations Supporting our Mechanism

   - Income grows $\rightarrow$ Demand shifts to high-income-elastic sectors.
   - Look at 8 broad sectors US economy (BEA, VA measures).
   - Measure Income elasticity parameters.

2. Wage bill share of high- and low-skill occupations concentrated in income-elastic sectors.
   - Demand shifts to high-income-elastic sectors $\rightarrow$ Relative demand of high- and low-skilled workers increases.
   - Classify occupations according to skill-intensity.
We Classify Occupations According to Skill Intensity

- Use Acemoglu and Autor (2011) classification.
- 3 levels: $H$ (high), $M$ (middle) and $L$ (low).
  - Use average wage from 1980 CPS (5th to 95th).
  - Ranking stable over time.
  - Ranking occupations by years of schooling very similar.
- AA group finer occupations by their skill level:
  - $H$: managerial, professional and technical occupations
  - $M$: sales, clerical and administrative support occupations; production, craft, repair and operative occupations; and
  - $L$: service occupations (food/cleaning, personal care, protective).
- Use employment shares from decennial census.
High-Skill 1980 Wage Bill Shares and Income Elasticity

- Education and Health Services
- Financial and Professional Services
- Manufacturing
- Wholesale, Retail and Transportation
- Agriculture, Mining and Utilities

Income Elasticity Parameters vs. Wage Bill Share within High Skill Employees

Sectors • Correlations • Wage Bill Growth Accounting • Factor Intensities
Low-Skill 1980 Wage Bill Shares and Income Elasticity

![Graph showing wage bill shares against income elasticity parameters for different sectors. The sectors are labeled: Agriculture Mining and Utilities, Wholesale, Retail and Transportation, Manufacturing, Financial and Professional Services, and Education and Health Services.](image_url)
Middle-Skill 1980 Wage Bill Shares and Income Elasticity

- Manufacturing
- Wholesale, Retail and Transportation
- Financial and Professional Services
- Agriculture, Mining and Utilities
- Education and Health Services
Outline

1. Documenting Two Facts.
2. **Bare-bones model and quantification of wage bill polarization.**
   - Production: CRS technology with only labor.
   - Representative household.
   - Allow for skill-biased tech. change and offshoring.
3. Full model: prices and quantities.
   - Production: Introduce capital, DRS in labor.
   - Occupational choice: wages and employment shares.
4. Extensions:
   - Trade.
   - Looking back and ahead, from 1950 to 2036.
5. Conclusion
General Equilibrium Structure of the Model

Households

Producers
General Equilibrium Structure of the Model

Households

Consume Goods

Supply Labor

$\text{Supply Labor}
\quad j \in \{H, M, L\}$

Producers

Sector $s$

Skill Intensity $\alpha_{st}$

Nonhom. CES $\sigma, \epsilon$
General Equilibrium Structure of the Model

Households

Supply Labor
$j \in \{H, M, L\}$

Consume Goods

Sector 1

... 

Sector $s$

... 

Sector $S$
General Equilibrium Structure of the Model

- Households
  - Supply Labor: $j \in \{H, M, L\}$
  - Consume Goods

- Sector 1
- Sector $s$
- Sector $S$

Skill Intensity $\alpha_{jst}$
General Equilibrium Structure of the Model

Households

Nonhom.CES $\sigma, \epsilon_s$

Supply Labor

$\quad j \in \{H, M, L\}$

Consume Goods

Sector 1

Sector $s$

Sector $S$

Skill Intensity $\alpha_{jst}$
Sectors Differ in Their Skill Intensity

- Economy with $S = \{1, \ldots, S\}$ distinct sectors.
- Representative firm in $s \in S$ produces output $Y_{st}$ at time $t$:
  \[ Y_{st} = A_{st} \prod_{j \in \{H,M,L\}} X_{jst}^{\alpha_{jst}} , \]

where $A_{st} > 0$ is a neutral technological change in sector $s$,

\[ \sum_{j \in \{H,M,L\}} \alpha_{jst} = 1 \]
Relative Sectoral Demand of Skill Depend only on \( \{ \alpha_{jst} \} \)

- Given a price of sectoral output \( p_{st} \) and wages \( w_{jt} \), the demand for occupation \( j \) in sector \( s \) at time \( t \) is
  \[
  w_{jt} X_{jst} = \alpha_{jst} p_{st} Y_{st}.
  \]

