Measuring the informativeness of economic actions and market prices[1]

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Abstract

In many circumstances economic agents are partially ignorant about payoff-relevant state variables, but must nonetheless decide what action to take. To give just one example, investment decisions must often be made without full knowledge of the rate of return. In many such cases, agents can potentially obtain useful information by observing either the actions of other agents, or market prices. However, the information content of such actions or prices is in general hard to order, outside of special cases in which linearity arises. This paper deploys Lehmann's (1988) information ordering to this class of problems. First, I relate Lehmann's ordering to supermodularity properties. Second, I obtain a tractable first-order version of Lehmann's ordering. Third, I relate this first-order condition to both individual optimization problems and to equilibrium pricing conditions.

1 Introduction

In many circumstances economic agents are partially ignorant about payoff-relevant state variables, but must nonetheless decide what action to take. To give just one example, investment decisions must often be made without full knowledge of the rate of return. In many such cases, agents can potentially obtain useful information by observing either the actions of other agents, or market prices. For example, one investing agent might observe how much other agents are investing in similar projects, and use this information to infer something about the likely rate of return. Alternatively, an investing agent might observe the market price of financial securities[1] whose payoff is tied to the rate of return.

The goal of this paper is to develop tools for analyzing how the informativeness of actions and prices responds to changes in the economic environment. In general, several approaches to this problem are possible. First, one can compute the full equilibrium, and in particular the expected payoff of agent(s) making use of the information, and use this as a measure of informativeness. Second, one can impose sufficient assumptions on the economic environment to ensure that actions and/or prices are linear in underlying state variables (see Example 1 below). This second approach has the advantage of avoiding an explicit calculation of expected payoffs, but has the disadvantage of restricting the class of problems that can be analyzed. Third, and related, one can make use of more general orderings of information content—and in particular, that of Blackwell (1953). Again, this avoids an explicit calculation of expected payoffs, but has the disadvantage that Blackwell's ordering is very incomplete, in the sense of failing to order many situations.[2]

In this paper, I follow the third of these approaches, but instead of using Blackwell's ordering I use Lehmann's (1988) statistical ranking of "experiments." First, I relate Lehmann's

[1] See the literature surveyed by Bond, Edmans and Goldstein (2012).

[2] See discussion in Lehmann (1988). A well-known example of the weakness of Blackwell's ordering is the following. Let ε be a normally distributed random available, and let δ be a non-normal random variable. Then an immediate consequence of Cramer's decomposition theorem is that the Blackwell ordering is unable to rank the informativeness of $X = \theta + \varepsilon$ and $Y = \theta + \delta$ with respect to θ, even as the variance ratio $\frac{var(\varepsilon)}{var(\delta)}$ grows either very large or very small.

measure to the log-supermodularity (or log-submodularity) of a monotone transformation of an agent's decision function. Second, I obtain a first-order version of Lehmann's condition. Third, I relate the first-order condition to both individual optimization problems and to equilibrium pricing conditions.

Example 1: When the informed agent's decision is a linear function of the underlying information and an "error" term, measuring informativeness is straightforward, as illustrated by the following example. Let θ and t be random variables, both privately observed by agent 1, and let ψ be a publicly observable "regime" parameter. Suppose agent 1 takes the action $a_1 = 1 - \psi + \psi\theta - t$. Agent 2 faces a decision problem for which θ is payoff-relevant but t is not. Agent 2 observes a_1.

From observing agent 1's action a_1, agent 2 can infer $\frac{a_1-(1-\psi)}{\psi} = \theta - \frac{t}{\psi}$. Consequently, the informativeness of action a_1 is summarized by ψ, since as this term increases agent 2 effectively sees a signal of θ that has less noise added. So in particular, an increase in ψ increases the informativeness of action a_1.

In this simple example, agent 1's action a_1 grows more informative as ψ changes when the action a_1 is supermodular in agent 1's information θ and the regime ψ. The results below establish the importance of supermodularity more generally.

Example 2: Agent 1 makes an investment a_1. The investment succeeds with probability θ, in which case it generates a cash flow of $v(a_1)$. In addition, if the investment fails, the government "bails out" agent 1 with probability $1-\psi$ and pays him $v(a_1)$. The cost of a_1 is $a_1 r(t)$, where $r(t)$ is agent 1's cost of funds, indexed by agent 1's type t. Both θ and t are privately observed by agent 1, while the government bail out policy ψ is public. Agent 2 is contemplating a similar investment, and so wishes to learn θ.

From the first-order condition, agent 1 chooses a_1 to satisfy $(\theta + (1-\psi)(1-\theta)) v'(a_1) - r(t) = 0$. It is non-obvious how a change in the bail out policy ψ affects the informativeness of a_1.

Example 3: Identical to Example 2, except now government bail outs are for the agents

who provide capital to agent 1. Because of these bail outs, agent 1's cost of a_1 is now $a_1 r(t,\psi)$. From the first-order condition, agent 1 chooses a_1 to satisfy $\theta v'(a_1) - r(t,\psi) = 0$. Again, it is non-obvious how a change in the bail out policy ψ affects the informativeness of a_1.

2 Model

There are two agents. Agent 1 privately observes a state variable $\theta \in \Re$. Agent 1 also has a privately observable "type" t. Agent 1 takes some publicly observable action, $a_1 \in A_1 \subset \Re$. The action depends on his type t, information θ, and a publicly observable "regime" parameter ψ according to the mapping η:

$$a_1 = \eta(t, \theta, \psi).$$

The distribution of type t admits a density function, and I assume it has full support over all relevant values. The type t and regime ψ belong to lattices T and Ψ respectively. The function η is strictly decreasing in t and strictly increasing in θ.

Note that both the state θ and the action a_1 are one-dimensional objects. These properties are needed to make use of Lehmann's ordering. In addition, unless otherwise stated all results in the paper assume:

Assumption 1 *The type $t \in \Re$.*

Agent 2 must make a decision $a_2 \in A_2 \subset \Re$. Agent 2 does not observe θ, and so must try to infer its realization from observing a_1. His payoff is given by $u_2(a_2, \theta)$. Consequently, the only way in which the regime ψ affects agent 2 is through its effect on the information content of a_1.

