Understanding Uncertainty Shocks
and the Role of the Black Swan

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1Disclaimer: The views expressed herein are those of the authors and do not necessarily reflect the position of the Board of Governors of the Federal Reserve or the Federal Reserve System
Introduction

- Many models explore effects of exogenous uncertainty shocks. But where do uncertainty shocks come from?
- Uncertainty: Stdev of a forecast error conditional on $I_t$.

$$U_{it} = \sqrt{E \left[ (y_{t+1} - E(y_{t+1}|I_t))^2 \right] | I_t}$$

- Typically: Uncertainty measurement assumes $I_t$ contains true $y_t$ distribution and its parameters.
- Our paper: Forecasters use real-time GDP data to learn about parameters.
- Finding: Learning about economic environment (esp. black swan risk) → large, counter-cyclical uncertainty shocks.
  - Also explains forecast bias puzzle.

**Message:** Rational expectations econometrics ignores most changes in uncertainty.
Forecasting Model

- We use real-time GDP data (1968-t) and Bayesian MCMC techniques to estimate

\[
\begin{align*}
y_t &= c + b \exp(-S_t - \sigma \varepsilon_t) \\
S_t &= \rho S_{t-1} + \sigma^S \xi_t
\end{align*}
\]

where \( \varepsilon_t \) and \( \xi_t \sim iid \ N(0, 1) \).

- In each quarter \( t \), the forecaster
  
  - observes \( y^t \) (with data revisions)
  - estimates distribution of \( \theta \equiv \{ c, b, \rho, \sigma, \sigma^S \} \) using particle filter.
  - forecasts \( E(y_{t+1}|y^t) = \int \int y_{t+1} f (y_{t+1}|\theta, y^t) g(\theta) d\theta dy_{t+1} \)
  - and computes \( U_t \equiv \sqrt{Var(y_{t+1}|y^t)} \)
Why this model?

- Start with a standard hidden state model, with homoskedastic shocks. Isolate changes in variance that come from skewness and parameter uncertainty.

- Linear model has tiny shocks and gets the average forecast wrong (> 0.5% too high).

- The nonlinear twist $\rightarrow$ skewed distribution of GDP growth. In data, skew $= -0.3$. Crises more likely than explosions.

- Forecaster can learn about skewness $(b, c) \rightarrow$ large uncertainty shocks because tail probabilities are sensitive to these parameters.

- Skewness is hard to learn in small samples, convergence is slow.
Uncertainty shocks are much larger with parameter learning.
## Normal and Skewed Model Results

<table>
<thead>
<tr>
<th>model:</th>
<th>normal</th>
<th>skewed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_t$</td>
<td>4.20%</td>
<td>4.53%</td>
</tr>
<tr>
<td>$V_t$</td>
<td>4.65%</td>
<td>5.41%</td>
</tr>
<tr>
<td><strong>Std deviation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_t$</td>
<td>0.48%</td>
<td>1.50%</td>
</tr>
<tr>
<td>$V_t$</td>
<td>0%</td>
<td>0.07%</td>
</tr>
<tr>
<td><strong>Autocorrelation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U_t$</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>$V_t$</td>
<td>0</td>
<td>0.92</td>
</tr>
</tbody>
</table>

### Detrended uncertainty/volatility

<table>
<thead>
<tr>
<th>Corr($\tilde{U}<em>t, E_t[y</em>{t+1}]$)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>-0.78</td>
<td></td>
</tr>
<tr>
<td>Corr($\tilde{V}<em>t, E_t[y</em>{t+1}]$)</td>
<td>0</td>
<td>-0.99</td>
</tr>
</tbody>
</table>

### Forecast properties

<table>
<thead>
<tr>
<th>data</th>
<th>normal</th>
<th>skewed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean forecast</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.29%</td>
<td>2.82%</td>
<td>2.27%</td>
</tr>
<tr>
<td>**Mean $</td>
<td>F\ Err</td>
<td>$**</td>
</tr>
<tr>
<td>1.87%</td>
<td>2.25%</td>
<td>2.51%</td>
</tr>
<tr>
<td><strong>Std forecast</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25%</td>
<td>1.17%</td>
<td>0.64%</td>
</tr>
<tr>
<td>**Std $</td>
<td>F\ Err</td>
<td>$**</td>
</tr>
<tr>
<td>1.46%</td>
<td>2.17%</td>
<td>2.39%</td>
</tr>
</tbody>
</table>
Black Swan Risk\(_t = \text{Prob}[y_{t+1} \leq -6.8\%|y^t]\)

Correlation\((BSw, U_t)\) is 75\%. 

Black Swan Risk and Uncertainty Shocks
Why Are Black Swan Probabilities Volatile?

- Extreme event probabilities are very sensitive to small revisions in skewness.
- Skewness keeps fluctuating because it is hard to learn.
What events trigger uncertainty shocks?