- The ratio of the wage bill accrued by two occupations, \( j \) and \( j' \), in sector \( s \), is
  \[
  \frac{w_{jt} X_{jst}}{w_{j't} X_{j'st}} = \frac{\alpha_{jst}}{\alpha_{j'st}}.
  \]

- Thus, in any sector \( s \), the wage bill of occupation \( j \) relative to occupation \( j' \) is entirely determined by the ratio of their wage bill shares, \( \alpha_{jst} \) and \( \alpha_{j'st} \).
We Compute the Total Wage Bill of Occupation $j$

The wage bill is $\alpha_{jst}$-Weighted Average of Production across Sectors $s$

- Let $X_{jt} = \sum_{s \in S} X_{jst}$.
- Nominal value added in sector $s$
  \[ VA_{st} = p_{st} Y_{st}. \]
- Aggregate wage bill of occupation $j$ is
  \[ w_{jt} X_{jt} = \sum_{s \in S} \alpha_{jst} VA_{st}. \]
Decompose Growth Rate of Wage Bill in $\Delta \alpha_{jst}$ and $\Delta VA_{st}$

- Relate change in VA (from 0 to $t$) to changes in wage bill.
- Rewrite $\alpha_{jst} VA_{st}$ as

$$\alpha_{jst} VA_{st} = (\alpha_{js0} + \Delta \alpha_{jst})(VA_{s0} + \Delta VA_{st}).$$

where $\Delta$ is the time difference operator between $t$ and 0.
Decompose Growth Rate of Wage Bill in $\Delta \alpha_{jst}$ and $\Delta VA_{st}$

- Growth in Wage Bill (as in introduction):

$$\frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{js0} \frac{\Delta VA_{st}}{VA_{s0}} + \sum_{s \in S} \gamma_{js0} \frac{\Delta \alpha_{jst}}{\alpha_{js0}}$$

Term 3: Covariance

$$+ \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} \right]$$

where $\gamma_{js0} \equiv \frac{\alpha_{js0}VA_{s0}}{\sum_{s \in S} \alpha_{js0}VA_{s0}}$ are wage bill shares (previous plots).
A One-Sector Model Imposes $\Delta VA_{st} = 0$

- If $S = 1$,
  \[
  \frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \frac{\Delta \alpha_{jt}}{\alpha_{j0}} + \frac{\Delta VA_t}{VA_0} + \frac{\Delta \alpha_{jt}}{\alpha_{j0}} \frac{\Delta VA_t}{VA_0}.
  \]

- Implying that the growth in the relative wage bill across occupations $j$ and $j'$ is
  \[
  \frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} - \frac{\Delta (w_{j't}X_{j't})}{w_{j'0}X_{j'0}} = \left( \frac{\Delta \alpha_{jt}}{\alpha_{j0}} - \frac{\Delta \alpha_{j't}}{\alpha_{j'0}} \right) \left( 1 + \frac{\Delta VA_t}{VA_0} \right).
  \]

- Variation in relative wage bill must come from variation in factor intensity \((\alpha_{jt}/\alpha_{j't})\).
- Good assumption in the data? Not so much!
US 1980-2016: Changes in Sectoral Composition are Key

\[
\frac{\triangle (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{js0} \frac{\triangle VA_{st}}{VA_{s0}} + \sum_{s \in S} \gamma_{js0} \frac{\triangle \alpha_{jst}}{\alpha_{js0}} + \sum_{s \in S} \gamma_{js0} \left[ \frac{\triangle VA_{st}}{VA_{s0}} \frac{\triangle \alpha_{jst}}{\alpha_{js0}} \right]
\]

<table>
<thead>
<tr>
<th></th>
<th>High</th>
<th>Mid</th>
<th>Low</th>
<th>H–M</th>
<th>L–M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Change</td>
<td>10.19</td>
<td>3.18</td>
<td>6.61</td>
<td>7.01</td>
<td>3.43</td>
</tr>
<tr>
<td>Term 1</td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td>Term 2</td>
<td>0.45</td>
<td>-0.22</td>
<td>0.00</td>
<td>0.67</td>
<td>0.22</td>
</tr>
<tr>
<td>Term 3</td>
<td>2.69</td>
<td>-1.26</td>
<td>-0.47</td>
<td>3.95</td>
<td>0.79</td>
</tr>
<tr>
<td>Contribution (\Delta VA_{st})</td>
<td></td>
<td></td>
<td></td>
<td>62%</td>
<td>82%</td>
</tr>
</tbody>
</table>

