Screening for good patent pools through price caps on individual licenses

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Abstract

Patent pools reduce prices when selling complementary inputs to technologies, but can also effectively cartelize markets when involving substitutes. Independent licensing, by reintroducing competition, ensures that only good pools form when there are two patent holders involved. For larger pools, independent licensing needs to be complemented by other policy tools. We propose to constrain the royalties for the patents individually licensed outside the pool with price caps replicating the pool's sharing rule. This information-free screening device works with asymmetries, even when licensors try to stabilize pools by readjusting the sharing rule in a way that may not reflect contributions.

Keywords: technology licensing, patent pools, substitutes and complements, independent licensing, price caps, joint marketing.

JEL: K11, K21, L12, L24, L41, M2.

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Introduction

Intellectual Property Rights (IPR) necessary to commercialize a product are frequently controlled by many firms. Uncoordinated pricing of complementary technological inputs leads to a total price for the final product that is higher than if they were all controlled by a single entity (Shapiro 2001).[1] This multiple marginalization problem can be solved through patent pools where holders of IPR come together to sell their respective innovations as a bundle. Pools also save transaction costs, and they avoid expensive litigation. Nevertheless, patent pools made of substitute patents are also paramount to cartels that eliminate competition and result in higher prices.

Whether patents are complements or substitutes depends on technological complementarities and on licensing fees. Intuitively, patents can be complements at low prices and substitutes at high prices. Naturally, the price effects of pools are also influenced by the licensees' preferences for the technologies they purchase through pools, i.e., the benefits they derive from them and the adoption costs they incur. Moreover, the nature of interactions between patents changes rapidly over time as markets and technologies evolve. New patents are issued and old ones expire frequently. Regulatory agencies have very limited information on all these characteristics. Therefore, direct assessment of pools on a case by case basis is very costly and leads to large legal uncertainty.

This is why the information-free screening mechanism for pools, i.e., one requiring no particular technical knowledge, proposed by Lerner and Tirole (2004), has such practical appeal. This seminal paper shows that requiring pool members to be free to independently license their patents outside the pool may ensure that only pools that reduce the total price of technology, i.e., welfare increasing pools, are actually created. This is a pathbreaking result that points out the importance of independent licensing in screening patent pools. However, the effectiveness of this screening device is guaranteed only when there are two IPR holders. In that case, independent licensing prevents the existence of an equilibrium with higher prices than

[1] Each patent holder does not take into account that an increase of his royalty reduces demand for the final product and thereby decreases profits of the other patent holders. This argument is often referred to as the Cournot effect.

in the absence of the pool.[2] When there are more than two IPR holders, Lerner and Tirole (2004) do establish the existence of an equilibrium with prices that do not exceed those in the absence of a pool, but do not rule out the existence of other equilibria with higher prices. Nevertheless, they do not show the existence of such equilibria. We demonstrate that these equilibria indeed exist. Moreover, we provide an enhanced screening device, still information-free, which destabilizes welfare reducing pools for sure.

We propose an augmented screening mechanism where royalties for the individual licenses are ex-ante constrained with price caps. These caps match the IPR holders' respective stakes in the pool's royalty.[3] For instance, if there are three IPR holders owning one patent each in a pool that redistributes its profits equally, their individual licenses cannot exceed a third of the pool's royalty. More generally, the cap simply reproduces the sharing rule of the pool, i.e., each licensor gets for his individual license the equivalent of the profit he would get from the pool. The distribution of revenues here neither has to be symmetric nor proportional to patent values, or their respective contributions to the pool. The cap effectively screens patent pools even with very general forms of asymmetries. We allow for asymmetric stand-alone patent valuations, complementarities between patents, sharing rules, as well as equilibrium royalties. The augmented screening destabilizes patent pools that increase the total price of technology and leaves the others unaffected.

Our findings have strong implications for policy. The European Commission, in the most recent revision of its IPR guidelines, already made the first step in recognizing the importance of constraining the royalties for the individual licenses when screening for pools consisting of complementary patents (European Commission 2014, paragraph 254). It is essential that other regulators acknowledge that independent licensing alone does not ensure that only welfare enhancing pools form. Courts have so far assigned disproportionately high importance to independent licensing in their assessment of pools. For example, in *Matsushita Electrical*, the U.S. District Court of Delaware held that the pool did not restrain trade because

[2] See annex E for a discussion of why the case of two IPR holders is a special one.

[3] We refer to individual licensing rather than independent licensing in the context of the cap because the cap would be determined in proportion to the pool's royalty.

"the antitrust plaintiff has the opportunity to license independently."[4] Similarly, the world's leading antitrust authorities, among them Europe and the United States, have long required that pool members remain free to license their respective technologies independently.[5] They even dissolved pools that did not meet this condition.[6] It should now be relatively easy to complement this legal practice with the mechanism we propose here.

The cap on individual licenses would restore competition without imposing additional costs on pools. Instead of safeguarding the freedom to independently license, pools administrators would need to ensure that the individual licenses are available at royalties not exceeding the cap. They could do so, for example, by making their sharing rule public. By informing customers about the royalty levels that do not exceed the cap, pools would ensure that their members do not price higher. Pool administrators could even administer these individual sales themselves, thereby saving transaction costs that would otherwise be incurred by customers and IPR owners in the process of bilateral transactions. In any event, as regulators can always request the sharing rule from the pool, the cap is an information-free screening device for them.

The analysis of patent pools in this paper has a broader applicability to other types of co-marketing agreements, where firms commercialize their respective products together.[7] Such alliances, like patent pools, when jointly marketing complementary products, have the potential of preventing multiple marginalization. Nevertheless, regulators normally do not have the relevant information to assess them directly.[8] Our findings would suggest a more lenient treatment of these agreements if their participants also market their products outside the common marketing scheme at prices not exceeding their respective stakes in the alliance.

Our results go beyond those in the recent paper by Rey and Tirole

[4] *Matsushita Electrical Industrial Co. versus Cinram International Inc.*, 299 F. Supp. 2d 370 (D. Del. 2004).

[5] European Commission (2004, paragraph 227 and 222b), U.S. Department of Justice and Federal Trade Commission (2007, p. 66).

[6] For example, in "*In re Summit Technology Inc.*, 127 F.T.C. 208 (1999), the FTC dissolved a pool made of two firms in part because the pool members granted each other the right to veto licenses" (Miller and Almeling 2007).

[7] See, e.g., European Commission (2011, chapter 6).

[8] See Rey and Tirole (2013) for recent examples of such joint marketing alliances.

(2013), who propose an additional requirement for pools, in the form of unbundling, to avoid tacit collusion. This reinforces the argument for augmenting the current requirement of independent licensing. First, we demonstrate that tacit collusion is not necessary for welfare reducing pools to be stable to independent licensing.[9] Second, Rey and Tirole (2013) show that their mechanism destabilizes welfare reducing pools with two asymmetric firms or an arbitrary large number of symmetric firms. Pools in practice involve many licensors that contribute patents of different values. Therefore, asymmetries with many players that we explore in this article are crucial in the context of pools. Our framework also allows for the generalization of the results in Rey and Tirole (2013) on tacit collusion. Given that with the cap customers would not buy from the pool in any period, tacit collusion is not possible even in the context of the general forms of asymmetries that we allow for.

In our paper, the pool has an additional degree of freedom to adjust the sharing rule in a way that may not reflect technological contributions. In practice, pools may try to counteract the effects of the policy by modifying the stakes of its members. Indeed, we find that a welfare reducing pool may change the sharing rule to remain stable in the presence of independent licensing. Here, owners of more valuable contributions to the bundle may be willing to accept smaller shares in the pool's revenue in order to ensure the stability of the profitable pool. Therefore, equal sharing rules, that are often observed in practice, may be favorable to the stability of bad pools even when they consist of asymmetric contributions. However, with the cap, it is no longer possible to counteract the effects of policy. In this case, there exists no sharing rule that allows bad pools to survive. At the same time, welfare increasing pools can always adjust the sharing rule to remain stable. In good pools with the cap, patents of equal values still choose their natural equal sharing rule, while with asymmetries, holders of patents with larger contributions receive more significant shares of the revenues.

We begin the paper with a simple example, with three IPR holders and one customer, to show that welfare reducing pools can be stable to independent licensing. Then, we characterize optimal pricing and sharing of pools stable to independent licensing in a general framework that allows

[9]Even in a static framework bad pools can be stable to independent licensing.

for any number of firms with asymmetries, as well as arbitrary sharing rules. Finally, we show that if the individual royalties are constrained with the cap, then only welfare increasing pools survive.

1 An example

Let us focus first on a simple example of three pure innovators, A, B and C, who license technology to one downstream firm, at royalties r_A, r_B and r_C. Consider a situation where in order to produce, the downstream firm needs to buy two distinct patents. If it gets a license for any two patents, it gets π, otherwise it gets a very small but strictly positive profit ϵ. That is, the value $V(m)$ generated by the purchase of m licenses is $V(1) = \epsilon$ and $V(2) = V(3) = \pi$. As patents are of equal values, the only thing that matters for customers is the number of patents acquired. In this situation, from a technological point of view, the first two patents are perfect complements, whereas any additional one is a perfect substitute. If IPR owners compete in prices, the ones with the (weakly) larger price will always undercut until reaching an uncoordinated royalty $r^u_A = r^u_B = r^u_C = 0$. However, if firms decided to sell the technologies jointly through a patent pool, they would effectively cartelize the market and set $R = \pi$. In this way, they would achieve greater profit at the expense of the customer.[10]

With independent licensing, it is immediate that offering these independent licenses at prices larger than π constitutes an equilibrium which does not destabilize the pool.[11] Customers then buy from the pool at π rather than buying any individual license, be it on a stand-alone basis or in combination. No IPR holder deviates because it can only sell its individual license on a stand-alone basis at a price not exceeding ϵ. More generally, for any price between the competitive ($R = 0$) and the monopoly level ($R = \pi$), there exist many associated equilibria of the independent licensing game that do not destabilize the pool. Suppose, for example, that the pool offers the bundle at R, and that firms offer individual licenses at roy-

[10]The pool shares its profits equally through dividends without lump-sum payment.

[11]In the first period, the pool sets its price, and then IPR holders simultaneously and uncooperatively set royalties for their independent licenses. Throughout the paper, we refer to this sub-game as the continuation game.

alties $r \geq R/2$. A profitable deviation $\widetilde{r}$ has to attract demand away from the pool, which requires $\widetilde{r} + r < R$ (assuming that indifferent customers buy from the pool). Therefore, the best deviation gives a profit that is arbitrarily close to but smaller than $R - r$. By selling through the pool instead, the firm would get $R/3$. Therefore, there is no profitable deviation as long as $R - r < R/3$ or $r > 2R/3$. That is, all firms charging a price $r > 2R/3$ constitutes an equilibrium of the independent licensing game that does not prevent the pool from selling the bundle at a total price R.

In this example, IPR owners would have incentives to set high prices for their independent licenses in order to ensure stability of the profitable pool. Only if we constrained these royalties with an adequate cap, we could ensure that this welfare reducing pool does not form. This intuition carries over to a very general setting, which we explore in the next sections of the paper.

