Abstract

Previous work has shown that, in a liquidity trap, aggressive government spending cuts can be self-defeating in the short-run due to a higher-than-normal multiplier. A potentially serious drawback of the existing literature is the use of linearized models. Recently, Braun, Koerber and Waki (2012) and others claim that in a liquidity trap, a model can behave qualitatively different depending on whether it has been linearized or not. We examine their claim with a focus on whether fiscal austerity can be self-defeating - i.e. austerity causes government debt to rise due to adverse effects on aggregate demand. Specifically, we compare the government debt and output effects due to changes in fiscal spending in linearized and nonlinear general equilibrium models.

We start with a variant of the simple benchmark model in Woodford (2003), which allows us to carefully parse out the differences between the linear and nonlinear solutions. Finally, we examine the robustness of our results in the workhorse model of Christiano, Eichenbaum and Evans (2005) augmented with a financial accelerator mechanism.

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1. Introduction

In this paper, we assess the implications of taking model non-linearities explicitly into account when calculating fiscal multipliers in an environment when the zero lower bound of nominal interest rates is binding.

Recent work on the effects of fiscal stimulus suggests that the fiscal spending multiplier can be much higher when monetary policy is constrained by the ZLB, see e.g. Eggertsson (2010), Davig and Leeper (2011), Christiano, Eichenbaum and Rebelo (2011), Woodford (2011), Coenen et al. (2012) and Erceg and Linde (2012). Erceg and Linde (2012) show that spending hikes can be associated with a “fiscal free lunch” in a sufficiently long-lived liquidity trap. The flip side of this finding is that it is hard to reduce government debt in the short-run through aggressive spending cuts.

The bulk of the existing literature has analyzed fiscal multipliers in models that are linearized around the steady state (apart from the ZLB constraint on the monetary policy rule). The implicit assumption with this procedure is that the linearized solution is accurate even far away from the steady state. But recent work (see e.g. Braun, Koerber and Waki, 2012) suggests that analyses based on linearized supply and demand schedules might produce misleading results at the zero lower bound. Essentially, Braun et al. are arguing that extrapolations of decision rules far away from the steady state are invalid.

In this paper we address the following question: can fiscal austerity be self-defeating in a liquidity trap in a fully nonlinear environment? We undertake a positive analysis of the effects of spending-based fiscal consolidations on output and government debt. The modeling starting point is a variant of the workhorse New Keynesian DSGE model of Woodford (2003). This model features monopolistic competition and Calvo sticky prices and the central bank follows a Taylor rule subject to the ZLB constraint on nominal rates. We rule out the well-known problems associated with steady state multiplicity emphasized by Benhabib, Schmitt-Grohe and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. We document and analyze the key differences between the linearized and fully nonlinear solutions of this model.

Next, we examine the differences in multiplier schedules in an empirically plausible model developed by Christiano, Eichenbaum and Evans (2005) which we augmented with the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism. Our analysis allows us to study potential fiscal free lunches in a liquidity trap in a model which has a spending multiplier in line
with the VAR evidence in times when monetary policy is unconstrained. [More literature to be discussed: Christiano and Eichenbaum (2012), Fernandez-Villaverde et al. (2012).] In our analysis, we compare fiscal multipliers on output and debt in nonlinear and linearized representations of the model. We focus on features which account for the discrepancies between the nonlinear and linearized solution. Relative to the existing literature, we focus on the implications for government debt in a model with real rigidities. In particular, we introduce real rigidities through the Kimball (1995) state-dependent demand elasticity for the intermediate goods firms which allows our model to simultaneously account for the macroeconomic evidence of a low linearized Phillips curve slope (0.01) and the microeconomic evidence of frequent price re-optimization (3-4 quarters) at the same time.

Our analysis points toward important quantitative differences between output and debt multipliers in linearized and nonlinear DSGE models when the model is calibrated to reflect microeconomic evidence on the frequency of price changes only. More importantly, when the model is calibrated to account for macroeconomic evidence of the slope of the Phillips curve and microeconomic evidence on the frequency of price changes jointly, the quantitative differences between the linear and nonlinear model appear to be much smaller. Another key finding is that linearization of pricing equations accounts for the bulk of the differences between nonlinear and linearized solutions.

While the results in the nonlinear variant of the stylized model suggest that the multiplier schedule is not that different from normal times, the results in the workhorse model augmented with the financial accelerator mechanism suggest that the multiplier may be twice as high in a long-lived liquidity trap. Accordingly, this model suggests that fiscal austerity that is expected to be transient may be self-defeating. [Remains to be written.]

The remainder of this paper is organized as follows. Section 2 presents the stylized New Keynesian model and discusses how the model is parameterized. Section 3 presents our benchmark results. Section 4 examines the implications of a more empirically-realistic model, and Section 5 concludes.

2. The Stylized New Keynesian Model

The simple model we study is very similar to the one developed in Erceg and Linde (2012), which in turn is closely related to the model studied by Eggertsson and Woodford (2003). We deviate from Erceg and Linde (2012) by allowing for a Kimball (1995) aggregator (with the standard Dixit-Stiglitz specification as a special case) as well as a discount factor shock. Below, we outline the
model. In the appendix we describe the linear and non-linear versions in greater detail.

2.1. The Model

2.1.1. Households

The utility functional for the representative household is

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \delta_t \left\{ \frac{1}{1-\frac{1}{\sigma}} \left( C_{t+j} - C_{\nu_{t+j}} \right)^{1-\frac{1}{\sigma}} - \frac{N_{t+j}^{1+\chi}}{1+\chi} + \mu_0 F \left( \frac{MB_{t+j+1}(h)}{P_{t+j}} \right) \right\}
\]  

(1)

where the discount factor \( \beta \) satisfies \( 0 < \beta < 1 \) and is subject to an exogenous component \( \delta_t \). The period utility function depends on the household’s current consumption \( C_t \) as deviation from a “reference level” \( C_{\nu_{t+j}} \), where the exogenous positive taste shock \( \nu_t \) raises this reference level and thus the marginal utility of consumption associated with any given consumption level. The period utility function also depends inversely on hours worked \( N_t \). Following Eggertsson and Woodford (2003), the subutility function over real balances, \( F \left( \frac{MB_{t+j+1}(h)}{P_{t+j}} \right) \), is assumed to have a satiation point for \( MB/P \). Hence, inclusion of money — which is a zero nominal interest asset — provides a rationale for the zero lower bound on nominal interest rates. However, we maintain the assumptions that money is additive and that \( \mu_0 \) is arbitrarily small so that changes in real money balances have negligible implications for seigniorage. Together, these assumptions imply that we can disregard the implications of money for government debt and output.

The household’s budget constraint in period \( t \) states that its expenditure on goods and net purchases of (zero-coupon) government bonds \( B_{G,t} \) must equal its disposable income:

\[
P_t \left( 1 + \tau_{C,t} \right) C_t + B_{G,t} + MB_{t+1} = (1 - \tau_{N,t}) W_t N_t + (1 + i_{t-1}) B_{G,t-1} + MB_t - T_t + \Gamma_t
\]

(2)

Thus, the household purchases the final consumption good (at a price of \( P_t \)) and subject to a sales tax \( \tau_{C,t} \). Each household earns after-tax labor income \( (1 - \tau_{N,t}) W_t N_t \) (\( \tau_{N,t} \) denotes the tax rate), pays a lump-sum tax \( T_t \) (this may be regarded as net of any transfers), and receives a proportional share of the profits \( \Gamma_t \) of all intermediate firms.

In every period \( t \), the household maximizes the utility functional (B.11) with respect to its consumption, labor supply and bond holdings. Forming the Lagrangian and computing the first-order conditions w.r.t. \( [C_t \ N_t \ B_{G,t}] \), we obtain the standard consumption Euler equation

\[
\frac{(C_t - C_{\nu_t})^{-\frac{1}{\sigma}}}{(1 + \tau_{C,t})} = \beta \delta_{t+1} E_t \frac{(1 + \delta_t)}{1 + \pi_{t+1}} \frac{(C_{t+1} - C_{\nu_{t+1}})^{-\frac{1}{\sigma}}}{(1 + \tau_{C,t+1})},
\]

(3)
where we have defined
\[
\delta_{t+1} = \frac{s_{t+1}}{s_t}
\]  
(4)
and introduced the notation \(1 + \tau_{t+1} = P_{t+1}/P_t\). We also have the following labor supply schedule
\[
\frac{N_t^\lambda}{(C_t - C_{t+1})^{-\frac{1}{\gamma}}} = \frac{(1 - \tau_{N,t}) W_t}{(1 + \tau_{C,t}) P_t}. 
\]  
(5)

Equations (3) and (5) are the key equations for the household side of the model.

2.1.2. Firms and Price Setting

**Final Goods Production** The single final output good \(Y_t\) is produced using a continuum of differentiated intermediate goods \(Y_t(f)\). Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is
\[
\int_0^1 G_Y \left( \frac{Y_t(f)}{Y_t} \right) df = 1. 
\]  
(6)

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), we assume that \(G_Y(.)\) is given by the following strictly concave and increasing function:
\[
G_Y \left( \frac{Y_t(f)}{Y_t} \right) = \left( \frac{\phi_p}{1 - (\phi_p - 1)\epsilon_p} \right) \left( \frac{\phi_p + (1 - \phi_p)\epsilon_p}{\phi_p} \right) \frac{Y_t(f)}{Y_t} + \left( \frac{\phi_p - 1 - \epsilon_p}{\phi_p} \right) \frac{1 - (\phi_p - 1)\epsilon_p}{1 - (\phi_p - 1)\epsilon_p}, 
\]  
(7)

where \(\phi_p \geq 1\) denotes the gross markup of the intermediate firms. The parameter \(\epsilon_p\) governs the degree of curvature of the intermediate firm’s demand curve. When \(\epsilon_p = 0\), the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When \(\epsilon_p\) is positive—as in SW07—the firm’s instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, we notice that \(G_Y(1) = 1\), implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index \(Y_t\), taking as given the price \(P_t(f)\) of each intermediate good \(Y_t(f)\). Moreover, final goods producers sell units of the final output good at a price \(P_t\), and hence solve the following problem:
\[
\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df 
\]  
(8)
subject to the constraint (6). Note that for \( \epsilon_p = 0 \), this problem leads to the usual expressions

\[
\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{\phi_p}{\phi_p - 1}}, \quad P_t = \left[ \int P_t(f) \frac{1}{\phi_p} df \right]^{1-\phi_p}
\]

**Intermediate Goods Production** A continuum of intermediate goods \( Y_t(f) \) for \( f \in [0, 1] \) is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in (8) that varies inversely with its output price \( P_t(f) \) and directly with aggregate demand \( Y_t \).