The payoff function u_2 has the single-crossing property (SCP), i.e., if $\tilde{a}_2 > a_2$ and $\tilde{\theta} > \theta$ then $u_2(\tilde{a}_2, \theta) - u_2(a_2, \theta) \geq (>)\, 0$ implies $u_2\left(\tilde{a}_2, \tilde{\theta}\right) - u_2\left(a_2, \tilde{\theta}\right) \geq (>)\, 0$. In addition, u_2

satisfies the following regularity condition:

$$\text{if } \tilde{a}_2 > a_2 \text{ then } u_2(\tilde{a}_2, \theta) - u_2(a_2, \theta) \text{ is weakly quasi-concave as a function of } \theta. \quad (1)$$

Note that quasi-concavity of the marginal benefit $u_2(\tilde{a}_2, \theta) - u_2(a_2, \theta)$ means that the marginal benefit is either monotone increasing or decreasing, or else is an increasing then decreasing function of θ. This regularity condition is used by Athey (2002); its role in the current paper will be made clear below.

3 Accuracy and supermodularity

3.1 Accuracy

Define the function $P(a_1, \theta, \psi)$ by

$$P(a_1, \theta, \psi) = \Pr(\eta(t, \theta, \psi) \leq a_1 | \theta).$$

Following Lehmann (1988) and Persico (2000), the "signal" a_1 is more *accurate*[3] in regime $\tilde{\psi}$ than regime ψ if and only if the function $h\left(a_1, \theta, \psi, \tilde{\psi}\right)$ defined by

$$P\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) = P(a_1, \theta, \psi) \quad (2)$$

is weakly increasing in θ for all a_1.[4] Lehmann (1988) and Quah and Strulovici (2009) establish conditions under which an increase in accuracy increases the payoff of a decision-maker. In particular, I make use of Quah and Strulovici's result below.

[3] "Accuracy" is Persico's term. The ordering is Lehmann's.

[4] To make the connection between this definition and Lehman and Persico's papers more explicit, define $P^{-1}(\gamma, \theta, \psi)$ as the inverse of P with respect to its first argument, i.e. $P^{-1}(P(a_1, \theta, \psi), \theta, \psi) = a_1$. Then the function h is given by

$$h\left(a_1, \theta, \psi, \tilde{\psi}\right) = P^{-1}\left(P(a_1, \theta, \psi), \theta, \tilde{\psi}\right).$$

To gain intuition for the accuracy ordering, it is useful to consider the case in which Assumption 1 holds. Given this assumption, define $\bar{t}(a_1, \theta, \psi)$ as the solution to

$$\eta\left(\bar{t}(a_1, \theta, \psi), \theta, \psi\right) = a_1. \tag{3}$$

In words, $\bar{t}(a_1, \theta, \psi)$ is the type of agent 1 such that, given information θ and regime ψ, he takes action a_1. So

$$P(a_1, \theta, \psi) = \Pr\left(t \geq \bar{t}(a_1, \theta, \psi)\right). \tag{4}$$

The analysis below makes heavy use of $\bar{t}$.

In this case, (2) is equivalent to

$$\Pr\left(t \geq \bar{t}\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right)\right) = \Pr\left(t \geq \bar{t}(a_1, \theta, \psi)\right),$$

and hence to

$$\bar{t}\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) = \bar{t}(a_1, \theta, \psi). \tag{5}$$

From this identity, one can see that $h\left(a_1, \theta; \psi, \tilde{\psi}\right)$ is the action that type $\bar{t}(a_1, \theta, \psi)$ takes when he has information θ and is in regime $\tilde{\psi}$. An increase in θ has two effects on $h\left(a_1, \theta; \psi, \tilde{\psi}\right)$. First, it directly increases the action, since $\eta_\theta \geq 0$. But second, it increases the type taking the action, since $\bar{t}_\theta > 0$, and this reduces the action. The function h is increasing if the first effect is stronger. Roughly speaking, the first effect is stronger if θ affects the action more in regime $\tilde{\psi}$ than regime ψ. This corresponds to a_1 being a more accurate signal of θ in regime $\tilde{\psi}$ than ψ.

Example 1, continued: In the example, $\bar{t}(a_1, \theta, \psi) = -a_1 + 1 - \psi + \psi\theta$ and $h\left(a_1, \theta; \psi, \tilde{\psi}\right) = \psi - \tilde{\psi} + \left(\tilde{\psi} - \psi\right)\theta + a_1$. So if $\tilde{\psi} > \psi$ then h is increasing in θ, and under the definition, a_1 is a more accurate signal in regime $\tilde{\psi}$.

3.2 Supermodularity

Proposition 1 *Consider $\tilde{\psi} \succeq \psi$. Action a_1 is a more (respectively, less) accurate signal of θ in regime $\tilde{\psi}$ than regime ψ if there exists a strictly increasing and positive function f such that $f \circ \eta$ is log-supermodular (respectively, log-submodular).*

Proof of Proposition 1: Fix $\tilde{\psi} \succeq \psi$, $\tilde{\theta} > \theta$, and a_1. By the definition of $\bar{t}$,

$$\begin{aligned} \eta\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \psi\right) &= a_1 \\ \eta\left(\bar{t}\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right), \theta, \tilde{\psi}\right) &= h\left(a_1, \theta, \psi, \tilde{\psi}\right). \end{aligned}$$

Combined with (5), these equalities imply that, for any function f,

$$\frac{f\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right)\right)}{f\left(a_1\right)} = \frac{f\left(\eta\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \tilde{\psi}\right)\right)}{f\left(\eta\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \psi\right)\right)}.$$

Similarly,

$$\frac{f\left(h\left(a_1, \tilde{\theta}, \psi, \tilde{\psi}\right)\right)}{f\left(a_1\right)} = \frac{f\left(\eta\left(\bar{t}\left(a_1, \tilde{\theta}, \psi\right), \tilde{\theta}, \tilde{\psi}\right)\right)}{f\left(\eta\left(\bar{t}\left(a_1, \tilde{\theta}, \psi\right), \tilde{\theta}, \psi\right)\right)}.$$

Hence if $f > 0$, then $f\left(h\left(a_1, \tilde{\theta}, \psi, \tilde{\psi}\right)\right) \geq f\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right)\right)$ is equivalent to

$$f\left(\eta\left(\bar{t}\left(a_1, \tilde{\theta}, \psi\right), \tilde{\theta}, \tilde{\psi}\right)\right) f\left(\eta\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \psi\right)\right) \geq f\left(\eta\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \tilde{\psi}\right)\right) f\left(\eta\left(\bar{t}\left(a_1, \tilde{\theta}, \psi\right), \tilde{\theta}, \psi\right)\right).$$

Since $\bar{t}\left(a_1, \tilde{\theta}, \psi\right) \geq \bar{t}\left(a_1, \theta, \psi\right)$, this inequality is equivalent to log-supermodularity of $f \circ \eta$ for a strictly increasing function f. The result for log-submodularity follows similarly. **QED**

4 Local conditions

4.1 Local accuracy condition

For the remainder of the paper, I assume:

Assumption 2 *The regime* $\psi \in \Re$.

