Inflation in the Great Recession and New Keynesian Models

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Abstract

It has been argued that existing DSGE models cannot properly account for the evolution of key macroeconomic variables during and following the recent great recession, and that models in which inflation depends on economic slack cannot explain the recent muted behavior of inflation, given the sharp drop in output that occurred in 2008-09. In this paper, we use a standard DSGE model available prior to the recent crisis and estimated with data up to 2008:Q3 to explain the behavior of key macroeconomic variables since the crisis. We show that as soon as the financial stress jumped in the fourth quarter of 2008, the model successfully predicts a sharp contraction in economic activity along with a modest and more protracted decline in inflation. The model does so even though inflation remains very dependent on the evolution of economic activity and of monetary policy. We conclude that while the model considered does not capture all short-term fluctuations in key macroeconomic variables, it has proven to be surprisingly accurate during the recent crisis and the subsequent recovery.


KEY WORDS: Great recession, fundamental inflation, DSGE models, Bayesian estimation.

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1 Introduction

As dramatic as the recent Great Recession has been, it constitutes a potential test for existing macroeconomic models. Prominent researchers have argued that existing DSGE models cannot properly account for the evolution of key macroeconomic variables during and following the crisis. For instance, Hall (2011), in his Presidential Address, has called for a fundamental reconsideration of models in which inflation depends on a measure of slack in economic activity. He suggests that all theories based on the concept of non-accelerating inflation rate of unemployment or NAIRU should have predicted more and more deflation as long as the unemployment rate has remained above a natural rate of, say, around 6 percent. Since inflation has declined somewhat in early 2009 and then remained contained for a few years, Hall (2011) concludes that such theories based on a concept of slack must be wrong. Most notably, he states that popular DSGE models based on the simple New Keynesian Phillips curve according to which prices are set on the basis of a markup over expected future marginal costs “cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack” as they rely on a NAIRU principle. Hall (2011) thus concludes that inflation behaves in a nearly exogenous fashion.

Similarly, Ball and Mazumder (2011) argue that Phillips curves estimated over the 1960-2007 period in the US cannot explain the behavior of inflation in the 2008-2010 period. Moreover, they conclude that the “Great Recession provides fresh evidence against the New Keynesian Philips curve with rational expectations.” They stress the fact that the fit of that equation deteriorates once data for the years 2008-2010 are added to the sample. One of the reasons for this is that the labor share, a proxy for firms marginal costs, declines dramatically during the crisis, resulting in a change in the comovement with other measures of slack, such as the unemployment rate.

A further challenge to the New Keynesian Phillips curve is raised by King and Watson (2012) who find a large discrepancy between the inflation predicted by a popular DSGE model, the Smets and Wouters (2007) model, and actual inflation. They thus conclude that the model can successfully explain the behavior of inflation only when assuming the existence
of large exogenous markup shocks. This is disturbing to the extent that such mark-up shocks are difficult to interpret and have small effects on variables other than inflation.

In this paper, we use such a standard DSGE model, which was available prior to the recent crisis and that is estimated with data up to 2008, to explain the behavior of key macroeconomic variables since the crisis. The model used is the Smets and Wouters (2007) model extended to include financial frictions as in Bernanke et al. (1999), Christiano et al. (2003), and Christiano et al. (forthcoming). We show that as soon as the financial stress jumped in the fall of 2008, the model successfully predicts a sharp contraction in economic activity along with a modest and more protracted decline in inflation. The model can explain the comovement of output and inflation in the aftermath of the Great Recession. The evidence that we provide is based on out-of-sample forecasts and makes sure that the model is not estimated to fit the post-2008 data. The forecast of output is quite weak (indeed somewhat weaker than actual), yet inflation is projected to remain in the neighborhood of 1%. The out-of-sample forecast of inflation does not perfectly capture the high frequency movements in inflation which were largely due to oil price fluctuations, but it does capture the overall contour of actual inflation data. These results contrast with the commonly held belief that such models are bound to fail to capture the broad contours of the Great Recession and the near stability of inflation. We will thus re-interpret the results of Hall (2011), Ball and Mazumder (2011), and King and Watson (2012), in the context of our model.

It is important to note that while inflation does not appear to have declined much even in the face of the sharp drop in real GDP in 2008 and 2009, it is in fact very dependent on the evolution of economic activity and of monetary policy, as we will show. The key to understand this is that inflation is more dependent on expected future marginal costs than on the current level of activity. Even though GDP growth contracted sharply, monetary policy has in fact been sufficiently stimulative to promise inflation in the future to remain near 2%. As a result, inflation expectations remained fairly stable. Well anchored inflation expectations imply that inflation did not need to decline much, even in the face of a big drop in economic activity.\footnote{Christiano et al. (2011) use the model in Altig et al. (2011), which is very similar to the one used here but without financial financial fictions, to generate simulations of the Great Recession taking the zero lower bound on interest rates into account.}
While King and Watson (2012) have shown that price mark-up shocks are crucial to explain inflation fluctuations in the Smets and Wouters (2007) model, our variant of that model does not suffer from that problem. A key reason for the difference is that our estimated model involves a higher degree of price rigidities than is the case in Smets and Wouters (2007). This allows our model to successfully explain inflation with much smaller mark-up shocks. Yet, while the slope of the short-run Phillips curve is lower in our model than in Smets and Wouters (2007), monetary policy still has an important effect on inflation.

Our analysis leads us to conclude that while the model considered, which again was available before the recent crisis, does not capture all short-term fluctuations in key macroeconomic variables, it has proven to be surprisingly accurate during the recent crisis and the subsequent recovery. The remainder of this paper is organized as follows. Section 2 presents the model used for the empirical analysis. Section 3 first addresses the King and Watson criticism and shows that in this model fundamental inflation captures the medium and low frequency fluctuations in inflation fairly well, and that this success depends on the degree of price rigidities. Next, we elaborate on why the price rigidity estimate in the model with financial frictions and spreads as observables, is higher than in the Smets and Wouters model. We then proceed with a discussion of the effects of monetary policy on inflation. Section 4 discusses the post-2008 forecasts of key macroeconomic variables, and decomposes the inflation forecast to explain how predicted inflation remained so moderate despite the large drop in economic activity. Section 5 concludes. Information on the construction of the data set used for the empirical analysis as well as detailed estimation results and supplementary tables and figures are available in the Online Appendix.

2 The DSGE Model

The model considered is the one used in Smets and Wouters (2007), which is based on earlier work by Christiano et al. (2005) and Smets and Wouters (2003). It is a medium-scale DSGE bound on interest rates into account. They also find that inflation declines only modestly in their model, in large part because, as is the case here, their estimated Phillips curve is flat. In their model the flatness of the Phillips curve partly results from the assumption of firm-specific capital.
model, which augments the standard neoclassical stochastic growth model with nominal price and wage rigidities as well as habit formation in consumption and investment adjustment costs. As discussed before, the model is augmented with financial frictions, as in Bernanke et al. (1999), Christiano et al. (2003), and Christiano et al. (forthcoming). All ingredients of the model were however publicly available prior to 2008. As such, the model does not include some of the features that may have been found to be relevant following the crisis.

2.1 The Smets-Wouters Model

We begin by briefly describing the log-linearized equilibrium conditions of the Smets and Wouters (2007) model. We follow Del Negro and Schorfheide (2013) and detrend the non-stationary model variables by a stochastic rather than a deterministic trend. Let $\tilde{z}_t$ be the linearly detrended log productivity process which follows the autoregressive law of motion

$$\tilde{z}_t = \rho \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \quad (1)$$

We detrend all non stationary variables by $Z_t = e^{\gamma t + \frac{1}{1-\alpha} \tilde{z}_t}$, where $\gamma$ is the steady state growth rate of the economy. The growth rate of $Z_t$ in deviations from $\gamma$, denoted by $z_t$, follows the process:

$$z_t = \ln \left( \frac{Z_t}{Z_{t-1}} \right) - \gamma = \frac{1}{1-\alpha} (\rho_z - 1) \tilde{z}_{t-1} + \frac{1}{1-\alpha} \sigma_z \varepsilon_{z,t}. \quad (2)$$

All variables in the following equations are expressed in log deviations from their non-stochastic steady state. Steady state values are denoted by $^*$-subscripts and steady state formulas are provided in the technical appendix of Del Negro and Schorfheide (2013). The consumption Euler equation is given by:

$$c_t = -\frac{(1 - he^{-\gamma})}{\sigma_c (1 + he^{-\gamma})} \left( R_t - \mathbb{E}_t [\pi_{t+1}] + b_t \right) + \frac{he^{-\gamma}}{(1 + he^{-\gamma})} (c_{t-1} - z_t)$$

$$+ \frac{1}{(1 + he^{-\gamma})} \mathbb{E}_t \left[ c_{t+1} + z_{t+1} \right] + \frac{(\sigma_c - 1)}{\sigma_c (1 + he^{-\gamma})} \frac{w^* L^*}{c^*} (L_t - \mathbb{E}_t [L_{t+1}]), \quad (3)$$

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2 This approach makes it possible to express almost all equilibrium conditions in a way that encompasses both the trend-stationary total factor productivity process in Smets and Wouters (2007), as well as the case where technology follows a unit root process.