Aggregate capital \((K)\) is assumed to be fixed, so that aggregate production of the intermediate good firm is given by

\[
Y_t(f) = K(f)^{\alpha} N_t(f)^{1-\alpha}.
\]  

(9)

Despite the fixed aggregate stock \( K \equiv \int K(f)df \), shares of it can be freely allocated across the \( f \) firms, implying that real marginal cost, \( MC_t(f)/P_t \) is identical across firms and equal to

\[
\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{(1-\alpha)K^\alpha N_t^{-\alpha}},
\]

(10)

where \( N_t = \int N_t(f)df \).

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm \( f \) faces a constant probability, \( 1 - \xi_p \), of being able to reoptimize its price \( P_t(f) \). The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price according to the following formula

\[
\tilde{P}_t = (1+\pi)P_{t-1},
\]

(11)

where \( \pi \) is the steady-state (net) inflation rate and \( \tilde{P}_t \) is the updated price.

Given Calvo-style pricing frictions, firm \( f \) that is allowed to reoptimize its price \( P_t^{opt}(f) \) solves the following problem

\[
\max_{P_t^{opt}(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \varsigma_{t+j} \Lambda_{t+j} \left[ (1+\pi)^j P_t^{opt}(f) - MC_{t+j} \right] Y_{t+j}(f)
\]

where \( \Lambda_{t+j} \) is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand \( Y_{t+j}(f) \) from the final goods
firms is given by:

\[ Y_{t+j}(f) = \frac{\phi_p}{\phi_p - (\phi_p - 1)\epsilon_p} \left( \frac{(1 + \pi)^j P_t^{opt}(f) \left( \frac{1}{\phi_p} \right)}{Y_{t+j}} \right) - \frac{((\phi_p - 1)\epsilon_p)}{\phi_p} \left( \frac{1 - \phi_p}{\phi_p} \right) Y_{t+j}, \]  

(12)

where \( \theta_{t+j} \) is the Lagrangian multiplier from the final good firms problem (8).

### 2.1.3. Monetary and Fiscal Policies

The evolution of nominal government debt is determined by the following equation

\[ B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau_{C,t} P_t C_t - \tau_{N,t} W_t N_t - T_t \]

(13)

where \( G_t \) denotes real government expenditures on the final good \( Y_t \). Scaling with \( 1/(P_t Y_t) \), we obtain

\[ \frac{B_{G,t}}{P_t Y_t} = \frac{(1 + i_{t-1}) B_{G,t-1}}{(1 + \pi_t)} + \frac{G_t}{P_{t-1} Y_t} - \tau_{C,t} \frac{C_t}{Y_t} - \tau_{N,t} \frac{W_t N_t}{P_t Y_t} - \frac{T_t}{P_t Y_t}. \]

(14)

Following the convention in the literature on fiscal multipliers, we start out by assuming that lump-sum taxes stabilize the evolution of government debt (as share of nominal trend GDP, \( b_{G,t} \equiv \frac{B_{G,t}}{P_t Y_t} \)). Specifically, we follow Erceg and Linde (2012) and assume that lump-sum taxes as share of nominal trend GDP, \( \tau_t \equiv \frac{T_t}{P_t Y_t} \), follow the simple rule

\[ \tau_t - \tau = \varphi_b \left( b_{G,t-1} - b_G \right) \]

(15)

Government spending, \( g_{y,t} \equiv \frac{G_t}{Y_t}, \tau_{C,t} \) and \( \tau_{N,t} \) are kept exogenous.

Turning to the central bank, it is assumed to adhere to a Taylor-type policy rule that is subject to the zero lower bound:

\[ 1 + i_t = \max \left( 1, (1 + i) \left[ \frac{1 + \pi_t}{1 + \pi} \right]^{\gamma_{\pi}} \left[ \frac{Y_t}{Y_t^{pot}} \right]^{\gamma_x} \right) \]

(16)

where \( Y_t^{pot} \) denotes the level of output that would prevail if prices were flexible, and \( i \) the steady-state (net) nominal interest rate, which is given by \( r + \pi \) where \( r \equiv 1/\beta - 1 \). In the linearized model, (16) is written

\[ i_t = \max (0, i + \gamma_{\pi} (\pi_t - \pi) + \gamma_x x_t) \]

where \( x_t \equiv \ln \left( Y_t / Y_t^{pot} \right) \).
2.1.4. The Aggregate Resource Constraint

We now turn to discuss the derivation of the aggregate resource constraint. Let $Y_{t}^{\text{sum}}$ denote the unweighted average (sum) of output for each firm $f$, i.e.

$$Y_{t}^{\text{sum}} = \int_{0}^{1} Y_{t}(f) df,$$

which from (9) and the fact that all firms have the same capital-labor ratio can be rewritten as

$$Y_{t}^{\text{sum}} = \int \left( \frac{K(f)}{N_{t}(f)} \right)^{\alpha} N_{t}(f) df$$
$$= \left( \frac{K}{N_{t}} \right)^{\alpha} \int N_{t}(f) df$$
$$= K^{\alpha} N_{t}^{1-\alpha} \tag{17}$$

Recalling that $Y_{t+j}(f)$ is given from (12), it follows that

$$Y_{t}^{\text{sum}} = Y_{t} \int_{0}^{1} \phi_{p} \left( \frac{P_{t}(f) \gamma_{t}}{P_{t}} \right)^{\frac{\phi_{p} - (\phi_{p} - 1)\gamma_{p}}{\phi_{p}} + \frac{(1-\phi_{p})\gamma_{p}}{\phi_{p}}} df,$$

or equivalently, using (17):

$$Y_{t} = (p_{t}^{*})^{-1} K^{\alpha} N_{t}^{1-\alpha}, \tag{18}$$

where

$$p_{t}^{*} \equiv \int_{0}^{1} \phi_{p} \left( \frac{P_{t}(f) \gamma_{t}}{P_{t}} \right)^{\frac{\phi_{p} - (\phi_{p} - 1)\gamma_{p}}{\phi_{p}} + \frac{(1-\phi_{p})\gamma_{p}}{\phi_{p}}} df.$$

In the technical appendix, we show how to develop a recursive formulation of the sticky price distortion term $p_{t}^{*}$.

Now, because actual output $Y_{t}$ is what is available for private consumption and government spending purposes, it follows that:

$$C_{t} + G_{t} \leq \underbrace{Y_{t}}_{\equiv Y_{t}^{\text{sum}}} \tag{19}$$

The sticky price distortion clearly introduces a wedge between input use and the output available for consumption (including by the government).\footnote{As the economy is assumed to be endowed with the fixed aggregate capital stock $K$ which does not depreciate, no resources is devoted to investment. An alternative formulation would have embodied a constant capital depreciation rate in which case output would have been used for $C_{t}$, $I$ and $G_{t}$.} Even so, this term vanishes in the log-linearized version of the model.
2.2. Parameterization

Our benchmark calibration – essentially adopted from Erceg and Linde (2012) – is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .01$ (i.e., four percent at an annualized rate). We set the intertemporal substitution elasticity $\sigma = 1$ (log utility), the capital share parameter $\alpha = 0.3$, the Frisch elasticity of labor supply $\frac{1}{\lambda} = 0.4$, and the steady state value for the consumption taste shock $\nu = 0.01$.\footnote{By setting the steady value of the consumption taste shock to a small value, we ensure that the dynamics for alternative shocks are roughly invariant to the presence of $-C\nu t$ in the period consumption utility function.} As a compromise between the low estimate of $p$ in Altig et al. (2011) and the higher estimated value by Smets and Wouters (2007), we set $\phi_p = 1.1$. This leaves us with two additional deep parameters to pin down; the price contract duration parameter $\xi_p$, and the Kimball elasticity demand parameter $\epsilon_p$. To pin down these parameters, our starting point is the New Keynesian Phillips Curve

$$\pi_t - \pi = \beta (E_t \pi_{t+1} - \pi) + \kappa_{mc} \bar{mc}_t, \tag{20}$$

which obtains in our model where $\bar{mc}_t$ denotes marginal cost as log-deviation from its steady state value. The parameter $\kappa_{mc}$, i.e. the slope of the Phillips curve, is given by

$$\kappa_{mc} \equiv \frac{(1-\xi_p)(1-\beta \xi_p)}{\xi_p} \frac{1}{1+\phi_p-1}\epsilon_p. \tag{21}$$

A large body of microeconomic evidence, see e.g. Klenow and Malin (2010) and Nakamura and Steinsson (2012) and the references therein, suggest that firms change their prices rather frequently, on average somewhat more often than once a year. Based on this micro evidence, we set $\xi_p = 0.667$, implying an average price contract duration of 3 quarters ($\frac{1}{1-0.667}$). On the other hand, the macroeconomic evidence suggest that the sensitivity of aggregate inflation to variations in marginal cost is very low, see e.g. Altig et al. (2011). To capture this, we adopt a value for $\epsilon_p$, so that the slope of the Phillips curve ($\kappa_{mc}$) – given our adopted values for $\beta$, $\xi_p$ and $\phi_p$ – equals 0.012.\footnote{The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolffson et al (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of 0.009 – .014.} This calibration allows us to match both the micro- and macroevidence on price setting behavior and is aimed at capturing the resilience of core inflation, and measures of expected inflation, during the recent global recession.