Assumption 3 *The function* η *is differentiable with respect to* θ *and* ψ.

I use Assumptions 2 and 3 to obtain a local, or first-order, formulation of Lehmann's accuracy condition for the case of small regime changes. Note that this formulation holds independently of Assumption 1.

Proposition 2 *A small increase in the regime* ψ *increases the accuracy of* a_1 *as a signal if and only if*

$$\frac{\partial}{\partial \theta} \frac{P_{a_1}(a_1, \theta, \psi)}{P_\psi(a_1, \theta, \psi)} \geq 0. \tag{6}$$

Proof of Proposition 2: Differentiating (2) with respect to θ gives

$$h_\theta\left(a_1, \theta, \psi, \tilde{\psi}\right) P_{a_1}\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) + P_\theta\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) - P_\theta(a_1, \theta, \psi) = 0.$$

Since $P_{a_1} \geq 0$, it follows that a_1 is a more accurate signal under regime $\tilde{\psi}$ than under regime ψ if and only if

$$P_\theta\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) - P_\theta(a_1, \theta, \psi) \leq 0$$

for all a_1 and θ. Since $h(a_1, \theta, \psi, \psi) = a_1$, it follows that a small increase in regime ψ increases the accuracy of a_1 as a signal if and only if

$$\frac{\partial}{\partial \tilde{\psi}} \left[P_\theta\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) - P_\theta(a_1, \theta, \psi)\right]\Big|_{\tilde{\psi}=\psi} \leq 0,$$

i.e., if and only if

$$P_{\theta a_1}(a_1, \theta, \psi)\, h_{\tilde{\psi}}(a_1, \theta, \psi, \psi) + P_{\theta\psi}(a_1, \theta, \psi) \leq 0.$$

From (2), $h_{\tilde{\psi}}$ satisfies

$$h_{\tilde{\psi}}\left(a_1, \theta, \psi, \tilde{\psi}\right) P_{a_1}\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) + P_\psi\left(h\left(a_1, \theta, \psi, \tilde{\psi}\right), \theta, \tilde{\psi}\right) = 0.$$

Substituting into the previous inequality, a small increase in regime ψ increases the accuracy of a_1 as a signal if and only if

$$-P_{\theta a_1}(a_1,\theta,\psi)\frac{P_\psi(a_1,\theta,\psi)}{P_{a_1}(a_1,\theta,\psi)}+P_{\theta\psi}(a_1,\theta,\psi)\leq 0,$$

which, since $P_{a_1}>0$, is equivalent to condition (6). **QED**

4.2 The relation between accuracy and the decision function η

Proposition 2 expresses Lehmann's accuracy condition in terms of the probability function P. The next result maps this into a condition on the decision function η.

The remainder of the paper deals with the case in which Assumptions 1-3 hold, along with:

Assumption 4 *The decision function η is differentiable with respect to type t.*

For this case, and using (4), condition (6) of Proposition 2 can be written as

$$\frac{\partial}{\partial\theta}\frac{\frac{\partial}{\partial a_1}\Pr\left(t\geq\bar{t}(a_1,\theta,\psi)\right)}{\frac{\partial}{\partial\psi}\Pr\left(t\geq\bar{t}(a_1,\theta,\psi)\right)}\geq 0,$$

and hence as simply

$$\frac{\partial}{\partial\theta}\frac{\bar{t}_a(a_1,\theta,\psi)}{\bar{t}_\psi(a_1,\theta,\psi)}\geq 0.$$

From (3), the derivatives of $\bar{t}$ are given by:

$$\eta_t\left(\bar{t}(a_1,\theta,\psi),\theta,\psi\right)\bar{t}_a(a_1,\theta,\psi) = 1 \tag{7}$$

$$\eta_t\left(\bar{t}(a_1,\theta,\psi),\theta,\psi\right)\bar{t}_\theta(a_1,\theta,\psi) = -\eta_\theta\left(\bar{t}(a_1,\theta,\psi),\theta,\psi\right) \tag{8}$$

$$\eta_t\left(\bar{t}(a_1,\theta,\psi),\theta,\psi\right)\bar{t}_\psi(a_1,\theta,\psi) = -\eta_\psi\left(\bar{t}(a_1,\theta,\psi),\theta,\psi\right). \tag{9}$$

Substituting into the above inequality, condition (6) of Proposition 2 can be written simply

as

$$\frac{\partial}{\partial \theta} \frac{1}{-\eta_{\psi}\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \psi\right)} \geq 0.$$

Manipulation of this condition delivers:

Proposition 3 *A small increase in the regime ψ increases the accuracy of a_1 as a signal if and only if*

$$\frac{\partial}{\partial \theta} \eta_{\psi}\left(\bar{t}\left(a_1, \theta, \psi\right), \theta, \psi\right) \geq 0, \tag{10}$$

or equivalently, if and only if

$$\frac{\partial}{\partial \psi} \left| \frac{\eta_{\theta}(t,\theta,\psi)}{\eta_t(t,\theta,\psi)} \right| \Bigg|_{t=\bar{t}(a_1,\theta,\psi)} \geq 0. \tag{11}$$

Condition (11) is intuitive. The regime change increases the accuracy of a_1 if it increases the sensitivity of a_1 to information θ relative to the sensitivity of a_1 to the type t, which from the perspective of agent 2 is noise.

In the special case in which agent 1's action η is a linear function of t (see Example 1), the condition of Proposition 3 reduces to $\eta_{\theta\psi} \geq 0$, i.e., the action is supermodular in θ and regime ψ.

Proof of Proposition 3: Condition (10) is immediate from the text above. See appendix for condition (11).

