Euro Area Business Cycle Network Training School

Term Structure Modeling and the Lower Bound Problem

By

Jens Christensen

(Federal Reserve Bank of San Francisco)

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Course Syllabus

Please note that, for each day, papers of particular importance are highlighted in bold.

Day 1: Term Structure Modeling in Normal Times

In the first day of the course, we first introduce the canonical affine term structure models as outlined by Dai and Singleton (2000) with particular focus on the canonical Gaussian models whose estimation is discussed in Joslin, Singleton, and Zhu (2011) and Hamilton and Wu (2012). Second, we analyze the class of arbitrage-free Nelson-Siegel models introduced in Christensen, Diebold, and Rudebusch (2011) and its extension to allow for stochastic volatility provided in Christensen, Lopez, and Rudebusch (2014a). Third, we will discuss the estimation of these models based on the Kalman filter and issues related to finite-sample bias, see Bauer, Rudebusch, and Wu (2012) and Christensen, Lopez, and Rudebusch (2015b).

Recommended Readings

Dai and Singleton (2000):

- This paper introduces the canonical classifications of affine dynamic term structure models.

Duffee (2002):

- This paper introduces the essentially affine risk premium specification for affine models.

Cheridito, Filipovi´c, and Kimmel (2007):

- This paper introduces the extended affine risk premium specification for affine models.

Joslin, Singleton, and Zhu (2011):

- This paper introduces a way to facilitate the estimation of Gaussian dynamic term structure models.

Hamilton and Wu (2012):

- This paper provides an alternative way to facilitate the estimation of Gaussian dynamic term structure models.

Note on Fisher and Gilles (1996):

- This note provides analytical formulas for the conditional and unconditional mean vector and covariance matrices in affine models.

Nelson and Siegel (1987):

- This is the founding paper that introduced the Nelson-Siegel yield curve. Note the short sample and maturity range!

Christensen, Diebold, and Rudebusch (2011):

- This paper introduces the affine arbitrage-free class of Nelson-Siegel (AFNS) term structure models.

Christensen, Diebold, and Rudebusch (2009):

- This paper generalizes the AFNS model class to allow for several slope and curvature factors as in Svensson (1995).

Christensen, Lopez, and Rudebusch (2014a):

- This paper generalizes the AFNS model class to allow for stochastic volatility.

Christensen, Lopez, and Rudebusch (2015b):

- This paper uses simulation exercises to analyze the efficiency of the Kalman filter for the estimation of AFNS models with and without stochastic volatility.

Bauer, Rudebusch, and Wu (2012):

- This paper analyzes finite-sample bias in Gaussian models and introduces a simulation-based method to adjust for it.

Day 2: Term Structure Modeling and the Lower Bound Problem

In the second day of the course, we focus on the problems arising from the fact that the key policy rates of the world's most prominent central banks have remained at or near their effective lower bounds for many years by now. We will study shadow-rate models and their estimation based on the extended Kalman filter in great detail since this is the leading solution to the lower bound problem. However, we will also analyze the novel stay-at-zero affine models developed by Monfort et al. (2015) and the linear-rational models introduced in Filipović, Larsson, and Trolle (2014).

Recommended Readings

Christensen and Rudebusch (2015a):

- This paper introduces the shadow-rate arbitrage-free Nelson-Siegel models and applies them to Japanese data.

Christensen and Rudebusch (2015b):

- This paper applies the shadow-rate AFNS model to U.S. Treasury yields since 1985 and studies its performance in both normal times and near the lower bound.

Christensen (2015):

- This paper uses simulation exercises to analyze the efficiency of the extended Kalman filter for the estimation of shadow-rate AFNS models.

Monfort, Pegoraro, Renne, and Roussellet (2015):

- This paper introduces a novel class of affine term structure models that respects a zero lower bound and can generate prolonged spells with the short rate stuck at its lower bound.

Filipović, Larsson, and Trolle (2014):

- This paper introduces the class of linear-rational term structure models that respects a lower bound and allows for unspanned stochastic volatility. The authors highlight the ability of this model class to price interest rate swaptions.

Day 3: Term Structure Modeling and Applications to Policy Questions

In the third day of the course, it is demonstrated how term structure models can be used to analyze problems relevant to monetary policy. First, the analysis in Christensen and Rudebusch (2012) and Christensen and Krogstrup (2015) are described. Both studies look into how quantitative easing (QE) affects long-term interest rates. Understanding how QE works is likely to be useful for how to handle the exit from the unconventional policies. Second, we will stress test the Fed's assets and income following Christensen, Lopez, and Rudebusch (2015a). In this case, the relevant policy questions are: What is the likelihood of large losses to the Fed's securities portfolio? What is the chance of a halt to the remittances to the U.S. Treasury? This research emphasizes the potential costs of QE and can also be used to stress test commercial banks' portfolios. Finally, we follow Christensen, Lopez, and Rudebusch (2010) and discuss a joint modeling of nominal and real yields in order to extract the inflation expectations embedded in Treasury yields.

Recommended Readings

Christensen and Rudebusch (2012):

- This paper analyzes the U.S. and U.K. experiences with QE. It uses real-time term structure model estimations to accurately decompose yield changes around QE announcements into changes to the expectations component and changes to the term premium component.

Christensen and Krogstrup (2015):

- This paper uses an approach similar to Christensen and Rudebusch (2012) to study the response of Swiss Confederation bond yields to announcements regarding the expansion of bank reserves undertaken by the Swiss National Bank in August 2011 in the weeks before it announced the peg of the Swiss franc to the euro on September 6, 2011. The paper emphasizes the role that bank reserves can play for the transmission of QE to long-term interest rates.

Christensen, Lopez, and Rudebusch (2015a):

- This paper uses the shadow-rate AFNS model analyzed in Christensen and Rudebusch (2015b) to stress test the assets and income of the Federal Reserve.

Christensen, Lopez, and Rudebusch (2010):

- This paper introduces a joint model of nominal and real yields (CLR TIPS model) and uses it to estimate the inflation expectations and risk premiums priced into nominal yields.

Christensen, Lopez, and Rudebusch (2012):

- This paper demonstrates how the CLR TIPS model can be used to assess deflation risk and value the deflation protection option embedded in the TIPS contract.

Christensen, Lopez, and Rudebusch (2015c):

- This paper modifies the CLR TIPS model to allow for stochastic volatility. This provides more accurate pricing of deflation risk. Also, the factors driving the deflation risk premium are analyzed in detail.

About the instructor

Jens Christensen is a research advisor in the Economic Research Department of the Federal Reserve Bank of San Francisco, which he joined in 2006 after receiving his PhD in finance from Copenhagen Business School. He also holds an MSc in economics from the University of Copenhagen. His research interests include credit risk modeling, credit risk management, and interest rate term structure modeling. His research in this area is widely cited and has been published in leading academic journals such as the Economic Journal, the Journal of Business and Economic Statistics, the Journal of Econometrics, the Journal of Financial Econometrics, Journal of Monetary Economics, and the Journal of Money, Credit, and Banking amongst others. Finally, he is a frequent presenter at international conferences on issues related to sovereign bond markets and monetary policy.

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