Blood and Money: Kin Altruism, Governance, and Inheritance in the Family Firm

*Thomas Noe (University of Oxford)
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Thomas Noe
Balliol College/Saïd Business School, Oxford
European Corporate Governance Institute

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Abstract

This paper develops a theory of governance and inheritance within family firms based on kin altruism (Hamilton, 1964). Family members weigh the payoffs to relatives in proportion to relatedness. The theory shows that family management entails both costs and benefits. The attenuated monitoring incentives associated with kin altruism produce a “policing problem” within family firms. This policing problem results in both increased managerial diversion and increased monitoring costs. Relatedness has conflicting effects on manager–owner compensation negotiations. On the one hand, owners are more willing to concede rents to family managers to increase total value. On the other hand, because family managers internalize the costs to the family from their rejection of owner demands, relatedness lowers managers’ reservation compensation level. Lower compensation leads to more diversion and costly monitoring. Firm founders anticipate the costs and benefits family control when designing their bequests. For typical family trees, kin altruism ensures that founders’ preferences are much more closely aligned with value maximization than any of their descendants’. Thus, founder bequests are designed to weaken closer relatives’ bargaining power in posthumous negotiations with more competent distant relatives.

Keywords: Corporate governance, entrepreneurship, kin altruism, contract theory

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1 Introduction

The organization of economic activity around family units is globally pervasive. The vast majority of businesses are controlled by families. As pointed out by Fukuyama (1995), outside the Anglo-sphere, Northern Europe, and Japan, the preponderance of non-state firms are family controlled. As documented by Porta, Lopez-de Silanes, and Shleifer (1999), 45% of publicly listed international firms are family controlled. Even in the U.S., the majority of firms with revenues less than $500 million are family controlled, and many very large firms are tied to families, e.g., Ford, Koch Industries, and Wallmart. While biological kinship is not a defining characteristic of a family given the possibility of adoption, biological kinship nevertheless is fundamental to defining the concept of family and is descriptive of the vast majority of family units. The aim of this paper is to develop a theory of family firms based on this fundamental property.

This theory is founded on the concept of inclusive fitness developed by Hamilton (1964). The inclusive fitness of a given agent is that agent’s own fitness summed with the weighted fitness sum of all other agents, the weights being determined by the other agents’ coefficient of relatedness, i.e., kinship, to the given agent. The logic behind kin altruism is that gene expression affects the number of copies of a gene in the gene pool both through its direct effect on the fitness of the agent expressing the gene and through its effect on the fitness of other agents sharing the gene. Because of relatedness, kin have a far higher than average probability of sharing any gene, including genes for altruistic behavior, than unrelated agents. Thus, a gene for kin altruism can increase in a population even if it is harmful to the fitness of the agent having the gene, provided that the costs to the agent are low relative to the benefits to kin. Selection of a kin altruistic gene requires that

\[ rB > C, \]

where \( r \) represents the coefficient of relatedness, \( B \) the benefit to the relative, and \( C \) is the cost to the altruistic agent. Our theory of the family firm identifies fitness with terminal wealth less any non-pecuniary effort costs and assumes that the all family members maximizes their inclusive fitness.\(^1\)

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\(^{1}\)Note this perspective is a reduced-form approximation to the conditions for natural selection favoring altruistic traits. First, selection requires only genetic similarity, such similarity need not be based of descent. Second, the coefficient of relatedness depends not only on an agent’s probability of sharing identical-by-descent genes with other agents but also on the kinship of the agent with himself, i.e., the degree to which the agent is inbred. These simplifications will not have any effect on the conclusions of the analysis.
Kin altruism in family firms is clearly a hypothesis worthy of consideration. The theoretical foundation of kin altruism is simple, its driving exogenous variable—kinship—is observable, and, as will be detailed below, in many scientific disciplines, its predictions are strongly supported. Moreover, basing the degree of altruism between agents on their genetic relationship forces the degree of altruism to satisfy the restrictions imposed by the calculus of heredity. We will show that these restrictions generate many of the prediction of the model. Thus, the kin-altruism hypothesis is testable.

The key characteristics of family altruism, motivated by the Hamiltonian concept of inclusive fitness, are that altruism is symmetric, limited, “harsh,” and governed by the calculus of relationship. The bonds of relation are symmetric because they are derived from a relatedness which is a symmetric relation in diploids such as humans.\(^2\) Altruism is limited because, except for identical twins and highly inbreed families, even the tightest kinship bonds, parent/offspring or sibling/sibling, produce relatedness sufficient to internalize roughly half of the effect of an agents’ actions on a relative. Moreover, this sort of altruism is harsh in that it aims only at maximizing total inclusive fitness and is thus not concerned with fair distribution of fitness across relations. For this reason, a family agent in my analysis is willing to sacrifice the welfare of a single relative to further his own genetic interests and/or the family’s as a whole.\(^3\) Because altruism is governed by the calculus of relationship, it is roughly based on the length of the path in the family tree that connects relatives. Thus, inclusive fitness predicts that ancestors will be more altruistic to descendants than the descendants linked through the ancestor will be to each other.\(^4\)

Family altruism is introduced into a standard principal/agent model of effort, monitoring, and diversion. First we consider the effect of kinship on monitoring and the diversion of output. Two family members have claims on the firm’s output. One family member called the “manager” actually observes the firm’s output. The other family member called the “owner” does not observe output. The claim of the manager on output might be fixed by negotiations between the owner and manager or might arise from an endowed equity stake in the firm. However, at the monitoring stage, these arrangements have been fixed and the process by which they were reached has no affect on the agents’ incentives going forward. Only the manager observes the cash flow. After observing the cash flow, the manager reports cash flows to the owner. The owner can either accept the manager’s report or verify the report. Verification

\(^2\)Diploids have two homologous copies of each chromosome, one from the mother and one from the father. Some social insects (e.g., bees, ants wasps) are haplodiploids—males carry one homologous copy and females two. For such haplodiploids, the brother–sister coefficient of relation is asymmetric.

\(^3\)For example, Mayer Amschel Rothchild’s decision to disinherit his daughters in order to keep family wealth in what he believed to be the more able hands of his sons is quite consistent with the specification used in this analysis. See Mayer (2012).

\(^4\)The “rough” qualification is required because of the possibility of inbreeding and multiple paths connecting family members. See Section 8.1 for more discussion.
is costly but perfect. As in, for example, Townsend (1979), the compensation received by the manager is a contracted function of the manager’s report and the results of verification (if verification occurs) and satisfies limited liability. The owner cannot, however, commit contractually to verify the cash flow. Rather, the verification decision is the owner’s best response to the manager’s report.

Our basic result is that kinship (the level of the coefficient of relation between the owner and manager) always exacerbates the monitoring problems by lowering likelihood of owner monitoring and increasing likelihood that the manager will falsely report low cash flows. The net effect of a lower likelihood of monitoring of low reports and more low reports is that the unconditional probability of monitoring increases with kinship as does the probability of successful diversion by the manager. This occurs because monitoring is costly and reduces total family welfare while the owner’s loss from diversion is partially offset by internalized gains to the related manager. Internalization makes the related owner a soft monitor. The owner’s softness is exploited by the related manager who increases his attempts to divert so much that the overall probability of monitoring and the attendant dissipation costs, always increase.

Next, we consider the effect of kinship on the production of cash flows. We assume that, in order for the project to produce output, effort on the part of the manager is required. Effort produces a stochastic cash flow. Effort is neither observable nor verifiable. The manager will work for the firm only if the expected utility from employment at least equals the utility from rejecting employment. If the manager accepts employment, the manager chooses the effort level that maximizes his utility. In this setting, we consider compensation negotiations between the owner and manager assuming that the owner acts as the principal in a standard principal-agent employment model. When the model is closed by fixing compensation, two new effects of kinship emerge: one favoring efficiency which we term the “bright-side” scenario and the other retarding efficiency, which we term the “dark-side” scenario. The bright-side scenario occurs when the incentive constraint for ex ante effort is binding. In this case, the owner selects from incentive compatible effort–compensation pairs to maximize his welfare. The owner’s tradeoff—at higher levels of compensation total family value is larger but the manager’s effort rents are also larger. Kin altruism has two effects on the owner’s choice. First, at any given level of compensation, because of the kin altruism of the manager, effort is higher, thus the rent concession required to induce a given level of effort is less. Second, the kin altruism of the owner means that the owner partially internalizes the manager’s effort rent. Thus, the owner is more willing to concede compensation to increase effort.
The two effects are reinforcing and lead to higher effort and thus total output. Hence, in the bright-side scenario, kinship’s adverse effect on the monitoring problem is mitigated and perhaps even outweighed by its favorable effect on the ex ante effort problem.

In contrast, in the “dark-side” scenario, the reservation constraint is binding. In this case, the endogenous determination of compensation increases the dissipative cost of kinship. When the manager is related to the owner, the manager’s walk-away value from rejecting an employment offer made by the owner will partially internalize the loss to the owner from the manager’s rejection. This makes the related manager’s minimum acceptable compensation less than the minimal acceptable compensation of an unrelated manager. Thus, the owner can offer the related manager less compensation than an equivalent unrelated manager would accept. We call this effect the loyalty holdup effect. Lower compensation encourages more diversion by the manager which in turn stimulates more monitoring by the owner but not enough increased monitoring to prevent diversion from increasing. In some parameterizations of the dark-side scenario, the manager’s value, which includes both compensation and expected diversion gains, is greatest for unrelated and highly related managers. Unrelated managers receive large compensation packages and divert little; closely related managers receive small compensation packages but divert a significant fraction of firm value. Firm value is greatest and the manager’s value is smallest for intermediate degrees of kinship: kinship which is close enough to extract significant compensation concessions but not so close that it leads to a significant reduction of owner monitoring incentives.

Finally, we consider the effect of kinship on inheritance. We model a single agent, called the “founder,” who has created the firm and has two relatives, one close and one distant, or perhaps even adopted, to whom she can bequeath the ownership rights in the firm. The founder’s preferences, like those of her descendants are governed by kin altruism. Thus, the founder’s bequest will trade off the payoffs to her close relative against the overall value of the family firm. We show that the calculus of relatedness implies that for normal family pedigrees, the founder’s preferences are more aligned with total family-value maximization than any of the descendants’ preferences. When the ability differences between the relations are sufficiently large, the founder will use her bequest to shield the more competent distant relative from hold ups by the less competent closer relative. Thus, laws which restrict founder bequests, which as Ellul, Pagano, and Panunzi (2010) point out are pervasive in non-Anglo-Saxon legal systems, have adverse productivity consequences for family firms.
1.1 Related literature

A paper centered on relatedness should fully reveal its own pedigree. This paper’s relatives come from two distinct families of research—economics/financial economics and evolutionary biology/psychology. The motivating idea for the theory—inclusive fitness—has been the subject of a great deal of research in both evolutionary biology and psychology. For example, Madsen, Tunney, Fieldman, Plotkin, Dunbar, Richardson, and McFarland (2007) provides experimental evidence that agents’ willingness to stand in an uncomfortable position for the sake of other agents is monotonically increasing in the agents’ degree of relatedness. Field experiments also seem to support the role of genetic relatedness in behavior. For example, Daly and Wilson (1988) find that the intra-family child homicide probability is 11 times higher for step parents than natural parents. Field evidence from other researchers shows that kinship relations increase political alliance ability (Dunbar, Clark, and Hurst, 1995), facilitate the assumption of group leadership (Hughes, 1988), and increase cooperation in catastrophic circumstances (Grayson, 1993). Evolutionary biology has produced overwhelming evidence for kin altruism in many species. For example, Dudley and File (2007) shows that a common plant, sea rocket, will grow its roots more aggressively when potted with unrelated plants than when potted with relatives. Even the policing problems associated with family altruism have close analogues in evolutionary biology. For example, Ratnieks (1988) finds evidence to support a negative association between relatedness and policing in a comparative study of Bumble Bee and Honey Bee behavior.

However, kin altruism is the only concept used in this work that is drawn from evolutionary theory. The rest of the paper’s pedigree can be found in the economics and financial economics literature. As pointed out by Bertrand and Schoar (2006), most of many family-firm theories are based on characteristics shared by many family firms—firm-specific human capital, capital constraints, private benefit extraction—which are not defining properties of family firms. These theories are not family-firm theories per se. If they offer a complete explanation of family firm behavior it would follow that there is

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5For a survey of a large body of related work see Cartwright (2000).
6The effect of kinship on policing opportunism modeled in this paper is, in a more generic and reduced-from framework also modeled by evolutionary biologists studying the problem of how “policing strategies,” can be favored by natural selection. Policing strategies involve imposing punishments on agents who engage in non-cooperative behavior even though imposing the punishment has adverse fitness consequences for the punisher. Gardner and West (2004) and El Mouden, West, and Gardner (2010) show that high levels of relatedness disfavor the selection of punishment strategies. From a different more general perspective, this paper is also connected to the emergent literature on the effects of genetics on economic and financial behavior. See, for example, Wallace, Cesarini, Lichtenstein, and Johannesson (2007) and Cesarini, Johannesson, Lichtenstein, Sandewall, and Wallace (2010).
7See, for example, Bennedsen, Nielsen, Perez-Gonzalez, and Wolfenzon (2007), Almeida and Wolfenzon (2006), and Burkart, Panunzi, and Shleifer (2003)
no need for a special theory of the family firm and that explanations of family firm behavior should be
subsumed in general contract theory. The theory of family firms that is most closely linked to defining
family characteristics is probably Fukuyama (1995). Fukuyama argues informally that, in the absence
of general social capital, important business relations must be based on intrafamily trust. Family trust
is based, in turn, on cultural norms related to family loyalty. From Fukuyama’s perspective, family
loyalty lowers agency costs in intrafamily business transactions while, at the same time, forming closed
networks and thus retarding the development of external capital and managerial labor markets. This
model’s perspective is rather different. In this paper, families control does not resolve the problem of
trust and in fact can lead to conflict because family altruism suppresses the policing of opportunistic
behavior. Moreover, rather than being substitutes for family firms, institutional and market development
strongly complement the relative performance of family firms, although such institutional development
might also reduce their incidence.

The empirical literature on family firms is more extensive than the theoretical literature. Most of this
research has focused on identifying the unconditional relation between family ownership and firm per-
formance. The results have been mixed and the definitions of family firms used by different researchers
have been inconsistent. For example, Bennedsen, Nielsen, Perez-Gonzalez, and Wolfenzon (2007) doc-
ument a negative effect of intra-family succession using Danish data. In contrast Anderson and Reeb
(2003) report a positive effect of family ownership on firm performance for U.S. firms and Sraer and
Thesmar (2007) report similar results for French firms. However, Miller, Breton-Miller, Lester, and
Cannella (2007) and Villalonga and Amit (2006) find, for U.S. firms, that, after controlling for the effect
of a founder/owner, family firms do not outperform non-family firms. Since this model does not predict
any uniform relation between family ownership and performance and the empirical literature produces
conflicting conclusions, it is not easy to use these papers to assess the kin altruism model.

