Integration, Delegation and Management in Industry Equilibrium*

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Abstract

We present a model in which managerial scarcity affects the joint determination of firm boundaries and internal decision-making structures in a competitive industry. Integration through asset sale grants authority to a top manager, who may choose to delegate decision making back to his subordinates, or to retain control for himself, depending on comparative advantage in coordination which is learned after contracting. Exogenous productivity and industry price are key determinants of both integration and delegation: across a heterogeneous population of firms, or with temporal variation in price, delegation and integration may co-vary, there is little integration and consequently lower aggregate productivity. Heterogeneity of integration structures is generic, even among ex-ante identical firms, and the degree of heterogeneity may be non-monotonically dependent on managerial scarcity. When enterprises can easily increase their scale, free entry into top management may lead to excessive integration, too many managers, and too little enterprise; when scale cannot easily be increased, there may be too little integration and management, even with free entry.

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1 Introduction

It is widely believed that management is essential to the functioning of firms and other organizations. At the same time, it is fair to say that consensus has yet to emerge on precisely why it matters, how it matters, or even how to define it. Nevertheless, for economists who study organizations, three facets of management can surely be viewed as crucial. First, managers allocate resources by authority. In private enterprise economies, authority emanates from property rights, and the scope of management is therefore constrained by ownership structure. Second, managers design the organization, particularly when and to whom to delegate decision rights. Owners will take account of the likely directions of delegation when choosing whether to sell their assets; the possibility that one might re-acquire decision making power softens the blow of giving up control. Hence, integration and delegation decisions are interdependent; firm boundaries limit delegation decisions, but the anticipation of delegation encourages the broadening of firm boundaries. Third, management is a scarce resource. The costs and benefits of integrating are affected by the price one pays for a manager to exercise authority competently, and the scarcity of managerial talent affects that price: management is not only constrained by the firm boundaries, it also constrains them.

In this paper we focus on the interactions among these three aspects of management: we study the relationship between firm boundaries and internal organization choices, how they respond to changes in the economic environment, especially the market for managers, and how they affect industry performance. Since managerial markets differ radically across countries, our analysis will have something to say about how the differential performance of firms and industries in developed and developing countries is tied to their differential ownership, organizational design, and management practices.

Our setting also provides an opportunity to underscore the important differences between outsourcing, akin to non-integration in our model, and delegation (or decentralization); in fact we will show that non-integration and delegation may move in opposite directions as the price of output or productivity changes.

Following Hart et al. (2010) and Legros and Newman (2013) we consider environments in which integration facilitates coordination between suppliers. The precise mechanism for this is due to a change of ownership following integration that gives authority to a central party who values only revenues, and will therefore try to coordinate activities in order to maximize expected output. It follows that both integration and non-integration involve costs and benefits: integration tends to put too

little weight on private costs which leads to over-coordination while non-integration tends to put too much weight on private costs which leads to too little coordination.

Top managers who have authority to make decisions in the firm may decide to delegate decision making if they realize that they do not have a comparative advantage, something they learn after contracting has taken place; this possibility of delegation mitigates the cost of excessive coordination under integration.

An important part of the analysis is that of the managerial market. The supply of managerial services is a function of the compensation managers can expect to receive in the market: if this compensation is high, they will also be more willing to expand their control to more enterprises, something akin to horizontal mergers, that will eventually define the horizontal scale of the integrated firm, and we suppose that top managers face a cost that is increasing in the horizontal scale. The demand for managerial services is a function of the surplus gain from integration, something that we show to be an inverted U. When the price is low or is high, there is little gain from integration: in the first case because revenues are small anyway, and in the second case because non-integration performs well in terms of output.

Within this model, we can address some key questions concerning the interaction between the product market, the managerial market and the performance of the industry.

Decentralization vs. Non-Integration Because management by delegation is subject to incentive compatibility conditions (Aghion and Tirole, 1997; Baker et al., 1999), the level and the sharing of firm revenues may affect the desire for top managers to delegate authority to subordinates and the desire of asset holders to integrate. As we argued in Legros and Newman (2013), the price level is a crucial determinant of organizational decisions since it modifies the trade-off between coordination and private costs. As the price increases in our model, the suppliers have good incentives to coordinate and it is therefore less likely that the top manager will have a comparative advantage: this makes delegation more likely when prices are high. In short, the opportunity cost of delegation decreases for a top manager since there is more coordination if the suppliers are delegated the right to make decision rights.

At the same time, we show that the gain from integration is an inverted U-shaped function of the price of output. In a short-run situation where the supply of top managers is fixed, as the price is small or is large, the payoff that suppliers are willing to give the top manager in order to integrate is low, implying a small scale

of integrated firms, and a low measure of integrated firms: there is heterogeneity in organizational form. As the price is intermediate, the equilibrium payoff is constant in order to satisfy the managerial market clearing condition. Hence in the short-run, there is an inverted U-relationship between the price and the horizontal scale of integration.

Since there is always more delegation as the price of output increases, the degrees of integration and delegation are positively related when prices are low and negatively related when prices are high. This highlights the significant difference between non-integration and delegation (or decentralization), or between outsourcing and delegation.

Is there too little or too much delegation? From the point of view of industry supply, there is the efficient level of delegation: the top manager compares his comparative advantage at coordinating activity to that of subordinates and therefore chooses optimally to delegate only if subordinates are better at coordinating and generate output. However, when we take into account the private costs of subordinates, we show that there can be too much centralization or too much delegation depending on the level of price. When price is low, the subordinates have low private cost when they decide, and value more delegation than the top manager but when output is high, the private costs are high and the subordinates value more the commitment benefit of centralization than the top manager.

Is there the right supply of managers? In particular, could output be increased by increasing or decreasing the supply of managers? In the short run, when the supply of managers is fixed, there is often too much supply of managerial services, that is the industry output would increase if the measure of managers decreases. In the long run when individuals can choose to become top managers or entrepreneurs, the equilibrium supply of top managers may be too low or too high depending, in particular, on the magnitude of the cost of extending horizontally the scale of integration. When this cost is high, like in developing countries, there is too little managerial supply. When it is low, like in developed countries, there is too much entry into the managerial activity; this also implies that integrated firms tend to be too small.

What is the Role of Finance? In the model, the key role of integration is to facilitate coordination among suppliers. If individuals have large cash holdings and no limited liability, they can solve their coordination problem through contracting,

and will not integrate.[1] Limited cash holdings is therefore a key determinant of integration, and scale of firms. Assuming that the suppliers on the long side of the market have no cash holding, the degree of integration in the industry will be an increasing function of the cash holdings of suppliers on the short side of the market, and integrated firms will also be less centralized. If there is borrowing to finance new capital, firms that are more leveraged will be more centralized: repayment is akin to a decrease in the price of output and makes coordination by subordinates less likely.

Links to the Literature Our model borrows from the literatures on firm boundaries and internal organization. These literatures have a lot in common but have evolved in parallel; papers focusing on firm boundary analyze the determinants and economic effects of integration versus non-integration choices by asset holders, while papers on internal organization focus on the organization of communication flows and the optimal mix of centralization and decentralization. Both non-integration and decentralization decisions tradeoff the benefit of having other parties, more informed or more able, in making decisions, versus the cost due to the loss of control when there is a conflict of interest among the parties.[2]

However, non-integration and decentralization decisions are quite different since in the former case owners and decision makers coincide, which is not the case in the case of decentralization (or delegation). As Aghion and Tirole (1997), Baker et al. (1999) have articulated, decisions that are delegated within a firm are subject to an incentive problem on the part of the owner, or the person who has authority, since he can overturn decisions, something that does not happen under non-integration.[3]

In our model, this incentive problem shapes the way integration contracts are designed, and in particular who should have authority within the integrated firm. We show that it is crucial for the manager to have authority in equilibrium; if this is not the case the initial asset holders cannot commit to coordinate and may as well retain ownership and not bring in a manager and integrate. Hence, while it may be reasonable to look at the internal organization problem when one, or both, initial asset holder(s) have authority, these situations are inconsistent with the fact that there would be integration to begin with. This illustrates the benefit of looking

[1] This is consistent with the view that managers or small entrepreneurs want a quiet life Bertrand and Mullainathan (2003) or more recently Hurst et al. (2011) who show by using a panel data of entrepreneurs, that most small businesses have little desire to grow.

[2] The literature is too large to survey here; see Aghion et al. (2014); Bolton et al. (2011); Dessein (2014); Legros and Newman (2014) for recent surveys on firm boundaries, internal organization, authority and the interplay between organizations and markets.

