Portfolio Choice and Partial Default in Emerging Markets: A Quantitative Analysis

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Motivation

- What are the causes and consequences of cross-border asset positions?

- One of the oldest questions in economics, but still relevant
  - is gross external debt too high? (Reinhart and Rogoff (2009))
  - savings/banking glut caused crisis? (Shin (2012), Caballero (2009))
  - too much short-term debt and too little equity finance? (Shiller)

- Goal: further our ability to model int’l capital flows, portfolios, and default
Primary Contributions and Results 1

- Integrate portfolio choice methods into model of int’l debt/default
  - introduce partial default into classic portfolio setting
  - apply to emerging market (EM) int’l finance

- Key: set-up allows me to exactly solve (numerically) an equil. model with
  1. endogenous haircuts and spreads
  2. many assets
  3. distinctions between gross/net debt, debt/equity, long/short maturity

- In contrast: current literature allows for only one or two assets
  - (or approximates solution)
Establish key properties of the model
- separation of portfolio choice from consumption/default decision
- rapid computation of equilibrium using analytic characterizations
- prove default increases as market prospects deteriorate

Quantitatively, address 4 elusive facts from EM int’l finance
- key fact: pro-cyclicality in gross flows and portfolio composition
- pro-cyclical portfolios stem from pro-cyclical int’l prices
- important since empirical lit. emphasizes gross flows and composition

Other facts: (link: 4 empirical regularities)
- partial default
- default in bad times
- high gross debt
Related Literature

**International Debt and Default**

**Empirical:** Tomz and Wright (2012), Sturzenegger and Zettelmeyer (2008), Moody’s (2008, 2009)


**Partial default:** Dubey, Geanakoplos, and Shubik (2005), D’Erasmo (2011), Benjamin and Wright (2009), Arellano, Mateos-Planas, and Ríos-Rull (2013), Tsomocos et al. papers

**Gross Capital Flows and Portfolios**


**Methodology (Pricing Kernels and Portfolio Choice)**

Model

- \( t = 0, 1, 2, \ldots \)

- Single numeraire consumption good

- Exogenous shock process: \( s_t \in S = \{1, \ldots, S\} \)
  - Markov: transition matrix \( \Pi = \{\pi_{ss'}\} \)
Model

- $s_t$ encompasses both emerging market and int’l shocks

- Shocks to the emerging market:
  - exogenous GDP growth
  - GDP level: $y' = g(s') y$
  - $g(s') \in \{g(1), ..., g(S)\}$

- Shocks to international state prices
  - represents financial market developments in, say, the U.S.
  - therefore, EM GDP growth correlated with foreign shocks
  - shocks to risk-free yield curve and equity premium
Continuum of ex ante identical agents in a small emerging market

- standard crra utility from consumption: \( E_0 \left[ \sum_{t>0} \beta^t \frac{(c_t)^{1-\sigma}}{1-\sigma} \right] \)
- discount factor \( \beta \)
- \( \sigma > 1 \)

Agents are anonymous and atomistic

- departure from Eaton-Gersovitz models
- Wright (2006)
- Jeske (2006)

\( \omega \) is the representative agent’s net worth

- suppose the aggregate state of the economy is just \( s \) (not \( (s, \omega) \))
- I prove this in paper
- In paper, I allow for idiosyncratic, agent specific wealth shocks
- Buy and sell 4 assets
  - emerging market equity
  - short-term EM and US bonds
  - long-term EM bonds (decaying perpetuities)
- My framework allows for many more assets
- Int’l bonds are risk-free, while emerging market bonds are defaultable

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<tr>
<th></th>
<th>Quantity</th>
<th>Price</th>
<th>Promise</th>
<th>Return</th>
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<tbody>
<tr>
<td>Equity (EM)</td>
<td>$a$</td>
<td>$P$</td>
<td>$y, y', ...$</td>
<td>$R'_a = \frac{P' + y'}{P}$</td>
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<tr>
<td>Short-term bond (EM)</td>
<td>$b_1$</td>
<td>$q_1$</td>
<td>1</td>
<td>$R'_{b_1} = \frac{1}{q_1}$</td>
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<tr>
<td>Long-term bond (EM)</td>
<td>$b_L$</td>
<td>$q_L$</td>
<td>$\delta, \delta^2, \delta^3, ...$</td>
<td>$R'_{b_L} = \frac{\delta(1+q'_L)}{q_L}$</td>
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<tr>
<td>Short-term bond (rf)</td>
<td>$B_1$</td>
<td>$Q_1$</td>
<td>1</td>
<td>$R'_{B_1} = \frac{1}{Q_1}$</td>
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</table>
International Investors