3 Available at http://economics.sas.upenn.edu/~schorf/research.htm.
where \( c_t \) is consumption, \( L_t \) is labor supply, \( R_t \) is the nominal interest rate, and \( \pi_t \) is inflation. The exogenous process \( b_t \) drives a wedge between the intertemporal ratio of the marginal utility of consumption and the riskless real return \( R_t - IE_t[\pi_{t+1}] \), and follows an AR(1) process with parameters \( \rho_b \) and \( \sigma_b \). The parameters \( \sigma_c \) and \( h \) capture the degree of relative risk aversion and the degree of habit persistence in the utility function, respectively. The following condition expresses the relationship between the value of capital in terms of consumption \( q^k_t \) and the level of investment \( i_t \) measured in terms of consumption goods:

\[
q^k_t = S'' e^{2\gamma (1 + \bar{\beta})} \left( i_t - \frac{1}{1 + \bar{\beta}} (i_{t-1} - z_t) - \frac{\bar{\beta}}{1 + \bar{\beta}} IE_t [i_{t+1} + z_{t+1}] - \mu_t \right),
\]

which is affected by both investment adjustment cost (\( S'' \) is the second derivative of the adjustment cost function) and by \( \mu_t \), an exogenous process called the “marginal efficiency of investment” that affects the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The exogenous process \( \mu_t \) follows an AR(1) process with parameters \( \rho_\mu \) and \( \sigma_\mu \). The parameter \( \bar{\beta} = \beta e^{(1 - \sigma_c)\gamma} \) depends on the intertemporal discount rate in the utility function of the households \( \beta \), the degree of relative risk aversion \( \sigma_c \), and the steady-state growth rate \( \gamma \).

The capital stock, \( \bar{k}_t \), evolves as

\[
\bar{k}_t = \left( 1 - \frac{i_s}{k_s} \right) (\bar{k}_{t-1} - z_t) + \frac{i_s}{k_s} i_t + \frac{i_s}{k_s} S'' e^{2\gamma (1 + \bar{\beta})} \mu_t,
\]

where \( i_s/k_s \) is the steady state ratio of investment to capital. The arbitrage condition between the return to capital and the riskless rate is:

\[
\frac{r^k_t}{r^k_s + (1 - \delta) IE_t[r^k_{t+1}]} + \frac{1 - \delta}{r^k_s + (1 - \delta) IE_t[q^k_{t+1}]} - q^k_t = R_t + b_t - IE_t[\pi_{t+1}],
\]

where \( r^k_t \) is the rental rate of capital, \( r^k_s \) its steady state value, and \( \delta \) the depreciation rate. Given that capital is subject to variable capacity utilization \( u_t \), the relationship between \( \bar{k}_t \) and the amount of capital effectively rented out to firms \( k_t \) is

\[
k_t = u_t - z_t + \bar{k}_{t-1}.
\]

The optimality condition determining the rate of utilization is given by

\[
\frac{1 - \psi}{\psi} r^k_t = u_t,
\]
where \( \psi \) captures the utilization costs in terms of foregone consumption. Real marginal costs for firms are given by

\[
mc_t = w_t + \alpha L_t - \alpha k_t, \tag{9}
\]

where \( w_t \) is the real wage and \( \alpha \) is the income share of capital (after paying markups and fixed costs) in the production function. From the optimality conditions of goods producers it follows that all firms have the same capital-labor ratio:

\[
k_t = w_t - r_k + L_t. \tag{10}
\]

The production function is:

\[
y_t = \Phi_p (\alpha k_t + (1 - \alpha) L_t) + \mathcal{I}\{\rho_z < 1\} (\Phi_p - 1) \frac{1}{1 - \alpha} \bar{z}_t, \tag{11}
\]

if the log productivity is trend stationary. The last term \( (\Phi_p - 1) \frac{1}{1 - \alpha} \bar{z}_t \) drops out if technology has a stochastic trend, because in this case one has to assume that the fixed costs are proportional to the trend. Similarly, the resource constraint is:

\[
y_t = g_t + \frac{c_s}{y_s} c_t + \frac{i_s}{y_s} i_t + \frac{k_s^k k_s}{y_s} u_t - \mathcal{I}\{\rho_z < 1\} \frac{1}{1 - \alpha} \bar{z}_t, \tag{12}
\]

where again the term \(-\frac{1}{1 - \alpha} \bar{z}_t\) disappears if technology follows a unit root process. Government spending \( g_t \) is assumed to follow the exogenous process:

\[
g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz} \sigma_z \varepsilon_{z,t}.
\]

Finally, the price and wage Phillips curves are, respectively:

\[
\pi_t = \frac{(1 - \zeta_p \bar{\beta})(1 - \zeta_p)}{(1 + \tau_p \bar{\beta})\zeta_p ((\Phi_p - 1)\epsilon_p + 1)} mc_t + \frac{\tau_p}{1 + \tau_p \bar{\beta}} \pi_{t-1} + \frac{\bar{\beta}}{1 + \tau_p \bar{\beta}} \mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \tag{13}
\]

and

\[
w_t = \frac{(1 - \zeta_w \bar{\beta})(1 - \zeta_w)}{(1 + \bar{\beta})\zeta_w ((\lambda_w - 1)\epsilon_w + 1)} (w_t^h - w_t) - \frac{1 + \tau_w \bar{\beta}}{1 + \bar{\beta}} \pi_t + \frac{1}{1 + \bar{\beta}} (w_{t-1} - z_t - \tau_w \pi_{t-1}) + \frac{\bar{\beta}}{1 + \bar{\beta}} \mathbb{E}_t [w_{t+1} + z_{t+1} + \pi_{t+1}] + \lambda_{w,t}, \tag{14}
\]

where \( \zeta_p, \tau_p, \) and \( \epsilon_p \) are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and \( \zeta_w, \tau_w, \) and \( \epsilon_w \) are the corresponding
parameters for wages. \( w^h_t \) measures the household’s marginal rate of substitution between consumption and labor, and is given by:

\[
w^h_t = \frac{1}{1 - he^{-\gamma}} \left( c_t - he^{-\gamma} c_{t-1} + he^{-\gamma} z_t \right) + \nu_t L_t, \tag{15}
\]

where \( \nu_t \) characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in absence of wage rigidities). The mark-ups \( \lambda_{f,t} \) and \( \lambda_{w,t} \) follow exogenous ARMA(1,1) processes

\[
\lambda_{f,t} = \rho_{\lambda_f} \lambda_{f,t-1} + \sigma_{\lambda_f} \varepsilon_{\lambda_f,t} + \eta_{\lambda_f} \sigma_{\lambda_f} \varepsilon_{\lambda_f,t-1}, \quad \text{and}
\]

\[
\lambda_{w,t} = \rho_{\lambda_w} \lambda_{w,t-1} + \sigma_{\lambda_w} \varepsilon_{\lambda_w,t} + \eta_{\lambda_w} \sigma_{\lambda_w} \varepsilon_{\lambda_w,t-1},
\]

respectively. Finally, the monetary authority follows a generalized feedback rule:

\[
R_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \psi_1 \pi_t + \psi_2 (y_t - y^f_t) \right) \\
+ \psi_3 \left( (y_t - y^f_t) - (y_{t-1} - y^f_{t-1}) \right) + r^m_t, \tag{16}
\]

where the flexible price/wage output \( y^f_t \) is obtained from solving the version of the model without nominal rigidities (that is, Equations (3) through (12) and (15)), and the residual \( r^m_t \) follows an AR(1) process with parameters \( \rho_{r,m} \) and \( \sigma_{r,m} \).

### 2.2 Adding Observed Long Run Inflation Expectations

In order to capture the rise and fall of inflation and interest rates in the estimation sample, we replace the constant target inflation rate by a time-varying target inflation. While time-varying target rates have been frequently used for the specification of monetary policy rules in DSGE model (e.g., Erceg and Levin (2003) and Smets and Wouters (2003), among others), we follow the approach of Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011) and include data on long-run inflation expectations as an observable into the estimation of the DSGE model. At each point in time, the long-run inflation expectations essentially determine the level of the target inflation rate. To the extent that long-run inflation expectations at the forecast origin contain information about the central bank’s objective function, e.g. the desire to stabilize inflation at 2%, this information is automatically included in the forecast.
Clark (2011) constructs a Bayesian VAR in which variables are expressed in deviations from long-run trends. For inflation and interest rates these long-run trends are given by long-horizon Blue Chip forecasts and the VAR includes equations that capture the evolution of these forecasts. Our treatment of inflation in the DSGE model bears similarities to Clark (2011)’s VAR.

More specifically, for the SW model the interest-rate feedback rule of the central bank (16) is modified as follows:

\[ R_t = \rho R_{t-1} + (1 - \rho_R) \left( \psi_1 (\pi_t - \pi^*_t) + \psi_2 (y_t - y^*_t) \right) + \psi_3 \left( (y_t - y^*_t) - (y_{t-1} - y^*_{t-1}) \right) + \epsilon^m_t. \]

The time-varying inflation target evolves according to:

\[ \pi^*_t = \rho_{\pi^*} \pi^*_{t-1} + \sigma_{\pi^*} \epsilon_{\pi^* t}, \]

where \( 0 < \rho_{\pi^*} < 1 \) and \( \epsilon_{\pi^* t} \) is an iid shock. We follow Erceg and Levin (2003) and model \( \pi^*_t \) as following a stationary process, although our prior for \( \rho_{\pi^*} \) will force this process to be highly persistent (see Panel II of Table A-1). The assumption that the changes in the target inflation rate are exogenous is, to some extent, a short-cut. For instance, the learning models of Sargent (1999) or Primiceri (2006) would suggest that the rise in the target inflation rate in the 1970’s and the subsequent drop is due to policy makers learning about the output-inflation trade-off and trying to set inflation optimally. We are abstracting from such a mechanism in our specification.