4.3 Relation with linearization of $a_1 = \eta\left(t, \theta, \psi\right)$

One way to understand condition (10) is a follows. Taking the first-order approximation, agent 1's action gives

$$a_1 = \left(\theta - E\left[\theta\right]\right) \eta_{\theta}\left(E\left[t\right], E\left[\theta\right], \psi\right) + \left(t - E\left[t\right]\right) \eta_t\left(E\left[t\right], E\left[\theta\right], \psi\right).$$

Similar to Example 1, the information content of observing agent 1's action is the same as observing

$$\theta + t\frac{\eta_t\left(E\left[t\right], E\left[\theta\right], \psi\right)}{\eta_\theta\left(E\left[t\right], E\left[\theta\right], \psi\right)} + \text{constant}.$$

Hence a change in regime ψ that increases the ratio $\left|\frac{\eta_\theta(E[t],E[\theta],\psi)}{\eta_t(E[t],E[\theta],\psi)}\right|$ increases the informativeness of observing a_1, since it effectively reduces the noise with which θ is observed. Consequently, Proposition 3 can be viewed as justifying this "naive" linearization approach to measuring informativeness.

4.4 Relation with log-supermodularity

It is also worthwhile to compare condition (11) to log-supermodularity of $f \circ \eta$. Expanding, (11) is equivalent to

$$\eta_{\theta\psi}\eta_t - \eta_\theta\eta_{t\psi} \leq 0, \tag{12}$$

evaluated at $t = \bar{t}\left(a_1, \theta, \psi\right)$. Define $\tilde{\eta} = f \circ \eta$. Log-supermodularity of $f \circ \eta$ implies that for any $\tilde{\theta} \geq \theta$ and $\tilde{t} \geq t$,

$$\frac{\partial}{\partial\psi}\ln\tilde{\eta}\left(\tilde{t}, \tilde{\theta}, \psi\right) \geq \frac{\partial}{\partial\psi}\ln\tilde{\eta}\left(t, \theta, \psi\right),$$

and hence that for any $\varepsilon_\theta \geq 0$ and $\varepsilon_t \geq 0$,

$$\varepsilon_\theta\frac{\partial^2}{\partial\theta\partial\psi}\ln\tilde{\eta}\left(t, \theta, \psi\right) + \varepsilon_t\frac{\partial^2}{\partial t\partial\psi}\ln\tilde{\eta}\left(t, \theta, \psi\right) \geq 0.$$

Evaluating, this last inequality is

$$\varepsilon_\theta\frac{\partial}{\partial\theta}\frac{\tilde{\eta}_\psi\left(t, \theta, \psi\right)}{\tilde{\eta}\left(t, \theta, \psi\right)} + \varepsilon_t\frac{\partial}{\partial t}\frac{\tilde{\eta}_\psi\left(t, \theta, \psi\right)}{\tilde{\eta}\left(t, \theta, \psi\right)} \geq 0,$$

i.e.,

$$\varepsilon_\theta\left(\tilde{\eta}_{\psi\theta}\tilde{\eta} - \tilde{\eta}_\psi\tilde{\eta}_\theta\right) + \varepsilon_t\left(\tilde{\eta}_{\psi t}\tilde{\eta} - \tilde{\eta}_\psi\tilde{\eta}_t\right) \geq 0,$$

i.e.,

$$\varepsilon_\theta\left(f'\eta_{\psi\theta}\eta+\left(f''-(f')^2\right)\eta_\theta\eta_\psi\right)+\varepsilon_t\left(f'\eta_{\psi t}\eta+\left(f''-(f')^2\right)\eta_\psi\eta_t\right)\geq 0.$$

Substituting in $\varepsilon_\theta = -\eta_t$ and $\varepsilon_t = \eta_\theta$ yields (12). In other words, log-supermodularity of $f \circ \eta$ implies condition (11), while it is easy to give examples satisfying (11) but not log-supermodularity.

5 Accuracy and agent 2's payoff

Propositions 2 and 3 give conditions under which a change in regime ψ renders a_1 a more accurate signal of θ. Since agent 2's objective u_2 has the SCP, Quah and Strulovici's (2009) Proposition 9 applies. In the notation of the current paper:

Proposition 4 *[Proposition 9, Quah and Strulovici (2009)] Let $\eta_2(a_1)$ be a weakly increasing function specifying agent 2's action as a function of a_1. If a_1 is a more accurate signal under regime $\tilde{\psi}$ than regime ψ, then there exists a function $\tilde{\eta}_2$ such that, for any realization of the state θ, agent 2's payoff $u_2\left(\tilde{\eta}_2\left(\eta_1\left(t,\theta,\tilde{\psi}\right)\right),\theta\right)$ first-order stochastically dominates $u_2(\eta_2(\eta_1(t,\theta,\psi)),\theta)$.*

Proposition 4 shows the power of Lehmann's accuracy ordering. In particular, and again following Quah and Strulovici, the following statement is an immediate corollary:

Corollary 1 *Let $\eta_2(a_1)$ be a weakly increasing function, and let a_1 be a more accurate signal under regime $\tilde{\psi}$ than regime ψ. Then there exists a function $\tilde{\eta}_2$ such that, for any increasing function v and any prior distribution of the state θ,*

$$E\left[v\left(u_2\left(\tilde{\eta}_2\left(\eta_1\left(t,\theta,\tilde{\psi}\right)\right),\theta\right)\right)\right]\geq E\left[v\left(u_2\left(\eta_2\left(\eta_1\left(t,\theta,\psi\right)\right),\theta\right)\right)\right].$$

Hence Corollary 1 implies that, provided agent 2's optimal action after observing a_1 is increasing in a_1 in regime ψ, then there is some strategy $\tilde{\eta}_2$ that delivers a higher expected

payoff to agent 2 under regime $\tilde{\psi}$ *regardless* of the prior distribution of θ. Moreover, in the important case in which u_2 takes the form of $v(c(a_2, \theta))$, i.e., in which u_2 is given by agent 2's utility v over consumption c, then $\tilde{\eta}$ also increases utility regardless of the form of v, provided only that v is strictly increasing (i.e., non-satiability).

Given these results, it is important to give conditions that justify the condition that $\eta_2(a_1)$ is a weakly increasing function. Both Lehmann (1988) and Quah and Strulovici (2009) achieve this by assuming that the signal a_1 satisfies the monotone likelihood ratio (MLR) property, i.e., $\frac{P_{a_1}\left(a_1, \tilde{\theta}, \psi\right)}{P_{a_1}(a_1, \theta, \psi)}$ is weakly increasing in a_1 whenever $\tilde{\theta} > \theta$. Together with certain properties on the payoff function u_2, the MLR property ensures that agent 2's optimal strategy η_2 is indeed weakly increasing in a_1. In particular, Quah and Strulovici give a condition for u_2 that nests both Lehmann's condition and the SCP.