- Pricing kernel which represents U.S. investors
- Chosen to match historical properties of U.S. yield curves and P/Ds
- Current literature: constant risk-free rate and flat risk-free yield curve
Debt and Default

- Based on Dubey, Geanakoplos, and Shubik (2005)
  - risky borrowing and lending occurs through anonymous pools
  - from the lender’s perspective, pools have delivery rate $d_{ss}$
  - borrowers take pool price as given and may default

- Equilibrium: expected $d$ must be consistent with actual default

- Departure from Eaton-Gersovitz models
  - but, allows me to include many assets and partial default
  - alternate interpretation: private portfolios, sovereign default
  - Moody’s (2008, 2009)
  - extension in progress
Agents may default $D \geq 0$

Utility cost $\lambda \omega^{-\sigma} D$
- minimum consumption level: $c = \lambda^{-1/\sigma} \omega$, $\lambda > 0$
- penalty declines proportionally with MU of consumption
- without growth, I could use just $\lambda D$

Tractable model of partial default
- reduced-form for loss in reputation, output, collateral, etc.
- $\omega^{-\sigma}$: technical assumption that makes model tractable

$\Rightarrow$ Debt After Default Figure
• Int’l investors take bond delivery rates $d_{ss'}$ as given

• In equilibrium, expected delivery rates are consistent with EM choices:

$$d' = 1 - \frac{D'}{-(b'_1 + \delta b'_L)}$$

• Default = missing coupons due
  • default spread equally across payments due
  • therefore, haircuts harsher on short-term debt
  • consistent with Sturzenegger and Zettelmeyer (2008)
Emerging Market Problem

\[ \nu(\omega; s) = \max_{c, a', B'_1, b'_1, b'_L, D} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \lambda \omega^{-\sigma} D + \beta E [\nu(\omega'; s') | s] \right\} \]

subject to

1. \[ c + Pa' + Q_1 B'_1 + q_1 b'_1 + q_L b'_L = \omega + D \]
2. \[ \omega' = (P' + y') a' + B'_1 + b'_1 + (1 + q'_L) b'_L \delta \]
3. \[ D \geq 0 \]
4. \[ B'_1, a' \geq 0 \]
5. \[ b'_1, b'_L \leq 0 \]
Emerging Market Problem: alternate formulation

\[ v(\omega; s) = \max_{c,D,\theta} \left[ \frac{c^{1-\sigma}}{1-\sigma} - \lambda \omega^{-\sigma} D + \beta E\left( v(\omega'; s') \mid s \right) \right] \]

subject to

1. \[ \omega' = R(\theta; s') (\omega + D - c) \]
2. \[ \theta \in \Theta, \text{ compact and convex and includes } \sum_j \theta_j = 1 \]
3. \[ D \geq 0 \]

- \[ \theta = (\theta_a, \theta_{B1}, \ldots) \] is the vector of portfolio weights
- \[ R(\theta; s') \] is the (random) return on portfolio \( \theta \):

\[ R(\theta; s') = R'_a \theta_a + R'_{B1} \theta_{B1} + \ldots \]
Proposition 1 (Portfolio Problem Solution)

There are $S$ constants $z_1, \ldots, z_S$ such that the Emerging Market Problem solution satisfies:

1. $v(\omega; s) = z_s \frac{\omega^{1-\sigma}}{1-\sigma}$

2. Consumption and default are proportional to $\omega$ and depend just on $\omega$ and market prospects $\mathcal{V}_s$:

   \[
   c(\omega, s) = \omega \max \left( \lambda^{-1/\sigma}, \mathcal{V}_s \right)
   \]

   \[
   D(\omega, s) = \omega \max \left( \lambda^{-1/\sigma} \frac{1}{\mathcal{V}_s} - 1, 0 \right)
   \]

3. Market prospects $\mathcal{V}_s$ are a monotonic transform of $\mathcal{U}_s$, utility from the optimal portfolio:

   \[
   \mathcal{U}_s = \max_{\theta \in \Theta} E \left[ z_{s'} \left( \frac{R(\theta; s')}{1-\sigma} \right)^{1-\sigma} \bigg| s \right]
   \]