- If uncertainty rises when skewness is believed to be more negative, what sort of data realizations trigger this?
  1. Negative outliers
  2. A string of mild positive realizations that increase mean make existing negative realizations lie farther in the tail.

★ Example: 7 quarters of mostly positive data in’70s double the black swan probability.

![Frequency and Density Graphs](image)
What triggers uncertainty shocks?

- Low-growth forecasts also raise uncertainty → counter-cyclical uncertainty.
- Why? Think of probability distribution as a having a normal pdf with a change-of-measure function $g^{-1}$. $y = g(x)$ where $x$ is normal.

**Lemma**

*Suppose that $y$ is a random variable with a probability density function $\phi(g^{-1}(y))$ where $\phi$ is a standard normal density and $g$ is an increasing, concave function. Then, $E[(y - E[y])^3] < 0$.***

- Then, by the Radon-Nikodym theorem,

$$Var[y_{t+1}|y^t] = E \left[ \left. \frac{dg}{dx} \right| x^t \right] Var[x_{t+1}|x^t] + cov\left( \frac{dg}{dx}, (x_{t+1} - E[x_{t+1}])^2 \right)$$

Since $g$ is concave, $dg/dx$ is a decreasing function. $Var[y_{t+1}|y^t]$ is higher when $E[x_{t+1}|x^t]$ is low.
Why Skewness + Parameter Uncertainty Lowers Forecasts

**Lemma**

Suppose \( y \sim f(y|\mu, \sigma) = N((g^{-1}(y) - \mu)/\sigma) \) where \( g \) is concave. If parameter distributions \( h(\mu') \) and \( k(\sigma') \) have means \( \mu \) and \( \sigma \), then the forecast, \( \hat{y} \equiv \int \int \int y f(y|\mu', \sigma') g(\mu') h(\sigma') dy d\mu' d\sigma' < \) the true mean \( \bar{y} \equiv \int y f(y|\mu, \sigma) dy \).

\[
E[y_{t+1}|y^t, \theta] \text{ is mean GDP growth} = 2.68\%.
\]

\[
E[y_{t+1}|y^t] \text{ is average growth forecast} = 2.29\% \text{ in data,} = 2.27\% \text{ in model.}
\]
How Does $U_t$ Compare with Common Measures?

Uncertainty Proxy Variables

- GARCH vol
- Forecast MSE
- Forecast disp
- VIX
- BBD policy unc

Corr $U_t$: GARCH 7%, MSE -3%, Disp 20%, VIX 36%, BBD 21%.
Conclusions

- Macro theories typically assume agents know the true model and its parameters. The only source of uncertainty is which draw from a known distribution. This limited view rules out important sources of uncertainty. Learning about skewness is one example.

- When we allow agents to learn about model skewness, they experience large uncertainty shocks.
  - Skewness is tough to learn in small samples, so new data causes revisions.
  - Small revisions cause large changes in the probability of extreme events (black swans).
  - Changes in black swan risk affect conditional variance → uncertainty shocks.
Conclusion

I'M UNCERTAIN ABOUT THE UNCERTAINTY OF THE ECONOMY. THIS, I AM CERTAIN OF...
Forecast and Uncertainty Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>θ known</th>
<th>L Model</th>
<th>NL Model</th>
<th>learn c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean forecast</td>
<td>2.24%</td>
<td>2.68%</td>
<td>3.06%</td>
<td>2.24%</td>
<td>2.21%</td>
</tr>
<tr>
<td>Mean</td>
<td>(FErr</td>
<td>)</td>
<td>2.20%</td>
<td>2.38%</td>
<td>2.31%</td>
</tr>
<tr>
<td>Mean (U_t)</td>
<td>–</td>
<td>2.91%</td>
<td>3.40%</td>
<td>5.79%</td>
<td>7.66%</td>
</tr>
<tr>
<td>Stdev (U_t)</td>
<td>–</td>
<td>0</td>
<td>0.20%</td>
<td>0.71%</td>
<td>1.60%</td>
</tr>
<tr>
<td>Correl((\tilde{U}_t),GDP)</td>
<td>–</td>
<td>0</td>
<td>13%</td>
<td>-90%</td>
<td>-34%</td>
</tr>
</tbody>
</table>

Results raise these questions

1. How does nonlinearity affect uncertainty? Why counter-cyclical?
2. Why does the model explain the forecast bias in the data?
3. What triggers large uncertainty shocks?
4. How does this uncertainty compare to commonly-used empirical measures?
## Full Results: Uncertainty and Volatility