Unfortunately, checking whether the MLR property is satisfied when the "signal" a_1 is an action is not easy. Specifically, the MLR property in this case is

$$\frac{\partial}{\partial a_1} \frac{\bar{t}_a\left(a_1, \tilde{\theta}, \psi\right)}{\bar{t}_a(a_1, \theta, \psi)} \geq 0,$$

and evaluating this does not yield a very tractable condition.

To circumvent this problem, I make use of Athey's (2002) Theorem 3, which establishes that $\eta_2(a_1)$ is increasing if the signal a_1 satisfies the monotone probability ratio (MPR) property, i.e., $\frac{P\left(a_1, \tilde{\theta}, \psi\right)}{P(a_1, \theta, \psi)}$ is weakly increasing in a_1 whenever $\tilde{\theta} > \theta$. The MPR property is necessary but not sufficient for the MLR property (Eeckhoudt and Gollier (1995)). The cost of relaxing the MLR property is that one needs more conditions on the payoff function u_2. This is the role of the regularity condition (1), which is imposed in Athey's Theorem 3.

To state the next result, it is convenient to adopt the following assumption, which is purely a normalization:[5]

Assumption 5 *The type t is distributed uniformly over one of* $[0,1]$, $(0,1]$, $[0,1)$, $(0,1)$.

[5] For an arbitrary continuously distributed random variable $\tilde{t}$ with distribution function F, the random variable $F\left(\tilde{t}\right)$ is uniformly distributed over one of $[0,1]$, $(0,1]$, $[0,1)$, $(0,1)$.

Lemma 1 *Assuming sufficient differentiability, the MPR property is equivalent to*

$$-\left(1-\bar{t}\left(a_1,\theta,\psi\right)\right)\frac{\partial}{\partial t}\ln\left|\frac{\eta_\theta\left(t,\theta,\psi\right)}{\eta_t\left(t,\theta,\psi\right)}\right|\Bigg|_{t=\bar{t}\left(a_1,\theta,\psi\right)}\leq 1. \tag{13}$$

Proof of Lemma 1: See appendix.

5.1 Main result

Together, the above results deliver the following, which relates when a small regime change increases agent 2's payoff to the properties of the ratio $\frac{\eta_\theta}{\eta_t}$.

Theorem 1 *If the ratio* $\frac{\eta_\theta}{\eta_t}$ *satisfies (13), then a small increase in the regime* ψ *increases agent 2's maximized payoff if and only if* $\frac{\eta_\theta}{\eta_t}$ *satisfies (11).*

Note that, parallel to Corollary 1, it is possible to state Theorem 1 in a stronger way, e.g., without any dependence on the prior distribution of the state θ.

6 Link to optimization problems

The results above are written in terms of the function η, which gives agent 1's action a_1 as a function of (t,θ,ψ). In many applications, a_1 emerges from an optimization problem.

Suppose now that agent 1's objective is $U\left(a_1,t,\theta,\psi\right)$, where $U_{aa}<0$. So η is given by the first-order condition

$$U_a\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)=0.$$

Hence $\eta_\theta U_{aa}=-U_{a\theta}$ and $\eta_t U_{aa}=-U_{at}$. I assume $U_{a\theta}>0$ and $U_{at}<0$, so that $\eta_\theta>0$ and $\eta_t<0$, as assumed earlier. Evaluating,

$$\frac{\eta_\theta\left(t,\theta,\psi\right)}{\eta_t\left(t,\theta,\psi\right)}=\frac{U_{a\theta}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{U_{at}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)},$$

which delivers the following corollary to Theorem 1:

Corollary 2 *If*

$$-\left(1-\bar{t}\left(a_{1},\theta,\psi\right)\right)\frac{\partial}{\partial t}\ln\left|\frac{U_{a\theta}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{U_{at}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}\right|\Bigg|_{t=\bar{t}\left(a_{1},\theta,\psi\right)}\leq 1, \tag{14}$$

then a small increase in the policy ψ increases agent 2's payoff if and only if for all a_1 and θ,

$$\frac{\partial}{\partial\psi}\left|\frac{U_{a\theta}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{U_{at}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}\right|\Bigg|_{t=\bar{t}\left(a_{1},\theta,\psi\right)}\geq 0. \tag{15}$$

7 Link to market prices

Next, I consider the case in which the observed "action" a_1 is a market price, rather than the action of a single agent.

Following Grossman and Stiglitz (1980), consider a setting in which a mass α of agents observe a signal θ and trade a risky asset. The risky asset has price a_1, and noisy supply $s(t)$, where s is an increasing function. Given price a_1 and signal θ, an agent's demand for the asset is given by some function $x(a_1,\theta,\psi)$. Unlike Grossman and Stiglitz, and the overwhelming majority of the subsequent literature, few preference and distributional assumptions are imposed here.[6]

For a given regime ψ, a (rational expectations) equilibrium is a price function $\eta(t,\theta,\psi)$ such that, for all realizations of the signal θ and supply shock t, the market clears, i.e.,

$$\alpha x\left(\eta\left(t,\theta,\psi\right),\theta,\psi\right)=s\left(t\right). \tag{16}$$

[6] A recent paper by Breon-Drish (2013) relaxes the normality assumptions on θ and $s(t)$, but maintains enough assumptions to ensure that demand of informed investors is linear in θ and some transformation of the price. The analysis below shares with Breon-Drish a heavy focus on the implications of the market-clearing condition.

Differentiation with respect to t, θ, ψ yields

$$
\begin{aligned}
\eta_t(t,\theta,\psi)\, x_a(\eta(t,\theta,\psi),\theta,\psi) - \frac{s'(t)}{\alpha} &= 0 \\
\eta_\theta(t,\theta,\psi)\, x_a(\eta(t,\theta,\psi),\theta,\psi) + x_\theta(\eta(t,\theta,\psi),\theta,\psi) &= 0 \\
\eta_\psi(t,\theta,\psi)\, x_a(\eta(t,\theta,\psi),\theta,\psi) + x_\psi(\eta(t,\theta,\psi),\theta,\psi) &= 0.
\end{aligned}
$$

Hence

$$\eta_\psi(\bar{t}(a_1,\theta,\psi),\theta,\psi) = -\frac{x_\psi(a_1,\theta,\psi)}{x_a(a_1,\theta,\psi)},$$

and so the accuracy-improvement condition (10) is

$$\frac{\partial}{\partial\theta}\frac{x_a(a_1,\theta,\psi)}{x_\psi(a_1,\theta,\psi)} \geq 0. \tag{17}$$

If demand x is linear in price a_1 and information θ, as is the case in the standard CARA-normal framework, this condition is simply $x_{\psi\theta} \geq 0$, i.e., supermodularity of demand in information θ and regime ψ.[7] In words, a change in ψ increases the accuracy of price as a signal if it increases the sensitivity of demand to information θ. This change then forces the equilibrium price to become more sensitive to θ to ensure that markets clear.

