Buyer Power and Dependency in a Model of Negotiations

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Abstract

We study bilateral bargaining between buyers and sellers in a framework that allows both sides flexibility to adjust trades during (temporary) disagreement. Our bargaining framework encompasses the outcome of auctions in truthful menus as limiting cases. Looking at the equilibrium per unit transaction prices, we find that following an horizontal merger this new larger buyer pays a lower price only when buyers' bargaining power in bilateral negotiations is sufficiently high, and a higher price otherwise—a similar opposite result holds for mergers of sellers. These ambivalent effect is explained by how size affects own dependency of a buyer on each seller and the dependency of each seller on a buyer. This suggests that size is not a substitute (but rather a complement) for bargaining power in bilateral negotiations. When sellers' bargaining power is lower, their payoff is more generally negatively affected when market shares of buyers become more asymmetric. The richer predictions of our model may help to explain the recent ambiguous empirical evidence on buyer size and inform empiricists, business strategists, and antitrust practitioners.

Keywords: Buyer power, dependency, bargaining.

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1 Introduction

This paper develops a bargaining model where prices arise endogenously from the ability of buyers to relocate their purchases to alternative sellers and from the ability of sellers to relocate their sales between different buyers. Prices are determined in bilateral bargaining between buyers and sellers, and disagreement payoffs are obtained from letting the respective seller and buyer temporarily adjust supplies to, or purchases from, other parties. The model proves tractable enough to accommodate various distributions of bargaining power and, for extreme distributions of bargaining power, nests those situations where buyers or suppliers bid in truthful menus in the respective auctions.

The model is applied to a question that has become increasingly important for researchers in Industrial Organization as well as antitrust and business strategy practitioners, namely whether size is of an advantage in vertical relationships. By both allowing for multiple sellers and buyers and interacting size with bilateral bargaining power from other sources, our proposed modelling approach to multilateral negotiations produces a rich set of predictions. In this model, and consistent with previous results, a larger buyer always obtains a lower price if there is a *single* seller. However this result does not extend to those (often more realistic) situations with *multiple* sellers of (imperfect) substitute goods. With several buyers and sellers, a large buyer, e.g., that forms after a merger, obtains better terms if and only if the bilateral bargaining power of sellers is sufficiently low - and, notably, when buyers bid in auctions. Remaining small generates an advantage when buyers have only limited bargaining power. The impact of size on per-unit prices depends further crucially on technology and in particular on whether this allows for an adjustment of bilateral trades when there is (temporary) bilateral disagreement with some buyer or seller. The intuition for these results hinges on our concepts of seller or buyer dependency, as discussed next.

Disagreement with a large buyer, who controls a larger fraction of potential demand, leaves a seller with few opportunities to replace his sales to alternative buyers. Instead, a disagreement with a small buyer displaces a smaller fraction of demand, but leaves the seller with many alternatives to turn to. In this sense a seller is more dependent on a larger buyer. However, when a large buyer finds himself in disagreement, it is equally less profitable for him to (temporarily) relocate demand to other sellers, as limits to capacity and more

generally increasing marginal costs at higher production volumes render it impossible or relatively more expensive for them to accommodate (off-equilibrium) a large required increase in sales.

Still we obtain clear-cut predictions. When buyers' bilateral bargaining power is low, e.g., because they are relatively less patient, a newly formed larger buyer obtains worse terms of supply. But when buyers' bargaining power is high, buyers are able to push sellers close to their inside options, resulting in better terms for the new, larger buyer. In particular, these observations yield unambiguous predictions for auctions, which are nested in our bargaining approach.

That relocation of demand can be more costly to a larger buyer has been largely overlooked by the literature. One reason for this is that the effect does not arise when there is a monopolistic buyer, and it is also overlooked when one considers only a single seller or an alternative source of supply that is outside the considered market, all of which are features that are shared by various previous contributions to the literature, as reviewed below. However, as our analysis shows, this countervailing effect of size on buyer power arises naturally from a parsimonious model that admits multiple sellers and buyers. Our results also support the view that relatively concentrated purchases are not per se conclusive of the existence of buyer power (in the sense of better terms of supply). Instead they suggest that practitioners and empiricists may also want to account for particular technological features of the specific industry, such as whether sellers in the market are able to accommodate large scale switching by adding capacity or utilizing existing capacity more extensively at a reasonable cost, or they may want to consider the specific organization of procurement processes (e.g., whether they are determined in buyer or seller auctions).

Overall, our work supports the view that buyer or seller power may be more industry specific than previously thought and that there is a richer relationship to size. This seems to accord well with facts. Recent evidence on the U.S. pharmaceutical industry indeed suggests that size alone is no guarantee to obtain discounts (see, for instance, Ellison and Snyder, 2010, Sorensen, 2003, and Grennan, 2013). From the detailed inquiry of the UK's Competition Commission into the contracting practices in the national grocery industry, we learn that while the buying practices of globally active supermarket chains ensure on average better terms and conditions, there is a wide variation of per unit prices across volumes (Competition Commission, 2008).

Our work can therefore provide guidance for antitrust and competition policy, for which the exercise of buyer power has become an increasing concern. From this perspective, our model seems to be particularly attractive for two reasons. First, it focuses on the concepts of ease of substitution and dependency and therefore seems to formalize concerns that have previously been informally expressed in the European Commission's guidelines on horizontal mergers as well as in recent sector inquiries.[1] In fact, as a side result we also establish a relationship between the HHI, as a common concentration measure, and dependency. Second, its predictions accord well with the views taken in several important court cases where doubts were cast on the presumption that larger buyers virtually always pay lower prices than smaller buyers.[2]

The rest of this paper is organized as follows. Section 2 relates our work to the literature. Section 3 introduces the economy. In Sections 4 and 5 we study a model of bargaining with temporary adjustments and also relate the equilibrium outcome to that of first-price menu auctions. In Section 6 we analyze conditions under which merging buyers pay either a higher or a lower per unit price. Section 7 extends the analysis, amongst other things to the complementary question of the formation of larger sellers. We conclude in Section 8. All proofs are collected in the Appendix.

[1] In the guidelines buyer power is defined as "the bargaining strength that the buyer has vis-à-vis the seller in commercial negotiations due to its size, its commercial significance to the seller and its ability to switch to alternative suppliers". This definition highlights that, in addition to size, an assessment of buyer power needs to take into account two additional considerations: one are the consequences to the supplier from loosing a particular buyer, another are the consequences to the buyer from loosing a particular supplier. Notably the German antitrust authority (report available at http://www.bundeskartellamt.de/Sektoruntersuchung_LEH.html) in its sector inquiry into supply relationships in the grocery market has expressed concerns about manufacturers' dependency vis-á-vis retailers that account for a considerable fraction of their overall sales. While the concept of such dependency is not made precise, let alone formal, it has far-reaching implications, as witnessed by its 2014 decision to impose fines on EDEKA, the largest German retailer, following the demand of better conditions after the take-over of the retailer PLUS in 2008 (document 03_07_2014_edeka.html available at http://www.bundeskartellamt.de/SharedDocs/Meldung/DE/Pressemitteilungen/2014/).

[2] In Hutchison/RCPM/ECT (2001) the container terminal operators involved argued that it would be easy for a carrier to switch large volumes to alternative ports and therefore large operators could still negotiate low prices. Yet the European Commission rejected that argument by stating that switching opportunities were limited for the largest operators since "there is currently a limited number of terminal operators able to accommodate the largest vessels being used" and "it becomes economically more difficult for the carrier to switch ports for a significant portion of its cargo". In yet another case concerning toilet tissue and kitchen towels, SCA/Metsa Tissue (2001), the European Commission observed that "buyer power can only be exercised effectively if the buyer has an adequate choice of alternative suppliers."

2 Related Literature

At the core of our approach to bilateral negotiations is the possibility of (temporary) adjustments to bilateral contracts, as these shape the alternative options of both buyers and sellers, i.e., their respective disagreement points. In Section 5 we motivate in detail our choices in terms of "inside options", i.e., of temporary adjustments. There, we also stress that the adjustments still honour the respective bilateral contracts that have (already) been concluded, while adjusting the level of trade, and that future agreement is still expected whenever this generates positive surplus. We offer also a non-cooperative foundation of the applied bargaining solution, where in case of delay in bilateral negotiations, the two affected parties can temporarily adjust their transactions with all other parties. Different assumptions on the outcome in case of disagreement underpin several other prominent solutions. In this section, we organize the discussion of the related literature around this theme.

A common approach is to assume that in case of a bilateral disagreement all other agreed terms (of linear supply contracts) remain unchanged. This approach was pioneered in Horn and Wollinsky (1988) and has been subsequently adopted in both theoretical and empirical work (e.g., Chipty and Snyder, 1999, Björnerstedt and Stennek, 2007, Crawford and Yurukoglu, 2012, Grennan, 2013). While the approach certainly offers the attraction of simplicity, it still has important limitations. For example, in situations like the one we consider here with decreasing returns to production, if one buyer fails to agree with one seller and that buyer can increase its order from the alternative sellers at the same unit price, then all those alternative sellers may realize a strictly negative payoff off equilibrium—to avoid this it is often assumed that marginal costs are constant, which as our work shows is not without significant loss of generality. Moreover, as we discuss in Section 6 in more detail, under this assumption mergers or the formation of buyer groups that do not generate horizontal concerns should have no impact on upstream (wholesale) contracts, which is however not the view of various antitrust authorities and also seems not to accord well with facts.

As we discuss further in Section 6, our modelling approach also suggests to incorporate more information both about suppliers' full cost function and their alternative options, notably as often also suppliers may be able to instigate (temporary) adjustments to de-

liveries. With respect to costs and technology, Section 7 analyses cases where a buyer's or sellers' adjustment options are restricted, e.g., as sellers have limited capacity, showing that then the impact of size may become unambiguous. This is notably different from the unambiguous effect of size that arises when a large buyer's advantage stems from the (more) credible threat to access an option that is outside the market (such as backward integration; cf. Katz, 1987). Then, fixed costs from accessing the alternative option generate increasing returns to scale from switching. Our setting seems more applicable when negotiations are shaped by the, often more credible, option to (temporarily) relocate demand across sellers in the same market (and with which a buyer has a relationship).

In the extant theoretical literature, a more elaborate approach assumes that each bilateral disagreement restarts all previously successful negotiations for all parties. It does however rule out the future formation of a "grand coalition" that would include the link on which agreement has not been reached—as in Stole and Zwiebel (1996) and deFontenay and Gans (2014). Also the assumption of ultimately non-binding contracts, from which parties can walk away whenever there is disagreement with *other* trading partners, may not be equally suitable across all applications. With such renegotiations the solution coincides with random order values, with the Shapley value being obtained when symmetry on bilateral bargaining power is imposed.