[3] See also Harris and Raviv (2010) who apply these ideas in a corporate governance context with asymmetry of information and moral hazard.

jointly at the determination of firm boundary and internal organization. Another illustration is provided by our result that while delegation is always increased when the price increases, integration is non-monotonic in the price level. As far as we know the first paper to analyze simultaneously the problem of firm boundary and delegation is Hart et al. (2010), but they do so by looking at two parties while we are also interested in the market determinants of firm boundaries and delegation decisions.

Our paper is also part of the literature on the market determinants of contractual or organizational forms.[4] We bring to this literature the idea that the characteristics of the managerial market and the joint determination of integration, and delegation decisions, are important determinants of the equilibrium scale of firms, and of its efficiency.

The rest of the paper is organized as follows. We first present the partial equilibrium model of integration and delegation, taking the price of the product and of the top manager as given. We then embed this model into a market equilibrium and investigate the determinants of integration and delegation, both in the short run and in the long run, and develop along the way the answers to the three questions we have sketched above.

2 A Model of Integration and Delegation

2.1 Basics

An *enterprise* is composed of assets U, D, and individuals, denoted also U, D, associated with these assets and specialized in making decisions on these assets. Production requires decisions u, d to be made on each asset. When the decisions are made, there are private costs d^2 and $(1-u)^2$ imposed on individuals working on assets U, D respectively.

If assets are owned by the individuals who work on them, we will call this non-integration, they choose u, d in a non-coordinated fashion, and the probability of obtaining an output of 2 is $\frac{1}{2}(1-(u-d)^2)$. This probability is bounded above by $1/2$, which is attained when $u = d$, that is when the two decisions coincide.

Alternatively, the assets could be purchased by a third party and the authority to decide given to a top manager M. This top manager has for unique role to coordinate activity in the firm but is often an outsider or at least less expert than U or D to

[4] See Antras (2014); Legros and Newman (2014), for surveys in international trade and industrial organization.

decide on the use the assets. We model these aspects by assuming that the top manager has ability μ to coordinate activities but that this ability is random, with continuous density f and cumulative F on $[0, 1]$, and the realization of μ happens after contracting and is private information to the top manager.[5]

If the top manager decides on u, d the probability of success is $\frac{\mu}{2}(1 - (u - d)^2)$. The top manager can choose to centralize decision making, in which case he will be making decisions u, d, or to decentralize, delegate, the decisions to U, D.

To summarize, defining:

$$y(u, d) \equiv 1 - (u - d)^2, \tag{1}$$

if decisions are u, d and the manager has ability μ, the expected output is:

$$\begin{cases} y(u, d) & \text{if } U, D \text{ decide} \\ \mu \times y(u, d) & \text{if } M \text{ decides.} \end{cases} \tag{2}$$

Enterprises behave competitively, facing a market price P for their product and having to offer an expected payoff of at least h to attract a top manager.

Contracts

Only output is contractible; a contract stipulates in addition to the asset ownership, the share of output that each party will obtain. Following standard arguments, it is sufficient to define the shares of revenue when output is equal to 2 as well as lump sum transfers at the time of contracting.

If there is non-integration, the transfers $\mathbf{t}$ and shares $\mathbf{s}$ involve U, D only, and when there is integration they involve U, D, M.[6] The sequence of events evolves as in figure 1. The grayed area represents the contracting stage; only M observes the realization of μ, and when there is integration u, d is either chosen non-cooperatively by U, D when there is delegation or by M when there is centralization

In terms of asset ownership,the other possibilities are that D or that U owns both U, D assets, but these situations are dominated by non-integration or integration.

[5] We comment at the end on the general support case at the end of the paper.

[6] It is standard to show that it is not worth borrowing for making lump sum transfers; a change in the share of output being a better instrument for transferring surplus than costly borrowing.

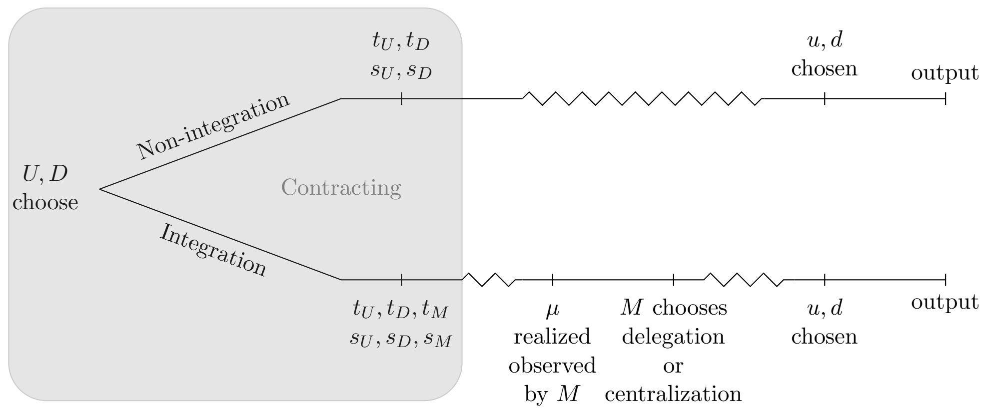

Figure 1: Timeline

Endowments and Outside Options

U has a zero outside option, and D has all the bargaining power. M, D have large cash holdings, while D has no cash holdings. Anticipating on the industry equilibrium, since the Us are in excess supply, they will have a zero expected payoff. We will make this assumption below to avoid useless generality.

2.2 Non-Integration

Under non-integration, U, D decide non-cooperatively on u, d. In this case, the two parties play a non-cooperative game and it is immediate to show (see Legros and Newman, 2013) that the expected output in the enterprise depends only on the total revenue accruing to U, D when output is equal to 2, but not on the way this revenue is allocated.

Precisely, under integration, U, D share the total revenue in case of success of $2P$, and it is convenient to write their shares as $s_U = 1 - \alpha, s_D = \alpha$. The equilibrium decisions are $d = \frac{\alpha P}{1+P}$ and $u = \frac{1+\alpha P}{1+P}$, implying that the expected output is equal to:

$$Q(P) \equiv 1 - \frac{1}{(1+P)^2}, \tag{3}$$

which is indeed independent of α. However, the equilibrium payoffs depend on the way revenue is allocated. D's expected payoff under non-integration, is given by the function:

$$\pi(P, \alpha) = \alpha Q(P)P - \alpha^2 C(Q(P)), \ \ i = U, D, \tag{4}$$

where the cost function C is:

$$C(Q) \equiv \left(1 - \sqrt{1-Q}\right)^2.$$

Since U's expected payoff is $\pi(P, 1-\alpha)$, and $\pi(P,\alpha)$ is concave in α, the sum of payoffs is maximum when $\alpha = 1/2$.

When the Us have a zero payoff, the optimal non-integration contract specifies $\alpha = 0$, in which case the expected payoff to D is equal to:

$$V^N(P) = Q(P)P - C(Q(P)). \tag{5}$$

Since D is effectively the unique decision maker ($u = 1$ for any P since U has a zero share of output), the envelop theorem implies that the marginal value of non-integration is equal to the expected output:

$$V_P^N(P) = Q(P). \tag{6}$$

2.3 Integration

With integration, M decides on centralization or decentralization after having observed μ. It is convenient to write $s_M = 1 - s$, and $s_U = (1-\alpha)s, s_D = \alpha s$.

M must have ownership under integration While M can decide on centralization or decentralization, if he does not have authority he must convince U, D to follow his instructions, a form of leadership. By contrast, with authority, he will be able to implement his preferred decisions.

We show that when $\mu \in [0,1]$, integration is beneficial to U, D, only if M has authority. Indeed, suppose that M does not have authority, a contract $(\mathbf{t}, s, \alpha)$ is chosen. M decides to centralize when $\mu \in \mathscr{C}$ by choosing decisions u^*, d^*. Since U, D have authority they will be able to choose individually which decision to take, and therefore U will choose his best response to d^*, and D his best response to u^*. Since they make the decisions the expected output is given by $y(u,d)$ instead of $\int_{\mu \in \mathscr{C}} \mu \frac{dF(\mu)}{F(\mathscr{C})} y(u^*, d^*)$. Since $\int_{\mu \in \mathscr{C}} \mu \frac{dF(\mu)}{F(\mathscr{C})} < 1$, the best response of U to d^* leads to a larger payoff than what U obtains by not overturning the decision. If there is an equilibrium it must therefore be the case than when there is centralization, U, D overturn and because they have a revenue in case of success of sP, the expected output is $Q(sP)$ in each state, implying that there is no benefit to integration for U, D.