We assume a government debt to annualized output ratio of 0.6 (consistent with U.S. pre-crisis federal debt level), implying a quarterly value for $bG = 2.4$. From (14), the steady labor income
tax rate $\tau_N$ equals
\[
\tau_N = \left( \frac{\phi_{\nu}}{1 - \alpha} \right) (r \times b_G + g_y - \tau_C (1 - g) - \tau).
\]
Under the additional assumptions that $\tau_C = 0$, the government consumption share of steady state output $g_y = 0.2$, and that net lump-sum taxes $\tau = 0$, the above steady state relationship implies $\tau_N = 0.33$, i.e. an average labor income tax of 33 percent. The parameter $\varphi_b$ in the tax rule (15) is set equal to 0.01, which implies that the contribution of lump-sum taxes to the response of government debt is extremely small in the first couple of years following a shock (so that almost all variation in tax revenue comes from fluctuations in labor tax revenues). For monetary policy, we use the standard Taylor (1993) rule parameters $\gamma_\pi = 1.5$ and $\gamma_x = .125$.

In order to facilitate comparison between the nonlinear and linear model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, the preference, discount and government spending shocks are assumed to follow AR(1) processes:
\[
\begin{align*}
\left( \frac{G_t - G}{G} \right) &= \rho_G \left( \frac{G_{t-1} - G}{G} \right) + \sigma_G \varepsilon_{G,t}, \\
(\nu_t - \nu) &= \rho_\nu (\nu_{t-1} - \nu) + \sigma_\nu \varepsilon_{\nu,t}, \\
\left( \frac{\delta_t - \delta}{\delta} \right) &= \rho_\delta \left( \frac{\delta_{t-1} - \delta}{\delta} \right) + \sigma_\delta \varepsilon_{\delta,t}.
\end{align*}
\]
Our baseline parameterization of these processes adopts a persistence coefficient of 0.95, so that $\rho_\nu = \rho_G = \rho_\delta = 0.95$ in (22). But following some prominent papers in the literature on fiscal multipliers, we also investigate the sensitivity of our results when the processes are assumed to be general Markov processes. Those results are reported in Appendix A.

2.3. Solving the Model

We confine ourself to study perfect foresight simulations, i.e. solutions where uncertainty about future shock realizations is irrelevant for the dynamics of the economy. While other papers — see for instance Adam and Billi (2006, 2007) within a linearized framework and Fernández-Villaverde et al. (2012) and Gust, López-Salido and Smith (2013) within a nonlinear framework — have shown that allowing for uncertainty can potentially have important implications for equilibrium dynamics, we nevertheless choose not to do so for the following two main reasons. First, because the bulk of the existing literature have used a perfect foresight approach, retaining this approach allows us to parse out the effects of going from a linearized to a nonlinear framework. Second, the perfect
foresight assumption allows us to readily study the robustness in a larger scale model with many state variables. So far, the solution algorithms used to solve models with shock uncertainty have typically not been applied to models with more than 4-5 state variables.\textsuperscript{4}

To solve the model, we feed the relevant equations in the nonlinear and log-linearized versions of the model to Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve nonlinear models with forward looking variables. For perfect foresight simulations like ours, Dynare uses a Newton-type algorithm, and the details of the algorithm used can be found in Juillard (1996). For the linearized model, we used the algorithm outlined in Hebden, Linde and Svensson (2012) to check for uniqueness. However, for the nonlinear version of the model, we cannot rule out the possibility that there exists other solutions in addition to the one found by Dynare. We note, however, that this problem pertains to all papers in the literature which study nonlinear models.

\section{Results for the Stylized Model}

In this section, we report our main results in the linearized and non-linear solution of the model outlined in the Section above. We start out by reporting how we construct the baseline scenarios and then report the marginal fiscal multipliers.

\subsection{Baseline Scenario}

As mentioned earlier, our aim is to compare fiscal spending multipliers in linearized and nonlinear versions of the model economy. Specifically, we seek to characterize how the difference between the multiplier in the linear and nonlinear frameworks varies with the expected duration of the liquidity trap.

To construct a baseline where the interest rate is bounded at zero for $\text{ZLB}_{DUR} = 1, 2, 3, ..., T$ periods, we follow the previous fiscal multiplier literature (e.g. Christiano, Eichenbaum and Rebelo, 2011) and assume that the economy is hit by a large adverse shock that triggers a deep recession and drives interest rates to zero. The larger value of $\text{ZLB}_{DUR}$ we want to have, the larger the adverse shock has to be. The particular shock we consider is a negative consumption taste shock $\nu_t$ (see 1 and 22) following Erceg and Linde (2014), but we present results in Appendix A when the recession is instead assumed to be triggered by the discount factor shock $\delta_t$ that was used in

\footnote{A recent paper by Judd, Maliar and Maliar (2011) provides a promising avenue to compute the stochastic solution of larger scale models efficiently.}
the seminal papers by Eggertsson and Woodford (2003), and Christiano, Eichenbaum and Rebelo (2011).

To provide clarity on how we pick the shock sizes, Figure 1 reports how the linear and nonlinear specifications react to the same negative taste shock (depicted in the bottom right panel). The economy is in the deterministic steady state in period 0, and then the shock hits the economy in period 1. As is evident from Figure 1, the same-sized shock has a rather different impact on the economy depending on whether the model is linearized or solved in its original nonlinear form. For instance, we see from panel 3 that while the nominal interest rate is bounded by zero from periods 1 to 8 in the linearized model, the equally-sized consumption demand shock (panel 9) only generates a two quarter trap in the nonlinear model. Hence, we need to subject the nonlinear model to a more negative consumption demand shock — as shown in panel 9 in Figure 2 — to generate $\text{ZLB}_{\text{DUR}} = 8$ for the interest rate (panel 3).

A lot of intuition about the differences between the linearized and nonlinear variants can be gained from Figures 1 and 2. Starting with Figure 1, we see from the fifth panel that the potential real rate falls roughly about the same in both models. Still, the linearized model generates a much longer liquidity trap because inflation and expected inflation falls much more (panel 2), which in turn causes the real interest rate (panel 4) to rise much more initially. The larger initial rise in the real interest rate triggers a larger fall in the output gap (panel 1) and consequently real GDP falls more in the linearized model as well (because the impact on potential GDP is about the same, as implied by the similarity of the potential real interest rate response).

Turning to Figure 2, we first note from the third panel that the paths for the policy rate are bounded at zero for 8 quarters and display a very similar path upon exit from the liquidity trap. Moreover, panel 9 shows that it takes a much larger adverse consumption demand shock in the nonlinear model to trigger a liquidity trap of the same expected duration as in the linearized model. This implies that the drop in the potential real rate and real GDP (panels 5 and 7) is much more severe in the nonlinear model. Even so, and perhaps most important, we see that inflation — panel 2 — falls substantially less in the nonlinear model. This suggests that the difference between the linearized and nonlinear version of the model too a large extent is driven by the linearization of the pricing block of the model.

---

5 Figure 2 also depicts a third line ("Nonlinear model with linearized price block"), which we will discuss further in Section 3.2.
3.2. Marginal Fiscal Multipliers

As previously noted, we are seeking to compare fiscal multipliers in liquidity traps of same expected duration in the linearized and nonlinear frameworks. Accordingly, we allow for differently sized shocks in the linearized and nonlinear models so that each model variant generates a liquidity trap with the same expected duration \( ZLB_{DUR} = 1, 2, 3, ..., T \). Let \( B_t^{\text{linear}}(\sigma_{\nu,i}^{\text{linear}}) \) and \( B_t^{\text{nonlin}}(\sigma_{\nu,i}^{\text{nonlin}}) \) denote vectors with simulated variables in the linear and nonlinear models, respectively. The baseline paths are functions of the size of the consumption demand shock \( \nu_t, \sigma_\nu \), which as explained in the previous section are set so that

\[
\sigma_{\nu,i}^{\text{linear}} \Rightarrow ZLB_{DUR} = i,
\]

and

\[
\sigma_{\nu,i}^{\text{nonlin}} \Rightarrow ZLB_{DUR} = i,
\]

where we consider \( i = 1, 2, ..., T \). In the specific case of \( i = 8 \), panel 9 in Figure 2 shows that \( \sigma_{\nu,8}^{\text{linear}} = -0.18 \) and \( \sigma_{\nu,8}^{\text{nonlin}} = -0.42 \).

To these different baseline paths, we add the fiscal response in the first period the ZLB binds; that is, the same period as the adverse shock hits (\( t = 1 \)). By letting \( S_t^{\text{linear}}(\sigma_{\nu,i}^{\text{linear}}, \sigma_G) \) and \( S_t^{\text{nonlin}}(\sigma_{\nu,i}^{\text{nonlin}}, \sigma_G) \) denote vectors with simulated variables in the linear and nonlinear models when both the negative baseline shock \( \sigma_\nu \) and the positive government spending shock \( \sigma_G \) hits the economy, we can compute the partial impact of the fiscal spending shock as

\[
I_j^t(ZLB_{DUR}) = S_t^j(\sigma_{\nu,i}^j, \sigma_G) - B_t^j(\sigma_{\nu,i}^j)
\]

for \( j = \{\text{linear}, \text{nonlin}\} \) and where we write \( I_j^t(ZLB_{DUR}) \) to highlight its dependence on the liquidity trap duration. Notice that the fiscal spending shock is the same for all \( i \) and is scaled so that \( ZLB_{DUR} \) remains unaffected. By setting the fiscal impulse so that the liquidity trap duration remains unaffected, we retrieve “marginal” spending multipliers in the sense that they show the impact of a “tiny” change in the fiscal instrument.\(^6\)

In Figure 3 we report the results of our exercise. The upper panels report results for the benchmark calibration with the Kimball aggregator. The lower panels report results under the Dixit-Stiglitz aggregator, in which case \( \epsilon_p = 0 \). This parametrization implies a substantially higher slope of the linearized Phillips curve (see 21) and thus a much stronger sensitivity of expected

\[\text{\footnotesize{(6) Had we considered a larger fiscal intervention that altered the duration of the liquidity trap, there would have been an important distinction between the average (i.e. the total response) and marginal (i.e. the impact of a small change in } g_t \text{ which leaves } ZLB_{DUR} \text{ unchanged) multiplier as discussed in further detail in Erceg and Linde (2014).}}\]
inflation to current and expected future marginal costs (and output gaps). We will first discuss the results under the Kimball parameterization, and then turn to the results under Dixit-Stiglitz.