2.3 Adding Financial Frictions

We now add financial frictions to the SW model building on the work of Bernanke et al. (1999), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (forthcoming). In this extension, banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to

\footnote{We follow the specification in Del Negro and Eusepi (2011), while Aruoba and Schorfheide (2008) assume that the inflation target also affects the intercept in the feedback rule.}
intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus be too low to pay back the bank loans. Banks protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of the entrepreneurs’ leverage and their riskiness. Adding these frictions to the SW model amounts to replacing equation (6) with the following conditions:

\[ E_t \left[ \tilde{R}^{k}_{t+1} - R_t \right] = b_t + \zeta_{sp,b} \left( q^k_t + \bar{k}_t - n_t \right) + \tilde{\sigma}_{\omega,t} \tag{19} \]

and

\[ \tilde{R}^k_t - \pi_t = \frac{r^k_*}{r^k_* + (1 - \delta)} r^k_t + \frac{(1 - \delta)}{r^k_* + (1 - \delta)} q^k_t - q^k_{t-1}, \tag{20} \]

where \( \tilde{R}^k_t \) is the gross nominal return on capital for entrepreneurs, \( n_t \) is entrepreneurial equity, and \( \tilde{\sigma}_{\omega,t} \) captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (forthcoming)) and follows an AR(1) process with parameters \( \rho_{\sigma_{\omega}} \) and \( \sigma_{\sigma_{\omega}} \). The second condition defines the return on capital, while the first one determines the spread between the expected return on capital and the riskless rate.\(^5\)

The following condition describes the evolution of entrepreneurial net worth:

\[ n_t = \zeta_{n,\tilde{R}^k} \left( \tilde{R}^k_t - \pi_t \right) - \zeta_{n,R} \left( R_{t-1} - \pi_t \right) + \zeta_{n,qK} \left( q^k_{t-1} + \bar{k}_{t-1} \right) + \zeta_{n,n} n_{t-1} - \frac{\zeta_{n,\sigma_{\omega}}}{\zeta_{sp,\sigma_{\omega}}} \tilde{\sigma}_{\omega,t-1}. \tag{21} \]

### 2.4 Fundamental Inflation

To understand the behavior of inflation, it will be useful to extract from the model-implied inflation series an estimate of “fundamental inflation” as in King and Watson (2012), and similarly to Galí and Gertler (1999) and Sbordone (2005). To obtain this measure, we rewrite the Phillips curve (13) as:

\[ \pi_t = \frac{\beta p}{1 + \beta p} \pi_{t-1} + \frac{\beta}{1 + \beta p} E_t[\pi_{t+1}] + \kappa m c_t + \lambda_{f,t}, \tag{22} \]

Note that if \( \zeta_{sp,b} = 0 \) and the financial friction shocks \( \tilde{\sigma}_{\omega,t} \) are zero, (19) and (20) coincide with (6).
where  $\kappa = \frac{(1 - \zeta_p \bar{\beta})(1 - \zeta_p)}{(1 + \zeta_p \bar{\beta}) \zeta_p ((\Phi_p - 1) \varepsilon_p + 1)}$. Quasi-differencing inflation by defining $\Delta_{i_p} \pi_t = \pi_t - i_p \pi_{t-1}$, one can simplify the expression for the Phillips curve as follows:

$$\Delta_{i_p} \pi_t = \bar{\beta} E_t[\Delta_{i_p} \pi_{t+1}] + (1 + i_p \bar{\beta}) (\kappa m c_t + \lambda_{f,t}).$$

This difference equation can be solved forward to obtain

$$\Delta_{i_p} \pi_t = (1 + i_p \bar{\beta}) \kappa \sum_{j=0}^{\infty} \bar{\beta}^j E_t[m c_{t+j}] + (1 + i_p \bar{\beta}) \sum_{j=0}^{\infty} \bar{\beta}^j E_t[\lambda_{f,t+j}].$$

The first component captures the effect of the sum of discounted future marginal costs on current inflation, whereas the second term captures the contribution of future mark-up shocks. Defining

$$S_t^\infty = \sum_{j=0}^{\infty} \bar{\beta}^j E_t[m c_{t+j}],$$

we can decompose inflation into

$$\pi_t = \tilde{\pi}_t + \Lambda_{f,t},$$

where

$$\tilde{\pi}_t = \kappa (1 + i_p \bar{\beta}) (1 - i_p L)^{-1} S_t^\infty,$$

$$\Lambda_{f,t} = (1 + i_p \bar{\beta}) (1 - i_p L)^{-1} \sum_{j=0}^{\infty} \bar{\beta}^j E_t[\lambda_{f,t+j}],$$

and $L$ denotes the lag operator. We refer to the first term on the right-hand-side of (26), $\tilde{\pi}_t$, as fundamental inflation. Fundamental inflation corresponds to the discounted sum of expected marginal costs.\(^6\) Thus, our decomposition removes the direct effect of mark-up shocks from the observed inflation. Note, however, that the summands in (26) are not orthogonal. Fundamental inflation still depends on $\lambda_{f,t}$ indirectly, through the effect of the markup shock on current and future expected marginal costs.

### 2.5 State-Space Representation and Estimation

We use the method in Sims (2002) to solve the log-linear approximation of the DSGE model. We collect all the DSGE model parameters in the vector $\theta$, stack the structural shocks in

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\(^6\)Our measure differs however slightly from the measure of Galí and Gertler (1999) and Sbordone (2005), who define fundamental inflation as $\tilde{\pi}_t = t_p \pi_t + \kappa (1 + i_p \bar{\beta}) S_t^\infty$. 

the vector $\epsilon_t$, and derive a state-space representation for our vector of observables $y_t$. The state-space representation is comprised of the transition equation:

$$s_t = T(\theta)s_{t-1} + R(\theta)\epsilon_t,$$

which summarizes the evolution of the states $s_t$, and the measurement equation:

$$y_t = Z(\theta)s_t + D(\theta),$$

which maps the states onto the vector of observables $y_t$, where $D(\theta)$ represents the vector of steady state values for these observables. The measurement equations for real output, consumption, investment, and real wage growth, hours, inflation, and interest rates are given by:

\begin{align*}
\text{Output growth} & = \gamma + 100\left(y_t - y_{t-1} + z_t\right) \\
\text{Consumption growth} & = \gamma + 100\left(c_t - c_{t-1} + z_t\right) \\
\text{Investment growth} & = \gamma + 100\left(i_t - i_{t-1} + z_t\right) \\
\text{Real Wage growth} & = \gamma + 100\left(w_t - w_{t-1} + z_t\right), \\
\text{Hours} & = \bar{l} + 100l_t \\
\text{Inflation} & = \pi_* + 100\pi_t \\
\text{FFR} & = R_* + 100R_t
\end{align*}

where all variables are measured in percent, where $\pi_*$ and $R_*$ measure the steady state level of net inflation and short term nominal interest rates, respectively and where $\bar{l}$ captures the mean of hours (this variable is measured as an index).

To incorporate information about low-frequency movements of inflation the set of measurement equations (31) is augmented by

$$\pi_t^{O.40} = \pi_* + 100\mathbb{E}_t \left[ \frac{1}{40} \sum_{k=1}^{40} \pi_{t+k} \right]$$

$$= \pi_* + \frac{100}{40} Z(\theta)(\pi_*) (I - T(\theta))^{-1} (I - [T(\theta)]^{40}) T(\theta)s_t,$$

where $\pi_t^{O.40}$ represents observed long run inflation expectations obtained from surveys (in percent per quarter), and the right-hand-side of (32) corresponds to expectations obtained from the DSGE model (in deviation from the mean $\pi_*$). The second line shows how to
compute these expectations using the transition equation (29) and the measurement equation for inflation. \(Z(\theta)_{(\pi, .)}\) is the row of \(Z(\theta)\) in (30) that corresponds to inflation. Finally, we add a measurement equation for the spread:

\[
\text{Spread} = SP_\ast + 100E_t \left[ \tilde{R}_{t+1} - R_t \right],
\]

where the parameter \(SP_\ast\) measures the steady state spread. The construction of the data set is summarized in Appendix A.

We use Bayesian techniques in the subsequent empirical analysis, which require the specification of a prior distribution for the model parameters. For most of the parameters we use the same marginal prior distributions as Smets and Wouters (2007). There are two important exceptions. First, the original prior for the quarterly steady state inflation rate \(\pi_\ast\) used by Smets and Wouters (2007) is tightly centered around 0.62% (which is about 2.5% annualized) with a standard deviation of 0.1%. We favor a looser prior, one that has less influence on the model’s forecasting performance, that is centered at 0.75% and has a standard deviation of 0.4%. Second, for the financial frictions mechanism we specify priors for the parameters \(SP_\ast, \zeta_{sp,b}, \rho_{\sigma_{\omega}}, \) and \(\sigma_{\omega}\). We fix the parameters corresponding to the steady state default probability and the survival rate of entrepreneurs, respectively. In turn, these parameters imply values for the parameters of (21).