This ties up with a small literature in cooperative game theory that has analyzed the possibility that owners of substitutable resources may lose from forming a monopoly in market games, which has been on identified in Postlewaite and Rosenthal (1974) for the Core, later studied by Gardner (1977), Guesnerie (1977) and Segal (2003) for the Shapley value and by Legros (1987) for the Nucleolus. Specifically, Segal (2003) shows that for random order values a pairwise merger may or may not be profitable depending on a complex chain of how each of the remaining players increases or decreases the substitutability of the two merging parties in each possible market configuration. In Section 6 we discuss in more detail how predictions differ, how we add to that debate and why our model may be better suited than alternative concepts to many of the applications of supplier-buyer relationships in industrial organization.

3 Buyers, Sellers, and Trades

There is a set of goods G, with $|G|$ denoting the number of goods. The cost of producing x units of each good $i \in G$ is $c(x)$, where c is a continuously differentiable, strictly increasing, and strictly convex function with $c(0) = 0$. Each seller is the only producer of a subset of goods. S is the partition of G, with $|S|$ elements, such that each element $I \in S$ contains exactly the $|I|$ goods produced by each seller I.

There are $|N|$ symmetric consumers in a set N. The utility of a consumer $j \in N$ is $u(a_j) + t$, where t is money and a_j is a generic vector in $R_+^{|G|}$ denoting the quantity of each of the $|G|$ goods consumed by j. The utility function u is symmetric, twice continuously differentiable, and strictly concave, with strictly positive first-order derivatives and strictly negative cross-partial derivatives, so that goods are substitutes and there are strictly decreasing benefits of consumption. Each buyer is a group of consumers that will bargain jointly with each seller. Formally, B denotes the partition of the set of consumers N, such that each of its $|B|$ elements contains exactly the $|J|$ consumers represented by each buyer $J \in B$. Throughout the paper we will generically refer to I as a seller and J as a buyer.

Trade in the economy is summarized by a trade matrix A with dimension $|N| \times |G|$ where each element a_{ij} is the quantity of good $i \in G$ delivered to consumer $j \in N$. The column vector a_j therefore represents the quantity of each good that is delivered and consumed by consumer j. The gross benefit of each buyer $J \in B$ at A is

$$v_J(A) = \sum_{j \in J} u(a_j),$$

and the total production cost to seller $I \in S$ at A is

$$C_I(A) = \sum_{i \in I} c(\sum_{j \in N} a_{ij}).$$

Total surplus is concave and is given by

$$\Pi(A) = \sum_{J \in B} v_J(A) - \sum_{I \in S} C_I(A).$$

The marginal cost $c'(0)$ is assumed to be sufficiently low and $c'(x)$ sufficiently large for large x such that the unique level of trade that maximizes the economy surplus is strictly positive and finite. Let A^* be the trade matrix that maximizes the economy surplus. Given symmetry and assumed convexity of costs and concavity of utility, for each good $i \in G$

and consumer $j \in N$ we have $a^*_{ij} = a^*$ with $c'(|N|\, a^*) = u_i(a^*_j)$, where a^*_j is a column vector with each element equal to a^* and u_i is the partial derivative with respect to good $i \in G$. (Hence, $|N|\, a^*$ is the production and consumption quantity for each good.) By imposing symmetry on consumers and goods we ensure that differences between buyers and sellers are accounted for only by differences in size. The characterization of our bargaining solution does, however, not rely on symmetry.

We study those situations where each seller and buyer pair $\{I, J\}$, with $I \in S$ and $J \in B$, try to reach an agreement specifying a $|I| \times |J|$ trade matrix A_{IJ}, with each element a_{ij} representing the quantities of each good $i \in I$ delivered to each consumer $j \in J$, and t_{IJ} a transfer received by seller I from buyer J.[3] The set of agreements between all buyers and sellers is summarized by the pair (A, T) where A is a trade matrix and T a $|B| \times |S|$ matrix of transfers. In the following sections we study how buyers and sellers come to these agreements. As a first step, and before introducing bargaining, it will be helpful to discuss the polar cases where either sellers or buyers are auctioneers in first-price menu auctions.

4 Auctions with Truthful Menus

In the first stage of an auction, bidders simultaneously submit a menu to each auctioneer. A menu specifies contingent payments for each possible bilateral trade. In the second stage, each auctioneer selects whether to accept each menu, picks a level of bilateral trade from each accepted menu, and the respective payments are made. We consider both auctions organized by sellers and those organized by buyers. These will also represent the polar cases of our subsequently analyzed bargaining game.

Truthfulness. Suppose first that all buyers conduct auctions simultaneously. In that case sellers compete with each other by bidding supply contracts. Rather than stipulating only a single transfer, the strategy of each seller $I \in S$ is to offer a menu $t(A_{IJ})$ to each buyer $J \in B$. These menus stipulate a transfer for each possible trade matrix A_{IJ}.[4] A strategy for each buyer J is then to choose a trade matrix A_{IJ} and to pay the respective

[3] We could likewise stipulate that a contract prescribes for each good an aggregate volume delivered to the respective buyer, which would then distribute it (efficiently) across the consumers that it represents.

[4] Note here that we abbreviate the notation somewhat as we write $t(A_{IJ})$ rather than also indexing the function accordingly, i.e., by writing instead $t_{IJ}(A_{IJ})$.

transfer, or alternatively to reject a menu. To abbreviate the following expressions, we capture the non-trade or rejection option by specifying $t(0) = 0$. For a particular strategy profile, seller I's payoff when buyers choose to trade A is then given by

$$\theta_I(A) = \sum_{J \in B} t(A_{IJ}) - C_I(A),$$

and buyer J's payoff is

$$\theta_J(A) = v_J(A) - \sum_{I \in S} t(A_{IJ}).$$

It is well known that even with a single auctioneer and complete information such auctions have in general a continuum of Nash equilibria, many of which are implausible. Bernheim and Whinston (1986) proposed the concept of truthful Nash equilibria, i.e., equilibria in which bidders use truthful strategies, as an attractive refinement of the equilibrium set. We now adopt this concept to our case with multiple auctioneers and bidders. Following Bernheim and Whinston (1986), seller I's menu offer $t(A_{IJ})$ is said to be truthful, now relative to a trade matrix A, if and only if for every other $A'_{IJ} \neq A_{IJ}$ and $A'_{IJ} \neq 0$ we have that

$$t(A'_{IJ}) - C_I(A') = t(A_{IJ}) - C_I(A), \tag{1}$$

where A' satisfies $a'_{kl} = a_{kl}$ if $k \notin I$ or $l \notin J$. This means that, having A as a reference, each menu $t(A_{IJ})$ allows buyer J to change trade with seller I at conditions such that the respective transfer is adjusted to exactly reflect the respective change in I's costs. It is in this sense that the seller's supply contract truthfully reflects his marginal costs.

Suppose now, instead, that sellers conduct auctions simultaneously. The strategy spaces are then swapped between buyers and sellers: buyers submit demand menus and sellers accept or reject and choose a level of trade from each menu. A buyer $J's$ menu $t(A_{IJ})$ is now said to be truthful relative to a trade matrix A if and only if for every other $A'_{IJ} \neq A_{IJ}$ and $A'_{IJ} \neq 0$ we have that

$$v_J(A') - t(A'_{IJ}) = v_J(A) - t(A_{IJ}), \tag{2}$$

where A' satisfies $a'_{kl} = a_{kl}$ if $k \notin I$ or $l \notin J$. Having A as a reference, $t(A_{IJ})$ allows seller I to change his trade with buyer J provided the transfer is adjusted to exactly reflect the respective change in the surplus of J. That is, when buyers bid, the transfer $t(A_{IJ})$ truthfully reflects the respective buyer's marginal valuation (which in turn reflects the aggregated marginal utilities of the respective consumers).

Definition 1. *Let Γ denote the set of all bidders' menus and A a trade matrix that summarizes the trades chosen by the auctioneers. Then a strategy profile (A, Γ) is said to be a Truthful Nash Equilibrium of a first-price menu auction if and only if it is a Nash Equilibrium and bidders' strategies are truthful relative to A.*

Equilibrium Construction. Consider the following trade matrix for a given buyer and seller pair:[5]

$$A^{IJ} = \arg\max \Pi(A) \text{ s.t.} \begin{cases} a_{ij} = 0 \text{ if } i \in I \text{ and } j \in J \\ a_{ij} = a^* \text{ if } i \notin I \text{ and } j \notin J \end{cases}.$$

That is, to obtain from the matrix of efficient trades A^* the trade matrix A^{IJ} the following steps are undertaken. First, all bilateral trades between these two parties are set equal to zero, $a_{ij}^{IJ} = 0$ if $i \in I$ and $j \in J$. Second, we leave all trades between any other buyers and sellers unchanged, so that $a_{ij}^{IJ} = a^*$, if both $i \notin I$ and $j \notin J$. The final step is to adjust the trades between seller I and all buyers other than J and between buyer J and all sellers other than I. These trades are adjusted so that total surplus is maximized, which by strict concavity of $\Pi(A)$ yields a unique solution.

Take now first the case where sellers are the auctioneers. When seller I rejects the bid of buyer J, the optimal reallocation of trades is represented by the respective row vectors a_i^{IJ} (of A^{IJ}) for goods $i \in I$. Note that when making these adjustments, from the truthfulness of all buyers' bids seller I can extract the full incremental valuation, which for a given buyer $J' \neq J$ equals $v_{J'}(A^{IJ}) - v_{J'}(A^*)$, while the difference between the seller's on equilibrium costs and his respective off equilibrium costs after this adjustment is $C_I(A^*) - C_I(A^{IJ})$. As he is just kept indifferent between acceptance and rejection, the difference between these two terms yields the transfer $t(A_{IJ}^*) = \varphi_{IJ}$, as reported in (3) of Proposition 1.[6] Uniqueness follows from the strict concavity of the total surplus function $\Pi(A)$ and as from (1) each seller chooses, holding all other trades constant, each respective trade a_{ij} so as to maximize total surplus.

The case when sellers bid is analogous, as now the reservation value of buyer J with respect to the offer of seller I is obtained from acceptance and optimal adjustment of the offers made by all other sellers $I' \neq I$. The respective transfers $\varkappa_{IJ}$ are reported in (4).

[5] That is, superscripts denote that these trades are obtained from adjustments.

[6] Note also that a seller can also not profitably deviate by rejecting the offers of more than one buyer, instead, and adjust trades to all other buyers. This observation follows immediately from our specification of decreasing returns.