Evidence which is perhaps more relevant to making a preliminary assessment of the plausibility kin
altruism as a driver of family firm behavior can be found in a smaller body of research which studies
specific decisions and behaviors of family firms rather than their overall performance. Ellul, Pagano,
and Panunzi (2010) show that restrictions on the rights of heirs to bequest firms to distant relatives has
a significant adverse impact on the value of family firms. This is consistent with the conclusions of kin
altruism in that founder bequests, although not unbiased, tilt toward total value maximization and thus

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8 See Miller and Le Breton-Miller (2006) for a comprehensive summary of these results and the various definitions of family
firm used in the empirical literature.
frustrating founders’ ability use bequests to protect the interests of competent distant relations destroys value. The model’s result that kinship increases monitoring costs and diversion is consistent with the observations of Bertrand and Schoar (2006) that cooperation between family members is frequently difficult to achieve. Such conflicts between family members have been even more extensively documented in the management literature (See for example (Davis and Harveston, 2001)). Clinical research by Karra, Tracey, and Phillips (2006) also seems consistent with the basic predictions of this paper. Karra, Tracy and Phillips find that family control increases cooperation is early stage firms but leads to conflicts as firms age and grow. Under the postulate that, in early stage firms, joint effort is key and liquid assets capable of being diverted are scarce while more mature firms generate large pools of liquid assets that can be diverted, their result is exactly what this paper’s analysis predicts.

Perhaps the closest relatives of this paper are two papers that do not model family firm behavior but do model general altruism motivated by friendship. Lee and Persson (2010) models the effect on an unrelated principal of delegated monitoring when a worker and the worker’s supervisor have altruistic preferences toward each other. In addition, both workers feel shame if they act opportunistically toward the principal. Their model has an important technical similarity to ours—like ours, in their model of altruism is symmetric and limited. However, in our analysis, agents are shameless, considering only the payoff effects of their decisions on themselves and their fellow family members. This lack of shame leads to a much darker perspective on the effect of altruism on monitoring efficiency. Lee and Persson (2012) develop a model of early-stage angel finance that incorporates paternalistic altruism. This paper, unlike ours, is not focused on monitoring or governance. Also, in this model, agents not only value the payoffs to friends they also value these payoffs to friends differently than the friends themselves value them. Thus, a risk-averse entrepreneur might prefer that his risk-neutral friend not finance his startup even though such financing benefits the entrepreneur and the friend is willing to provide financing because, evaluated using the entrepreneur’s risk-averse preferences, the deal provided the friend is unfavorable. In our analysis, such effects will not occur because all agents value the cash flow streams to a given agent in the same fashion. Altruism only effects the weights that the place on these valuations. In addition, friendship-based altruism does not restrict the degree of altruism connecting agents from different generations while the kinship calculus does. Thus, our results on inheritance, which rely on the founder being more altruistic toward collateral descendants than the descendants are toward other, have no analog in these papers.9

9Another stream of related literature is Becker (1974) and its descendants such as Bergstrom (1989). Like this paper, the
2 Model

2.1 Overview

In the basic model, the world lasts for one period, bracketed by dates 0 and 1. There are two agents in the basic model: an owner and a manager. The owner has monopoly access to a project which we will call a firm. The owner can only operate the project if he secures the efforts of the manager and the owner and manager are related by kinship. We will refer an owner/manager pair who are kin simply as the “owner” and “manager.” Collectively, the family owner and family manager are called “family agents” and the total value received by these agents is called the “family” value. We assume that consanguinity between agents leads them to partially internalize the effects of their actions on the payoffs to other family members. The specific mechanism governing this internalization, borrowed from the theory of kin selection, is presented later.

In the basic model, the owner hires the manager by making a first-and-final compensation offer to which the manager can either accept or reject. If the offer is accepted, the manager makes an unobservable effort decision, which produces a cash flow. The cash flow is only observed by the manager. The manager then decides on the cash flow to report to the owner. After observing the manager’s report, the owner decides whether to monitor the cash flow. Monitoring is costly but perfectly reveals the actual cash flow. Based on the realized cash flow, the report, and possibly information produced by monitoring, the cash flows of the firm are divided between the family agents. In Section 8 we extend the basic model analysis to consider a bequest of ownership rights by a founder one of two potential heirs. At least one of the potential heirs is related to the founder. The bequest, made at date -1, will determine the owner of the firm at date 0. The heir-owner will then have the option of hiring the other potential heir to manage the firm. In which case, the heir assumes the role of owner and the non-heir assumes the role of manager within the basic model setup.

The timing of events is summarised in Figure 1.
3 Specifics

3.1 Preferences

All agents are risk neutral and patient. The kin altruism preferences of family agents is reflected in their utility function, $u$:

$$u^{Own} = v^{Own} + hv^{Relative}, \quad 0 \leq h \leq 1/2.$$  \hfill (2)

where $v^{Own}$ represents the agent’s own value and $v^{Relative}$ represents the relatives’s value. The scalar $h$ represents kinship, the strength of the relation, or family ties, between the family agents. Note that agents are not altruistic in the sense of preferring relatives’ gains to their own. If asked how they would split a fixed amount of money with a relative, each relative’s preferred choice is to take everything for herself. However, relatives may abstain from such transfers when the transfers are highly dissipative, i.e., the transfer to one from one to another significantly reduces total family value. This observation is most apparent if we rewrite the utility function using to equivalent formulations. Let $v^{Family} = v^{Own} + v^{Relative}$ represent total family value. Then we can also express the utility function of a family agent in the following three forms:

$$u^{Own} = v^{Family} - (1 - h)v^{Relative},$$  \hfill (3)

$$u^{Own} = (1 + h)v^{Family} - u^{Relative},$$  \hfill (4)

$$u^{Own} = hv^{Family} + (1 - h)v^{Own}.$$  \hfill (5)

These reformulations of the utility function show that across choices that leave the either the utility or the payoff of kin fixed, agents prefer the decision that maximizes family value. This is not very surprising observation, but it will prove useful in the subsequent analysis.
3.2 Reporting and monitoring

After the cash flow is generated, the manager sends a message to the owner. This message is only observed by the owner. By the revelation principle, we can assume that the message is a report a cash flow of 0 or a cash flow of \( \bar{x} \). We call this report, “reported income.” If the owner does not monitor, reported income equals income. If the owner monitors, income equals the actual cash flow. Monitoring cannot be verified and the report is observed only by the owner. Income is transferred by the manager to the owners, and then distributed to the owners and the manager. Because reported income must be transferred, reported income can never exceed cash flows. Thus, when the firm’s cash flow is 0, the manager always reports 0. If the cash flow is \( \bar{x} \) the manager can report either 0 or \( \bar{x} \). The manager’s compensation is based on reported income. Because payments to the manager satisfy limited liability in reported income, the payment received after a report of 0 always equals 0. If the manager report 0 and the actual cash equals \( \bar{x} \) and the owner does not monitor, the manager diverts \( \bar{x} \) to personal consumption. If the manager reports 0 and the actual cash equals \( \bar{x} \), and the owner monitors, the manager cannot divert \( \bar{x} \) to personal consumption and, since the payment received by the manager given a report of 0 is 0, the owner receives the entire cash flow. If the owner verifies a report of 0, regardless of whether the report is truthful or not, the owner pays a monitoring cost of \( c \). If the manager reports \( \bar{x} \), the owner knows that the report is truthful. In this case, cash flows are divided between the manager and the owner based on reported cash flows with the manager receiving \( w \) and the owner receiving \( \bar{x} - w \). From the perspective of the reporting and monitoring model, it does not matter how \( w \) is fixed. However, when we analyze manager-owner compensation negotiations later, we will fix \( w \) using the managers’ reservation and incentive compatibility conditions.

The payoffs, to the manager and firm, excluding managerial effort costs, for a fixed contracted payment , \( w \), to the manager are depicted in Figure 2.

3.3 Effort

The random cash flow from the project, \( \bar{x} \), has the following distribution

\[
\bar{x} = \begin{cases} 
\bar{x}, & \text{w.p. } p \\
0, & \text{w.p. } 1 - p
\end{cases}
\]  

(6)
The manager selects \( p \in [0, \bar{p}] \). The manager’s choice of \( p \) imposes a non-pecuniary effort cost of \( K(p) \) on the manager, where \( K \) is a weakly increasing function of \( p \). Effort is not observable by any agent accept the manager. If the firm fails to operate the project produces a payoff of 0, and the manager receives a payoff of \( v_R \), the manager’s reservation payoff.

### 3.3.1 Parameter restriction

Throughout the analysis, we impose the following parameter restrictions:

\[
\max_{p \in [0, \bar{p}]} \ p \bar{x} - (v_R + K(p) + c) > 0, \tag{7}
\]

\[
(1 - h) \bar{x} - c > 0. \tag{8}
\]

(7) implies that the expected cash flow to the project exceeds the cost of effort, monitoring, and the manager’s reservation payoff. Thus, absent any kinship between the agents, undertaking the project is optimal even if undertaking the project requires the owner to incur the monitoring expenditure, \( c \). The second restriction implies that the owner’s utility benefit from monitoring, which equals the gain from transferring a concealed cash flow of \( \bar{x} \) from the manager to the owner, \( (1 - h) \bar{x} \), exceeds the cost of
monitoring, $c$. If this assumption were violated, the owner would never monitor and the manager would divert the entire cash flow.

## 4 Kinship and monitoring

In this section, we analyze the monitoring/reporting problem in the case where the game is not trivial, i.e., when the cost of monitoring $c$ is positive. In this case, monitoring is costly and will only be undertaken when the gains from monitoring exceed its cost. The gain from monitoring depends on the likelihood managers attempt diversion by underreporting. Managerial underreporting will depend, in turn, on the likelihood of monitoring. In equilibrium, monitoring and underreporting will be simultaneously determined. Our aim is to investigate the effect of kinship the diversion and monitoring.

The manager’s reporting policy decision is whether to report $\bar{x}$ or 0 when the cash flow is equal to $\bar{x}$. The manager cannot commit to a reporting policy but rather makes the reporting decision after observing the cash flow. The owner’s policy decision is whether to monitor the manager’s report of a zero cash flow. The owner cannot commit to a monitoring decision ex ante but rather must choose a monitoring policy which is a best reply to the manager’s reporting policy.

### 4.1 Incentives to underreport

When the cash flow equals $\bar{x}$ and the manager reports $\bar{x}$, he receives $w$ and the owner receives $\bar{x} - w$. If the manager reports 0, and the owner does not monitor, the manager receives $\bar{x}$ and the owner receives 0. If the owner monitors, the manager receives 0 and the owner receives $\bar{x} - c$. Thus, conditioned on underreporting, the manager’s utility is

$$u_{M, \text{Underreport}}^M = (1 - m)\bar{x} + hm(\bar{x} - c), \quad (9)$$

and conditioned on truthfully reporting $\bar{x}$, the manager’s utility is

$$u_{M, \text{NotUnderreport}}^M = w + h(\bar{x} - w). \quad (10)$$

Thus, the manager’s best reply is to divert if $m < m^*$, not divert if $m > m^*$, and, both diversion and non-diversion are best responses if $m = m^*$, where $m^*$ is determined by equating (9) and (10), which
produces
\[ m^* = \frac{(1-h)(\bar{x} - w)}{ch + (1-h)\bar{x}}. \]  \hspace{1cm} (11)

### 4.2 Incentives to monitor

Let \( \rho \) represent the owner’s posterior assessment of the probability that the cash flow is \( \bar{x} \) conditioned on the manager reporting 0. Later we will determine this posterior using Bayes rule. If the owner monitors, the owner’s will receive \(-c\) if the cash flow is 0 and \(\bar{x} - c\) if the cash flow is \(\bar{x}\). Thus, the owner’s payoff from monitoring is
\[ \rho \bar{x} - c. \]  \hspace{1cm} (12)

If the owner decides not to monitor, his payoff is 0. Now consider the manager’s payoff conditioned on a report of 0. If the cash flow is actually 0, the manager’s payoff is 0 regardless of the owner’s monitoring decision, if the cash flow is \(\bar{x}\), the manager receives \(\bar{x}\) if the owner does not monitor, and 0 in the owner monitors. Thus, the utility to the owner from monitoring, reflecting both his payoff and the manager’s payoff as specified in (2). It is given by
\[ u_{\text{Mon.}}^O = \rho \bar{x} - c. \]  \hspace{1cm} (13)

If the owner does not monitor, the owner’s utility is given by
\[ u_{\text{NotMon.}}^O = h \rho \bar{x}. \]  \hspace{1cm} (14)

Thus, the owner’s best reply is to monitor if \( \rho > \rho^* \); not monitor if \( \rho < \rho^* \); both monitoring and not monitoring are best replies if \( \rho = \rho^* \), where
\[ \rho^* = \frac{c}{(1-h)\bar{x}}. \]  \hspace{1cm} (15)

Let \( \sigma \) represent the probability of the manager reporting 0 conditioned on the cash flow being \( \bar{x} \). The cash flow distribution under effort (which is given by (6)) and Bayes rule imply that \( \rho \), the probability that the cash flow equals \( \bar{x} \) conditioned on a report of 0, is given by
\[ \rho = \frac{\sigma p}{\sigma p + (1-p)}. \]  \hspace{1cm} (16)
4.3 Monitoring/reporting equilibrium

To determine the equilibrium, we first impose the following parametric restriction:

\[(1 - h)\bar{x} p > c. \tag{17}\]

Assumption (17) is simply imposed to ensure that, the probability of success, \(p\), is sufficiently high to ensure that monitoring is a best reply to a managerial strategy of always attempting diversion by under-reporting cash flows. To determine the equilibrium level of monitoring and reporting, first note that no equilibrium exists in which monitoring occurs with probability 1: if monitoring did occur with probability 1, then the manager would never underreport. In which case, monitoring would not be a best response for the owner. Next, note that the highest possible value of \(\rho\), produced by the conjecture that the manager always underreports, is \(p\). Thus, assumption (17) ensures that for a sufficiently high probability of underreporting, the owner will monitor. If assumption (17) were not satisfied, then the owner will never monitor and the equilibrium solution would be for the manager to divert with probability 1. Thus, there is a unique mixed strategy equilibrium in which (16), (11) and (15) are all satisfied. The equilibrium probabilities of underreporting, \(\sigma^*\), and monitoring reports of 0, \(m^*\), in this mixed strategy equilibrium are given by

\[
\sigma^* = \frac{c (1 - p)}{p (\bar{x}(1 - h) - c)}, \quad m^* = \frac{(1 - h)(\bar{x} - w)}{ch + (1 - h)\bar{x}}. \tag{18}
\]

We see from equation (18), that monitoring is decreasing in kinship, \(h\), while managerial underreporting is increasing in \(h\). This implies that diversion is larger when kinship is higher. At first glance this result seems odd, why should managers steal more when kinship is high? The result follows from the nature of diversion. Diversion is an intra-family transfer and monitoring is costly to the family as it lowers the family’s total value. As kinship increases, the owner internalizes more of the manager’s gain from underreporting and, thus, the level of expected managerial underreporting required to induce monitoring also increases. At the same time, underreporting, by triggering monitoring, increases expected monitoring costs, and the manager internalizes these monitoring cost in proportion to kinship. Thus, as kinship increases, the level of monitoring required to deter diversion falls.