A summary of the priors is provided in Table A-1 in Appendix B. Most of the results presented below are based on the version of the DSGE model that includes the time-varying target inflation rate (see Section 2.2) and financial frictions (see Section 2.3). We refer to this model as SWFF. In some instances we provide comparisons to the basic Smets-Wouters model (see Section 2.1), denoted by SW, and the Smets-Wouters model with time-varying inflation target, SW\(\pi\). Importantly, all the results obtained with the SWFF model, including the forecasts, are based on the modal pre-Great Recession estimates of the model parameters, that is, on data from 1964:Q1 to 2008:Q3 (Table A-2 in the Appendix reports the modal estimates). For comparison with King and Watson, we use the original SW estimates for all results involving the SW model (the posterior mode obtained re-estimating the model up to 2008:Q3 are quite similar). In terms of notation, we refer to the sample from 1964:Q1
to 2008:Q3 on which the models are estimated as $Y_{1:T} = \{y_1, \ldots, y_T\}$, and to the forecast period (2008:Q4 to 2012:Q3) as $Y_{T+1:T_{full}}$.

3 New Keynesian Models and Inflation

We begin with a detour from the analysis of the (post-) Great Recession data and study how the New Keynesian model described in the previous section explains the dynamics of inflation in the whole sample. Galí and Gertler (1999) and Sbordone (2005) introduce the concept of fundamental inflation, which measures the present discounted value of future marginal costs. We referred to this variable as $\tilde{\pi}_t$ and provided a formal definition in (27).

The forward-looking New Keynesian Phillips Curve (NKPC) is considered successful if fundamental inflation broadly captures the dynamics of inflation. Recently, King and Watson (2012), henceforth KW, report a measure of fundamental inflation based on the posterior mode estimates of Smets and Wouters (2007). KW strongly criticize the NKPC in Smets and Wouters (2007)’s model for its inability to generate a plausible measure of fundamental inflation, and argue that inflation is largely explained by exogenous markup shocks.

In contrast to King and Watson (2012), as we show below, the model-generated $\tilde{\pi}_t$ tracks the low frequency component of actual inflation quite well (Section 3.1) in the model with financial frictions. We then show that the ability to track inflation relies on a larger estimate of nominal price rigidity than the one reported in Smets and Wouters (2007), and explain why the model with financial frictions and spreads as an additional observable yields higher estimates of the price rigidity parameter than in Smets and Wouters (2007). Last, we examine whether a flatter short-run Phillips curve implies that monetary policy loses some of its ability to stabilize inflation (Section 3.3).

3.1 Inflation and Fundamental Inflation

We first reproduce the KW estimate of fundamental inflation before presenting the fundamental inflation associated with our DSGE model. We will show that the estimate of $\tilde{\pi}_t$ is closely related to actual inflation if the estimated price rigidity, which is governed by the
parameter $\zeta_p$, is high enough. The SWFF model generates a larger estimate of $\zeta_p$, which in turn produces a $\tilde{\pi}_t$ that closely tracks the low-frequency dynamics of inflation. We defer a discussion of why our DSGE model is associated with a relatively large estimate of $\zeta_p$ to Section 3.2.

Figure 1 depicts actual GDP-deflator inflation (solid black line) and fundamental inflation. The dashed purple line depicts the $\tilde{\pi}_t$ series obtained from the SW model, and essentially reproduces the estimate reported by KW (formally, we reporting the smoothed estimates of fundamental inflation $E_{T_{fut}}[\tilde{\pi}_t]$). The discrepancy with actual inflation is staggering. In the first part of the sample, the SW measure grossly overestimates actual inflation, whereas in the second part of the sample it underestimates GDP-deflator inflation, in particular since 2007. Were inflation to coincide with the Smets and Wouters (2007)/KW fundamental inflation, it would be of the order of -12% annualized.

We now turn to the measure of fundamental inflation that corresponds to our SWFF model, described in Section 2. Its path is given by the solid blue line in Figure 1. The solid blue line indicates that $\tilde{\pi}_t$ from the SWFF model is able to track actual inflation very well.

\footnote{We use a different vintage of data, so the smoothed series are not identical, but this makes little difference.}
Table 1: Posterior Mode for Selected Parameters

<table>
<thead>
<tr>
<th>Nominal rigidities Policy</th>
<th>SWFF</th>
<th>SW[07]</th>
<th>SWFF</th>
<th>SW[07]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_p$</td>
<td>0.87</td>
<td>0.65</td>
<td>$\psi_1$</td>
<td>1.37</td>
</tr>
<tr>
<td>$\zeta_w$</td>
<td>0.89</td>
<td>0.73</td>
<td>$\psi_2$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>0.23</td>
<td>0.22</td>
<td>$\psi_3$</td>
<td>0.24</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>0.42</td>
<td>0.59</td>
<td>$\pi^*$</td>
<td>0.77</td>
</tr>
</tbody>
</table>

and essentially captures its low and mid-frequency variation. The difference between our estimate of fundamental inflation and the KW estimate is mainly driven by the degree of price rigidity $\zeta_p$ reported in Table 1. If we replace our modal estimate of $\hat{\zeta}_p = 0.87$ with Smets and Wouters (2007)’s estimate of $\hat{\zeta}_p = 0.65$ we obtain very similar results as KW. The effect of all other coefficients is much more muted.

Our model’s estimate of price rigidities may appear as surprisingly large when compared to microeconomic evidence about the frequency of price changes reported, e.g., in Bils and Klenow (2004) or Nakamura and Steinsson (2008). However, it is important to remember that prices change in every quarter in our model, as prices that are not re-optimized are

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8Figure A-2 in the Appendix also shows the core PCE inflation (since a core measure for the GDP Deflator is not available) as well as a measure of fundamental inflation stripped of the indirect effect that arises from the impact of markup shocks on the evolution of marginal costs ($\pi_t^{no mkup}$). $\tilde{\pi}_t$ and $\pi_t^{no mkup}$ are close to one another and track core inflation even better than they track the GDP deflator inflation. The comparison between core and actual inflation is also revealing: differences between core and headline inflation usually reflect abrupt changes in commodity prices, as in the latest period or in the mid-2000, and these changes are captured by mark-up shocks in the model. But mark-up shocks also have an effect on marginal costs: positive mark-up shocks depress economic activity and make fundamental inflation lower, opening a gap between $\tilde{\pi}_t$ and $\pi_t^{no mkup}$.

9There are two differences between SWFF and SW, as discussed in Section 2: the addition of long-run inflation expectations as an observable and a time-varying inflation target (model SW$\pi$), and the addition of financial frictions and spreads as observables. The first difference is of little import for fundamental inflation. Figure A-1 in the Appendix shows that fundamental inflation for the SW$\pi$ model is just as off the mark in terms of capturing the behavior of inflation as that of the SW model.
indexed to past inflation.\footnote{Our estimate of $\hat{\zeta}_p = 0.87$ implies that prices are re-optimized on average every $1/(1 - 0.87) = 7.7$ quarters. Furthermore, as argued by Boivin et al. (2009), while individual or sectoral prices may vary frequently in response to sector-specific disturbances, they appear much more sluggish in response to aggregate shocks, which are arguably more relevant for our purposes. Finally, as shown in Woodford (2003) and Christiano et al. (2011), a relatively flat slope of the NKPC can alternatively be obtained without large price rigidities by assuming that firms use firm-specific capital, or by assuming a larger curvature parameter in the Kimball aggregator for prices, $\epsilon_p$.}

Why is fundamental inflation so sensitive to the level of price rigidity? Fundamental inflation is the sum of discounted future marginal costs, multiplied by the slope of the Phillips curve $\kappa$ (see equation (27), which we repeat below):

$$\tilde{\pi}_t = \kappa(1 + t_p \bar{\beta})(1 - t_p L)^{-1} \sum_{j=0}^{\infty} \bar{\beta}^j E_t [mc_{t+j}] .$$

The Calvo parameter $\zeta_p$ affects fundamental inflation through two channels. First, a larger value of $\zeta_p$ implies a flatter Phillips curve, i.e., a smaller slope coefficient $\kappa$, as more rigid prices are less responsive to given changes in marginal costs. Hence, holding $S^\infty$ fixed, an increase in nominal rigidities, tends to dampen the fluctuations in $\tilde{\pi}_t$. Second, and equally important, the parameter $\zeta_p$ indirectly affects the marginal costs equilibrium dynamics and therefore $S^\infty$.

To understand the second channel, suppose that the prices were close to fully flexible. In this case, firms would set their prices at a mark-up over the nominal marginal costs, where the mark-up would be exogenously time-varying in response to price mark-up shocks.\footnote{This can be seen by taking the limit as prices become fully flexible in the linearized NKPC (13). Noting that $\lambda_{f,t}$ is a renormalized version of the mark-up shock (i.e., $\lambda_{f,t} = \kappa \tilde{\lambda}_{f,t}$, following SW), this equation implies that $0 = mc_t + \tilde{\lambda}_{f,t}$, as $\zeta_p \rightarrow 0$.} As a consequence, real marginal costs would not move, except in response to exogenous mark-ups shocks. To the extent that exogenous mark-up movements are not very persistent, real marginal costs revert quickly to steady state when prices are relatively flexible, so that $S^\infty$ essentially coincides with current real marginal costs $mc_t$. In contrast, if prices were very rigid, a persistent increase in the demand for goods would be met by firms with an increase
in the supply of goods, which would result in a persistent increase in marginal costs, even in the absence of mark-up shocks.