Proposition 1 *A menu auction where either buyers bid or sellers bid has an unique Truthful Nash Equilibrium, and trade is given by the efficient trade matrix A^*, while equilibrium transfers between each seller $I \in S$ and each buyer $J \in B$ are characterized as follows in the two cases:*
i) Seller auction (buyers bid):

$$t_{IJ} = \varphi_{IJ} \equiv \sum_{J' \in B \setminus J} (v_{J'}(A^{IJ}) - v_{J'}(A^*)) - (C_I(A^{IJ}) - C_I(A^*)). \tag{3}$$

ii) Buyer auction (sellers bid):

$$t_{IJ} = \varkappa_{IJ} \equiv (v_J(A^*) - v_J(A^{IJ})) - \sum_{I' \in S \setminus I} (C_{I'}(A^*) - C_{I'}(A^{IJ})). \tag{4}$$

Discussion. It can be easily checked that all bilateral transfers are strictly higher when sellers bid instead of when buyers bid, i.e., $\varphi_{IJ} < \varkappa_{IJ}$. How the transfers compare across buyers and sellers as a function of size will be analyzed later. Before moving on, note that both the difference between φ_{IJ} and $\varkappa_{IJ}$ and variations across seller-buyer pairs are due to the fact that each buyer and seller enjoys some "market power" in the following sense. Denote $\mu = c'(|N|\, a^*) = u_i(a_j^*)$ and recall that $|N|\, a^*$ is the total production volume of each product and a_j^* the consumption vector of each consumer. We can show that both transfers, φ_{IJ} and $\varkappa_{IJ}$, converge to the limit $|I|\,|J|\,\mu a^*$, where $|I|$ denotes the number of goods produced by seller I and $|J|$ the number of consumers represented by buyer J, when we replicate our economy more and more often. This competitive benchmark is intuitive as then, in the limit, at disagreement the responding side could fully replace the respective trades by turning to (infinitely many) other producers or consumers of this good. By truthfulness a disagreeing buyer would then have to pay, in the limit, to each of the remaining producers only the marginal costs $c'(|N|\, a^*)$ and a disagreeing seller could extract from each of the respective alternative consumers the marginal utility $u_i(a_j^*)$.[7]

5 Bargaining

We now take a bargaining approach. The previously characterized auctions in truthful menus will represent the limiting cases when all bargaining power lies with sellers or with buyers. We provide a formulation of the bargaining problem both in terms of applying the

[7] A proof is contained in the online appendix.

asymmetric Nash bargaining solution to all bilateral negotiations and as an equilibrium of a non-cooperative bargaining game. After deriving our key comparative results, in relation to buyer power, we will discuss how the characterized bargaining solution differs from other solutions proposed in the literature, both in the underlying assumptions and implications.

Bilateral (Axiomatic) Nash Bargaining. For the bilateral bargaining problem between each seller and buyer pair $\{I, J\}$, with $I \in S$ and $J \in B$, the generic payoff for each side is $\theta_I(A)$ and $\theta_J(A)$. Denote further the net disagreement payoffs by d_{IJ} and d_{JI} for I and J respectively. Below we will discuss in detail the choice of disagreement payoffs.

Given that utility is transferable, each trade A_{IJ} that is consistent with bilateral Nash bargaining, i.e., that maximizes the Nash product, will maximize the bilateral gains from agreement between I and J, which are given by

$$g_{IJ}(A) = \theta_I(A) + \theta_J(A) - d_{IJ} - d_{JI}.$$

The asymmetric Nash solution captures differences in bilateral bargaining power through the respective choice of the sharing rule. The outcome of Nash bargaining with seller power ρ awards seller I a share ρ of the gains from bilateral agreement plus his disagreement payoff d_{IJ} and buyer J the remainder of those gains plus his disagreement payoff d_{JI}. This yields[8]

$$\theta_I(A) = d_{IJ} + \rho g_{IJ}(A) \text{ and } \theta_J(A) = d_{JI} + (1-\rho) g_{IJ}(A). \tag{5}$$

It is noteworthy that this approach would allow also to apply a different sharing rule to each bilateral negotiation.[9]

Truthful Disagreement Points. An important modelling choice concerns the selection of the disagreement payoffs. The non-cooperative bargaining literature has typically invoked two main motives that induce players to reach an agreement. A first motive to come to an agreement is the fear that prolonged negotiations may eventually lead negotiations to break down, as one party may randomly withdraw permanently from the bilateral bargaining process. In this case the disagreement payoff of a player is a so called

[8] Generally, a larger exponent of a player in the Nash product is interpreted as representing a higher bargaining power of that player in bilateral negotiations. The corresponding sharing rule is obtained as utilities are linear in payments.

[9] In the interest of clarity this generalization is made only in the online appendix.

"outside option": his payoff in the event of a permanent breakdown of that particular bargaining process. Bargaining may however take place in a more deterministic manner, rather than being subject to the risk of breakdown. In that case players' main motivation to reach an agreement is to avoid losses associated with delays, as the process of preparing and exchanging offers is time consuming and they forego a mutually beneficial trading opportunity during delay (cf. our strategic model of negotiations below). In this case, the disagreement payoff of a player is a so called "inside option": the net benefit accruing to that player in the course of the dispute while an agreement is delayed. Our choice of disagreement payoffs rests on the concept of such "inside options".[10] We are now more specific about this.

Take a disagreement between seller I and buyer J. Reflecting the temporary nature of disagreement—as also captured in our strategic game below—we stipulate that only those players that failed to reach an agreement are presently aware of it and will therefore want to make temporary adjustments in their transactions with all other players. We propose a variant of the truthfulness criterion, as introduced in our discussion of buyer or seller auctions, as a parsimonious way to capture such "local" off-equilibrium adjustments. While the respective adjustments will not be used on-equilibrium, their use off-equilibrium pins down the exact distribution of total surplus—just as in the auction case. This approach thus offers a solution that is tailored to modelling bargaining situations where disagreements are expected to be only temporary. Precisely, we stipulate that contracts allow each side to require an adjustment of trade that makes the other side just indifferent between accepting the adjustment or trading according to the (equilibrium) contract point. This is the case if and only if the respective adjustment of the transfer truthfully reflects the incremental valuation or incremental cost of the counterpart.[11] We note, however, that all results hold as well when the side that proposes the adjustments can only extract a strictly positive share $\gamma > 0$ of the resulting incremental bilateral surplus.

Definition 2. *Take the model of simultaneous bilateral Nash bargaining. The disagreement points d_{IJ} and d_{JI} for seller I and buyer J respectively are said to be truthful relative to*

[10] The preceding discussion thus reflects some of the fundamental ideas in Binmore et al. (1986). On a broader discussion, notably of such "outside options" and "inside options", see also Muthoo (1999).

[11] This does not entail that agreements can be re-opened unilaterally. The case where both sides to an agreement would want to adjust the contract in this way will not be relevant for the construction of the equilibrium (cf. also the discussion in the subsequent game of strategic negotiations). For completeness we may stipulate, however, that then the original trades persist.

a trade matrix A if and only if they represent the option for each side to adjust their trade with all their other trading partners on a truthfulness basis: Seller I can request from some buyer $J' \neq J$ an adjustment of trade from the contractually agreed trade A_{IJ} to another $A'_{IJ'}$ at an incremental transfer $v_{J'}(A'_{IJ'}) - v_{J'}(A_{IJ'})$ and buyer J can request from some seller $I' \neq I$ an adjustment from the contractually agreed trade $A_{I'J}$ to another $A'_{I'J}$ at an incremental transfer $C_{I'}(A'_{I'J}) - C_{I'}(A_{I'J})$.

Bargaining Solution. We now consider bargaining outcomes that have the following properties:

Definition 3. *A bargaining outcome with "two-sided truthfulness" and seller power ρ is a pair (A, T) of a trade and a transfer matrix that gives raise to payoffs that are consistent with bilateral Nash bargaining where i) for each pair of buyer $J \in B$ and seller $I \in S$ the sharing rule is such that the seller obtains the share ρ of the net gains from trade and ii) disagreement points d_{IJ} and d_{JI} are truthful relative to A (according to Definition 2).*

We now apply the sharing rule (with weights ρ and $1-\rho$, as in expression (5)) and Definition 2 for the disagreement points to a generic pair I and J. For given (A, T) this uniquely ties down the transfer. As $A = A^*$ follows as the bilateral surplus is maximized in each transaction and as industry profits are strictly concave, we have:

Proposition 2 *The pair (A^*, T), with*

$$t_{IJ} = (1-\rho)\varphi_{IJ} + \rho\varkappa_{IJ}, \tag{6}$$

forms the unique bargaining outcome with "two-sided truthfulness" and seller power ρ, where φ_{IJ} and $\varkappa_{IJ}$ are the equilibrium payments of the Truthful Nash Equilibrium of menu auctions, where respectively buyers or sellers bid (cf. Lemma 1).

An important implication of Proposition 2 is that by studying the outcome of truthful auctions in Proposition 1, we also learn the most and the least that buyers may pay to sellers when they bargain bilaterally and both sides are allowed to make truthful adjustments. The measure of seller power in bilateral negotiations ρ pins down where the payment lies in that interval.

Strategic Negotiations. We conclude this section by proposing a strategic bargaining game that supports as an equilibrium the allocation in Proposition 2. For brevity's sake we relegate much of the formal exposition to the Appendix. We consider an alternating offer bargaining game in the spirit of Rubinstein (1982) and Binmore et al. (1986). Bargaining takes place in periods $\tau = 0, .., \infty$. The time between each period is $z > 0$. We stipulate that bargaining proceeds simultaneously in pairwise negotiations between agents of each buyer and seller pair $\{I, J\}$, with seller agents making the first offers in $\tau = 0$ and buyers' agents making offers in all odd periods. Buyers have a discount rate $r_b > 0$ and sellers $r_s > 0$.

In each negotiation, the respective agents form rational expectations about the outcomes in all other (still proceeding) negotiations and seek to maximize the payoff of the respective buyer or seller they represent. Once an agreement is reached, negotiations in the respective pair stop. The chosen game form where agents of each buyer and seller negotiate bilaterally is shared with much of the literature.[12] As we show in the online Appendix, however, we can also support the characterized outcome as a sequential equilibrium of a game where first players make simultaneous offers. Then, in case the offers do not match, sellers and buyers take turns with each side making simultaneous offers to all counterparties without matching offers, which these also either accept or reject simultaneously each received offers. If they reject, then they may ask for adjustments from those counterparties with whom they already have an agreement with. **(Add the detailed description of procedure, and propose to only keep this one. Add Appendix.)**

In line with the preceding specifications, a bilateral agreement consists of a contract (A_{IJ}, t_{IJ}) that still gives parties the option to adjust purchases and sales as long as this makes the other side not worse off (Definition 2). Each period, after these potential adjustments have been made, sellers produce and buyers make consumption decisions.[13] Hence, when there is delay in any bilateral relationship, this does not forestall production and the respective trades and consumption under existing agreements. To model this, we let the respective payoffs and transfers, arising from costs and consumption, represent flows. We relegate a full derivation of payoffs and the equilibrium to the proof of the

[12] This holds in particular for the (structural) empirical literature that is referenced in the Introduction as well as further below.