Since kinship both (a) increases underreporting, (b) reduces the probability that zero reports will be monitored, the combined effect of (a) and (b) determines kinship’s effect on the unconditional probabil-
The fall in $m^*$ induced by an increase in kinship implies that a report of zero is less likely to be monitored. At the same time, the increase in $\sigma^*$, also induced by an increase in kinship, implies that as kinship increases the probability of a report of 0 increases. Since monitoring occurs if and only if a report of zero is made and the report is monitored, the effect of kinship on the probability of monitoring is not obvious at first glance. However, explicit calculation of the equilibrium total probability of monitoring, given by $PM^* = m^*(1 - p(1 - \sigma^*))$, shows that

$$PM^* = m^*(1 - p(1 - \sigma^*)) = \frac{(1 - h)^2(1 - p)x(x - w)}{((1 - h)x - c)(ch + (1 - h)x)},$$

which is an increasing function of $h$. These observations establish the following proposition.

**Proposition 1.** There is a unique equilibrium level of monitoring and underreporting conditioned on a given contractual payment to the manager, $w$. In this equilibrium, the probability of monitoring zero reports, $m^*$, and underreporting, $\sigma^*$ are given by equation (18). In the equilibrium, the probability of

- Underreporting is increasing and convex in kinship, $h$,
- Monitoring of zero reports is decreasing and concave in kinship $h$.
- The total probability of monitoring occurring is increasing in kinship, $h$.
- The probability of successful diversion is increasing in kinship, $h$.

Thus, at any fixed compensation level, kinship increases both diversion and costs of monitoring, which are proportional to the probability of monitoring. This result follows because increasing kinship reduces the welfare loss to the owner from diversion of firm resources by his kin—the manager. This weakens monitoring incentives. Weaker monitoring incentives lead to more underreporting and thus more reports of low cash flows. Since monitoring only occurs after low reports, this leads to a higher total probability of monitoring even though conditional on a low report being made, the probability of monitoring is lower. Since the only dissipative cost faced by the family as a whole is monitoring costs, increasing kinship lowers total family payoffs. These observations are illustrated in Figure 3 below.

### 5 Intrafamily compensation negotiations: The bright-side scenario

As will become more apparent as we develop the model fully, kinship has a number of distinct effects on firm efficiency, as well as manager and owner welfare. In the most general formulation of the model, these effects are difficult to disentangle. Thus, we will begin our analysis by imposing parametric
Figure 3: The probability of underreporting, $\sigma^*$ (represented by the solid line), monitoring reports of 0, $m^*$ (represented by the thick dashed line), managerial diversion, $Div.$, (represented by dotted line), and monitoring occurring, $PMon.$ (represented by the thin-dashed line), as a function of kinship, $h$. In the graph, $c = 0.2$, $p = 0.5$, $x = 1.0$, and $w = 0.4$.

assumptions that isolate the positive and negative effects of family control. We first consider the case where the reservation wage constraint is not binding and monitoring is costless. This is the “bright side” scenario for family control. Next, we consider the opposite case, where the monitoring problem is predominant and limited liability and the managerial reservation constraint fix compensation. This is, as we shall see, the “dark-side scenario” for family control. Finally, primarily through numerical simulations, we explore the general case where monitoring is a significant problem and but the effort incentive compatibility conditions fix managerial compensation. We will see that the results in the general case will depend on whether the positive or negative effects dominate.

To implement the bright-side scenario model, we assume that $c = 0$ and $v_R = 0$. Under these assumptions, monitoring is costless and thus the monitoring/reporting problem has a trivial solution—the owner monitors with probability 1 and the manager does not underreport; incentive computability and limited liability are the binding constraints. We also assume in this section, that the output/effort relation takes the following simple form:

$$K(p) = \frac{1}{2}k p^2, \quad p \in [0, \bar{p}], \bar{p} \leq \bar{x}/k. \quad (20)$$

$\bar{p}$ is not required to obtain any of the results. The bound simply rules out effort levels that are so high that they lower total family welfare. Such effort levels are never optimal. By ruling out these effort levels we shorten some of the proofs.
If the manager works for the firm, the wage level is $w$, and success probability is $p$, the value to the manager $v_M$ and owner $v_O$ are given as follows:

$$
\begin{align*}
  v_M &= pw - \frac{1}{2}k p^2, \\
  v_O &= p(\bar{x} - w),
\end{align*}
$$

$p \in [0, \bar{p}]$ and $w \geq 0$. \hfill (21)

The utilities of the manager and owner are given by

$$
\begin{align*}
  u_M &= pw - \frac{1}{2}k p^2 + h(p(\bar{x} - w)), \\
  u_O &= p(\bar{x} - w) + h(pw - \frac{1}{2}k p^2)
\end{align*}
$$

$p \in [0, \bar{p}]$ and $w \geq 0$. \hfill (22)

If the manager refuses to accept employment, the firm cannot operate and thus the payoffs to the manager and owner both equal 0 and thus their utilities equal 0.

The manager’s effort problem is given by

$$
\max_{p \in [0, \bar{p}]} u_M.
$$

The solution to this problem is

$$
p^*_M(w) = \min \left[ \frac{w(1-h)+\bar{x}h}{k}, \bar{p} \right].
$$

Define $p_0$ by

$$
p_0 = p^*_M(0) = \frac{\bar{x}h}{k}.
$$

Note that $p_0$ is the probability of success, i.e., realising $\bar{x}$, given the lowest possible level of managerial compensation, 0. The function $p^*_M$ is strictly increasing thus we can define the inverse map

$$
w^*_M(p) = \frac{k p - h \bar{x}}{1-h}, \quad p \in [p_0, \bar{p}].
$$

Provided $[p_0, \bar{p}]$ is non empty, we can thus formulate the owner’s problem as choosing, $p, w^*_M(p)$ combinations to maximize the owner’s utility subject to the manager’s reservation constraint. However, given our assumption that the reservation compensation is 0, the reservation constraint is not binding. Thus,
using expressions (26) and (22), we can write the owner’s problem as

$$\max_{p \in [p_0, \bar{p}]} p(\bar{x} - w^*(p)) + h(p w^*(p) - \frac{1}{2}k p^2).$$

(27)

Note that if we evaluate the derivative of the objective function, given in (27) we obtain:

$$(h + 1) \bar{x} - (h + 2)k p.$$

(28)

If we evaluate this expression at $p_0$ we see that it is positive for $h \leq \frac{1}{2}$ thus the optimal choice of $p$ will either be in the interior of $[p_0, \bar{p}]$ or will equal the upper endpoint, $\bar{p}$. The fact that the first-order condition is positive at $p_0$ and the definition of $p_0$ given by expression (25) combined with the fact that $\bar{p} \leq \bar{x}/k$ implies that the optimal solution to the owner’s problem, $p^*_O$ satisfies the following inequalities:

$$p^*_O k - \bar{x} \leq 0 \quad \text{and} \quad p^*_O k - h \bar{x} \geq 0.$$  

(29)

Solving the first-order condition shows that inside owner’s optimal choice, $p^*_O$, if the interval $[p_0, \bar{p}]$ is not empty, is given by

$$\min \left[ \frac{(h + 1)\bar{x}}{(h + 2)k}, \bar{p} \right] \quad h \in [0, 1/2].$$

(30)

The interval is empty if and only if $\bar{p} < p_0$. If the interval is empty, the highest feasible $p$ is attained at a 0 compensation level. Thus, the optimal solution is to set $p = p_0$. Combining this observation with expression (30) shows that the optimal choice for the owner is defined by

$$p^*_O = \max \left[ p_0, \min \left[ \frac{(h + 1)\bar{x}}{(h + 2)k}, \bar{p} \right] \right], \quad h \in [0, 1/2].$$

(31)

If the owner managed the firm himself, and thus internalized all of the benefits and costs of undertaking the project, the owner would set $p = \min[\bar{x}/k, \bar{p}]$. This first-best level of effort is never attained under management by the manager as can be seen through inspecting (30). However, it is equally apparent that kinship leads to a higher level of output than would be attained if the manager was not kin and that the level of output is increasing in kinship.\(^\text{11}\) There are two drivers of the welfare gains from kinship.

\(^{11}\)In fact, inspecting the equation seems to indicate that even if $h = 1$, the probability of success might fall below the first best level. However, this conjecture is not correct as it fails to take into account the parameter restriction, $h < 1/2$. If we evaluate equation (28) at $p_0$ as defined in (25), we see that for all $h > 1/2(\sqrt{5} - 1) \approx 0.618$ (28) is negative at $p_0$, implying that the optimal solution to the owner’s problem calls for $p = p_0 = h(\bar{x}/k)$ which indeed converges to the first best solution as $h \to 1$. 

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The first, which has a standard analog the principal-agent literature, is that the manager internalizes firm gains through kinship, in a sense the manager has an implicit ownership stake in the owner’s profits engendered by kinship. Thus, the manager even without explicit performance based compensation, acts as if he is a partial owner of the firm. The other effect, which is somewhat different than that found in standard agency models is that the owner acts as if he partially “owns” the manager’s pay check. Increasing $p$ increases total output at the cost of raising compensation and thus increasing the fraction of output captured by the manager. Thus, the partial internalization of the manager’s compensation gain induces the owner to be more willing to sacrifice firm profits for the sake of increasing overall output. This observation is illustrated in Figure 4 Given the intuition developed above the following results are

![Figure 4: The effect of kinship on output and compensation in the costless monitoring framework. In the figure, equilibrium levels of compensation, $w$, and success probability, $p$, are plotted for family and non-family firms. The gray lines represent the non-family firm, i.e., the case where $h = 0$. The black lines represent the family firm, i.e., $h > 0$. The dashed lines represent the combinations of $p$ and $w$ which are incentive compatible. The solid lines represent the indifference curves of the owner. The point labeled “Fam” represents the solution for the family firm; The point labeled “NotFam” represents the solution for the non-related firm.](image)

not too surprising.

**Proposition 2.** In the bright-side scenario, total output and the value of the firm are weakly increasing in kinship and are strictly increasing whenever the success probability $p^*_O$ is in the interior of the feasible range. The manager’s value is weakly decreasing in kinship and is strictly decreasing whenever $p^*_O$ is in the interior of the feasible region.

**Proof.** See appendix.
6 Intrafamily compensation negotiations: The dark side

In the bright-side scenario we abstracted from the monitoring problem by assuming that monitoring costs equaled 0 but effort was costly. This produced the most favorable case for family ownership from an overall welfare perspective—the bright-side scenario. In this section, we abstract from the ex ante effort problem by assuming that effort costs are 0 but monitoring is costly. We call this case the “dark-side scenario.” The specific parametric assumptions we impose are as follows:

\[ K(p) = 0, \]  
\[ \nu_R > 0, \]  
\[ (1 - h) \bar{x} \bar{p} > c, \]  
\[ \bar{x} \bar{p} - \nu_R > c. \]

Equation (32) sets effort costs to 0 and inequality (33) ensures the reservation value of the manager is positive. Under these assumptions, the manager will choose the highest feasible success probability, \( \bar{p} \).

Inequality (34) insures that the general condition (17) is satisfied and thus the owner’s monitoring costs are not so high that no monitoring will occur. Inequality (35) ensures that the general condition (7) is satisfied and thus the project has positive value.

6.1 Compensation

In this section, we determine the equilibrium level of compensation. As specified in Section 3.3, no output can be produced without managerial effort. Thus, the owner will always offer sufficient compensation to ensure effort and retain the manager. Since effort is costless in the dark-side scenario being analyzed here, the manager will always exert effort if he accepts employment.

If the manager accepts employment, the cash flow to the manager either equals \( \bar{x} \) or 0. If the cash flow equals \( \bar{x} \), the manager’s utility is as given in monitoring/reporting subgame defined in Section 4.3.

If the realized cash flow is 0, the manager’s payoff is 0 and the owner’s payoff equals the losses from monitoring the manager’s 0 cash flow report, given by \( -m^*c \). Thus, the manager’s utility is

\[ u^*_M = \bar{p}(w + h(\bar{x} - w)) - (1 - \bar{p})h m^*c. \]
The owner’s utility is determined in like fashion. The owner’s utility is given by the expectation over the two possible reports 0 and \( \bar{x} \). If 0 is reported, the owner’s utility is given by the mixed strategy equilibrium in the monitoring/reporting subgame. Since the utility to the owner is the same whether he monitors or does not monitor in the subgame, the utility of the owner after a report of 0 equals the owner’s utility after a report of 0 given the owner does not monitor. This is given by \( h^* \rho^* \bar{x} \). The probability of a zero report is \( 1 - (1 - \sigma^*) \bar{p} \). If the manager reports \( \bar{x} \), which occurs with probability \( (1 - \sigma^*) \bar{p} \), the owner’s utility is \( \bar{x} - (1 - h)w \). Thus, the utility of the owner is given by

\[
(1 - (1 - \sigma^*) \bar{p})(h^* \bar{x}) + ((1 - \sigma^*) \bar{p})(\bar{x} - (1 - h)w).
\]

Using equation (16) we can simplify this expression to

\[
u^*_O = \bar{p} ((1 - \sigma^*) (\bar{x} - (1 - h)w) + \sigma^* h \bar{x}).
\]

From (18) and (38), and (34) it is clear that, despite kinship, the owner’s utility is decreasing in the level of managerial compensation. For this reason the owner will never set compensation higher than the level required to satisfy the problem’s constraints. If the manager does not work for the firm he earns \( v_R \) and the owner’s payoff is 0. Thus, minimum managerial compensation that satisfies the reservation constraint satisfies

\[
\bar{p} (w + h(\bar{x} - w)) - (1 - \bar{p}) h m^* c = v_R.
\]

Solving this equation for \( w \), yields the minimal compensation to the manager required to ensure the reservation constraint is satisfied:

\[
\frac{v_R}{\bar{p}} = \frac{h((\bar{p} \bar{x} - v_R)((1 - h)(\bar{p} \bar{x} - c) + c \bar{p}))}{(1 - h)\bar{p}(ch + (1 - h)\bar{p} \bar{x})}.
\]

There is an additional constraint on compensation, limited liability, which requires a positive payment to the manager. Thus, in order to obtain the equilibrium level of compensation we need only impose the limited liability condition:

\[
w^*_M = \max \left[ \frac{v_R}{\bar{p}} - \frac{h((\bar{p} \bar{x} - v_R)((1 - h)(\bar{p} \bar{x} - c) + c \bar{p}))}{(1 - h)\bar{p}(ch + (1 - h)\bar{p} \bar{x})}, 0 \right].
\]

As long as the limited liability constraint is not binding, increasing kinship, reduces the equilibrium
compensation level, $w^*_M$. This result is recorded and demonstrated below.

**Proposition 3.** Compensation, $w^*_M$, is weakly decreasing in kinship, $h$ and, whenever $w^*_M > 0$, $w^*_M$ is a smooth strictly decreasing convex function of $h$.

*Proof.* See the Appendix.

The negative effect of kinship on compensation results from a “loyalty hold-up.” Because the management skills for the project are firm specific, if the manager refuses to work for the family firm, project cash flows are lost, which harms the family as a whole. The manager internalizes the family’s losses and thus will be reticent to reject even low salary offers from the owner. Thus, the firm-specific nature of the skills required to run the firm actually weaken the bargaining position of the family member possessing these unique skills.

### 6.2 Efficiency

In Section 4.3 we showed that total monitoring increases with kinship at a fixed wage. In Section 6.1 we showed that increased kinship leads to lower compensation. Reductions in compensation, absent diversion attempts by the manager, increase the size of the owner’s residual claim, $x - w$. The gain from diversion relative to non-diversion is exactly this residual share. Thus, lowered compensation makes underreporting more attractive at a any fixed monitoring policy. Hence, reductions in compensation, require increases in monitoring to deter diversion. Combining these two observations makes the logic behind the following proposition apparent.