By contrast if M has authority and decides to delegate when $\mu \leq Q(sP)$, he will not want to overturn U, D's decisions when there is delegation since $\mu \leq Q(sP)$. Therefore when μ has support on $[0,1]$, giving authority to M is the only way for U, D to commit to an effective centralization of decisions and benefit from integration.

As we show in the Appendix, this result holds for any distribution F with any support, hence even if the expectation of μ is greater than 1. We also show there that other ownership structures, like D owning both assets, or M owning U asset, are dominated by M ownership.[7]

Proposition 1. *If there is integration, M has ownership of the U, D assets.*

The optimal contract under integration. Since M has authority on decisions, if he chooses centralization, he will choose to perfectly coordinate the decisions $u = d$, generating an expected output of μ. Because the top manager is indifferent among all these coordinated decisions, we assume that he chooses the cost minimizing decision $u = d = 1/2$.[8]

If M decides to delegate, he anticipates that U, D will generate an expected output $Q(sP)$ since U, D have the same incentives as in the non-integration case when they face a revenue of sP and have shares $\alpha, 1-\alpha$. Therefore as long as $1-s$ is positive, M chooses to centralize whenever $\mu \geq Q(sP)$.

Optimal ex-ante Contracting

Assuming that U have a zero outside option and no cash, the maximum surplus a D asset holder can have under non-integration is obtained when $s_D = 1$:

$$\begin{aligned}\pi(P,1) &= Q^N(P)P - C(Q^N(P)) \\ &= \frac{P^2}{1+P} \\ &= V^N(P).\end{aligned}$$

For integration, the optimal contracting problem reduces to choosing (s, α) and transfers t_U, t_D to solve:

[7] If the conditional expected mean of μ when μ is greater than $Q(P)$ is large enough, it may be the case that U, D retaining authority leads to an outcome at most equal to that under M ownership. Under centralization, instructions by M are obeyed because he has a large expected comparative advantage.

[8] See Appendix A for an extension of the model that allows for biased manager.

$$\max_{s,\alpha,t_U,t_D} \int_{Q(sP)}^{1} \left(\mu\alpha sP - \frac{1}{4}\right) dF(\mu) + F(Q(sP))\pi(sP,\alpha) + t_D$$

$$\text{s.t.} \int_{Q(sP)}^{1} \left(\mu(1-\alpha)sP - \frac{1}{4}\right) dF(\mu) + F(Q(sP))\pi(sP,1-\alpha) + t_U \geq 0 \quad (7)$$

$$\left(\int_{Q(sP)}^{1} \mu dF(\mu) + F(Q(sP))Q(sP)\right)(1-s)P - t_U - t_D \geq h \quad (8)$$

$$t_U \geq 0. \quad (9)$$

The first constraint (7) is the participation constraint of U, the second constraint (8) is the participation constraint of M. The objective function is increasing in s and in α. In (7), setting $\alpha = 1$ requires $t_U = \frac{1-F(Q(sP))}{4}$. In (8), the constraint binds, and therefore the maximum payoff to U when $\alpha = 1$ is

$$\int_{Q(sP)}^{1} \left(\mu P - \frac{1}{2}\right) dF(\mu) + F(Q(sP))\left(Q(sP)P - C(Q(sP))\right) - h,$$

which is increasing in s. It is therefore optimal to set s as close to 1 as possible, and to have $t_D = -h - t_U = -h - \frac{1-F(Q(sP))}{4}$. This assumes that D has enough cash to make this ex-ante transfer, and we will assume this for now. An interpretation of the optimal integration contract is that D contributes to the purchase of U's asset, but foregoes ownership in order to give authority to the top manager (CEO).

The case $s = 1$ is a knife edge case since M is indifferent between centralization and decentralization; however because his preferences are strict as long as $1 - s$ is positive, we can consider the case $s = 1$ as a limit case when the cash holding of D becomes large enough to make a transfer equal to $h + \frac{1-F(Q(P))}{4}$ to M.[9]

Proposition 2. *Suppose that D has large cash holdings, then the surplus under integration is:*

$$V^I(P,h) = \int_{Q(P)}^{1} \left(\mu P - \frac{1}{2}\right) dF(\mu) + F(Q(P))V^N(P) - h,$$

Integration leads to a lottery where with probability $F(Q(P))$ there will be delegation and the expected payoff in this case is the same as under non-integration;

[9] In Legros and Newman (2013), the value of s did not affect the total surplus since there was always centralization. In the current paper, a higher share to M makes decentralization less likely and decreases the total surplus from integration. In order to have a strictly positive share, one could assume that a top manager has to exert a small effort to make decisions, however this would complicate the analysis without changing the main qualitative results.

with the complementary probability there will be full coordination by M. A necessary condition for integration to be beneficial is therefore that the expected value of coordination when $\mu > Q(P)$ is greater than $V^N(P)$.

At $\mu = Q(P)$, the difference between the expected payoff under centralization and decentralization is equal to $C(Q(P)) - \frac{1}{2}$, which is positive only if $P > \sqrt{2} + 1$. Hence, when $P < \sqrt{2} + 1$, there are states leading to centralization where U, D end up worse off with coordination than if they had the authority to choose the decision. In this sense, there is too much centralization.

If $P > \sqrt{2}+1$, it is always the case that coordination improves on decentralization, but there is too little centralization from the point of view of U, D who would like to have M coordinate for values of μ smaller than $Q(P)$.

Having too much centralization at low prices and too little centralisation at high prices is reflected in the comparison between the marginal value of integration and the expected output under integration. Indeed, a corollary of Proposition 2 is that at low prices, the marginal value of integration is larger than the expected output, because when P increases, U, D avoid more often inefficient centralization. The opposite is true for large prices where price increases make it more likely that there is not sufficient centralization.

Corollary 1. *If $P < \sqrt{2} + 1$, the marginal value of integration with respect to price $V_P^I(P, h)$ is greater than the expected output, and if $P > \sqrt{2} + 1$ the marginal value of integration with respect to price is lower than the expected output.*

Proof. By differentiating the Lagrangian with respect to P (details are in the Appendix), the marginal payoff from integration as P varies is equal to:

$$V_P^I(P, h) = \int_{Q(P)}^1 \mu dF(\mu) + F(Q(P))Q(P) + Q'(P)f(Q(P)) \left(\frac{1}{2} - C(Q(P))\right).$$

The result follows since the last term on the right-hand sum is non-negative only if $P \leq \sqrt{2} + 1$. □

Comparing Integration and Non-Integration

A necessary condition for integration to be chosen is that it is chosen when $h = 0$, that is when

$$\Delta(P) \equiv \int_{Q(P)}^1 \left(\mu P - \frac{1}{2} - V^N(P)\right) dF(\mu) \tag{10}$$

is non-negative. Since $P - \frac{1}{2} = V^N(P)$ at $P = 1$, if $P \leq 1$, $\Delta(P) \leq 0$, and integration will not be chosen. If $P > \sqrt{2}+1$, $\Delta(P)$ is positive, and therefore there exists $P \in (1, \sqrt{2}+1)$ for which $\Delta(P) = 0$; we show that such a value is unique.

Lemma 1. *There exists a unique price $P_0 \in (1, \sqrt{2}+1)$ such that $\Delta(P)$ is positive if, and only if, the price is larger than P_0.*

Proof. Let $\delta(\mu, P) \equiv \mu P - \frac{1}{2} - V^N(P)$. From the arguments in the text, for all $P \geq \sqrt{2}+1$, $\delta(\mu, P) > 0$ and we necessarily have $\Delta(P) > 0$ for $P_0 > \sqrt{2}+1$. Now, for $P \in (1, \sqrt{2}+1)$, the sign of $\delta(\mu, P)$ is "increasing" in P since by (6) $\delta_P(\mu, P) = \mu - Q(P)$ is positive for $\mu \geq Q(P)$; therefore the sign of $\Delta(P)$ is increasing in P. The result follows. □

While $\Delta(P)$ is positive for $P > P_0$, it converges to zero as $P \to \infty$. Therefore, when the price of managers is positive, there are at least two price levels such that $\Delta(P) = h$. To simplify the exposition, we restrict ourselves to distribution functions such that $\Delta(P)$ is quasi-concave. For instance, power distributions $F(\mu) = \mu^a$ where $a \geq 1$ have this property.

Assumption 1. *$\Delta(P)$ is quasi-concave in P.*

Figure 2 and the following proposition summarize our discussion.