The left panels report the output-spending multiplier on impact, i.e. simply

\[ m_i = \frac{1}{g_y} \frac{\Delta Y_{t,i}}{\Delta G_{t,i}} \]

where the \( \Delta \)-operator represents the difference between the scenario with the spending change and the baseline without the spending change. We compute \( m_i \) for \( \text{ZLB}_{Dur} = 1, \ldots, 12 \), but also include results for the case when the economy is at the steady state, so that \( \text{ZLB}_{Dur} = 0 \).

As the linear approximation should be more accurate the closer the economy is to the steady state, it is not surprising that the difference between the “linear” and “nonlinear” multiplier increases with the duration of the liquidity trap. For a three year liquidity trap, the recorded multiplier is more than twice as large in the linearized model relative to the nonlinear model. For shorter-lived liquidity traps, the differences are notably more modest, and in the special case when the economy is in the steady state (\( \text{ZLB}_{Dur} = 0 \) in the figure) we note that the multipliers are identical (as they should) in both economies. The difference in government debt (as share of actual annualized GDP) response after 1 year, shown in the upper right panel, largely follows the pattern for \( m_i \) and increases with \( \text{ZLB}_{Dur} \).

The substantial differences in the output and debt responses begs the question of which factors account for them. The middle upper panel, which shows the response of the one-period ahead expected annualized inflation rate (i.e., \( 4E_t \pi_{t+1} \)), sheds some light on this. As can be seen from the panel, expected inflation responds much more in a long-lived trap in the linearized model than in the nonlinear model. The sharp increase in expected inflation triggers a larger reduction in real rates in the linearized model, and thereby induces a more favorable response of private consumption which helps to boost output relative to the nonlinear model.

Turning to the Dixit-Stiglitz case shown in the lower panels, we see that they are qualitatively similar to the results under the Kimball aggregator, although the differences between the linear and nonlinear models are even more pronounced in this case, with the multiplier in an 8-quarter trap being over 5 times larger than in the nonlinear model. This result is to a large extent driven by the fact that we are in effect allowing a substantially higher slope (i.e. \( \kappa_{mc} \)) of the New Keynesian Phillips curve in eq. (20) in the Dixit-Stiglitz case. Taken together, the results in Figure 3 suggest

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7 For ease of interpretability, we have normalized the response of debt and inflation so that they correspond to a initial change in government spending (as share of steady state output) by one percent.

8 We only show results up to 8 quarters with the Dixit-Stiglitz aggregator to be able to show the differences more clearly in the graph.
that the findings of the papers in the previous literature which relied on linearized models were more distorted to the extent that they relied on a calibration with a higher slope of the Phillips curve and thus a larger sensitivity of expected inflation.

The key question is then why expected inflation responds so much more in the linearized economy, and particularly so in the Dixit-Stiglitz case? To shed light on this, we simulated two additional variants of the nonlinear model. In the first, we linearized the pricing equations of the model, e.g. replaced all pricing equations in the nonlinear model with the standard linearized Phillips curve. In the second, we linearized all the pricing equations and removed the price distortion term from the aggregate resource constraint (19). Following the approach with the linear and nonlinear models, we construct baseline scenarios for the two additional variants of the model as described in section 3.1 for $ZLB_{DUR} = 1, ..., 12$. The blue dash-dotted line in Figure 2 depicts the eight quarter liquidity trap baseline in the variant with linearized pricing equations and resource constraint (second additional variant described above). Clearly, the simulated paths in this model are very similar to those in the linearized model. Hence, it is perhaps not that surprising that the results in Figure 4 for this model (blue dashed-dotted line, referred to as “Linearized Resource Constraint and NKPC”) also display a striking similarity with the linearized model. Hence, we draw the conclusion that it is the linearization of the resource constraint and the Phillips curve (20), and not the aggregate demand part of the model, which account for the bulk of the differences between the effects of fiscal spending in a long-lived liquidity trap in the linear and nonlinear models under Kimball. In fact, as suggested by the green dash-dotted line in the upper panels of Figure 4, it is sufficient to only linearize the NKPC to account for most of the discrepancy between the linearized and nonlinear solution under the Kimball aggregator.

Even so, we see from the lower panels in Figure 4 that under the Dixit-Stiglitz parameterization of the model, log-linearization of the New Keynesian Phillips curve only is not sufficient to explain the large discrepancies between the linear and nonlinear solutions. Also accounting for the price distortion in the aggregate resource constraint is necessary (i.e. log-linearizing the constraint). The reason for the difference between the Kimball and the Dixit-Stiglitz specifications is that the price distortion variable moves much more for the latter specification, as the re-optimizing firms will adjust their prices much more under Dixit-Stiglitz compared to Kimball when we just change $\epsilon_p$, and thus implicitly give the firms the incentive to adjust prices more when they re-optimize under the Dixit-Stiglitz aggregator (see 21). So in a world where firms adjust prices a lot when they re-optimize, the bulk of the differences is driven by the price distortion, whereas in a world where
firms adjust prices by little, the bulk of the differences is driven by the pricing equations directly. Braun, Körber and Waki (2013) suggest that the key is accounting for the price distortion, and their claim is quite reasonable given that they are considering a model closer to our Dixit-Stiglitz parameterization.

To further tease out the difference between Kimball vs. Dixit-Stiglitz variants of the model, Panel A in Figure 5 compares outcomes when the sticky price parameter $\xi_p$ is adjusted in the Dixit-Stiglitz version so that the slope of the linearized Phillips curve (20) is the same as in our benchmark calibration although the Kimball elasticity $\epsilon_p = 0$. Both the Kimball and Dixit-Stiglitz versions hence feature a linearized Phillips curve with an identical slope coefficient ($\kappa_{mc} = 0.012$, see 21), but the Dixit-Stiglitz version of the model achieves this with a substantially higher value of $\xi_p$ (0.90). However, since only the value of $\kappa_{mc}$ matters in linearized versions of the model the multiplier schedules for the linearized models are invariant w.r.t. the mix of $\xi_p$ and $\epsilon_p$ that achieves a given $\kappa_{mc}$, and thus given given by the solid black line in the upper panel in Figure 3. But in the nonlinear versions of the model, shown in Panel A in Figure 5, the results differ. In particular, we see that when the Dixit-Stiglitz aggregator implies that expected inflation and output multiplier responds more when the duration of the liquidity trap increases relative to the benchmark Kimball variant of the model. Thus, when the Kimball parameter $\epsilon_p$ is reduced, the more will expected inflation and output multiplier respond when ZLB$_{DUR}$ increases; conversely, increasing $\epsilon_p$ and lowering $\xi_p$ flattens the output multiplier schedule even more. Our intuition behind these results is that a higher value of $\epsilon_p$ induces the elasticity of demand to vary more with the relative price differential among the intermediate good firms; and this price differential increases when the economy is far from the steady state. Thus, intermediate firms which only infrequently are able to re-optimize their price will optimally choose to respond less to a given fiscal impetus far from the steady state when price differentials are larger as they may experience a much larger impact on their demand for a given change in their relative price. As a result, aggregate current and expected inflation are less affected far from the steady state relative to the Dixit-Stiglitz case for which the elasticity of demand is independent of relative price differentials. This suggests within a nonlinear framework, explicit modeling of price frictions may be important, at least when nominal wages are flexible.

So far, we have followed the convention in the literature and assumed that non-optimizing firms index their prices w.r.t. the steady state inflation rate, see eq. (11). While this is a convenient modelling assumption as it simplifies the analysis by resulting in no steady state price distortions,
there is an important issue to what extent it matters, especially in the nonlinear model. To examine this, we respecify the model, assuming no indexation among the non-optimizing firms, i.e.

$$\bar{P}_t = P_{t-1}. \quad (23)$$

In Panel B in Figure 5, we report the results when comparing the nonlinear baseline model (black solid line, which features indexation) with the nonlinear variant without indexation for the non-optimizing firms (red dotted line). From the panels, we see that abandoning the conventional assumption of full indexation results in a somewhat steeper multiplier schedule, partly explained by the higher sensitivity of expected inflation in the “no-indexation” model. [Need to sort out whether this finding is driven by (a) the firms having stronger incentives to respond to variations in marginal costs far off the steady state under no-indexation, or (b), a stronger response of marginal cost itself under a no-indexation scheme, or (c), perhaps a combination of both (a) and (b). Remains to be written.]

4. Results in a Workhorse New Keynesian Model

The benchmark model studied so far is useful for highlighting many of the key factors likely to shape how a change in fiscal stance would affect the economy. However, to the extent that the benchmark model featured a very low multiplier in normal conditions when the economy is not far off the steady state (about 0.25, as can be seen from Figure 3), it may well understate the aggregate effects of fiscal policy due to the exclusion of Keynesian accelerator effects on household and business spending. A consequence is that the aggregate multiplier remains relatively modest even in a long-lived liquidity trap unless inflation rises significantly.