**Proposition 4.** (a) Whenever $w^*_M > 0$, the probability that the owner will monitor the manager’s report of a zero cash flow is strictly increasing in kinship.

(b) The total probability of monitoring is increasing in kinship

(c) Total family value is decreasing in kinship

*Proof.* See the Appendix.

Note that both when the payment to the manager is fixed, the case considered in section 4.3, and in the analysis of this section where the payment is negotiated, kinship increases the unconditional probability of monitoring. However, in the fixed payment case, the probability of monitoring conditioned on a report of zero falls as kinship increases. The increase in the total probability of monitoring occurs in the fixed payment case because the reduced conditional probability of monitoring zero reports is
swamped by the increase in the likelihood of zero reports. However, when the payment to the manager is negotiated and the manager’s reservation constraint binds, compensation varies with kinship and the loyalty hold up effect ensures that even the conditional probability of monitoring increases with kinship. Thus, while kinship increases the probability of inefficient monitoring even when the manager’s payment is fixed, the probability of monitoring will be much more responsive to increases in kinship when compensation is negotiated and the reservation constraint binds, i.e., the loyalty hold up is effected.

6.3 Value

Next, we consider the value effects of kinship. The manager does not aim to maximize the value of his employment relation with the firm nor does the owner aim to maximize the firm’s value, rather both of these family agents aim to maximize their utility which partially internalizes the gains to fellow family members. However, the actions of the owner and manager, which vary with kinship, have valuation effects. The effect of kinship on valuation depends both on kinship’s effect on efficiency was well at its effect on the distribution of value between the manager and the owner. Because of these distributional effects, in the dark-side scenario, kinship may increase firm value even though it lowers total family value.

6.3.1 Firm value

Equations (41), (18), and (38), determine the owner’s equilibrium value, $v_O^*$, which is given by

$$v_O^* = \bar{p} \left( m^* \bar{x} \sigma^* + (1 - \sigma^*) (\bar{x} - W_M^*) \right) - cm^*(1 - \bar{p} (1 - \sigma^*)).$$

(42)

where $\sigma^*$ is defined by (18) and $W_M^*$ by (41).

From Propositions 1 and 1, we see that increasing kinship will (i) lower compensation, (ii) increase underreporting, and (iii) increase monitoring. Effect (i) increases firm value while effects (ii) and (iii) lower firm value. For this reason, the relation between firm value and kinship is, in general, neither monotone nor concave. However, the relation between kinship and value is strictly quasiconcave. Hence, the relation is always unimodal. Whether the value-maximizing level of kinship is interior depends on the degree of uncertainty regarding firm cash flows and the costs of monitoring relative to the total expected operating cash flows. These observations are formalized in Proposition 5.

**Proposition 5. a.** The value of the firm is a quasiconcave function of kinship, $h$. 
b. Let $\gamma = c/(\bar{p}\bar{x})$, then

i. if

$$(1 - \gamma)^2 - \gamma^3 (((1 - \bar{p}) - \gamma)(\bar{p} - \gamma) + (1 - \gamma)\gamma) > 0,$$  \hspace{1cm} (43)

There exists a unique positive level of kin relatedness, which maximizes the value of the firm.

ii. If

$$(1 - \gamma)^2 - \gamma^3 (((1 - \bar{p}) - \gamma)(\bar{p} - \gamma) + (1 - \gamma)\gamma) \leq 0,$$  \hspace{1cm} (44)

then the value of the firm is maximized at when the manager is not related to the owner, i.e., $h = 0$

The region of the parameter space over which kinship increases value is presented in Figure 5.

![Figure 5: The horizontal axes represents $\gamma$, monitoring cost as a fraction of total firm value. The vertical axis represents $\bar{p}$, the probability of a high cash flow given effort.](image)

Proposition 5 shows that the answer to the question of whether kinship can increase value depends only on the cost of monitoring as a proportion of the total expected operating ash flows, $\gamma$ and the *uptick* probability, the probability of a positive cash flow given effort, $\bar{p}$. The fact that some level of kinship increases value does not preclude higher levels of kinship from destroying value. If fact, a sufficiently a high level of kinship may make credible monitoring impossible and thus prevent owners from extracting any value from the project. However, despite these limitations, the proposition does reveal that family firms may have higher values than otherwise identical non-family firms even when family firms are inefficient. In these cases, the firm’s gain in value from the loyalty holdup exceeds its loss from inefficient monitoring. Figure 6 presents fairly representative illustrations of the possible
relations between firm value and kinship under the dark-side scenario.

Figure 6: Effect of kinship on firm value at a sample of admissible model parameters. In each of the figures, kinship is plotted on the horizontal axis, $h$, and firm value is plotted on the vertical axis. The region labeled $\partial v^*_O/\partial h > 0$ represents the region where firm value is increasing with in kinship between the owner and manager. The region labeled $\partial v^*_O/\partial h < 0$ represents the region where firm value is decreasing in kinship and the managerial compensation exceeds 0. The region labeled $w^*_M = 0$ represents the region where managerial compensation equals 0. As the graphs suggest, the relation between kinship and firm value is quasiconcave.

6.3.2 The manager’s value

Because, in the dark-side scenario, there are no effort costs, the manager’s value is just the expected cash flow received by the manager. It is given by

$$
 v^*_M = \bar{\rho} \left( \left( 1 - m^* \right) \sigma^* \bar{x} + \left( 1 - \sigma^* \right) w^*_M \right).
$$

The effect of kinship on the manager’s value function is somewhat subtle. Recall, that the reservation constraint is always binding at the equilibrium compensation contract if the limited liability constraint
can be satisfied at a compensation level that makes this constraint bind. However, this condition only ensures that the manager’s utility from accepting employment is constant. Since, utility incorporates internalized family gains, it is not identical to value. The manager’s value is produced by two components: compensation and diversion. Increasing kinship always weakly lowers compensation, but at the same time, it also increases diversion. Thus, the effect of kinship on the manager’s value is determined by the balance of these two effects. When kinship is low, the compensation reduction effect always dominates, and increasing kinship lowers the manager’s value. However, increasing kinship from a sufficiently high starting point can actually increase the manager’s value. This reversal can occur for two reasons, one fairly obvious and the other subtle. The obvious reason is that compensation has been lowered so much by kinship that it has hit the limited liability boundary. In which case, the manager clearly earns positive rents as kinship increases because his compensation cannot fall and equilibrium diversion increases. But this is not the only mechanism through which increasing kinship increases the manager’s value. Increases in value can also occur when the reservation constraint is binding. The condition for such increases is that the value of the project, measured by $\bar{p}\bar{x} - v - c$ is sufficiently low. The logic behind the reversal is that although an increased degree of kinship increases the fraction of the family’s gain the manager internalizes and thus uses to satisfy his reservation constraint, the increase in kinship also increases monitoring and thus lowers the total family gain. Thus, at a higher degree of kinship, the manager has less family gain to internalize into his utility function. Hence, to keep his utility constant, his direct gains from diversion must increase. These results are recorded in the following proposition:

**Proposition 6.**

(a) The manager’s value is strictly quasiconvex in kinship, $h$ and thus there is a unique degree of kinship that minimizes the manager’s value.

(b) For all $h$ sufficiently close to 0, increases in kinship lower the manager’s value.

(c) The manager’s value is maximized either at the lowest or highest admissible degree of kinship.

(d) If the manager’s value is maximized at the highest degree of kinship, the limited liability constraint must be binding.

(e) If $\bar{p} > \frac{1}{1-\gamma}$, where $\gamma = c/(\bar{p}\bar{x})$ and $\bar{p}\bar{x} - v - c$ and $\gamma = c/(\bar{p}\bar{x})$ and $\bar{p}\bar{x} - v - c$ and $\gamma = c/(\bar{p}\bar{x})$ and $\bar{p}\bar{x} - v - c$ and $\gamma = c/(\bar{p}\bar{x})$ and $\bar{p}\bar{x} - v - c$ are sufficiently close to 0, then increases kinship, increase the manager’s value.

**Proof.** See the Appendix.

Examples of parameters which do and do not satisfy (e) are presented in Panels A and B of Figure 7.
respectively. The quasiconvexity of the manager’s value in kinship, developed in Proposition 6 and Figure 7: Effect of kinship on the manager’s value. In each of the figures, kinship is plotted on the horizontal axis and the manager’s value is plotted on the vertical axis. In both cases, part (e) of Proposition 6 is verified to hold, i.e., at low levels of kinship, the manager’s value falls with increased kinship. In Panel A, the conditions of (d) in Proposition 6 are satisfied and, at high levels of kinship, further increases in kinship increase the manager’s value while in Panel B, conditions of (d) in Proposition 6 are not satisfied and the manager’s value is monotonically decreasing in kinship.

Figure 7, is easiest to understand if we consider extreme cases. First eliminate the kinship effect by supposing that $h = 0$. In this case, monitoring is strict and compensation large, reflecting the manager’s reservation value, and thus the manager’s value is large. Next, consider the case were $h$ is large. In this case, the incentive to monitor is weak but, at the same time, compensation is low because of the loyalty holdup. Because of low compensation, the manager has a strong propensity to divert. Thus, the level of monitoring is high despite the owner’s kinship-attenuated propensity to monitor. Excessive diversion and monitoring reduce total family gains. Thus, such gains can make only a limited contribution to meeting the manager’s reservation utility. To compensate, the manager’s value must be large, just as in the $h = 0$ case.

7 Shades of gray: Combining dark-side and bright side effects of kinship

In this section, we consider the family firm when both monitoring and effort effects are present. The analysis in these cases is much less straightforward. We will assume that monitoring costs, $c$, are positive, the reservation constraint is not binding, i.e., $v_R = 0$, and that output/effort relation is as specified in equation (20) in the bright-side scenario. We can solve the model in an analogous fashion to the development in sections 5 and 6. Monitoring and reporting probabilities will be the same as those derived
In Section 4.3, i.e.,

\[ \sigma^* = \frac{c(1 - p)}{p(\bar{x}(1 - h) - c)}, \quad \text{(46)} \]

\[ m^* = \frac{(1 - h)(\bar{x} - w)}{ch + (1 - h)\bar{x}}. \quad \text{(47)} \]

In order to induce the manager to expend sufficient effort to produce success probability, \( p \) it must be

the case that, given compensation \( w \), \( p \) is an optimal choice for the manager. From equation (36) we see

that this condition can be expressed as

\[ p \in \text{Argmax}\{ p \in [0, \bar{p}] : p(w + h(\bar{x} - w)) - \frac{1}{2}(1 - p)h m^*(w)c p^2 \}. \quad \text{(48)} \]

The owner will always prefer to induce effort at the smallest level of compensation consistnt the in-

centive compatibility condition give by (48) and the limited liability constraint. Define this level of

compensation as \( w_M^*(p) \). Following the same approach as was followed in Section 5, we can solve for

\( w_M^*(p) \) which yields

\[ w_M^*(p) = \frac{kp}{(1 - h)^2} - \frac{h}{\bar{x}} \left( \frac{(kp(\bar{x} - c) + \bar{x}(c + (1 - h)\bar{x}))}{(1 - h)^2} \right). \quad \text{(49)} \]

The owner maximizes utility over feasible choices of \( p \in [0, \bar{p}] \), were as in (38), the owner’s utility is
given by

\[ \bar{p} \left( (1 - \sigma^*(p))(\bar{x} - (1 - h)w_M^*(p)) + \sigma^*(p)h\bar{x}\right). \quad \text{(50)} \]

Using the approach developed in Section 5, this yields an optimal choice of \( p \), denoted by \( p_O^* \) and given

given by

\[ p_O^* = \min[\max[p_{O}^{\text{Int}}, p_{O}^{\text{Zero}}], \bar{p}], \quad \text{(51)} \]

where \( p_{O}^{\text{Int}} \) is the value of \( p \) that solves the first-order condition for the manager’s optimization problem
given compensation of \( w_M^*(p) \) and \( p_{O}^{\text{Zero}} \) is \( p \) the manager will select when compensation is set to the
limited liability boundary, 0. A bit of algebra yields explicit forms for \( p_{O}^{\text{Int}} \) and \( p_{O}^{\text{Zero}} \). These are provided
below:

\[
\begin{align*}
P^\text{int}_O &= \frac{c^2hk + c(1-h)k\bar{x} + (1-h)^2(1+h)\bar{x}^3}{(1-h)k\bar{x}((1-h)(2+h)\bar{x} + ch)}, \\
P^\text{Zero}_O &= h\left(\frac{\bar{x}}{k}\right)\left(1 + \frac{(1-h)c}{ch + (1-h)\bar{x}}\right). 
\end{align*}
\] (52)

Substituting \(P^*_O\) from (51) into the equilibrium, wage, \(w^*_M\), monitoring, \(m^*\), and underreporting, \(\sigma^*\), given by (49), (47), and (46), respectively can be used to determine the effects of kinship in the gray scenario developed in this section.

Unfortunately, as can be seen by inspecting the equations just developed, the gray scenario is somewhat opaque. This is not surprising as there are a number of effects in play. Now the owner’s tradeoffs are significantly more complex than the ones he faced in the bright-side or dark-side scenarios. An increase in kinship will (i) increase the owner’s own willingness to raise compensation, at the cost of owner’s own payoff, if it will increase total family payoff, (ii) make the manager willing to exert the more effort at any given level of compensation, and (iii) make the owner less willing to monitor low reported cash flows Effect (ii) will tend to increase compensation when increased compensation increases firm output at the cost of increasing the rents captured by the manager. However, increased compensation will itself lower the owner’s incentive to monitor. The incentive to monitor will also be directly lowered by closer kinship. Because of effect (iii), the manager’s gains from diversion will be higher. Since diversion gains can only be reaped if cash flows to divert are produced, these diversion gains will themselves improve the manager’s effort incentives. So, even at a fixed or reduced compensation level the manager’s value might increase as kinship is increased. Such an outcome becomes more likely when monitoring costs are high. Thus, when output is highly responsive to effort, as in the bright-side case, and monitoring costs are significant, as in the dark-side case, an increase in the degree kinship may associated with greater manager payoffs but lower compensation, with closely related managers extracting a significant portion of their value from diversion. In contrast, if managerial value-creation is not very pay sensitive at the equilibrium level of compensation, then the increase in the manager’s willingness to exert effort for the good of the family caused by an increase in kinship, will lead the owner to simply reduce compensation. This reduction in compensation will itself increase managerial diversion incentives. In addition, the increased level of family altruism, by discouraging monitoring, will also increase diversion incentives. Thus, compensation will fall steeply in kinship, while diversion and monitoring will rise.
Essentially the same result as observed in the dark-side scenario. Which scenario will emerge depends on the specifics of the parameter choices in a highly nonlinear fashion. Thus, we will address this issue numerically rather than through algebraic analysis.

Having fixed on a numerical approach, the next question is which variables should be the focus of analysis. Since neither non-pecuniary costs nor agent utility are observable, we will not simulated changes in these variables. Rather, we will focus on the observable effects of kinship on monitoring, the owner’s (i.e., firm) value, and the manager’s monetary value. The manager’s monetary value is the manager’s value gross of non-pecuniary effort costs. Notationally, we represent the manager’s monetary value by superscripting “$” to $v_M$. The family’s monetary value is the sum of the owner’s value and the manager’s monetary value, also represented by superscripting family value $V$ with $$. 