Turning to condition (13), observe that

$$\frac{\eta_\theta(t,\theta,\psi)}{\eta_t(t,\theta,\psi)} = -\frac{\alpha}{s'(t)} x_\theta(\eta(t,\theta,\psi),\theta,\psi),$$

so that (13) is

$$-\left(1-\bar{t}(a_1,\theta,\psi)\right)\left(\tfrac{x_{\theta a}(a_1,\theta,\psi)\eta_t(\bar{t}(a_1,\theta,\psi),\theta,\psi)}{x_\theta(a_1,\theta,\psi)} - \tfrac{s''(\bar{t}(a_1,\theta,\psi))}{s'(\bar{t}(a_1,\theta,\psi))}\right) \leq 1,$$

[7] Recall that $x_a < 0$, i.e., demand is decreasing in price.

which after substitution is

$$-\left(1-s^{-1}\left(\alpha x\left(a_1,\theta,\psi\right)\right)\right)\left(\frac{x_{\theta a}(a_1,\theta,\psi)}{x_\theta(a_1,\theta,\psi)x_a(a_1,\theta,\psi)}\frac{s'(\bar{t}(a_1,\theta,\psi))}{\alpha}-\frac{s''(\alpha x(a_1,\theta,\psi))}{s'(\alpha x(a_1,\theta,\psi))}\right)\leq 1.$$

Using $s'\left(t\right)=\frac{1}{(s^{-1})'(s(t))}$, this inequality is equivalent to

$$-\frac{\left(1-s^{-1}\left(\alpha x\left(a_1,\theta,\psi\right)\right)\right)}{\alpha\left(s^{-1}\right)'\left(\alpha x\left(a_1,\theta,\psi\right)\right)}\frac{x_{\theta a}(a_1,\theta,\psi)}{x_\theta(a_1,\theta,\psi)x_a(a_1,\theta,\psi)}+\left(1-s^{-1}\left(\alpha x\left(a_1,\theta,\psi\right)\right)\right)\frac{s''(\alpha x(a_1,\theta,\psi))}{s'(\alpha x(a_1,\theta,\psi))}\leq 1. \quad (18)$$

Corollary 3 *If (18) holds, then a small increase in the regime ψ increases agent 2's maximized payoff if and only if (17) holds.*

An important feature of conditions (17) and (18) is that both can be checked without solving for the equilibrium price function η.

8 Feedback from decisions to prices

In Section 7, a_1 is a market price, and I ask the question: how much information does the price a_1 provide to an agent 2. Importantly, the action chosen by agent 2 has no effect on the price a_1?

In this section, I consider instead the case in which there is a "feedback" effect from the action a_2 of agent 2 to the price. This is the case in which prices both affect decisions and reflect decisions.[8]

Formally, consider a firm with a share price a_1. Some economic actor—agent 2—associated with the firm (a manager, an employee, an activist shareholder, a regulator etc) takes an action a_2 that responds to a_1. Among the reasons a_2 may respond to a_1 is that a_1 contains information about θ, and θ affects agent 2's preferred action. The information

[8]See Bond, Edmans and Goldstein (2012) for a literature survey.

content of a_1 with respect to θ depends on the regime ψ. Accordingly, I write

$$a_2(a_1, \psi)$$

for the action chose by agent 2.

Investors (but not agent 2) observe θ and t, and these variables potentially impact the share price. Moreover, given a share price a_1, investors who trade the firm's shares can anticipate how agent 2 will act, and this also affects the firm's share price. Formally, the pricing equation is[9]

$$g(a_1, t, \theta, \psi) = f(a_2(a_1, \psi), t, \theta, \psi). \tag{19}$$

The price is function of θ and t, observed by investors, along with the regime ψ. Hence the price is $\eta(t, \theta, \psi)$, defined as a solution to

$$g(\eta(t, \theta, \psi), t, \theta, \psi) = f(a_2(\eta(t, \theta, \psi), \psi), t, \theta, \psi).$$

(One concern in the literature has been that the possibility of feedback generates multiple equilibria in the pricing stage, see, e.g., Bond, Goldstein and Prescott (2010). Here, I abstract from this complication.)

Differentiation with respect to θ and t yields

$$\begin{aligned}
& g_{a_1}(\eta(t, \theta, \psi))\, \eta_\theta(t, \theta, \psi) + g_\theta(\eta(t, \theta, \psi), t, \theta, \psi) \\
= \quad & f_{a_2}(a_2(\eta(t, \theta, \psi), \psi), t, \theta, \psi)\, a_{2,a_1}(\eta(t, \theta, \psi), \psi)\, \eta_\theta(t, \theta, \psi) \\
+ \quad & f_\theta(a_2(\eta(t, \theta, \psi), \psi), t, \theta, \psi)
\end{aligned}$$

[9]The advantage of writing the pricing equation in the form of (19) as opposed to the equivalent but seemingly simpler form $a_1 = f(a_2(a_1, \psi), t, \theta, \psi)$ is to facilitate the application of earlier results.

and

$$
\begin{aligned}
& g_{a_1}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)\eta_t\left(t,\theta,\psi\right)+g_t\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right) \\
= \quad & f_{a_2}\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)a_{2,a_1}\left(\eta\left(t,\theta,\psi\right),\psi\right)\eta_t\left(t,\theta,\psi\right) \\
+ \quad & f_t\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right),
\end{aligned}
$$

or equivalently,

$$
\begin{aligned}
\eta_\theta\left(t,\theta,\psi\right) &= \frac{f_\theta\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),\theta,t,\psi\right)-g_\theta\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{g_{a_1}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)-f_{a_2}\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)a_{2,a_1}\left(\eta\left(t,\theta,\psi\right),\psi\right)} \\
\eta_t\left(t,\theta,\psi\right) &= \frac{f_t\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)-g_t\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{g_{a_1}\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)-f_{a_2}\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)a_{2,a_1}\left(\eta\left(t,\theta,\psi\right),\psi\right)}.
\end{aligned}
$$