Hence, we move in this section to a substantive analysis with the aim of examining the multiplier schedule in a more quantitatively realistic model environment. Specifically, we consider a workhorse New Keynesian model with endogenous investment that closely follows the seminal model of Christiano, Eichenbaum and Evans (2005). This model has been successfully estimated by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007). In the following, we sketch the model and then turn to discussing the results. Appendix B provides a detailed exposition of the model and its parameterization.

From now on, we focus exclusively on the nonlinear model, as the previous analysis indicated substantial differences between the linear and nonlinear model for longer-lived liquidity traps.
4.1. Model

Following CEE, the model includes both sticky nominal wages and prices, allowing for some intrinsic persistence in both components; habit persistence in consumption; and embeds a $Q-$theory investment specification modified so that changing the level of investment (rather than the capital stock) is costly. However, our model departs from this earlier literature in two substantive ways. First, we introduce consumption (VAT), labor income and capital income taxes to get a realistic calibration of fiscal flows in the model. This enables us to study the impact changes in fiscal spending has on the evolution of government debt. Second, we incorporate a financial accelerator using Christiano, Motto and Rostagno’s (2008) variant of the Bernanke, Gertler and Gilchrist (1999) financial accelerator mechanism.\(^9\) These features boost the natural real interest rate following a spending shock and hence tend to amplify the spending multiplier in normal times even if inflation doesn’t respond very much. And consistent with the findings of many influential papers in the empirical literature on government spending multipliers, e.g. Blanchard and Perotti (2002), we work with a parameterization of the model that implies a multiplier of about unity in the short term.\(^10\)

4.2. Results

Analogously with how we proceeded in the benchmark model, the analysis with the workhorse model starts by feeding it with a set of baseline shocks which generate a baseline scenario where the interest rate is bounded at zero for $\text{ZLB}_{\text{Dur}} = 1, 2, 3, ..., T$ periods. In addition to using a negative consumption demand shock ($v_t$), we also feeded the large scale model with a negative net worth (entrepreneur survival) and a positive bond risk-premium shock. The consumption demand shock exhibits a correlation of 0.85, whereas the financial market shocks were assumed to be white noise. Together, these shocks exert a significant adverse impact on the economy, in which economy activity dampens, interest rate spreads rise, and inflation falls. As a result, the policy rate is driven towards its lower bound when the magnitude of these shocks are increased.

Figure 6 depicts the specific baseline for an 8-quarter liquidity trap in the nonlinear specification of the model. We think about an 8-quarter trap as roughly corresponding to the situation United

\(^9\) CMR assumes the entrepreneurs borrow nominal debt, whereas BGG assumed all debt were real.

\(^{10}\) To generate a unit multiplier in the model in the short-term, we work with a relatively high degree of habit formation in consumer preferences and large investment adjustment costs relative to the empirical evidence. An alternative avenue to generate a higher multiplier would be to assume that a fraction of the households are “Keynesian”, and simply consume their current after-tax income. Galí, López-Salido and Vallés (2007) show that the inclusion of non-Ricardian households helps account for structural VAR evidence indicating that private consumption rises in response to higher government spending.
States were facing in the first half of 2009. As can be seen from the figure, output and consumption fall by roughly 8 percent, and investment by twice as much (relative to trend). Consistent with the U.S. episode, interest rate spreads rise roughly 3 percentage points before receding towards their steady state value. Government debt as share of GDP rises about 10 percent. Core inflation falls to almost 0 percent, but consistent with the U.S. evidence the model avoids deflation for a protracted period. Even so, amid the negative output gap and low inflation, the policy interest rate is driven to nil for eight quarters starting in period 8 (in the figure, the economy starts out in the steady state in period 0).

Against this background, we consider a marginal change in fiscal spending announced and undertaken the first period the ZLB binds (in the 8-quarter liquidity trap case described in Figure 6, the change in fiscal spending occurs in period 8) for all ZLB_{\text{DUR}} = 1, 2, 3, ..., T. As before, we can then construct impulse responses by deducting the scenario with the change in fiscal spending with the baseline with no change in fiscal spending.

The results of this exercise for the equivalent of a spending cut equal to 1 percent of trend GDP are shown in Figure 7 for the case of 12-quarter liquidity trap (red dashed line) and when the economy starts out in the steady state.\footnote{As we want to consider the effects of a marginal change in spending which does not change the duration of the liquidity trap, we use a spending cut of 0.01 percent of GDP in the scenario and once the results are obtained we resize the impulses to be the equivalent of a 1 percent change in spending. Had we used a full 1 percent spending cut in the scenario, we would have extended the duration of the liquidity trap, and obtained a larger negative average multiplier (see Erceg and Lindé, 2014, for a thorough discussion of the distinction between the average and marginal multiplier in a liquidity trap).} Although the spending hikes are implemented in different periods (period 1 in the steady state case with no baseline shocks, and in period 6 in the 12-quarter ZLB case), we adjust the starting point in the figure so that the results can be easily compared (i.e. the spending cut occurs in period 0 in both cases, see the lower right panel). As shown in the figure, investment is crowded in the liquidity trap, whereas it is crowded out in the normal case when monetary responds by cutting nominal interest rates immediately.\footnote{Notice that the figure plots the policy rate as deviation from baseline, which implies that the policy rate can become negative in the ZLB case after lift-off occurs in the baseline (implying the interest rate in the scenario is kept lower than in the baseline).} Private consumption, however, is crowded in both cases, albeit to a lower extent at the zero lower bound. Output therefore responds much more during the first year (about $-1.4$ percent) in the liquidity trap case compared to normal times, in which case output falls less than one 1 percent. And as a consequence of the larger fall in output at the ZLB, government debt as share of GDP rises for a protracted period with about 1 percent, and does not start to decline until after 4 years. This contrasts sharply to the results under usual conditions, in which case debt falls already within a
As shown by the black solid line in the figure (which report results for the benchmark specification with Kimball aggregator in the price setting block of the model), the results are qualitatively similar to the results with the stylized model in Figure 3. However, an important difference is that the inflation response is now an increasing function of the liquidity trap duration, as opposed to the results in the simple model in which case expected inflation responded less in a liquidity trap. As a result, the output multiplier schedule is more convex in the workhorse model and the government debt multiplier therefore turns negative in a sufficiently long-lived trap.

The red-dotted line shows results for a calibration of the model with the Dixit-Stiglitz aggregator variant of the model when the slope of the Phillips curve is kept unchanged at 0.012, corresponding to the results in Panel A of Figure 5 for the stylized model. As can be seen from the figure, the results under the DS aggregator differ very little from those obtained with Kimball aggregator. This finding differs somewhat from the analysis in the stylized model, in which case the output multiplier was about twice as high with DS aggregator in a 12-quarter liquidity trap. While we need to analyze our model more to understand fully the results, we suspect that the key driver behind the difference in results is that wages were fully flexible in the stylized model, whereas wage stickiness is a key channel in the workhorse model.

From a substantive perspective, the results in the workhorse model paint a quite different picture than the stylized model. Importantly, the larger scale model implies that the multiplier might be substantially elevated in a longer-lived liquidity trap, so that front-loaded austerity intended to reduce government debt might actually be self-defeating, at least for a substantial period of

year of the implementation of austerity.

In Figure 8 we show the corresponding marginal multiplier as function of liquidity trap duration. As before we show results for $ZLB_{DUR} = 1,...,12$ but also include the impact at steady state ($ZLB_{DUR} = 0$). The output multiplier is the 1-year cumulated multiplier, calculated as the sum of output responses during the first year divided by sum of spending cuts during the same period. In the middle panel, we show expected inflation during the first year, calculated as $\ln P_{t+4}/\ln P_t$ and normalized with the change in spending in the initial period (so that the impact is always positive regardless of whether we consider the effects of a small cut or hike in spending). Finally, the government debt multiplier in the right panels is computed as the level of debt after 3 years divided by the absolute average change in spending during this period, so that a positive value implies that debt falls following a cut in spending (whereas a negative value implies that debt rises at this horizon).
time. This result was never obtained in the nonlinear variant of the stylized model, for which the
government debt multiplier always remained positive. In fact, an aggressive spending cut that is
perceived to be transient may be self-defeating at all horizons in a protracted liquidity trap, as
shown by the results in Figure 9. In the figure, we plot the effects in normal times and in a 12-
quarter liquidity trap of a uniform decrease in spending for 12-quarters (i.e. as long as the ZLB is
expected to last). As can be seen from the figure, this transient spending cut elevates the marginal
multiplier even further (to about 2 in the ZLB case), and causes debt to converge back to nil from
above.

A key mechanism in the model that generates large and elevated multiplier schedules is the
financial accelerator mechanism embedded into the model. To see this, the (blue) dash-dotted line
in Figure 8 report the multiplier schedules when the financial accelerator mechanism is omitted
from the model.\textsuperscript{13} As can be seen by comparing the no financial accelerator variant with the
benchmark specification (black solid), the output multiplier and inflation responses are shifted
down and not amplified to the same extent as the duration of the liquidity trap is extended. This
implies that the government debt multiplier does not change sign even in a long-lived trap in the
no accelerator model, implying that a cut in government spending reduces debt after 3 years in
the no-financial accelerator variant of the model for a persistent cut in spending (recalling that all
results in Figure 8 are conditioned on an AR(1) process for government spending with persistence
0.95). For a more transient cut in government spending in a long-lived liquidity trap (expected to
last 12-quarters, corresponding to the experiment in Figure 9), Figure 10 shows that the adverse
impact is substantially mitigated in the variant without the financial accelerator as investment (and
consumption) is not crowded-out. As a result, government debt falls eventually and there is no
“fiscal free lunch” for the treasury as in the benchmark model.

5. Conclusions

[Remains to be written.]