Figure 2: Price Rigidities and Forecasts of Marginal Costs

Notes: The solid black line corresponds to smoothed estimates of marginal costs in the aftermath of the Great Recession using the SWFF model. The solid red lines are the forecasts of marginal costs actual GDP deflator inflation; the dashed purple line is fundamental inflation computed from the SW/KW model; the solid blue line is $\tilde{\pi}_t$ associated with the SWFF model.

In Figure 2 we compare marginal cost forecasts for two choices of $\zeta_p$: the estimate obtained from the SWFF model and the estimate reported by Smets and Wouters (2007). The solid black line in the figure corresponds to smoothed estimates of marginal costs in deviations from steady state $E_{T_{full}}[mc_t]$ obtained using the SWFF model (estimates using the SW model are very similar), shown for the post-Great Recession period. The red lines departing at each point in time from the black line are the projected path of future marginal costs (that is, $E[mc_{t+h}|s_t|T_{full}]$, where $s_t|T_{full} = E_{T_{full}}[s_t]$ is the smoothed state of the economy for period $t$). Solid red lines are the forecasts for marginal costs obtained using the SWFF estimates of price rigidities. Dashed red lines are the forecasts obtained using the SW estimates of price rigidities. The dashed red lines show that when prices are relatively flexible, as in SW, marginal costs revert quickly to steady state.

\footnote{All other parameters are as in SWFF, but as pointed out above the values of the other parameters makes little difference – see Figure A-3 in the Appendix).}
Notes: The dashed black line corresponds to smoothed estimates of marginal costs using the SW model and the purple line is SW estimates of fundamental inflation. Both SW fundamental inflation and marginal costs are normalized (demeaned and divided by their respective standard deviation).

It follows that the lower estimated degree of price rigidities in the SW model has two implications. First, the present value of marginal costs, $S^\infty_t$, and hence fundamental inflation, are highly correlated with current marginal costs, as shown in Figure 3. Second, since movements in marginal costs are only weakly correlated with inflation most of the fluctuations in inflation have to be explained by the NKPC residual, namely price mark-up shocks.\(^{13}\)

Conversely, under the high $\zeta_p$ estimate associated with the SWFF model, mark-up shocks are still important in matching high frequency movements in inflation, but play a small role in driving fluctuations in fundamental inflation. For this model, fluctuations in marginal costs, and especially in $S^\infty$, are mostly explained endogenously by changes in economic activity,\(^{13}\)

\(^{13}\)Another way to make this point is to use the SW implied estimates of the present discounted value of marginal costs $S^\infty_t$ in equation (27), keeping the value of $\kappa$ the same as in SWFF, and show that the estimates of fundamental inflation $\tilde{\pi}$ have little in common with actual inflation (see Figure A-3 in the Appendix). Note that $S^\infty_t$ is relatively small when prices are more flexible because expected future marginal costs (in deviations from steady state) revert to zero quickly. The higher slope coefficient $\kappa$ in SW works in the opposite direction however, so that fundamental inflation turns out to be more volatile in SW than in SWFF.
rather than by exogenous changes in mark-ups shocks (see Figure A-6 in the Appendix). In sum, with a higher estimate of the degree of price rigidities, the SWFF model provides a unifying explanation for activity and inflation.

3.2 Why Does the Financial Frictions Model Have Higher Price Rigidities?

Figure 4: Demand Shocks and the slope of the Phillips Curve (AS)

We have seen that the estimate of the price rigidity parameter $\zeta_p$ makes all the difference between our results and those of KW. But why does the model with financial frictions and spreads as an observable yields a higher estimate of $\zeta_p$? The argument is quite intuitive, and can be explained using Figure 4.

Imagine that we knew for sure that we just observed a negative demand shock (a leftward shift in the AD curve), and that as a result of this shock output dropped a lot but inflation fell only a little. A model with a steep Phillips curve (AS curve) would have to rationalize this chain of events with a joint shift of the AD and the AS curve (left panel), the latter caused by a positive mark-up shock — given that an AD shift only would cause a large fall in prices. Conversely, a model with a flat Phillips curve would have no problem explaining
this with a simple shift in the AD curve only (right panel). The first explanation would involve a positive correlation of demand and mark-up shocks — one that is at odds with the model’s assumptions. The second explanation would be more natural, in that the outcomes can be explained as the result of one shock only.

Now, if we knew that a good portion of business cycle fluctuations is explained by such demand shocks, we would arguably be more comfortable with the second, simpler, explanation (flat AS curve) than with the first one (steep AS curve and correlated mark-up shocks). This is precisely the argument we are making. Since in this class of models movements in spreads are by and large associated with demand shocks — either “discount rate” \( b \) or “spread” \( \sigma_w \) shocks — using the spreads as observables makes demand shocks more important from an econometrician’s point of view.

What evidence do we have for our story? First, we can use spreads as an observables in the SW model, and see whether the estimated degree of price rigidities increases.\textsuperscript{14} Indeed the estimate of \( \zeta_p \) jumps to 0.81 — not quite the 0.86 in SWFF, suggesting that the endogenous part of the financial friction mechanism may also play a role, but certainly higher than the 0.65 in SW.\textsuperscript{15}

Second, we can point to the rolling window estimates of \( \zeta_p \) obtained by increasing the end-of-sample date from 1991:Q3 to 2010:Q3 (see Figure A-4 in the Appendix). Two facts emerge from the recurve estimation. First, the posterior estimate of \( \zeta_p \) is always significantly higher for the SWFF model than for the SW model. Second, and most importantly, for both models the posterior estimates jump upward after the Lehman crisis, but the increase is particularly

\textsuperscript{14}Note that equation (6) implies that in the SW model any spread between the return on capital and the riskless rate would be entirely explained by the \( b \) shock.

\textsuperscript{15}It has been documented, e.g., in Schorfheide (2008) that DSGE model-based estimates of the slope of the Phillips curve can vary widely across studies. The variation can be caused by a combination of model specification, data set, and choice of the prior distribution. Del Negro and Schorfheide (2008) document that reasonable changes in the prior distribution can generate estimates of \( \zeta_p \) ranging between 0.54 and 0.84, well within the range reported here. In addition, Herbst and Schorfheide (forthcoming) document that under a diffuse prior the SW model can generate a posterior distribution with two modes, one with \( \zeta_p = 0.59 \) and another one with \( \zeta_p = 0.70 \). Including the spread data as an additional observable contributes to shifting the posterior mass from one modal region to another.
large for the SWFF model. Why this increase? The Lehman event is interpreted by both models as largely driven by a demand shock, with the SWFF model being even more certain of that as it observes the dramatic increase in spreads. The posterior estimates are updated with the post-Lehman data precisely the way we described — by making the AS flatter.\textsuperscript{16}

Last, we computed the correlation between demand and markup shocks in the SW and SWFF models. As argued in describing Figure 4, this correlation is negative for the SW model (-0.37), implying that adverse demand shocks are associated with positive mark-up shocks, and slightly positive for the SWFF model (0.18).

3.3 The Slope of the Phillips Curve and Policy Effectiveness for Inflation Stabilization

Our finding that the NKPC in the SWFF model is quite flat raises natural questions: doesn’t a flatter Phillips curve imply that monetary policy looses some of its ability to stabilize inflation? Doesn’t this lead credence to Hall (2011)’s assumption of a nearly exogenous inflation rate?

The answer is no. To illustrate this point, the left panel of Figure 5 plots again the fundamental inflation $\tilde{\pi}_t$ (in black) estimated in our baseline model. The other two lines show $\tilde{\pi}_t$ using the same slope of the Phillips curve, $\kappa$, and the same estimated exogenous disturbances, but assume a different monetary policy response to inflation deviations from the inflation target. Specifically, while the policy coefficient $\psi_1$ is 1.37 in our baseline case, the green dashed line plots $\tilde{\pi}_t$ in the case that we set $\psi_1$ to 1.1, and the blue dashed line plots the case that $\psi_1$ is set to 2.0. Clearly, a stronger/weaker policy response to inflation fluctuations results to substantially smaller/larger fluctuations in fundamental inflation.