- Term 2 generates little variation alone! Multi-sector is key.
Preferences Drive Sectoral Reallocation of Production

- **Representative household** earns all wages.
- **Nonhomothetic CES Preferences**, implicitly defined:

\[
\sum_{s=1}^{I} \left( \zeta_s C_t^{\varepsilon_s} \right)^{\frac{1}{\sigma}} c_{st}^{\frac{\sigma-1}{\sigma}} = 1,
\]

- \( \zeta_s > 0 \) constant taste parameter for \( i = 1, \ldots, I \).
- \( \sigma \) is the elasticity of substitution.
- \( \varepsilon_i \) governs nonhomotheticity of \( i \).
- If \( \varepsilon_i = 1 - \sigma \), we recover homothetic CES.
Preferences Drive Sectoral Reallocation of Production

- **Representative household** earns all wages.
- **Nonhomothetic CES Preferences**, implicitly defined:

\[
\sum_{s=1}^{I} \left( \zeta_s C_t^{\epsilon_s} \right)^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} c_{st}^{\frac{1}{\sigma}} = 1,
\]

- \( \zeta_s > 0 \) constant taste parameter for \( i = 1, \ldots, I \).
- \( \sigma \) is the elasticity of substitution.
- \( \epsilon_i \) governs nonhomotheticity of \( i \).
- If \( \epsilon_i = 1 - \sigma \), we recover homothetic CES.

\[
\sum_{i=1}^{S} \left( \zeta_i C_t^{1-\sigma} \right)^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} c_{it}^{\frac{1}{\sigma}} = 1
\]
Preferences Drive Sectoral Reallocation of Production

- **Representative household** earns all wages.
- Nonhomothetic CES Preferences, implicitly defined:

\[ \sum_{s=1}^{I} \left( \zeta_s C_t^{\varepsilon_s} \right)^{\frac{1}{\sigma}} c_{st}^{\frac{\sigma-1}{\sigma}} = 1, \]

- \( \zeta_s > 0 \) constant taste parameter for \( i = 1, \ldots, I \).
- \( \sigma \) is the elasticity of substitution.
- \( \varepsilon_i \) governs nonhomotheticity of \( i \).
- If \( \varepsilon_i = 1 - \sigma \), we recover homothetic CES.
- Parameter restriction (Hanoch, 75): \( \zeta_i > 0, \sigma > 0, \varepsilon_i > 0 \) if \( \sigma \in (0,1) \), \( \varepsilon_i < 0 \) if \( \sigma > 1 \).
- Preferences defined up to scaling factor in
  1. nonhomotheticity: \( \tilde{\varepsilon}_i \equiv \tilde{\xi}\varepsilon_i \),
  2. taste parameter: \( \hat{\zeta}_i \equiv \hat{\Omega}\zeta_i \).
Sectoral Demand is Log-Linear

- HH facing prices \(\{p_{st}\}\) with budget constraint \(\sum_s p_{st} c_{st} \leq E_t\).
- Demand (Hicksian)

\[
c_{st} = \zeta_s \left( \frac{p_{st}}{P_t} \right)^{-\sigma} C_t^{\varepsilon_s}.
\]
Sectoral Demand is Log-Linear

- HH facing prices \( \{p_{st}\} \) with budget constraint \( \sum_s p_{st} c_{st} \leq E_t \).
- In terms of observables (marshallian)

\[
c_{st} = \zeta_s \left( \frac{p_{st}}{P_t} \right)^{-\sigma} \left( \frac{E_t}{P_t} \right)^{\varepsilon_s}
\]

and

\[
P_t = \left[ \sum_{s \in S} \left( \zeta_s p_{st}^{1-\sigma} \right)^{\chi_s} \left( x_{st} E_t^{1-\sigma} \right)^{1-\chi_s} \right]^{\frac{1}{1-\sigma}}
\]

where \( x_{st} = p_{st} c_{st} / E_t \) and \( \chi_s \equiv (1 - \sigma) / \varepsilon_s \).
Sectoral Demand is Log-Linear

- HH facing prices \( \{p_{st}\} \) with budget constraint \( \sum_s p_{st} c_{st} \leq E_t \).
- In terms of observables (marshallian)

\[
c_{st} = \zeta_s \left( \frac{p_{st}}{P_t} \right)^{-\sigma} \left( \frac{E_t}{P_t} \right)^{\epsilon_s}
\]

and

\[
P_t = \left[ \sum_{s \in S} \left( \zeta_s p_{st}^{1-\sigma} \chi_s \left( \chi_{st} E_t^{1-\sigma} \right)^{1-\chi_s} \right) \right]^{\frac{1}{1-\sigma}}
\]

where \( \chi_{st} = \frac{p_{st} c_{st}}{E_t} \) and \( \chi_s \equiv \frac{1 - \sigma}{\epsilon_s} \).