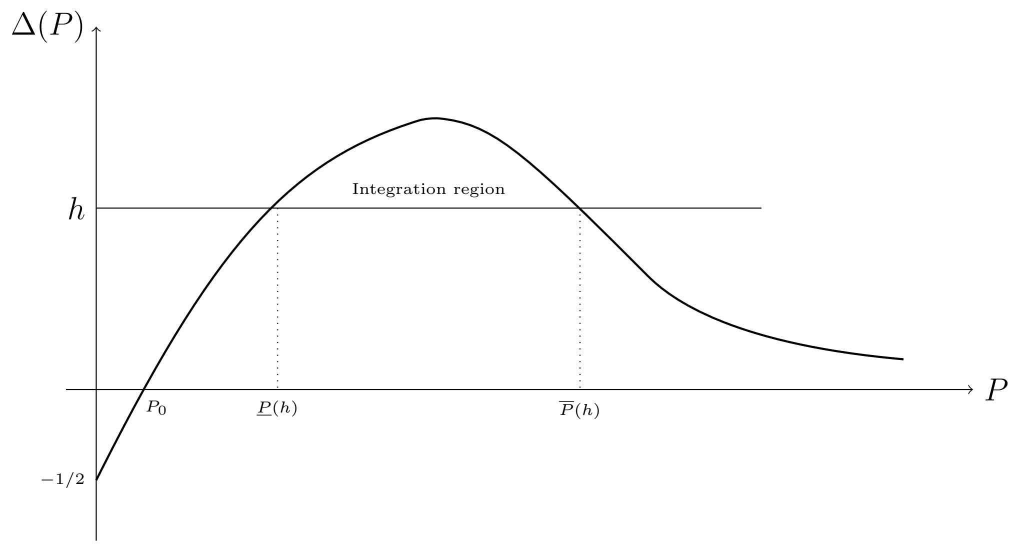

Figure 2: Relative value of integration

Proposition 3. *(i) If $h > \max_P \Delta(P)$, there is no integration.*

(ii) *If* $h \leq \max_P \Delta(P)$*, there exist finite prices* $\underline{P}(h), \overline{P}(h)$*, with* $\underline{P}(h) > 1$*, and* $\overline{P}(h) \geq \max(\underline{P}(h), \sqrt{2}+1)$ *such that there is integration if, and only if,* $P \in [\underline{P}(h), \overline{P}(h)]$*, .*

(iii) $\underline{P}(h)$ *is an increasing function of* h *and* $\overline{P}(h)$ *is a decreasing function of* h*, these two bounds coincide at the price maximizing* $\Delta(P)$*, and* $\underline{P}(0) = P_0$*,* $\overline{P}(0) = \infty$*.*

First-Order Shifts in F. If top managers are of better quality, in the sense that $F(\mu)$ shifts in a first-order stochastic sense, one would expect that integration will be favored more often. While correct, this conclusion is not immediate because when P belongs to $[1, \sqrt{2}+1]$, there is too much centralization from U, D's perspective, something that will be reinforced when F shifts in a first order stochastic way.

Lemma 2. *Suppose that* $\Delta(P)$ *is positive for some distribution* F*. Then as* F *increases in the first-order,* $\Delta(P)$ *increases.*

Proof. We can write

$$\Delta(P) = (1 - F(Q(P))) \left[\int_{Q(P)}^{1} (\mu P - 1/2) \frac{dF(\mu)}{1 - F(Q)} - V^N(P) \right]$$

If $\hat{F}$ dominates F in the first order, then the conditional $\frac{\hat{F}(\mu)}{1-\hat{F}(Q(P))}$ dominates $\frac{F(\mu)}{1-F(Q(P))}$ on $\mu \in [Q(P), 1]$. It follows that $\int_{Q(P)}^{1} (\mu P - 1/2) \frac{dF(\mu)}{1-F(Q)}$ increases with F. Because $1 - F(Q)$ increases with F, the result follows since the overall expression increases when the term in brackets is positive. □

While of interest, this result does not immediately imply that industries where managers are more able, in the sense that F increases in the first-order, will have more integration. Indeed, the expected output of integrated firms increases with F, and therefore we should expect a decrease in the price, a force that will tend to favor non-integration.

Exogenous Productivity.

If firms have different productivity levels θ and if the price in the industry is P, Proposition 3 implies that firms with $\theta \in \left[\frac{\underline{P}(h)}{P}, \frac{\overline{P}(h)}{P}\right]$ will be integrated, and among these firms, the higher θ is the less centralization there will be since the probability of delegation is $F(Q(\theta P)\theta)$, increasing in θ.

Corollary 2. *Among integrated firms, more productive firms are less centralized.*

The Role of Cash Endowments Let us continue to assume that U has no cash holding, but suppose now that D has limited cash holding $L_D < h$.[10]

Under integration, D benefits from transferring cash ex-ante to M: this has the effect of softening the constraint (8) and therefore to increase the value of s that would bind (8). Hence, the optimal integration problem is similar to a problem where U, D have no cash holdings and the right hand side of (8) is replaced by $h - L_D$, and where there is the addition constraint that $t_D \geq 0$. In this class of problems, the transfer constraints bind, $t_U = t_D = 0$ since for any transfer that M would make to U, D, the share s will have to decrease, implying a decrease in the total surplus. There is a well defined maximum share $\overline{s}(h, P)$ that binds (8), and given this share, a well defined α binding (7). It is immediate to show that these shares are decreasing in $h - L_D$, hence increasing in L_D.

Contrary to the case of large cash holdings, where $V^I(P, h) = V^I(P, 0) - h$, limited cash holdings of D imply that $V^I(P, h) < V^I(P, 0) - h$: transferring one extra utility payoff to M requires a decrease in s or α and an extra loss of surplus for D. For the same reason, an increase in D's cash holding has a multiplier effect on the surplus from integration. Since D's cash holding plays no role under non-integration, integration is more likely as L_D increases.

Corollary 3. *Suppose that U has no cash holding. As the cash endowment of D increases, the integration contract specifies a larger share s and a larger relative share α to D. This is a force towards integration and delegation.*

Outside Financing If integration involves a capital cost K that has to be financed from an outside lender, the integration contract must satisfy an additional repayment constraint to the outside lender. This will lead to a decrease in the share s going to U, D and therefore like in Jensen (1986) a cost of capital greater than K. As K increases, the share s decreases and therefore the probability of delegation $F(Q(sP))$ will decrease.

Corollary 4. *More leveraged integrated firms are more centralized.*

[10] The cash holding of U plays a role both under non-integration and under integration, while the cash holdings of D play a role only under integration. Under non-integration, U can transfer some cash to D in exchange for a bigger share of output. In the limit, when U has large cash holdings, the two parties can minimize their cost of production by using an equal share of output, and D's total surplus under non-integration is then equal to $Q(P)P - \frac{1}{2}C(Q(P))$. Since the maximum surplus under integration and centralization, $P - 1/2$, is always inferior to $Q(P)P - \frac{1}{2}C(Q(P))$, there will not be integration in the industry.

3 Industry Equilibrium

The industry equilibrium will be affected by the nature of the managerial market in the given industry, in particular on whether enterprises face an elastic or inelastic supply of managerial services. We will consider two situations. In the short-run, m is exogenous but h is endogenous; in the long-run, both m, h are endogenous.

To make the comparison between the short and long run easier, we will assume that the total measure of Ds and Ms is equal to 1. In the short-run, if there is a measure m of Ms, there is a measure $1-m$ of Us, and in the long run m is endogenously determined. We will focus on the case where $m < 1/2$, which is a property of the long run equilibrium.

We introduce the possibility of horizontal scale for firms, that is the possibility of a firm to be consituted of multiple U, D enterprises, all under the same authority. hence, enterprises consist of a pair U, D but a top manager can have authority on n entreprises. We assume that there is a limit to horizontal integration, and we model this as a private cost $\frac{\gamma}{2}n^2$ born by the top manager when he expands the scale of his authority. We interpret γ as a proxy for the difficulties an entreprise faces when it merges horizontally, e.g., because of antitrust oversight, financial constraints, or increased difficulty to exercice authority among different entreprises.

Hence, if a manager has price h per entreprise, he will control

$$n(h) = \frac{h}{\gamma} \tag{11}$$

enterprises.

The output under non-integration is equal to $Q(P)$; we will denote the output under integration by $Y(P)$:

$$Y(P) \equiv Q(P) + \int_{Q(P)}^{1} (\mu - Q(P))dF(\mu), \tag{12}$$

It is clear that $Y(P) > Q(P)$ for each value of P.

