Target's Learning in M&A Negotiations*

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Abstract

Do targets' stock markets affect mergers and acquisitions (M&A) negotiations? In this paper, we study targets' learning from runups to develop a theory of stock markets' effects on M&A negotiations. Existing estimations of the premium-runup relation are consistent with our model's predictions under different market noise levels, so they cannot reject the stock market's causal effects hypothesis. We further derive two testable predictions: market noises reduce premiums for high value deals, and the ratio of the target's gain to the perceived M&A value strictly increases in the runup. Finally, when the market is sufficiently noisy, publicizing the ongoing M&A negotiation may jeopardize a value-creating M&A deal.

Keywords: M&A negotiation, learning, premium-runup relation, feedback effect, real efficiency

JEL Classification: G14, G34, G38

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1 Introduction

There are two salient phenomena in almost all mergers and acquisitions (M&A) negotiations. First, bidders on average end up paying very high acquisition premiums (the difference between final deal values and targets' pre-negotiation stock prices) upon announcement. For example, according to Betton, Eckbo, and Thorburn (2008), bidders acquisition premiums are nearly 50% of targets' pre-negotiation stock prices. Second, announcements of M&A negotiations' outcomes are usually preceded by substantial (around 15%) increases in targets' stock prices (runups). As a result, a natural question is: do higher runups raise acquisition premiums? Or, more generally, do targets' stock markets impact M&A negotiations' outcomes?

Since the seminal paper by Schwert (1996), the question whether runups raise acquisition premiums has attracted great attention in empirical studies (see, for example, the recent important papers by Betton, Eckbo, and Thorburn (2009) and Betton et al. (2014)). Yet, the above questions also have great theoretical interests and corporate governance implications. For example, how does the target's stock market potentially affect the M&A negotiation? After all, there are no cash flows between the target and its secondary stock market. In addition, the stock market starts to react only after the information about the ongoing M&A negotiation is leaked. Releasing more information to the market about firms' material activities, such as M&A negotiations, can usually increase market efficiencies, but does leaking the information about the ongoing M&A negotiations always benefit value-creating deals?

Despite the importance of the above questions, current empirical conclusions about the premium-runup relation seem mixed.[1] Schwert (1996) develops the "markup pricing" hypothesis, under which the target receives a fixed markup, and thereby the acquisition premium increases with the runup by a one-to-one ratio. The estimated premium-runup relation in Schwert (1996) is around one, supporting the markup pricing hypothesis, which favors the conclusion that higher runups raise acquisition premiums.

[1]We will review hypotheses and empirical results in this literature in more details in Section 6.

Betton et al. (2014) reconsider the markup pricing within a rational deal anticipation framework, which is named "costly feedback loop" hypothesis. Under such a hypothesis, a one-dollar increase in the runup leads to a one-dollar increase in the premium, which in turn causes higher runups due to deal anticipation. This feedback is discounted by the decrease in the deal probability, so the loop will stop eventually, and the premium-runup relation must be strictly greater than one. Conversely, if the runup does not affect the premium, the deal anticipation also leads to a nonlinear premium-runup relation, which could be any positive number, depending on the runup. The estimated premium-runup relation is 0.76 in Betton et al. (2014), which strongly rejects the costly feedback loop hypothesis. This empirical evidence implies that higher runups may not cause higher acquisition premiums.

However, existing studies mainly focus on one mechanism through which the runup affects the M&A negotiation: the entire runup will be added to the amount the bidder will pay. But there may be other mechanisms at work, which could provide alternative interpretations of the existing empirical evidence. In addition, while the current mechanism may suffice for analyzing the runup's effect on the acquisition premium, it overlooks other stock market characteristics that may affect the M&A negotiation outcomes, and thus constrain future studies. For example, in both the "markup pricing" hypothesis and the "costly feedback loop" hypothesis, conditional on the runup, the acquisition premium is independent of the target's stock market noise (or the "ex-ante" noise trading volume in the target's stock market).

In this paper, we develop a theory that endogenizes the formations of the acquisition premium and the runup. In our theory, the target's stock market affects the M&A negotiation because the target learns from the runup. Central to our arguments is the assumption that the target does not have precise information about the bidder's private value from the acquisition, and speculators in the target's stock market possess noisy signals about it. The information asymmetry between the bidder and the target is realistic, and it allows value-creating deals to fail endogenously.[2] Because the target does not precisely know the bidder's

[2]Betton, Eckbo, and Thorburn (2009) document that 23% of merger negotiations fail. However, in any negotiation with symmetric information, two strategic parties will always reach an agreement, as long as

private value from the acquisition, even though the target may have much more precise information than speculators, it still has strong incentives to learn from the runup.[3] This is similar to the informativeness principle in Holmstrom (1979).

Explaining our theory in more detail, we model an M&A negotiation in which the bidder's private value from the acquisition is only known to the bidder. After the information about the ongoing M&A negotiation is leaked to the stock market, a continuum of investors, each receiving a private signal about the bidder's private value, submit price-contingent demands to a competitive market maker. The market maker, who receives demands from investors and observes an exogenously stochastic supply, sets a price to clear the stock market. Hence, the stock price partially aggregates investors' signals and provides the target with an informative signal about the bidder's private value, which the target will then use to bargain over the acquisition premium with the bidder.

In the equilibrium, the acquisition premium increases in the runup, but the value of the premium-runup relation is determined by the target's stock market noise. Specifically, for any value between zero and one, there is one market noise level at which the premium-runup relation takes that value. The intuition of this equilibrium property is as follows. For a fixed large runup, when the target's stock market is extremely noisy, the runup becomes uninformative, and therefore the target will mainly rely on the common prior belief, which incorporates all publicly available information, to bargain with the bidder. Consequently, the target's stock market will not affect the deal premium, and the premium-runup relation should be zero. At the other extreme, the exogenous shock to the target's stock market becomes deterministic, and so the runup approaches a perfectly informative signal to the target. As a result, the premium-runup relation becomes one. Then, by the continuity of the premium-runup relation on the market noise, the premium-runup relation can take any

they expect the M&A deal to be value-creating.

[3] As early as in Boot, and Thakor (1997) and Subrahmanyam and Titman (1999), public trading are shown to allow firms to infer information from their stock prices and use it to improve their corporate decisions. The theoretical literature is carefully surveyed by Bond, Edmans, and Goldstein (2012). Empirical studies, for example, Luo (2005), Edmans, Goldstein, and Jiang (2012), and Rajamani (2013) have shown that after the M&A deals have been announced, bidders and targets will use the information from stock prices when completing the deals.

value between zero and one, depending on the market noise. However, the premium-runup relation is not bounded between zero and one. If we fix a low runup, the premium-runup relation may vary with the market noise, from zero to infinity.

Existing estimations of the premium-runup relation are all consistent with the above equilibrium property, because the premium-runup relation can take any value in the equilibrium between zero and one. Since the existing empirical tests do not control for the market noise, the different estimations (around 1 in Schwert (1996) and 0.76 in Betton et al. (2014)) may be due to different market noise levels. Since the runup affects the acquisition premium through the target's learning, the current empirical evidence cannot reject the assertion that the target's stock market affects the M&A negotiation.

We then attempt to derive new empirical predictions about the target's learning in M&A negotiations. First, for any fixed large runup that indicates a high bidder's private value, a lower market noise level increases the acquisition premium. This is a direct effect of the market noise level on the acquisition premium, and it is not through the runup. A less noisy market will increase the target's estimation of the bidder's private value, because the target will put more weight on the information revealed by the runup when learning. A less noisy market will also make the target more confident in its estimation. Such a direct effect of the market noise level does not exist in a benchmark model wherein the target does not learn from the runup for exogenous reasons, and thus it provides us with a testable prediction about the target's learning.

Second, the informational role of the target's stock market is also embodied in the runup's effect on the strategic interactions between the target and the bidder in the M&A negotiation. In particular, we show that because the target needs to balance the acquisition premium and the probability of the success of the M&A negotiation, the ratio of the target's gain to the perceived total surplus generated by the M&A deal strictly increases in the runup.[4] Such an equilibrium property provides us with a new test of whether the target's stock market affects

[4]This property does not imply that the bidder's gain is negatively correlated to the runup in a cross-sectional analysis. On the contrary, when we don't control for the surplus generated by the M&A deal in the learning model, the bidder's gain increases in the runup, which is consistent with the empirical evidence in Betton et al. (2014).

the M&A negotiation. In the benchmark model without target's learning, the acquisition premium is independent of the runup because it is solely determined by target's prior belief. Therefore, as the runup increases, the target's gain does not change while the perceived total surplus generated by the M&A deal increases since it is positively related to the runup. The ratio of the target's gain to the perceived total surplus generated by the M&A deal can be considered as a normalized target's gain, which can largely mitigate the missing variable problem (the surplus generated by the M&A deal) in an estimation of the premium-runup relation.

The runup in our model plays the informational role through learning as well as the allocation role through informed trading. In an otherwise identical model that abstracts away the informed trading and simply substitutes the price with an aggregated public signal, the premium-runup relation is increasing and convex. However, due to the allocation role, the premium-runup relation in our model is neither bounded nor globally convex.

The fact that the financial market may greatly impact M&A negotiation outcomes sheds light on corporate governance policies about disclosing these negotiations. The current law neither requires disclosure of ongoing M&A negotiation, nor bans it. Since the M&A negotiation is a material corporate activity, disclosing it can increase the amount of information incorporated in the stock price, promoting financial market efficiency. However, the impact of the information disclosure on social welfare is not always positive. For any value-creating deal, it is socially optimal if the M&A succeeds. We show that if the runup is an accurate signal about the bidder's private value, disclosing information about an ongoing M&A negotiation to the market can increase the deal completion probability. In such a case, the parties in the M&A negotiation should be required to disclose such a material corporate activity. However, if the target's stock market is sufficiently noisy, and the prior of bidder's private value is low, disclosure (or leakage) of an ongoing M&A negotiation will imperil a value-creating M&A deal. Consequently, in this case, the government should restrict any disclosure of the ongoing M&A negotiation and intensively monitor to prevent information leakage.

Our paper relates to the literature of how the secondary market and its participants

influence M&A deal completion. Edmans, Goldstein, and Jiang (2012) document that firms whose stocks suffer from exogenous mutual fund redemption become more likely to be acquired in subsequent periods. Liu (2012) argues that bidder firms strategically overbid in order to signal high deal quality to the market and thus lower deal financing costs. Moeller, Schlingemann, and Stulz (2007) and Chatterjee, John, and Yan (2012) empirically test the impact of opinion divergence on acquirers' abnormal returns and acquisition premium, respectively. Our paper contributes to this strand of literature by showing how the stock price is formed by informed trading and how information impounded into the stock price affects M&A negotiation outcomes through managerial learning.

Our paper also relates to the broad literature of managerial learning and feedback from financial markets (Subrahmanyam and Titman (1999, 2001), Chen, Goldstein, and Jiang (2007), Bond, Edmans, and Goldstein (2012)). Several empirical papers have tested managerial learning during M&As. Luo (2005) shows that the market reaction upon the takeover announcement predicts the final deal consummation probability, thereby supporting the hypothesis of managerial learning. Kau, Linck, and Rubin (2008) find that agency problems affect such managerial learning behaviors. Ouyang and Szewczyk (2012) report that acquirers learn from their own market prices when determining the size of the merger investment. The contribution of our paper to this literature is two-fold. First, we look at a time window before the deal announcement and one specific informational source from which managers can learn, that is, the target's stock runups. Second, we derive empirical implications that are consistent with a managerial learning hypothesis as well as a new empirical strategy to test this hypothesis.

The conclusion that ongoing M&A negotiation leaks may jeopardize a value-creating M&A deal also contributes to the recent discussions of the possible tension between the financial market efficiency and real efficiency (see, for example, Bond, Edmans, and Goldstein (2012), Edmans, Heinle, and Huang (2015), and Goldstein and Yang (2014)).

The rest of the paper is organized as follows. Section 2 lays out the model. Before solving the model, in Section 3, we first analyze a benchmark no-learning model where for exogenous reasons the target does not learn from the runup. We then characterize a

monotone equilibrium of the model in Section 4. We show the target's stock market plays an important information role in the M&A negotiation in Section 5. Section 6 intensively discuss existing empirical evidence, and how it relates to our theory, and we also delineate some new empirically-testable predictions. In Section 7, we discuss the social welfare of the target's learning and discuss policy implications. Section 8 concludes. The appendix contains all proofs not in the main text.

2 Model

One target and one bidder are negotiating an M&A deal.[5] There are three periods, $t = 0, 1, 2$. At the beginning of period $t = 0$, the negotiation is initiated, and the information about the M&A negotiation is leaked to the target's stock market. In period $t = 1$, speculators in the target's stock market trade on their private information about the bidder's private value from the acquisition. In period $t = 2$, after observing the stock price, the target and the bidder formally bargain over the acquisition price. At the end of period $t = 2$, the outcome of the M&A negotiation is publicly announced.