- Expenditure elasticity:

\[
\frac{\partial \ln c_{st}}{\partial \ln E_t} = \sigma + (1 - \sigma) \frac{\epsilon_s}{\sum_s \chi_{st} \epsilon_s}.
\]
We Close the Model Imposing Market Clearing

• The representative household spends all its income

\[ E_t = \sum_s \sum_j w_{jt} X_{jst}. \]

• Goods consumed in each sector need to be produced,

\[ VA_{st} = \zeta_s (p_{st}/P_t)^{-\sigma} (E_t/P_t)^{\epsilon_s}. \] \hspace{1cm} (1)

• Wage Bill for occupation \( j \) is

\[ w_{jt} X_{jt} = \sum_{s \in S} \alpha_{jst} VA_{st} = \sum_{s \in S} \alpha_{jst} \zeta_s E_t^{\sigma + \epsilon_s} p_{st}^{1-\sigma} P_t^{-\epsilon_s}. \] \hspace{1cm} (2)
We Close the Model Imposing Market Clearing

- The representative household spends all its income

\[ E_t = \sum_{s} \sum_{j} w_{jt} X_{jst}. \]

- Goods consumed in each sector need to be produced,

\[ VA_{st} = \zeta_s (p_{st}/P_t)^{-\sigma} (E_t/P_t)^{\varepsilon_s}. \] (1)

- Wage Bill for occupation \( j \) is

\[ w_{jt} X_{jt} = \sum_{s \in S} \alpha_{jst} VA_{st} = \sum_{s \in S} \alpha_{jst} \zeta_s E_t^{\sigma+\varepsilon_s} p_{st}^{1-\sigma} P_t^{-\varepsilon_s}. \] (2)

We use Equations (1) and (2) for quantifying bare-bones model
Initial Values

- Demand parameters $\{\epsilon_s, \sigma\}$ from CEX (as discussed).
- Demand parameters $\{\zeta_s\}$ to match VA shares 1980 (BEA).
- $\{\alpha_{s,1980}\}$ inferred from wage bill.
  - Hours worked from Census, wages from CPS.

Shock the 1980 Economy with 2016 Values

- Uniform increase in productivity, match increase in real PCE:
  - Compute same way as in BEAs with Fisher price indeces.
  - Hold relative sectoral prices to 1980.
- Change prices $\{p_{st}\}$ to match change in relative prices (BEA).
- Hold $\{\alpha_{s,1980}\}$ for now.
Quantification: Match 1980, then shock model to 2016

Initial Values

- Demand parameters \( \{\epsilon_s, \sigma\} \) from CEX (as discussed).
- Demand parameters \( \{\zeta_s\} \) to match VA shares 1980 (BEA).
- \( \{\alpha_{s,1980}\} \) inferred from wage bill.
  - Hours worked from Census, wages from CPS.

Shock the 1980 Economy with 2016 Values

- Uniform increase in productivity, match increase in real PCE:
  - Compute same way as in BEAs with Fisher price indeces.
  - Hold relative sectoral prices to 1980.
- Change prices \( \{p_{st}\} \) to match change in relative prices (BEA).
- Hold \( \{\alpha_{s,1980}\} \) for now.
We Are Allowing only Variation in $\Delta VA_{st}$

Back to the Wage Bill Decomposition

$$\frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{js0} \frac{\Delta VA_{st}}{VA_{s0}} + \sum_{s \in S} \gamma_{js0} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} + \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} \right]$$

where $\gamma_{js0} \equiv \frac{\alpha_{js0} VA_{s0}}{\sum_{s \in S} \alpha_{js0} VA_{s0}}$ are wage bill shares.
We Are Allowing only Variation in $\Delta VA_{st}$