Consequently,

$$
\frac{\eta_\theta\left(t,\theta,\psi\right)}{\eta_t\left(t,\theta,\psi\right)}=\frac{f_\theta\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)-g_\theta\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}{f_t\left(a_2\left(\eta\left(t,\theta,\psi\right),\psi\right),t,\theta,\psi\right)-g_t\left(\eta\left(t,\theta,\psi\right),t,\theta,\psi\right)}.
$$

8.1 Application

A firm has a project succeeds with probability θ, producing output $v\left(a_2\right)$. If the project fails (an event with probability $1-\theta$), the government bails out the firm's shareholders with probability $1-F\left(t,\psi\right)$, where $F_\psi>0$, so that higher values of ψ correspond to less likely bailouts. The variable t represents investor perceptions about the probability of a bailout, where we normalize t so that $F_t>0$, i.e., higher values of t correspond to less likely bailouts.

As discussed above, investors trading the firm's shares observe t and θ, while the firm attempts to learn θ from the share price. The share price a_1 of this firm is

$$
a_1=\theta v\left(a_2\right)+\left(1-\theta\right)\left(1-F\left(t,\psi\right)\right)v\left(a_2\right).
$$

Note that t is not relevant for the manager's decision, since it is shareholders rather than

the firm itself that are bailed out. So from the manager's perspective, variation in investor perceptions about the bailout probability adds noise to the price's information about θ.

In this setting,

$$\begin{aligned} f\left(a_2, t, \theta, \psi\right) &= \theta v\left(a_2\right) + \left(1-\theta\right)\left(1 - F\left(t, \psi\right)\right) v\left(a_2\right) \\ g\left(a_1, t, \theta, \psi\right) &= a_1. \end{aligned}$$

Given uniqueness of the price function η,

$$g_{a_1}\left(\eta\left(t, \theta, \psi\right), t, \theta, \psi\right) - f_{a_2}\left(a_2\left(\eta\left(t, \theta, \psi\right), \psi\right), t, \theta, \psi\right) a_{2,a_1}\left(\eta\left(t, \theta, \psi\right), \psi\right) > 0.$$

Note that

$$\begin{aligned} sign\left(\eta_t\right) &= sign\left(f_t\right) < 0 \\ sign\left(\eta_\theta\right) &= sign\left(f_\theta\right) > 0, \end{aligned}$$

so that the required assumptions on η are satisfied.

Evaluating,

$$\frac{\eta_\theta\left(t, \theta, \psi\right)}{\eta_t\left(t, \theta, \psi\right)} = -\frac{v\left(a_2\right) F\left(t, \psi\right)}{\left(1-\theta\right) v\left(a_2\right) F_t\left(t, \psi\right)} = -\frac{1}{1-\theta} \frac{F\left(t, \psi\right)}{F_t\left(t, \psi\right)} = -\frac{1}{1-\theta}\left(\frac{\partial}{\partial t} \ln F\left(t, \psi\right)\right)^{-1}.$$

Hence

$$sign\left(\frac{\partial}{\partial \psi}\left|\frac{\eta_\theta\left(t, \theta, \psi\right)}{\eta_t\left(t, \theta, \psi\right)}\right|\right) = -sign\left(\frac{\partial^2}{\partial t \partial \psi} \ln F\left(t, \psi\right)\right).$$

Hence a reduction in bailouts (higher ψ) leads the price to be more (less) informative if F is log-submodular (log-supermodular). In particular, when F is log-submodular, more generous bailouts lead firms to make worse investment decisions.

Economically, log-submodularity corresponds to the reduction in bailout probability hav-

ing most effect on firms that investors think are most likely to be bailed out. This reduces variance in investor perceptions about bailout probability, making the price more informative about θ.

8.2 Irrelevance of bailout regime under some circumstances

As a separate example, suppose instead that the price is

$$p(\theta, t)\, v(a_2) + (1 - p(\theta, t))\,(1 - F(\psi))\, v(a_2)\,.$$

One interpretation of this case is that investors perceive θ only with noise, and asses the success probability as $p(\theta, t)$. Since the firm chooses a_2, the price has the same information as $p(\theta, t)$, and so bailout probability has no effect on price informativeness. This can also be seen from the fact that

$$\frac{f_\theta}{f_t} = \frac{p_\theta}{p_t},$$

which is independent of ψ

9 Applications

9.1 Information content of an investment

Both Examples 2 and 3 are special cases of agent 1 choosing a_1 to maximize a payoff of the form

$$U(a_1, t, \theta, \psi) = B(t, \theta, \psi)\, v(a_1) - \kappa(t, \theta, \psi) c(a_1)\,, \tag{20}$$

where v is an increasing and concave function, c is an increasing and convex function, and the cost-benefit ratio $\frac{B}{\kappa}$ is increasing in θ and decreasing in t.

The first-order condition can be written as

$$\frac{c'(a_1)}{v'(a_1)} = \frac{B(t,\theta,\psi)}{\kappa(t,\theta,\psi)}.$$

Hence, by Proposition 1, a change in regime ψ—which in these two examples is a change in bail out policy—increases the accuracy of a_1 as a signal of θ if $\frac{B}{\kappa}$ is log-supermodular, and decreases accuracy of $\frac{B}{\kappa}$ is log-submodular.

For Example 2,

$$\frac{B}{\kappa} = (\theta + (1-\psi)(1-\theta))(r(t))^{-1},$$

which is log-supermodular since

$$\frac{\partial^2}{\partial\psi\partial\theta}\ln(\theta + (1-\psi)(1-\theta)) = \frac{\partial}{\partial\psi}\frac{\psi}{\theta + (1-\psi)(1-\theta)} > 0.$$

So in Example 2, less generous government bail outs (higher ψ) increase the accuracy of a_1.