2 The model

We assume that there exist n IPR holders, each offering one patent, which may vary in importance, to a continuum of customers. If $\mathbf{x} = (x_i)_{i \in \{1,\dots,n\}} \in \{0;1\}^n$ is the vector such that $x_i = 1$ when a customer buys patent i, and $x_i = 0$ if he does not, buying the basket $\mathbf{x}$ provides the customer of type θ with utility $V(\mathbf{x}) + \theta$.[12] This is true for any basket $\mathbf{x}$ that is not empty, as otherwise customers derive no utility.

The surplus function V is not user specific. Its shape reflects the complementarities between the different patents. The idiosyncratic parameter θ represents heterogeneity in the benefits that licensees derive from the technology, and in their costs to adopt it, as compared to alternative technologies. It has a density function f and a cdf F. We denote by $\mathbf{1}$ the vector corresponding to the full basket of patents, and therefore all the

[12] Our model is more general than the asymmetric case in Lerner and Tirole (2004), who assume that each patent might have a different contribution n_i, such that $\|\mathbf{n}\| = \sum_{i=1}^{n} n_i = n$, and therefore buying $\mathbf{x}$ provides the customer of type θ with utility $V(\mathbf{x} \cdot \mathbf{n}) + \theta$. Lerner and Tirole (2004) is a simple way to generate asymmetry for patent valuations, but it does not allow for asymmetric complementarities between patents. If IPRs 1 and 2 provide the same stand-alone utility, then $V(n_1) = V(n_2) \Leftrightarrow n_1 = n_2$ and they both have the same complementarity with IPR 3: $V(n_1 + n_3) = V(n_2 + n_3)$.

patents together provide utility $V(\mathbf{1}) + \theta$. We denote by $\mathbf{r}$ the vector of royalties and $\|\mathbf{r}\| = \sum_{i \in \{1,\dots,n\}} r_i$ is the total royalty for the full basket of n patents. Moreover, we denote by $\mathbf{1_{-i}}$ the basket of all the IPR except for IPR i. Then, $\mathbf{r}_{-i}$ is the vector of all the royalties apart from the royalty of IPR i. Buying $\mathbf{1_{-i}}$ costs $\|\mathbf{r}_{-i}\|$. Last, we denote by $\boldsymbol{\delta}_\mathbf{i}$ the basket including IPR i only, which costs r_i.

Because of the additive separability of the random taste parameter θ, for a given set of royalties, all customers have the same preferred basket $\mathbf{x}^*, \|\mathbf{x}\| \geq 1$. All customers have the same returns from basket $\mathbf{x}$ versus $\mathbf{k}$ patents because θ drops in this comparison (with $\|\mathbf{x}\| \geq 1$ and $\|\mathbf{k}\| \geq 1$). We assume that if, for a given vector of royalties, customers are indifferent between two baskets, they choose the one with the largest number of patents. This optimal basket sold at $\mathbf{x}^* \cdot \mathbf{r}$ provides customers with net utility $V(\mathbf{x}^*) - \mathbf{x}^* \cdot \mathbf{r}$.

We assume that θ has a wide support, such that there always exist some customers who buy this optimal basket, even when $V(\mathbf{x}^*) - \mathbf{x}^* \cdot \mathbf{r}$ is arbitrarily small. This wide support condition ensures interior solutions. We also assume that the distribution of θ has a strictly increasing hazard rate $\frac{f}{1-F}$, which is satisfied by most common distributions. As is well known (see lemma 2.1) this ensures a unique maximum in all the maximization programs. Overall, the demand D for a basket of $\mathbf{x}$ patents sold at $\mathbf{x} \cdot \mathbf{r}$ is given by: $D(\mathbf{x} \cdot \mathbf{r} - V(\mathbf{x})) = 1 - F(\mathbf{x} \cdot \mathbf{r} - V(\mathbf{x}))$.

We only make one assumption regarding V. Each IPR brings a positive value to a basket. This means that $\forall \mathbf{x}, x_i = 0, V(\mathbf{x} + \boldsymbol{\delta}_\mathbf{i}) > V(\mathbf{x})$. Here, $V(\mathbf{x} + \boldsymbol{\delta}_\mathbf{i})$ can be arbitrarily close to $V(\mathbf{x})$. Therefore, we can be arbitrarily close to perfect complementarity or substitutability.[13] This assumption rules out any equilibrium where an IPR holder cannot profitably sell because the royalty for another perfectly complementary IPR is too high. With this assumption, an IPR holder can always attract demand for his own IPR by lowering his royalty.[14] This assumption also implies that the full basket gives the highest utility to customers: $\max_{\mathbf{x} | \|\mathbf{x}\| \geq 1} V(\mathbf{x}) = V(\mathbf{1})$. This means that the technology can be implemented with fewer IPRs but

[13] $V(\mathbf{x} + \boldsymbol{\delta}_\mathbf{i}) \simeq V(\mathbf{x} + \boldsymbol{\delta}_\mathbf{j}) \simeq V(\mathbf{x})$ and only $V(\mathbf{x} + \boldsymbol{\delta}_\mathbf{i} + \boldsymbol{\delta}_\mathbf{j})$ is larger than $V(\mathbf{x})$.

[14] If for a given vector of royalties $\mathbf{r}$, the optimal basket $\mathbf{x}^*$ is such that $x_i^* = 0$, then IPR holder i can make positive profit by lowering its price to $V(\mathbf{x}^* + \boldsymbol{\delta}_\mathbf{i}) - V(\mathbf{x}^*) > 0$.

is optimal with all the patents. In the context of standards, this assumption could be limiting. As explained in Lerner and Tirole (2013), a bulkier standard might be more costly to implement, or it may raise compatibility issues. However, licensees who purchase patents through pools do not generally need to implement all the patents. Therefore, our assumption seems reasonable in the context of pools.

We assume that IPR holders bear no marginal costs of production. This is a standard assumption in the context of innovation, where normally only the fixed R&D costs are relevant. They can sell their patents uncooperatively or form patent pools, with or without independent licensing. When they form pools, the holder of IPR i is assumed to be entitled to a share α_i of the pool's profit, with the condition that $\|\boldsymbol{\alpha}\| = 1$. Pools here have an additional degree of freedom to adapt the sharing rule in a way that may not reflect the patents' contributions. They might try to use this degree of freedom to stabilize the pool.

2.1 Uncoordinated royalties

Let us assume a candidate uncoordinated equilibrium in royalties $\mathbf{r^u}$. Any equilibrium is such that each IPR holder sells. Otherwise, due to our assumption that any IPR brings a positive value to a basket, the IPR holder that does not sell would necessarily decrease its price until his patent is included. Therefore, IPR holders have to price in such a way as to be included in the optimal, full, basket of patents. We will refer to this constraint, like in Lerner and Tirole (2004), as the *competition margin.*

Some IPR holders might be effectively constrained by their competition margin. They price r_i^u to satisfy:

$$V(\mathbf{1}) - \|\mathbf{r^u}\| = \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r^u}\}.$$

These IPR holders set the highest royalty possible with which they are not excluded from the optimal basket chosen by customers, i.e., they match the utility from the best package without them.

For some IPR holders, this could lead to royalty levels higher than what would maximize their profits. These IPR holders then maximize:

$r_i D\left(\left\|\mathbf{r^u_{-i}}\right\| + r_i - V(\mathbf{1})\right)$. We will refer to these IPR holders as constrained by their *demand margin*, like in Lerner and Tirole (2004). Making the change of variable, $R = r_i + \|\mathbf{r_{-i}}\|$, we have $r_i D(r_i + \|\mathbf{r_{-i}}\| - V(\mathbf{1})) = (R - \|\mathbf{r_{-i}}\|)D(R - V(\mathbf{1}))$. Therefore, the maximization program is equivalent to selling the full bundle $\mathbf{1}$ with the implicit cost $\|\mathbf{r_{-i}}\|$, corresponding to the price of the other licenses. We can show in the following Lemma 2.1 that the total royalty R for any basket $\mathbf{x}$ is increasing in the implicit cost R' incurred by the IPR holder who sets R:

Lemma 2.1. *For all $\mathbf{x} \in \mathbb{R}^n$, residual profit $\pi(R, R') = (R - R')D(R - V(\mathbf{x}))$ is supermodular in R and R'. Therefore, the optimal value R is increasing in R'.*

Proof. See annex A. □

It follows from Lemma 2.1 that the maximization program when the demand margin binds leads to an optimal royalty larger than the monopoly's royalty $argmax_R RD\left(R - V(\mathbf{1})\right)$ as soon as $\|\mathbf{r}_{-i}\| > 0$. In other words, when the demand margin binds, patents are complements, and IPR holders face a classical multiple marginalization problem.

Moreover, IPR holders for whom the demand margin binds all have the same maximization program, and therefore they all set the same price. To see that, assume $\mathbf{r^u}$ is an uncoordinated equilibrium such that for both i and j the demand margin binds. Assume by the absurd that $r^u_i > r^u_j$, and therefore $\left\|\mathbf{r^u_{-j}}\right\| > \left\|\mathbf{r^u_{-i}}\right\|$. Then, due to supermodularity as in Lemma 2.1:

$$\begin{aligned} & \left\|\mathbf{r^u_{-j}}\right\| > \left\|\mathbf{r^u_{-i}}\right\| \\ \Rightarrow \quad & argmax_R \left\{ \left(R - \left\|\mathbf{r^u_{-j}}\right\|\right) D(R - V(x)) \right\} > \\ & \qquad\qquad argmax_R \left\{ \left(R - \left\|\mathbf{r^u_{-i}}\right\|\right) D(R - V(x)) \right\} \end{aligned}$$

In other words, the maximization program of j leads to a total royalty larger than the maximization of i, which is a contradiction.

Overall, IPR holders are constrained by either of the two margins when setting their uncoordinated prices. The demand margin binds for a licensor if it could individually raise its price without triggering an exclusion of its patent from the basket chosen by customers, otherwise it is the competition margin that binds. These conditions determine equilibria in royalties,

which do not have to be unique. Therefore, we can show the following proposition about the set of uncoordinated equilibria $\mathcal{R}^u$:

Proposition 2.2. *Uncoordinated equilibria*

1. *The set of uncoordinated equilibria $\mathcal{R}^u$ is non-empty.*

2. *For a given equilibrium $\mathbf{r^u} \in \mathcal{R}^u$, IPR holders are either constrained by:*

   (a) *their competition margin and set r_i^u that satisfy:*

$$V(\mathbf{1}) - \|\mathbf{r^u}\| = \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r^u}\},$$

   (b) *or by their demand margin and all set the same price*

$$argmax_{r_i} r_i D\left(\left\|\mathbf{r^u_{-i}}\right\| + r_i - V(\mathbf{1})\right).$$

Proof. As we have seen in the text, conditions in *(a)* and *(b)* characterize a Nash equilibrium. Such an equilibrium exists because the best response function:

$$\begin{array}{rccl} g: & [0,V(\mathbf{1})]^n & \to & [0,V(\mathbf{1})]^n \\ & (r_i)_{i\in[1,n]} & \to & \left(\begin{array}{c} \min\{\, V(\mathbf{1}) - \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} - \|\mathbf{r}_{-i}\|, \\ \operatorname{argmax}\{r_i D(\|\mathbf{r}\| - V(\mathbf{1}))\}\} \end{array}\right)_{i\in[1,n]} \end{array}$$

is a continuous function from a compact space to the same compact space. Indeed, we can see that $[0,V(\mathbf{1})]^n$ is stable. For all $i\in\{1,\ldots,n\}$, we have that for all $\mathbf{x}$ such that $x_i = 0$ and $\|\mathbf{x}\|\geq 1$:

$$\begin{array}{rrcl} & V(\mathbf{1}) - Z & \geq & V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{z} \\ \Rightarrow & V(\mathbf{1}) - V(\mathbf{x}) & \geq & (\mathbf{1}-\mathbf{x})\cdot\mathbf{z} \\ & & \geq & z_i \text{ (as } x_i = 0). \end{array}$$

Therefore, $\min_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{1}) - V(\mathbf{x})\} \geq z_i$, from which we derive that $z_i \leq V(\mathbf{1})$ and $[0,V(\mathbf{1})]^n$ are stable.