3.1 Short-Run

We first consider a short-run situation where the measure of top managers is exogenously given, and we assume that there is a measure m strictly less than $1/2$ of top managers and a measure $1-m$ of D asset holders.

A *short-run equilibrium* is defined by prices (P, h) and organizational (integration

or non-integration together with shares of output) decisions for each enterprise such that:

- For each enterprise, U, D are better off choosing the equilibrium organization.
- Managerial market clearing: three is no excess supply or demand for top managers.
- Product market clearing: total industry supply equals demand.

There is also a supplier market clearing condition that there is no excess demand or supply of Us, but this reduces to the condition that the Us get a zero equilibrium payoff.

From Proposition 3, if the top manager have a payoff of h, there is a demand for managerial services and integration only if P belongs to the interval $[\underline{P}(h), \overline{P}(h)]$.

As $P < P_0$, there is no demand for integration and managerial services since the net payoff under integration is less than that under non-integration for any value of h. For these prices, firms are non-integrated and total industry output is $(1-m)Q(P)$.

As $P > P_0$, there is a demand for integration and managerial services as long as $h \leq \Delta(P)$. Since the total supply of managerial services at price h is equal to mh/γ, and since $h \leq \Delta(P)$, we can have two configurations, depending on P.

- If $m\Delta(P)/\gamma > 1-m$, there is excess supply of managerial services, and the managerial payoff must decrease up to the point where $mh = (1-m)\gamma$. In this case, all enterprises are integrated, the managerial payoff clears the managerial market, that is $h = \frac{1-m}{m}\gamma$, and industry output is equal to $(1-m)Y(P)$. Note that $\Delta(P) > \frac{1-m}{m}\gamma$ whenever P belongs to the interval

$$\mathcal{P} \equiv [\underline{P}((1-m)\gamma/m), \overline{P}((1-m)\gamma/m)] \tag{13}$$

- Finally, if $m\Delta(P)/\gamma < 1-m$, there is excess demand for managerial services and it must be the case that the managerial payoff is maximum and equal to $\Delta(P)$. In this case, Us are indifferent between integration and non-integration since $\Delta(P) - h = 0$, and we have heterogeneity of organizational forms. The measure of integrated enterprises is equal to $m\Delta(P)/\gamma$ and the measure of non-integrated enterprises is equal to $1 - m - m\Delta(P)/\gamma$, implying that the total industry output is equal to $\frac{m\Delta(P)}{\gamma}Y(P) + (1-m-m\Delta(P)/\gamma)Q(P)$, which can be written as $(1-m)Q(P) + \frac{m\Delta(P)}{\gamma}\int_{Q(P)}^{1}(\mu - Q(P))dF(\mu)$.

It follows that the short-run OAS for the industry is:

$$S(P;m) = \begin{cases} (1-m)Q(P) & \text{if } P \leq P_0 \\ (1-m)\left[Q(P) + \int_{Q(P)}^{1}(\mu - Q(P))dF(\mu)\right] & \text{if } P \in \mathcal{P} \\ (1-m)Q(P) + \frac{m\Delta(P)}{\gamma}\int_{Q(P)}^{1}(\mu - Q(P))dF(\mu) & \text{if } P \notin \mathcal{P} \text{ and } P \geq P_0. \end{cases}$$

As Figure 3 illustrates, at high prices the marginal value of integration, as measured by $\Delta(P)$ decreases, hence when $\Delta(P) = h$ happens on the decreasing part of $\Delta(P)$, h has to decrease when P increases in order to keep the managerial market in equilibrium. This requires that $h = \Delta(P)$, is decreasing in P for high prices, and therefore that the scale of firms decreases, leaving some firms non-integrated. As P becomes large, $\Delta(P)$ becomes negligeable and most firms are non-integrated.

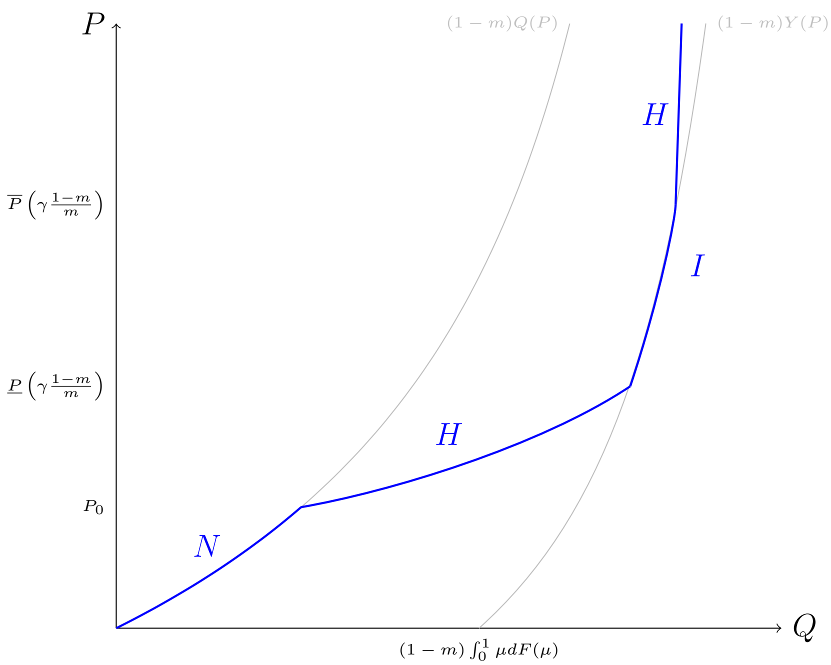

Figure 3: Short Run OAS

Along the H and I parts of the OAS, the measure of integrated firms is constant and equal to m The OAS binds backward at prices greater than $\overline{P}((1-m)\gamma/m)$ because as the price P increases, the willingness to pay for integration decreases and the benefit of integration decreases, implying that there is a larger proportion of non-integrated firms. The per-enterprise compensation of a top manager is equal to

$$\min\left\{\Delta(P), \frac{1-m}{m}\gamma\right\},$$

which is therefore non-monotonic in P. Since the horizontal scale of an integrated is proportional to the per-enterprise compensation, it is also non-monotonic in P.

Hence contrary to the case where h is fixed, the relationship between the degree of integration, that is the number of integrated enterprises $m\frac{h^*P}{\gamma}$, and the prevalence of delegation, measured by $F(Q(P))$, changes for low and high prices. At prices lower than $\overline{P}((1-m)m\gamma)$, as the price increases there is more integration and more delegation. For prices greater than $\overline{P}((1-m)m\gamma)$, price increases lead to less integration but still more delegation.

As m decreases, there are two effects at play. There is first a scale effect since as m decreases there are more enterprises in the industry, and the industry output levels under integration and non-integration increase. There is also an organizational effect since as m decreases the price region over which integration is strictly preferred by D shifts upward: both $\underline{P}((1-m)\gamma/m)$ and $\overline{P}((1-m)\gamma/m)$ increase. Hence, we should expect that, for high enough prices, decreasing the number of top managers is output enhancing; we show that this can also be the case for low prices.

This is indeed the case for $P \in \mathcal{P}$ since $\frac{\partial S(P;m)}{\partial m} = -Y(P)$. This is also the case for P large enough since when $P > \overline{P}((1-m)\gamma/m)$, the variation of $S(P;m)$ with respect to m is equal to $-Q(P) + \frac{\Delta(P)}{\gamma}\int_{Q(P)}^{1}(\mu - Q(P))dF(\mu)$, and since $\Delta(P)$ is a decreasing function of $P \geq \overline{P}((1-m)\gamma/m)$, there exists P large enough such that the variation is negative. Since at P_0, $\Delta(P_0) = 0$, the variation of $S(P_0;m)$ is equal to $-Q(P_0)$ and therefore decreasing the measure of top managers is also output improving.

Corollary 5. *For prices close to P_0, for high enough prices greater than $\overline{P}((1-m)\gamma/P)$ and prices in $\mathcal{P}$, the short run supply would increase if the measure of top managers decreases.*

This "too many top managers" result complements the usual view that there is a scarcity of managerial talent. This begs however the question of whether such excess supply of managerial talent would arise in the long-run, when individuals can choose their occupation anticipating the demand for integration by enterprises. We turn to this case now.

3.2 Long Run OAS

We model the long run by assuming that out of a measure 1, a proportion m of individuals decide to become managers and a measure $1-m$ decide to become entrepreneurs with asset holding D. There is still a measure larger than 1 of U

suppliers. A long-run equilibrium specifies the measure m together with the short-run equilibrium corresponding to m, as derived in the previous section.