These effects are illustrated in Figures 8, 9, and 10. In these figures, the variable of interest is plotted against kinship, $h$ for two cases, high effort costs, represented by $K$ in the graph, and low effort costs, represented by $k$. Because the absolute payoffs in the two cases are very different, and thus difficult to present on the same graph, we graph the percentage difference of the variable’s value from its $h = 0$ value. In Panel A. of Figure 8, we see that increasing kinship lowers total family value when effort costs are low and increases total family value when effort costs are high. Firm value, plotted in Panel B, has an interior maximum in kinship in the low effort case and is increasing in kinship in the high effort-cost case.

![Graph A: Family monetary value](image)

![Graph B: Firm value](image)

Figure 8: The % effect of kinship, $h$, on total family monetary value, $V^*$, and firm value, $v_O^*$, for high, $K$, and low, $k$, levels of effort cost.

The drivers of these results are plotted in Figure 9. Panel A of this figure shows that compensation is decreasing in kinship in the high effort-cost case and low effort-cost case but that the rate of decrease is much faster when effort costs are low. When effort costs are low, the owner will opt for a high level of effort regardless of the strength of kinship. As kinship increases, high level of effort will be attainable at
lower levels of compensation. Leading compensation to fall dramatically. In contrast, when effort costs are high, the owner faces a tradeoff with respect to compensating effort between using the higher degree of kinship to motivate more effort or to lower compensation at the same level of effort. The ability of the owner to raise effort reverses the fall in compensation, leading compensation to rise with kinship in the high effort-cost case. The increase in compensation coupled with the positive incentive effects of kinship for effort leads the large increase in the probability of the high cash flow, \( \bar{x} \) illustrated in Panel B. of Figure 9.

Figure 9: The % effect of kinship, \( h \), on managerial compensation, \( w^*_M \), and the probability of success, \( p^* \), for high, \( K \), and low, \( k \), levels of effort cost.

Through its effect on compensation and owner monitoring incentives, kinship also affects monitoring and the manager’s monetary value. Compensation is negatively associated with underreporting because the larger the fraction of output the manager can legitimately claim through compensation, the smaller the owner’s claim and thus the smaller the gain from diverting the owner’s claim. Kinship also is negatively associated with monitoring. In the case of high effort costs, the small decrease in compensation triggered by increasing kinship leads to only a small increase in underreporting. Thus, expected monitoring and thus monitoring costs are much less sensitive to kinship in the high effort-cost case. In contrast, when effort costs are low, and thus compensation is falling rapidly in kinship, the increase in the incentive to underreport is dramatic. This incentive is produced both directly from laxer monitoring and indirectly through the sharp drop in compensation. Thus, as illustrated in Panel A of Figure 10, expected monitoring increases rapidly in kinship in the low effort-cost case but only slowly in the high effort-cost case. The manager’s monetary value, plotted in Panel B of Figure 10, is composed both of gains from compensation and gains from diversion. In the low effort-cost case, the manager’s monetary value falls until the limited liability constraint is reached. At which point compensation is fixed and the
reduced monitoring associated with further increases in kinship caused monetary value to increase. In the high effort-cost case, the increase in compensation caused by increasing kinship amplifies the increased gains from diversion caused by more relaxed monitoring and the manager’s monetary value increases dramatically with kinship.

![Graph A: Probability of monitoring](image1.png)

![Graph B: Manager’s value](image2.png)

**Figure 10:** The % effect of kinship, $h$, the probability of monitoring, $PM^*$, and on the manager’s monetary value, $v_M^*$, for high, $K^*$, and low, $k$, levels of effort cost.

### 8 Inheritance from the founder and the birth of family firms

In the previous sections we examined the behavior of family firms—firms in which ownership rights, control and managerial human capital are concentrated within a family. In this section, we examine the birth of family firms from the bequest of a founder. We assume that founder has developed a positive NPV project which we will call the “firm,” and has also developed, amongst her relatives, the human capital required to manage this project after the her death. The firm is the founder’s only asset and, as her life draws toward its end, she needs draw up a bequest transferring ownership rights to surviving family members. These family members have differing degrees of relatedness to the founder also have differing endowments of human capital. The founder has nepotistic preferences favoring enriching closer relations at the expense of some reduction in overall family-firm value. However, as we will show, kin altruism based on genetic relatedness implies that, for typical family pedigrees, the founder’s nepotistic preference for closer relations will not be as strong as those relations own selfish preferences for their own payoffs relative to rival family members. The founder’s bequest problem arises because the founder cannot directly control the policies her relatively “selfish” descendants adopt after she dies. However, the founder can use her bequest to shape the descendants’ intra-family negotiations and thus influence the posthumous governance and management of the family firm.
8.1 Inheritance model

Consider the problem of an founder at date -1 making a bequest of the firm. The founder knows she will die between date -1 and date 0. Thus, the founder derives no direct payoff the bequest decision. The founder’s decision will maximize the founder’s inclusive fitness based on kin altruism as measured by the coefficient of relationship, i.e., kinship, between the founder and her potential heirs. There are two potential heirs: $S$ and $N$. We represent the kinship between the founder and $S$ with $h_S$, the kinship between founder and $N$ by $h_N$, and the kinship between $N$ and $S$ by $h_{NS}$. We assume that

$$0 \leq h_N < h_S \leq \frac{1}{2} \text{ and } 0 \leq h_{NS} \leq \frac{1}{2}.$$  

(54)

For the most part, these assumptions simply extend the parametric assumptions imposed in the earlier section to the case of three related agents. The key assumption is that $h_N < h_S$, i.e., the kinship between the founder and $S$ is greater than the kinship between the founder and $N$. An important special case of the model is when the two potential heirs are collaterally related, with only $S$ being a direct descendant of the founder. For example, assuming no inbreeding, and that $S$ is the son of the founder and $N$ is the founder’s nephew, then $h_S = \frac{1}{2}, h_N = \frac{1}{4}$, and $h_{NS} = \frac{1}{8}$. Another special case is when $S$ is the founder’s daughter and $N$ has been adopted into the family or is the founder’s son-in-law (or perhaps both as would be the case under Japanese Mukoyoshi adoption). In this case, $h_S = \frac{1}{2}, h_N = 0$, and $h_{NS} = 0$. The potential heirs have (perhaps) different reservation payoffs, $v^S_R$ for $S$ and $v^N_R$ for $N$. The upside payoff $\bar{x}$ in the earlier sections of the paper) the potential heirs can attain from managing the firm also varies and equals $x_S$ under $S$’s management and equals $x_N$ under $N$’s management.

Although the cash flows to income can be divided between the heirs through contracts, control of the firm is not divisible. Only one of the potential heirs can assume control. We call potential heir to whom the control of the firm is bequeathed the heir and call the potential heir to whom it is not bequeathed the non-heir. If some cash flows rights are bequeathed to the non-heir, through for example, a minority equity stake or a debt claim, we call the non-heir’s stake a noncontrolling stake. When the founder’s bequest does not specify a noncontrolling stake, we call the bequest simple. Thus, a bequest is an assignment of control rights to one of the potential heirs and (possibly) the assignment of a noncontrolling ownership stake to the other potential heir. After receiving the bequest, the heir decides on the firm’s employment and compensation policies. The founder’s bequest cannot bind the heir with respect to these choices. We will say that a bequest implements a given payoff profile $(v_N, v_S)$ if, under
the bequest, posthumous negotiations between the potential heirs produces payoffs \((v_N, v_S)\) to \(N\) and \(S\) respectively.

The negotiations between the potential heirs evolves as specified in the family-firm model developed earlier. The heir, being the owner of the firm and thus having control over the firm’s policies, chooses the firm’s employment and compensation policies. If the heir does manage the firm himself he become the “heir-manager.” If the bequest is simple and the heir manages the firm, the non-heir obtains his reservation payoff regardless of the management decisions make by the heir. Thus, kin altruism does not affect the heir-manager’s effort decision and the heir-manager, as sole owner, internalizes the entire effect of his effort decision. Thus, a heir-manager’s effort level is first-best, i.e., it maximizes total firm value net of effort costs. If the heir delegates management to the non-heir, we model the division of ownership and management between them mutatis mutandis using the family owner-manager model developed in the earlier sections of the paper. Thus, in general, in this case, the firm’s effort and monitoring policies will not be first best.

The tension in the founder’s bequest decision arises because expected net output at any given level of effort is highest when the firm is managed by the founder’s more distant relative, \(N\), i.e.,

\[ x_N - v_R^N > x_S - v_R^S. \] (55)

Inequality (55) implies that total payoffs to the family are highest if the founder makes a simple bequest to \(N\). In addition to putting the firm in the strongest hands, allocating all control and ownership to \(N\) eliminates the agency costs associated with the division of ownership and control. However, the founder values payoffs to \(N\) less than payoffs to \(S\). Even if, \(S\) inherits and hires the more efficient \(N\) to manage the firm, the agency costs associated with the separation of ownership and control will still lower total output relative to management, control, and ownership by \(N\). In fact, a simple bequest to \(S\) may or may not lead to higher total payoffs than management by \(N\) and ownership by \(S\) depending on whether the higher productivity of \(N\) can compensate for the increased agency costs associated with the separation of ownership and management.

The founder, when making her bequest, anticipates these effects and chooses her bequest that maximizes her utility. The founder’s utility is the relationship-weighted sum of \(N\) and \(S\)’s anticipated payoffs.
Thus, the founder chooses his bequest to maximize

\[ u_F = h_S v_S^c + h_N v_N^c, \]  

(56)

where \( v_S^c \) and \( v_N^c \) represent the anticipated payoffs to \( N \) and \( S \) resulting from the bequest. Next note that

the founder’s utility function is only unique up to increasing affine transformations. Thus, we can and will express the founder’s utility in the following equivalent form

\[ u_F = v_S^c + \frac{h_N}{h_S} v_N^c. \]  

(57)

As in the family firm model developed earlier. The potential heirs, \( S \) and \( N \) will each maximize their own inclusive utility. Thus, \( S \) chooses the policies that maximize

\[ u_S = v_S^c + h_{NS} v_N^c, \]  

(58)

and \( N \) chooses policies that maximize

\[ u_N = v_N^c + h_{NS} v_S^c. \]  

(59)

8.2 Founder benevolence

For all family members, family governance decisions trade off total family welfare against selfish gain. This is a bit more obvious if we rewrite the utility functions of the family members using expression (5). This yields

\[ u_S = h_{NS} v^c + (1 - h_{NS}) v_S^c, \]  

(60)

\[ u_N = h_{NS} v^c + (1 - h_{NS}) v_N^c. \]  

(61)

\[ u_F = \frac{h_N}{h_S} v^c + \left( 1 - \frac{h_N}{h_S} \right) v_S^c, \]  

(62)

where \( v^c = v_S^c + v_N^c \) represents the total anticipated payoff to both potential heirs. Thus, for the founder, the ratio \( h_N/h_S \) represents the degree to which the founder’s preferences are aligned with total value maximization while \( h_{NS} \) represents the potential heirs’ degree of alignment. This observation motivates the following definition.
Definition 1.

- If \( \frac{h_N}{h_S} > \frac{h_{NS}}{h_S} \) then we will term the founder’s preferences relatively benevolent;
- if \( \frac{h_N}{h_S} = \frac{h_{NS}}{h_S} \) then we will term the founder’s preferences aligned;
- if \( \frac{h_N}{h_S} < \frac{h_{NS}}{h_S} \) then we will term the founder’s preferences relatively discriminatory.

If the founder preferences are relatively benevolent, they tilt more toward total family value maximization as opposed to the maximization of the payoffs to any particular family member. Since all value in our model is captured by some family member, a founder’s tilt toward total family value maximization implies a tilt towards total value maximization. Thus, if the founder is relatively benevolent, the founder’s preferences are also more pro-social.

In some cases, founder and heir preferences are automatically aligned. For example, when \( N \) is not related to either or \( S \) or the founder. In which case, \( \frac{h_N}{h_S} = \frac{h_{NS}}{h_S} = 0 \). Thus, if \( S \) is the heir, the founder’s preferences are aligned with \( S \). This implies that if \( S \) inherits, \( S \) will make the same policy choices as the founder would have made had she been alive.

Lemma 1. If the less related potential heir, \( N \), is in fact not related to either the founder or the other potential heir, then the founder’s preferences aligned with \( S \)’s regarding posthumous policies. Thus, a simple bequest of the entire firm to \( S \) will always implement the founder’s preferred hiring and compensation policy.

Another case of alignment, although one of less practical significance, is where the founder is not inbreed, \( S \)’s spouse is not related to \( S \) and \( N \) is \( S \)’s child, i.e., \( N \) is \( S \)’s grandchild and \( S \) is her child. In this case, the alignment condition is satisfied because \( h_S = \frac{1}{2}, h_S = \frac{1}{4}, \) and \( h_{NS} = \frac{1}{2} \). However, because of age differences, children and grandchildren of the founder will rarely be competing for inheritance. Even if preferences are not aligned, for many specific policy choices, the founder’s preferences and \( S \)’s do not clash. For example, for any two polices that produce the same expected payoff to \( N \). Over such policies both the founder and \( S \) will prefer the policy that maximizes total value. Finally, using expression (4) we also see that, whenever the two polices produce the same utility to \( N \), the founder’s and \( S \)’s preferences are also the same, and, again, both will prefer the policy that maximizes total value. However, when \( S \) hires \( N \) it is often the case that total payoffs can be increased by increasing the rents earned by \( N \). In this case, unless the alignment condition is satisfied, the policy chosen by \( S \) will generally not coincide with the policy the founder would have chosen were she to have made the choice. In which direction will the
founder’s bias fall? The logic of kinship-based altruism provides us with a fairly definitive answer to this question—typical family structures imply relative founder benevolence.

**Proposition 7.** The following conditions are sufficient for the founder relative benevolence condition (given by Definition 1) to be satisfied:

i. The founder is not inbreed, $S$ is the son of the founder, $N$ is not a descendant of $S$, and the coefficient of relation between the founder’s spouse and $N$, represented by $h'_N$, is less then three times the coefficient or relation between the founder and $N$, $h_N$.

ii. The founder is not inbreed, $N$ is not a direct descendant of either the founder or founder’s spouse but is related to the founder, and the family tree is unilineal, i.e., all indirect lines of descent between collateral relatives pass through only one of each relatives’ parents.

In case ii, the founder’s benevolence exceeds $S$’s by a considerable margin because

\[
\frac{h_N}{h_S} \geq 4 h_{NS}. \tag{63}
\]

**Proof.** First consider condition i. Note that by Malécot’s formula (see Malécot (1948) or Chapter 5 of Lange (2002)),

\[
h_{NS} = \frac{1}{2} \left( h_{N} + h'_{N} \right). \tag{64}
\]

Because the founder is not inbreed and $S$ is her son, $h_S = \frac{1}{2}$. Using this fact and equation (64) we have that

\[
\frac{h_{NS}}{h_N} / h_S = \frac{1}{4} \left( 1 + \frac{h'_N}{h_N} \right), \tag{65}
\]

and the result follows.