2.1 The M&A Negotiation

Right after the beginning of the M&A negotiation, the bidder gets to know her acquisition value, denoted by v.[6] The bidder's private value v consists of synergies, improved efficiencies from replacing the management team, future investments upon the acquisition, and so on. v is also net of the acquisition costs and negotiation costs. Hence, the private value v is tied to the particular bidder. As a result, we assume that the value v is the bidder's private knowledge.

[5] We assume that there is no agency problem, such that managers in the bidder firm and the target firm will maximize their own shareholders' benefits. Therefore, we refer to the bidder (target) firm and the manager of the bidder (target) firm as the bidder (target) for brevity.

[6] We assume that the bidder's private value v is drawn by nature after the M&A negotiation is initiated. Such an assumption abstracts away the signaling effect of the initiation of the negotiation, which makes the model very tractable. It is also plausible because the bidder often gets to know his value only after interacting with the target.

While it is commonly known that v is drawn from the prior distribution $\mathcal{N}(v_0, \eta^{-1})$, the target does not know the realization of the bidder's private value. The common prior distribution summarizes all public information about the bidder's private value but cannot incorporate speculators' private signals, which are aggregated by the target's stock price runup P. Therefore, even though the target may a signal that is more precise than other signals in the economy, she still attempts to learn from the runup. This is emphasized in the literature on feedback effects between the financial market and corporate decisions (e.g., Subrahmanyam and Titman (1999), Bond, Edmans, and Goldstein (2012), and Goldstein, Ozdenoren, and Yuan (2013)).

After observing the stock price runup, the target proposes an acquisition premium b. The bidder then decides whether to accept such an offer or not. In the former case, the M&A deal goes through; but the M&A negotiation fails in the later case. We use such a take-it-or-leave-it bargaining protocol mainly for simplicity; we actually only need the target to be able to affect the negotiation outcome, especially the acquisition premium, so that she has incentives to learn from the runup. In particular, a take-it-or-leave-it offer is strategically similar to but much simpler than many other mechanisms of selling a company, such as sealed-bid auctions with a reserved price. In fact, the optimal take-it-or-leave-it offer is the same as the reserve price in a first- or second-price sealed-bid auction, regardless of the number of bidders. The optimal take-it-or-leave-it offer is also one of equilibria in a double auction, and could also be viewed as the final round of bidding in a finite horizon bargaining game.[7]

We normalize the stand-alone values of both firms at zero. The take-it-or-leave-it negotiation implies that the M&A deal will go through if and only if $v \geq b$. Therefore, the payoffs to the bidder and the target are, respectively,

$$\pi_B = (v - b) \cdot \mathbb{1}_{(v \geq b)} \quad \text{and} \quad \pi_T = b \cdot \mathbb{1}_{(v \geq b)},$$

[7]The static take-it-or-leave-it offer rules out the possibility that the target learns in the negotiation in a dynamic bargaining process, so the target has no private information about v. However, in an online appendix, we relax this assumption by allowing the target to have imperfect private information about v. The results are essentially the same.

where $\mathbb{1}_{(v \geq b)}$ is the indicator function that takes the value of 1 if $v \geq b$ and zero otherwise.

2.2 Speculative Trading

There is a continuum of speculators with measure one, who are trading shares in the target's stock market. The speculators are uniformly distributed over $[0, 1]$ and are indexed by i. Denote speculator i's ex-post demand by d_i, then his payoff π_i is

$$\pi_i = d_i(\pi_T - P).$$

Besides the public information incorporated in the common prior belief about the bidder's private value, each speculator i receives a private signal $s_i = v + \epsilon_i$ about v, where $\epsilon_i \sim \mathcal{N}(0, \gamma^{-1})$ is independent across speculators.

The stock price P is set by the competitive market maker to clear the target's stock market. Each speculator i, based on his private signal s_i and the publicly available price P,[8] submits a price-contingent demand schedule $d_i(\cdot) \in [-1, 1]$. The trading restriction is imposed because of the risk neutrality of speculators in our model and, in practice, can be justified by margin, capital, and liquidity constraints. Individual speculator i's trading strategy, as a result, is a mapping $d : \mathbb{R}^2 \to [-1, 1]$ from signal-price pairs (s_i, P) into feasible trading positions. Aggregating speculators' individual orders leads to the total demand by speculators $D(v, P) = \int_0^1 d(s, P)\, \mathrm{d}\Phi(\sqrt{\gamma}(s - v))$, where $\Phi(\cdot)$ is the cumulative distribution function (cdf) of a standard normal distribution and $\Phi(\sqrt{\gamma}(s - v))$ represents the cross-sectional distribution of private signals s_i conditional on the realization of v.[9] There is an exogenous stochastic supply of the target's stock, denoted by $S(\xi)$. We assume that

$$S(\xi, P) = 1 - 2\Phi(\xi), \tag{1}$$

[8] We assume that speculators can condition on prices when making their trading decisions, as in Grossman and Stiglitz (1980).

[9] We assume that the Law of Large Numbers applies to the continuum of speculators so that conditional on v the cross-sectional distribution of signal realizations ex post is the same as the ex-ante distribution of speculators' signals.

where $\xi \sim \mathcal{N}(0, \beta^{-1})$ represents liquidity shocks. The variance of ξ, β^{-1}, measures the target's stock market noise. The specific structure of the exogenous random supply $S(\xi, P) = 1 - 2\Phi(\xi)$ is inelastic and contains a noise term to prevent the price from fully revealing the true fundamental. It follows Albagli, Hellwig, and Tsyvinski (2013) for tractability, and it is similar to the exogenous random supply function employed by Hellwig, Mukherji, and Tsyvinski (2006) and Goldstein, Ozdenoren, and Yuan (2013). Hence, P is set such that $D(v, P) = S(\xi, P)$.[10]

Such an explicit modeling of the runup formation follows our key assumption that there is information in the market that the target does not know. As emphasized in the feedback effects between corporate decisions and financial market (for example, see the survey by Bond, Edmans, and Goldstein (2012)), the runup aggregates speculators' trading behaviors and thus speculators' private information. Because the target has incentives to learn from the runup, speculators, when submitting orders to the market maker, will take into account their trading behaviors' effect on the acquisition premium the target will propose. Therefore, the runup is endogenously determined, and it cannot be replaced by an exogenous function of publicly available information. In particular, by explicitly modeling the runup formation, we can analyze how the characteristics of the financial market (for example, the exogenous supply shock) affect the M&A negotiation. Furthermore, this setting also helps us to analyze the effects of the target's learning. In particular, we analyze the financial efficiency and the real efficiency of the ongoing M&A negotiation disclosure in Section 7. This not only contributes to the growing literature about the tension between financial efficiency and real efficiency (Edmans, Heinle, and Huang (2015) and Goldstein and Yang (2014)), but it also sheds light on the policy debate about whether the parties in the M&A negotiation should be required to disclose the negotiation information.

2.3 Timing and Equilibrium

The timing is summarized in Figure 1 below.

[10] In an early version of the paper, we assumed an upward-sloping supply function as $S(\xi, P) = 1 - 2\Phi(\xi - P)$ and all of the results remained the same.

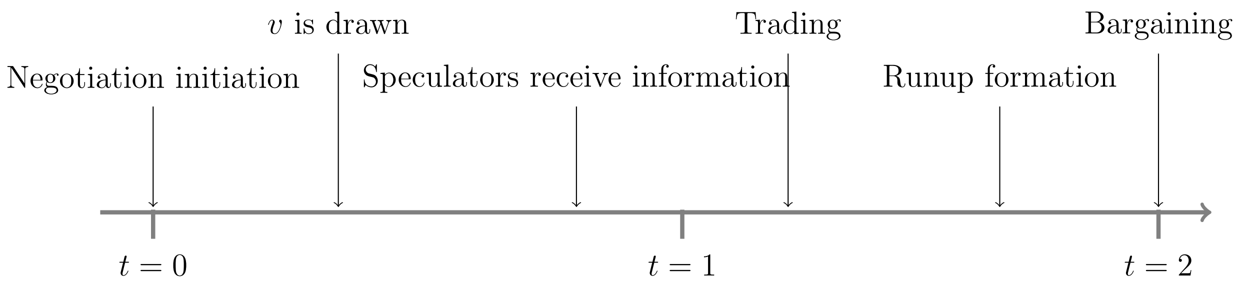

Figure 1: Timing

We now formally define a perfect Bayesian equilibrium as the solution concept of this model.

Definition 1 (Equilibrium) *An asking acquisition premium by the target* $b^*(P)$*:* $\mathbb{R} \to \mathbb{R}$*, a symmetric trading strategy* $d(s, P)$*:* $\mathbb{R}^2 \to [-1, 1]$*, and a price function* $P(v, \xi)$ *constitute a perfect Bayesian equilibrium if*

1. *for the target,* $b^*(P) \in \text{argmax}_b \, \mathbb{E}\left[b \cdot \mathbb{1}_{(v>b)} | P\right]$*;*

2. *for any speculator* $i \in [0, 1]$*,* $d(s_i, P) \in \text{argmax}_{d\in[-1,1]} \, \mathbb{E}\left[\left(b^* \cdot \mathbb{1}_{(v>b^*)} - P\right) \cdot d | s_i, P\right]$*;*

3. *market clears:* $D(v, P) = S(\xi, P)$*; and*

4. $\mathbb{E}[\cdot|P]$ *and* $\mathbb{E}[\cdot|s_i, P]$ *are calculated w.r.t. posterior probability measures of* v *using Bayes' rule.*

3 Benchmark Model: No Learning

As a benchmark, we first assume that the target does not learn from the runup. Specifically, the target does not update her belief about the bidder's private value v based on the runup, and this is common knowledge in the game. This scenario may occur if the manager is commonly known to be extremely overconfident.

Intuitively, because the target does not learn from the runup, she will set the optimal asking premium b based only on the prior of v. Formally, the target's optimization problem

is

$$\max_b \mathbb{E}\left[b \cdot \mathbb{1}_{(v>b)}\right], \tag{2}$$

leading to the first order condition

$$b \cdot \frac{f_v(b)}{1 - F_v(b)} = 1, \tag{3}$$

where $f_v(\cdot)$ and $F_v(\cdot)$ are the prior pdf and the prior cdf of v. We denote the solution to (3) by b^N.

Given the target's strategy in the M&A negotiation, speculators will trade on their private signals. Because the takeover premium is given, speculators' private signals only affect their inferences of the probability that the M&A negotiation succeeds. As a result, the market equilibrium can be solved using the same method employed by Albagli, Hellwig, and Tsyvinski (2013). In particular, speculators employ a cutoff strategy with the threshold $g^N(P)$, such that, given the runup P, any speculator i will buy one share if his private signal lands above $g^N(P)$ and will sell one share if his private signal lands below $g^N(P)$. Thus, the indifference condition of the marginal speculator is

$$b^N \mathbb{E}\left[\mathbb{1}_{(v>b^N)} | s_i = g^N(P), P\right] - P = 0. \tag{4}$$

Then the aggregate demand for any given bidder's private value v is

$$D^N(P, v) = 1 - 2\Phi\left[\sqrt{\gamma}\left(g^N(P) - v\right)\right].$$

Set it equal to $S(P, \xi)$, we have

$$g^N(P) = v + \frac{\xi}{\gamma}. \tag{5}$$

While more details of the solution to the benchmark model will be presented in an online Appendix, Proposition 1 below characterizes a monotone equilibrium of the benchmark.

Proposition 1 (The Monotone Equilibrium) *Suppose the target does not learn from*

the runup, there is a monotone equilibrium in which

1. *the optimal acquisition premium* b^N *proposed by the target is the unique solution to the target's maximization problem and is determined by* (3)*;*
2. *the speculators employ a symmetric cutoff trading strategy with the threshold* $g^N(P)$ *for any given* P*, where* $g^N(P)$ *is uniquely determined by* (4)*; and*
3. *the market price* P^N *is uniquely determined by* (5)*.*

Proposition 1 shows that when the target does not learn from the runup, the relation between the acquisition premium and the runup coincides with the pure substitution hypothesis in Schwert (1996): if the M&A negotiation is successful, the runup will have no effect on the takeover premium. Denote the derivative of the premium with respect to the runup by $\Lambda(P)$, we have Corollary 1 below.

Corollary 1 (Markup-Runup Relation without Learning) *When the target does not learn from the runup, the runup does not affect the acquisition premium; that is,* $\Lambda(P) = 0$*. Furthermore, this premium-runup relation is independent of the target's stock market noise, so* $\Lambda(P)$ *is independent of* β*.*

When the target does not learn from the runup, because the acquisition premium does not depend on the runup, the target's payoff does not depend on the runup. On the other hand, the runup is positively correlated to the bidder's private value. As a result, as the runup increases, the fraction of the value generated by the M&A deal taken by the target decreases. Denote the ratio of the target's payoff to the expected total gain of the M&A deal by

$$\Gamma(P^N) = \frac{b^N}{\mathbb{E}(v|P^N, v > b^N)},$$

we have Corollary 2 below.