When the managerial market is active, and a top manager has a compensation of h per enterprise he controls, his total payoff is $hn(h) - \gamma n(h)^2 = \frac{h^2}{2\gamma}$. By contrast the payoff of a D asset holder is $V^I(P, h)$ when there is integration; therefore if there are integrated firms, and therefore a market for top managers, it must be the case that the following occupational indifference condition holds:

$$V^I(P, h) = \frac{h^2}{2\gamma},$$

and for a given P there exists a unique $h^o(P)$ solving this quadratic equation:

$$h^O(P, \gamma) \equiv -\gamma + \sqrt{\gamma^2 + 2\gamma V^I(P, 0)}. \tag{14}$$

This function is equal to 0 when $V^I(P, 0) = 0$, something that must happen at $P < P_0$ since $V^I(P_0, 0) = V^N(P_0) > 0$, and is always smaller than $V^I(P, 0)$. $h^O(P)$ is the indifference occupational condition: individuals are indifferent between being top manager or a U entrepreneur when $h = h^O(P, \gamma)$.

If the D asset holders are indifferent between integration and non-integration, it must be the case that $h = \Delta(P)$, which is the indifference organizational condition for enterprises.

As γ increases, $h^O(P, \gamma)$ becomes steeper. When γ is large, the curve $h^O(P, \gamma)$ is above $\Delta(P)$. Hence there is no demand for management and in the long run $m = 0$ and total supply is equal to $Q(P)$. This situation characterizes developing countries where a variety of market imperfections prevent the increase in the scale of firms. In this case, subsidizing entry into management, or facilitating the increase in the scale of firms will lead to an improvement in the long run output. This is a case of too little management.

For low enough value of γ, $h^O(P, \gamma)$ will intersect $\Delta(P)$ at least twice, see figure 4. If γ is small enough, there will be only two prices at which the curves intersect, and we will focus on this case, illustrated in Figure 4.

We have three possible regimes:

- When the price is below P_L, there is either no demand for or no supply of managers and therefore all individuals become U-entrepreneurs and enterprises are not integrated.

- When the price belongs to the interval $[P_L, P_H]$, there is demand for integration

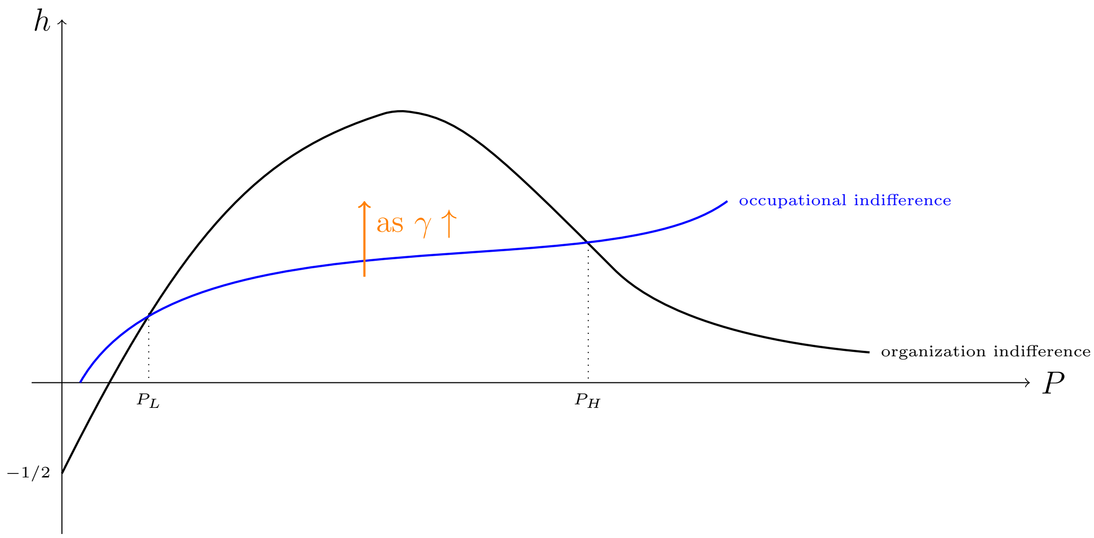

Figure 4: Occupational and Organizational Indifference Conditions

and all enterprises are integrated. The compensation per enterprise is equal to $h^O(P)$ and the measure of managers solves the managerial market clearing condition $1 - m = m\frac{h^O(P)}{\gamma}$, that is:

$$m = \frac{\gamma}{\gamma + h^O(P)},$$

which is the measure of integrated firms and is a decreasing function of P, each having an horizontal scale of $\frac{h^O(P)}{\gamma}$. Hence along the integration region, as P increases, there are fewer integrated firms, but each firm has a larger scale. The measure of integrated enterprises U, D increases since it is equal to $1 - m = \frac{h^O(P)}{\gamma + h^O(P)}$.

- Finally when P is greater than P_H, the managerial market is inactive, $m = 0$ and all firms are non-integrated.

It follows that the long-run OAS of the industry is:

$$S(P) = \begin{cases} Q(P) & \text{if } P \notin [P_L, P_H] \\ \frac{h^O(P,\gamma)}{\gamma + h^O(P,\gamma)} Y(P) & \text{if } P \in [P_L, P_H] \end{cases} \tag{15}$$

Depending on the parameters, in particular the distribution function F and the cost parameter of horizontal scale γ, two possibilities arise in the long run. Either the integration supply, equal to $\frac{h^O(P)}{\gamma + h^O(P)} Y(P)$, is greater than the non-integration supply

in the interval $[P_L, P_H]$, or the two curves intersect in this interval.

In the first case, the long run supply will be backward bending, since at price P_H, output decreases from $\frac{h^O(P,\gamma)}{\gamma+h^O(P,\gamma)}Y(P)$ to $Q(P)$.[11] In the second case, the long run supply will be monotonic increasing, as illustrated in Figure 5.

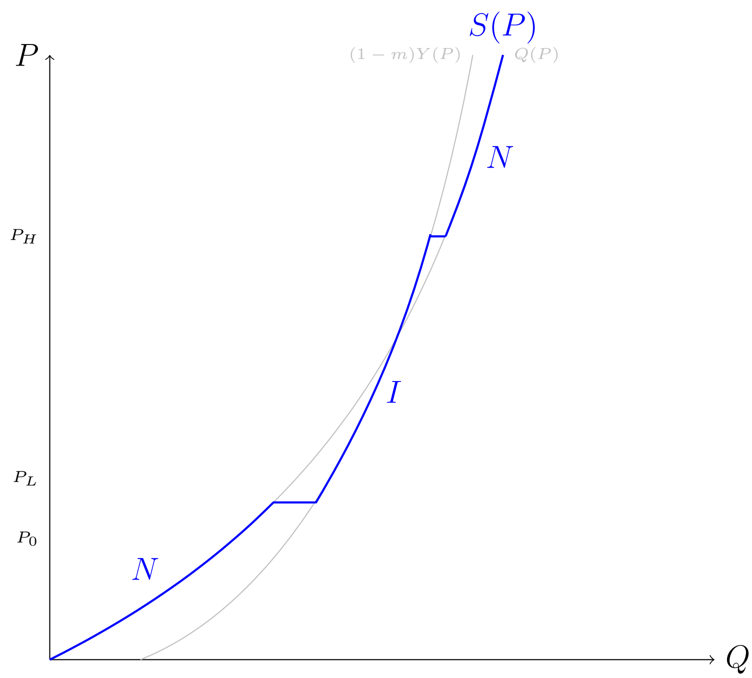

Figure 5: Long Run OAS

Is there too much or too little management in the long-run? If one is concerned with growth and focus on output, the answer is unambiguously that there is too much management since the supply under integration is a decreasing function of m. If m decreases to $\hat{m} < m$, the total demand for integration $1-\hat{m}$ is larger than the total supply of managerial services $\hat{m}\frac{h^O(P)}{\gamma}$, and there will be competitive pressure to increase h beyond $h^O(P)$, implying a per-entreprise compensation of $\hat{h} = \gamma\frac{1-\hat{m}}{\hat{m}}$. It is clear from Figure 4 that there always exists a small enough variation $m - \hat{m}$ such that $\hat{h} < \Delta(P)$, and therefore where limiting the entry into the managerial occupation is output increasing.