\textsuperscript{13} In this variant of the model, we replace the negative net worth shock with a negative investment-specific
technology shock to generate a qualitatively similar baseline in the no financial accelerator model as plotted for the
benchmark model in Figure 6.
References


Figure 1: Baselines in Linear and Nonlinear Models for an Equally-Sized Consumption Demand Shock

1. Output Gap
2. Yearly Inflation ($\ln(P_t/P_{t-4})$)
3. Nominal Interest Rate (APR)
4. Real Interest Rate (APR)
5. Potential Real Interest Rate (APR)
6. Price Dispersion
7. Real GDP
8. Government Debt to GDP
9. Consumption Demand Shock
Figure 2: Baselines for 8-Quarter Liquidity Trap

1. Output Gap

2. Yearly Inflation ($\ln(P_t/P_{t-4})$)

3. Nominal Interest Rate (APR)

4. Real Interest Rate (APR)

5. Potential Real Interest Rate (APR)

6. Price Dispersion

7. Real GDP

8. Government Debt to GDP

9. Consumption Demand Shock

Legend:
- **Black**: Linear Model
- **Red**: Nonlinear Model
- **Blue**: Nonlinear Model with Linear NKPC and Res. Con.
Figure 3: Marginal Multipliers in Stylized Model

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Percentage Points</th>
<th>Percentage Points</th>
<th>Percentage Points</th>
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<tr>
<td>Impact Spending Multiplier</td>
<td>Benchmark Calibration</td>
<td>Expected Inflation ($4E_t\pi_{t+1}$)</td>
<td>Govt Debt to GDP (After 1 Year)</td>
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<td>Linear Model</td>
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<td>Linear Model</td>
<td>Nonlinear Model</td>
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Impact Spending Multiplier

Expected inflation ($4E_t\pi_{t+1}$)

Govt Debt to GDP (After 1 Year)

Benchmark Calibration

Alternative Calibration: Dixit Stiglitz
Figure 4: Marginal Multipliers in Stylized Model: Why They Differ

Benchmark Calibration

Impact Spending Multiplier
Expected Inflation ($4 \cdot E_t \pi_{t+1}$)
Govt Debt to GDP (After 1 Year)

Alternative Calibration: Dixit Stiglitz

Impact Spending Multiplier
Expected Inflation ($4 \cdot E_t \pi_{t+1}$)
Govt Debt to GDP (After 1 Year)
Figure 5: Marginal Impact of Changes in Spending: Sensitivity Analysis in Nonlinear Model

Panel A: Kimball ($\xi_p = 0.667; \varepsilon > 0$) vs. Dixit-Stiglitz ($\xi_p = 0.9; \varepsilon = 0$)

Panel B: Impact of Indexation Assumption for Non-Optimizing Firms
Figure 6: Baseline for an 8-quarter Liquidity Trap in Workhorse Model

- GDP (% dev from SS)
- Inflation (4-quarter change)
- Federal Funds Rate (APR)

- Private Consumption (% dev from SS)
- Investment (% dev from SS)
- Hours Worked Per Capita (% dev from SS)

- Capital Utilization (% dev from SS)
- Net Worth (% dev from SS)
- Bankruptcy Rate (APR, dev from SS)

- External Finance Premium (APR, dev from SS)
- Govt Debt (Share of GDP, dev from SS)
- Govt Consumption (Share of trend GDP, dev from SS)
Figure 7: Impulses to a Persistent Cut in Govt Spending in Normal Times and in a Long–Lived Liquidity Trap
Figure 8: Marginal Multipliers in Workhorse Model

Benchmark Model: Kimball vs. Dixit-Stiglitz Aggregator in Price Setting

Assessing the Role of the Financial Accelerator Mechanism
Figure 9: Impulses to a Transient Cut in Govt Spending in Normal Times and in a Long–Lived Liquidity Trap
Figure 10: Impulses to a Transient Cut in Govt Spending: Assessing the Role of the Fin. Acc. Channel

- GDP (%)
- Inflation (4–quarter change)
- Federal Funds Rate (APR, dev from baseline)
- Private Consumption (%)
- Investment (%)
- Hours Worked Per Capita (%)
- Capital Utilization (%)
- Net Worth (%)
- Bankruptcy Rate (APR)
- External Finance Premium (APR)
- Govt Debt (Share of GDP)
- Govt Consumption (Share of trend GDP)

Legend:
- Red dashed line: Benchmark Model
- Blue solid line: No Financial Accelerator Variant of Model
Appendix A. Additional Results for the Stylized Model

In this appendix, we state the log-linearized variant of the stylized model and present some additional results.

A.1. The Log-linearized Stylized Model

As shown in the technical appendix (available upon request from the authors), the equations of the log-linearized model can be written as follows.

[Remains to be written.]

A.2. Sensitivity Analysis in the Simple Model

Figure A.1 report results for an alternative shock driving the baseline in Figures 2. The upper panels in the figure confirm the results in by Erceg and Lindé (2014) by showing that the fiscal spending multiplier at the margin in the linearized version is independent of the shock driving the baseline, as long as it generates an equally long-lived ZLB episode. So our choice to work with the consumption demand shock in $\nu_{t+j}$ instead of the conventional discount factor shock in $\gamma_t$ in (1) to generate the baseline path underlying Figures 3 to 5 has no consequence for the results with linearized model. However, the results for the non-linear variant may differ. However, the lower panels in Figure A.1 show that the results are very similar even in the nonlinear, so our choice of baseline appears immaterial for our results.

Another aspect we study to understand how our results differ from Braun et al. (2012, 2013) due to our AR(1) assumption for government spending instead of the MA-process they work with. Figure A.2 assess this issue by comparing results under our AR(1) process with persistence $.95$ against the MA process for which $G_t$ is cut in an uniformal fashion as long at the policy rate is bounded at zero for $ZLB_{DUR} = 1, 2, 3, ..., T$ and set at its steady state value otherwise. Apart from that our solution procedure does not allow for future shock uncertainty, this way of modeling government spending is identical to Braun et al. who in turns follow Eggertsson (2010).

As can be seen from the upper panels of Figure A.2, the MA-process specification increases the marginal spending multiplier substantially for the ZLB durations we consider, as increases in government spending has very benign effects on the potential real interest rate when the duration of the spending hike equals the expected duration of the liquidity trap (see e.g. Erceg and Lindé, 2014). For a one quarter liquidity trap it equals unity, as shown by Woodford (2011). Our fairly persistent
AR(1) process tends to dampen the multiplier schedule as a relatively large fraction comes on line when the ZLB is not binding. This feature explains why the AR(1) multiplier is substantially lower in a short lived. However, the AR(1) process is also associated with a substantially lower multiplier even in a fairly long-lived trap compared to the MA process because its has less benign effects on the potential real rate.

All this is well-known from the literature on linearized models. However, the results on the non-linear model, shown in the lower panels, are less explored. We have already discussed the AR(1) case at length in the text. What we see is that the results for the MA process are quite different for longer ZLB durations, because in constrast to the MA multiplier schedule for the linearized model the MA schedule for the nonlinear model stays essentially flat at unity, in line with the findings of Braun et al. (2013). Hence, our results for the linear and nonlinear models in Figure A.2 are in line with the results in the existing literature.
Figure A.1: Marginal Multipliers: Sensitivity With Respect to Baseline Shock

<table>
<thead>
<tr>
<th>Linearized Model</th>
<th>Nonlinear Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Spending Multiplier</td>
<td>Impact Spending Multiplier</td>
</tr>
<tr>
<td>Expected inflation (4E_t \pi_{t+1})</td>
<td>Expected Inflation (4E_t \pi_{t+1})</td>
</tr>
<tr>
<td>Govt Debt to GDP (After 1 Year)</td>
<td>Govt Debt to GDP (After 1 Year)</td>
</tr>
</tbody>
</table>

- **Consumption Demand Shock**
- **Discount Factor Shock**

---

**Consumption Demand Shock**

- Impact Spending Multiplier
- Expected Inflation \(4E_t \pi_{t+1}\)
- Govt Debt to GDP (After 1 Year)

**Discount Factor Shock**

- Impact Spending Multiplier
- Expected Inflation \(4E_t \pi_{t+1}\)
- Govt Debt to GDP (After 1 Year)
Figure A.2: Marginal Multipliers: Sensitivity With Respect to Specification of Spending Process

- **AR(1) Process for Spending (Benchmark)**
- **MA Process for Spending**

**Linearized Model**

1. Impact Spending Multiplier
2. Expected inflation ($4 \cdot E_{t+1} \pi_t$)
3. Govt Debt to GDP (After 1 Year)

**Nonlinear Model**

1. Impact Spending Multiplier
2. Expected Inflation ($4 \cdot E_{t+1} \pi_t$)
3. Govt Debt to GDP (After 1 Year)
Appendix B. The Workhorse New Keynesian Model


Below, we first describe the firms’ and households’ problem in the model, and state the market clearing conditions. Some parts will be repetitive w.r.t. the baseline model in Section 2, but we nevertheless include a comprehensive description to make it self-contained. However, given that the mechanics underlying the financial accelerator are well-understood, we simplify our exposition by focusing on a special case of our model which abstracts from a financial accelerator. We conclude our model description with a brief description of how the model is modified to include the financial accelerator (Section B.5). Then we provide details on the parameterization of the model and how it is solved.