One way to understand this is to look at the projections of marginal costs. The right panel of Figure 5 reports again the estimated path of marginal costs (black line), and the red solid lines plot, for three different dates, the expected future marginal costs according to our baseline model (these are the same objects shown in Figure 2). The red dashed lines show

\textsuperscript{16}As we discussed, we only use pre-Great recession estimates of the parameters for our analysis.
Figure 5: Counterfactual Fundamental Inflation and Marginal Cost Evolution Under Different Policy (Response to Inflation)

\[
\text{Counterfactual } \tilde{\pi}_t \quad \text{Counterfactuals Marginal Costs Evolution}
\]

\[\begin{array}{c}
\text{1965} \quad 1970 \quad 1975 \quad 1980 \quad 1985 \quad 1990 \quad 1995 \quad 2000 \quad 2005 \quad 2010 \quad 2015 \\
-0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5
\end{array}\]

\[\begin{array}{c}
\text{fund. infl.} \quad \text{fund. infl. w/ } \psi_1 = 2.0 \quad \text{fund. infl. w/ } \psi_1 = 1.1
\end{array}\]

\[\begin{array}{c}
\text{2005} \quad 2010 \quad 2015 \quad 2020 \quad 2025 \quad 2030 \quad 2035 \quad 2040 \quad 2045
\end{array}\]

\[\begin{array}{c}
-10 \quad -8 \quad -6 \quad -4 \quad -2 \quad 0 \quad 2
\end{array}\]

\[\begin{array}{c}
\text{mc} \quad \text{future mc} \quad \text{future mc } \psi_1 = 1.1
\end{array}\]

Notes: Left panel: Fundamental inflation (black) and counterfactual fundamental inflation using different values of the inflation response in the policy rule \(\psi_1\). Right panel: The black and red lines are the same as in Figure 2. The dashed red “hairs” depict forecasted paths of future marginal costs obtained under \(\psi_1 = 1.1\).

the expected future marginal costs that would have obtained if the monetary policy rule had a lower response to inflation (\(\psi_1 = 1.1\)). The figure shows that in that case, marginal costs are expected to return to steady state more slowly than with the baseline policy. Since future marginal costs tend to wander off much away from their long-run steady state, with a lower \(\psi_1\) inflation expectations are less anchored.

4 DSGE Forecasts of the Great Recession

We examine the forecast performance of the New Keynesian DSGE model introduced in Section 2 during the 2007-2009 recession. We show that this model predicts a deep recession, and a subsequent weak recovery, just as observed in the data, and yet it does not predict deflation (Section 4.1).
4.1 Forecasts of the Economic Activity and Inflation

A DSGE forecaster on January 1st 2009 would have data from 1964:Q1 to 2008:Q3 (what we referred to as $Y_{1:T}$), but would also have access to 2008:Q4 information on the federal funds rate and the spread. We denote this information as $y_{1,T+1}$. Conditional on $(Y_{1:T}, y_{1,T+1})$ we generate multi-step forecasts. The output growth forecasts (quarter-on-quarter percentages and cumulative growth rates starting from the forecast origin) are depicted in Figure 6. Similar forecasts as well as a detailed description on how to compute them were reported in Del Negro and Schorfheide (2013).\footnote{The forecasts in Del Negro and Schorfheide (2013) were based on real-time data, whereas the forecasts in this paper are based on the 2012:Q3 vintage of data. Although revised and unrevised data are somewhat different as of 2008:Q3, the forecasts turn out to be very similar (see Figure A-5 in the Appendix).} When made aware – via the spread data – of the financial consequences of the Lehman default, the DSGE model with financial frictions predicts a sharp drop in GDP growth and a very sluggish recovery.\footnote{This is consistent with the findings of Gilchrist and Zakrajsek (2012) who use a reduced form approach (and a different measure of spreads).} The right panel of Figure 6 shows the forecast of cumulative growth rates. Arguably, the forecasts for real

Notes: The solid black lines depict actuals up to the forecast origin; the solid red lines indicate the forecast paths; the dashed black lines correspond to the actual paths.
activity are remarkably accurate: As of mid 2012 the model’s prediction for the level of output are almost perfectly in line with the data.

We now turn to the inflation forecasts, depicted in Figure 7. Inflation rates are also reported in terms of quarter-on-quarter percentages. The DSGE model forecast misses the deflation in 2009:Q1 and slightly under-predicts the average inflation rate between 2009:Q2 and 2012:Q3. The second panel of Figure 7 shows the cumulative inflation rates, that is, the change in the log price level relative to the forecast origin. This plot highlights that the model initially overpredicts and subsequently underpredicts inflation. The third panel of Figure 7 shows the DSGE model-implied output gap, that is the gap between actual output and counterfactual output in an economy without nominal rigidities and markup shocks. Since the counterfactual output is unobserved, the output gap is a latent variable and we plot the path of the smoothed gap prior to 2008:Q4. The model foresees large and persistent gaps, up to -7%. To summarize, using information available in the Fall of 2008, the DSGE model is able predict the drop in output growth as well as the subsequent recovery, it predicts that the output gap falls well below -6% until 2013. However, unlike Hall (2011)’s and Ball and
Mazumder (2011)’s conjecture, the model-implied Phillips curve does not generate negative inflation forecasts.

One may wonder whether the fact that inflation does not fall below zero is due to the fact that the corresponding interest rate forecasts grossly violates the zero lower bounds. Figure 10 shows that this is not the case. The figure shows that the SWFF interest rate forecast hit zero in 2009 but then rise. It also shows that these forecasts were roughly in line with the Blue Chip forecasts for the federal funds rate made at the time. Of course, ex post these forecasts were very wrong: the interest rate has not yet risen from the zero lower bound. We will discuss this issue later.

The left panel of Figure 8 shows the marginal costs $mc_t$. As in the case of the output gap, marginal costs are a latent variable and not directly observed. Thus, we are replacing actual values by smoothed values. More precisely, the solid black line corresponds to $E_{T+1}[mc_t]$ for $t = 1, \ldots, T$, the solid red line depicts forecasts conditional on $T$ information, and the dashed black line marks ex-post smoothed values $E_{T_{full}}[mc_t]$. The DSGE model grossly over-predicts marginal costs.$^{19}$

At first sight, Figure 8 presents damning evidence against this New Keynesian model: Even if the model captured the decline in activity, it did not forecast the decline in marginal costs. If it had, its forecasts of inflation surely would have been substantially lower. This is essentially the point made by Ball and Mazumder (2011): Feeding into the NKPC post-recession measures of the “gap” (either unemployment, or the labor share) would surely result in deflation. The right panel shows that this is not quite the case. This panel shows the forecasts of inflation (red line) obtained “hard-conditioning” on the ex-post marginal costs shown in the left panel. These forecasts are lower than those shown in Figure 7, but not dramatically so. For comparison, we also show the forecasts of inflation from a backward-looking Phillips curve obtained by feeding in actual realizations of unemployment.$^{20}$

$^{19}$Half of the forecast errors can be explained by to markup shocks. See Figure A-7 in the Appendix.

$^{20}$Our version of the backward looking Phillips curve is taken from Stock and Watson (2008) (equation (9), with four lags for both inflation and unemployment and no other regressor), estimated with quarterly data on the GDP deflator and unemployment up to 2008:Q3. We also tried a version of the Phillips curve in differences (equation (10) in Stock and Watson (2008), again with four lags), and obtained very similar
Figure 8: Marginal Costs ($mc_t$) and Inflation Forecast Conditional on realized $mc_t$ vs. Backward-Looking Phillips Curve Projections Conditional on Realized Unemployment

Notes: The left panel shows the smoothed value of (demeaned) marginal costs as of $Y_{T+}$ (black solid), the forecasts of marginal costs made with $Y_{T+}$ information (red solid), and the smoothed value of marginal costs as of $Y_{T+full}$ (dashed black line), i.e. the realized marginal costs. The right panel shows the inflation forecast conditional on these realized marginal cost (red dash), the inflation forecast from a reduced-form Phillips curve conditional on realized unemployment (blue dash), and actual inflation (solid black) up to $Y_{T+}$ and the realized path of inflation after the forecast origin (black dash).

backward looking Phillips curve does forecasts deflation (about -2% annualized), which may not be surprising given the level of unemployment. Marginal costs are also well below steady state, yet the NKPC’s forecasts are not nearly as much at odds with ex-post outcomes as those from the backward looking Phillips curve.

Ironically, it is precisely the forward looking nature of the NKPC that keeps its forecasts afloat. Figure 2 illustrates this point. The red lines in Figure 2 – the projections for marginal costs – present the following pattern: the more current marginal costs differ from steady state, the faster the projections revert to steady state. Why is that the case? For policy to deliver on its promise (made implicitly via the Taylor rule) that inflation is kept close to the long run target $\pi^*$, it better be that inflation expectations remain anchored. Since inflation expectations are the present discounted value of future marginal costs, anchored inflation expectations means that the latter cannot fluctuate too much. This implies that whenever current marginal costs are very low, future marginal costs must rise to compensate (indeed, results.
The black line corresponds to smoothed marginal costs; the red “hairs” depict forecasted paths of future marginal costs which determine fundamental inflation.

In sum, the counterargument to Ball and Mazumder (2011)’s critique of the NKPC is that it is not the current slack in the economy, i.e., the current value of $m_c t$, that matters for inflation in the NKPC, but the entire projected path of the slack. If policy is bent on inflation stabilization the latter must remain anchored. The next section will further elaborate on the role of policy.

This counterargument may not convince a skeptic, possibly for good reasons. First, it relies on the promise of better times in the distant future to make things better in the present – and one would wonder whether real-life agents are as forward looking as the model assumes. Second, why should agents trust promises of marginal costs (and hence real activity) reverting to steady state when this has not happened in the past five years?