To prove condition ii, first note that, by assumption $N$ is not a direct descendant of the founder or the founder’s spouse. Thus, all lines of decent connecting $N$ and $S$ are indirect. By the assumption that the family tree is unilineal and that the founder and $N$ are related, all indirect lines of decent connecting $S$ and $N$ pass through the founder. Thus, each of these lines of decent also connect the founder to $N$. Thus, for each path from $S$ to $N$, there exists a path from the founder to $N$, which is shorter by at least one arc. Thus, by Wright’s formula for the coefficient of relationship (Wright, 1922), we see that the contribution of a path from $S$ to $N$ to relatedness is at most half of the corresponding path from the founder to $N$. Therefore, the coefficient of relationship between $N$ and $S$, $h_{NS}$, which is the sum of
all the path contributions by the Wright formula, is at most one half of the coefficient of relationship between the founder and \( N \), \( h_N \), i.e., \( h_{NS} \leq h_N/2 \). Because, the founder is not inbreed, \( h_S \leq 1/2 \). Thus, \( h_N/h_S \geq 2h_N \). The result follows. \( \square \)

As Proposition 7 demonstrates, the conditions for founder benevolence will be satisfied by “typical” family pedigrees. In such pedigrees, collateral relatives of the founder are only indirectly connected to each other through a common ancestor of the founder. In this case, the paths connecting the founder to \( N \) are shorter than the paths connecting \( N \) and \( S \) to each other. The geometric decline of the level of kin altruism implied by the inclusive fitness model, leads to the founder having relatively more benevolent disposition toward \( N \) than \( S \). Hence, the founder, although biased toward his closer relative \( S \) is more willing to sacrifice \( S \)’s welfare for the sake of overall value creation than \( S \) himself. As shown by the more general condition i, kinship relationships on the founder spouse’s side between \( N \) and \( S \) must be very strong to reverse founder benevolence. For example, assuming no inbreeding and that \( S \) is the son of the founder, even if \( N \) and \( S \) are double cousins, and thus the unilinearity constraint of condition ii of Proposition 7 is violated, we have \( h_S = 1/2 \), \( h_N = h_{NS} = 1/4 \). In which case, the founder benevolence condition is still satisfied as \( h_N/h_S = 2h_{NS} > h_{NS} \). However, it is possible for kinship links between the founder’s spouse and \( N \) to reverse the relative benevolence condition. An example is provided below in Figure 11.

![Figure 11: Counterexample: Founder’s preferences are more discriminatory than S’s.](image)

In the figure, ancestors who are source nodes are assumed to be unrelated and not inbred. Directed arcs connecting nodes indicate that the source node is the parent of the sink node. In the graph, \( S \) and \( N \) are half siblings with common mother \( M \). \( F \), the founder is the half-uncle of \( N \). \( N \) and \( S \) share two common ancestors, \( M \), and \( F \)’s grandmother, \( GM \). The founder and \( N \) have only one common ancestor, \( GM \). Thus, the relationship coefficient \( h \) between the two collateral potential heirs \( S \) and \( N \), \( h_{NS} = 5/16 \), while the relationship coefficient between the founder and \( N \) is only \( h_N = 1/8 \). Hence, \( h_N/h_S = 1/4 < h_{NS} = 5/16 \). Hence, the founder’s preferences are relatively discriminatory.
Because founder relative benevolence seems to be the far more typical state of affairs, we will focus on this case in the subsequent analysis. Even if the founder is relatively benevolent, and thus has policy preferences which conflict with those of the potential heirs, a simple bequest may still be sufficient to ensure that the founder’s preferred policy is followed posthumously. For example, if the relationship between \( N \)’s rents and firm value is linear under delegated management, then the founder will either prefer to choose the compensation policy that minimizes \( N \)’s rents or the policy that maximize total firm value. However, total firm value is maximized by eliminating the agency problem entirely by setting \( N \)’s compensation so that \( N \) captures the full value of the project. However, this policy produces identical payoffs to founder and the potential heirs as the policy of handing ownership and control to \( N \). If the founder prefers to the policy that minimize \( N \) rents at the expense of lowering total value then, by the assumption of relative benevolence, so will \( S \). Thus, as long as \( S \) prefers delegated management, bequeathing the firm to \( S \) will lead to the implementation of the founder’s preferred policy.

Thus, when there is a linear relation between \( N \)’s rents and total firm value, the only case in which a simple bequest will not implement the founder’s preferred policy is the case where the founder prefers delegated management but \( S \), if bequeathed the firm, does not hire \( N \) and instead manages the firm himself. However, as long as compensation under delegated management provides no rents to \( N \) then we see that \( S \)’s and the founder’s preferences regarding delegation relative to sole management by \( S \) will be the same. In the dark-side scenario of section 6, total firm value is, in fact, linear in the manager’s reservation payoff. Moreover, increasing the manager’s reservation payoff is equivalent to leaving reservation payoff fixed and increasing the manager’s rent. Also in the dark side scenario, equilibrium compensation always leaves the manager on his reservation constraint provided that compensation meeting the reservation constraint satisfies limited liability. Thus, we have established the following sufficient conditions for the optimality of simple bequests.

**Lemma 2.** As long as \( N \) reservation payoff is sufficiently large so that \( N \)-reservation constraint cannot be satisfied at 0 compensation, in the dark-side scenario where relatedness does not increase firm value, founder-preferred policies can always be attained through simple bequests of the entire firm to one of the two potential heirs.
8.3 Implementation: an extended example

8.3.1 Unrestricted bequests

When family altruism generates value, the conclusion of Lemma 2 no longer holds. As we will show, divided bequests can increase total value at the expense of lowering $S$ payoff. Because the founder’s preferences are relatively benevolent, the founder will often prefer such divided bequests. To develop these results, a monitoring problem is not required. Thus, to minimize the algebraic burden, we develop the analysis in simplest possible setting—the bright-side scenario: $c = 0$, the reservation payoff of the potential heirs is 0, and the relation between managerial effort and output is as specified in equation (20).

We also assume that $x_N < k$. This assumption ensures that the optimal effort levels is interior and limits the number of cases we have to consider. Otherwise, it has no material effect on the result. It is clear from simple continuity arguments that all of our qualitative results can be obtained with positive reservation payoffs and positive monitoring costs so long as the effort-incentive rather than the reservation constraint is binding. In order to distinguish between payoffs and utilities under the different potential control and management configurations for firm, we represent the payoffs and utilities as follows: when $S$ controls the firm and hires $N$ to manage, we superscript with $n$; when $S$ controls and manages the firm, we superscript with $S$; when $N$ controls and manages, we superscript with $N$.

First, consider the founder’s ability to implement preferred allocations though a control bequest to $N$ combined with a minority ownership stake bequested to $S$. Because $N$ manages the firm, firm cash flows equal either 0 or $x_N$. Any limited liability claim on the firm will pay 0 when the cash flow is 0 and some amount less than or equal to $x_N$ when the cash flow equals $x_N$. Thus, the we can specify the noncontrolling stake by simply by stating the cash flow that stake in the event that the firm’s cash flow equals $x_N$. We represent this cash flow with $o$. The stake could take different legal forms: e.g., preferred stock, dual class stock, debt, or, if the promised cash flow was sufficiently small, of a minority equity stake. Perhaps a noncontrolling stake could also be implemented by contracts requiring the firm to purchase inputs from a company controlled by $S$.

Next, consider the relation between $N$’s effort, parameterized by the uptick probability $p$, and the size of the minority stake $o$. Specializing equation (59) to the parametric assumptions in this section, we see that the utility of $N$ given an noncontrolling stake of $o$ held by $S$ and an uptick probability of $p$ is given by

$$p \left( (x_N - o) + h_{NS}o - \frac{kp^2}{2} \right) - (1 - p) \frac{kp^2}{2}. \quad (66)$$
The first-order condition for effort ensures that all effort incentive-compatible combinations of $p$ and $o$ are related by

$$o = \frac{x_N - kp}{1 - h_{NS}}.$$  \hspace{1cm} (67)

If we substitute (67) into equation (66) we obtain the utility of $S$ as a function of the uptick probability $p$:

$$u_N^S(p) = \frac{1}{2}p (2(1+h_{NS})x_N - (2+h_{NS})kp).$$  \hspace{1cm} (68)

Thus, this concave function of $p$ attains its maximum at

$$p_N^S = \frac{(1+h_{NS})x_N}{(2+h_{NS})k},$$  \hspace{1cm} (69)

which, using expression (67), implies a noncontrolling stake of

$$o_N^S = \frac{x_N}{2 - h_{NS}(1+h_{NS})}.$$  \hspace{1cm} (70)

Next, consider the founder’s preferred choice of $p$. This is determined using in exactly the same way as $S$’s. Thus, the founder’s utility under $N$ control and management and the founder’s choice of optimal uptick probability and noncontrolling stake, $p_F^N$ and $o_F^N$ are given as follows:

$$v^N(p) = \left( px_N - \frac{1}{2}kp^2 \right),$$  \hspace{1cm} (71)

$$u_F^N(p) = (h_N - h_Sh_{NS}) v^N(p) + (h_S - h_N) u_S^N,$$  \hspace{1cm} (72)

$$p_F^N = p_N^S \left( 1 + \frac{h_N - h_{NS}h_S}{(1 + h_{NS})(h_S - h_N) + (h_S - h_Nh_{NS})} \right),$$  \hspace{1cm} (73)

$$o_F^N = \frac{x_N - kp_F^N}{1 - h_{NS}}.$$  \hspace{1cm} (74)

Relative benevolence implies that the founder’s preferred incentive-compatible uptick probability is higher than $S$’s, and thus the noncontrolling stake which the founder bequests to $S$ is less than $S$ preferred stake, i.e., $p_F^N > p_N^S$ and $o_F^N < o_N^S$. Because $o_F^N < o_N^S$, a bequest of control to $N$ combined with a noncontrolling stake of $o_F^N$ to $S$, will not be renegotiated by $N$ and $S$. To see this, note that any reduction in the stake would lower the utility of $S$ and any increase in the stake would lower the utility of $N$. Since the total payoff is lower under $S$ management, $N$ and $S$ will not negotiate a transfer of management to $S$. Thus, by bequeathing control to $N$ and providing $S$ with a minority stake, the founder can implement
any policy involving \( N \) management. Amongst all policies involving \( S \) management, the founder always prefers a simple bequest to \( S \). This follows because, conditioned on \( S \) managing the firm, increasing \( S \) ownership share always increases both total value and the payoff to \( S \). Thus, increasing \( S \)'s ownership always increases the founder's welfare. Since the firm will choose the first best level of effort when \( S \) is the sole owner and manager, and thus set \( p = x_S/k \), the total value of the firm when \( S \) is the sole manager and owner \( v^S \), and the payoffs to \( S \) and \( N \) are given by

\[
v^S = \frac{x_S^2}{2k}, \quad v^S_S = \frac{x_S^2}{2k}, \quad v^S_N = 0. \tag{75}
\]

We claim that whenever the founder’s preferred policy is \( S \) ownership and management, \( S \) will in fact not choose to hire \( N \) to manage the firm. For \( S \) to prefer hiring \( N \) to managing the firm himself subsequent to a simple bequest, it must be the case that \( u^S_F \geq u^N_F(p^N_S) \); using the utility functions provided by equation (3), we see that this inequality is equivalent to

\[
v^S_S > v^N(p^N_S) - (1 - h_N) v^N_N(p^N_S). \tag{76}
\]

By relative founder benevolence, this implies that

\[
v^S_S > v^N(p^N_S) - (1 - h_{NS}) v^N_N(p^N_S). \tag{77}
\]

Which implies that \( u^S_F \geq u^N_F(p^N_S) \). Hence, whenever the founder’s optimal policy is for is a simple bequest to \( S \), \( S \) will manage the firm himself. Combining all the cases considered thus far, we note that using a combination of control bequests and noncontrolling ownership bequests to \( N \) the founder can always implement any allocation of value between \( S \) and \( N \) that satisfies effort incentive compatibility.

**Proposition 8.** When the founder is free to divide control and cash flow rights in any way the founder sees fit, and the benevolence condition is satisfied, then the founder, perhaps by combining the allocation of control with an allocation of a noncontrolling interest to the closer relative, \( S \), can implement posthumously her preferred incentive compatible allocation.

The choice between a simple bequest to \( S \) and bequeathing control to \( N \) and a minority stake to \( S \) is a simple tradeoff between the gain in total value from placing control and management in the hands of \( N \) versus the increase in \( S \)'s share of the total value induced by handing the firm to \( S \). In Figure 12 these
Figure 12: Optimal bequests in the absence of bequest restrictions. In the figure, the green region labeled $S$ represents the region of the parameter space where the founder makes a simple bequest of the firm to $S$. The blue region labeled $N - nc$ represents the region where the founder bequests a noncontrolling stake to $S$ and control to $N$. The horizontal axis represents the relative ability of $S$ compared with $N$ and the vertical axis represents their relative degree of relatedness to founder. In the figure, $h_S/h_N = 4 h_{NS}$, which corresponds to a unilineal relation between $N$ and $S$.

tradeoffs are illustrated for the $h_S/h_N = 4 h_{NS}$ case which corresponds to a unilineal relation between $N$ and $S$.

As Figure 12 shows, absent bequest restrictions, founder frequently eschews bequests to less competent close relations in favor of control bequests to more competent but more distant relations. The interests of the close relative are protected by noncontrolling stakes in the firm. Only when the less competent but closer relative’s ability is fairly close to the distant relatives and/or the genetic distance between the founder and the distant relative is very large, will the founder make a value-destroying bequest to the closer relative. However, the noncontrolling stake provided the close relative itself, by attenuating the controlling relatives effort incentives, will reduce output below first-best levels.

8.3.2 Primogeniture restrictions

As we have seen, when founder bequests are unrestricted, founders can implement their preferred outcomes through either a simple bequest to $S$ or by bequeathing control to $N$ and a noncontrolling share to $S$. However, in practice, bequests are restricted in most legal jurisdictions. These restrictions limit the ability of founders to prefer distant over close relatives.\footnote{For a complete discussion of the restrictions on inheritance see Ellul, Pagano, and Panunzi (2010)} Moreover, even when bequests are unre-
stricted, rules regulating security design, e.g., laws that prevent the separation of cash flow and control rights might block some bequests. It is not possible to model all possible bequest restrictions. So, we will consider a simple, and rather classic restriction—primogeniture. Under primogeniture, control of the family business must pass to the closest relative, \( S \), a noncontrolling stake can be passed to \( N \). \( S \) will then exercise his control rights by either hiring \( N \) to manage the firm or by managing the firm himself. Our formulation of inheritance restrictions as primogeniture restrictions imposes a fairly weak restriction on bequests. It simply requires that control of family business be handed down to the closer relative without restricting the ability of the founder to pass cash flow rights to more distant relatives. Thus, if we can identify welfare costs from our primogeniture restriction, stronger inheritance restrictions, which might, for example, also restrict cash flow transfers to distant relatives, will also entail welfare costs.