Such an equilibrium is not necessarily unique. The set of the uncoordi-

nated equilibria is:

$$\mathcal{R}^u = \left\{ \begin{array}{l} \mathbf{r^u} \in \mathbb{R}_+^n, \forall i \in \{1, \ldots, n\}, \\ r_i^u = \min \left\{ \begin{array}{l} V(\mathbf{1}) - \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r^u}\} - \left\|\mathbf{r^u_{-i}}\right\|, \\ \text{argmax}\{r_i^u D(\|\mathbf{r^u}\| - V(\mathbf{1}))\} \end{array} \right\} \end{array} \right\}.$$

□

2.2 Patent pool

Consider now a situation where IPR licensors come together to license their technologies jointly through a patent pool. The pool only sells the full bundle of patents. In the model, this is a consequence of the additive separability of θ. This seems reasonable given that pools normally do not offer menus of options (Lerner, Strojwas and Tirole 2007).

Without independent licensing, a patent pool would be facing no competition. Therefore, it would price in a way that maximizes its profits given the overall demand that is only driven by the distribution of θ. It would always set the aggregate royalty $\overline{R} = argmax_R R \cdot D\left(R - V(\mathbf{1})\right)$.

Because independent licensing imposes an additional constraint on patent pools, they might need to decrease their price below $\overline{R}$ with independent licensing. However, a pool will never have any incentive to increase its total royalty beyond $\overline{R}$. This would necessarily decrease its profits and create more scope for deviation. Therefore, we restrict our attention to pools choosing their total royalty $R \in [0, \overline{R}]$.

If all patents are of equal value, there exists a unique uncoordinated equilibrium (see annex E). Then, pools are welfare reducing if they increase the aggregate price of technology, as compared to the uncoordinated price setting, and welfare increasing otherwise.[15]

With asymmetries, the set of uncoordinated equilibria $\mathcal{R}^u$ is a closed subset of $[0, V(\mathbf{1})]^n$ so that it is a compact. As $\|.\|$ is continuous, we can define $\overline{R}^u = \max\{\|\mathbf{r^u}\|, \mathbf{r^u} \in \mathcal{R}^u\}$, as well as $\underline{R}^u = \min\{\|\mathbf{r^u}\|, \mathbf{r^u} \in \mathcal{R}^u\}$. Naturally, if a pool implements a price between the lowest uncoordinated

[15]Pools that do not change the aggregate price are assumed to be welfare increasing. Moreover, we assume that if the total price for the independent licenses equals the price of the pool, then customers prefer to buy from the pool. These assumptions are in line with the fact that pools in practice also reduce transaction costs.

equilibrium $\underline{R}^u$ and the highest uncoordinated equilibrium $\overline{R}^u$, it is not possible to assess its consequences on welfare. Therefore, we can define welfare reducing and welfare increasing pools;

Definition 2.1. *Welfare reducing pool*
A patent pool is welfare reducing if it is selling at the aggregate price R such that $R > \overline{R}^u$.

Definition 2.2. *Welfare increasing pool*
A patent pool is welfare increasing if it is selling at the aggregate price R such that $R \leq \underline{R}^u$.

What directly follows from the double marginalization problem, discussed earlier, is that if IPR holders play an equilibrium where the demand margin binds for at least one IPR holder in the absence of the pool, then the patent pool without independent licensing is welfare increasing.

Hence, if there exists an uncoordinated equilibrium in which the demand margin binds for at least one IPR holder, even without independent licensing, the pool cannot be welfare reducing. By contraposition, a welfare reducing pool, with or without independent licensing, can only occur if the competition margin would bind for all IPR holders in the absence of the pool, and the demand margin would not bind for any IPR holder. Therefore, the total uncoordinated price $Z = \|\mathbf{z}\|$ is such that:

$$\forall i \in \{1, \ldots, n\}, V(\mathbf{1}) - Z = \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{z}\}.$$

Let $\mathcal{Z}$ be the set of royalties where the competition margin binds for every IPR holder (and therefore the set of Nash equilibria if the demand margin does not bind for anyone):

$$\mathcal{Z} = \left\{ \mathbf{z} \in \mathbb{R}_+^n, \forall i \in \{1, \ldots, n\}, V(\mathbf{1}) - Z = \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{z}\} \right\}.$$

Such a set is not empty for the same reason that $\mathcal{R}^u$ is not empty.[16] Overall, we have the following proposition 2.3:

Proposition 2.3. *If a pool is welfare reducing, then $\mathcal{R}^u = \mathcal{Z}$.*

[16] Another proof can be found using Lemma D.1 starting from zero royalties.

Proof. We show this result by contraposition. If $\mathcal{R}^u \neq \mathcal{Z}$, there exists an uncoordinated equilibrium where the demand margin binds for at least one IPR holder, which leads to a total royalty larger or equal to $\overline{R}$. Therefore, $\overline{R} \leq \overline{R}^u$. Hence, by contraposition, if $\overline{R} > \overline{R}^u$ then $\mathcal{R}^u = \mathcal{Z}$. Moreover, we limit ourselves to pools pricing $R \in [0, \overline{R}]$. Hence, if the pool is welfare reducing, then $R > \overline{R}^u$, which implies that $\overline{R} > \overline{R}^u$ and $\mathcal{R}^u = \mathcal{Z}$. $\square$

3 Stability of welfare reducing pools

We now turn to the study of stability of welfare reducing patent pools to independent licensing where each firm independently sets a rate r_i for its patent. The pool first chooses the royalty R for its bundle. Then, in the continuation game, IPR owners non-cooperatively and simultaneously set prices for their individual licenses. As stability depends on the sharing rule, we define a patent pool by its royalty R and its sharing rule $\boldsymbol{\alpha}$. We then define stability as follows:

Definition 3.1. *Stability*
A patent pool $(R, \boldsymbol{\alpha})$ *is* stable *to independent licensing if there exists an equilibrium of the continuation "independent licensing" game* $\mathbf{r}^*$ *such that customers buy from the pool at price* R.[17]

Pools are stable if, for a given set of independent royalties $\mathbf{r}^*$, customers buy from the pool, and no IPR holder wants to deviate from $\mathbf{r}^*$. The first condition is straightforward. However, imagine that at $\mathbf{r}^*$ the pool is selling. A deviating firm needs to undercut the pool to attract demand. By decreasing its royalty further, an IPR holder would attract even more demand than the pool. However, to attract demand away from the pool, a deviating IPR holder needs to pay an implicit cost. Therefore, due to supermodularity of its objective function, as in Lemma 2.1, the demand margin of a deviating IPR holder binds at a total royalty exceeding $\overline{R}$. It is therefore not profitable to undercut the pool further, and the optimal deviation of an IPR holder is to just undercut the pool:

[17]Lerner and Tirole (2004) referred to this form of stability as weak.

Lemma 3.1. *If a patent pool sets the aggregate price R, and all the other IPR holders set the independent licenses $\mathbf{r}^*$, the optimal deviation of an IPR holder is to just undercut the pool and set its royalty $\widetilde{r}$ marginally under $R + \max_{\mathbf{x}|x_i=1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^* + r_i^*\} - V(\mathbf{1})$.*

Proof. See annex B □

Then, using Lemma 3.1, deviation is not profitable if the optimal deviation leads to a smaller profit than what could be achieved in the pool. This leads to the following proposition, which characterizes stable pools:

Proposition 3.2. *Stability to independent licensing*

i) The set of stable pools $(R, \boldsymbol{\alpha})$ is non-empty.

ii) The aggregate price R of a stable pool satisfies:

$$R \leq R_{max}, \tag{3.1}$$

where

$$\begin{aligned} R_{max} &= min\left\{\overline{R}, \min_{i\in\{1,\ldots,n\}}\left\{\tfrac{V(\mathbf{1})-V(\boldsymbol{\delta_i})}{1-\alpha_i}\right\}\right\} \\ \overline{R} &= argmax_R R\cdot D(R - V(\mathbf{1})). \end{aligned}$$

iii) The equilibrium independent royalties of the continuation game $\mathbf{r}^*$ that do not destabilize the pool satisfy:

$$\begin{aligned} &\forall i \in \{1,\ldots,n\}, \forall \mathbf{x} \in \mathbb{R}^n | x_i = 1, \|\mathbf{x}\| \geq 2, \\ &\mathbf{x}\cdot\mathbf{r}^* - r_i^* \geq V(\mathbf{x}) - V(\mathbf{1}) + (1-\alpha_i) R, \end{aligned} \tag{3.2}$$

and

$$\forall \mathbf{x} \in \mathbb{R}^n | \, \|\mathbf{x}\| \geq 1, \quad \mathbf{x}\cdot\mathbf{r}^* \geq V(\mathbf{x}) - V(\mathbf{1}) + R. \tag{3.3}$$

Proof. First, a stable pool has to sell despite independent licenses $\mathbf{r}^*$. This requires:

$$\max_{\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^*\} \leq V(\mathbf{1}) - R,$$

from which we directly derive condition 3.3.

Moreover, facing $\mathbf{r}^*$ and R, a deviating firm has to beat the patent pool to attract demand. It is clear from the inequality above that by

beating the pool which sells, a deviating firm is also preferred than any offer consisting of the other independent licenses. As shown in Lemma 3.1, due to supermodularity of his objective function, the deviator does not want to undercut the pool more than just marginally. When just undercutting the pool, the deviator prices below his unconstrained optimal royalty, so undercutting further would decrease his profit.

Then, the patent pool that effectively sells is stable to unilateral deviations if and only if just undercutting the pool by pricing (just below) $\widetilde{r}^* = R + \max_{\mathbf{x}|x_i=1}\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^* + r_i^*\} - V(\mathbf{1})$ is less profitable than selling through the pool, that is, if:

$$\begin{aligned} & \widetilde{r}^* D\left(\widetilde{r}^* - \max_{\mathbf{x}|x_i=1}\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^* + r_i^*\}\right) &\leq&\ \alpha_i R D(R - V(\mathbf{1})) \\ \Leftrightarrow\quad & \widetilde{r}^* D(R - V(\mathbf{1})) &\leq&\ \alpha_i R D(R - V(\mathbf{1})). \end{aligned}$$

Given that due to the wide support assumption on θ, $D(R - V(\mathbf{1})) > 0$, the last condition is equivalent to $\max_{\mathbf{x}|x_i=1}\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^* + r_i^*\} - V(\mathbf{1}) + R \leq \alpha_i R$.[18]

This condition for one patent then constitutes the second element of the minimum in equation (3.1), and for all the other baskets, the second element in equation (3.2). All elements with $\mathbf{r}^*$ appear on one side of equation (3.2), and the remaining variables on the other.