Corollary 2 (Target's Payoff Ratio without Learning) *When the target does not learn from the runup, the ratio of target's payoff to the expected total gain generated by the M&A deal globally decreases in the runup.*

4 Equilibrium Characterization with Learning

We now analyze a more realistic case where the target learns from the runup. Managerial learning has been emphasized by Bond, Edmans, and Goldstein (2012) and empirically documented by Luo (2005), Edmans, Goldstein, and Jiang (2012), and so on. In this section, we first characterize our core model's equilibrium.

The most important feature of the model is the informational feedback effects between the M&A negotiation and the stock market. On one hand, before proposing the acquisition premium, the target makes inferences from the runup, which imperfectly aggregates speculators' dispersed beliefs about the bidder's private value. Because of the target's learning, the runup greatly impacts the M&A negotiation outcomes. On the other hand, speculators take into account the target's equilibrium behavior in the M&A negotiation when submitting their orders to the market maker. Therefore, the negotiation outcomes and the market trading are interdependent and need to be solved simultaneously. To solve the model, we first propose an equilibrium pricing function, based on which the target makes inferences for any given realized price. Then, we solve the optimal acquisition premium proposed by the target as a function of the realized price. Finally, we go back to the stock market to solve speculators' equilibrium trading strategies and verify that the proposed pricing function is the equilibrium pricing function.

4.1 Negotiation Outcome

Assume that a public signal z can be inferred from the price runup P in the equilibrium and it takes the form as $z = g(P) = v + \xi/\sqrt{\gamma}$, which is derived from the speculators' cutoff trading strategy characterized by $g(P)$ that we will verify later. Recall that $\xi \sim \mathcal{N}(0, \beta^{-1})$

represents the exogenous supply shock. We show in the appendix that z strictly increases in P in the equilibrium, so z is a sufficient statistic of P. Using z to form posterior beliefs, the target's optimization problem is

$$\max_b \mathbb{E}\left[b \cdot \mathbb{1}_{(v>b)}|z\right]. \tag{6}$$

Note, the target's information set in Equation (6) is finer than that in Equation (2). When making an offer, the target faces a trade-off between the acquisition premium and the deal success probability: an increase in the proposed premium b will increase the payoff if the deal goes through but reduce the probability of the success.

The first-order condition of the target's optimization problem characterizes this trade-off and can be simplified as

$$b \cdot f_v(b|z) = 1 - F_v(b|z), \tag{7}$$

where $f_v(\cdot|z)$ and $F_v(\cdot|z)$ are the posterior probability distribution function (pdf) and the cdf of v conditional on the signal z, respectively. Equation (7) has a very intuitive interpretation: at the optimum, for a unit increase in b, the marginal increase in the expected premium due to an increase in the proposed premium, $1 - F_v(b|z)$, is equal to the marginal decrease in the expected premium due to the decrease of the probability of a successful M&A deal, $bf_v(b|z)$. Although the objective function in (6) is not globally concave, we show in the appendix that the objective function has a single peak. Therefore, the first-order condition is both sufficient and necessary to characterize the target's proposed optimal premium.

We conclude this subsection with a lemma characterizing the properties of the optimal b^* that solves (7) under normal distributions.

Lemma 1 (Optimal Acquisition Premium) *Suppose the equilibrium price of the stock market is in the form $\sqrt{\gamma}\,(g(P) - v) = \xi$. For each realized z, the acquisition premium proposed by the target, b^*, is characterized by equation (7). In particular,*

1. *for any z, b^* is unique, and it is global maximum to (6);*

2. *b^* is increasing and convex in z with the slope $\frac{\partial b^*}{\partial z} \in (0, \frac{\gamma\beta}{\eta+\gamma\beta})$, and $\lim_{z\to\infty} \frac{\partial b^*}{\partial z} = \frac{\gamma\beta}{\eta+\gamma\beta}$;*

and

3. $b^* > 0$*, and there exists* $\tilde{z}$ *such that* $b^* < \frac{v_0\eta + z\gamma\beta}{\eta + \gamma\beta}$ *for all* $z > \tilde{z}$*.*

A large z is indicative of a high value of v and motivates the target to propose a high acquisition premium. Moreover, the speed at which the proposed acquisition premium increases, $\partial b^*/\partial z$, is always smaller than the speed at which the target updates the value of v, $\gamma\beta/(\eta + \gamma\beta)$. As z becomes large enough, the adverse impact on the deal success probability of increasing the asking premium diminishes, so the target can reap almost the entire incremental surplus implied by one unit increase in z, and $\partial b^*/\partial z$ approaches $\gamma\beta/(\eta + \gamma\beta)$ in the limit.

$b^* > 0$ is intuitive, since the target firm's reservation value is zero. When expected synergy is high (z exceeds a threshold $\tilde{z}$), the target firm will propose the acquisition premium less than its posterior estimate of v, just to insure that the deal is more likely to go through.

4.2 Trading in the Stock Market

We restrict our attention to a monotone equilibrium in the trading market, where each speculator i follows a cutoff trading strategy characterized by $g(P)$ as below:

$$d(s_i, P) = \begin{cases} 1, & \text{if } s_i > g(P) \\ \in [-1, 1] & \text{if } s_i = g(P) \\ -1, & \text{if } s_i < g(P) \end{cases} \tag{8}$$

The cutoff trading strategy follows from the market microstructure and the target's equilibrium strategy. Given a price P, when s_i is high, speculator i believes that the private value to the bidder is high; thus both the proposed premium and the deal completion probability are high. As a result, speculator i would like to buy the target share. Similarly, when s_i is low, speculator i would like to hold a short position of the target share. Therefore, given a price, speculators will use a cutoff trading strategy, and the speculator with a private signal equal to the cutoff point will be indifferent between buying a share and selling a share. The

threshold point $g(P)$ is price-contingent, because when the price changes, the private signal, with which the speculator is indifferent between buying and selling, also changes.

Given the demand function in (8), we can calculate the aggregate demand from informed speculators:

$$D(v,P) = \int_{g(P)-v}^{\infty} \phi\left(\sqrt{\gamma}\epsilon\right) \, d\epsilon - \int_{-\infty}^{g(P)-v} \phi\left(\sqrt{\gamma}\epsilon\right) \, d\epsilon = 1 - 2\Phi\left(\sqrt{\gamma}(g(P)-v)\right), \tag{9}$$

where $\phi(\cdot)$ is the standard normal pdf. From the market clearing condition that $D(v,P) = S(\xi,P)$, it follows that, as we proposed,

$$g(P) = v + 1/\sqrt{\gamma}\xi. \tag{10}$$

However, this is still an implicit expression of P, since we have not yet solved $g(P)$.

With the bid premium function b^* given by (7), we can now write speculator i's expected payoff from buying one unit of the target's shares given its information set:

$$\mathbb{E}\left[b^* \cdot \mathbb{1}_{(v>b^*)} - P|s_i, P\right]. \tag{11}$$

A speculator will choose to buy the asset if and only if (11) is positive. From the proposed cutoff trading strategy in (8), a speculator with a private signal equal to $g(P)$ is indifferent to buying or selling the stock, which implies that (11) is equal to 0 if we plug in $s_i = g(P)$. Formally,

$$\mathbb{E}\left[b^* \cdot \mathbb{1}_{(v>b^*)} - P|s_i = g(P), P\right] = 0. \tag{12}$$

Or, using sufficient statistic z, we have

$$P = \mathbb{E}\left[b^* \cdot \mathbb{1}_{(v>b^*)}|s_i = z, z\right] = b^*\mathbb{P}\left[v > b^*|s_i = z, z\right]. \tag{13}$$

The price runup P in our model is determined by the marginal trader's expectation of the target payoff, conditional on his own private signal $s_i = z$ and public signal z.

The proposition below shows that there exists a monotone equilibrium as we have described and characterized it.

Proposition 2 (The Monotone Equilibrium) *The model has a monotone equilibrium, in which*

1. *the optimal premium b^* proposed by the target is the unique solution to the equation* (7)*;*

2. *speculators employ a symmetric cutoff trading strategy, described in Equation* (8)*, with the threshold $g(P)$ uniquely determined by Equation* (12)*; and*

3. *the market price P is uniquely determined by* (13)*.*

5 Runups' Informational Effects

In this section, we analyze how the target's stock market noise affects M&A negotiation outcomes. In particular, we study the effects of the market noise on both the premium-runup relation (or the markup-runup relation) in the equilibrium and how the runup affects the target's gain as a fraction of the perceived value created by the M&A deal. The empirical implications of these equilibrium properties will be discussed in Section 6.

5.1 Impacts of Market Noise

Because there is information in the market that is unknown to the target, the target is incentivized to learn from the runup, which partially aggregates such information. The target's learning then links the target's stock market to the M&A negotiation. Hence, the runup plays an important informational role in the M&A negotiation; as a result, the market noise that determines the runup's informativeness will significantly affect the M&A negotiation outcomes. In this section, we explore how the target stock market noise affects the premium-runup relation and the acquisition premium.

Let's first analyze how the runup plays an informational role in the M&A negotiation. In the equilibrium, the target's optimal asking premium b^* is characterized by Equation (7), where the signal z is the key variable determining b^*. Recall that

$$z \equiv g(P) = v + \frac{1}{\sqrt{\gamma}}\xi.$$

Hence, the runup will affect the optimal asking premium through the signal z; that is, the runup will have causal effects on the takeover premium because the target is learning about the bidder's private value. For a given function of $g(\cdot)$, an exogenous shock Δ to the runup P will lead the target to infer the signal z is increased by $g'(P) \cdot \Delta$ and raise the optimal asking price b^* by $\frac{\partial b^*}{\partial z} \cdot g'(P) \cdot \Delta$. Proposition 3 below summarizes this causal effect of the runup on the takeover premium in the equilibrium.

Proposition 3 (Premium-runup relation) *When the target learns from the runup, the equilibrium premium-runup relation has following properties:*

1. b^* *strictly increases in* P*;*

2. b^* *is a nonlinear function of* P*; and*

3. b^* *is not globally convex in* P*, and* $\partial b^*/\partial P$ *can have a value in* $(a, +\infty)$*, where* $a \in (0, 1)$ *and is determined by other parameters in the model.*

The first part of Proposition 3 directly follows from Lemma 1 and the fact that z strictly increases in P. That is, the target perceives a larger bidder's private value from a higher runup, and thus proposes a higher acquisition premium. The second part of Proposition 3 is due to the effects of an increase in the runup on speculators' trading decisions. When P increases, the cost of buying the target's shares increases, discouraging speculators from buying. On the other hand, given other speculators' trading strategies described by Equation (8), an increase in P suggests a higher payoff from holding the target's share. Then the indifference condition (12) can only be retained if $g(P)$ is increasing. However, the interaction between these two effects gives rise to the nonlinearity of $g(P)$, which, together with the convexity of b^* on $g(P)$, implies that b^* is also nonlinear in P.

Part three of Proposition 3 is due to both the runup's information role and allocation role. When the runup P is close to zero, z is extremely low, implying that the target's and speculators' beliefs about the private value v and the expected deal probability are both low. In order for the market to clear and therefore for the indifference condition (13) to hold when the expected deal probability is close to 0, a marginal increase in P must be coupled by a non-trivial increase in b^*. That is, when $P \to 0$, $\partial b^*/\partial P \to \infty$ and thus $\partial b^*/\partial P$ is decreasing in P when P is very small. On the contrary, when $P \to \infty$, the deal success probability approaches 1, and Equation (13) implies that b^* increases with P in a one-for-one ratio. Combined with the fact that $b^*(P)$ approaches the 45-degree line from above (because b^* is always greater than P), we show that $\partial b^*/\partial P$ must be increasing in P when P is large. Therefore, as P increases from zero to infinity, $\partial b^*/\partial P$ first decreases and ultimately increases in P, implying that b^* is neither globally concave nor globally convex in P. Furthermore, because $\partial b^*/\partial P$ decreases from infinity when P is small and increases to 1 when P is large, the value of $\partial b^*/\partial P$ ranges from $[a, +\infty)$, where a is a number smaller than 1.

Part three of Proposition 3 differentiates our model from a model in which the runup is treated as an exogenous signal. The latter can be developed by replacing the target's stock market by the sufficient statistic z. The difference can be seen by comparing the effects of z on the premium in Lemma 1 with those of the runup P in Proposition 3, and is summarized in Corollary 3 below.