For Example 3,

$$\frac{B}{\kappa} = \frac{\theta}{r(t,\psi)},$$

and so whether this ratio is log-supermodular or sub-modular is determined by the sign of

$$-\frac{\partial^2}{\partial t\partial\psi}\ln r(t,\psi) = -\frac{\partial}{\partial t}\frac{r_\psi}{r}.$$

Using the same normalization as in Example 2 that higher ψ corresponds to less generous bailouts, $r_\psi > 0$. Hence if the change in government policy has the greatest effect for high-t agents (i.e., low r), the ratio $\frac{B}{\kappa}$ is log-submodular, and less generous government bail outs (higher ψ) reduce the accuracy of a_1. This case is likely to arise when bail outs are driven by "too big to fail" concerns. Conversely, if the tightening of government policy affects mostly low t agents (i.e., high r), then this increases the accuracy of a_1.

9.2 The effect of risk-aversion on the information content of financial securities

Consider now the framework of Section 7. As a benchmark, consider first the standard CARA-normal framework: specifically, suppose that traders have an absolute risk aversion γ, and the asset being traded pays $\theta + \varepsilon$, where ε is normally distributed. Let $\psi = (\gamma var(\varepsilon))^{-1}$. In this case, demand is $x(a_1, \theta, \psi) = \psi(\theta - a_1)$, and from the market-clearing condition (16), the equilibrium price function is

$$\eta(t, \theta, \psi) = \theta - \frac{1}{\alpha\psi} s(t).$$

From this expression, one can see that informativeness is increasing in ψ, i.e., decreasing in both risk aversion γ and residual variance $var(\varepsilon)$.

As an application of Corollary 3, I show that these comparative statics hold in a considerably wider class of models, i.e., ones that relax CARA preferences and/or the assumption that cash flows are normally distributed conditional on the signal θ. Specifically, the accuracy improvement condition (17) holds under the following three properties:

Property 1: The payoff of the asset being traded is $\theta + \varepsilon$, where ε is independent of θ.

Property 2: $x_{\theta\psi} \geq 0$.

Property 3: $x \geq 0 \Longrightarrow x_{\theta\theta} \leq 0$ and $x_\psi \geq 0$, while $x \leq 0 \Longrightarrow x_{\theta\theta} \geq 0$ and $x_\psi \leq 0$.

Property 1 implies that demand $x(a_1, \theta, \psi)$ is a function of $\theta - a_1$ only, since demand is derived as the solution to a utility maximization problem of the type $\max_x E[U(W - a_1 x + x(\theta + \varepsilon))]$.

Property 2 generalizes the property of the CARA-normal model, that the sensitivity of demand to the signal θ is increasing in ψ.

Property 3 simply says that when the trader takes a long (respectively, short) position, demand is concave (convex) in his signal, and the size of the position increases in ψ.

The combination of these three properties is likely satisfied in many cases. Together, they imply the accuracy improvement condition (17), as follows. Expanding, and making

use of Property 1, the accuracy improvement condition (17) is

$$x_{a\theta}x_{\psi} - x_a x_{\theta\psi} = -x_{\theta\theta}x_{\psi} + x_{\theta}x_{\theta\psi} \geq 0.$$

Since certainly $x_\theta > 0$, Properties 2 and 3 ensure this is satisfied.

10 Conclusion

In many circumstances economic agents are partially ignorant about payoff-relevant state variables, but must nonetheless decide what action to take. To give just one example, investment decisions must often be made without full knowledge of the rate of return. In many such cases, agents can potentially obtain useful information by observing either the actions of other agents, or market prices. However, the information content of such actions or prices is in general hard to order, outside of special cases in which linearity arises. This paper deploys Lehmann's (1988) information ordering to this class of problems. First, I relate Lehmann's ordering to supermodularity properties. Second, I obtain a tractable first-order version of Lehmann's ordering. Third, I relate this first-order condition to both individual optimization problems and to equilibrium pricing conditions.

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A Appendix

Proof of Proposition 3: Condition (10) expands to

$$\eta_{\psi\theta}\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)+\bar{t}_\theta\left(a_1,\theta,\psi\right)\eta_{\psi t}\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)\geq 0,$$

i.e.,

$$\eta_{\psi\theta}\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)-\frac{\eta_\theta\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)}{\eta_t\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)}\eta_{\psi t}\left(\bar{t}\left(a_1,\theta,\psi\right),\theta,\psi\right)\geq 0,$$

which is equivalent to condition (11). **QED**

Proof of Lemma 1: Assuming sufficient differentiability, the MPR property is equivalent to

$$P_{a_1}\left(a_1,\tilde{\theta},\psi\right)P\left(a_1,\theta,\psi\right)-P\left(a_1,\tilde{\theta},\psi\right)P_{a_1}\left(a_1,\theta,\psi\right)\geq 0 \text{ if } \tilde{\theta}>\theta.$$

By a further round of differentiation, the MPR property is equivalent to

$$P_{a_1\theta}(a_1,\theta,\psi)\,P(a_1,\theta,\psi)-P_\theta(a_1,\theta,\psi)\,P_{a_1}(a_1,\theta,\psi)\geq 0.$$

(Note that this inequality is equivalent to $\frac{\partial^2 \ln P}{\partial\theta\partial a_1}\geq 0$, which is the differential formulation of P being log-supermodular, which is in turn is how the condition is stated in Athey (2002).)

In this paper's setting, $P(a_1,\theta,\psi)=1-\bar{t}(a_1,\theta,\psi)$. From (7),

$$\begin{aligned}(\eta_{tt}(\bar{t}(a_1,\theta,\psi),\theta,\psi)\,\bar{t}_\theta(a_1,\theta,\psi)+\eta_{t\theta}(\bar{t}(a_1,\theta,\psi),\theta,\psi))\,\bar{t}_a(a_1,\theta,\psi)&\\ +\eta_t(\bar{t}(a_1,\theta,\psi),\theta,\psi)\,\bar{t}_{a\theta}(a_1,\theta,\psi)&=0\end{aligned}$$

Hence the MPR property is equivalent to

$$(1-\bar{t})\,\frac{1}{\eta_t}\,(\eta_{tt}\bar{t}_\theta+\eta_{t\theta})\,\bar{t}_a\geq\bar{t}_a\bar{t}_\theta,$$

and hence (using $\bar{t}_a<0$, $\eta_t<0$ and (8)) to

$$(1-\bar{t})\left(-\eta_{tt}\frac{\eta_\theta}{\eta_t}+\eta_{t\theta}\right)\geq-\eta_\theta,$$

and hence to

$$(1-\bar{t})\left(\frac{\eta_{tt}}{\eta_t}-\frac{\eta_{t\theta}}{\eta_\theta}\right)\leq 1,$$

and hence finally to (13). **QED**