Back to the Wage Bill Decomposition

\[
\frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{js0} \frac{\Delta VA_{st}}{VA_{s0}} + \sum_{s \in S} \gamma_{js0} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} + \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} \right]
\]

where $\gamma_{js0} \equiv \frac{\alpha_{js0} VA_{s0}}{\sum_{s \in S} \alpha_{js0} VA_{s0}}$ are wage bill shares.
We Are Allowing only Variation in $\Delta VA_{st}$

Back to the Wage Bill Decomposition

$$\frac{\Delta (w_{jt}X_{jt})}{w_{j0}X_{j0}} = \sum_{s \in S} \gamma_{js0} \frac{\Delta VA_{st}}{VA_{s0}} + \sum_{s \in S} \gamma_{js0} \frac{\Delta \alpha_{jst}}{\alpha_{js0}} + \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \right] p_s$$

where $\gamma_{js0} \equiv \frac{\alpha_{js0} VA_{s0}}{\sum_{s \in S} \alpha_{js0} VA_{s0}}$ are wage bill shares.

Zoom in Term 1: How important is Nonhomotheticity?

$$\sum_{s \in S} \gamma_{js0} \frac{\Delta VA_{st}}{VA_{s0}} = \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \right] E + \sum_{s \in S} \gamma_{js0} \left[ \frac{\Delta VA_{st}}{VA_{s0}} \right] p_s$$
## Quantification of the Mechanism

### Sectoral Growth Predictions

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>H−M</th>
<th>L−M</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Value Added growth</strong></td>
<td>11.53</td>
<td>3.53</td>
<td>7.20</td>
<td>7.60</td>
<td>3.68</td>
</tr>
<tr>
<td><strong>VA only growth (Term 1)</strong></td>
<td>7.47</td>
<td>4.95</td>
<td>8.13</td>
<td>2.52</td>
<td>3.19</td>
</tr>
<tr>
<td><strong>Predicted change of...</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Estimated Model</strong></td>
<td>7.11</td>
<td>3.92</td>
<td>7.89</td>
<td>3.18</td>
<td>3.97</td>
</tr>
<tr>
<td><strong>Increase in $E_t$</strong></td>
<td>5.57</td>
<td>4.02</td>
<td>5.93</td>
<td>1.55</td>
<td>1.91</td>
</tr>
<tr>
<td><strong>Growth in $p_{st}$</strong></td>
<td>0.16</td>
<td>-0.05</td>
<td>0.24</td>
<td>0.21</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Covariance</strong></td>
<td>1.38</td>
<td>-0.05</td>
<td>1.72</td>
<td>1.43</td>
<td>1.77</td>
</tr>
</tbody>
</table>
## Quantification of the Mechanism

### Sectoral Growth Predictions

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>H−M</th>
<th>L−M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Value Added growth</td>
<td>10.19</td>
<td>3.18</td>
<td>6.61</td>
<td>7.01</td>
<td>3.43</td>
</tr>
<tr>
<td>VA only growth (Term 1)</td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td>Predicted growth of...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation with (\uparrow E_t, p_{st})</td>
<td>6.20</td>
<td>3.63</td>
<td>7.16</td>
<td>2.57</td>
<td>3.53</td>
</tr>
</tbody>
</table>
## Quantification of the Mechanism

### Sectoral Growth Predictions

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>H–M</th>
<th>L–M</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Value Added growth</strong></td>
<td>10.19</td>
<td>3.18</td>
<td>6.61</td>
<td>7.01</td>
<td>3.43</td>
</tr>
<tr>
<td><strong>VA only growth (Term 1)</strong></td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td><strong>Predicted growth of...</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Simulation with ↑ E&lt;sub&gt;t&lt;/sub&gt;, p&lt;sub&gt;st&lt;/sub&gt;</strong></td>
<td>6.20</td>
<td>3.63</td>
<td>7.16</td>
<td>2.57</td>
<td>3.53</td>
</tr>
<tr>
<td><strong>↑ E&lt;sub&gt;t&lt;/sub&gt; Only</strong></td>
<td>5.64</td>
<td>3.91</td>
<td>6.24</td>
<td>1.73</td>
<td>2.33</td>
</tr>
<tr>
<td><strong>↑ p&lt;sub&gt;st&lt;/sub&gt; Only</strong></td>
<td>0.08</td>
<td>-0.06</td>
<td>0.16</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>Covariance</strong></td>
<td>0.48</td>
<td>-0.22</td>
<td>0.76</td>
<td>0.70</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Quantification of the Mechanism  ▶ Sectoral Growth Predictions