[13] As previously, we need not be specific about what happens when both sides to an agreement request an adjustment at the same time. For completeness only we may stipulate that if, for a given contract, a request is simultaneously made by the respective seller and buyer, then no adjustment is made.

subsequent result:[14]

Proposition 3 *Consider the strategic game with alternating offers in each bilateral negotiation. Contracts specify the respective trades and transfers (A_{IJ}, t_{IJ}) together with the option for each party to temporarily adjust purchases or sales as long as the other side is not worse off ("two-sided truthfulness"). Then we obtain for each length of time between two consecutive offers z a sequential equilibrium that leads to immediate and efficient trade such that, along the respective sequence of equilibria, for $z \to 0$ transfers converge to those of the Nash bargaining outcome ((6) in Proposition 2), where $\rho = r_B/(r_B + r_S)$ and where we interpret costs and utilities as flows.*

6 Buyer Power

6.1 Dependencies

Take now a merger of any two buyers J_1 and J_2 to form a larger buyer of size $|J_3| = |J_1| + |J_2|$. Observe first that this will not affect the traded quantities, which are still given by A^*. An immediate consequence of this is that the transfers paid by any other buyers $J \notin \{J_1, J_2\}$ are still the same. For the newly formed larger buyer J_3, the respective per unit prices are lower than the average prices paid by J_1 and J_3 before if and only if

$$(1-\rho)\varphi_{IJ_3} + \rho\varkappa_{IJ_3} < (1-\rho)(\varphi_{IJ_1} + \varphi_{IJ_2}) + \rho(\varkappa_{IJ_1} + \varkappa_{IJ_2}). \tag{7}$$

To determine when (7) holds, it is instructive to first consider separately the two benchmark cases with $\rho = 0$ and $\rho = 1$. In either case transfers are such that the payoffs of one side of the market are reduced to the value of the respective alternatives, as expressed by the adjusted trade matrix A^{IJ}. We thus need to ask how the larger purchasing volume of J_3 affects the value of these alternatives for both sides. We frame this in terms of the two sides' relative dependency.

Seller Dependency. The effect on seller dependency can be summarized as follows:

Lemma 1 *Suppose buyers J_1 and J_2 form a buyer J_3 with size $|J_3| = |J_1| + |J_2|$. Then*

$$\varphi_{IJ_3} < \varphi_{IJ_1} + \varphi_{IJ_2} \tag{8}$$

[14] With transferable utility, the proof follows the very transparent exposition in Sutton (1986).

holds, implying that when buyers hold all the bilateral bargaining power ($\rho = 0$), the unit price paid by J_3 is always strictly lower than the average per unit price paid jointly by J_1 and J_2.

Off equilibrium any seller I has the opportunity to (temporarily) increase the sales to all remaining buyers. A disagreement with the larger buyer leaves the seller with less alternative consumers to sell to. As consumers have decreasing marginal utility for the seller's products, the alternative to adjust sales therefore becomes less valuable after disagreement with the larger buyer. In this sense a seller becomes more dependent on the newly formed larger buyer J_3 than he was on either J_1 or J_2 before. It is worthwhile noting that this is however no longer the case when costs are linear, simply as in this case there are no profitable adjustments to be made by the respective seller when there is disagreement with either a smaller or a larger buyer. Trades with other buyers are already such that the respective marginal utilities equal marginal costs.

Lemma 1 derives also from a second effect, which - as further discussed below - would also be present when adjustments of trades were not possible. This is the "incremental cost" effect that has already been isolated in the literature:[15] To put it succinctly, J_3 negotiates for the larger trading volume less "at the margin" and more "inframarginally" and as marginal costs are increasing, the per-unit incremental cost that J_3 generates at any seller I and for which the buyer must compensate I is strictly lower than for either J_1 or J_2.

Buyer Dependency. The effect of the merger on buyer dependency can be summarized as follows:

Lemma 2 *Suppose buyers J_1 and J_2 form a buyer J_3 with size $|J_3| = |J_1| + |J_2|$. If $|I| \neq |G|$, so that there is more than one seller,*

$$\varkappa_{IJ_3} > \varkappa_{IJ_1} + \varkappa_{IJ_2} \tag{9}$$

holds, implying that when sellers hold all the bilateral bargaining power ($\rho = 1$) the unit price paid by J_3 is always higher than the average per unit price paid jointly by J_1 and J_2. If $|I| = |G|$, then $\varkappa_{IJ_3} = \varkappa_{IJ_1} + \varkappa_{IJ_2}$, so that for $\rho = 1$ the merger has no effect.

[15] Cf. the survey in Snyder (2012). With large fixed costs, the opposite result has been shown to hold (Raskovich, 2003).

Recall that by purchasing more from other sellers off equilibrium, a buyer can reduce the loss associated with the failure to reach an agreement with any particular seller. As marginal costs are increasing, however, the per-unit incremental costs of temporarily increasing the trade level with other sellers is higher for larger quantities, as demanded by J_3. In this sense a larger buyer becomes more dependent on each seller.

This negative effect of size, as it increases the per-unit price, relies crucially on the possibility to adjust trades and is thus not present when there is only one seller (as frequently assumed in the theoretical literature). It is also not present when contracts or technology rule out such adjustments (cf. the further analysis in Section 7.2). In sum, in our model the formation of a larger buyer can thus be said to increase both the dependency of any seller on the newly formed buyer (Lemma 1) and the dependency of the large buyer on all sellers (Lemma 2).

6.2 Large Buyer Advantage or Disadvantage?

Recall that the unit price paid by the large buyer J_3 is strictly lower than the average price paid by J_1 and J_2 before if and only if (7) holds. Rearranging terms, this condition becomes

$$-\frac{[\varkappa_{IJ_3} - (\varkappa_{IJ_1} + \varkappa_{IJ_2})]}{[\varphi_{IJ_3} - (\varphi_{IJ_1} + \varphi_{IJ_2})]} > \frac{1-\rho}{\rho} \tag{10}$$

Lemmas 1 and 2 allow us to sign the left-hand side, which is always strictly positive and invariant in ρ, unless there is a single seller in which case is exactly zero. The right-hand side is monotonically decreasing in ρ, from ∞ when $\rho = 0$ to 0 when $\rho = 1$. We obtain the following result:

Proposition 4 *Suppose buyers J_1 and J_2 form a buyer J_3 with size $|J_3| = |J_1| + |J_2|$. The larger buyer pays a strictly lower per unit price if and only if either*
i) there is only a single seller, i.e., $|I| = |G|$ for some $I \in S$ and $\rho < 1$ (as for $\rho = 1$ the single seller always extracts all surplus);
ii) or there are multiple sellers, i.e., $|I| \neq |G|$ for every $I \in S$, and ρ is lower than some critical level that lies in $(0, 1)$.

This result confirms and extends the current understanding of buyer power in the literature. It confirms because when there is a single seller, a larger buyer will indeed pay a lower per unit price for any bargaining power in bilateral negotiations. However that

simple result no longer holds when there are multiple sellers of substitute products and sellers have decreasing returns. Then a larger buyer can still obtain lower per unit prices, but only if buyers have a sufficient advantage in bilateral bargaining, i.e., ρ is sufficiently low. In that case buyers are able to push sellers close to their alternative option, which as we saw becomes relatively less favorable as the larger buyer controls more of all purchases. However, when sellers have the advantage in bilateral bargaining a larger buyer pays a higher per unit price. In that case sellers are able to push buyers close to their alternative option, which we saw is relatively less favorable for a larger buyer because it becomes more costly to shift a larger volume to alternative sellers.

Buyers' incentives to merge are thus in general higher when bargaining power arising from other channels is already high. In other words, bargaining power as represented by the sharing rule ρ and size have a mutually reinforcing role in bringing down the per unit price. This also implies that size is not a good substitute for the lack of bilateral bargaining power, as expressed by the sharing rule. In what follows we derive from these results further implications that may, in particular, inform future empirical work. In the subsequent section we then provide additional formal results.

Discussion of Implications. As noted in the introduction, our approach allows to encompass both findings where larger buyers enjoy a discount as well as contrasting findings in the empirical literature where larger buyers seem to pay a premium, offering for this an explanation that is based on the ability buyers and sellers have of adjusting trade in case of temporary disagreement (which is in turn based on the fundamentals, preferences and technology) rather than heterogeneity in firm-specific bargaining abilities—as the latter could also be perceived as a circular argument.

It also yields predictions on when we should observe mergers, as well as the formation of buyer groups, that are motivated by their effect on buyer power. Relying first on the interpretation in terms of seller and buyer auctions, the implications depend on the procurement format that prevails in the industry.[16] When buyers run procurement auctions, size could be a disadvantage as it increases dependency on any individual seller. Next, whether size bestows an advantage depends, more generally, on the sharing rule ρ.[17] The

[16] We are aware that this may itself be an endogenous variable, unless the particular format was determined by technological requirements or efficiency considerations.

[17] As noted above, our approach as well as the comparative results, can be extended to the case where

literature has identified, both empirically and theoretically, various determinants of such a bilateral sharing rule. Notably, this may reflect both sides' relative impatience to come to an agreement, which may in turn depend on their financial flexibility and the importance of the respective profits for their overall revenues. Empirically, the former may relate to firms' leverage while conglomerates or notably multi-product retailers may depend less on the respective revenues than more focused firms.[18] Ceteris paribus, for buyers with more financial flexibility, when this translates into a larger sharing rule, size should be of (additional) advantage.

We also noted above that technology should determine which of our isolated effects is stronger. Here, our analysis suggests the importance of incremental costs at "inframarginal" units of production versus incremental costs at "supramarginal" units of production (that is, above the equilibrium production levels). These may again be features of the industry that are either observable to empiricists or known by practitioners. Furthermore, buyer size affects buyer dependency only when there is indeed scope (off-equilibrium) to temporarily adjust purchases from other sellers, which should again depend on technology. We continue to analyze the role of technology more formally in Section 7.2.

We conclude our brief discussion of implications with a closer comparison to the recent empirical literature that has shown increasing interest in markets where prices are determined through bilateral negotiations and, more specifically, in the determinants of buyer power. Several structural empirical papers adopt a pairwise negotiations framework, as we do in this model, albeit they restrict consideration to linear costs (possibly, with additional fixed costs).[19] When remedies imposed by antitrust authorities on a buyer merger or the particular structure of a buyer group ensure that downstream competition was not affected, then in these models the terms of trade would *not* depend either on a buyer's absolute size or on its importance for a particular seller. As we discussed in the introduc-

the sharing rule differs between negotiations (ρ_{IJ} with $I \in S$ and $J \in B$). A formal extension is contained in the online appendix.