Corollary 3 (Difference between a Runup and a Public Signal) *The effect of the runup (P) on the acquisition premium (b^*) differs from that of an exogenous public signal (z). In particular,*

1. *b^* is globally convex in z, but not in P; and*

2. *$\partial b^*/\partial z \in (0, 1)$ for any z, but $\partial b^*/\partial P$ is unbounded.*

These different predictions are due to the fact that z affects speculators' payoffs through target's learning only, while on the top of the informational effects, the runup P directly

affects speculators' net returns.[11]

We are now ready to study the impact of the target's stock market noise. Part three of Proposition 3 implies that the premium-runup relation, $\Lambda(P)$, may have a range of $(0, +\infty)$, depending on the noise of the exogenous supply shock, β.[12] Let's first consider the case that β is extremely small; that is, β goes to zero. In this extreme case, the runup becomes uninformative, because given any runup, the conditional distribution of the bidder's private value is uniform over the whole real line. This can be seen by rearranging terms in Equation (10),

$$v = g(P) - \frac{\xi}{\sqrt{\gamma}}.$$

When $\beta \to 0$, $\frac{\xi}{\sqrt{\gamma}}$ converges to an improper uniform distribution over $\mathbb{R}$. Therefore, neither the target nor the speculators learn from the runup; consequently, the target will make the offer based only on the prior distribution of the bidder's private value. As a result, the runup will have no impact on the M&A negotiation, so $\partial b^*/\partial P$ converges to zero for all P. Note that this limiting runup is different from that in the benchmark model, where the target does not learn from the runup for exogenous reasons. In the benchmark model, the runup P is correlated with the bidder's value and the probability that the deal will go through; but here, in the limiting model, the runup P is indeed independent of the bidder's value.

Now, let's assume β diverges to positive infinity. Because the information aggregation is complete, the target's learning becomes perfect. In this case, the relation between the acquisition premium and the runup depends on the signal z, which is revealed from the equilibrium runup P. Let's fix $z > 0$ first. Since the target's learning is perfect, the target knows that the bidder's private value is z. So the target will propose an acquisition premium

[11] The runup P in Equation (12) differs the speculators' expected payoffs from holding a long position using all available public information, which is $\mathbb{E}\left[b^* \cdot \mathbb{1}_{(v>b^*)}|z\right]$. Such a difference is termed as "informational aggregation wedge," and its implications are thoroughly studied in Albagli, Hellwig, and Tsyvinski (2013). Because of such a wedge, in order to analyze the effects of the runup on the M&A negotiation, we need to formally model the runup formation, or the information aggregation, in the stock market.

[12] The precision of speculators' private signals, γ, also affects the informativeness of the runup. Surprisingly, the limit of $\Lambda(P)$ as γ goes to zero is very different from that when β goes to zero. This is a very interesting theoretical point, in our opinion. However, since we are analyzing the impact of the stock market noise, the limiting result of β suffices for the purpose of this paper. Therefore, we present this interesting limiting result of γ in Appendix.

z, which will surely be accepted by the bidder. Then the equilibrium runup must also be z, otherwise the stock market cannot be cleared. As a result, a unit increase in the runup leads to a unit increase in the signal z, which in turn causes a unit increase in the acquisition premium.

Conversely, if the revealed signal z is negative, the derivative of b^* with respect to P goes to ∞. That is, a marginal increase in the runup will lead to a nontrivial increase in the optimal asking premium. This seemingly counter-intuitive asymptotic result is rooted in the target's optimal bargaining strategy and the market clearing condition. The perceived deal probability approaches 0 when the market become increasingly sure that the deal is value-destroying. By (13), the market can only be cleared if marginal increase in the price is met by a non-trivial increase in the bid premium. Hence, as β diverges to infinity, $\partial b^*/\partial P \to \infty$. Therefore, the relation between the takeover premium and the runup has a range between 1 and ∞ when the target stock market's noise level is extremely low.

The above analysis is summarized in Proposition 4 below and has great empirical implications.

Proposition 4 (Markup and Stock Market Characteristics) *The equilibrium premium-runup relation, $\Lambda(P)$, is determined by the target's stock market noise, β^{-1}. In particular,*

1. *when $\beta \to 0$, $\Lambda(P) \to 0$ for any given P; and*

2. *when $\beta \to \infty$,*

$$\Lambda(P) \to \begin{cases} 1, & \textit{for any} \quad z > 0; \\ \infty, & \textit{for any} \quad z < 0. \end{cases}$$

For any fixed runup P, $\Lambda(P)$ is continuous in β, so $\Lambda(P)$ may has a value between 0 and ∞, varying by β.

Furthermore, in our model, speculators take into account the target's learning during trading, so the target's strategy affects the runup's formation. On the other hand, when the target designs the strategy in the M&A negotiation, the target will also consider the effect of its strategy on the runup formation, which is affected by stock market's characteristics,

such as the market noise. Therefore, the market noise is an important determinant of the target's equilibrium strategy; that is, β directly determines b^*. Proposition 5 below shows how the market noise affects the equilibrium takeover premium.

Proposition 5 (Price Informativeness and Bid Premium) *When the target learns from the runup, the market noise measured by β^{-1}, directly affects the takeover premium for any fixed z. In particular, there exists $\bar{z}$ and $\underline{z}$ (with $\bar{z} > \underline{z}$), such that*

$$\frac{\partial b^*}{\partial \beta} \begin{cases} > 0, & \textit{when} \quad z > \bar{z}; \\ < 0, & \textit{when} \quad z < \underline{z}. \end{cases}$$

In particular,

$$\lim_{\beta \to +\infty} \frac{\partial b^*}{\partial \beta} = 0.$$

The intuition behind Proposition 5 is as follows. When the signal z derived from the runup is sufficiently large, the perceived private value of the bidder is also very large. On the one hand, as the precision of z increases, the mean of posterior v increases, as the target puts more weights on the signal z. On the other hand, the target optimally chooses an asking premium lower than the perceived value, because an increase in the asking premium may lead to a huge decrease in the deal completion probability. Then, as β increases, the precision of the sufficient statistic z increases, and the target is more confident in its perception and thus proposes a higher acquisition premium. Conversely, if the sufficient statistic z is small, the asking premium will be larger than the perceived bidder's value for a better conditional payoff. Then, an increase in β will cause the target to decrease the asking acquisition premium to increase the deal completion probability.

Proposition 5 distinguishes the target's learning hypothesis from the existing hypotheses, thereby providing a method to test the hypothesis that the target learns from the runup. Under the substitution hypothesis and the deal anticipation hypothesis, the runup does not affect the M&A negotiation, and the market characteristics can affect only the runup; consequently, the market noise does not affect the M&A negotiation outcomes. Under the

markup hypothesis and the costly feedback loop hypothesis, the whole runup will be added to the final premium, so the market noise will not affect the M&A negotiation once we control for the runup. Therefore, if the variation of the market noise directly contributes to the variation of the proposed premium (conditional on the runup), the learning hypothesis is supported.

5.2 Dividing the M&A's Value

An important feature of our model is that the negotiation outcomes are determined by the strategic interaction between the target and the bidder. While the acquisition premium may capture both the bargaining outcomes and the value of the M&A, the ratio of the target's gain to the total surplus generated by the M&A deal is more likely to mainly reflect the strategic interaction results. Such a ratio can be formally defined as

$$\Gamma(P) = \frac{b^*}{\mathbb{E}[v|P, v > b^*]}.$$ [13]

That is, suppose the M&A deal goes through, the ratio in our model is calculated by dividing the equilibrium acquisition premium by the perceived bidder's private value. In this section, we analyze how the runup affects such a ratio, when the target learns from the runup.

From Proposition 3, conditional on that the M&A negotiation goes through, the target's gain b^* strictly increases in P. Hence, the numerator of $\Gamma(P)$ strictly increases in the runup P. In fact, Proposition 6 below shows that because the probability of a successful M&A deal increases in the runup, the target's interim payoff (conditional on the runup, but not on a successful M&A deal) also increases in the runup.

Proposition 6 (Runup Effects on Target's Payoffs) *In the equilibrium, the deal success probability* $\mathbb{E}\left[\mathbb{1}_{(v>b^*)}|P\right]$ *increases in the runup* P*, and the target's interim payoff* $\mathrm{E}\left[b^* \cdot \mathbb{1}_{(v>b^*)}|P\right]$ *also increases in the runup* P*.*

The denominator of $\Gamma(P)$ is the perceived total surplus generated by the M&A deal,

[13] This ratio is calculated conditional on that M&A negotiation is successful. In the proof of Proposition 8, we show that the unconditional ratio $(\mathbb{E}[b^*\mathbb{1}_v|P]/\mathbb{E}[(v-b^*)\mathbb{1}_{(v>b^*)}|P)$ is the same as the conditional ratio.

which is the sum of the target's gain and the bidder's gain. As shown in Proposition 7 below, the perceived bidder's gain is strictly increasing in the runup, so the denominator of $\Gamma(P)$ also strictly increases in P.

Proposition 7 (Runup Effects on Bidder's Gain) *In the equilibrium, conditional on successful M&A deals, the perceived bidder's gain,* $\mathbb{E}\left[(v-b^*)|P, v > b^*\right]$*, strictly increases in* P*.*

The intuition of Proposition 7 is that a higher runup means a higher M&A surplus, which dominates the higher acquisition premium the bidder needs to pay to acquire the target. This happens because the target balances the trade-off between the higher conditional payoff and the deal probability when maximizing its expected payoff. The higher the perceived surplus (due to a higher run-up P), the higher the marginal adverse effect of an increase in the acquisition premium. Therefore, as the runup increases, the acquisition premium b^* increases but at a lower speed than that of the increase in the perceived surplus $\mathbb{E}[v|P]$.

Because both the numerator and the denominator of $\Gamma(P)$ strictly increase in P, it is not straightforward to tell whether $\Gamma(P)$ is increasing or decreasing in P. Fortunately, the target's equilibrium acquisition premium provides us with some clues. Recall that the first order condition of the target in the negotiation is

$$1 - b^*\sqrt{\eta+\gamma\beta}\lambda(G) = 0,$$

where λ is the inverse Mills Ratio. And $G = \sqrt{\eta+\gamma\beta}\left(b - \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)$. Then the expected bidder's gain $\mathbb{E}[v-b^*|P, v > b^*] = \frac{v_0\eta+g(P)\gamma\beta}{\eta+\gamma\beta} - b^* + (\eta+\gamma\beta)^{-1/2}\lambda\left(G\right)$, where $(\eta+\gamma\beta)^{-1/2}\lambda\left(G\right)$ measures the increase in the mean of the bidder's value due to the condition that the M&A

deal is successful. Then we have

$$
\begin{aligned}
\Gamma(P) &= \frac{b^*}{b^* + \mathbb{E}[v - b^*|P, v > b^*]} \\
&= \frac{b^*}{b^* + \left\{\frac{v_0\eta + g(P)\gamma\beta}{\eta+\gamma\beta} - b^* + (\eta+\gamma\beta)^{-1/2}\lambda\left(G\right)\right\}} \\
&= \frac{\sqrt{\eta+\gamma\beta}b^*}{\sqrt{\eta+\gamma\beta}b^* - G + \lambda\left(G\right)} \\
&= \frac{1}{1+\lambda(G)\left(\lambda\left(G\right) - G\right)} \qquad (14) \\
&= \frac{1}{1+\lambda'(G)}, \qquad (15)
\end{aligned}
$$

where Equation (14) is due to the target's first order condition, and Equation (15) is from the property of the inverse Mill's Ratio. Consequently, we have

$$\frac{\partial \Gamma(P)}{\partial P} = \frac{\partial \Gamma}{\partial G} \cdot \frac{\partial G}{\partial z} \cdot \frac{\partial z}{\partial P}.$$

The first term is negative, because the inverse Mill's Ratio is strictly convex; the second term is also negative, because as in Proposition 7, the higher value created by the M&A implied by a stronger signal overweighs the higher takeover premium due to the stronger signal; the third term is positive, since a higher runup indicates a higher bidder's private value. Therefore, $\Gamma(P)$ strictly increases in P; that is, when the target learns from the runup, the higher the runup, the larger fraction that the target will get of the value created by the M&A deal. Intuitively, the property of the monotonicity of $\Gamma(P)$ is driven by the speed at which its numerator increases, relative to the speed at which its denominator increases. In our model, this relative speed increases in P (or z) precisely because of the convexity of $b^*(z)$.

Proposition 8 (Target Gains versus Bidder Gains) *When the target learns from the runup, the ratio* $\Gamma(P)$ *strictly increases in* P *and is strictly greater than* $1/2$*.*

It may appear from Proposition 8 that, as the runup increases, the target obtains a larger share of the total surplus created by the M&A deal, and, consequently, the target's gain and

the bidder's gain are negatively correlated. However, this is not the case in our model. We have shown that both b^* and $\mathbb{E}[v - b^*|P, v > b^*]$ are increasing functions of P (or z), then the covariance $cov\,(b^*, \mathbb{E}[v - b^*|P, v > b^*]) \geq 0$,[14] which is consistent with the evidence in Betton et al. (2014). The intuition lies in the fact that the ratio $\Gamma(P)$ is like a normalized gain to the target, which to a large extent eliminates the effect of the positive correlation between the runup and the total surplus created by the M&A deal. Yet the covariance of the target's gain and the bidder's gain is cross-sectional: even though the runup helps the target to bargain an increasingly larger portion of the total gain, the size of total gain is positively correlated with runup in the cross sections.