Figure 9 shows for the full sample the same objects as Figure 2 – smoothed historical marginal costs in deviations from steady state ($E_{T_{full}}[m_c t]$, solid black lines) and the projected

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Note that while the estimated model uses long run inflation expectations as an observable, the projections in both Figures 7 and 8 do not condition on ex-post long run inflation expectations. Hence the reason why inflation forecasts do not tank is not that long run inflation expectations are forced to remain in the neighborhood of 2% annualized.
path of future marginal costs ($E[mc_{t+h}|s_{t|T_{full}}]$, where $s_{t|T_{full}} = E_{T_{full}}[s_{t}]$ is the smoothed state of the economy for period $t$ – these are the red “hairs” departing at each point in time from current marginal costs). The figure shows that agents in the model had made correlated forecast errors before for long stretches of time (say, the mid-1990s), but that eventually they had it right: marginal costs did revert to steady state. For the sake of assessing more formally whether the marginal costs forecasts coming out from the model are “crazy”, we compare their forecast errors with those of two reduced form alternatives: a random walk model, and an AR(2) model. Figure A-8 in the Appendix compares the performance of these three models for the period 1989Q4 to 2012Q3 measured in terms of the root mean square error (RMSE) of the forecasts (note that the sample includes the recent period). While the three models produce very different forecasts for marginal costs, their forecast performance is very similar, with the AR(2) model performing slightly worse. In sum, even if the marginal costs forecasts for past few years have so far been well off the mark, it is not clear that from an historical point of view the model’s forecasts can be readily dismissed.

4.2 Forecasts of the Federal Funds Rate [PRELIMINARY]

Figure 10 shows the forecasts for the federal funds rate. As anticipated above, these forecasts do not fall below zero, implying that our forecasts of inflation and economic activity presented above do not rely on bringing the short-term interest rate below its zero lower bound (ZLB). Also, the interest-rate forecasts made at the end of 2008 are broadly in line with the Blue Chip median forecasts (blue diamonds): in this sense it is not so odd that the model’s forecasts bounce back fairly rapidly from the ZLB — private forecasters were anticipating this as well. Yet, these interest rate forecasts raise two issues.

First, the ex-post realized path of the federal funds rate is quite different from the forecasts, unlike inflation and economic activity which lined up much more clearly with forecasts.

---

22 The AR(2) is estimated recursively using data on $E_{T_{full}}[mc_{1:t}]$.

23 Figure 9 also shows the presence of a clear downward trend in marginal costs, as pointed out in King and Watson (2012). This downward trend, possibly caused by demographic factors, hurts the model in terms of explaining and forecasting inflation. How to address this trend is an important topic for future research.
What shocks may have happened after the Lehman crisis that yielded roughly the predicted outcomes for output and inflation, but a much lower path for interest rates? Figure 11 shows the inflation forecasts along with the path that inflation would have taken if, respectively, only mark-up shocks (left panel), policy shocks (middle panel), and all other shocks combined (right panel) had realized the post-Lehman US economy. The figure shows that after 2008, the economy experienced generally negative shocks that pushed inflation (and activity, not shown) down. At the same time, monetary policy pushed in the opposite direction, deviating from the historical rule by providing more stimulus and pushing inflation above the earlier forecast shocks. While the effects on output and inflation roughly netted out, the monetary stimulus resulted in a much lower interest rate path than initially forecasted.

Second, the fact that the interest rate forecasts does not dip below zero given the rather pessimistic real activity and inflation forecasts may raise questions about the estimates policy rule. At first impression, one may think that in the estimated policy rule the responses to inflation and/or the output gap are sufficiently small for the federal funds rate to deviate only moderately from its steady state. Figure 12 shows that the opposite may be the case. The Figure shows a counterfactual forecast under a policy rule that has a slightly lower response to inflation ($\psi_1 = 1.2$) than in the estimate rule. Under this alternative policy rule the interest rate forecast would fall below the ZLB for a few quarters. We therefore solve

---

**Figure 10: Forecasts of the Federal Funds Rate**

Notes: The solid black lines depict actuals up to the forecast origin; the solid red lines indicate the forecast paths; the dashed black lines correspond to the actual paths; the blue diamonds correspond to the Blue Chip forecasts.
Figure 11: Contributions of Ex-post Shocks for Inflation

Figure 12: Counterfactual forecasts with lower policy response to inflation ($\psi_1 = 1.2$)

the model taking the ZLB ($R = 0.12/4$) into account, that is, imposing that

$$R_t = \max \left\{ -R_s + R, \, \rho_R R_{t-1} + (1 - \rho_R) \left( \psi_1 (\pi_t - \pi^*_t) + \psi_2 (y_t - y^f_t) \right) \right\}$$

and compute the forecasts. We find that output and inflation forecasts are only marginally weaker than in our baseline forecast despite the fact that the interest rate is forecast to be
lowered to the ZLB for several quarters.

Why does a rule with lower response to inflation generate a lower interest rate forecast? It is because by responding less to inflation deviations from target, this rule brings inflation back to target more slowly. This results in lower lower inflation expectations (consistently with Figure 5) and lower economic activity, and hence lower short term rates in equilibrium. Figure 12 therefore provides an alternative explanations for ex-post outcomes: What the model interprets as negative shocks to real activity might alternatively be viewed as the result of agents losing confidence in the policy authorities’ ability to control inflation — that is, to fulfill the promise of higher marginal costs in the future implicit in keeping inflation expectations anchored. Under this interpretation, forward guidance and quantitative easing (which is not modeled here) can be seen as a way of compensating for this loss of confidence.

5 Conclusions

In this paper we examined the behavior of inflation forecasts generated from a Smets-Wouters style DSGE model, augmented by a time-varying target inflation rate and a financial friction mechanism. The model embodies a New Keynesian Phillips curve which relates current inflation to future expected real marginal costs. Several authors recently argued that the Phillips curve relationship seemed to have broken down during the Great Recession. The basis for this argument is the observation that real activity dropped sharply without generating a corresponding drop of inflation. We debunk this argument by showing that this observation can be reconciled with a standard DSGE model in which inflation is determined by expectations of future marginal costs. As of 2008:Q3 our DSGE model is able to predict a sharp decline in output without forecasting a large drop in inflation. The model predicts marginal costs to revert back to steady state after the crisis, which, through the forward-looking Phillips curve, prevents a prolonged deflationary episode. While the underlying optimistic marginal cost forecasts turned out to be inaccurate ex post, we show that forecasts from a recursively estimated AR(2) model are less accurate than model-implied marginal costs forecasts over the past two decades. We also document that our DSGE model generates a plausible
measure of fundamental inflation for the post-1964 era which captures the low- to medium-frequency fluctuations of inflation and tracks core PCE inflation. The markup shocks, which are often interpreted as a wedge or misspecification of the Phillips-curve relationships, are only needed to explain high-frequency fluctuations of inflation.

References


Online Appendix for

*Inflation in the Great Recession and New Keynesian Models*

Marco Del Negro, Marc Giannoni, and Frank Schorfheide

A  Data

Real GDP (GDPC), the GDP price deflator (GDPDEF), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are constructed at a quarterly frequency by the Bureau of Economic Analysis (BEA), and are included in the National Income and Product Accounts (NIPA). Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment (CE16OV), and civilian noninstitutional population (LNSINDEX) are produced by the Bureau of Labor Statistics (BLS) at the monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary (ESS). Since our models are estimated on quarterly data, we take averages of the monthly data. Compensation per hour for the nonfarm business sector (COMPNFB) is obtained from the Labor Productivity and Costs (LPC) release, and produced by the BLS at the quarterly frequency. All data are transformed following Smets and Wouters (2007). Let $\Delta$ denote the temporal difference operator. Then:

\[
\begin{align*}
\text{Output growth} &= 100 \times \Delta \ln \left( \frac{GDPC}{LNSINDEX} \right) \\
\text{Consumption growth} &= 100 \times \Delta \ln \left( \frac{PCEC}{GDPDEF} \right) \\
\text{Investment growth} &= 100 \times \Delta \ln \left( \frac{FPI}{GDPDEF} \right) \\
\text{Real Wage growth} &= 100 \times \Delta \ln \left( \frac{COMPNFB}{GDPDEF} \right) \\
\text{Hours} &= 100 \times \ln \left( \frac{AWHNONAG \times CE16OV}{100} \right) \\
\text{Inflation} &= 100 \times \Delta \ln \left( GDPDEF \right).
\end{align*}
\]

The federal funds rate is obtained from the Federal Reserve Board’s H.15 release at the business day frequency. We take quarterly averages of the annualized daily data and
divide by four. In the estimation of the DSGE model with financial frictions we measure *Spread* as the annualized Moody’s Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity. Both series are available from the Federal Reserve Board’s H.15 release. Like the federal funds rate, the spread data is also averaged over each quarter and measured at the quarterly frequency. This leads to:

\[
FFR = \left(\frac{1}{4}\right) \times FEDERAL \ FUNDS \ RATE
\]

\[
Spread = \left(\frac{1}{4}\right) \times (BaaCorporate \ - \ 10yearTreasury)
\]

The long-run inflation forecasts used in the measurement equation (32) are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters (SPF) available from the FRB Philadelphia’s Real-Time Data Research Center. Long-run inflation expectations (average CPI inflation over the next 10 years) are available from 1991:Q4 onwards. Prior to 1991:Q4, we use the 10-year expectations data from the Blue Chip survey to construct a long time series that begins in 1979:Q4. Since the Blue Chip survey reports long-run inflation expectations only twice a year, we treat these expectations in the remaining quarters as missing observations and adjust the measurement equation of the Kalman filter accordingly. Long-run inflation expectations $\pi_t^{O,40}$ are therefore measured as

\[
\pi_t^{O,40} = \left(10\text{-YEAR AVERAGE CPI INFLATION FORECAST} - 0.50\right)/4.
\]

where .50 is the average difference between CPI and GDP annualized inflation from the beginning of the sample to 1992. We divide by 4 since the data are expressed in quarterly terms.