[18] The role of financial assets and thus financial flexibility has been analyzed theoretically and empirically in the labour literature on union-firm bargaining (cf. Cramton and Tracy 2002 for a survey), though the respective theory models are typically based on a screening motive or involve particular assumptions on how outside options develop over time (e.g., Hart 1989). This literature also includes an analysis of negotiations with (deeper-pocket) conglomerates. The relationship between leverage and a firm's trade-off between profits today and tomorrow has been analyzed in the financing literature (e.g., in Chevalier and Scharfstein 1996).

[19] E.g., Grennan (2013) or Govrisankara et al. (2014) on health markets and Crawford and Yurukoglu (2012) on cable television.

tion, these possible determinants of buyer power are however of key concern to antitrust authorities.[20] Our model shows how nonlinear costs at suppliers and decreasing returns at buyers can generate size effects and how other determinants of bargaining power, as expressed by ρ, interact to then account for better or worse terms of supply.

A few exceptions in the empirical literature analyze also non-linear supply contracts. As researchers typically do not observe wholesale contracts, the assumption of a particular form of wholesale contracts is part of the identifying restrictions and different models are then compared in terms of their overall fit with the data.[21] Our approach suggests broadly to extend these models to allow also suppliers more flexibility in reacting to (temporary) disagreements. Here, the imposition of bilateral truthfulness may both account for the relatively fluid form of agreements in some markets and allow to incorporate more information, in particular about manufacturers' alternative sales options at other retailers.[22]

6.2.1 Relation to other Solution Concepts (to add discussion)

Introduce parts of letter and prepare the remainder, with solutions, as online appendix

Our approach is clearly geared towards the considered application in Industrial Organization, notably to vertical negotiations. There, disagreement in bilateral negotiations should often not render void any other agreements or exclude any of the firms from other agreements (such as by partitioning the set of all buyers and sellers accordingly). Further, individual contracts may allow for the flexible adjustments of quantities and may thereby still react to disagreements with other firms. While we certainly do not claim that our approach provides a full description of any particular real-world negotiation,[23] incorporating these features may still add realism in many circumstances and, what seems more important, it provides additional structure to generate novel implications. We explore in

[20] An example in a merger context is the take-over of the large grocery retailer PLUS by Germany's largest grocery retailer EDEKA in 2008. As is common, this was only cleared after the divestiture of outlets in regional markets where concentration would otherwise have increased. Still, this merger has observably put large downward pressure on supply contracts, which were objected by a later decision in 2014 (cf. the reference in footnote 1).

[21] For the grocery market see, e.g., Draganska et al. (2010) and Bonnet et al. (2013). Again under the assumption of linear costs, a prominent such restriction is to impose marginal cost pricing for suppliers (in exchange for the payment of a fixed fee).

[22] In practice, applied to the grocery market, this may be achieved through manufacturer instigated promotions.

[23] In the language of one of our referees, our model thus provides a particular "parable" of such negotiations.

Section 7.2 how the scope for adjustments, which should depend on technology, generates disadvantages of size that would otherwise be absent. In Section 6.2 we showed also how the interaction between the sharing rule and buyer size generates novel predictions, which are further extended to seller size in Section 7.1. As we discussed in Section 6.2, our approach thus differs notably from other contributions that apply the Nash bargaining solution to bilateral negotiations.

As noted in the Introduction, we share however our interest in the implications of mergers with another strand of the literature, which applies cooperative solution concepts to general (N-person) bargaining problems. We argue next that also in light of this literature our approach provides novel insights. Our characterized equilibrium is also not generally contained in the Core.

As noted in the Introduction, various other approaches have lead to the Shapley value, which essentially includes or excludes players to calculate their respective incremental contribution to various coalitions. Applying this to our setting, where in each coalition production and consumption is chosen so as to maximize joint surplus, we first note that the results do not coincide. Precisely, there exists no value ρ for which our approach yields the Shapley value. It is also noteworthy that under the Shapley value convex costs together with submodularity of consumption are not sufficient to unambiguously sign the impact of a merger. This holds as well when we apply generalized random-order bargaining values, as in Segal (2003).[24] Besides these evident differences, as our discussion of implications shows, the close-knit relationship to the Industrial Organization application ensures that we can isolate various effects, notably relating to buyer and seller dependency, and relate them to potentially testable implications.

7 Extensions of the Analysis

7.1 The Case of Seller Power

We now turn our attention to the mirror case of how seller size affects the average unit price paid by each buyer. In analogy to how we proceeded in the case of buyer power, let there thus be two sellers I_1 and I_2 and consider the effect of a merger so that we have a seller I_3 with size $|I_3| = |I_1| + |I_2|$. In equilibrium each individual consumer still receives

[24] Even there it can be shown that, for any given random-order bargaining value, we can not find a value ρ so that the outcomes coincide.

the same quantity of each good, before and after the merger. The large seller I_3 receives a strictly higher per unit price from some buyer J if and only if

$$(1-\rho)\varphi_{I_3 J} + \rho \varkappa_{I_3 J} > (1-\rho)(\varphi_{I_1 J} + \varphi_{I_2 J}) + \rho(\varkappa_{I_1 J} + \varkappa_{I_2 J}).$$

Like above, we focus in turn on the effect of the merger on seller and buyer dependency.

Lemma 3 *Suppose sellers I_1 and I_2 form a buyer I_3 with size $|I_3| = |I_1| + |I_2|$. If $|J| \neq |N|$, so that there is more than one buyer, then*

$$\varphi_{I_3 J} < \varphi_{I_1 J} + \varphi_{I_2 J} \tag{11}$$

holds, implying that when buyers hold all the bilateral bargaining power ($\rho = 0$) the unit price received by I_3 is always strictly lower than the average per unit price received jointly by I_1 and I_2. If $|J| = |N|$, then $\varphi_{I_3 J} = \varphi_{I_1 J} + \varphi_{I_2 J}$, so that for $\rho = 0$ the merger has no effect.

All proofs for the results in this section are relegated to the online appendix as they proceed largely analoguously to those with a buyer merger. Since buyers have decreasing returns, in case of disagreement with any buyer the larger seller I_3 will find it harder to sell his relatively larger number of goods to the same set of alternative buyers without depressing prices too much.[25] Thus, as a seller's size increases, so does his dependency on any particular buyer. Note that this is analogous to the interaction of buyer dependency and buyers' size. There, we observed that as a larger buyer finds it relatively more costly to reallocate purchases, given sellers' convex costs, a buyer merger increases dependency on any individual seller.

In the case where $\rho = 1$ suppose first that goods are perfect substitutes so that we can write $u(a_j) = u\left(\sum_{i \in G} a_{ij}\right)$.[26] Then, concavity of $u(\cdot)$ mirrors convexity of $c(\cdot)$ and it is intuitive that again the result mirrors that of a buyer merger. When goods are however not perfect substitutes, then we need an additional assumption for this to hold, namely that when a consumer loses access to some good, then this reduces his utility by more when he is already more constrained in that he has access to fewer good. Writing this in a continuous fashion, take thus some consumption vector a_j and consider the derivative

[25] Note that this presumes strictly convex costs so as to make such adjustments optimal.

[26] As then the efficient allocation does not prescribe how all bilateral trades are allocated across seller-buyer pairs, we consider here the symmetric equilibrium outcome.

with respect to some element a_{ij}, $u_i(a_j)$, representing (in absolute terms) the marginal loss when consumption of good i must be reduced. Consider then any other two goods z and y and suppose that we further tilt consumption towards, say, good z, as already $a_{zj} > a_{yi}$ and as we consider a further shift $da_{zj} = -da_{yi} = \Delta > 0$. We suppose that the impact on $u_i(a_j)$ satisfies[27]

$$\frac{d}{da_{zj}}u_i(a_j) \geq \frac{d}{da_{yj}}u_i(a_j). \tag{12}$$

Lemma 4 *Suppose sellers I_1 and I_2 form a seller I_3 with size $|I_3| = |I_1| + |I_2|$. Then with condition (12)*

$$\varkappa_{I_3J} > \varkappa_{I_1J} + \varkappa_{I_2J} \tag{13}$$

holds, implying that when sellers hold all the bilateral bargaining power ($\rho = 1$), the unit price received by I_3 is always strictly higher than the average per unit price received jointly by I_1 and I_2.

Finally, taking together the results from Lemmas 3 and 4, we immediately obtain the following result that is analoguous to that for the formation of a larger buyer in Proposition 5 .

Proposition 5 *Suppose sellers I_1 and I_2 form a seller I_3 with size $|I_3| = |I_1| + |I_2|$ and (12) holds. The larger seller receives a strictly higher per unit price if and only if either*
i) there is only a single buyer, i.e., $|J| = |N|$ for some $J \in B$ and $\rho > 0$ (as for $\rho = 0$ the single buyer always extracts all surplus);
ii) or there are multiple buyers, i.e., $|J| \neq |N|$ for every $J \in B$, and ρ is higher than some critical threshold that lies in $(0, 1)$.

7.2 Technology

Our bargaining approach presumes that bilateral (temporary) adjustments of trades are feasible. A particular case where these adjustments may however *not* be feasible could be that were the technology is such that adjustments would involve additional and sufficiently large lump-sum costs, which we did not consider in our model (or assumed to be sufficiently small).[28] Formally, we could capture this by so-called "quantity-forcing" contracts that do

[27] Then the total change to $u_i(a_j)$ equals $\Delta\left(\frac{d}{da_{zj}}u_i(a_j) - \frac{d}{da_{yj}}u_i(a_j)\right) > 0$.

[28] When incorporating such fixed adjustment costs in future work, it is notable that if these are not prohibitively high, then this would generate economies of scale "off equilibrium".

not allow for adjustments. For analyzing both the case of auctions and that of bargaining, we focus on the equilibrium that supports the efficient trade matrix A^*.[29] Therefore, at disagreement trades are now given by A^{IJ} with $a_{ij}^{IJ} = 0$ if $i \in I$ and $j \in J$ and $a_{ij}^{IJ} = a^*$ otherwise. For the two auction games we can adopt Proposition 1 and obtain, when buyers bid ($\rho = 0$), the transfers

$$\varphi_{IJ} = |I| \left[c\left((|N|\, a^* \right) - c\left((|N| - |J|) a^* \right) \right] \tag{14}$$

and, when sellers bid ($\rho = 1$), the transfers

$$\varkappa_{IJ} = |J| \left[u(a_j^*) - u(a_{IJ}^{IJ}) \right]. \tag{15}$$

Also Proposition 3 extends with quantity-forcing contracts, so that the outcome of bilateral negotiations is again a convex combination of φ_{IJ} and $\varkappa_{IJ}$. With respect to buyer size, we now no longer have two conflicting sources (dependencies). When adjustments are not possible, only the well-known "incremental cost" effect remains, which - as discussed after Lemma 1 - still ensures that condition (8) holds, now with the respective transfers from (15), while with the transfers from (14) condition (9) no longer holds. Likewise, condition (13) holds, but without adjustments there is again no negative countervailing effect of seller size. A formal statement of this is part of Proposition 6 below.