5.3 Market Noise and Negotiation Outcomes

We have shown that market noise affects the target's learning from the runup, and therefore directly affects the final bid premium and premium-runup relation. We now study its direct effect on other deal outcomes, which is summarized in the proposition below.

Proposition 9 (Market Noise and Negotiation Outcomes) *When the target learns from the runup, the market perceived deal probability $\mathbb{E}[\mathbb{1}_{(v>b^*)}|P]$ and the ratio between the target gain and total perceived gain $\Gamma(P)$ all increase (decrease) in β when $z > v_0$ ($z < v_0/2$); expected bidder's gain $\mathbb{E}[(v - b^*)|P, v - b^* > 0]$ is decreasing in β when $z < v_0/2$.*

An increase in β allows the target to learn more precise information. When $z > v_0$, it also means higher perceived synergy, implying the cost of losing the deal is higher if the target bargains more aggressively. These two effects combined indicate the increase in b^* should be less than the increase in the posterior mean of synergy v if β increases. The distance between b^* and the posterior mean $\mu_{v|z}$ therefore increases with β when $z > v_0$, leading to a higher deal probability $\mathbb{E}[\mathbb{1}_{(v>b^*)}|P]$. However, as β becomes large, its impact on b^* and $\mu_{v|z}$ slows down and approaches zero (as one can tell from Proposition 5). Thus, the speed at which b^* increases ($\partial b^*/\partial\beta$) relative to the speed at which $\mu_{v|z}$ increases ($\partial\mu_{v|z}/\partial\beta$) increases

[14] See Behboodian (1994) for proof.

in β, resulting in an increasing ratio $\Gamma(P)$. The prediction on the perceived bidder's gain is unclear. On the one hand, increase in the distance between b^* and $\mu_{v|z}$ implies the mean of untruncated distribution of $v - b^*$ conditional on z shifts to the right, therefore indicating a higher mean when the distribution is truncated below at 0. On the other hand, an increase in β decreases the conditional variance of $v - b^*$, lowering the truncated mean.

However, when z is small, the perceived synergy is lower when β increases. When the target adjusts the optimal asking price based on its F.O.C. in (7) as β increases, the marginal benefit of increasing b becomes lower than its marginal cost in the trade-off, leading to not only a lower b^* but also a more aggressive bargaining behavior. As a result, predictions reverse signs. As for the perceived bidder's gain, the aforementioned two effects now work in the same direction. Therefore, the bidder's gain decreases with β.

6 Existing Evidence and New Empirical Predictions

Our model shows the important informational effects of the target's stock market on the M&A negotiation. In this section, we first discuss the existing empirical evidence related to the runup's effects on the M&A negotiation and, importantly, how it relates to the predictions of our theoretical analysis in Section 5. We then delineate several new empirical predictions that stem from our model.

6.1 Existing Evidence of the Runup's Effects

The central question about the runup's effect is whether the runup raises the acquisition premium. Schwert (1996) presents two competing hypotheses, the pure substitution hypothesis and the markup hypothesis, and implements the first set of empirical tests. Under the pure substitution hypothesis, the runup does not affect the takeover premium, so premium-runup relation should be 0. By contrast, if the markup hypothesis is true, the target will always maintain a fixed markup, so a one-dollar increase in the runup will increase acquisition premium by one dollar. In a linear regression of the premium on the runup, the coefficient of the

runup should be 0 if the pure substitution hypothesis is correct, whereas such a coefficient shouldn't be significantly different from 1 if the markup hypothesis is true. Schwert (1996) estimates premium-runup relation in a linear regression model, and the estimate is around 1. This empirical evidence supports the markup hypothesis and the conclusion that the runup raises the bidder's acquisition costs.

Betton et al. (2014) revisits this question by carefully analyzing a rational deal anticipation model, which provides us with new understandings of the two hypotheses proposed by Schwert (1996). Their key insight is that strong takeover signals not only imply greater value created by the M&A deal and a higher deal probability, but also leads to a higher runup. They assume that the target takes a predetermined fraction of the value created by the M&A deal, so the acquisition premium is positively correlated to the runup. Hence, under the deal anticipation hypothesis (analogous to the pure substitution hypothesis in Schwert (1996)), the premium-runup relation is nonlinear and may have a value in the range of $(0, +\infty)$. However, if the costly feedback loop hypothesis is true (analogous to the markup hypothesis in Schwert (1996)), the linear estimation of the premium-runup relation must be positive. Betton et al. (2014) implement the test employing a new data set. They find that the linear estimation of the premium-runup relation is about 0.76, which rejects the costly feedback loop hypothesis, implying that the runup may not increase the acquisition premium.

These estimations are consistent with the predictions of our model, especially those in Proposition 4. In particular, given the fact that runups are usually high (which implies positive signals), when the target's stock market's noise level is extremely low ($\beta \to +\infty$), the premium-runup relation converges to 1, while when the informativeness of the runup is moderate, the premium-runup relation can be less than 1. Hence, the estimate in Schwert (1996) could be the estimation from a sample with extremely low market noise, while the estimate in Betton et al. (2014) may be an estimation from a sample with moderate runup informativeness. However, in our model, for any $\beta > 0$, the runup raises the acquisition premium. Hence, the existing empirical evidence cannot reject the assertion that higher runups cause higher acquisition premiums.

Another way to test whether the runup raises the acquisition premium is to directly test

whether the bidder's payoff decreases in the runup. In our model, the acquisition premium the bidder pays increases in the runup, because the target learns from it. Therefore, with the bidder's private value from the acquisition fixed, his gain, conditional on the M&A negotiation is successful ($v - b^*(P)$), would decrease in the runup. This is also Schwert (1996)'s argument under markup hypothesis. However, the cross-sectional estimation of the runup's effect on the bidder's gain is positive, as documented by Betton, Eckbo, and Thorburn (2009), Betton et al. (2014), and Rajamani (2013).

The distinction of these two observations stems from different information sets, conditional on which we analyze the bidder's payoff. If we fix one bidder's value, because his acquisition costs increase in the runup, his gain decreases in the runup. However, as we assume in our model, the bidder's value is his own private information, and thereby we cannot control it in a cross-sectional analysis. Hence, cross-sectionally, the runup's effect on the bidder's gain also includes the positive correlation between the runup and the bidder's private value. We have shown, in Proposition 7, that the bidder's gain, whether conditional or unconditional on a successful M&A deal, increases in the runup in a cross-sectional analysis, without controlling the bidder's private value from the acquisition. This prediction is consistent with empirical findings in Betton, Eckbo, and Thorburn (2009), Betton et al. (2014), and Rajamani (2013), in which the bidders' private values are not controlled.

The empirical literature documents that target shareholders extract the majority of the value created by takeover (Martynova and Renneboog (2006)). Ahern (2012) considers the effects of bargaining powers on how the target and the bidder divide the value created by an M&A deal, and also show that the target obtains at least half of the value. The prediction of Proposition 8 that the ratio of the target's gain to the total gain is always greater than 1/2 is consistent with these empirical evidence, but our result is instead due to the informational effect of the runup.

6.2 New Empirical Predictions

As argued above, the theory that we have developed in this paper is consistent with the existing empirical evidence. However, there are several new predictions that can be drawn from our model. We delineate them as follows.

Prediction 1 *If the target learns from the runup, the target's stock market noise significantly affects both the premium-runup relation and the acquisition premium. Specifically, when the runup is high, the acquisition premium is strictly decreasing in the market noise level.*

The most important feature of our model is that the target learns from the runup, because there is information possessed by speculators in the market, but unknown to the target. Such a learning links the financial market to the M&A negotiation. Therefore, the characteristics of the target's stock market will hugely impacts the equilibrium M&A negotiation outcomes, especially the acquisition premium, even when we keep the runup constant. Let's compare the predictions of our benchmark model in Section 3, and those of our core model where the target learns from the runup. In Corollary 1, because the target does not learn from the runup, the takeover premium (b^N) and the premium-runup relation ($\Lambda(P)$) are independent of the target's stock market noise (β^{-1}). But when the target learns from the runup, we show that β determines the value of $\Lambda(P)$ for any fixed P in Proposition 4 and that β directly affects the acquisition premium b^* in Proposition 5.

Prediction 2 *If the target learns from the runup, the ratio of the target's gain to the perceived total surplus created by the M&A deal increases in the runup.*

This is directly from Proposition 8. Conversely, in the benchmark model where the target does not learn from the runup, because the acquisition premium is not affected by the runup, and the a higher runup indicates a larger value of the M&A deal, as shown in Corollary 2, the ratio of the target's gain to the bidder's gain strictly decreases. Therefore, this prediction also provides us with a test whether the target is learning from the runup, and whether the stock market affects the M&A negotiation.

7 Real Effects of the Target's Learning

In our model, the information about the ongoing M&A negotiation is leaked to the target's stock market after the M&A negotiation is initiated. This public information enables the stock price to incorporate more information about the target, and therefore improve the market efficiency (Diamond (1985), Diamond and Verrecchia (1991), Kanodia (1980), and Fishman and Hagerty (1989)). However, as a general rule, until the parties reach an agreement, federal securities regulations neither require the involving parties to publicly disclose the ongoing M&A negotiation, nor do they require them to keep their confidentiality, even if the negotiations are material. If the regulator aims to maximize the social welfare, the lack of requirement of disclosing or keeping confidential ongoing M&A negotiations implies that there is no clear conclusion as to whether the market efficiency will increase or decrease the real efficiency.[15]

Generally, the regulator is concerned about the total social welfare. In our model, if the surplus of the M&A is positive, then it is socially optimal if the M&A deal goes through; if the private value is negative, the M&A deal will not go through endogenously in the equilibrium, because the acquisition premium proposed by the target is always positive. Therefore, if disclosing the information about the ongoing M&A negotiation may decrease the ex-ante probability of a successful M&A deal conditional on the surplus $v > 0$, the regulator should ban such a disclosure. Proposition 10 below states that when the ex-ante mean of the private value v is negative and the stock price is sufficiently uninformative, disclosure of the information of the ongoing M&A negotiation will jeopardize the probability of the success of M&A deals with a positive surplus.

Proposition 10 (Ex-ante Deal Probability and Value Creation) *In the equilibrium,*

1. *for each* $v_0 < 0$, *there exists* $\underline{\gamma\beta}(v_0) > 0$, *such that* $\frac{\partial \mathbb{E}[\mathbb{1}_{(v > b^*)}]}{\partial \gamma\beta} < 0$ *for* $\gamma\beta \in \big(0, \underline{\gamma\beta}(v_0)\big)$*;*
2. *for every* $v_0 > 0$, *there exits* $\overline{\gamma\beta}(v_0) > 0$, *such that* $\frac{\partial \mathbb{E}[\mathbb{1}_{(v > b^*)}]}{\partial \gamma\beta} > 0$ *for* $\gamma\beta \in \big(0, \overline{\gamma\beta}(v_0)\big)$.

[15] For a long time, actions that can increase the market efficiency were thought to increase the real efficiency. Recently, however, Edmans, Heinle, and Huang (2015) and Goldstein and Yang (2014) argue that, under some circumstances, actions that increase the market efficiency may lead to lower real efficiency.

The adverse effects of the disclosure are due to the very noisy stock market and the very pessimistic prior belief. If the disclosure is banned, the target will propose an acquisition premium solely based on the prior belief. Since the prior belief of a positive private value is low without learning, the target will propose a very low acquisition premium. In the case in which the information about the ongoing negotiation is leaked to the market, when the market is very noisy, the stock price could be high with a fairly large probability. As a result, the target may propose a too high premium, which decreases the probability of a successful M&A deal with a positive surplus. On the other hand, the stock price can be low with an equally large probability. However, since the acquisition premium is lower-bounded by zero, its impact on the deal probability is highly limited. On average, with a pessimistic prior and noisy stock market, negotiation disclosure destroys the ex-ante deal probability. By contrast, with optimistic priors, and if the disclosure is banned, the acquisition premium based solely on the prior is larger than the realized value of v with a certain probability. Such a probability can be reduced by supplementing the target with a public signal, even it is noisy.

Our model, therefore, has an important policy implication. When the market is very noisy and the prior of the M&A surplus is sufficiently pessimistic, the regulator should closely monitor the information leakage so that the ongoing negotiation can be kept confidential, and thus a socially desirable M&A deal can go through with higher probability.

8 Conclusion

Our paper provides a theory in which the target's stock market affects the M&A negotiation outcomes because the target learns from the runup. We show that the premium-runup relation may have a value in $(0, \infty)$, depending on the target's stock market noise. We argue that the current empirical findings are consistent with this theoretical result. Since the target will learn from the runup if the runup is informative, we conclude that the existing empirical evidence cannot reject the assertion that the runup does not raise the bidder's acquisition costs. Other predictions about the runup's effects on the M&A negotiation outcomes are

also consistent with existing empirical findings.