It is trivial that the set of stable pools $(R, \boldsymbol{\alpha})$ is non empty. For instance, if R satisfies (3.1), then $\mathbf{r}^*$ is such that $r_i^* = R$ does not destabilize the pool. $\square$

Proposition 3.2 demonstrates that there is an upper limit on the total royalty a stable pool can implement. Stability also requires independent licenses to be high enough, as conditions (3.2) and (3.3) show.[19] Last, note that all things equal, a lower α_i makes deviation for the IPR holder i more appealing.

Corollary 3.3 immediately follows from proposition 3.2, and describes

[18] Even without this wide support assumption, a pool would never choose a total royalty such that $D(R - V(\mathbf{1})) = 0$, where there is no demand, and so the profit is null.

[19] There can exist an equilibrium where the pool is destabilized: if an IPR holder expects the other IPR holders to have low, e.g., competitive, independent royalties, then, as in Lerner and Tirole (2004), he would price his own independent license low enough to sell outside the pool as well.

the pricing of a stable pool which maximizes its aggregate profits:

Corollary 3.3. *For a given sharing rule $\boldsymbol{\alpha}$, the profit maximizing stable pool's royalty R_{max} is given by:*

i) $R_{max} = \overline{R}$ *if* $\forall i \in \{1, \ldots, n\}, V(\boldsymbol{\delta}_\mathbf{i}) < V(\mathbf{1}) - (1-\alpha_i)\overline{R}$,

ii) $R_{max} = \min_{i\in\{1,\ldots,n\}} \left\{ \frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i} \right\}$ *otherwise.*

The condition $i)$ in Corollary 3.3 depends on the relative importance of a stand-alone patent compared to the full basket of patents, and the maximum price that the pool can implement $\overline{R}$. In reality, it is necessary to combine many IPR to bring a significant value to the market, and therefore acquiring one technology alone should normally bring a very small value in the context of pools. However, $\overline{R}$ could be much higher than $V(\mathbf{1})$. For such large values, condition $i)$ is bound to fail and the patent pool with independent licensing necessarily meets its competition margin.[20] $\overline{R}$ very large, and in particular much larger than $V(\mathbf{1})$, would require the distribution of θ to show a thick tail for large values of θ, such that the average valuation counts less than the outliers for the pricing of the pool. In this case, the pricing of the patent pool would be targeted to exploit the customers with very large valuations for the technology. In the presence of high enough royalties for the independent licenses, the only option for these customers would be to purchase the patent of the deviating IPR holder only. Therefore, independent licensing limits the exploitation of these customers by the patent pool, but generally does not eliminate the harm.

Moreover, if the pool can choose its sharing rule, it will select one that ensures stability and maximizes its profit;

Proposition 3.4. *Sharing rule*

From the perspective of a stable pool, the total profit maximizing sharing

[20]The condition necessarily fails as soon as $\exists i, \overline{R} \geq \frac{1}{1-\alpha_i} V(\mathbf{1}) \Leftrightarrow \overline{R} \geq \frac{1}{1-\min_i\{\alpha_i\}} V(\mathbf{1})$. However, $\min_i\{\alpha_i\} \leq \frac{1}{n} \underset{n\to\infty}{\longrightarrow} 0$. Therefore, the right side of the condition tends to $V(\mathbf{1})$ as n tends to infinity. Then, the condition is likely to fail if $\overline{R}$ is larger than $V(\mathbf{1})$. This never happens if θ is negative, considered to be a cost only, as in Rey and Tirole (2013) and Lerner and Tirole (2013). In this case, $\overline{R} \leq V(\mathbf{1})$, which implies that $V(\mathbf{1}) - (1-\alpha_i)\overline{R} \geq \alpha_i V(\mathbf{1})$. Then, condition $i)$ is always satisfied as soon as $\forall i, V(\boldsymbol{\delta}_i) < \alpha_i V(\mathbf{1})$, which should be the case in practice, unless the sharing rules are very unequal.

rule is:

$$\alpha_i^* = 1 - (n-1)\frac{V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{i})}{\sum_{j=1}^n V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{j})}. \tag{3.4}$$

Proof. The pool has incentives to maximize $\min_{i\in\{1,\ldots,n\}}\left\{\frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i}\right\}$. This can be achieved by equating all these quantities to each other. The pool needs to solve $\frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i} = \frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{j})}{1-\alpha_j}$, under the constraint that $\|\boldsymbol{\alpha}\| = 1$. This is a linear system of n independent equations and n unknowns, which has at most one solution. The pool needs to find $\forall i, 1-\alpha_i = \gamma(V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{i}))$. Then, summing these n equalities, one finds that $n-1 = \gamma\sum_{j=1}^n V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{j})$, and therefore $1-\alpha_i = (n-1)\frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{\sum_{j=1}^n V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{j})}$. $\square$

There exist stable welfare reducing pools if and only if $R_{max} > \overline{R}^u$. Proposition 3.5 shows that, if pools can choose the sharing rules in order to maximize their profits, pools that were welfare reducing without independent licensing are generally also welfare reducing with independent licensing.

Proposition 3.5. *If a patent pool without independent licensing would be welfare reducing, i.e., $\overline{R} > \overline{R}^u$, then the profit maximizing stable pool $(R_{max}, \boldsymbol{\alpha}^*)$ is such that $R_{max} \geq \overline{R}^u$.*

Proof. If $\forall i \in \{1,\ldots,n\}, V(\delta_\mathbf{i}) < V(\mathbf{1}) - (1-\alpha_i^*)\overline{R}$, with independent licensing, the pool can still effectively sell at $R_{max} = \overline{R} > \overline{R}^u$.

Otherwise, it can effectively sell at $R_{max} = \min_{i\in\{1,\ldots,n\}}\left\{\frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i^*}\right\}$. We can then show that $R_{max} \geq \overline{R}^u$ and find necessary and sufficient conditions for the equality. First, given that $\overline{R} > \overline{R}^u$, $\mathcal{R}^u = \mathcal{Z}$. Moreover, let $\mathbf{z} \in \mathcal{Z}$. Then we have:

$$\begin{aligned} & \forall i, V(\mathbf{1}) - \|\mathbf{z}\| \geq V(\boldsymbol{\delta}_\mathbf{i}) - z_i \\ \Rightarrow \quad & \|\mathbf{z}\| \leq \tfrac{1}{n-1}\textstyle\sum_{j=1}^n V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{j}) = \frac{V(\mathbf{1})-V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i^*}. \end{aligned}$$

From this we deduce that $R_{max} \geq \overline{R}^u$, and the inequality is strict unless:

$$\forall \mathbf{z} \in \mathcal{Z}, \|\mathbf{z}\| = \min_{i\in\{1,\ldots,n\}}\left\{\frac{V(\mathbf{1}) - V(\boldsymbol{\delta}_\mathbf{i})}{1-\alpha_i^*}\right\}.$$

$\square$

Proposition 3.4 shows that in the case of symmetric patents, the sharing rule that maximizes total profits of a stable pool is equal sharing. Moreover, if each individual patent brings a very small value to the pool, the profit maximizing sharing rule is also very close equal sharing. It is only when one patent has a much larger stand-alone value than the others that the pool will depart from the equal sharing rule in favor of this particular IPR. Last, the optimal sharing rule does not necessarily reflect the contribution of each patent to the pool, but rather its stand-alone value $V(\boldsymbol{\delta}_{\mathbf{i}})$.

Naturally, sharing rules also have to reflect other considerations, such as IPR holders' incentives for joining the pool. These are determined by their options outside the pool, from which we abstract in this analysis. However, what Proposition 3.5 shows is that if pools can manipulate the sharing rule, they will normally be able to stabilize welfare reducing pools. Given that IPR holders have high incentives to join pools as they increase their profits, they may accept a sharing rule that does not reflect their contribution in order to stabilize the pool. For example, let us assume that an IPR holder i would set price r_i absent the pool, for instance, because there exists only one uncoordinated equilibrium. Then, abstracting from coalition formation issues, an IPR holder would join the pool if $\alpha_i RD(R - V(\mathbf{1})) \geq r_i D(\|\mathbf{r}\| - V(\mathbf{1}))$. If the pool allows to cartelize the market consisting of rather substitute technologies, aggregate profits are much larger with the pool, and then it is indeed plausible that $RD(R - V(\mathbf{1}))/n \geq r_i D(\|\mathbf{r}\| - V(\mathbf{1}))$ for all i. Therefore, even without side payments, there is room to adjust the sharing rule in order to remain stable to independent licensing, and give incentives to IPR holders to join.

Moreover, as shown in Proposition 3.4, because the stand-alone value of each and every patent is likely to be limited in practice, the sharing rule that maximizes the stable pool's total profits is likely to be very close to equal sharing. Of course, this rule could have been chosen for other reasons, such as simplicity, and not necessarily in order to stabilize a welfare decreasing pool. For instance, in the case of fairly symmetric patents, equal sharing is very natural. Nevertheless, Proposition 3.4 and 3.5 indicate that equal sharing rules, which are prevalent in reality, are also generally favorable to the stability of welfare decreasing pools, even with patents that bring asymmetric contributions to the pool.

4 Royalty cap on the individual licenses

Given that the stability of welfare reducing pools requires high enough independent royalties, one could envisage an ex-ante screening mechanism focusing on the royalty level for the individually licensed patents.[21] It is very natural to propose a cap at $\alpha_i R$, where α_i is the share that the IPR holder i gets from the pool's profits. This share is directly observable. Hence, the cap is also information-free from the regulator's point of view, i.e., requires no particular technical knowledge.[22] As we show in Proposition 4.2, this mechanism ensures that welfare decreasing pools cannot be stable. This is true for any repartition rule $\boldsymbol{\alpha}$. We demonstrate that welfare decreasing pools can never stabilize the pool by changing the sharing of their profits. Conversely, a welfare increasing pool can always choose a repartition rule that stabilizes the pool.[23] This sharing rule gives a larger share of profits to more valuable patents.

Before turning to this proposition and its proof, it is important to show one intermediary result, which explains the best responses of IPR holders (Lemma 4.1). This result will be used at several instances in the proof of the proposition.[24]

Assume that for a given vector of royalties $\mathbf{r}$, IPR holder i is not constrained by the competition margin, i.e., $\max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}^*\} < V(\mathbf{1})-\|\mathbf{r}^*\|$. Then, abstracting from the demand margin, i's best response is to increase its price. Let us describe how such a price increase would affect the best response of a competitor j. If $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}^*\} < V(\mathbf{1})-\|\mathbf{r}^*\|$, then, before i's price increase, IPR holder j can also increase its royalty without risking to be excluded from the optimal basket.