Adjustments may also be made infeasible or unprofitable by the shape of the cost function. In the remainder of this section we explore the role of capacity constraints. We suppose first that capacity is limited in that $c(x)$ is well-behaved until some K, while $c(x) = \infty$ when $x > K$. The capacity constraint is binding with $a^* = K/|N|$, which now deprives buyers of their option to adjust trades in case of disagreement, so that $\varkappa_{IJ}$ is still given by (15). The adjustment option however now still persists for sellers, as reflected in the originally derived transfer φ_{IJ} in (3), so that only for φ_{IJ} the previously derived negative impact of own size holds.[30] With limited capacity but otherwise convex costs, size is then an unambiguous advantage only for buyers.

In a mature industry, firms may have adjusted to a predictable production level, e.g., by employing sufficient staff, so that marginal costs may increase only above $x = |N|\, a^*$,

[29] As noted already in Section 4, equilibria with only quantity-forcing contracts can support a multiplicity of inefficient outcomes.

[30] Strictly speaking, we would need to slightly modify the proof to take into account the corner solutions for a_{IJ}^{IJ}.

e.g., by only then requiring more expensive overtime. For this case, we may thus suppose that $c(x) = c$ holds up to $x = |N|\, a^*$, while $c''(x) > 0$ for $x > |N|\, a^*$. As marginal costs are thus constant at "inframarginal" production levels, as already noted, optimally sellers will not use their adjustment option, so that with $c(x) = c$ expression (14) further simplifies to $\varphi_{IJ} = c\,|I|\,|J|\,a^*$. Then, through this channel ("seller dependency") a seller's size can not be a disadvantage. On the other hand, as costs are still strictly convex for adjustments above the equilibrium production level, though not prohibitively high to rule out any adjustments, the "buyer dependency" effect is now still present. For the following summarizing statement we simplify the exposition and restrict consideration to the case with more than one seller and more than one buyer. The results follow immediately from the discussed comparative analysis of φ_{IJ} and $\varkappa_{IJ}$.

Proposition 6 *We compare the terms of a large buyer J_3 with size $|J_3| = |J_1| + |J_2|$ to the average terms of the smaller buyers J_1 and J_2 and the terms of a large seller I_3 with size $|I_3| = |I_1| + |I_2|$ to the average terms of the smaller buyers I_1 and I_2. In contrast to the preceding main analysis, consider the following change in adjustment possibilities:*
i) ("Quantity-forcing" contracts): Suppose no adjustments are feasible. Then the larger buyer always pays a lower unit price (and strictly so when $\rho < 1$) and the larger seller always receives a higher unit price (and strictly so when $\rho > 0$).
ii) (Binding capacity constraints): Suppose $c(x) = \infty$ when $x > K$, with $a^* = K/\,|N|$. Then the larger buyer always pays a lower unit price (and strictly so when $\rho < 1$), while the larger seller receives a higher unit price when ρ is higher than some critical threshold that lies in $(0, 1)$ and a lower unit price otherwise.
iii) (Adjusted capacity): Suppose $c(x) = c$ for $x \leq |N|\, a^*$ and $c''(x) > 0$ for $x > |N|\, a^*$. Then the larger buyer always pays a larger unit price (and strictly so when $\rho > 0$) and the larger seller always receives a higher unit price (and strictly so when $\rho < 1$).

7.3 Asymmetries and Concentration

So far our attention was focused on a comparison of the terms of trade of larger and smaller sellers or buyers, notably through considering the implications of a merger. As discussed in the Introduction, this was motivated also by recent antitrust interest in supply markets and the exertion of buyer power. In the more standard analysis of mergers and

market power vis-a-vis consumers,[31] however, what matters for authorities as well is the asymmetry in market shares. Most prominently, this is captured by the HHI, which is the sum of the square roots of individual market shares and which is probably the most important first-phase screening criterion for mergers, as used in, for instance, the EU's 2004 horizontal guidelines and the respective US guidelines (revised 2010). Interestingly, the latter acknowledge that, just as in the present paper, buyer power may be exerted in bilateral negotiations rather than through withholding demand (monopsony), but still note the following: "To evaluate whether a merger is likely to enhance market power on the buying side of the market, the Agencies employ essentially the framework described above for evaluating whether a merger is likely to enhance market power on the selling side of the market." Our subsequent results provide a foundation for this approach.

In light of our present interest in market concentration, we now take the (limit) case where products are perfect substitutes, so that $u(a_j) = u\left(\sum a_{ij}\right)$. We compare market structures that differ in the respective sizes of two buyers: While the joint size of J_1 and J_2 remains unchanged at $|J_1| + |J_2| = M$, starting from a situation where already $|J_1| \geq |J_2|$, we let them trade assets (i.e., consumers) so that subsequently $\left|\widehat{J}_1\right| = |J_1| + m$ and $\left|\widehat{J}_2\right| = |J_2| - m$ with $m > 0$.[32] Bilateral relationships are left unchanged as we consider the outcome with $a_{ij} = a^*$. This reorganization of sellers' market increases the respective HHI, that is[33]

$$HHI_B = \sum_{J \in B} \left(\frac{|J|}{|N|} \right)^2 .$$

We show in the proof of Proposition 7 that the considered increase in asymmetry reduces each seller's profits when $\rho = 0$.[34] As HHI_B also increases after a buyer merger, together with Lemma 1 we have the following result:

Proposition 7 *Consider either a merger of two buyers or a trade of assets (consumers) between them so that concentration increases (higher HHI_B). Then this increases seller*

[31] More precisely, the defining characteristics are typically non-negotiated and thus also non-differentiated terms that are accessible to all consumers ("uniform pricing").

[32] The case where $m = |J_2|$ is covered, as a limit, by the subsequent analysis, though given the subsequent restrictions on substitutability, the preceding analysis is not made redundant.

[33] In fact, the respective increase equals the difference of $\left(\frac{|J_1|+m}{|N|}\right)^2 - \left(\frac{|J_1|}{|N|}\right)^2$ and $\left(\frac{|J_2|}{|N|}\right)^2 - \left(\frac{|J_2|-m}{|N|}\right)^2$, which is strictly positive as $|J_1| \geq |J_2|$ and $m > 0$.

[34] In light of our subsequent discussion we show this both for our baseline case and when there is a binding capacity constraint (case ii) in Proposition 6).

dependency: For each I, $\sum_{J \in B} \varphi_{IJ}$ strictly decreases.

Proposition 7 already informs us about the total impact on sellers' profits for different scenarios, notably when capacity is constrained (cf. the discussion preceding Proposition 6), when contracts are determined through auctions where buyers bid (cf. Proposition 1), or in negotiations when buyers can extract a sufficiently large share of the bilateral surplus (low ρ). Defining next from buyers' perspective

$$HHI_S = \sum_{I \in S} \left(\frac{|I|}{|G|} \right)^2,$$

we have the following mirror image to Proposition 7.

Proposition 8 *Consider either a merger of two sellers or a trade of assets (production facilities) between them so that concentration increases (higher HHI_S). Then this increases buyer dependency: For each J, $\sum_{I \in S} \varkappa_{IJ}$ strictly increases.*

Propositions 7 and 8 are silent about the impact of increased concentration on the dependency of the respective side of the market, e.g., the dependency of two buyers J_1 and J_2 that trade assets. As is intuitive from our preceding analysis, unless adjustment options are restricted (e.g., by capacity constraints, cf. Proposition 6), the "joint" dependency of the involved parties, here the two buyers, indeed increases, as captured by an increase in $\sum_{I \in S}(\varkappa_{IJ_1} + \varkappa_{IJ_2})$.[35] While the impact of greater asymmetry on payoffs is thus generally ambiguous, it is precisely for an asymmetric distribution of bargaining power, i.e., relatively low or high values of ρ, that we obtain clear-cut predictions, which should be indeed the cases of greatest interest to antitrust authorities.

8 Conclusion

In this paper we proposed a framework to model bargaining in a bilateral oligopoly where adjustments are possible but are not expected to be permanent. Our approach nests as special cases auctions in truthful menus, both among sellers and among buyers. In fact, the outcomes from these two more standard mechanisms are obtained at the extremes where either sellers or buyers have all bargaining power. Even if one side has all bargaining power,

[35] A proof is contained in the online appendix.

our approach ensures that the other side's payoff does not go to zero, as long as there are alternative buyers or sellers that stand ready to buy more or sell more of (imperfect) substitute goods. In case of delay in bilateral bargaining, when a strategic approach is taken, it allows the two affected parties to temporarily adjust their transactions with all other parties, without presuming that all parties must adjust and change the terms of trade with one another—i.e., adjustments are "local" rather than "global".

We then used our approach to study the determinants of buyer power. A key result of this paper is that buyer size can be both an advantage or a disadvantage when trying to secure a low price. We expressed this result in terms of buyer and seller dependency. Size affects either buyer or seller dependency differently. Which effect becomes stronger and then determines the overall effect of a merger depends on the distribution of bargaining power, i.e., how net surplus is shared in bilateral transactions. Seller dependency on a particular buyer captures the ease with which a seller can profitably adjust sales to other buyers when there is (temporarily) no agreement with the buyer. Buyer dependency captures likewise the profitability of a buyer's adjustment of his purchases from all other sellers. The size of a given player can increase the dependency of his counterparties but it can also increase his own dependency, as switching becomes a less attractive alternative for both sides.

By considering different technologies and contracts, varying in the degree to which adjustments are feasible or optimal, we make transparent the different channels through which size advantages or disadvantages arise. Also, when a buyer's bargaining power from other sources is weak, the negative effect of size dominates, given that then the option to substitute a seller becomes more important for the overall outcome of negotiations. Technology as well as the distribution of bargaining power, as also expressed in the procurement format, thus determine whether the overall effect of size, but also that of asymmetry in market concentration, are driven by the effect on seller or buyer dependency.

We assumed that a change in buyer size does not change the nature of the procurement process. In that respect we have taken the prevalent view that otherwise "it would be too easy to obtain a theory of the costs and benefits of integration if it were supposed that the bargaining process changes under integration" (Hart, 1995). If however, in addition to the effects discussed here, size also shifts bilateral bargaining power in more subtle ways (for example a larger buyer may find it profitable to hire a more skilled procurement team),

this may also change the way buyer power would go.