Our theory provides several new empirically-testable predictions. In particular, we show that the ratio of the target's gain to the total surplus created by the M&A deal strictly increases in the runup. This provides a new test about whether the target's stock market affects the M&A negotiation. Such a test afford great theoretical interest: even though the bargaining protocol and thus the bargaining power, are fixed, the decrease in the information asymmetry will dramatically change the bargaining result.

Appendix

We first present useful properties of the Inverse Mill's Ratio.

Corollary 4 *Denote* $\lambda(z) = \frac{\phi(z)}{1-\Phi(z)}$ *as the Inverse Mills Ratio, then* $0 < \lambda'(z) < 1$ *for all* z*.*

Proof.

$$\lambda'(z) = \lambda(z) \cdot (\lambda(z) - z) \tag{A.1}$$

Using Mill's Ratio inequality derived from Birnbaum (1942) and Sampford (1953) that $\frac{\sqrt{z^2+8}+3z}{4} < \lambda(z) < \frac{\sqrt{z^2+4}+z}{2}$ for $z > 0$ and the fact that $\lambda(z)$ is convex in z, it is straightforward to show that $0 < \lambda(z) \cdot (\lambda(z) - z) < 1$ for all z. ■

Proof of Lemma 1.

The target firm's optimization problem is

$$\begin{aligned} &\max_b \mathbb{E}\left[b \cdot \mathbb{1}_{(v>b)} | z\right] \\ =&\max_b b \cdot \left[1 - \Phi\left(\frac{b - \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]. \end{aligned} \tag{A.2}$$

Its first-order condition is

$$\left[1 - \Phi\left(\frac{b - \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right] \cdot \left[1 - b \cdot \sqrt{\eta+\gamma\beta} \cdot \lambda\left(\frac{b - \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right] = 0. \tag{A.3}$$

Next, we show that there is a unique solution to the target's F.O.C. (A.3) and that it is also the global maximum to its objective function in (A.2).

Existence, Uniqueness, and Global Maximum

Since the first term in (A.3) is positive, the F.O.C. can be simplified as

$$1 - b \cdot \sqrt{\eta+\gamma\beta} \cdot \lambda\left(\frac{b - \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right) = 0. \tag{A.4}$$

The LHS of (A.4) is strictly positive when $b \leq 0$ and approaches $-\infty$ as $b \to \infty$. So it crosses 0 at least once – only when $b > 0$. The derivative of the LHS of equation (A.4) is equal to

$$-\sqrt{\eta+\gamma\beta}\cdot\lambda\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)-b(\eta+\gamma\beta)\cdot\lambda'\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right), \tag{A.5}$$

which is negative when $b > 0$. Since (A.4) is strictly decreasing when $b > 0$, it crosses 0 only once. Therefore, a unique solution to (A.4), denoted by $b^* > 0$, is guaranteed. It also follows that (A.4) is positive when $b < b^*$ and negative when $b > b^*$, which means that the objective function in (A.2) is a single-peaked function at $b = b^*$. The single-peaking directly implies global maximum.

Monotonicity and Convexity

Applying implicit function theorem to (A.4), we have

$$\begin{aligned}\frac{\partial b^*}{\partial z}&=\frac{b^*\gamma\beta\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)}{b^*(\eta+\gamma\beta)\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)+\sqrt{\eta+\gamma\beta}\lambda\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)}\\&=\frac{b^{*2}\gamma\beta\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)}{b^{*2}(\eta+\gamma\beta)\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)+1}.\end{aligned} \tag{A.6}$$

Using (A.1), it follows that $0 < \frac{\partial b^*}{\partial z} < \frac{\gamma\beta}{\eta+\gamma\beta} < \frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta}$. From (A.6), we know $\frac{\partial b^*}{\partial z}$ is increasing in $b^{*2}\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)$. Using F.O.C. in (A.4) and (A.1),

$$b^{*2}\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)=(\eta+\gamma\beta)^{-1/2}\,b^*\left[\lambda\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)-\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]. \tag{A.7}$$

Since

$$\frac{\partial\left[\lambda\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)-\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]}{\partial z}=\left[\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)-1\right]\cdot\left[\frac{\frac{\partial b^*}{\partial z}-\frac{\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right]>0$$

and

$$\frac{\partial b^*}{\partial z}>0,$$

their product, $b^{*2}\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)$, is also increasing in z. As a result, $\frac{\partial b^*}{\partial z}$ is also increasing in z, and b^* is convex in z. Moreover, both $\left[\lambda\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)-\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]\to+\infty$ and $b* \to +\infty$ as $z\to+\infty$. It then follows that, as $z\to+\infty$, $b^{*2}\lambda'\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\to+\infty$ and, from (A.6), $\frac{\partial b^*}{\partial z}\to\frac{\gamma\beta}{\eta+\gamma\beta}$.

Last, as b^* increases with z, $\lambda\left(\frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)=\frac{1}{b^*\cdot\sqrt{\eta+\gamma\beta}}$ is decreasing in z and approaches 0 as z goes to $+\infty$. Let $\tilde{z}$ be such that $b^*(\tilde{z})-\frac{v_0\eta+\tilde{z}\gamma\beta}{\eta+\gamma\beta}=0$, then $b^*<\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}$ for all $z>\tilde{z}$. Moreover, it can be easily verified that $\bar{z}=\frac{\sqrt{\eta+\gamma\beta}-2v_0\eta}{2\gamma\beta}$. ■

Proof of Proposition 2.

We used a proposed pricing function to derive b^*. Next, we confirm that, with b^*, investors' follow cutoff strategies characterized by $g(P)$, which lead to the pricing function as we initially proposed. Investor i's expected payoff from buying one unit of the target's shares given its information set is

$$\begin{aligned}&\mathbb{E}\left[b^*\cdot\mathbb{1}_{(v>b^*)}-P|s_i,P\right]\\=&b^*\cdot\left(1-\Phi\left(\frac{b^*-\frac{v_0\eta+s_i\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\right)-P.\end{aligned}\tag{A.8}$$

We first show that (A.8) is increasing in s_i. Taking the first-order derivative with respect to s_i, we have

$$b^*\phi\left(\frac{b^*-\frac{v_0\eta+s_i\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\frac{\gamma}{\sqrt{\eta+\gamma+\gamma\beta}}>0.\tag{A.9}$$

So investor i's profit of holding one share of target stock is increasing with his signal s_i. It thus supports the cutoff strategies we proposed in (8). $g(P)$ is determined through the marginal investor's indifference condition; that is, when $s_i=g(P)$, (A.8) is equal to 0. However, the existence of $g(P)$ is yet to be proved. Before we do that, we first show that price P is bounded.

We now consider two limiting cases. First, all informed investors buy. This means (A.8) is positive when $s_i\to-\infty$, and therefore $P>0$. Second, all informed investors sell. This means (A.8) is negative when $s_i\to+\infty$, and therefore $P<b^*$. In sum, the price runup in our model is bounded: $P\in(0,b^*)$.

The indifference condition is

$$b^* \cdot \left(1 - \Phi\left(\frac{b^* - \frac{v_0\eta + g\gamma + g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\right) - P = 0. \tag{A.10}$$

We first show that the LHS of (A.10) is increasing in g. Taking the first-order derivative with respect to g, we have

$$\begin{aligned} &b^*\phi\left(\frac{b^* - \frac{v_0\eta+g\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\sqrt{\eta+\gamma+\gamma\beta}\left(\frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta} - \frac{\partial b^*}{\partial z}\frac{\partial z}{\partial g}\right) \\ &+ \frac{\partial b^*}{\partial z}\frac{\partial z}{\partial g}\left(1-\Phi\left(\frac{b^* - \frac{v_0\eta+g\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\right) > 0. \end{aligned} \tag{A.11}$$

As $\frac{\partial z}{\partial g} = 1$, (A.11) is positive since $0 < \frac{\partial b^*}{\partial z} < \frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta}$. So the LHS of (A.10) is strictly increasing in g. Moreover, using the boundaries on P, the LHS of (A.10) approaches $b^* - P > 0$ when $g \to +\infty$ and $-P < 0$ when $g \to -\infty$. The existence and uniqueness of $g(P)$ are therefore guaranteed.

The last step to complete the proof is to derive the pricing function as in (13), which is followed directly from (8), (9), and (1). ■

Proof of Proposition 3.

Taking the total differentiation of equation (A.10) with respective to P, we have

$$\begin{aligned} &b^*\phi\left(\frac{b^* - \frac{v_0\eta+g\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\sqrt{\eta+\gamma+\gamma\beta}\left(\frac{\gamma g'(P)+\gamma\beta g'(P)}{\eta+\gamma+\gamma\beta} - \frac{\partial b^*}{\partial z}g'(P)\right) \\ &+ \frac{\partial b^*}{\partial z}g'(P)\left(1-\Phi\left(\frac{b^* - \frac{v_0\eta+g\gamma+g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}\right)\right) - 1 = 0. \end{aligned} \tag{A.12}$$

Rearrange and we have

$$g'(P) = \frac{1}{b^*\phi\left(K\right)\sqrt{\eta+\gamma+\gamma\beta}\left(\frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta} - \frac{\partial b^*}{\partial z}\right) + \frac{\partial b^*}{\partial z}\left(1-\Phi\left(K\right)\right)}, \tag{A.13}$$

where $K \equiv \frac{b^* - \frac{v_0\eta + g\gamma + g\gamma\beta}{\eta+\gamma+\gamma\beta}}{(\eta+\gamma+\gamma\beta)^{-1/2}}$. Then

$$\begin{aligned}\frac{\partial b^*}{\partial P} &= \frac{\partial b^*}{\partial z} g'(P) \\ &= \frac{\partial b^*}{\partial z} \cdot \frac{1}{b^* \phi\left(K\right) \sqrt{\eta+\gamma+\gamma\beta}\left(\frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta} - \frac{\partial b^*}{\partial z}\right) + \frac{\partial b^*}{\partial z}\left(1-\Phi\left(K\right)\right)} > 0, \end{aligned} \tag{A.14}$$

where the last inequality follows from $0 < \frac{\partial b^*}{\partial z} < \frac{\gamma\beta}{\eta+\gamma\beta} < \frac{\gamma+\gamma\beta}{\eta+\gamma+\gamma\beta}$.

From Equation (A.14), $\partial b^*/\partial P$ is a function of P. Therefore, b^* is a nonlinear function of P.

To study the asymptotic behavior of (A.14), we first plug (A.6) into (A.14), through simplification, we have:

$$\frac{\partial b^*}{\partial P} = \frac{\gamma\beta b^{*2}\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}}{b^*\phi\left(K\right)\left(\gamma+\gamma\beta+\gamma\eta b^{*2}\lambda'(G)\right) + \gamma\beta b^{*2}\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}\left(1-\Phi\left(K\right)\right)}. \tag{A.15}$$

When $P \to 0$, we have $z \to -\infty$, $b^* \to 0$, $G \to \infty$ and $K \to \infty$.. Divide the denominator of (A.14) by its numerator, we have

$$\begin{aligned} &\frac{\phi\left(K\right)\left(\gamma+\gamma\beta+\gamma\eta b^{*2}\lambda'(G)\right)}{\gamma\beta b^*\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}} + \left(1-\Phi(K)\right) \\ =&\frac{\phi\left(K\right)\lambda(G)\left(\gamma+\gamma\beta+\gamma\eta b^{*2}\lambda'(G)\right)\sqrt{\eta+\gamma\beta}}{\gamma\beta\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}} + \left(1-\Phi(K)\right) \end{aligned} \tag{A.16}$$

Since $\lambda'(G) \to 1$ and $\phi(K) \to 0$ at exponential speed of z and $\lambda(G) \to \infty$ at the speed of $\mathrm{O}\left(z\right)$, it is easy to show that the term above goes to 0.

When $P \to \infty$, we have $z \to \infty$, $b \to \infty$, $G \to -\infty$. Using the expression that $b^* = \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta} - \frac{\sqrt{2logb^* + log(\eta+\gamma\beta) - 2log(1-\Phi(G)) - log2\pi}}{\sqrt{\eta+\gamma\beta}}$, it can be shown that $K \to \mathrm{O}\left(-z\right)$ and $G \to \mathrm{O}\left(\sqrt{2logz}\right)$. Apply these results to (A.16) and realizing that $\lambda'(G) = \lambda(G) \cdot (\lambda(G) - G)$, the term goes to 1 from below.

Last, by continuity, as P increases, $\partial b^*/\partial P$ first decreases from infinity to some positive value a that is smaller than 1 and then increases to 1 asymptotically. ■

Proof of Proposition 4.