[21] When speaking about the cap, we refer to individual rather than independent licensing because the cap will depend on the pool's price.

[22] Antitrust agencies can always request this information from pool administrators.

[23] Note that we refer here to a more general form of stability than in Definition 3.1 because with the cap there exists no equilibrium in individual royalties that destabilizes a welfare increasing pool. Lerner and Tirole (2004) referred to this form of stability as strong.

[24] This lemma is also used to show how equilibria in royalties can be reached by a best response algorithm (Lemma D.1).

We assume that j effectively wants to increase its price because we abstract from the demand margin. This is still the case after a small price increase from i, which, a fortiori, does not force j to decrease its price. Conversely, if $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}^*\}=V(\mathbf{1})-\|\mathbf{r}^*\|$, then r_j^* is the best response to $\mathbf{r}^*_{-\mathbf{j}}$. After i's price increase, it could be the case that $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\widetilde{\mathbf{r}}\}>V(\mathbf{1})-\|\widetilde{\mathbf{r}}\|$. This would occur in case $\|\widetilde{\mathbf{r}}\|$ increases more than the total royalty of the best outside option without j, in particular, if i is not part of the best basket without j after i's price increase. In the case where $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\widetilde{\mathbf{r}}\}>V(\mathbf{1})-\|\widetilde{\mathbf{r}}\|$, the increased royalty of i would trigger a decrease of the royalty of j, who would otherwise be excluded from the optimal basket. Lemma 4.1 shows that this is not the case, and i can increase its price without triggering any price decrease from j. This result is important to show that there is no strategic substitutability;

Lemma 4.1. *Let* $\mathbf{r}\in\mathbb{R}^n$ *and* i *and* j *such that*

$$\begin{array}{rcl}\max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}\} & < & V(\mathbf{1})-\|\mathbf{r}\| \\ \max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}\} & = & V(\mathbf{1})-\|\mathbf{r}\|.\end{array}$$

Then, i *remains part of the optimal basket for* $x_j=0$ *if its royalty is smaller than* $\widetilde{r}_i$*, solving* $V(\mathbf{1})-\|\mathbf{r}\|+r_i-\widetilde{r}_i=\max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1}\{V(\mathbf{x})-\mathbf{x}\cdot\mathbf{r}\}$*, and the best response of* j *is unchanged by* i*'s price increase.*

Proof. See annex C □

We can then show the following proposition, which is the key contribution of this paper:

Proposition 4.2. *When a patent pool is selling at a total royalty* R *and its members are under the obligation to offer individual licenses at royalties not exceeding* $\alpha_i R$*:*

(i) there exists no $\boldsymbol{\alpha}$ *such that a welfare reducing pool* $(R,\boldsymbol{\alpha})$ *is stable,*

(ii) there exist $\boldsymbol{\alpha}$ *such that a welfare increasing pool* $(R,\boldsymbol{\alpha})$ *is stable.*

Proof. (i) Let us assume that a pool is pricing R, and denote by $\mathbf{r}^*$ the royalties offered by IPR holders in the equilibrium of the continuation individual licensing game with the cap, where $r_i^*\leq\alpha_i R$. Assume that this pool

$(R, \boldsymbol{\alpha})$ is welfare reducing and stable, despite the cap, with the continuation equilibrium $\mathbf{r}^*$. Because the pool is welfare reducing, we necessarily have $\mathcal{R}^u = \mathcal{Z}$ (see proposition 2.3). Therefore, a sufficient condition for $\mathbf{r}^* \in \mathcal{R}^u$ is that $\forall i \in \{1, \ldots, n\}, \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*\} = V(\mathbf{1}) - \|\mathbf{r}^*\|$. At these royalties, the demand margin does not bind for any IPR holder, and they have no incentives to decrease their royalties.

As the pool is stable, due to condition 3.3, with $\mathbf{x} = \mathbf{1}$, we have $\|\mathbf{r}^*\| \geq R$. Because $r_i^* \leq \alpha_i R$, we necessarily have:

$$\|\mathbf{r}^*\| = R$$

and

$$\forall i \in \{1, \ldots, n\}, r_i^* = \alpha_i R.$$

Since to be stable the pool has to sell, we have:

$$\begin{aligned} & V(\mathbf{1}) - \|\mathbf{r}^*\| \geq \max_{\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*\} \\ \Rightarrow \quad & \forall i \{1, \ldots, n\}, V(\mathbf{1}) - \|\mathbf{r}^*\| \geq \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*\}. \end{aligned}$$

Therefore, we can directly use Lemma D.1 starting from these inequalities to show that there exists $\mathbf{z} \in \mathcal{Z}$ with $\|\mathbf{z}\| \geq \|\mathbf{r}^*\| = R$.

The intuitions of these results are the following. Starting from a situation where some IPR holders are constrained by their competition margin and some others are not, by a process of sequential price increases, we can construct an equilibrium where the competition margin binds for everyone. We do so through a best response algorithm in Lemma D.1 in the appendix. We assume here that there exists no exogenous constraint on the royalties that the IPR holders can choose, i.e., there is no cap, because we are looking for an uncoordinated equilibrium. Then, if all the other players price $\mathbf{r}$, an IPR holder i, for whom the competition margin does not bind, can increase its price starting from r_i without being excluded from the optimal basket of individual licenses. This means that, abstracting from the cap and the demand margin, starting from $\mathbf{r}$, IPR holders would increase their royalties until reaching an equilibrium in royalties where the competition margin binds for every IPR holder. Since in the end we reach an equilibrium where the demand margin does not bind for anyone, it is

irrelevant whether during the virtual process of the algorithm the demand margin binds or not. This uncoordinated equilibrium will lead to an aggregate royalty higher than $\|\mathbf{r}\|$ because, as shown in Lemma 4.1, throughout the process of sequential price increases, no IPR holder ever unilaterally decreases its price.

As the pool is welfare reducing, $\mathcal{Z} = \mathcal{R}^u$ and then $\mathbf{z} \in \mathcal{R}^u$. Therefore, we have a vector of royalties that belongs to $\mathcal{Z}$, for which the demand margin does not bind for any IPR holder (as $\mathcal{R}^u = \mathcal{Z}$). However, because the pool is welfare reducing $R > \overline{R}^u$. Then, overall, we would have $\mathbf{z} \in \mathcal{R}^u$ and $\|\mathbf{z}\| > \overline{R}^u$, which is a contradiction. This means that if a welfare reducing pool sells despite the cap, there exists an uncoordinated equilibrium (without the pool) where the competition margin binds for everyone, leading to an even higher total royalty. This is a contradiction.

(ii) Consider a welfare increasing pool (R, α) and a continuation equilibrium $\mathbf{r}^*$, for which one of the three constraints binds: the competition margin, the demand margin or the cap. We first show that if the cap binds for every IPR holder, then $\mathbf{r}^*$ is such that the pool sells, and no IPR holder wants to undercut the pool. Then, we show that the pool can always modify its sharing rule such that the cap binds for every IPR holder.

If the cap binds for all IPR holders, then $\forall i \in \{1, \ldots, n\}, r_i^* = \alpha_i R$ and $V(\mathbf{1}) - R = V(\mathbf{1}) - \|\mathbf{r}^*\|$. Moreover, if the cap binds for everyone, the competition margin does not strictly bind for anyone, i.e., does not bind alone. Then customers prefer to buy all the patents rather than any smaller basket of IPRs, and the pool sells.

As the pool sells, we have

$$\max_{\mathbf{x} | \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*\} \leq V(\mathbf{1}) - \|\mathbf{r}^*\| = V(\mathbf{1}) - R.$$

From this we deduce:

$$\forall i \in \{1, \ldots, n\}, \max_{\mathbf{x} | x_i = 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*\} \leq V(\mathbf{1}) - \|\mathbf{r}^*\| = V(\mathbf{1}) - R.$$

And then, we indeed verify that undercutting the pool is not profitable

because the optimal deviation is:

$$\underbrace{\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*\right\}-V(\mathbf{1})+R}_{\leq 0}+r_i^*\leq\alpha_i R.$$

Overall, if the cap binds for every IPR holder, the pool is stable. To prove stability of welfare increasing pools, it is then sufficient to show that there always exists a sharing rule $\boldsymbol{\alpha}$ such that it is the case that the cap binds for every IPR holder.

Let us assume that for a given sharing rule $\boldsymbol{\alpha}$, there exists a continuation equilibrium $\mathbf{r}^*$ where the pool does not sell. As we have just shown, it has to be that the cap does not bind for at least one IPR holder. This could be because at least one of the competition or the demand margins binds. It is immediate to show that the demand margin cannot bind for any IPR holder unless the cap also binds. Due to the supermodularity of the objective function, the demand margin can indeed only bind for each IPR holder at a total royalty exceeding (or equal to) $\overline{R}$, and therefore at the royalty for an individual license higher than (or equal to) $\alpha_i R$:

$$\widehat{r}_i+\sum_{j\neq i}r_j^*\geq\overline{R}\geq R\Rightarrow\widehat{r}_i\geq R-\sum_{j\neq i}r_j^*\geq R-\sum_{j\neq i}\alpha_j R=\alpha_i R.$$

Therefore, the pool can only be unstable if the competition margin binds for at least one IPR holder, but the cap does not bind. We refer to this situation as the competition margin being strictly binding (as explained, the demand margin cannot be strictly binding). The IPR holders, for whom the competition margin strictly binds, individually license strictly below the cap, and the pool does not sell. In this situation, let us consider the following disjoint-union of $\{1,\ldots,n\}=\mathcal{I}_1\cup\mathcal{I}_2\cup\mathcal{I}_3$, where $\mathcal{I}_1$, $\mathcal{I}_2$ and $\mathcal{I}_3$ are respectively the sets of IPR holders for whom the cap strictly binds, for whom the competition margin strictly binds, and for whom both the cap and the competition margin bind.

An important property of this disjoint-union is that if $\mathcal{I}_2\neq\emptyset$, then $\mathcal{I}_1\neq\emptyset$. Otherwise, as the competition margin binds for everyone (and not the demand margin, as explained earlier), $\mathbf{r}^*$ is an uncoordinated equilibrium. However, as there exists some IPR holders for whom the cap does not

bind, we necessarily also have $\|\mathbf{r}^*\| < R$. This is not possible as $R \leq \underline{R}^u$. Therefore, if there are some IPR holders for whom the competition margin strictly binds, there also are IPR holders for whom the cap strictly binds.

Then, if the pool does not sell, this is because $\mathcal{I}_2$ is non-empty, and therefore $\mathcal{I}_1$ is also non-empty. IPR holders in $\mathcal{I}_1$ are bound by the cap (and not by the competition margin). They have a large contribution and would like to increase their royalties. Those in $\mathcal{I}_2$ are not bound by the cap and are strictly bound by their competition margin. They have a small contribution and would be excluded from the basket if they would price at the cap. The problem of the pool is therefore that the cap (and the share of profits) attributed to the second group is too large. The pool would need to decrease the shares of profits (and hence the caps) given to this second group and increase the shares (and caps) of the first group.