B.1. Firms and Price Setting

Final Goods Production As in the benchmark model, the single final output good \( Y_t \) is produced using a continuum of differentiated intermediate goods \( Y_t(f) \). Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

\[
\int_0^1 G_Y \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \tag{B.1}
\]

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007) we assume that \( G_Y(\cdot) \) is given by a strictly concave and increasing function:

\[
G_Y \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\phi_p}{1-(\phi_p-1)\epsilon_p} \left( \frac{\phi_p+(1-\phi_p)\epsilon_p}{\phi_p} \right) \frac{Y_t(f)}{Y_t} + \frac{(\phi_p-1)\epsilon_p}{\phi_p} \frac{1-(\phi_p-1)\epsilon_p}{\phi_p} + \left[ 1 - \frac{\phi_p}{1-(\phi_p-1)\epsilon_p} \right], \tag{B.2}
\]

where \( \phi_p \geq 1 \) denotes the gross markup of the intermediate firms. The parameter \( \epsilon_p \) governs the degree of curvature of the intermediate firm’s demand curve. When \( \epsilon_p = 0 \), the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When \( \epsilon_p \) is positive the firms instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. Notice that \( G_Y(1) = 1 \), implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both the product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of
the output index $Y_t$, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price $P_t$, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f) df,$$  \hspace{1cm} (B.3)

subject to the constraint (B.1). The first order conditions for this problem can be written

$$\frac{Y_t(f)}{Y_t} = \frac{\phi_p}{\phi_p - (\phi_p - 1)\epsilon_p} \left[ \frac{P_t(f)}{P_t} \frac{1}{\Lambda_t} \right]^{-\frac{\phi_p - (\phi_p - 1)\epsilon_p}{\phi_p}} + \frac{(1 - \phi_p)\epsilon_p}{\phi_p},$$  \hspace{1cm} (B.4)

$$P_t \Lambda_t^p = \left[ \int P_t(f) \frac{1 - (\phi_p - 1)\epsilon_p}{\phi_p - 1} df \right]^{-\frac{\phi_p - 1}{1 - (\phi_p - 1)\epsilon_p}},$$

$$\Lambda_t^p = 1 + \frac{(1 - \phi_p)\epsilon_p}{\phi_p} - \frac{(1 - \phi_p)\epsilon_p}{\phi_p} \int \frac{P_t(f)}{P_t} df,$$

where $\Lambda_t^p$ denotes the Lagrange multiplier on the aggregator constraint (B.1). Note that for $\epsilon_p = 0$ and $\Lambda_t^p = 1$ in each period $t$, the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions

$$\frac{Y_t(f)}{Y_t} = \left[ \frac{P_t(f)}{P_t} \right]^{-\frac{\phi_p - 1}{\phi_p - 1}}, P_t = \left[ \int P_t(f) \frac{1}{1 - \phi_p} df \right]^{1 - \phi_p}.$$  \hspace{1cm} (B.5)

*Intermediate Goods Production* A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in eq. (B.4) from the final goods firms through the solution to the problem in (B.3), which varies inversely with its output price $P_t(f)$ and directly with aggregate demand $Y_t$.

Each intermediate goods producer utilizes capital services $K_t(f)$ and a labor index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = K_t(f)^\alpha \left[ \gamma^\ell L_t(f) \right]^{1 - \alpha} - \gamma^\ell \Phi,$$  \hspace{1cm} (B.6)

where $\gamma^\ell$ represents the labour-augmenting deterministic growth rate in the economy, $\Phi$ denotes the fixed cost (which is related to the gross markup $\phi_p$ so that profits are zero in the steady state).

Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital $R_{K,t}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.
The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm \( f \) faces a constant probability, \( 1 - \xi_p \), of being able to reoptimize its price \( P_t(f) \). The probability that any firm receives a signal to re-optimize its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., \( P_t(f) = (1 + \pi_{t-1})^{\epsilon_p} P_{t-1}(f) \) where \( 0 \leq \epsilon_p \leq 1 \) and \( \pi_{t-1} \) denotes net inflation in period \( t - 1 \). [Check: We should relax the indexation assumption, and work with a variant without indexation, i.e. \( \epsilon_p = 0 \).] A positive value of \( \epsilon_p \) introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

\[
\max_{P_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \tilde{P}_{t+j} P_t \left( \Pi_{s=1}^j (1 + \pi_{t+s-1})^{\epsilon_p} (1 + \pi)^{1-\epsilon_p} \right) - MC_{t+j} Y_{t+j}(f), \quad (B.7)
\]

where \( \tilde{P}_t(f) \) is the newly set price. Notice that with our assumptions all firms that re-optimize their prices actually set the same price.

**B.2. Households and Wage Setting**

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household’s labor services \( L_t(h), h \in [0, 1] \), as imperfect substitutes for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregated labor index \( L_t \) has the well-known Dixit-Stiglitz form:

\[
L_t = \left[ \int_0^1 \left( L_t(h) \right)^{1/\phi_w} dh \right]^{\phi_w^{-1}}, \quad 1 \leq \phi_w < \infty. \quad (B.8)
\]

The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household’s wage rate \( W_t(h) \) as given, and then sells units of the labor index to the production sector at their unit cost \( W_t \):

\[
W_t = \left[ \int_0^1 W_t(h)^{-1/\phi_w} dh \right]^{-\phi_w}. \quad (B.9)
\]

which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator’s demand for the labor hours of household \( h \)—or equivalently, the total demand for this household’s
labor by all goods-producing firms—is given by

$$L_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{\frac{\sigma_w}{1+\sigma_w}} L_t. \quad (B.10)$$

The utility function of a typical member of household $h$ is

$$E_t \sum_{j=0}^{\infty} \beta^j \xi_{t+j} \left[ \ln \left( C_{t+j}(h) - \tau C_{t+j-1}(h) - C\nu_{t+j} \right) - \left( \frac{\chi_0}{1+\sigma_l} L_{t+j}(h)^{1+\sigma_l} \right) + \mu_0 F \left( \frac{MB_{t+j+1}(h)}{P_{t+j}} \right) \right], \quad (B.11)$$

where the discount factor $\beta$ satisfies $0 < \beta < 1$. The period utility function depends on household $h$’s current and lagged consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence. The period utility function also depends inversely on hours worked $L_t(h)$. Finally, the period utility function also depends on the households end-of-period real money balances, $\frac{MB_{t+1}(h)}{P_t}$, a savings shock $\xi_t$, and a consumption demand shock, $\nu_t$. The subutility function $F(.)$ over real balances is assumed to have a satiation point to account for the possibility of a zero nominal interest rate; see Eggertsson and Woodford (2003) for further discussion.

Household $h$’s budget constraint in period $t$ states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_t (1 + \tau_{C,t}) C_t(h) + P_t I_t(h) + \frac{BG_{t+1}(h)}{\varepsilon_{B,t}R_t} + \int \xi_{t+1}B_{D,t+1}(h) - B_{D,t}(h) \quad (B.12)$$

$$= B_{G,t}(h) + (1 - \tau_{L,t}) W_t(h) L_t(h) + (1 - \tau_{K,t}) \left[ R_{K,t}Z_t(h) K^p_t(h) - (a(Z_t(h)) + \delta) K^p_t(h) \right] + \Gamma_t(h) - T_t(h).$$

Thus, the household purchases part of the final output good (at a price of $P_t$), which it chooses either to consume $C_t(h)$ (subject to a VAT tax $\tau_{C,t}$) or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans (2005), investment augments the household’s (end-of-period) physical capital stock $K^p_{t+1}(h)$ according to

$$K^p_{t+1}(h) = (1 - \delta)K^p_t(h) + \varepsilon_{I,t} \left[ 1 - S \left( \frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h), \quad (B.13)$$

where $\varepsilon_{I,t}$ is an “investment-specific” technology shock (see Fisher, 2006) that is assumed to follow the process:

$$\ln \varepsilon_{I,t} = \rho_I \ln \varepsilon_{I,t-1} + \eta_{I,t}, \eta_{I,t} \sim N(0, \sigma_I). \quad (B.14)$$

---

B.1 Note that we deviate slightly from the notation in CEE by using $h$ to index households and using $\kappa$ to denote the degree of habit formation.

B.2 For simplicity, we assume that $\mu_0$ is sufficiently small that changes in the monetary base have a negligible impact on equilibrium allocations, at least to the first-order approximation we consider. It therefore does not affect the Government’s seigniorage, see eq. (B.19).

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Thus, the extent to which investment by each household $h$ turns into physical capital is assumed
to depend on the investment-specific shock $\varepsilon_{I,t}$ and how rapidly the household changes its rate of
investment according to the function $S\left(\frac{I_t(h)}{I_{t-1}(h)}\right)$, which we specify as

$$S(x_t) = \frac{1}{2} (x_t - \gamma)^2. \quad (B.15)$$

Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$.

In addition to accumulating physical capital, households may augment their financial assets
through increasing their government nominal bond holdings $(B_{G,t+1})$, from which they earn an
interest rate of $R_t$. The return on these bonds is also subject to a risk-premium shock, $\varepsilon_{B,t}$, which follows

$$\ln \varepsilon_{B,t} = \rho_B \ln \varepsilon_{B,t-1} + \eta_{B,t}, \eta_{B,t} \sim N(0, \sigma_B). \quad (B.16)$$

We assume that agents can engage in frictionless trading of a complete set of contingent claims
to diversify away idiosyncratic risk. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net pur-
chases of these state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state-dependent price,
and $B_{D,t+1}(h)$ the quantity of such claims purchased at time $t$.