We first plug (A.6) into (A.14), through simplification, we have:

$$\frac{\partial b^*}{\partial P} = \frac{\gamma\beta b^{*2}\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}}{b^*\phi\left(K\right)\left(\gamma+\gamma\beta+\gamma\eta b^{*2}\lambda'(G)\right)+\gamma\beta b^{*2}\lambda'(G)\sqrt{\eta+\gamma+\gamma\beta}\left(1-\Phi\left(K\right)\right)}. \tag{A.17}$$

When $\beta \to 0$, $b^* \to b_0$, where b_0 is the optimal asking price using only the prior information on v. Let $G \equiv \frac{b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}$. As $\beta \to 0$, $G \to G_0 = \frac{b_0-v_0}{\eta^{-1/2}}$ and $K \to K_0 = \frac{b^*-\frac{v_0\eta+g\gamma}{\eta+\gamma}}{(\eta+\gamma)^{-1/2}}$. Substitutes these results into (A.17) it can be shown that $\frac{\partial b^*}{\partial P} \to 0$.

When $\gamma \to 0$, $b^* \to b_0$, $G \to G_0$ and $K \to G_0$. These results instead imply that $\frac{\partial b^*}{\partial P} \to \frac{\beta b_0^2\lambda'(G_0)\sqrt{\eta}}{b_0\phi(G_0)(1+\beta+\eta b^{*2}\lambda'(G_0))+\beta b_0^2\lambda'(G_0)\sqrt{\eta}(1-\Phi(G_0))} > 0$.

When $\gamma\beta \to \infty$, there are two cases.

Case I: $z > 0$

When $z > 0$, $b^* \to^- z$ as $\beta \to \infty$. By (A.4), we know $\lambda(G) = \frac{1}{b^*\sqrt{\eta+\gamma\beta}} \to \mathrm{O}\left(\beta^{-1/2}\right)$ since b^* is bounded by z. And $\lambda(x) \to \mathrm{O}\left(e^{-\frac{1}{2}\pi x^2}\right)$ as $x \to -\infty$, it then follows that $G \to \mathrm{O}\left(-\sqrt{log(\beta)}\right)$. Using the expression that $b^* = \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta} - \frac{\sqrt{2logb^*+log(\eta+\gamma\beta)-2log(1-\Phi(G))-log2\pi}}{\sqrt{\eta+\gamma\beta}}$, it can shown that $K \to \mathrm{O}\left(-\sqrt{log(\beta)}\right)$. Therefore, $\phi(K) \to \mathrm{O}\left(\beta^{-1/2}\right)$ and $1-\Phi(K) \to \mathrm{O}\left(1\right)$. Using (A.7), $b^{*2}\lambda'(G) = (\eta+\gamma\beta)^{-1/2}b^*\left[\lambda(G)-G\right] \to (\mathrm{O}\left(\sqrt{log(\beta)}(\eta+\gamma\beta)^{-1/2}\right)$. Plug these results into (A.17), it is easy to show that $\frac{\partial b^*}{\partial P} \to 1$.

As $\gamma \to \infty$, the proof remains the same.

Case II: $z < 0$

When $z < 0$, $b^* \to^+ 0$ as $\beta \to \infty$. $G \to \mathrm{O}\left(\beta^{1/2}\right)$ and $\lambda(G) - G \to \mathrm{O}\left(G^{-1}\right) = \mathrm{O}\left(\beta^{-1/2}\right)$ (using the fact that $\frac{\sqrt{z^2+8}+3z}{4} < \lambda(z) < \frac{\sqrt{z^2+4}+z}{2}$ for $z > 0$). And $b^* = \frac{1}{\sqrt{\eta+\gamma\beta}\lambda(G)} \to \mathrm{O}\left(\beta^{-1}\right)$. By (A.7), $b^{*2}\lambda'(G) = (\eta+\gamma\beta)^{-1/2}\frac{1}{\sqrt{\eta+\gamma\beta}\lambda(G)}\left[\lambda(G)-G\right] \to \mathrm{O}\left(\beta^{-5/2}\right)$. Plug it into (A.14), it is easy to show that $\frac{\partial b^*}{\partial z} \to \mathrm{O}\left(\gamma^{-1}\right)$.

Next, through similar calculations, $K \to \mathrm{O}\left(\beta^{1/2}\right)$. It is important to note that now $\phi(K) \to \mathrm{O}\left(e^{-\frac{1}{2}\pi\beta}\right)$ and $1-\Phi(K) \to \mathrm{O}\left(\beta^{1/2}\, e^{-\frac{1}{2}\pi\beta}\right)$. Again, apply these asymptotic results to (A.17), we have $\frac{\partial b^*}{\partial P} \to \infty$.

As $\gamma \to \infty$, the proof remains identical. ■

Proof of Proposition 5.

From (A.4), we have:

$$\frac{\partial b^*}{\partial \gamma\beta} = \frac{\sqrt{\eta+\gamma\beta} b^* \lambda'(G)\left[\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2} - \frac{1}{2}(\eta+\gamma\beta)^{-3/2}G\right] - \frac{1}{2}(\eta+\gamma\beta)^{-3/2}}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G) + \lambda(G)} \tag{A.18}$$

$$= \frac{\frac{1}{2}(\eta+\gamma\beta)^{-3/2}\left[G(G-\lambda(G))-1\right] + (\lambda(G)-G)\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}}{2\lambda(G)-G}, \tag{A.19}$$

where the last equation follows from $\sqrt{\eta+\gamma\beta} b^*\lambda'(G) = \lambda(G) - G$. Now, $G(G-\lambda(G))-1 > 0$ if and only if $G < c < 0$, where c is a constant value. Moreover, since $G(G-\lambda(G))-1$ is increasing in z and unbounded, then for $\forall\gamma\beta$, there exists z^u such that when $z > z^u$, $G(G-\lambda(G))-1 > 0$ and $G < c$. In other words, z^u is the solution to $G = c$. We differentiate $G = c$ or $\left(\frac{b^* - \frac{v_0\eta + z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right) = c$ w.r.t. z^u and we have:

$$\frac{\partial \gamma\beta}{\partial z^u} = \frac{\frac{\gamma\beta}{\eta+\gamma\beta} - \frac{\partial b^*}{\partial z^u}}{\frac{\partial b^*}{\partial\gamma\beta} - \frac{\eta(z^u - v_0)}{(\eta+\gamma\beta)^2} + \frac{G}{2(\eta+\gamma\beta)^{2/3}}}. \tag{A.20}$$

Its numerator is positive, and the denominator can be simplified by using (A.19) as

$$\frac{\partial b^*}{\partial\gamma\beta} - \frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2} + \frac{G}{2(\eta+\gamma\beta)^{2/3}} = \frac{\frac{1}{2}(\eta+\gamma\beta)^{-3/2}\left[G\lambda(G)-1\right] - \lambda(G)\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}}{2\lambda(G)-G}. \tag{A.21}$$

Since $G = c < 0$, (A.20) and (A.21) is negative if $z^u \geq v_0$. Define $\mathbb{A} := \{\gamma\beta | z^u(\gamma\beta) = v_0\}$ and $\Delta^u = \sup_{\gamma\beta\in\mathbb{A}} \gamma\beta$. $\mathbb{A}$ is not an empty set since G is unbounded in z. Then $\frac{\partial\gamma\beta}{\partial z^u} < 0$ for $\forall\gamma\beta \in [\Delta^u, +\infty)$, or equivalently, $\forall z^u \in [v_0, +\infty)$. It follows that, $\frac{b^*}{\partial\gamma\beta}|_{\gamma\beta>\Delta^u} > 0$ for $\forall z \in [v_0, +\infty)$.

Next, note that the numerator of (A.18) is bounded for any given z and $\gamma\beta \in (0, \Delta^u]$ but is unbounded and increasing in z when it is positive. Then $\exists \tilde{z}(\Delta^u)$ s.t. $\frac{b^*}{\partial\gamma\beta}|_{0<\gamma\beta<\Delta^u} > 0$ for $\forall z \in [\tilde{z}(\Delta^u), +\infty)$. Now let $\bar{z} = \max(v_0, \tilde{z}(\Delta^u))$. When $z > \bar{z}$, $\frac{b^*}{\partial\gamma\beta} > 0$ for $\forall\gamma\beta \in (0, +\infty)$.

Similarly, let $c_1 > 0$ be the value such that $c_1\lambda(c_1) = 1$. It follows that if $G > c_1$, then $G(G-\lambda(G)) - 1 < 0$ and $G\lambda(G) - 1 > 0$. Let z^d be the solution to $G = c_1$ for $\forall\gamma\beta$, $\mathbb{B} := \{\gamma\beta | z^d(\gamma\beta) = v_0\}$, and $\Delta^d = \inf_{\gamma\beta\in\mathbb{B}} \gamma\beta$. By the same logic, $\frac{b^*}{\partial\gamma\beta}|_{\gamma\beta>\Delta^d} < 0$ for $\forall z \in (-\infty, v_0]$. It is also easy to verify that for a given $\gamma\beta$, the numerator of (A.18) approaches 0 from below when $z \to -\infty$. Combined with the fact that it is first decreasing in z and then increasing in z, there exists $\underset{\sim}{z}(\gamma\beta)$ for $\forall\gamma\beta \in [0, \Delta^d]$, such that $\frac{b^*}{\partial\gamma\beta}|_{0\leq\gamma\beta\leq\Delta^d} < 0$ for $\forall z \in (-\infty, \underset{\sim}{z}(\gamma\beta)]$. Let

$\underline{z} = \min\left(v_0, \min_{0\leq\gamma\beta\leq\Delta^d}(\underline{z}(\gamma\beta))\right)$. Then $z < \underline{z}$, $\frac{b^*}{\partial\gamma\beta} < 0$ for $\forall\gamma\beta \in (0, +\infty)$.

Now, when $\gamma\beta \to +\infty$, $b^* \to^- z$ if $z > 0$ and $b^* \to^+ 0$ if $z < 0$. That is, in the limit, b^* approaches z from left if $z > 0$ and b^* approaches 0 from right if $z < 0$. In the case when $z > 0$, G is negative and $G \to \mathrm{O}\left(-log(\gamma\beta)\right)$, and $\lambda(G) \to \mathrm{O}\left((\gamma\beta)^{-1/2}\right)$. Then the first term of the numerator of (A.19) goes to 0 from above and its rate of convergence is $\mathrm{O}\left((\gamma\beta)^{-3/2}\log(\gamma\beta)\right)$. The second term of the numerator also converges to 0 from above if $z > v_0$ or from below if $z < v_0$, but its rate of convergence is $\mathrm{O}\left((\gamma\beta)^{-2}\log(\gamma\beta)\right)$. When $z > v_0$, the whole numerator is positive; when $z < v_0$, the first term dominates the second term, and their sum still converges to 0 from above. As a result, regardless of whether $z > v_0$ or $z < v_0$, as long as $z > 0$, $\frac{\partial b^*}{\partial\gamma\beta} \to^- 0$ as $\gamma\beta \to +\infty$.

In the case when $z < 0$, G is positive and $G \to \mathrm{O}\left((\gamma\beta)^{1/2}\right)$. Through straightforward comparison, both the first and the second term of the numerator of (A.18) are negative and go to 0 from below. ■

Proof of Proposition 6.

Conditional on P (or equivalently z), the probability of a successful deal is

$$\mathrm{E}\left[\mathbb{1}_{(v>b^*)}|P\right] = 1 - \Phi\left(\frac{b^* - \frac{v_0\eta + g(P)\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right) \tag{A.22}$$

$$\frac{\partial\mathrm{E}\left[\mathbb{1}_{(v>b^*)}|P\right]}{\partial P} = \phi(G)\left(\frac{\gamma\beta}{\eta+\gamma\beta} - \frac{\partial b^*}{\partial z}\right)g'(P) > 0.$$

The expected payoff conditional on P is the product of b^* and $\mathrm{E}\left[\mathbb{1}_{(v>b^*)}|P\right]$. Since both b^* and $\mathrm{E}\left[\mathbb{1}_{(v>b^*)}|P\right]$ are positive and increasing in P, their product is also increasing in P. ■

Proof of Proposition 7.

The bidder's expected gain conditional on the price P and the deal going through is:

$$\begin{aligned}
&\mathrm{E}\left[v - b^*|v > b^*, P\right] \\
=&\frac{v_0\eta + g(P)\gamma\beta}{\eta+\gamma\beta} - b^* + (\eta+\gamma\beta)^{-1/2}\lambda\left(\frac{b^* - \frac{v_0\eta+g(P)\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right).
\end{aligned}$$

Its derivative w.r.t. P is

$$\begin{aligned}\frac{\partial \mathrm{E}\left[v-b^*|v>b^*,P\right]}{\partial P} =& \frac{\partial \mathrm{E}\left[v-b^*|v>b^*,P\right]}{\partial z}\cdot\frac{\partial z}{\partial P}\\ =& \left(\frac{\gamma\beta}{\eta+\gamma\beta}-\frac{\partial b^*}{\partial z}\right)\left(1-\lambda'(G)\right)g'(P)>0.\end{aligned}$$

The inequality follows since $\frac{\partial b^*}{\partial z}<\frac{\gamma\beta}{\eta+\gamma\beta}$ and $\lambda'(K)<1$.