A further avenue for future work that we briefly discussed is the analysis of the equilibrium market structure, rather than considering an exogenously chosen merger or trade of assets between seller or buyers. Hart and Kurtz (1983, 1984) introduced several concepts of stability for this type of analysis, which has been pursued by many others in particular in environments with externalities (see, for instance, the survey by Bloch, 2002). Assume in our setting that only players of the same type, buyers or sellers, can form groups that will bargain jointly but are otherwise unconstrained in what groups they can form (this is unlikely to be the case in most applications). It follows from our results that for extreme values of bilateral bargaining power, i.e., for ρ close to 0 or 1, the players on the strong side benefit from forming a monopoly. Then the players of the other type, anticipating that they will face a monopoly, also benefit from forming themselves a monopoly. Thus in those cases a bilateral monopoly should emerge in equilibrium. The problem becomes significantly more complex for balanced distributions of the bilateral bargaining power, and in particular and it becomes dependent on the way the concept of stability specifies how the members of a group will organize if one if its subgroups deviates.

9 Appendix

Proof of Proposition 3. To support an equilibrium, suppose that the efficient trade matrix was implemented and that bilateral transfers are given by t_{IJ}^S, where the superscript shall denote that the respective offers were made by sellers (e.g., in $\tau = 0$). Still stipulating symmetric discount rates for all buyers, $r_b > 0$, and symmetric discount rates for all sellers, $r_s > 0$, we then have the following discounted payoffs:

$$\begin{aligned} W_I &= \frac{1}{r_S}\left(\sum_{J\in B} t_{IJ}^S - C_I(A^*)\right), \\ V_J &= \frac{1}{r_B}\left(v_J(A^*) - \sum_{I\in S} t_{IJ}^S\right). \end{aligned}$$

Now suppose instead that one agreement, namely that between I and J, has not yet been concluded. In the present period, their adjustments to all other trades are given by

A^{IJ}, so that over the period of length z their discounted payoffs are respectively

$$
\begin{aligned}
D_{IJ} &= \frac{1-e^{-zr_S}}{r_S}\left[\sum_{J'\in B\setminus J}\left(t^S_{IJ'}+v_{J'}(A^{IJ})-v_{J'}(A^*)\right)-C_I(A^{IJ})\right],\\
D_{JI} &= \frac{1-e^{-zr_B}}{r_B}\left[v_J(A^{IJ})-\sum_{I'\in S\setminus I}(t^S_{I'J}+C_{I'}(A^0_{IJ})-C_{I'}(A^*))\right].
\end{aligned}
$$

Suppose next that I and J reach an agreement in the following period, leading to the efficient trade matrix and with respective transfers (from the buyer's offer) t^B_{IJ}, so that from then onwards their continuation payoffs are

$$
\begin{aligned}
W^B_I &= \frac{1}{r_S}\left(t^B_{IJ}+\sum_{J'\in B\setminus J}t^S_{IJ'}-C_I(A^*)\right),\\
V^B_J &= \frac{1}{r_B}\left(v_J(A^*)-t^B_{IJ}-\sum_{I'\in S\setminus I}t^S_{I'J}\right).
\end{aligned}
$$

To now make buyer J in $\tau=0$ just indifferent between acceptance and rejection, it must hold that

$$V_J = D_{JI}+e^{-zr_B}V^B_J. \tag{16}$$

Likewise, provided that there was disagreement between I and J and $\tau=1$ is reached, for the respective offer of the buyer to now make seller I just indifferent, it must hold that

$$W^B_I = D_{IJ}+e^{-zr_S}W_I. \tag{17}$$

We show next that the respective equations (16) and (17) yield a unique solution for (t^S_{IJ}, t^B_{IJ}), which also does not depend on all other transfers. After substitution, these two equations transform to

$$
\begin{aligned}
t^S_{IJ}-e^{-\Delta r_B}t^B_{IJ} &= (1-e^{-\Delta r_B})\left[v_J(A^*)-v_J(A^{IJ})-\sum_{I'\in S\setminus I}(C_{I'}(A^*)-C_{I'}(A^{IJ}))\right]\\
&= (1-e^{-\Delta r_B})\varkappa_{IJ}
\end{aligned}
$$

and

$$
\begin{aligned}
t^B_{IJ}-e^{-\Delta r_S}t^S_{IJ} &= (1-e^{-\Delta r_S})\left[\sum_{J'\in B\setminus J}\left(v_{J'}(A^{IJ})-v_{J'}(A^*)\right)+C_I(A^*)-C_I(A^{IJ})\right]\\
&= (1-e^{-\Delta r_S})\varphi_{IJ},
\end{aligned}
$$

respectively. Substituting out for t^B_{IJ}, this yields

$$t^S_{IJ} = \frac{(1-e^{-\Delta r_B})\varkappa_{IJ} + e^{-\Delta r_B}(1-e^{-\Delta r_S})\varphi_{IJ}}{1-e^{-\Delta(r_B+r_S)}}.$$

As $\Delta \to 0$ this converges to

$$t^S_{IJ} = \frac{r_B\varkappa_{IJ} + r_S\varphi_{IJ}}{r_B + r_S} = \rho\varkappa_{IJ} + (1-\rho)\varphi_{IJ}.$$

Finally, to fully characterize the equilibrium, we make the following specifications. We specify that in $\tau = 0$ all agents of sellers make the respective offers (A^*_{IJ}, t_{IJ}), which buyers' agents accept. Further, if there was so far no agreement in one particular relationship (I, J), then it is expected that there will be immediate agreement in the next period, namely to the respective contract offered by the buyer's agent (A^*_{IJ}, t^B_{IJ}) in odd periods and to the respective contract (A^*_{IJ}, t^S_{IJ}) in even periods. By construction of transfers, given these expectations, as well as by construction of the efficient trade matrix, the respective strategies indeed constitute an equilibrium. **Q.E.D.**

Proof of Lemma 1. Given that in equilibrium the total quantity purchased by J_1 and J_2 will be equal to the quantity purchased by J_3 alone, we only have to compare the respective transfers:

$$\varphi_{IJ} = \sum_{J' \in B \setminus J} (v_{J'}(A^{IJ}) - v_{J'}(A^*)) - (C_I(A^{IJ}) - C_I(A^*)).$$

In what follows, we will first transform this expression. For this note that $C_I(A^*) = |I|\, c(|N|\, a^*)$ and that $\sum_{J' \in B \setminus J} v_{J'}(A^*) = (|N| - |J|)u(a^*_j)$, where a^*_j is a column vector with each element equal to a^*. Recall as well that $u_i(a_j)$ denotes the partial derivative of u with respect to the consumption of good $i \in G$. Recall that a^{IJ}_j denotes the adjusted consumption vector for j. It is convenient to write all elements $i \notin I$ as

$$a^{IJ}_{ij} = \frac{|N|\, a^* - |J|\, z_{IJ}}{|N| - |J|},$$

where we still need to determine the (symmetric) adjustments z^{IJ}. From the first-order condition we have

$$c'(|N|\, a^* - |J|\, z^{IJ}) = u_i(a^{IJ}_j), \tag{18}$$

so that with strict convexity we have immediately $z^{IJ} \in (0, a^*)$ (while, for future reference, note that $z^{IJ} = 0$ when costs are linear). With this notation, using also that $C_I(A^{IJ}) =$

$|I|\, c((|N|\, a^* - |J|\, z^{IJ})$ and that $\sum_{J' \in B \setminus J} v_{J'}(A^{IJ}) = (|N| - |J|)u(a_j^{IJ})$, we can rewrite all φ_{IJ}, notably also

$$\varphi_{IJ_3} = (|N| - |J_1| - |J_2|)(u(a_j^{IJ_3}) - u(a_j^*)) - |I|\,(c(|N|\, a^* - |J_3|\, z^{IJ_3}) - c(|N|\, a^*)). \quad (19)$$

We proceed as follows. To establish the assertion, we derive boundaries for the left-hand and right-hand side of expression (8). For this let next $\widehat{a}_j^{IJ_y}$ be the column vector with each element equal to a^* for $i \notin I$ and

$$\widehat{a}_{ij}^{IJ_y} = \frac{|N|\, a^* - |J_y|\, z^{IJ_3}}{|N| - |J_y|} \quad (20)$$

for $i \in I$, where in the latter case we have $\widehat{a}_{ij}^{IJ_y} < a_{ij}^{IJ_3}$ for $y = 1, 2$. (Formally, differentiating expression (20) with respect to $|J_y|$ the respective sign is determined by $a^* - z^{IJ_3} > 0$, which we established to hold strictly with strictly convex costs.) We then have for $y = 1, 2$ that

$$\varphi_{IJ_y} > (|N| - |J_y|)(u(\widehat{a}_j^{IJ_y}) - u(a_j^*)) - |I|\,(c(|N|\, a^* - |J_y|\, z^{IJ_3}) - c(|N|\, a^*)), \quad (21)$$

since we did not use the optimal adjustment $|J_y|\, z_{IJ_y}$ but instead $|J_y|\, z_{IJ_3}$. For condition (8) to hold, using (19) and (21) it is sufficient that the following two conditions hold:

$$\begin{aligned} &(c(|N|\, a^* - |J_3|\, z^{IJ_3}) - c(|N|\, a^*)) - (c(|N|\, a^* - |J_1|\, z^{IJ_3}) \\ &-c(|N|\, a^*)) - (c(|N|\, a^* - |J_2|\, z^{IJ_3}) - c(|N|\, a^*)) \\ > \;& 0, \end{aligned} \quad (22)$$

and

$$\begin{aligned} &(|N| - |J_1| - |J_2|)(u(a_j^{IJ_3}) - u(a_j^*)) - (|N| - |J_1|)(u(\widehat{a}_j^{IJ_1}) - u(a_j^*)) \\ &-(|N| - |J_2|)(u(\widehat{a}_j^{IJ_2}) - u(a_j^*)) \\ \leq \;& 0. \end{aligned} \quad (23)$$

We confirm (22) and (23) separately. Take thus first condition (22), which transforms to

$$c(|N|\, a^*) - c(|N|\, a^* - |J_1|\, z^{IJ_3}) > c(|N|\, a^* - |J_2|\, z^{IJ_3}) - c((|N|\, a^* - (|J_1| + |J_2|)z^{IJ_3}),$$

so that this is indeed strictly satisfied when c is strictly convex. Next, for condition (23) observe that the left-hand side is zero when $z^{IJ_3} = a^*$ (which it is not). Note as well that for $y = 1, 2$ and $z^{IJ_3} \in (0, a^*)$, we have that

$$\frac{|N|\, a^* - |J_y|\, z^{IJ_3}}{|N| - |J_y|} < \frac{|N|\, a^* - |J_3|\, z^{IJ_3}}{|N| - |J_3|}.$$