**B Additional Tables and Figures**

Table A-1 summarizes the prior distribution.

Table A-2 summarizes the posterior mode for selected model parameters.
Table A-1: Priors

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(Note $\beta = (1/(1 + r_*/100))$)

$\rho_s$, $\sigma_s$, and $\eta_s$

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<th>Density</th>
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Panel II: Model with Long Run Inflation Expectations (SW$\pi$)

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Panel III: Financial Frictions (SWFF)

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Notes: Smets and Wouters (2007) original prior is a Gamma(62, .10). The following parameters are fixed in Smets and Wouters (2007): $\delta = 0.025$, $g_s = 0.18$, $\lambda_w = 1.50$, $\varepsilon_w = 10$, and $\varepsilon_p = 10$. In addition, for the model with financial frictions we fix the entrepreneurs’ steady state default probability $F_s = 0.03$ and their survival rate $\gamma_s = 0.99$. The columns “Mean” and “St. Dev.” list the means and the standard deviations for Beta, Gamma, and Normal distributions, and the values $s$ and $\nu$ for the Inverse Gamma (InvG) distribution, where $p_{\text{IG}}(\sigma^2) \propto \sigma^{-(\nu+1)} e^{-\nu \sigma^2 / 2 \beta^2}$. The effective prior is truncated at the boundary of the determinacy region. The prior for $l$ is $N(-45, 5^2)$. 
Table A-2: Posterior Mode for DSGE Parameters

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Figure A-1: Fundamental Inflation in SW, SWFF, and SWπ

Notes: The solid black line corresponds to actual GDP deflator inflation; the dashed purple line is fundamental inflation computed from the SW/KW model; the solid blue line is $\tilde{\pi}_t$ associated with the SWFF model.
Figure A-2: Inflation, Fundamental Inflation, Counterfactual Inflation without Mark-up Shocks, and Core Inflation

Notes: The solid black line corresponds to actual GDP deflator inflation; the solid green line is core PCE inflation; the solid blue line is $\tilde{\pi}_t$ associated with the SWFF model; the dashed blue line is counterfactual inflation without markup shocks, $\pi_t^{no\ mkup}$. 
Figure A-3: SWFF Fundamental Inflation vs Fundamental Inflation using SW Price Rigidities in the Evolution of Marginal Costs
Figure A-4: Recursive Estimation of $\zeta_p$

Notes: The black and blue lines corresponds to the posterior mean and mode respectively, and the dashed line to the 90% bands. These estimates are obtained using real time data for each vintage.
Figure A-5: SWFF Output, Inflation, and Interest Rates Forecasts Using Real-Time Data available on Jan 9, 2009, and Corresponding Blue-Chip Forecasts (from Del Negro and Schorfheide (2013))

Notes: The panels show for each model/vintage the available GDP deflator data (black line), the DSGE model’s multi-step (fixed origin) mean forecasts (red line) and bands of its forecast distribution (shaded blue areas; these are the 50, 60, 70, 80, and 90 percent bands, in decreasing shade), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the May 2011 vintage (black dashed line). All the data are in percent, Q-o-Q.
Figure A-6: Movements in Fundamental Inflation $\tilde{\pi}_t$ Attributable to Mark-up Shocks

Notes: The solid purple line in the left panel is fundamental inflation computed from the SW model and the solid blue line in the right panel is fundamental inflation computed from the SWFF model. The solid green lines in the left and right panels are the movements in fundamental inflation attributable to mark-up shocks computed from the SW model and SWFF model, respectively.

Figure A-7: Effect of Markup Shocks

Notes: See Figure 6. The solid green lines correspond to forecast that condition on future values of the markup shock.
Figure A-8: RMSE of Forecasts of Marginal costs: DSGE vs RW vs AR(2)

Notes: The figure shows the RMSE of forecasts of marginal costs in the SWFF model (DSGE), an AR(2) model estimated recursively on past marginal cost data, and a random walk model, for the period 1989Q4-2012Q3.
C Solving linear rational expectation methods with anticipated policy changes.


\[
\tilde{\Gamma}_2 I_t[z_{t+1}] + \tilde{\Gamma}_0 z_t = \tilde{\Gamma}_c + \tilde{\Gamma}_1 z_{t-1} + \tilde{\Psi}_t\varepsilon_t, \tag{A-1}
\]

where \(z_t\) includes all endogenous and exogenous variables, but unlike the \(s_t\) vector in \texttt{gensys}, it \textit{does not} include expectations. The matrices \(\tilde{\Gamma}_c, \tilde{\Gamma}_0, \tilde{\Gamma}_1,\) and \(\tilde{\Psi}\) correspond to the \(\Gamma_c, \Gamma_0, \Gamma_1,\) and \(\Psi\) matrices in \texttt{gensys}, except that the rows associated with the expectational equations are cut off, and the matrix \(\tilde{\Gamma}_2\) contains the coefficients on the expectations (which in \texttt{gensys} are part of \(\Gamma_0\)). The expectational errors \(\eta_t\) and the corresponding matrix \(\Pi\) are of course missing from this formulation. Write the solution as

\[
z_t = C + T z_{t-1} + R \varepsilon_t. \tag{A-2}
\]

Note that \(C, T,\) and \(R\) are not quite the \(C, T,\) and \(R\) in \texttt{gensys} solution, because \(z_t\) is of a different size relative to \(s_t\). More below on how to map the \(C, T\) and \(R\) into \(T\) and \(R\).

The model is

\[
\tilde{\Gamma}_{2,2} I_t[z_{t+1}] + \tilde{\Gamma}_{0,2} z_t = \tilde{\Gamma}_{c,2} + \tilde{\Gamma}_{1,2} z_{t-1} + \tilde{\Psi}_{2} \varepsilon_t, \text{ for } t \geq H + 1 \tag{A-3}
\]

and

\[
\tilde{\Gamma}_{2,1} I_t[z_{t+1}] + \tilde{\Gamma}_{0,1} z_t = \tilde{\Gamma}_{c,1} + \tilde{\Gamma}_{1,1} z_{t-1} + \tilde{\Psi}_{1} \varepsilon_t, \text{ for } t = 1, \ldots, H. \tag{A-4}
\]

Write the (time-varying) solution of this model as

\[
z_t = C_t + T_t z_{t-1} + R_t \varepsilon_t. \tag{A-5}
\]

(Note that by writing the solution this way we have ruled out sunspots solutions by assumption). We need to compute \(C_t, T_t,\) and \(R_t\). First, compute the (time invariant) solution for model 2:

\[
z_t = C_2 + T_2 z_{t-1} + R_2 \varepsilon_t. \tag{A-6}
\]
Since for $\bar{H}$ onward only model 2 applies, then

$$C_t = C_2, \quad T_t = T_2, \quad R_t = R_2, \quad \text{for } t \geq \bar{H} + 1.$$  \hfill (A-7)

For $t \leq \bar{H}$ we will compute $C_t, T_t, \text{ and } R_t$ recursively. Substitute for $IE_t[z_{t+1}]$ the expectations generated from (A-5):

$$\tilde{\Gamma}_{2,1} C_{t+1} + \tilde{\Gamma}_{2,1} \tilde{T}_{t+1} z_t + \tilde{\Gamma}_{0,1} z_t = \tilde{\Gamma}_{c,1} + \tilde{\Gamma}_{1,1} z_{t-1} + \tilde{\Psi}_1 \varepsilon_t, \quad \text{for } t = 1, \ldots, \bar{H},$$  \hfill (A-8)

implying

$$z_t = \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \left( \tilde{\Gamma}_{c,1} - \tilde{\Gamma}_{2,1} C_{t+1} \right) + \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \tilde{\Gamma}_{1,1} z_{t-1}$$

$$+ \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \tilde{\Psi}_1 \varepsilon_t, \quad \text{for } t = 1, \ldots, \bar{H}.$$  \hfill (A-9)

Equating (A-5) and (A-9) we have:

$$C_t = \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \left( \tilde{\Gamma}_{c,1} - \tilde{\Gamma}_{2,1} C_{t+1} \right),$$

$$T_t = \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \tilde{\Gamma}_{1,1},$$

$$R_t = \left( \tilde{\Gamma}_{2,1} T_{t+1} + \tilde{\Gamma}_{0,1} \right)^{-1} \tilde{\Psi}_1, \quad \text{for } t = 1, \ldots, \bar{H}.$$  \hfill (A-10)

We start the recursion at $t = \bar{H}$ with $T_{t+1} = T_2, C_{t+1} = C_2$

Last, on how to map the the $C, T$ and $R$ in gensys into $C, T$ and $R$. Write the state $s_t$ in gensys as $s'_t = [s'_{1, t}, s'_{2, t}]'$ where $s_{1, t} = z_t$ and $s_{2, t}$ collects the expectation terms $IE_t[z_{t+1}]$.

Partition

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \quad R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}.$$  \hfill (A-10)

If $T_{12} = 0$, then $T = T_{11}, C = C_1$ and $R = R_1$. 