This excess entry into the managerial occupation is reminiscent of the excessive variety result in spatial competition models. As in Chamberlin's monopolistic com-

[11]This is the case for instance when $F(P) = P^4$, $\gamma = 3 \times 10^{-4}$. As far as we know there are few documented instances of backward bending supply curves. We provide in (Legros and Newman, 2014) an example constructed from a partial equilibrium in the spirit of (Schmidt, 1997).

petition type of models, firms tend to be too numerous, and their scale too small.

Proposition 4. *If there is integration in the long run equilibrium, there is excess entry into the managerial activity; restricting entry and increasing the compensation of managers will increase output.*

Costly Access to Management If there is a fixed cost E for becoming a manager, the occupational incentive compatible condition becomes $V^I(P, 0) = h + \frac{h^2}{2\gamma} - E$; the managerial compensation $h(P, E)$ is an increasing function of E. It follows that the occupational indifference curve in Figure 4 shift upward. The range of prices for which firms are integrated $[P_L(E), P_H(E)]$ decreases; the lower bound increases and the upper bound decreases At the same time, in the range where there is integration, since the managerial compensation increases, firms have a higher scale; but since the number of managers decreases, the measure of integrated firms increases. To summarize:

- As E increases, integration is less likely
- As E increases, when the managerial market is active, industry supply increases.

The second effect suggests that it may be output enhancing to increase E, but the first effect is a countervailing force. To illustrate the policy problem, consider an increase of the cost from E_0 to $E_1 > E_0$. Denote the industry supply under integration by $S(P, E) := \frac{h(P,E)}{h(P,E)+\gamma} Y(P)$, where $h(P, E)$ is the compensation that solves the occupational indifference condition $V^I(P, 0) = h + \frac{h^2}{2\gamma} - E$.

Let P_0 the long run equilibrium price when the cost is E_0. Integration is a long-run equilibrium outcome when $P \in [P_L(E), P_H(E)]$, and the interval is decreasing in E. In particular, $P_L(E_0) < P_L(E_1) < P_H(E_1) < P_H(E_0)$. Assume that the managerial market is active when the cost is E_0, that is $P_0 \in [P_L(E_0), P_H(E_0)]$. Under these conditions, we have:

Proposition 5. *The long run equilibrium price is lower with E_1 than with $E_0 < E_1$ if and only if $P_0 > P_L(E_1)$.*

Proof. Suppose that $P_0 \in (P_L(E_1), P_H(E_0)]$. By the argument in the text, and the definition of the supply curve in (15), $S(P, E_1) \geq S(P, E_0)$ for $P > P_L(E_1)$. The result follows.

The two supply curves cross at $P_L(E_1)$, and therefore if $P_1 \in [P_L(E_0), P_L(E_1)]$, $S(P, E_0) > S(P, E_1)$, and the equilibrium product market price increases when E increases. $\square$

4 Conclusion

A benefit of developing a single framework for analyzing firm boundaries and internal organization is to clarify the difference between outsourcing decisions and delegation decisions. In our framework, if integration is prevented, all enterprises outsource part of their supply, but the degree of delegation in integrated firms is a function of the level of price, since it correlates with the efficiency of delegation. Since the price level also determines the desire of asset holders to integrate, there may be a covariation between integration and delegation: more integrated firms, defined as having a larger horizontal scale $n(h)$, are also those in which the top manager delegates more. This effect is not due to the fact that it is more costly to centralize when the horizontal scale increases: the costs and benefit of delegation are independent of the horizontal scale and depend only of the price level in our model. The causality runs from the level of price to both the horizontal scale and the degree of delegation.

Another benefit of this integrated approach is to draw a distinction between environments where increasing the scale of firms is difficult and those where it is relatively easy. As we have argued in our long-run analysis, if we interpret γ as an index of the difficulty to increase the scale of integration, developing countries are likely to have a high γ and to be characterized by too little entry into management. By contrast when γ is low, policies that restrict entry into the managerial occupation are likely to generate a higher industry supply, and one may view these economies as being characterized by too much, rather than too little, top managerial talent.

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A Extensions

A.1 Horizontal Differentiation: a Rationale for the Cost of Horizontal Integration

Top managers have lexicographic preferences; they first maximize their revenue and then minimize the total cost that their decision imposes on stakeholders. There is a measure m strictly less than 1 of top managers.

The upstream segment has a measure greater than 1 of suppliers with common preference $(1-u)^2$ for decisions. The downstream segment has a measure 1 of producers with differentiated preferences; producer $i \in [0,1]$ has preference $(i-d)^2$ for decisions. We continue to assume that suppliers do not have access to finance and cannot make lump sum payments to producers at the time of contracting.

Because of excess supply of upstream suppliers, their equilibrium payoff will be equal to zero; to simplify we assume that this is the case when doing the contracting analysis in partial equilibrium.

Consider a vertical relationship between an upstream supplier and a downstream producer of type i. If they are non integrated, the supplier has a zero share of output, as in the text, and therefore the decisions are $u = 1$ producer chooses $d = \frac{P+i}{P+1}$, implying decisions

$$u = \frac{1+(1-s)P}{P+1} + \frac{isP}{P+1},\ d = \frac{(1-s)P}{P+1} + i\frac{1+sP}{P+1}$$

hence

$$u - d = \frac{1-i}{P+1}$$

and the expected output is

$$Q(P,i) := Q(P,0) + \frac{i(2-i)}{(P+1)^2}.$$

Hence the bigger i is the greater is the expected output.

Since $Q(P,i)$ is independent of s, the net payoff to the producer is maximum when $s = 0$. Hence,

$$V^N(P,i) := V^N(P,0) + i(2-i)\frac{P}{P+1}. \tag{16}$$

Let $\mathcal{I} \subset [0,1]$ the set of vertical chains that a top manager controls. Given a realization of μ, the manager delegates to chain $k \in \mathcal{I}$ whenever $Q(P,k) > \mu$ and does

not otherwise. Let $\mathcal{C}(\mu; I) := \{k \in \mathcal{I} : Q(P; k) \leq \mu\}$. If there is centralization, the manager chooses the decision $\delta(\mu, \mathcal{I})$ that minimizes the cost

$$\delta(, \mu, \mathcal{I}) := \arg\min_{\delta} \int_{k \in \mathcal{C}(\mu;\mathcal{I})} \left[(k-\delta)^2) + (1-\delta)^2\right] dk \tag{17}$$

Hence, from the point of view of the $k \in \mathcal{I}$ chain, the value of integration is

$$V^I(P, k; \mathcal{I}) := \int_{\mu \geq Q(P,k)} (P-(k-\delta(\mu,\mathcal{I}))^2-(1-\delta(\mu,\mathcal{I})^2) dF(\mathcal{C}(\mu;\mathcal{I}))+(1-F(\mu;\mathcal{I}))V^N(P,k). \tag{18}$$

From the point of view of an individual chain that is

A.2 Biased Managers

In the model, managers are indifferent among all decisions that maximize expected revenue. Suppose however that a manager can be biased in the following sense. He has lexicographic preferences, with first element the revenue, and with second element the average cost to U, D from decisions, where the weight on U's cost is β and the weight on D's cost is $1-\beta$. $\beta \in [0, 1]$ is realized after contracting takes place.

Hence when the manager makes a decision, he internalizes with probability β the cost of U and with probability $1-\beta$ the cost of D. Our basic model is similar to a situation where $\beta = 1/2$ for then when the top manager makes a decision, he chooses a unique decision $u = d$ in order to maximize revenues, but then chooses $u = d = 1/2$ in order to minimize $1/2(1-u)^2 + 1/2d^2$.

Decentralization choices are not affected by the bias. Indeed, the top manager makes this choice on the basis of the expected revenue that he will get. For the resulting equilibrium choice of decisions (u, d) under decentralization, the top manager does not want to overturn these decisions since by doing so he can get at most a revenue of μ.

Hence biases will change the value of integration for U, D since it will modify the expected cost under centralization.

The decision under centralization is to minimize the total cost, that is $d^*(\beta) := \arg\min \beta(1-d)^2 + (1-\beta)d^2$, that is $d^*(\beta) = \beta$. Therefore the total expected cost under bias is

$$\int_0^1 [(1-\beta)^2 + \beta^2] dG(\beta). \tag{19}$$

which is greater than $1/2$ for any non trivial distribution G. Hence, biased managers decrease the likelihood of integration with respect to our basic model.