The question is whether the pool can do so without triggering any decrease of an individual royalty throughout the process, and therefore without new IPR holders being bound by their competition margin only. If there exists $j \in \mathcal{I}_2$, i.e., such that $V(\mathbf{1}) - \|\mathbf{r}^*\| = \max_{x_j=0,\|\mathbf{x}\|\geq 1} V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*$, then we know that there also exists $i \in \mathcal{I}_1$, i.e., $V(\mathbf{1}) - \|\mathbf{r}^*\| > \max_{x_i=0,\|\mathbf{x}\|\geq 1} V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^*$. Lemma 4.1 shows that if the pool increases the cap for i, then i will increase its royalty and this will not trigger a decrease of j's royalty (nor from any other IPR holder for whom the competition margin would bind). Therefore, the pool can increase the cap of i and decrease the cap of j accordingly, until either the cap binds for j ($j \in \mathcal{I}_3$), or the competition margin binds for i ($i \in \mathcal{I}_3$). Any element of $\mathcal{I}_3$ stays in $\mathcal{I}_3$ during this process: both its cap and its competition margin remain unchanged (see Lemma 4.1).

The pool can then adjust its sharing rule by an iterative process until $\mathcal{I}_2 = \emptyset$. As long as $\mathcal{I}_2 \neq \emptyset$, the pool can indeed pick $i \in \mathcal{I}_1$ and $j \in \mathcal{I}_2$ and adjust the sharing rule until either $i \in \mathcal{I}_3$ or $j \in \mathcal{I}_3$. This process can be used iteratively at most n times, given that at each step either $\mathcal{I}_1$ or $\mathcal{I}_2$ loses an element to $\mathcal{I}_3$ (and elements of $\mathcal{I}_3$ stay in $\mathcal{I}_3$). Given that $\mathcal{I}_1$ cannot be empty before $\mathcal{I}_2$ is empty, the process stops when $\mathcal{I}_2 = \emptyset$. The sharing rule at the end of the process is such that the cap binds for all IPR holders, and therefore the pool is stable.

$\square$

Proposition 4.2 has two important components. First, it shows that in the presence of independent licensing with the cap, any welfare reducing pool is unstable, and the sharing rule does not allow to stabilize the pool. The proof relies on the fact that if the pool sells despite the individual licenses, i.e., if there exists no smaller basket that beats the pool, the pool's royalty cannot exceed the maximum total royalty of an uncoordinated equilibrium. Then, the pool cannot be welfare reducing. By contraposition, when the pool is welfare reducing, with all the royalties set at the cap, customers prefer to buy a smaller basket of the individual licenses rather than the full bundle from the pool. Overall, there exists no welfare reducing pool that sells with the obligation to offer individual licenses at royalties not exceeding $\alpha_i R$. This is true for any sharing rule $\boldsymbol{\alpha}$, which means that welfare reducing pools cannot use their sharing rules to remain stable.

Second, a welfare increasing pool can always stabilize itself by an adequate sharing rule. Nevertheless, not every sharing rule allows to stabilize a welfare increasing patent pool. The cap creates an additional constraint, one that was not present with the independent licensing alone. However, a pool, with or without independent licensing and the cap, also faces other constraints, related, for instance, to IPR holders' participation and incentives to innovate. As shown in the proof of Proposition 4.2, a sharing rule that destabilizes the pool is such that one IPR holder receives too much of the pool's profit.[25] Then, his competition margin is violated when all the other IPR holders are pricing at the cap. As a consequence, one IPR holder receives too little of the pool's profit, and the pool is not stable. While the full characterization of feasible sharing rules is beyond the scope of this paper, we will use a symmetric case to show that the constraints indeed exist, but still allow a wide set of sharing rules.

[25]We assume in this discussion, as we implicitly did in the whole paper, that there are no ex-ante transfers or lump-sum payments. Remuneration is solely made through the redistribution of the pool's profits. This assumption reflects what pools do in practice.

5 Feasible sharing rules for good pools

We consider the case of symmetry, studied in annex E, where we already know that welfare increasing pools are stable with equal sharing rules (and the corresponding caps). In this case, equal sharing is the fair sharing rule. We propose to analyze further the sharing rules that make welfare increasing pools stable. Here we consider that the pool is stable if $r_i^* = \alpha_i R$ is an equilibrium.[26]

We focus on a symmetric case with n firms. However, we assume that firms might receive different shares of the pools' profits (and therefore face different caps). Without loss of generality, we assume that $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_n$. Then, as we have seen earlier, the only reason why $r_i^* = \alpha_i R$ might not be an equilibrium with the cap is because the competition margin is violated for at least one IPR holder. If all royalties are set at the cap, there exists a smaller basket (of size $k < n$) that is preferred by customers to the full basket with the total royalty R (or with independent licenses that lead to the same total price). Given that we have ranked the shares, this basket of size k is made of the first cheapest k patents, which costs $\sum_{i \leq k} \alpha_i R \geq k\alpha_1 R$. If the competition margin is violated for one IPR holder at the cap, it is necessarily also violated for the IPR holder n who has the largest royalty.

Therefore, we have:

$$V(n) - R < \max_{k<n} \max_{\|\mathbf{x}\|=k} V(k) - (\mathbf{x} \cdot \boldsymbol{\alpha})\, R.$$

Hence, there exists $k < n$ patents such that $V(n) - R < V(k) - k\alpha_1 R \Leftrightarrow (1 - k\alpha_1)R > V(n) - V(k)$. As we explained earlier, the problem of the pool emerges from the fact that the rule is too unequal, i.e., $(1 - k\alpha_1)$ is "large". Here, there is one player that gets too much of the pool's profit ("IPR holder n"), which has the consequence that at least another one ("IPR holder 1") receives too little of the total surplus. This goes against the stability of the pool.

[26] This is a broad definition of stability as there could possibly also exist equilibria in independent licensing with the cap where the pool is destabilized. However, pricing exactly at the share of the pool is an obvious focal point to coordinate and to stabilize the pool.

We can qualify this claim further. The pool has been assumed to be welfare increasing. Therefore:

$$R \leq R^u,$$

where R^u is the total royalty at the uncoordinated equilibrium. Moreover, this total royalty is smaller than the total royalty when the competition margin binds nz:

$$\begin{aligned} R^u &\leq nz \\ &\leq n \min_{i<n} \tfrac{V(n)-V(i)}{n-i} \\ &\leq n \tfrac{V(n)-V(k)}{n-k}. \end{aligned}$$

As $(1-k\alpha_1)R > V(n) - V(k)$ and $V(n) - V(k) \geq (n-k)R^u/n$, we have:

$$\begin{aligned} (1-k\alpha_1)R > \tfrac{(n-k)}{n} R^u \quad &\Leftrightarrow \quad \alpha_1 < \tfrac{1}{n}\left(1 - \tfrac{n-k}{k}\tfrac{R^u - R}{R}\right) \\ &\Rightarrow \quad \alpha_1 < \tfrac{1}{n}\left(1 - \tfrac{1}{n-1}\tfrac{R^u - R}{R}\right). \end{aligned}$$

Overall, we can conclude that if $r_i^* = \alpha_i R$ is not an equilibrium, then $\alpha_1 < \frac{1}{n}\left(1 - \frac{1}{n-1}\frac{R^u-R}{R}\right)$. By contraposition, if $\min \alpha_i \geq \frac{1}{n}\left(1 - \frac{1}{n-1}\frac{R^u-R}{R}\right)$, then the welfare increasing pool is stable. This is a sufficient condition, not a necessary one. We find here, as before, that equal sharing stabilizes the pool. However, we also see that the fact that the pool is welfare increasing, i.e., leads to a smaller total royalty, gives more room for unequal or unfair sharing to be sustainable by the pool. The more the pool is welfare increasing, i.e., $(R^u - R)/R$ is large, the more there is scope for unfair rules. Stability of welfare increasing pools puts a constraint on the degree of the unfairness of the sharing rule, but this constraint can be fairly lax for very welfare increasing pools.

6 Concluding remarks

The first contribution of this paper is showing that if firms offer independent licenses, there exists a continuum of equilibria in independent licensing where patent pools still exploit their market power and harm customers. Consequently, the current requirement of the freedom to independently

license does not ensure that only welfare increasing pools form.[27]

To prevent formation of welfare reducing pools, we propose an ex-ante mechanism where the royalties for the individual licenses cannot exceed the respective patents' shares in the pool's royalty. The freedom to independently license would be replaced by an obligation to publicly offer the individual licenses at royalties not exceeding the cap. Instead of contractually safeguarding the freedom to license, pool administrators would now need to require from their members to publicly offer individual licenses at royalties not exceeding their corresponding stakes in the pool's royalty. They could do so, for instance, by publishing the pool's sharing rule, thereby setting the maximum individual royalties that licensors can charge. To save transaction costs on both sides, these sales could be administered by the pool itself. This screening mechanism is information-free for the regulators and does not involve any additional enforcement costs for them. Moreover, it does not impose any new obligation and administrative burdens on pools.

Our results hold with asymmetric stand-alone patent values and complementarities between them, arbitrary sharing rules, and asymmetric equilibria in royalties. For that matter, this paper can apply to situations when IPR holders contribute unequal bundles of patents to the pool. Moreover, our analysis allows for pools to have an additional degree of freedom to adapt the sharing rule, also in a way that may not reflect contributions, in order to counteract the effects of policy. With the cap, only welfare increasing pools survive. They can always remain stable by assigning higher shares to more valuable contributions, which is also what is required for firms to innovate and participate in pools in the first place.

The augmented screening device we propose would give firms legal certainty and thereby stimulate formation of good pools. The strength of this legal certainty depends on the trust that the competition authorities have in the new rule. Therefore, it is important to reflect on issues that may make such caps distortionary. For example, caps may affect participation in pools, while the pool's size is also an important determinant of consumer welfare. However, we show that the sharing rule which ensures the stability

[27]Note that the requirement to ensure the IPR holders' freedom to independently license is a weaker requirement than forcing them to do so, which as we show is already insufficient.

of good pools in the presence of the cap is also the one that gives higher rewards to high value patents. Consequently, the sharing rule that emerges with the cap should incentivize participation in good pools. Intuitively, appropriate rewards for important discoveries and profits increased through good pools should also have a positive impact on innovation. Welfare increasing pools increase consumer welfare and also increase firms' profits. Therefore, the incentives to innovate are higher when welfare increasing pools are allowed. As welfare increasing pools are stable with the cap, the mechanism we propose preserves incentives to innovate.

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Appendix

A Proof of Lemma 2.1

Consider the following profit $\pi(R,c) = (R-c)D(R-V(\mathbf{x}))$. Then, $\partial\pi/\partial R = ((R-c)\cdot D'/D+1)\cdot D = (1-(R-c)\cdot f/(1-F))\cdot D$. Because of the wide support of θ, $D>0$. Moreover, $1-(R-c)\cdot f/(1-F)$ is continuous and strictly decreasing in R. It equals 1 for $R=c$ and tends to $-\infty$ when R tends to infinity. Therefore, $\partial\pi/\partial R$ equals zero exactly once in $\widehat{R}(c)$ such that $\partial\pi/\partial R(\widehat{R}(c),c)=0$. Moreover, $\pi(R,c)$ is increasing before $\widehat{R}$ ($\partial\pi/\partial R>0$), and it is decreasing after ($\partial\pi/\partial R<0$). Therefore, for a given c, $\pi(R,c)$ has exactly one global maximum in $\widehat{R}(c)$.