The bidder's expected gain conditional on P is

$$\mathbb{E}\left[(v-b^*)\cdot\mathbb{1}_{(v>b^*)}|P\right]=\mathbb{E}\left[v-b^*|v>b^*,P\right]\cdot\mathbb{E}\left[\mathbb{1}_{(v>b^*)}|P\right].$$

Since both $\mathbb{E}\left[v-b^*|v>b^*,P\right]$ and $\mathbb{E}\left[\mathbb{1}_{(v>b^*)}|P\right]$ are positive and increasing in P, their product is also increasing in P. ■

Proof of Proposition 8.

We first show that the ratios are the same whether conditioning on the deal going through or not.

$$\begin{aligned}&\frac{\mathbb{E}[b^*\mathbb{1}_{(v>b^*)}|P]}{\mathbb{E}[(v-b^*)\mathbb{1}_{(v>b^*)}|P]}\\ =&\frac{\left(1-\Phi(G)\right)b^*}{\left(1-\Phi(G)\right)\cdot\left\{\frac{v_0\eta+g(P)\gamma\beta}{\eta+\gamma\beta}-b^*+(\eta+\gamma\beta)^{-1/2}\lambda\left(G\right)\right\}}\\ =&\frac{b^*}{\left\{\frac{v_0\eta+g(P)\gamma\beta}{\eta+\gamma\beta}-b^*+(\eta+\gamma\beta)^{-1/2}\lambda\left(G\right)\right\}}\\ =&\frac{\mathbb{E}[b^*|P,v>b^*]}{\mathbb{E}[(v-b^*)|P,v>b^*]}\end{aligned}\tag{A.23}$$

Using (A.1) and (A.3), $\Gamma(P)$ can be further simplified as

$$\Gamma(P)=\frac{b^*}{b^*+\left(-G+\lambda\left(G\right)\right)(\eta+\gamma\beta)^{-1/2}}=\frac{1}{1+\lambda(G)\cdot\left(\lambda\left(G\right)-G\right)}=\frac{1}{1+\lambda'(G)}.$$

Take first-order derivative w.r.t. P,

$$\begin{aligned}\frac{\partial \Gamma}{\partial P} &= \frac{\partial \Gamma}{\partial G} \cdot \frac{\partial G}{\partial z} \cdot \frac{\partial z}{\partial P} \\ &= \frac{-\lambda''(G)}{\lambda'^2(G)} \sqrt{\eta + \gamma\beta} \left(\frac{\partial b^*}{\partial z} - \frac{\gamma\beta}{\eta + \gamma\beta} \right) g'(P) \\ &> 0,\end{aligned}$$

where the inequality follows from the convexity of $\lambda(\cdot)$ and Lemma 1. Moreover, since $\lambda'(\cdot) \in (0, 1)$, then $R > 1$ for $\forall z$ (or P). And $\lim_{z\to-\infty} G = +\infty$ and $\lim_{z\to+\infty} G = -\infty$, combined with the convexity of $\lambda(\cdot)$, we have $\lim_{z\to-\infty} R = 1$ and $\lim_{z\to+\infty} R = +\infty$. ■

Proof of Proposition 9.

All these deal outcomes can be expressed in terms of $G = \frac{b^* - \frac{v_0\eta + g(P)\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}$:

$$\mathrm{E}\left[\mathbb{1}_{(v>b^*)}|P\right] = 1 - \Phi(G) \tag{A.24}$$

$$\mathrm{E}\left[v - b^*|v > b^*, P\right] = (\eta + \gamma\beta)^{-1/2} \left(\lambda(G) - G\right) \tag{A.25}$$

$$\Gamma(P) = \frac{1}{1 + \lambda'(G)} \tag{A.26}$$

We next calculate that:

$$\begin{aligned}\frac{\partial G}{\partial \beta} &= \sqrt{\eta + \gamma\beta} \left(\frac{\partial b^*}{\partial \gamma\beta} - \frac{\eta(z - v_0)}{(\eta + \gamma\beta)^2} + \frac{G}{2(\eta + \gamma\beta)^{2/3}} \right) \\ &= \sqrt{\eta + \gamma\beta} \frac{\frac{1}{2}(\eta + \gamma\beta)^{-3/2}\left[G\lambda(G) - 1\right] - \lambda(G)\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}}{2\lambda(G) - G} \qquad \text{(A.27)} \\ &= \frac{1}{2}(\eta + \gamma\beta)^{-3/2} \frac{\sqrt{\eta + \gamma\beta}\left[G\lambda(G) - 1\right] - 2\lambda(G)\eta(z - v_0)}{2\lambda(G) - G} \\ &= \frac{1}{2}(\eta + \gamma\beta)^{-3/2} \frac{-\lambda(G)\left[(2\eta + \gamma\beta)z - 2\eta v_0\right]}{2\lambda(G) - G}\end{aligned}$$

where (A.27) is due to (A.21). It then follows that if $z > v_0$, $\partial G/\partial\beta < 0$ for all $\beta > 0$, and if $z < v_0/2$, $\partial G/\partial\beta > 0$ for all $\beta > 0$.

For (A.25), $(\eta + \gamma\beta)^{-1/2}$ is decreasing in β and $\lambda(G) - G$ is decreasing in β if $z < v_0/2$. As a result, (A.25) decreases with β if $z < v_0/2$. However, when $z > v_0$, the result is not clear. ■

Proof of Proposition 10.

The ex-ante deal probability is

$$\begin{aligned}\mathbb{E}[\mathbb{1}_{(v>b^*)}] &= \mathbb{E}\left[\mathbb{E}[\mathbb{1}_{(v>b^*)}|z]\right] \\ &= \int_{-\infty}^{+\infty}\left[1-\Phi\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right] f_z(z)\,\mathrm{d}z,\end{aligned}$$

where $f_z(z)$ is the pdf of z and $f_z(z)=\frac{1}{\sqrt{\eta^{-1}+(\gamma\beta)^{-1}}}\phi\Big(\frac{z-v_0}{\sqrt{\eta^{-1}+(\gamma\beta)^{-1}}}\Big)$. The derivative of the ex-ante deal probability is

$$\begin{aligned}\frac{\partial\,\mathbb{E}[\mathbb{1}_{(v>b^*)}]}{\partial\gamma\beta} &= \int_{-\infty}^{+\infty}\left[1-\Phi\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]\frac{\partial f_z(z)}{\partial\gamma\beta}\,\mathrm{d}z \\ &+\int_{-\infty}^{+\infty}\frac{\partial\left[1-\Phi\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]}{\partial\gamma\beta} f_z(z)\,\mathrm{d}z. \qquad \text{(A.28)}\end{aligned}$$

We now evaluate the above derivative at $\gamma\beta=0$, then the first term is

$$\left[1-\Phi\big(\sqrt{\eta}(b_N-v_0)\big)\right]\int_{-\infty}^{+\infty}\frac{\partial f_z(z)}{\partial\gamma\beta}\,\mathrm{d}z = C\cdot\int_{-\infty}^{+\infty}\left[\frac{(z-v_0)^2}{\sigma_z^2}-1\right]f_z(z)\,\mathrm{d}z=0, \qquad \text{(A.29)}$$

where b_N is the solution to (A.30) below and C is a quantity that is independent of z

$$1-b_N^*\sqrt{\eta}\lambda\left(\frac{b_N^*-v_0}{\eta^{-1/2}}\right)=0. \qquad \text{(A.30)}$$

Next, we calculate the second term:

$$\begin{aligned}\frac{\partial\left[1-\Phi\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\right]}{\partial\gamma\beta} &= -\,\phi\left(\frac{b-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}}{(\eta+\gamma\beta)^{-1/2}}\right)\left\{\sqrt{\eta+\gamma\beta}\left(\frac{\partial b^*}{\partial\gamma\beta}-\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}\right)\right. \\ &\left.+\frac{1}{2\sqrt{\eta+\gamma\beta}}\left(b^*-\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)\right\}. \qquad \text{(A.31)}\end{aligned}$$

To simplify the representation, we now define $G = \sqrt{\eta+\gamma\beta}\left(b - \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)$ and $H = \sqrt{\eta+\gamma\beta}\left(\frac{\partial b^*}{\partial\gamma\beta} - \frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}\right) + \frac{1}{2\sqrt{\eta+\gamma\beta}}\left(b^* - \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)$. Differentiate (A.4) w.r.t. $\gamma\beta$, we have

$$\begin{aligned}
&\frac{\partial b^*}{\partial\gamma\beta}\lambda(G) + b^*\lambda'(G)\left[\sqrt{\eta+\gamma\beta}\left(\frac{\partial b^*}{\partial\gamma\beta} - \frac{\eta(z-v_0)}{(\eta+\gamma\beta)^2}\right) + \frac{1}{2\sqrt{\eta+\gamma\beta}}\left(b^* - \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)\right] \\
=& -\frac{1}{2}(\eta+\gamma\beta)^{-3/2}.
\end{aligned}$$

Rearrange and we have

$$\frac{\partial b^*}{\partial\gamma\beta} = \frac{b^*\lambda'(G)\left[\sqrt{\eta+\gamma\beta}\frac{\partial b^*}{\partial\gamma\beta} - H\right] - \frac{1}{2}(\eta+\gamma\beta)^{-3/2}}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G) + \lambda(G)}.$$

Plug it into (A.31) and through simplification, we have

$$\begin{aligned}
&\frac{\partial\left[1-\Phi(G)\right]}{\partial\gamma\beta} \\
=&\frac{-\phi(G)}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)}\cdot\left\{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)\left(\sqrt{\eta+\gamma\beta}\frac{\partial b^*}{\partial\gamma\beta} - H\right)\right. \\
&\left. -\frac{1}{2}(\eta+\gamma\beta)^{-1} - \left(\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)\right)\left(\sqrt{\eta+\gamma\beta}\frac{\partial b^*}{\partial\gamma\beta} - H\right)\right\} \\
=&\frac{-\phi(G)}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)}\cdot\left\{-\frac{1}{2}(\eta+\gamma\beta)^{-1} - \lambda(G)\left(\sqrt{\eta+\gamma\beta}\frac{\partial b^*}{\partial\gamma\beta} - H\right)\right\} \\
=&\frac{\phi(G)}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)}\cdot\left\{\frac{1}{2}(\eta+\gamma\beta)^{-1} + \lambda(G)\left[\frac{\eta(z-v_0)}{(\eta+\gamma\beta)^{3/2}} - \frac{1}{2}\frac{1}{\sqrt{\eta+\gamma\beta}}\left(b^* - \frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right)\right]\right\} \\
=&\frac{\phi(G)}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)}\cdot\frac{1}{\sqrt{\eta+\gamma\beta}}\lambda(G)\left[\frac{\eta(z-v_0)}{\eta+\gamma\beta} + \frac{1}{2}\frac{v_0\eta+z\gamma\beta}{\eta+\gamma\beta}\right] \qquad \text{(using (A.4))} \\
=&\frac{\phi(G)}{\sqrt{\eta+\gamma\beta}\, b^*\lambda'(G)+\lambda(G)}\cdot\frac{1}{2\sqrt{\eta+\gamma\beta}}\lambda(G)\left[\frac{\eta(z-v_0)}{\eta+\gamma\beta} + z\right],
\end{aligned}$$

which, when evaluated at $\gamma\beta = 0$, is

$$\left.\frac{\partial\left[1-\Phi(G)\right]}{\partial\gamma\beta}\right|_{\gamma\beta=0} = \frac{\phi(G_N)}{\sqrt{\eta}\, b^*\lambda'(G_N)+\lambda(G_N)}\cdot\frac{1}{2\sqrt{\eta}}\lambda(G_N)\left[(z-v_0)+z\right],$$

where $G_N = \sqrt{\eta}(b_N - v_0)$ and is independent of z. Then

$$\int_{-\infty}^{+\infty}\left.\frac{\partial\left[1-\Phi\left(G\right)\right]}{\partial\gamma\beta}\right|_{\gamma\beta=0} f_z(z)\,\mathrm{d}z = \frac{\phi(G_N)}{\sqrt{\eta}\, b^*\lambda'(G_N)+\lambda(G_N)}\cdot\frac{1}{2\sqrt{\eta}}\lambda(G_N)\cdot v_0.$$

Combined with (A.28) and (A.29), it follows that $\frac{\mathbb{E}\left[\mathbb{1}_{(v>b^*)}\right]}{\partial\gamma\beta}\Big|_{\gamma\beta=0} > 0$ if $v_0 > 0$ and $\frac{\mathbb{E}\left[\mathbb{1}_{(v>b^*)}\right]}{\partial\gamma\beta}\Big|_{\gamma\beta=0} < 0$ if $v_0 < 0$. Last, by continuity of $\frac{\mathbb{E}\left[\mathbb{1}_{(v>b^*)}\right]}{\partial\gamma\beta}$ in $\gamma\beta$, each of the above results must hold in a neighborhood of $\gamma\beta = 0$, respectively. ■

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