In this framework, the founder’s flexibility is limited to choosing the noncontrolling stake to assign to \( N \). The key effect of bequeathing noncontrolling ownership to \( N \) has is on \( N \) is through its effect on \( N \)’s compensation as determined by the posthumous negotiations between \( S \) and \( N \). \( N \)’s compensation is affected by \( N \)’s noncontrolling stake because this stake affects the payoff to \( N \) if \( N \) rejects an employment offer made by \( S \). After \( N \)’s rejection, \( S \) will manage the firm with \( N \) receiving a payment proportional to his noncontrolling ownership stake. As in the previous section, we can specify this stake simply as the payment, \( o \), received by the noncontrolling owner when the firm’s cash flow equals \( x_S \).

Noncontrolling bequests to \( N \) are aimed only at increasing bargaining power of \( N \) in negotiations with \( S \). The founder will only make such bequests when the founder anticipates that \( S \) will hire \( N \) to manage the firm. The follows because a noncontrolling state held by \( N \) when \( S \) manages the firm lowers firm value through agency conflicts caused by divided ownership. Because both total firm value and \( S \)’s payoff will be lower when \( S \) manages and \( N \) holds a noncontrolling state than they would be if \( S \) were the sole owner, the founder’s bias toward \( S \) ensures that the founder prefers the a simple bequest to \( S \) over \( S \) management and divided ownership.

Because of the founder’s relative benevolence and because greater managerial bargaining power leads to greater total firm value, granting \( N \) a noncontrolling stake to use as a bargaining chip in employment negotiations may increase the founder’s utility. However, as we will show, bequests of noncontrolling stakes to \( N \) are a much less effective mechanism for implementing founder-preferred allocations than direct control allocations to \( N \). The problem with noncontrolling stakes is that they affect payoffs only through their ability to increase \( N \)’s’ reservation demands. However, \( N \)’s reservation demands
depend on the payoff that \( N \) would have attained as a noncontrolling owner under \( S \) control. When \( S \) is sufficiently incompetent, \( N \)'s reservation value from a noncontrolling stake is too low to affect employment negotiations. In this case, a noncontrolling stake cannot implement founder preferences and thus, primogeniture restrictions, by frustrating the founder’s benevolent dispositional preferences, cause significant welfare loses.

To initiate the analysis, first consider the reservation payoffs to \( S \) and \( N \). These payoffs are realized if \( N \) rejects a compensation proposal of \( S \) and thus \( S \) manages the firm. Given a noncontrolling stake of \( o \) for \( N \), if \( S \) manages the firm himself, he will receive \( x_S - o \) with probability \( p \) and 0 with probability \( 1 - p \). Following the development in Section 8.1, \( S \) will pick \( p \) to maximize the kinship weighted average of his payoff and \( N \)'s payoff. This yields an optimal effort level for \( S \) conditioned on the noncontrolling stake \( o \) defined as follows.

\[
P_S^o(o) = \frac{x_S - (1 - h_{NS})o}{k}.
\]

The payoffs to the firm under \( S \) management determine the utility to \( N \) from rejecting \( S \)'s employment offer. The definition of \( N \)'s utility given by equation (60) and \( S \)'s optimal effort choice, given by (78), yields the the utility to \( N \) from rejecting an employment offer from \( S \), \( u^\text{Rej.}_N \), which is defined below:

\[
u^\text{Rej.}_N(o) = o \cdot P_S^o(o) + h_{NS} \left( P^o_S(o)(x_S - o) - \frac{1}{2}kP^o_S(o)^2 \right) = \frac{(x_S - (1 - h_{NS})o)((2 - h_{NS} - h_{NS}^2)o + h_{NS}x_S)}{2k}.
\]

The utility of \( S \) given that \( S \) picks effort level \( P^o_S(o) \) is given by

\[
u^\text{Rej.}_S(o) = \frac{(x_S - (1 - h_{NS})o)^2}{2k}.
\]

\( u^\text{Rej.}_N \) is maximized at

\[
o^+ = \frac{x_S}{(1 - h_{NS})(2 + h_{NS})}.
\]

Raising \( o \) above \( o^+ \) lowers the reservation demands of \( N \) and thus cannot strengthen \( N \)'s bargaining position. Thus, when searching for an optimal level of noncontrolling ownership, we can restrict attention to \( o \in [0, o^+] \).

Now consider the effort decision of \( N \), if \( N \) agrees to accept a payment of \( w \) in exchange for surren-
dering his noncontrolling ownership stake in the firm to $S$, then $N$’s utility will be given as follows:\textsuperscript{13}

$$ p \left( (\omega + h_{NS}(x_N - w)) - \frac{1}{2} kp^2 \right) + (1 - p) \left( -\frac{1}{2} kp^2 \right). \quad (82) $$

Differentiating expression (82) with respect to $p$ shows that the first-order condition for an optimal choice of $p$ can only be satisfied if

$$ w = \frac{kp - h_{NS}x_N}{1 - h_{NS}}. \quad (83) $$

The utility of the owner, $S$, is given by

$$ p (x_N - w) + h_{NS} \left( pw - \frac{1}{2} kp^2 \right). \quad (84) $$

Substituting the effort incentive compatibility function (83) into expressions (84) and (84), yields the utility and payoffs of the owner and manager, conditional on $S$ hiring $N$ to manage the firm. We represent the utility and payoffs of $S$ under $N$’s management with $u^S_n(p)$, and $v^S_n(p)$ respectively and represent the utility and payoffs of $N$ under $N$ management with $u^N_n(p)$ and $v^N_n(p)$ respectively.

$$ u^S_n(p) = (2 + h_{NS}) p \left( \frac{1 + h_{NS}}{2 + h_{NS}} x_N - \frac{1}{2} kp \right), \quad (85) $$

$$ v^S_n(p) = \frac{p(x_N - pk)}{1 - h_{NS}}, \quad (86) $$

$$ u^N_n(p) = \frac{kp^2}{2}, \quad (87) $$

$$ v^N_n(p) = \frac{p((1 + h_{NS})kp - 2h_{NS}x_N)}{2(1 - h_{NS})}. \quad (88) $$

If $S$ is bequeathed control, and $S$ decides to attempt to hire $N$, $S$ makes a first-and-final offer to $N$. Because of the monotone relation between compensation and $p$, we can think of this offer as being a proposed uptick probability which fixes a compensation level rather than a compensation level that fixes an uptick probability. Thus, we can express $S$’s compensation determination problem as being

$$ \max_{p \in [0,1]} u^S_n(p) \quad \text{s.t. } u^S_n(p) \geq u^N_{Rej}(o). \quad (89) $$

\textsuperscript{13}Of course, it would, in practice, be more usual for $N$ to receive a payment, say $a$, in addition to his ownership stake for managing the firm. However, a dollar from compensation has exactly the same incentive effect as a dollar from noncontrolling ownership. Thus, to simply to simplify exposition and make as obvious as possible the parallel between our analysis in this section and the analysis in the earlier sections of the paper, we represent the total payment to $N$ as $w$ rather than $a + o$. 

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Now consider the unconstrained solution to problem (89). Simple calculus shows that the unconstrained solution chosen by $S$ is given by

$$p^u_S = \frac{(1 + h_{NS})x_N}{(2 + h_{NS})k}. \quad (90)$$

The first-best level of effort, which maximizes the total value is given by

$$p^f_{fb} = \frac{x_N}{k}. \quad (91)$$

By the assumption that the founder’s preferences are relatively benevolent, the founder has no incentive to implement a probability less than $p^u_S$. This follows because implementing a probability above the first best would lower total payoffs as well as lower $S$’s payoff relative to first best. Thus, we restrict attention to $p \in [p^u_S, p^f_{fb}]$. If the constraint in problem (89) binds, i.e. $u^f_N(p^u_S) < u^\text{Rej.}_N(o)$, then because $N$’s utility is strictly increasing in $p$ and $S$’s utility is strictly decreasing in $p > p^u_S$, $S$, conditioned on hiring $N$, will increase $p$ to level where constraint is exactly satisfied. If, at this $p$, $S$ is weakly better off hiring $S$ rather than managing the firm himself, then, given $o$, hiring $N$ is incentive compatible. Thus, we will say that $o > 0$ implements probability $p_o$ if (a) $u^f_N(p_o) = u^\text{Rej.}_N(o)$ and $u^u_S(p_o) \geq u^\text{Rej.}_S(o)$. Next note that by equation (4) (a) and (b) imply the following equivalent conditions for implementation

$$u^f_N(p_o) = u^\text{Rej.}_N(o), \quad (92)$$

$$v^u_N(p_o) \geq v^\text{Rej.}_N(o). \quad (93)$$

The reservation-demand increase generated by a noncontrolling stake depends on the payoff to $N$ from rejecting $S$’s offer. This payoff depends on the value of the firm under $S$’s control and management. Only if $S$ is sufficiently competent will $N$ reservation demands be increased by a noncontrolling stake sufficiently to affect $N$’s negotiated compensation.

**Lemma 3.** It is possible for the founder to implement a probability in excess of $S$’s optimal choice if and only if value under $S$’s management is sufficiently large, i.e.,

$$\frac{x_S}{x_N} > \frac{1}{\sqrt{2 + h_{NS}}}. \quad (94)$$

The necessity of condition (94) is illustrated in Figure 13. In the figure, $S$ represents the curve in $u_N$–$v$ space traced out by varying the noncontrolling ownership stake of $N$ from 0 to $o^+$ under the
Figure 13: When $S$’s ability is so low that a noncontrolling stake held by $N$ has no effect on posthumous firm policies.

assumption that the firm is managed by $S$. We see that increasing $N$’s noncontrolling ownership stake increases $N$’s utility conditioned on rejecting $S$’s employment offer and thus increases his reservation demands. $N$ represents the curve in $u_N-v$ space traced out by varying $p$ from $S$’s unconstrained choice, $p_n^s$, to the first-best choice, $p_{fb}^n$, under the assumption that $S$ delegates management to $N$. As $p$ increases, both the utility of $S$ and the total payoff increase. Implementation requires that the noncontrolling stake affects either the compensation or delegation policy of $S$ when $S$ is bequeathed controlling ownership. However, as Figure 13 shows, even the highest reservation payoff for $N$ which noncontrolling ownership can induce, given by $u_N^n$, is less than the payoff produced by $S$’s unconstrained compensation choice, $p_n^s$. In this case, the output of the firm under $S$’s management is so low that the payoff to of $N$ from being a noncontrolling owner with $S$ managing the firm is so low that the threat to opt for this payoff does not improve $N$’s bargaining position.

At higher levels of $S$’s relative ability, $N$’s noncontrolling stake will improve $N$’s bargaining position. This improvement will lead to higher compensation for $N$ and thus a higher uptick probability and increased total family welfare.

The sufficiency of (94) for implementation is illustrated in Figure 14. In this figure, $N$’s reservation
payoff at his optimal minority ownership stake under $S$’s management, $o = o^+$ exceeds $N$’s payoff from $S$’s unconstrained choice of $p$ given $N$’s management. Thus, if the founder bequeaths ownership stake $o^+$ to $N$, $S$ will only be able to hire $N$ if he chooses $p \geq p^+$. Given that $p^+$ exceeds $S$’s unconstrained optimal choice of $p$, $S$ will set $p = p^+ > p^S_N$ when he chooses to hire $N$. $S$ in fact will choose to hire $N$ because given minority stake $o^+$, if $S$ manages the firm himself, $N$’s utility will be the same as it would have been had he been hired by $S$ and because the $N$-curve is below the $S$-curve, total value would be lower. Total value being lower and $N$’s utility being the same implies that $S$’s utility is lower when he manages rather than delegates management to $N$. We can see from Figure 14, that the founder can implement all $p$ between $p_S^n$ and $p^+$. This characteristic of Figure 14 is generally true provided that $S$’s ability, measured by $x_S$, is not too great. This result is recorded below.

**Proposition 9.** If

$$\frac{1}{\sqrt{2 + h_{NS}}} < \frac{x_S}{x_N} \leq \sqrt{\frac{3 + 4h_{NS} + h_{NS}^2}{(2 + h_{NS})^2}},$$

(95)

then $p$ can be implemented by some noncontrolling interest $o$ if and only if $p \in [p_S^n, p^+]$, where $p^+$, given
is the solution to $u^N_N(p) = u^\text{Rej}_N(o^+)$. 

Figure 15: Using a noncontrolling stake to enforce management by $N$ when $S$ ability is close to $N$’s

When $S$’s ability is very close to $N$’s, in the absence of a minority stake, $S$ will prefer managing the firm himself rather than delegating management to $N$. In this case, bequeathing a noncontrolling stake to $N$, by lowering $S$’s utility from managing the firm himself can be used to induce $S$ to hire $N$. This case is illustrated in Figure 15. Note that at $S$’s preferred choice of $p$, $p_S^*$, the total value of the firm is higher under $S$ management than under delegated management. However, as $p$ increases the value of the firm under delegated management increases. At $p = p^+$—the value of $p$ which exactly satisfies $N$’s reservation constraint when $o = o^+$—total value is higher under delegated management. This implies, because $N$’s utility under delegated and $S$-management is the same, that $S$’s utility is higher under delegated management. As is apparent from Figure 14, values of $p$ less than $p^+$ can also be implemented through noncontrolling stakes. However, those values of $p$ that map into points on the $N$-curve that lie below the $S$-curve cannot be implemented because at these values of $p$, $S$ prefers managing the firm himself to delegation. This observation is generalized in Proposition 10.

**Proposition 10.** If

$$\sqrt{\frac{(3 + 4h_{NS} + h_{NS}^2)}{(2 + h_{NS})^2}} < \frac{x_S}{x_N} < 1,$$  

(97)
there exist parameterizations of the model under which, without implementation through a noncontrolling interest, it is not possible to attain any policy under which S hires N, under these parameterizations, only a proper closed upper interval of \([p^0_S, p^+]\) can be implemented.

The above results characterize the conditions under which the founder can implement delegated management and/or affect the terms of employment when management is delegated through minority ownership bequests. When will the founder prefer the delegation over a simple bequest? To answer this question, we first consider the founder’s optimal choice with noncontrolling bequest option and then identify the conditions for a noncontrolling bequest to be strictly optimal. To do this, first suppose that founder could implement any choice of \(p \in [p^0_S, p^+]\). As stated in Section 8.1 the founder will maximize the weighed sum of the two heir’s payoffs, with the weights given by the coefficient of relationship. Using equations (86) and (88), and solving this optimization problem yields,

\[ p^F_n = \frac{(h_S - h_N h_{NS}) x_N}{((h_S - h_N h_{NS}) + (h_S - h_N)) k}. \]  

The founder’s objective function is concave. Thus, the optimal choice of \(p\) subject to the constraint that \(p \in [p^0_S, p^+]\) is given by

\[ p^c_F = \text{mid}[p^0_S, p^F_n, p^+]. \]  

where \(\text{mid}(\cdot)\) is the median operator which selects the median of the three probabilities, \(p^0_S, p^F_n,\) and \(p^+\).

Next note that if \(p \in [p^0_S, p^+]\) and \(p\) is not implementable, then \(p \neq p^c_F\). This follows because those \(p \in [p^0_S, p^+]\) which are not implementable are not implementable only because, under the candidate implementing minority ownership stake for \(N\), \(S\) prefers not to hire \(N\). This implies that \(S\) management, even given the noncontrolling state \(o\) held by \(N\), yields both a higher total payoff and a higher utility to \(S\) than hiring \(N\) at \(N\)’s reservation payoff. However, bequeathing the firm to \(S\) must always yield even greater utility to \(S\) and higher total value than bequeathing the firm to \(S\) with a noncontrolling stake granted to \(N\). Thus, delegation outcomes producing a non-implementable \(p \in [p^0_S, p^+]\), produce lower founder utility than a simple bequest to \(S\) and thus are not an optimal choice for the founder. For this reason, the optimal policy choice for the founder will be the same in the relaxed problem, where the founder choses over all \(p \in [p^0_S, p^+]\) as it is when the founder chooses only over implementable allocations. This argument establishes the following result.