Moreover, $\partial^2\pi/\partial R\partial c = -D' > 0$. Therefore, the profit is supermodular in R and c. From that, it directly follows that $\widehat{R}(c)$ is strictly increasing in c as $\widehat{R}'(c) = -(\partial^2\pi/\partial R\partial c)/(\partial^2\pi/(\partial R)^2) > 0$.

B Proof of Lemma 3.1

A deviating firm has the demand and the competition margin. Its maximization problem (in the limit) is the following:

$$\begin{aligned} &\widetilde{r}^* = argmax_{\widetilde{r}}\left\{\widetilde{r}D\left(\widetilde{r}-\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}\right)\right\} \\ u.c.\quad &\widetilde{r} < R+\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}-V(\mathbf{1}). \end{aligned}$$

Using an argument similar to the one showing that the price of the patent pool is smaller than the uncoordinated royalty when the demand margin binds, we can show that it is always the competition with the pool that binds here. The profit of the deviating firm, with the change of variable $\widetilde{R} = \widetilde{r}-\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}+V(\mathbf{1})$, is:

$$\begin{aligned} \pi &= \max_{\widetilde{r}}\left\{\widetilde{r}D\left(\widetilde{r}-\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}\right)\right\} \\ &= \max_{\widetilde{R}}\left\{\left(\widetilde{R}+\max_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}-V(\mathbf{1})\right)D\left(\widetilde{R}-V(\mathbf{1})\right)\right\} \\ &= \max_{\widetilde{R}}\left\{\left(\widetilde{R}-\min_{\mathbf{x}|x_i=1}\left\{V(\mathbf{1})-V\left(\mathbf{x}\right)+\mathbf{x}\cdot\mathbf{r}^*-r_i^*\right\}\right)D\left(\widetilde{R}-V(\mathbf{1})\right)\right\}. \end{aligned}$$

We denote by $\mathbf{x}^* = argmax_{\mathbf{x}|x_i=1}\left\{V\left(\mathbf{x}\right)-\mathbf{x}\cdot\mathbf{r}^*+r_i^*\right\}$ the optimal basket

of independent licenses for customers (for this given $\mathbf{r}^*$). Then, we can see that $\widetilde{R} = \widetilde{r} - r_i^* + \mathbf{x}^* \cdot \mathbf{r}^* + V(\mathbf{1}) - V(\mathbf{x}^*)$ is the total implicit price for a subset of $\mathbf{x}^*$ independent licenses compared to $\mathbf{1}$.

As $\min_{\mathbf{x}|x_i=1} \{V(\mathbf{1}) - V(\mathbf{x}) + \mathbf{x} \cdot \mathbf{r}^* - r_i^*\} > 0$, the deviating firm has to take into account an additional implicit cost compared to the patent pool. Because of supermodularity, we then have $\widetilde{R}^* > \overline{R}$, where:

$$\begin{aligned}\widetilde{R}^* &= argmax_{\widetilde{R}} \left\{ \left(\widetilde{R} - \min_{\mathbf{x}|x_i=1} \{V(\mathbf{1}) - V(\mathbf{x}) + \mathbf{x} \cdot \mathbf{r}^* - r_i^*\} \right) \right. \\ &\qquad \left. \cdot D\left(\widetilde{R} - V(\mathbf{1})\right) \right\} \\ \overline{R} &= argmax_R R \cdot D(R - V(\mathbf{1})).\end{aligned}$$

If we define

$$\widetilde{r}^* = argmax_{\widetilde{r}} \left\{ \widetilde{r} D \left(\widetilde{r} - \max_{\mathbf{x}|x_i=1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^* + r_i^*\} \right) \right\},$$

we have $\widetilde{r}^* = \widetilde{R}^* + \max_{\mathbf{x}|x_i=1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^* + r_i^*\}$ and as $\widetilde{R}^* > \overline{R} \geq R$. Then,

$$\widetilde{r}^* \geq R + \max_{\mathbf{x}|x_i=1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^* + r_i^*\} - V((\mathbf{1}).$$

Hence, the competition margin with the pool binds and the optimal deviation is to price (just below) $\widetilde{r} = R + \max_{\mathbf{x}|x_i=1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^* + r_i^*\} - V(\mathbf{1})$.

C Proof of Lemma 4.1

Let i be such that $\max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} < V(\mathbf{1}) - \|\mathbf{r}\|$. Conversely, j is such that $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} = V(\mathbf{1}) - \|\mathbf{r}\|$. Then, we have:

$$\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} > \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\}.$$

This means that necessarily the max for $x_j = 0$ is reached for a vector $\widetilde{\mathbf{x}}$ such that $\widetilde{x}_i = 1$. Then:

$$\begin{aligned}\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} &= \max_{\mathbf{x}|x_j=0,x_i=1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} \\ &> \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\}.\end{aligned}$$

We also have:

$$\max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} \geq \max_{\mathbf{x}|x_i=0,x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\}.$$

Therefore, for a basket that does not include j, the price increase of IPR holder i is smaller than the incremental utility of acquiring technology i:

$$\begin{aligned}\widetilde{r}_i - r_i &= V(\mathbf{1}) - \|\mathbf{r}\| - \max\nolimits_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} \\ &= \max\nolimits_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} - \max\nolimits_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} \\ &\leq \max\nolimits_{\mathbf{x}|x_j=0,x_i=1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} - \max\nolimits_{\mathbf{x}|x_j=x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\}.\end{aligned}$$

This means that if i increases its royalty up to $\widetilde{r}_i$ such that $V(\mathbf{1}) - \|\mathbf{r}\| + r_i - \widetilde{r}_i = \max_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\}$, then even at this royalty, he will not be excluded from the basket for $x_j = 0$.

Let us now show that i's price increase does not change j's optimal royalty. If j is such that $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} = V(\mathbf{1}) - \|\mathbf{r}\|$, by increasing the total aggregate price of technology, we may arrive to a situation where $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \left\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^{(1)}\right\} > V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\|$ if at the same time $\max_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \left\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^{(1)}\right\}$ is reduced by less than $V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\|$.

However, this is not the case. To see this, let us define the following vector $\mathbf{r}^{(1)}$ after i's price increase:

$$\begin{aligned} r_i^{(1)} &= V(\mathbf{1}) - \max\nolimits_{\mathbf{x}|x_i=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} + r_i - \|\mathbf{r}\| \\ r_j^{(1)} &= r_j \text{ if } j \neq i. \end{aligned}$$

Let us call $\mathbf{x^b}$ the optimal basket without j before i's price increase:

$$\begin{aligned} \mathbf{x^b} &= argmax_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\} \\ &= argmax_{\mathbf{x}|x_j=0,x_i=1,\|\mathbf{x}\|\geq 1} \{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}\}. \end{aligned}$$

Because even after the price increase i is included in the optimal basket without j, then:

$$\max\nolimits_{\mathbf{x}|x_j=0,\|\mathbf{x}\|\geq 1} \left\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^{(1)}\right\} = \max\nolimits_{\mathbf{x}|x_j=0,x_i=1,\|\mathbf{x}\|\geq 1} \left\{V(\mathbf{x}) - \mathbf{x}\cdot\mathbf{r}^{(1)}\right\}.$$

We have:

$$\begin{aligned} & \forall \mathbf{x} | x_j = 0, x_i = 1, \|\mathbf{x}\| \geq 1, V\left(\mathbf{x^b}\right) - \mathbf{x^b} \cdot \mathbf{r} & \geq & V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r} \\ \Rightarrow & \forall \mathbf{x} | x_j = 0, x_i = 1, \|\mathbf{x}\| \geq 1, V\left(\mathbf{x^b}\right) - \mathbf{x^b} \cdot \mathbf{r}^{(1)} & \geq & V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)}. \end{aligned}$$

Then:

$$\begin{aligned} V\left(\mathbf{x^b}\right) - \mathbf{x^b} \cdot \mathbf{r}^{(1)} &= \max_{\mathbf{x} | x_j = 0, x_i = 1, \|\mathbf{x}\| \geq 1} \left\{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)} \right\} \\ &= \max_{\mathbf{x} | x_j = 0, \|\mathbf{x}\| \geq 1} \left\{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)} \right\}. \end{aligned}$$

Moreover, as we had $V\left(\mathbf{x^b}\right) - \mathbf{x^b} \cdot \mathbf{r} = V(\mathbf{1}) - \|\mathbf{r}\|$, we also have $V\left(\mathbf{x^b}\right) - \mathbf{x^b} \cdot \mathbf{r}^{(1)} = V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\|$. Overall, we can therefore conclude that $V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\| = \max_{\mathbf{x} | x_j = 0, \|\mathbf{x}\| \geq 1} \left\{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)} \right\}$. Competition margin still binds for j after i's price increase, and therefore its optimal royalty does not change.

This result is true for any i for which the inequality is strict and any j for which there is equality.

D Lemma D.1

Lemma D.1. *If there exists a vector* $\mathbf{r} \in \mathbb{R}^n$ *such that:*

$$\forall i \in \{1, \ldots, n\}, \max_{\mathbf{x} | x_i = 0, \|\mathbf{x}\| \geq 1} \{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r} \} \leq V(\mathbf{1}) - \|\mathbf{r}\|,$$

then there exists $\mathbf{z} \in \mathcal{Z}$ *and* $\|\mathbf{z}\| \geq \|\mathbf{r}\|$.

Proof. If $\forall i \in \{1, \ldots, n\}, \max_{\mathbf{x} | x_i = 0, \|\mathbf{x}\| \geq 1} \{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r} \} = V(\mathbf{1}) - \|\mathbf{r}\|$, then $\mathbf{r} \in \mathcal{Z}$.

Otherwise, $\exists i \in \{1, \ldots, n\}, \max_{\mathbf{x} | x_i = 0, \|\mathbf{x}\| \geq 1} \{ V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r} \} < V(\mathbf{1}) - \|\mathbf{r}\|$. Then, assuming that there exists no exogenous constraint on the royalties that IPR holders can choose, if all the other players price $\mathbf{r}$, IPR holder i can increase his price, starting from r_i, without being excluded from the optimal basket of independent licenses. We assume he wants to do so because we abstract from the demand margin. We will show that it is possible for one IPR holder (the one closer to the equality) to increase his price and still have the (non strict) inequality hold for all IPR holders. This will allow us to construct $\mathbf{z}$ through an iterative process.

First, let us denote by $\mathcal{I}$ the set of IPR holders not bound by their competition margin:

$$\mathcal{I} = \left\{ i \in \{1, \ldots, n\} \, | V(\mathbf{1}) - \|\mathbf{r}\| > \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} \right\}.$$

Then, let us define:

$$\widetilde{k} \in argmax_{i \in \mathcal{I}} \left\{ \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} \right\}.$$

Among all the IPR holders for which the inequality is strict, $\widetilde{k}$ is the closest (or one of the closest if there are several) from the equality, and hence it would increase its price the least.