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>H–M</th>
<th>L–M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Value Added growth</td>
<td>10.19</td>
<td>3.18</td>
<td>6.61</td>
<td>7.01</td>
<td>3.43</td>
</tr>
<tr>
<td>VA only growth (Term 1)</td>
<td>7.05</td>
<td>4.66</td>
<td>7.09</td>
<td>2.39</td>
<td>2.43</td>
</tr>
<tr>
<td>Predicted growth of...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation with ( \uparrow E_t, p_{st} )</td>
<td>6.20</td>
<td>3.63</td>
<td>7.16</td>
<td>2.57</td>
<td>3.53</td>
</tr>
<tr>
<td>( \uparrow E_t ) Only</td>
<td>5.64</td>
<td>3.91</td>
<td>6.24</td>
<td>1.73</td>
<td>2.33</td>
</tr>
<tr>
<td>( \uparrow p_{st} ) Only</td>
<td>0.08</td>
<td>-0.06</td>
<td>0.16</td>
<td>0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>Covariance</td>
<td>0.48</td>
<td>-0.22</td>
<td>0.76</td>
<td>0.70</td>
<td>0.99</td>
</tr>
<tr>
<td>% Accounted, ( \uparrow ) in ( E_t )</td>
<td>95%</td>
<td>105%</td>
<td>92%</td>
<td>81%</td>
<td>80%</td>
</tr>
</tbody>
</table>

- If we assign half of covariance to \( E_t \), \( \uparrow E_t \) accounts for
  - 81% of H–M,
  - 80% of L–M.
Outline

1. Documenting Two Facts.
2. Bare-bones model and quantification of wage bill polarization.
   ▶ Production: CRS technology with only labor.
   ▶ Representative household.
   ▶ Allow for skill-biased tech. change and offshoring.
3. Full model: prices and quantities.
   ▶ Production: Introduce capital, DRS in labor.
   ▶ Occupational choice: wages and employment shares.
4. Extensions:
   ▶ Trade.
   ▶ Looking back and ahead, from 1950 to 2036.
5. Conclusion
Production Technologies

- Generalize production to

\[
Y_{st} = A_{st} K_{st}^{1-\beta_{st}} \left( \prod_{j \in \{H,M,L\}} \tilde{X}_{jst}^{\alpha_{jst}} \right)^{\beta_{st}},
\]

where \( \tilde{X}_{jst} \) denotes the number of efficiency units of labor are employed in occupation \( j \) in sector \( s \) in year \( t \).

- Demand is now

\[
\tilde{w}_{jt} \tilde{X}_{jst} = \beta_{st} \alpha_{jst} p_{st} Y_{st},
\]

\[
r_{t} K_{st} = (1 - \beta_{st}) p_{st} Y_{st}.
\]

- Total wage bill in sector \( s \) is

\[
\sum_{j=1}^{J} \tilde{w}_{jt} \tilde{X}_{jst} = \beta_{st} p_{st} Y_{st} \sum_{j=1}^{J} \alpha_{jst}.
\]
Demand Side

- Continuum of households indexed by $h$ from $(0,1)$.
- Each household inelastically supplies a unit of labor to one of the three occupations.
- Household income is composed of the labor income plus the rental income accrued from the capital it owns ($K_{ht}$).
- We assume that capital is evenly distributed across households.
- Every period household expenditure, $E_{ht}$, equals household income.
Preferences and Aggregate Demand

- Each household has nonhomothetic CES preferences as before.

\[ \sum_{s \in S} \left( \zeta_s U_{ht}^{\varepsilon_s} \right)^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} c_{ht} = 1. \]

- Aggregate demand for sectoral output is

\[ C_{st} = \int \zeta_s E_{ht}^{\sigma+\varepsilon_s} p_{st}^{-\sigma} P_{ht}^{-\varepsilon_s} dh. \]
Occupational Choice

- Each HH draws a vector \((\eta_L, \eta_M, \eta_H)\) of efficiency units in each occupation
  - Draws from iid log-normal.
- Price for each unit of skill: \((\tilde{w}_L, \tilde{w}_M, \tilde{w}_H)\).
- The optimal choice of the agent is to select occupation s.t.
  \[
  \max_{j \in \{L, M, H\}} \{\eta_j \tilde{w}_j\}.
  \]
Equilibrium and Overview of Quantification