On the income side, each member of household $h$ earns after-tax labor income $(1 - \tau_{L,t}) W_t(h) L_t(h)$, after-tax capital rental income of $(1 - \tau_{K,t}) R_{K,t} Z_t(h) K^p_t(h)$ after paying a utilization cost of the
physical capital equal to $(1 - \tau_{K,t}) a(Z_t(h)) K^p_t(h)$ where $Z_t(h)$ is the capital utilization rate, so
that capital services provided by household $h$, $K_t(h)$, equals $Z_t(h) K^p_t(h)$. The capital utilization
adjustment function $a(Z_t(h))$ is assumed to be given by

$$a(Z_t(h)) = \frac{r^k}{\tilde{z}_1} \left[ \exp \left( \tilde{z}_1 (Z_t(h) - 1) \right) - 1 \right], \quad (B.17)$$

where $r^k$ is the steady state net real interest rate $(\bar{R}_{K,t}/\bar{P}_t)$. Notice that the adjustment function satisfies $a(1) = 0$, $a'(1) = r^k$, and $a''(1) \equiv r^k \tilde{z}_1$. Finally, each member also receives an aliquot share
$\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period $t$, each member of household $h$ maximizes the utility function (B.11) with
respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization
rate, bond holdings, and holdings of contingent claims, subject to its labor demand function (B.10),
budget constraint (B.12), and transition equation for capital (B.13).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar
to the price contracts described previously. Thus, the probability that a household receives a signal
to re-optimize its wage contract in a given period is denoted by $1 - \xi_w$. For those households that do not get a signal to re-optimize, we specify the following dynamic indexation scheme for the adjustment of the wages: $W_t(h) = (\gamma (1 + \pi_{t-1})^{\xi_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\bar{W}_t(h)} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\bar{z}_{t+j}}{\bar{z}_t} \left[ \bar{W}_t(h) \left( \Pi_{s=1}^j (1 + \pi_{t+s-1})^{\xi_w} (1 + \pi)^{1-\xi_w} - W_{t+j} \right) L_{t+j}(h) \right],$$

where $\bar{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that reoptimize their wages will actually set the same wage.

### B.3. Fiscal and Monetary Policy

Government purchases $G_t$ are exogenous, and the process for government spending is adopted from the benchmark model (see eq. 22). As before, government purchases have no direct effect on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$B_{G,t} = (1 + i_{t-1}) B_{G,t-1} + P_t G_t - \tau_C t P_t C_t - \tau_L t W_t L_t - \tau_K t [R_{K,t} Z_t - (a (Z_t) + \delta)] K_t^p - T_t.$$  

In comparison to the evolution of government debt in the benchmark model (see eq. 13), the constraint in eq. (B.19) add receipts from VAT and capital income taxes. However, the tax rates $\tau_C, \tau_L$, and $\tau_K$, are kept constant. Instead, following the bulk of the literature on fiscal multipliers we assume that the fiscal authority use lump-sum taxes (as share of nominal trend GDP, $\tau_t \equiv \frac{T_t}{Y_t^p}$), to stabilize debt according to the simple rule

$$\tau_t - \tau = \varphi_b (b_{G,t-1} - b_G),$$

where $b_{G,t-1} = B_{G,t}/(4\bar{P}_t \bar{Y}_t)$

Turning to the central bank, it is assumed to follow a linearized variant of (16) subject to some smoothing on the notional interest rate $i_t^{not}$:

$$i_t^{not} = (1 - R_t) i_t + R_t (\pi_t - \pi) + \sigma_t x_t + \rho R_t i_{t-1}^{not},$$

$$i_t = \max (0, i_t^{not}).$$

where $x_t \equiv \ln \left( Y_t/Y_t^{pot} \right)$, $Y_t^{pot}$ denotes the level of output that would prevail if prices and wages were flexible, and $i$ the steady-state (net) nominal interest rate, which is given by $r + \pi$ where $r \equiv 1/\beta - 1$.
B.4. Market Clearing Conditions

Total output of the final goods sector is used as follows:

\[ Y_t = C_t + I_t + G_t + a(Z_t)K_t^P, \]  

(B.21)

where \( a(Z_t) \) is the capital utilization adjustment cost.

Finally, we need to specify the aggregate production constraint. To do that, we note that the unweighted sum of the intermediate firms’ output equals

\[ Y_{t}^{\text{sum}} = \int_{0}^{1} Y_t(f) \, df, \]  

(B.22)

which from eq. (B.6) can be rewritten as

\[ Y_{t}^{\text{sum}} = \int_{0}^{1} \left[ K_t(f)^{\alpha} [\gamma^tL_t(f)]^{1-\alpha} - \gamma^t\Phi \right] \, df \]  

(B.23)

where the second equality follows from the fact that every firm’s capital-labor ratio will be the same in equilibrium.

From the first-order conditions to the final goods aggregator problem (B.4), it follows that

\[ Y_{t}^{\text{sum}} = Y_t \int_{0}^{1} \frac{\phi_p}{\phi_p^{-(\phi_p-1)\phi_p}} \left( \left[ \frac{p_t(f)}{p_t} \right]^{1-\alpha} \right) \, df, \]  

(B.24)

so that

\[ \left( \frac{K_t}{\gamma^tL_t} \right)^{\alpha} \gamma^t \int_{0}^{1} L_t(h) \, dh - \gamma^t\Phi = Y_t \int_{0}^{1} \phi_p \left( \left[ \frac{p_t(f)}{p_t} \right]^{1-\alpha} \right) \, df. \]  

By inserting the expression for the unweighted sum of labor, \( \int_{0}^{1} L_t(h) \, dh \) from eqs. (B.8) and (B.10) into this last expression, we can finally derive the aggregate production constraint which depends on aggregate technology, capital, labor, fixed costs, as well as the price and wage dispersion terms.

B.5. Production of capital services

The model is amended with a financial accelerator mechanism into the model following the basic approach of Bernanke, Gertler and Gilchrist (1999). Thus, the intermediate goods producers rent capital services from entrepreneurs (at the price \( R_{K,t} \)) rather than directly from households.
Entrepreneurs purchase physical capital from competitive capital goods producers (and resell it back at the end of each period), with the latter employing the same technology to transform investment goods into finished capital goods as described by equations B.13 and B.15). To finance the acquisition of physical capital, each entrepreneur combines his net worth with a loan from a bank, for which the entrepreneur must pay an external finance premium (over the risk-free interest rate set by the central bank) due to an agency problem. Banks obtain funds to lend to the entrepreneurs by issuing deposits to households at the risk-free interest rate set by the central bank, with households bearing no credit risk (reflecting assumptions about free competition in banking and the ability of banks to diversify their portfolios). In equilibrium, aggregate shocks induce endogenous variations in entrepreneurial net worth – i.e., the leverage of the corporate sector – and thus fluctuations in the corporate finance premium.\footnote{We follow Christiano, Motto and Rostagno (2008) by assuming that the debt contract between entrepreneurs and banks is written in nominal terms (rather than real terms as in Bernanke, Gertler and Gilchrist, 1999). For further details about the setup, see Bernanke, Gertler and Gilchrist (1999), and Christiano, Motto and Rostagno (2008). An elaborate exposition is also provided in Christiano, Trabandt and Walentin (2007).} Moreover, we allow for an exogenous net worth shock, which through its effect on the net worth of entrepreneurs affect the corporate risk premium.

\section*{B.6. Calibration and Solution}

In Table A.1, we state all the parameters used in the model. These parameters are all standard, apart from the fact that we use somewhat larger-than-normal degrees habit formation ($\varepsilon = 0.95$) and investment adjustment cost ($\varphi = 10$) to generate a government spending multiplier close to unity in the short-term. We do not allow for time-varying capital utilization and working capital in the current calibration, and we have full indexation to lagged inflation in price- and wage-setting for non-optimizers. \footnote{Remains to be written.} [\textbf{NOTE: We should check sensitivity of our results to these choices.}] We solve numerically for the flex price-wage version of the model, and in this variant everything is identical with the exception that the probability for re-optimizing prices (firms) and wages (households) equals unity in each period.

\[\text{[Remains to be written.]}\]
Table A.1: Parameters in Workhorse Model.

Panel A: Households

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<th>Description</th>
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Panel B: Firms and Entrepreneurs

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</tr>
<tr>
<td>$\iota_p$</td>
<td>Ind. for non-opt. prices</td>
<td>1.00</td>
<td>$\gamma$</td>
<td>Steady state gross growth</td>
<td>1.0082</td>
</tr>
<tr>
<td>$\rho_{nw}$</td>
<td>Net worth shock</td>
<td>0.00</td>
<td>$\eta$</td>
<td>Working capital share</td>
<td>0.00</td>
</tr>
<tr>
<td>$F_{w}$</td>
<td>Bankruptcy prob. of entrep.</td>
<td>0.005</td>
<td>$NW/K$</td>
<td>Net worth to Capital Ratio in SS</td>
<td>0.667</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Bank monitoring cost</td>
<td>0.20</td>
<td>$ENT_s$</td>
<td>Entrep. Endowment to GDP Ratio</td>
<td>0.002</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Entrep. survival prob.</td>
<td>0.995</td>
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</tbody>
</table>

Panel C: Fiscal and Monetary Policy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_y$</td>
<td>Gov't $G/Y$ ss-ratio</td>
<td>0.20</td>
<td>$\pi$</td>
<td>Steady state net infl. rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Gov’t spending per.</td>
<td>0.95</td>
<td>$r_{\pi}$</td>
<td>Long-term $\pi$ coeff. in MP rule</td>
<td>2.50</td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>VAT tax rate</td>
<td>0.05</td>
<td>$r_x$</td>
<td>Long-term gap coeff. in MP rule</td>
<td>0.125</td>
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<tr>
<td>$\tau_L$</td>
<td>Labor income tax rate</td>
<td>0.30</td>
<td>$\rho_R$</td>
<td>Smoothing coeff. in MP rule</td>
<td>0.50</td>
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<tr>
<td>$\tau_K$</td>
<td>Capital income tax rate</td>
<td>0.25</td>
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<tr>
<td>$B_C/(4PY)$</td>
<td>Govt debt to GDP</td>
<td>0.90</td>
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<tr>
<td>$\varphi_b$</td>
<td>Lumpsum debt tax coeff.</td>
<td>0.04</td>
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</tbody>
</table>

Notes: Our parameterization implies a steady state annualized nominal interest rate of 3.75 percent and an inflation rate of 2 percent at an annualized rate. The flex-price wage equilibrium imposes identical parameter, except that $\xi_w^{**} = \phi_p^{**} = 0$. 