Then, it is possible to define the following vector $\mathbf{r}^{(1)}$:

$$\begin{array}{rcl} r_{\widetilde{k}}^{(1)} & = & V(\mathbf{1}) - \max_{\mathbf{x}|x_{\widetilde{k}}=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} + r_{\widetilde{k}} - \|\mathbf{r}\| \\ r_j^{(1)} & = & r_j \text{ if } j \neq \widetilde{k}. \end{array}$$

$\mathbf{r}^{(1)}$ is the updated vector of royalties where only $\widetilde{k}$ has increased its price to reach the equality. We want to show that after this price increase, we still have:

$$\forall k \in \{1, \ldots, n\}, \max_{\mathbf{x}|x_k=0, \|\mathbf{x}\| \geq 1} \left\{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)}\right\} \leq V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\|.$$

For all the other $i \in \mathcal{I}$ for which the inequality was strict, we have:

$$\begin{array}{rcl} V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\| & = & \max_{\mathbf{x}|x_{\widetilde{k}}=0, \|\mathbf{x}\| \geq 1} \left\{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)}\right\} \\ & = & \max_{\mathbf{x}|x_{\widetilde{k}}=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} \\ & \geq & \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}\} \\ & \geq & \max_{\mathbf{x}|x_i=0, \|\mathbf{x}\| \geq 1} \left\{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)}\right\}. \end{array}$$

Therefore, because we have updated the price of the IPR holder who wanted the smallest price increase, the inequality still holds for all the other IPR holders who wanted to increase their prices. Moreover, as shown in Lemma 4.1, this price increase does not change the optimal royalty of the IPR holders who were bound by their competition margin before the price increase. Therefore, the (weak) inequality still holds after the price increase

for all the IPR holders: $\forall k \in \{1, \ldots, n\}, \max_{\mathbf{x}|x_k=0, \|\mathbf{x}\|\geq 1} \left\{V(\mathbf{x}) - \mathbf{x} \cdot \mathbf{r}^{(1)}\right\} \leq V(\mathbf{1}) - \left\|\mathbf{r}^{(1)}\right\|$.

Because of this inequality, we can repeat the same algorithm: if the inequality with $\mathbf{r}^{(1)}$ is strict for one $i \in \{1, \ldots, n\}$, we can define $\mathbf{r}^{(2)}$ as before (by updating the royalty i for which the inequality is strict and which has the largest outside option for customers). This process stops after at most n iterations at the vector $\mathbf{r}^{(lim)}$. The only fixed points of this algorithm are the equilibria where the competition margin binds for all IPR holders. However, the algorithm necessarily stops at a fixed point. Thus $\mathbf{r}^{(lim)} \in \mathcal{Z}$. Given that at each iteration the total royalties increase, $\left\|\mathbf{r}^{(lim)}\right\| \geq \|\mathbf{r}^*\|$.

□

E Symmetric case

If all patents are of equal values, the symmetric uncoordinated equilibrium royalty r^u has to be such that customers (weakly) prefer to buy n patents at royalty r rather than any subset m, $m \in \{1, \ldots, n-1\}$. This requires that for all $m \in \{1, \ldots, n-1\}$, $V(n) - nr \geq V(m) - mr \Leftrightarrow r \leq \frac{V(n)-V(m)}{n-m}$. Hence, the symmetric equilibrium royalty when the competition margin binds is $z = min_{m\in\{1,\ldots,n-1\}} \frac{V(n)-V(m)}{n-m}$. The symmetric equilibrium royalty when the demand margin binds is $\widehat{r} = argmax_r \left\{rD(r + (n-1)\widehat{r} - V(n))\right\}$. Overall, as IPR holders are constrained by either of the two margins, the (unique) symmetric uncoordinated equilibrium royalty without the pool is $r^u = min\left\{z, \widehat{r}\right\}$.

Without independent licensing, a pool would charge a total royalty $\overline{R} = argmax_R RD(R - V(n))$, each member receiving a share $\alpha_i = 1/n$ of the pool's profits. A pool, with and without independent licensing, is welfare increasing when the demand margin binds in the absence of the pool. By contraposition, a pool can only be welfare reducing if it is the competition margin that binds. Overall, a patent pool is welfare reducing if and only if $r^u = z$, and the aggregate price R satisfies $R/n > z$.[28]

[28]If the demand margin binds, then the patent pool is welfare increasing. Therefore, if the patent pool is welfare reducing, the competition margin binds and $r^u = z$. Moreover, $R/n \leq \overline{R}/n \leq \widehat{r}$. Therefore, $R/n > z \Rightarrow z \leq \widehat{r}$, and the competition margin binds.

In this symmetric context, we can simplify stability conditions 3.2 and 3.3 as follows:

$$r^* \geq \max \begin{Bmatrix} \max_{m\in[2,\dots,n]} \frac{1}{m-1}\left(V(m)-V(n)+\frac{(n-1)R}{n}\right), \\ \max_{m\in\{1,\dots,n\}} \frac{1}{m}\left(V(m)-V(n)+R\right) \end{Bmatrix}. \tag{E.1}$$

Then, a pool would be welfare reducing absent independent licensing if $\overline{R}/n > z$. With independent licenses that do not destabilize the pool, if $V(1) < V(n) - \frac{(n-1)\overline{R}}{n}$, the stable pool still implements $R_{max} = \overline{R}$, which is welfare reducing. Otherwise, it implements $R_{max} = n.\frac{V(n)-V(1)}{n-1}$. However, $\frac{V(n)-V(1)}{n-1} \geq min_{m\in\{1,\dots,n-1\}} \frac{V(n)-V(m)}{n-m} = z$. Therefore, a stable pool is welfare reducing unless $z = \frac{V(n)-V(1)}{n-1}$, as then it is welfare neutral.

In this symmetric case, it is easy to see that the game with two IPR holders is a special one. For a pool to be welfare increasing, the competition margin has to bind in the absence of the pool, such that the total uncoordinated equilibrium royalty equals $2(V(2)-V(1))$. Overall, a patent pool is therefore welfare reducing if and only if $2(V(2)-V(1)) < R$ (we note that in any event $R < 2\widehat{r}$, so the previous inequality implies that the competition margin binds). The patent pool is welfare reducing if the complementarity between the two patents is limited.[29] However, the patent pool is stable only if $R \leq 2(V(2)-V(1))$. Therefore, with two IPR holders, there are no welfare reducing pools that are stable to independent licensing. This is consistent with the result in Lerner and Tirole (2004), who show that with two IPR holders, any of the two IPR holders would unilaterally deviate from a welfare reducing pool (IPR holders would necessarily deviate, see Proposition 3.2).

Notice that the situation is very different with more than two IPR holders. Without pools, the competition margin is determined by the competition from baskets of 1 to $n-1$ patents. The total price of technology

Then, $R/n > r^u$ and the pool is welfare reducing.

[29]This interpretation is even clearer when θ has a degenerated distribution and is for sure null. Then, the pool prices $V(2)$, which is also the total royalty when the demand margin binds. Moreover, the pool is welfare reducing if and only if $2(V(2)-V(1)) < V(2) \Leftrightarrow V(1) > \frac{V(2)}{2}$. Then the competition margin binds if the two patents are not very strong complements, i.e., the returns from acquiring the technology are decreasing. If patents are very strong complements, i.e., the returns from acquiring the technology are increasing, the demand margin binds and the patent pool is welfare neutral.

has to be smaller than both $n \cdot min_{m\in\{1,\ldots,n-1\}} \frac{V(n)-V(m)}{n-m}$ and $n \cdot \widehat{r}$. The only competition that remains for a patent pool with independent licensing, provided that r^* exceeds the threshold in equation E.1, is the competition from each individual IPR holder. The total price of technology for stable patent pools has to be smaller than $n\frac{V(n)-V(1)}{n-1}$ and $\overline{R}$ (with $\overline{R} \leq n \cdot \widehat{r}$). The patent pool with independent licensing removes any competition from baskets of 2 to $n-1$ patents. With 2 IPR holders, the only constraint without the pool is already determined by each individual IPR holder. Therefore, here, the pool with independent licensing does not remove any source of competition. Conversely, with at least three firms, the pool with independent licensing does remove significant sources of competition.

However, the cap set at $\alpha_i = 1/n$ would destabilize all welfare reducing pools here and leave the others unaffected. First, the proof that the cap destabilizes welfare decreasing pools is immediate here. A stable patent pool sells in the presence of independent licenses constrained by the cap when: $V(n) - nr^* \geq \max_{m\in\{1,\ldots,n-1\}} \{V(m) - mr^*\}$. Hence, customers (weakly) prefer to buy all the patents rather than any smaller subset of patents and so the competition margin is not violated. This inequality implies that $r^* \leq z \Leftrightarrow R \leq nz$, where z is the price when the competition margin binds. Therefore, a welfare reducing pool, i.e., one that prices $R > nz$, cannot sell, and thus cannot be stable, with the cap at R/n.

Assume now that the pool is welfare increasing, i.e., $R/n \leq z$. We can show that the competition and the demand margin bind at a royalty that is larger than or equal to R/n, which exceeds the cap. Indeed, as discussed before, as far as the demand margin is concerned, the best response of an individual IPR holder leads to a total royalty that exceeds or equals $\overline{R}$, which is larger or equal to R:

$$\widehat{r} + (n-1)r^* \geq \overline{R} \geq R.$$

As with the cap $r^* \leq R/n$, the price for $n-1$ patents is smaller than or equal to $(n-1)R/n$. Then, for whatever positive or zero individual royalties of the other IPR holders, the demand margin binds at a royalty

larger or equal to R/n:

$$\widehat{r} \geq R - (n-1)r^* \geq \frac{R}{n}.$$

Moreover, as there is only one symmetric equilibrium where the competition margin binds, and it is at $z \geq R/n$ (the pool here is welfare increasing), the competition margin binds at royalties larger than or equal to the cap. Therefore, the cap binds.

Overall, neither the demand nor the competition margin can bind for a candidate equilibrium r^* of the continuation game with the cap set at R/n unless the cap also binds, and then indeed the constraint of the cap always binds. The pool therefore necessarily sells and $r^* = R/n$.

We have shown uniqueness, we now need to show existence, i.e., that if $r^* = R/n$ then no IPR holder wants to undercut the pool. Indeed, the difference between the profit of the best deviation and the profit derived from the pool is lower than or equal to zero:

$$\begin{aligned} R + V(m^*) - (m^*-1)r^* - V(n) - \tfrac{R}{n} &= (n-m^*)\tfrac{R}{n} - (V(n) - V(m^*)) \\ &\leq 0, \end{aligned}$$

where $V(m^*) - (m^*-1)r^* = \max_{m \in \{1, \ldots n\}} V(m) - (m-1)r^*$ and $\frac{R}{n} \leq z \leq \frac{V(n)-V(m^*)}{n-m^*}$. Therefore, no IPR holder wants to deviate and $r^* = R/n$ is indeed a continuation equilibrium in which the pool sells. And as this is the only equilibrium, the pool is stable to independent licensing with the cap.

Note that in this symmetric case a welfare increasing pool does not need to manipulate its natural equal sharing rule to remain stable to independent licensing with the cap.