$\Rightarrow$ Regularity Condition and Proof Sketch
Proposition 1 (comments)

- Separation of portfolio choice from consumption/default decision
  - \(c/\omega\) and \(D/\omega\) depend just on the exogenous state
  - the optimal portfolio does not depend on \(\omega\), \(c\), or \(D\)
  - effectively removes \(\omega\) as a state variable

- Easy-to-compute, even with many states and assets

- \(z\)'s determined by recursion that converges quickly in practice
Corollary (Default in Bad Times)

\[ \mathcal{V}_s \leq \mathcal{V}_{s'} \implies \]

\[ c(\omega; s) \leq c(\omega; s') \]
\[ D(\omega; s) \geq D(\omega; s') \]

Comments:
- When market prospects are grim in utility terms, the agent
  - consumes less
  - saves more
  - defaults more
- This holds in spite of the \( \omega^{-\sigma} \) term
- In contrast with models with exclusion punishment
Definition of Equilibrium

1. Delivery rates: $d_{ss'}$
2. Pricing Kernel: $\mu_{ss'}$
3. Prices: $q_1(s), P_D(s), ...$
4. Policy functions: $c(\omega; s), \theta(\omega; s), D(\omega, s), \omega' = w(\omega; s)$

such that

1. Given prices, policy functions are optimal for emerging market agents
2. Given delivery rates, prices are consistent with pricing kernel
3. Emerging market choices generate delivery rates $d$:

$$d_{ss'} = 1 - \frac{D(w(\omega; s), s')}{-(b_1(\omega_i, s) + \delta b_L(\omega_i, s))}$$
### Exogenous GDP Growth Grid

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<thead>
<tr>
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<th>EM GDP Growth</th>
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<td><strong>US GDP Growth</strong></td>
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- \( S = \{BB, BM, BG, MB, \ldots\} \), \( S = 9 \), int’l investor is first letter
- Estimate \( \Pi \) by maximum likelihood
  - annual frequency
  - World Bank data, Latin American Aggregate
  - 1970 - present
Having many assets, I need to model stock prices and yield curves.

I find that price fluctuations greatly impact portfolios and default.

Therefore, I need plausible, no-arbitrage prices in the model.

Build pricing kernel around US GDP process.
Calibration

- Default Cost: choose $\lambda$ so that default occurs in worst 2 states
  - GB (probability 0.78%), GM (probability 16.93%)
  - default coincides with low EM growth and high risk-free rates
  - $\lambda^{-1/\sigma} = c/\omega = 0.0391$

- Pricing kernel: realistic US yield curves and P/Ds
Equilibrium Haircuts

\[ h_1 (s, s') = 1 - d (s, s') \]
\[ h_L (s, s') = \frac{1 - d (s, s')}{{1 + q_L (s')}} \]

- Average realized haircut in model: (GB,GM) = (22%,3%)
- IQR for 7 largest Latin American Economies = 2-33%
- Overall prob. of being in default is 18%
  - Tomz and Wright (2012): 1.8-19%, for sovereign default
  - Probability of big default in model \( \approx 1\% \)
- Default occurs when risk-free rates are high and growth is low
Equilibrium Portfolios

Optimal Portfolio Weights

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1. Within U.S. states, portfolios are similar
   - int’l prices drive the composition of flows
   - int’l prices also drive default
   - market prospects are grim when stock prices and interest rates are high
2. Agents only use long-term debt in US-M states
   - because US-B is unlikely, this hints at pro-cyclicality in short-term debt
   - intuition: in US-G, long-term debt is very risky
   - intuition: in US-B, the yield curve is steep
Equilibrium Portfolios: 4 observations

Optimal Portfolio Weights

GG
GM
GB
MG
MM
MB
BG
BM
BB

Asset

a b1 bL B1

-1 0 1

%
3. In US-M, gross borrowing/lending pro-cyclical but net debt relatively flat
   - the agents are borrowing long and lending short
   - this trade becomes less attractive as EM growth declines:
   - risk-free rates fall and bond price increases become more likely
Equilibrium Portfolios: 4 observations