- Study competitive equilibrium.
- Demand elasticities $\{\epsilon_s, \sigma\}$ estimated from HH expenditure survey (CEX).
- Use moments in the data for 1980 to set the values of the model parameters.
  - Sectoral prices and sectoral value added in 1980 come from the BEA.
  - $\{\zeta_s\}$ is set to match sectoral consumption in 1980.
- $\{\alpha_{st}, \beta_{st}\}$ that is set to match the sectoral wage bill in each sector in year $t$. 
Quantification

Initial Values

- Demand parameters $\{\epsilon_s, \sigma\}$ from CEX
- Demand parameters $\{\zeta_s\}$ to match VA shares 1980.
- Variance of log-normal for $M$ and $H$ to match relative wages in 1980.
Quantification

Initial Values

- Demand parameters $\{\epsilon_s, \sigma\}$ from CEX
- Demand parameters $\{\zeta_s\}$ to match VA shares 1980.
- Variance of log-normal for $M$ and $H$ to match relative wages in 1980.

Changes to the 1980 Economy

- Explore how different shocks bring us to 2016.
- Uniform increase in labor productivity to match increase in personal consumption expenditure:
  - Compute same way as in BEAs PCE with Fisher price indices.
- $\{\alpha_{st}, \beta_{st}\}$ change by period and sector from the data.
### Table 6: Full Quantitative Model

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_L$</th>
<th>$W_M$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\sum W_i L$</th>
<th>$\sum W_i M$</th>
<th>$\sum W_i H$</th>
<th>Exercise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.80</td>
<td>1.49</td>
<td>0.129</td>
<td>0.488</td>
<td>0.383</td>
<td>0.088</td>
<td>0.421</td>
<td>0.491</td>
</tr>
<tr>
<td>Model</td>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.095</td>
<td>0.653</td>
<td>0.252</td>
<td>0.068</td>
<td>0.630</td>
<td>0.302</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.86</td>
<td>1.44</td>
<td>0.133</td>
<td>0.543</td>
<td>0.324</td>
<td>0.101</td>
<td>0.483</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.77</td>
<td>1.41</td>
<td>0.095</td>
<td>0.582</td>
<td>0.323</td>
<td>0.066</td>
<td>0.524</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>2016</td>
<td>0.87</td>
<td>1.57</td>
<td>0.125</td>
<td>0.499</td>
<td>0.376</td>
<td>0.091</td>
<td>0.416</td>
<td>0.493</td>
</tr>
<tr>
<td>Fraction of observed change$^1$</td>
<td></td>
<td>2.17</td>
<td>1.32</td>
<td>0.88</td>
<td>0.93</td>
<td>0.95</td>
<td>1.15</td>
<td>1.02</td>
<td>1.01</td>
</tr>
<tr>
<td>Contribution of $E$</td>
<td></td>
<td>0.85</td>
<td>0.55</td>
<td>1.13</td>
<td>0.63</td>
<td>0.5</td>
<td>1.26</td>
<td>0.6</td>
<td>0.51</td>
</tr>
<tr>
<td>Contribution of $\alpha + \beta$</td>
<td></td>
<td>0.15</td>
<td>0.45</td>
<td>-0.13</td>
<td>0.37</td>
<td>0.5</td>
<td>-0.26</td>
<td>0.4</td>
<td>0.49</td>
</tr>
</tbody>
</table>

(1) Fraction of the change produced by the full model, with changes in the level of expenditures, factor intensities and in the sectoral labor shares relative to total changed observed in the data.

International Financial Statistics. $^{24}$

### 3.4 Results

The results from our baseline analysis are reported in Table 6. The first two rows report the actual values of the key variables in the data. As before, we study the wage bills of the three occupations but also the relative wages and the share of hours worked in each of the three occupations. Row 3 reports the model simulations for 1980 which, by design, match the data. Rows 4 to 6 report the values of the variables of interest for 2016 produced by the model in different simulations. We consider three exercises. The neutral increase in expenditure (row 4), a simultaneous increase in the sectoral factor intensities and in the sectoral labor shares as the one observed in the data (row 5), and the effect of conducting simultaneously both exercises (row 6). Row 7 reports the fraction of the observed change in each of the variables from 1980 to 2016 that the model produces. Rows 8 and 9 report the contributions of the neutral increase in expenditure and the change in factor intensities and labor shares in the evolution of each variable using the approach detailed in section (C) in the appendix.$^{25}$

$^{24}$We conduct robustness checks using the initial and final values as well as allowing the rental rate to vary over time.