<table>
<thead>
<tr>
<th>model</th>
<th>linear (1)</th>
<th>nonlinear (2)</th>
<th>learn c (3)</th>
<th>signals (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$U_t$</td>
<td>3.38%</td>
<td>5.79%</td>
<td>7.65%</td>
</tr>
<tr>
<td></td>
<td>$V_t$</td>
<td>2.91%</td>
<td>6.82%</td>
<td>6.82%</td>
</tr>
<tr>
<td>Std deviation</td>
<td>$U_t$</td>
<td>0.21%</td>
<td>0.71%</td>
<td>1.60%</td>
</tr>
<tr>
<td></td>
<td>$V_t$</td>
<td>0%</td>
<td>0.37%</td>
<td>0.37%</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>$U_t$</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>$V_t$</td>
<td>0</td>
<td>0.47</td>
<td>0.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>detrended data moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std deviation</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Corr($\tilde{U}_t$, $y_t$)</td>
</tr>
<tr>
<td>Corr($\tilde{V}_t$, $y_t$)</td>
</tr>
<tr>
<td>Corr($\tilde{U}<em>t$, $y</em>{t+1}$)</td>
</tr>
<tr>
<td>Corr($\tilde{V}<em>t$, $y</em>{t+1}$)</td>
</tr>
</tbody>
</table>
RGDP growth and forecasted growth

![Graph showing RGDP growth and forecasted growth from 1970 to 2010. The graph includes lines for GDP growth, linear forecast, and nonlinear forecast.]

- GDP growth
- Linear forecast
- Nonlinear forecast

The graph illustrates the fluctuation of GDP growth over time, with forecasts for linear and nonlinear scenarios.
Parameter Estimates from Normal Shocks Model

\begin{figure}
\centering
\includegraphics[width=\textwidth]{parameter_estimates}
\end{figure}
Are uncertainty shocks volatility shocks?

\[ VOL_{it} = \sqrt{E \left[ (y_{t+1} - E (y_{t+1} | y^t_i, \theta, M))^2 \right | y^t_i, \theta, M]} \]

\[ U^{(h)}_{it} = \sqrt{E \left[ (y_{t+h} - E (y_{t+h} | l_t))^2 \right | l_t]} \]

\[ MSE_{t+1} = \sqrt{\frac{1}{N} \sum_i [y_{t+1} - E (y_{t+1} | l_t)]^2} \]

- If many forecasters, with indep errors, then \( MSE_{t+1} = U_t \).

<table>
<thead>
<tr>
<th>Proxy</th>
<th>Mean</th>
<th>Coeff Var</th>
<th>Autocorrel</th>
<th>Correl w/GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>2.64</td>
<td>0.58</td>
<td>0.48</td>
<td>0.04</td>
</tr>
<tr>
<td>GARCH vol</td>
<td>3.65</td>
<td>0.37</td>
<td>0.9</td>
<td>0.06</td>
</tr>
</tbody>
</table>

- Series differ greatly! Small sample and error correlation do not fully explain the difference (see paper).

*Uncertainty shocks do not seem to be fully explained by volatility shocks.*
Comparison with proxies (detrended) uncertainty

Uncertainty Proxy Variables, Detrended

GARCH vol
Forecast MSE
Forecast disp
VIX
BBD policy unc
Plot estimated $c$ parameter
Isn’t Forecast Dispersion a ”Model-free” Uncertainty Measure?

A general orthogonal decomposition:

\[ y_{t+1} = E (y_{t+1} | I_t) + \eta_t + \epsilon_{it} \]

Then, uncertainty and forecast dispersion are

\[ U_{it}^2 = E \left[ (\eta_t + \epsilon_{it})^2 | I_t \right] = \text{Var} (\eta_t | I_t) + \text{Var} (\epsilon_{it} | I_t) \]

\[ D_t^2 = \frac{1}{N} \sum_i \left( E (y_{t+1} | I_t) - \bar{E}_t \right)^2 = \frac{1}{N} \sum_i \text{Var} (\epsilon_{it} | I_t) \]

Dispersion measures uncertainty with the following model assumptions:

1. \( \text{Var} (\eta_t | I_t) = 0 \)
2. \( \text{Var} (\epsilon_{it} | I_t) = \text{Var} (\epsilon_{jt} | I_t) \) for all \( i, j, t \).
Is there any relationship between forecast dispersion and model uncertainty?

Same hidden state model with \( I_t = \{ M, y^t, z^t \} \).

\[
z_{it} = y_{t+1} + \sigma_\xi \xi_{it} + \sigma_\varepsilon \varepsilon_t
\]

Calibrate \( \sigma_\xi \) and \( \sigma_\varepsilon \) to match forecast dispersion and average forecast error in the SPF.

Findings

- Generates forecast dispersion and avg forecast error (by construction).
- **But despite changes in \( U_t \), no changes in dispersion!**
- Gets close to \( \text{corr}(\text{forecast, GDP}): 71\% \) in data \( 77\% \) in model (30\% baseline).
- Lowers uncertainty (2.85\%) and dampens the uncertainty shocks (0.12\% std).