Optimal Portfolio Weights

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Asset %

a b1 bL B1
-1 0 1

Kieran Walsh (Darden)
4. Equity share is counter-cyclical
   - home-bias constraint binds in US-G
   - highest in US-B
   - intuition: agents exploit pro-cyclicality in P/Ds
Equilibrium Portfolios: 4 observations

Optimal Portfolio Weights

GG

GM

GB

MG

MM

MB

MG

MM

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BM

BB

Asset

Asset

Asset

Kieran Walsh (Darden)

March 7, 2015 31 / 38
### Equilibrium Portfolios and Flows

#### Cyclical Properties of Gross Flows and Portfolios

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<tbody>
<tr>
<td>Equity Inflow*</td>
<td>.20</td>
<td>.42</td>
<td>.11</td>
<td>.31</td>
<td>.01</td>
<td>.21</td>
</tr>
<tr>
<td>Short-term Debt Share</td>
<td>.29</td>
<td>.22</td>
<td>-.09</td>
<td>.01</td>
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<tr>
<td>Gross Capital Inflow*</td>
<td>.12</td>
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Data Sources: Model, Lane and Milesi-Ferretti (2007), WDI (World Bank)


- I have not targeted these moments
- Inflow: coming from equity and long-term debt
- Outflow: pro-cyclical lending in US-M states

*Pro-cyclical portfolios stem from pro-cyclical int’l prices*
Equilibrium Gross Debt

- Gross Debt/GNI is volatile and too high on average
  - because GDP traded, debt on the order of $P$ not $y$
  - robustness of papers without equity liabilities?

- However, the 10th and 50th percentiles are 14% and 170%

- Debt is too high because of gross positions in US-M states
  - quantity side of asset pricing puzzles
  - high $\sigma$ and the home-bias constraint mitigate this to some degree
  - with borrowing constraints, can match average Debt/GNI
4 Facts

1. Gross Flows and Portfolios
   - gross flows pro-cyclical $\approx 10\%$ ✓
   - equity inflow pro-cyclical $\approx 20\%$ ✓
   - short-term debt share pro-cyclical (too much so)

2. Partial Default
   - lots of variation in haircuts
   - haircuts $\approx 20\%$ in large defaults ✓

3. Default in Bad Times
   - property of model ✓
   - quantitatively, default coincides with low growth

4. High Debt
   - average debt too high
   - able to distinguish between gross and net debt
   - about half of time, debt/gni in 14-170% range
Conclusion

- Developed a new quantitative model of portfolio choice
  - allowed for partial default
  - allowed for int’l price fluctuations
  - applied to emerging market int’l finance

- Established properties of the model
  - separation of portfolio problem from consumption/default choice
  - rapid computation
  - default increases as market prospects deteriorate

- Explored quantitative implications for Latin America
  - pro-cyclicality in gross flows, equity inflow, and short-term debt
  - default coincides with low EM growth and high risk-free rates
  - plausible endogenous haircuts and high debt
Tomz (2007): little evidence for coordinated market exclusion
Public and Private Gross External Debt (WDI)
Reinhart and Rogoff (2009) dates
Proposition 1

Define

\[ U_s (z_1, ..., z_S) = \max_{\theta \in \Theta(s)} E \left[ z_{s'} \frac{(R(\theta; s, s'))^{1-\sigma}}{1-\sigma} \right] \]

The regularity condition is:

\[ \beta U_s (1) (1 - \sigma) < 1. \]

Sketch of Proof of Proposition

- Guess form of value function
- Solve for c and D in terms of z’s and \( \omega \)
- Verify \( \omega \) guess and derive a recursion for the z’s
- prove the recursion operator has a unique solution

⇒ Back
4 Facts in Emerging Market (EM) International Finance

1. Level and composition of gross capital flows ⇒ Back
   - as important as current accounts in understanding risk and crises
   - pro-cyclical: gross inflow/outflow, equity liabilities, short debt
   - theory literature mostly silent on equity and gross debt

2. Usually, default is only partial (across and within different bonds)
   - In contrast, in classic theoretical models:
     - market exclusion as punishment
     - full default

3. Default generally occurs in bad economic times
   - especially true in post-1970 Latin America
   - exclusion punishment implies lots of boom-time default:
     - persistence means punishment is least effective in booms

4. Gross external debt levels in excess of 50% of GNI
   - most policy and empirical work is about gross debt
   - in contrast, classic theoretical models get less than 10%
   - and, these models focus on net debt