$^{25}$Note that these contributions are relative to the change induced by the model when all the exogenous variables change.
Extensions

1. Introduce trade.
   ▶ Most action comes from services, which are non-traded.
   ▶ Correct total demand for sectoral net exports.

   ▶ Account for the rise of middle-class.
   ▶ Manufacturing was more of a luxury good in that period.

3. Other OECD countries.
   ▶ How much differences in levels of income account for different polarization experiences?

4. Depart from Cobb-Douglas with varying shares, \( \{\alpha_{st}\} \), use CES.
   ▶ Estimate sector specific ES across skills \( \sigma_s \)

\[
Y_s = \left( \gamma_1 H^\frac{\sigma_s - 1}{\sigma} + \gamma_2 M^\frac{\sigma_s - 1}{\sigma} + \gamma_3 L^\frac{\sigma_s - 1}{\sigma} \right) \frac{\sigma_s}{\sigma s - 1}.
\]

▶ IV: Bartik with immigrants in CZ and sector.
▶ Key Finding: Negative correlation between \( \{\sigma_s\} \) and \( \{\epsilon_s\} \).
Conclusions

- Sectoral growth between 1980-2016 is highly correlated with the distribution of employment in high and low skill occupations.
- One consequence of this new empirical finding is that changes in sectoral composition of output induced by increase in expenditures are a major driver of labor market polarization.
- Our mechanism explains around 100% of the changes in relative wage bill of low vs. medium occupations and 50% of high vs. medium.
- Extensions of basic analysis of workers captures substantial part of changes in employment and wages by occupation.
Thank you!
## Backward Exercise: 1950-1980

<table>
<thead>
<tr>
<th>year</th>
<th>$\frac{W_L}{W_M}$</th>
<th>$\frac{W_H}{W_M}$</th>
<th>$L_s$</th>
<th>$M_s$</th>
<th>$H_s$</th>
<th>$\frac{W_L L}{\sum_k W_k K}$</th>
<th>$\frac{W_M M}{\sum_k W_k K}$</th>
<th>$\frac{W_H H}{\sum_k W_k K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.087</td>
<td>0.664</td>
<td>0.248</td>
<td>0.062</td>
<td>0.640</td>
<td>0.298</td>
</tr>
<tr>
<td>1950</td>
<td>0.68</td>
<td>1.14</td>
<td>0.100</td>
<td>0.739</td>
<td>0.161</td>
<td>0.069</td>
<td>0.745</td>
<td>0.186</td>
</tr>
<tr>
<td>Sim.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.74</td>
<td>1.24</td>
<td>0.087</td>
<td>0.664</td>
<td>0.248</td>
<td>0.062</td>
<td>0.640</td>
<td>0.298</td>
</tr>
<tr>
<td>1950</td>
<td>0.65</td>
<td>1.14</td>
<td>0.059</td>
<td>0.729</td>
<td>0.212</td>
<td>0.038</td>
<td>0.721</td>
<td>0.240</td>
</tr>
<tr>
<td>1950</td>
<td>0.76</td>
<td>1.13</td>
<td>0.104</td>
<td>0.701</td>
<td>0.195</td>
<td>0.079</td>
<td>0.700</td>
<td>0.221</td>
</tr>
<tr>
<td>1950</td>
<td>0.70</td>
<td>1.04</td>
<td>0.087</td>
<td>0.755</td>
<td>0.159</td>
<td>0.062</td>
<td>0.770</td>
<td>0.168</td>
</tr>
</tbody>
</table>

*TFP*, $\alpha$, $\beta$, $\alpha$
## Sectoral Growth Predictions

<table>
<thead>
<tr>
<th></th>
<th>Neutral Increase in Expenditures</th>
<th>Increase in Relative Prices</th>
<th>Increase in Prices and Expenditures</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of data variance(^1)</td>
<td>0.73</td>
<td>0.058</td>
<td>1.06</td>
</tr>
<tr>
<td>% of model variance(^2)</td>
<td>0.82</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>

(1) Calculated as the covariance between the model generated growth rates and the growth rates observed in the data, relative to the variance of the growth rates observed in the data.

(2) Calculated as the covariance between the growth rates generated in the partial exercise and the growth rates generated in the full exercise, relative to the variance of the growth rates generated in the full exercise.
Sectoral Growth Predictions