Differentiate now the left hand side of (23) with respect to z^{IJ_3}, which by symmetry can be simplified to

$$|I|\left[|J_1|\,u_i(\widehat{a}_j^{IJ_1}) + |J_2|\,u_i(\widehat{a}_j^{IJ_2}) - (|J_1| + |J_2|)u_i(a_j^{IJ_3})\right] \tag{24}$$

for any $i \in I$. Recall now that $\widehat{a}_{ij}^{IJ_y} < a_{ij}^{IJ_3}$ for $y = 1, 2$ and $i \notin I$. As the second-order derivatives are strictly negative for each $i, j \in G$, we then have that $u_i(a_j^{IJ_3}) < u_i(\widehat{a}_j^{IJ_y})$, so that the derivative (24) is positive. Finally, when $z^{IJ_3} \in (0, a^*)$ holds strictly, as we just verified for strictly convex costs, the left hand side of (23) is increasing in that interval and equal to zero when $z^{IJ_3} = a^*$, so that (23) holds (strictly) when costs are (strictly) convex. This completes the proof. **Q.E.D.**

Proof of Lemma 2. Again, as the total quantity purchased by J_1 and J_2 will be equal to the quantity purchased by J_3 alone, we only have to compare the respective transfers

$$\varkappa_{IJ} = (v_J(A^*) - v_J(A^{IJ})) - \sum_{I' \in S \backslash I} (C_{I'}(A^*) - C_{I'}(A^{IJ})),$$

as used in condition (9). As $v_J(A^*) = |J|\,u(a_j^*)$, these elements cancel in (9), which we can therefore rewrite as

$$\begin{aligned} & v_J(A^{IJ_3}) - \sum_{I' \neq I} C_{I'}(A^{IJ_3}) + \sum_{I' \neq I} C_{I'}(A^*) \\ \leq \;& v_J(A^{IJ_1}) - \sum_{I' \neq I} C_{I'}(A^{IJ_1}) + v_J(A^{IJ_2}) - \sum_{I' \neq I} C_{I'}(A^{IJ_2}) + 2\sum_{I' \neq I} C_{I'}(A^*). \end{aligned} \tag{25}$$

We proceed as in the proof of Lemma 1 by first deriving properties of the optimal reallocation of purchases. Let Δ^{IJ} denote the (symmetric) increase in the purchases from $i \in G \backslash I$, so that for these elements $a_{ij}^{IJ} = a^* + \Delta^{IJ}/\,|J|$, and note that $c'(|N|\,a^* + \Delta^{IJ}) = u_i(a_j^{IJ})$. Note that $\Delta^{IJ} > 0$ as the cross partial derivatives of $u(\cdot)$ are strictly negative. Using that

$$v_J(A^{IJ}) - \sum_{I' \neq I} C_{I'}(A^{IJ}) = |J|\,u(a_j^{IJ}) - (|G| - |I|)c(|N|\,a^* + \Delta^{IJ}). \tag{26}$$

and $\sum_{I' \neq I} C_{I'}(A^*) = (|G| - |I|)c(|N|\,a^*)$, we can further rewrite condition (25) as follows:

$$\begin{aligned} & |J_3|\,u(a_j^{IJ_3}) - (|G| - |I|)c(|N|\,a^* + \Delta^{IJ_3}) - (|G| - |I|)c(|N|\,a^*) \\ \leq \;& |J_2|\,u(a_j^{IJ_2}) - (|G| - |I|)c(|N|\,a^* + \Delta^{IJ_2}) + |J_1|\,u(a_j^{IJ_1}) - (|G| - |I|)c(|N|\,a^* + \Delta^{IJ_1}). \end{aligned} \tag{27}$$

If there is a single seller, i.e., $|I| = |G|$, then Δ^{IJ} is zero and therefore both sides of (27) are equal. Suppose for the remainder that $|I| \neq |G|$. We proceed as in the proof of Lemma 1 by deriving boundaries. Inequality (27) will then be satisfied if a more stringent inequality is satisfied, which is obtained by replacing in the inequality above the optimizers Δ^{IJ_2} and Δ^{IJ_1} respectively by $\Delta^{IJ_3}\frac{|J_2|}{|J_3|}$ and $\Delta^{IJ_3}\frac{|J_1|}{|J_3|}$. But then the respective column vectors used as arguments in $u(\cdot)$ in (27) are all the same, namely equal to $a_j^{IJ_3}$ (that is, each element equal to $a^* + \Delta^{IJ_3}/|J_3|$ for each good that is not produced by I and 0 for each good produced by I). The respective utilities in (27) then cancel out and after dividing by $(|G| - |I|)$ we are left with the condition

$$c\left(|N|\, a^* + \Delta^{IJ_3}\right) - c\left(|N|\, a^* + \Delta^{IJ_3}\frac{|J_1|}{|J_3|}\right) - c\left(|N|\, a^* + \Delta^{IJ_3}\frac{|J_2|}{|J_3|}\right) + c\left(|N|\, a^*\right) \geq 0.$$

This is finally equivalent to

$$c(x+y) - c(x+\lambda y) - c(x+(1-\lambda)y) + c(x) \geq 0,$$

where $x = |N|\, a^*$, $\lambda = \frac{|J_1|}{|J_3|}$ and $y = \Delta^{IJ_3}$. We can then further rewrite this as

$$\int_0^{(1-\lambda)y} \left[c'(x+\lambda y+s) - c'(x+s)\right] ds \geq 0,$$

which holds strictly by strict convexity of $c(\cdot)$. **Q.E.D.**

Proof of Proposition 7. We consider first the baseline case with strictly convex costs and no capacity constraints. As goods are now perfect substitutes, we can simplify expressions as follows. Considering a disagreement between I and J, we generically denote by a the choice of (adjusted) trade of seller I with all other buyers $J \neq I$, so that

$$\varphi_{IJ} = \max_a \left[(|N| - |J|)\, u\left((|G| - |I|)\, a^* + |I|\, a\right) - |I|\, c\left((|N| - |J|)\, a\right)\right] - \left[(|N| - |J|)\, u\left(|G|\, a^*\right) - |I|\, c\left(|N|\, a^*\right)\right] \tag{28}$$

Denote the respective expressions for the two buyers by φ_{IJ_1} and φ_{IJ_2}. As described in the main text, we consider an increase in $|J_1|$ and a corresponding decrease in $|J_2|$. Though $|J|$ takes on only integer values, note that expression (28) is defined also generally for real-valued $|J|$. To make this transparent, denote the respective expression by a function $\widetilde{\varphi}_I(x = |J|)$. Letting $x_2 = M - x_1$, we thus have from application of the envelope theorem that

$$\begin{aligned} \frac{d(\widetilde{\varphi}_I(x_1) + \widetilde{\varphi}_I(x_2))}{dx_1} &= \left[u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_2\right) - u\left((|G| - |I|)\, a^* + |I|\, \widehat{a}_1\right)\right] \qquad (29) \\ &\quad - |I| \left[\widehat{a}_2 c'\left((|N| - x_2)\, \widehat{a}_2\right) - \widehat{a}_1 c'\left((|N| - x_1)\, \widehat{a}_1\right)\right], \end{aligned}$$

where $\widehat{a}_y$ (for $y = 1, 2$) solves (after dividing by $(|N| - x_y)|I|$)

$$u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_y\right) - c'\left(\left(|N| - x_y\right)\widehat{a}_y\right) = 0, \tag{30}$$

so that from strict convexity of $c(\cdot)$ and strict concavity of $u(\cdot)$ we have $\widehat{a}_1 > \widehat{a}_2$ as long as $x_1 > x_2$. Substituting (30) into (29), this expression is strictly negative if

$$\begin{aligned} & u\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_1\right) - u\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_2\right) \\ > \;& |I|\,\widehat{a}_1 u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_1\right) - |I|\,\widehat{a}_2 u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_2\right). \end{aligned} \tag{31}$$

Using strict concavity of $u(\cdot)$ and $\widehat{a}_1 > \widehat{a}_2$, (31) surely holds if, on the right-hand side, we replace $u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_2\right)$ by $u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_1\right)$, which yields the following sufficient requirement:

$$\frac{u\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_1\right) - u\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_2\right)}{|I|\left(\widehat{a}_1 - \widehat{a}_2\right)} > u'\left(\left(|G| - |I|\right)a^* + |I|\,\widehat{a}_1\right).$$

This holds from strict concavity of $u(\cdot)$. The assertion in the proposition then follows as, first, by leaving $|J_1| + |J_2| = M$ constant, all other φ_{IJ} for $J \notin \{J_1, J_2\}$ are not affected, and, second, we can express

$$\left(\varphi_{I\widehat{J}_1} + \varphi_{I\widehat{J}_2}\right) - \left(\varphi_{IJ_1} + \varphi_{IJ_2}\right) = \int_{|J_1|}^{|J_1|+m} \frac{d(\widetilde{\varphi}_I(x_1) + \widetilde{\varphi}_I(M - x_1))}{dx_1} dx_1 < 0.$$

It remains to prove the assertion for the case where adjustments are not given by the first-order condition (30) but are (optimally) equal to a binding capacity constraint, given that $c(x) = \infty$ when $x > K$, while $a^* = K/|N|$. For this case, we have, using that goods are perfect substitutes,

$$\varphi_{IJ} = \left(|N| - |J|\right)\left[u\left(\left(|G| - |I|\right)a^* + |I|\,\frac{K}{|N| - |J|}\right) - u\left(|G|\,a^*\right)\right].$$

Then, with $x_2 = M - x_1$, we have in this case

$$\begin{aligned} \frac{d(\widetilde{\varphi}_I(x_1) + \widetilde{\varphi}_I(x_2))}{dx_1} = & \left[u\left(\left(|G| - |I|\right)a^* + |I|\,\frac{K}{|N| - x_2}\right) - u\left(\left(|G| - |I|\right)a^* + |I|\,\frac{K}{|N| - x_1}\right)\right] \\ & - |I|\left[\frac{K}{|N| - x_2}u'\left(\left(|G| - |I|\right)a^* + |I|\,\frac{K}{|N| - x_2}\right) - \frac{K}{|N| - x_1}u'\left(\left(|G| - |I|\right)a^* + |I\right.\right. \end{aligned}$$

To show that this is strictly negative, using now $\frac{K}{|N|-x_1} > \frac{K}{|N|-x_2}$, we can proceed in complete analogy to the previous (interior adjustment) case. **Q.E.D.**

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