B Proofs

Proof of Corollary 2 (i)

Let us denote the Lagrange coefficients of the constraints (7), (8) and (9) by λ, ϕ, ξ, the functions on the right hand sides of (7) and (8) by $v_U(s, \alpha, t_U, P)$ and $v_M(s, \alpha, t_U, t_D, P)$, the objective function by $v_D(s, \alpha, t_D, P)$. Then the Lagrangian is:

$$\mathcal{L}(s, \alpha, \mathbf{t}, P) = v_D(s, \alpha, t_D, P) + \lambda v_U(s, \alpha, t_U, P) + \phi v_M(s, \alpha, t_U, t_D, P) + \xi t_U. \quad (20)$$

It follows that:

$$\begin{aligned} \frac{d\mathcal{L}}{dP} &= \frac{\partial \alpha}{\partial P}\mathcal{L}_\alpha + \frac{\partial s}{\partial P}\mathcal{L}_s + \frac{\partial t_U}{\partial P}\mathcal{L}_{t_U} + \frac{\partial t_D}{\partial P}\mathcal{L}_{t_D} \\ &\qquad + \frac{\partial \lambda}{\partial P}\mathcal{L}_\lambda + \frac{\partial \phi}{\partial P}\mathcal{L}_\phi + \frac{\partial \xi}{\partial P}\mathcal{L}_\xi \\ &= \mathcal{L}_P. \end{aligned}$$

The third equality is due to the fact that α, s are constant and therefore have zero variation, that $\mathcal{L}_{t_U} = \mathcal{L}_{t_D} = 0$ by optimality and interiority of t_U, t_D, and that $\mathcal{L}_\lambda = \mathcal{L}_\phi = 0$ since the two constraints bind. Finally, since $t_U > 0$, $\xi = 0$. Note that $\mathcal{L}_{t_D} = 1 - \phi$ and therefore $\phi = 1$. Now, $\mathcal{L}_{t_U} = \lambda - \phi - \xi$, and since $\xi = 0$ it follows that $\lambda = \gamma = 1$. Now,

$$\mathcal{L}_P = \frac{\partial v_D(s, \alpha, t_D, P)}{\partial P} + \lambda \frac{\partial v_U(s, \alpha, t_U, P)}{\partial P} + \frac{\partial v_M(s, \alpha, t_U, t_D, P)}{\partial P}$$

Hence,

$$\begin{aligned} V_P^I(P, h) &= \frac{d}{dP}\left[\int_{Q(P)}^1 (\mu P - \frac{1}{2})\right] dF(\mu) + F(Q(P)V^N(P) \\ &= Q(P) + \int_{Q(P)}^1 (\mu - Q(P)) dF(\mu) \end{aligned}$$

as claimed since $\frac{dV^N(P)}{dP} = Q(P)$.

Proof of Proposition 1

Consider any distribution $F(\mu)$, and assume that the upper bound of the support is greater than 1. To simplify, let us continue to assume that U has no cash endowment and D has large cash endowments.

U, D **retain ownership** Let us suppose that U, D retain ownership and that M does not have authority. To simplify, let us focus on the case where $\alpha = s = 1$. In this case, U optimally will choose $u = 1$, and therefore no other decision can be imposed on him, and We can effectively restrict attention to the game between M and D.

Since the payoff to M is increasing in μ, whenever he chooses to centralize at a realization μ, he will also choose to centralize for larger values. Hence his strategy to centralize or delegate is reduced to choosing a cutoff value μ_0. If there is delegation, the optimal decision of D maximizes $(1-d^2)P - d^2$, leading to a payoff of $V^N(P)$. If there is centralization for states greater than μ_0, D believes that the state is greater than this cutoff and therefore D's expected payoff from following the instructions of M is equal to:

$$\int_{\mu \geq \mu_0} \left(\mu(1-d_0^2)P - d_0^2\right) \frac{dF(\mu)}{1-F(\mu_0)}. \tag{21}$$

By disobeying D's instruction and implementing his preferred decision, D loses the benefit of μ but may have lower cost. Hence, the best decision from M's point of view that will be followed by D is to choose d_0 in such a way that D is indifferent between obeying and disobeying, hence, (21) must be equal to $V^N(P)$.[12]

Now, anticipating this, M chooses μ_0, that is when to delegate, in order to maximize $\int_{\mu \geq \mu_0} \mu dF(\mu) + F(\mu_0)Q(P)$, that is:

$$\mu_0(1-d_0^2) = Q(P), \tag{22}$$

implying that $\mu_0 > Q(P)$. However, since (21) is equal to $V^N(P)$, the expected payoff to D is equal to:

$$\int_{\mu \geq \mu_0} \left(\mu(1-d_0^2)P - d_0^2\right) dF(\mu) + F(\mu_0)Q(P) = V^N(P)$$

and there is no benefit from integration when M does not have authority.

If when U, D retain ownership and $s < 1$ while $\alpha = 1$, the previous reasoning

[12] Full coordination is not possible since (21) is negative at $d_0 = 1$; hence for any μ_0, P there indeed exists $d_0 < 1$ such that (21) is equal to $V^N(P)$.

applies directly since U always choose $u = 1$. But then the maximum totol surplus is equal to $V^N(sP) - h$, which is less than $V^N(P)$.

If $\alpha < 1$, the equilibrium payoff of $i = U, D$ when they decide non-cooperatively are given by $\pi(P, 1-\alpha)$ and $\pi(P, \alpha)$ respectively, where these functions are defined in (4). If M chooses centralization for states greater than μ_0, let us denote the conditional expected value of μ by

$$\mathcal{E}_{\mathcal{C}} := \int_{\mu \geq \mu_0} \mu \frac{dF(\mu)}{1 - F(\mu_0)}.$$

Then, M will give instructions u, d in order to minimize the difference $|u - d|$ while satisfying the two "no-disobeying" incentive constraints:

$$\alpha(1 - (u-d)^2)(\mathcal{E}_{\mathcal{C}} P) - d^2 \geq \pi(P, \alpha)$$
$$(1-\alpha)(1 - (u-d)^2)(\mathcal{E}_{\mathcal{C}} P) - (1-u)^2 \geq \pi(P, 1-\alpha).$$

If it is possible to satisfy the two incentive compatibility conditions, the maximum surplus is at most equal to the surplus when M has ownership. Indeed, under centralization the maximum surplus is $\mathcal{E}_{\mathcal{C}} P - 1/2$ as when M has authority and M chooses centralization whenever $\mu \geq Q(P)$, as under M ownership.

Suppose now that the two incentive compatibility conditions cannot be satisfied when $u = d$ and assume, without loss of generality; that $u > d$. We show that both constraints must bind. If for instance the first constraint binds but the second does not; it is then possible to decrease u slightly without violating the second constraint and the first constraint becomes slack. Since $u - d$ decreases, M is better off. Now, if the two constraints must bind for any $\mathcal{E}_{\mathcal{C}}$, it follows that in the equilibrium the maximum payoff to D is $\pi(P, \alpha)$, and there is no benefit from integration.

D is the full owner Suppose that D has full ownership of the two assets and authority. Assume without loss of generality that $s = 1$. When M delegates, D can impose a decision $u = 0$ on U and chooses $d = 0$; this leads to full coordination and minimizes the cost to D. Hence D expects to have a payoff of αP when he makes decisions, and U expects a payoff of $(1-\alpha)P - 1$. Since there is full coordination under delegation, a top manager will find it optimal to centralize only if μ is greater than 1, and will choose decisions $u = d$ in such a way that $\mathcal{E}_{\mathcal{C}} P - d^2 \geq P$, or $d^2 \leq (\mathcal{E}_{\mathcal{C}} - 1)P$. This condition is not binding when $\mathcal{E}_{\mathcal{C}}$ or P is large, and the maximum surplus that

can be attained is therefore:

$$F(1)(P-1)+\int_{\mu\geq 1}(\mu P-1/2)\,dF(\mu) \tag{23}$$

and this is smaller than the surplus $V^I(P,0)$ under integration when M has ownership when:

$$\int_{Q(P)}^{1}(\mu P-1/2)dF(\mu)\geq F(1)(P-1)-F(Q(P))V^N(P).$$

This condition holds since $\int_{Q(P)}^{1}(\mu P-1/2)dF(\mu)>(F(1)-F(Q(P))(Q(P)P-1/2)$ and both $P-1$ and $V^N(P)$ are inferior to $Q(P)P-1/2$.

It follows that having full ownership is not optimal for D.

Another possibility is for U to sell its asset to M, and D retain ownership of his asset, but this case is equivalent to the case where U has full ownership (since M always favor full integration), since under delegation the unique equilibrium is $u=d=0$.[13]

[13]It is now M who decides on d, but since he does not have a cost of choosing u while D will always want to deviate slightly form $u=d$, the unique equilibrium is indeed $u=d=0$.