**Proposition 11.** The option of bequeathing a minority ownership stake to the more distant relative, \(N\),
will increase the welfare of the founder if and only if the optimal uptick probability, $p_{ncF}$, in the relaxed delegation problem defined by equation (99) satisfies the following conditions.

$$p_{ncF} > p_{nS}^c \text{ and } u_n^F(p_{ncF}) > \max[u_n^S, u_n^N].$$  

(100)

This proposition is illustrated in Figure 16. In the region of the graph labeled $S$, the founder bequests the firm to $S$ and $S$ manages the firm. In this region, the ability differential between $S$ and $N$ is small. Primogeniture restrictions have little effect because the ability differential between $N$ and $S$ is less than the founder's bias towards $S$. In the region labeled $n - ne$, the founder uses a bequest of a noncontrolling stake to $N$ to increase $N$'s bargaining power in hiring negotiations. This bequest both encourages $S$ to hire $N$ to manage the firm and forces $S$ to pay $N$ higher compensation than $S$ would otherwise choose. Hence, the noncontrolling bequest increases total value. This region is characterized by moderate differences in the ability of $N$ and $S$. In the region labeled $n$, $S$'s ability is much lower than $N$'s. Thus, $N$'s reservation compensation demand, fixed by the value of $N$'s stake under $S$ management, is not increased by a noncontrolling stake. In this case, a simple bequest of the firm to $S$ is weakly optimal under the primogeniture restriction. After the simple bequest, $S$ will hire $N$ using the unconstrained $S$-preferred optimal contract. As can be seen by comparing this figure to Figure 12, absent primogeniture constraints, the founder would have bequeathed control to $N$ and granted $S$ a noncontrolling stake in the firm. The unrestricted bequest would have produced to higher total value as well as higher founder utility. Thus, even if inheritance restrictions favoring close relatives allow unlimited allocations of noncontrolling stakes to more distant relatives and close relatives have the option of hiring the more competent distant relatives, when close relatives are sufficiently incompetent, primogeniture restrictions will impose welfare costs and lower the value of family businesses.

9 Directions for future research

This paper is a first attempt to integrate intra-family altruism into the corporate finance paradigm. Since corporate finance theory is a very rich and well developed field of intellectual inquiry, the scope of a single research paper cannot hope to extract all, or even all of the important implications, of kin altruism. In fact, even listing all of the interesting directions in which this analysis might be extended is not an easy task. However, I will try to map out a few paths for future exploration.
Figure 16: *Optimal bequests under the primogeniture constraint.* In the figure, the green region labeled $S$ represents the region of the parameter space where the founder makes a simple bequest of the firm to $S$ and $S$ manages the firm. The dark blue region labeled $n - nc$ represents the region where the founder bequests a noncontrolling stake to $N$, control to $S$ and $S$ delegates management to $N$. The light blue region labeled $n$ represents the region where the founder makes simple bequest to $S$ and $S$ delegates management to $N$. The horizontal axis represents the relative ability of $S$ compared with $N$ and the vertical axis represents their relative degree of relatedness to founder. In the figure, $h_S/h_N = 4h_{NS}$, which corresponds to a unilineal relation between $N$ and $S$.

The extension that is closest to the current model development but probably not the most interesting is relaxing the assumption that owners have all bargaining power in compensation negotiations. To understand the effect of reallocating bargaining power, first consider the effect of a complete reversal of the baseline model assumptions, i.e., grant all bargaining power to the manager. In this case, because the skills required to run the firm are manager specific, the owner’s reservation payoff is 0. Thus, if all bargaining power were to be assigned to the manager, the manager would be able to capture the entire payoff from the project. The family manager would in essence become the family owner/manager and family ownership would be resolved into a sole proprietorship. Thus, just as in the standard principal–agent model with unrelated agents, reversing the bargaining power would eliminate the agency problem and lead to the first-best solution. A non-trivial division of bargaining power that modeled bargaining using a standard efficient bargaining model (e.g., Nash bargaining solution) between the owner and manager would lead to a solution intermediate between the solution in the baseline model and the first-best solution, with the solution approximating the baseline model when the manager’s bargaining power was small and approximating first-best solution when the manager’s bargaining power was large. Since this
is same result which standard non-kinship principal–agent models yield, from the perspective of theory, it is not a very exciting extension. However, such an extension would produce a perhaps important, but not very surprising empirical implication—increasing managerial bargaining power reduces the effect of kinship on value. However, as long as the manager has insufficient bargaining power to capture all of the firm’s cash flows, a manager–owner monitoring problem would remain. Moreover, this problem would still be affected by kinship.

This paper has deliberately focused on self-contained family firms in which both capital and labor are provided by family members. This seems like a logical first step in the analysis of the effect of kinship on governance and value. Focusing on the self-contained family firm has the advantage of isolating the problems of family firms qua family firms from the more general governance questions surrounding small capital-constrained firms administered by managers with firm-specific human capital. However, the analysis of the self-contained family firm is just a first step in addressing the effects of kinship on economic activity. Most large family firms, especially in more developed economies, use a mix of family and non-family labor and capital. Thus, the next step this analysis should be to analyze the effect of kinship on the use of external labor and capital markets.

The most obvious motivation for accessing external capital is capital constraints. In this paper, the family firm’s investment in the project is made by the family and thus the project is fully owned by the family. If the family firm were capital constrained, it would have to resort to external capital to finance investment. If this capital was passive and thus the family owner retained the controlling interest, then external capital would both (a) reduce the manager’s effort incentives and (b) the owner’s monitoring incentives. Effect (a) is specific to family firms while effect (b) is a general agency cost of external finance. Hence, external finance would be even more costly for family firms than other capital-constrained firms. This wedge between the cost of inside and outside capital would lead to even more severe underinvestment incentives for family firms than those faced by non-family capital-constrained firms.

In contrast to passive external capital, active external capital capable of monitoring management, e.g., private equity capital, might be employed by family firms even in the absence of capital constraints. When monitoring is costly kinship can increase expected monitoring costs. Thus, delegating monitoring to an external (to the family) agent may increase value by increasing monitoring credibility. Delegation requires that outsiders be given sufficient ownership stakes to monitor and this increase in external own-
ership has two effects on managerial incentives. (i) Outside ownership attenuates the family manager’s incentive to provide effort and (ii) it reduces the potency of the loyalty holdup because part of the loss caused by the failure to operate is absorbed by extra family parties. (i) causes an efficiency loss but (ii) simply transfers value to the owner. In the dark-side scenario, effect (i) is turned off. Thus, the potential value gains from injections of active external capital are greatest in this setting. In this case, family owners would have an incentive to sell family firms to active external financiers in order to capitalize the increase in total value accruing from eliminating kinship’s adverse effect on monitoring.

Access to external labor markets would also affect the governance of family firms. In this paper, the family firm’s project requires firm-specific human capital only possessed by family members. If general human capital provided by professional managers could substitute for firm-specific human capital, family owners would need to decide whether to operate the firm with kin-managers or outside professional managers. Even in cases where family firms did not hire professional managers, the option to hire a professional manager would reduce the loss to the family from the family manager rejecting an employment offer from the family owner. This effect would reduce the power of the loyalty holdup and thus increase the bargaining power of the family manager. As discussed above, increasing the bargaining power of family manager increases efficiency. Thus, the presence a pool of professional managers would increase the efficiency of family firms.
Appendix

Proof of Proposition 2. The proof is straightforward and follows from explicit computation. By inspection, we see that \( p \to w_M^\ast \) is increasing and thus order preserving. Substitution of expression (31) into the function \( w_M^\ast \), defined by equation (26), thus yields

\[
w_O^\ast = w_M^\ast(p_O) = w_M^\ast \left( \max \left[ 0, \min \left[ \frac{(h+1)x}{(h+2)k}, \bar{p} \right] \right] \right) = \max \left[ w_M^\ast(0), \min \left[ w_M^\ast \left( \frac{(h+1)x}{(h+2)k} \right), w_M^\ast(\bar{p}) \right] \right] \max \left[ \min \left[ \frac{x}{2} - h\frac{h^2 - h}{1-h}, \frac{h\bar{p} - hx}{1-h} \right], 0 \right]. \tag{101}
\]

Now consider firm value which equals \( p_O^\ast (\bar{x} - w_O^\ast) \). Because \( w_O^\ast \) is weakly decreasing in \( h \) and strictly decreasing at \( p_O^\ast \in (p_0, \bar{p}) \) and \( (p_O^\ast \) is weakly increasing and strictly increasing at \( p_O^\ast \in (p_0, \bar{p}) \) we see that the assertion regarding firm value is true. Next, consider the manager’s value, given by

\[
p_O^\ast w_M^\ast(p_O^\ast) - \frac{1}{2} kp_O^2. \tag{102}
\]

Consider the function \( \Gamma \) defined by

\[
\Gamma : p \to pw_M^\ast(p) - \frac{1}{2} kp^2. \tag{103}
\]

From (102) we see that the manager’s value is given by \( \Gamma[p_O^\ast] \). Moreover \( \Gamma \) is increasing for \( p \geq p_0 \) and thus order preserving. Hence, the manager’s value is given by

\[
\Gamma[p_O^\ast] = \Gamma \left( \max \left[ p_0, \min \left[ \frac{(h+1)x}{(h+2)k}, \bar{p} \right] \right] \right) = \max \left[ \Gamma(p_0), \min \left[ \Gamma \left( \frac{(h+1)x}{(h+2)k} \right), \Gamma(\bar{p}) \right] \right]. \tag{104}
\]

Applying the definition of \( \Gamma \) and \( p_O^\ast \) given by equations (103) and (31) respectively yields

\[
\max \left[ -\frac{h^2 \bar{x}^2}{2k}, \min \left[ \frac{(1+h)(1-2h-h^2)\bar{x}^2}{2(2+h)^2k(1-h)}, \frac{\bar{p}((1+h)kp-2h\bar{x})}{2(1-h)} \right] \right]. \tag{105}
\]

The component functions of (105) are all weakly decreasing in \( h \) and all components except the 0 term are strictly decreasing. Thus, the manager’s value is weakly decreasing in \( h \) and strictly decreasing whenever compensation is positive.

Proof of Proposition 3. Managerial compensation, when the limited liability constraint is not binding,
i.e., $w_M^* > 0$, is given by (40). The expression in (40) is a rational function without an zero in the denominator within the acceptable range of parameters. Thus, the function is smooth. Rather straightforward, but very tedious, differentiation and simplification of this function shows that when $w_M^* > 0$

$$w_M^*(h) = -\frac{(\bar{p}\bar{x} - v_R) \left( ((1-h)^2(\bar{x} - c) + 2c(1-h)) (\bar{p}\bar{x} - c) + c^2 \right)}{(1-h)^2(ch + (1-h)\bar{p}\bar{x})^2}.$$  \hspace{1cm} (106)$$

Because $w_M^*$ is negative and decreasing the result follows for the case of $w_M^* > 0$. Since $w_M^*$ is a continuous non-negative function of $h$ and is decreasing whenever $w_M^* > 0$, $w_M^*$ is weakly decreasing for all $h \in [0, 1/2]$.

\textbf{Proof of Proposition 4.} To prove (a), let $M_+$ represent the equilibrium probability of monitoring conditioned on the equilibrium level of compensation when compensation is positive, i.e.

$$M_+ = m^*(w_M^*), \quad w_M^* > 0$$  \hspace{1cm} (107)$$

Let let $M_0$ represent the equilibrium probability of monitoring conditioned on the equilibrium level of compensation when compensation is 0, i.e.,

$$M_0 = m^*(w_M^*), \quad w_M^* = 0.$$  \hspace{1cm} (108)$$

The equilibrium monitoring probability is then given by $M$ defined by

$$m^*(w_M^*) = \begin{cases} M_+ & \text{if } w_M^* > 0, \\ M_0 & \text{if } w_M^* = 0. \end{cases}$$  \hspace{1cm} (109)$$

To find the explicit form of $M_+$, substitute the equilibrium compensation when compensation is positive, defined in expression (41), into the probability of monitoring a report of 0 given the equilibrium compensation, defined by (18). This yields

$$M_+(h) = \frac{\bar{p}\bar{x} - v_R}{ch + (1-h)\bar{p}\bar{x}}.$$  \hspace{1cm} (110)$$
Differentiating this expression with respect to $h$ yields

$$M'_+(h) = \left(\frac{\bar{p}\bar{x} - v_R}{(ch + (1-h)\bar{p}\bar{x})^2}\right) > 0. \tag{111}$$

This establishes (a). To prove (b), first note that the total probability of monitoring equals the probability that the manager reports 0 times the probability that the owner monitors a report of 0. Thus the total probability of monitoring is given by

$$m^*(w^*_M)(1 - \bar{p}(1 - \sigma^*)), \tag{112}$$

where $\sigma^*$ is defined by (18). As shown in Proposition (1), $\sigma^*$ is increasing in kinship. As shown by (b), $M$ is increasing when $w^*_M > 0$, thus the result is established for the case of $w^*_M > 0$. Proposition 1 shows that for any fixed compensation level, and thus for $w = 0$, the total probability of monitoring is increasing. Now consider (c). First note that the total family value of the is equal the expected operating cash flow, $\bar{p}\bar{x}$ less expected monitoring costs. Thus, total family value is given by

$$\bar{p}\bar{x} - c m^*(w^*_M)(1 - \bar{p}(1 - \sigma^*)). \tag{113}$$

Thus, the fact that the probability of monitoring is increasing in kinship implies that the expected cost of monitoring is increasing in kinship. Since kinship does not affect expected operating cash flows, $\bar{p}\bar{x}$, and increases monitoring costs, it must decrease family value. \qed

Proof of proposition 5. Let $\bar{h}$ be defined as

$$\bar{h} = \max\{h \in [0, 1 - c/(\bar{p}\bar{x})] : w^*_M(h) \geq 0\}. \tag{114}$$

After considerable algebraic simplification, we can express the value of the family firm as a function of $h$, restricted to the domain $[0, \bar{h}]$ as follows:

$$v'_\Omega(h) = (\bar{p}\bar{x} - v_R)\frac{N(h)}{D(h)}, \tag{115}$$

$$N(h) = \left(\left(\frac{c^2}{(1-h)\bar{x}(\bar{p}\bar{x} - c)} - \frac{c^2}{1-h}\right)\right), \tag{116}$$

$$D(h) = ((1-h)\bar{x} - c)(hc + (1-h)\bar{p}\bar{x}). \tag{117}$$
The functions, $h \to N(h)$ and $h \to D(h)$ are both positive under the assumptions given in (34) and (35). The term $\bar{p} x - \nu_R$ is a positive and constant in $h$ and thus can be ignored in the subsequent derivation. Because the functions $N$ and $D$ are smooth over their domain, and the second derivative of $N$ is negative while the second derivative of $D$ is positive, $N(\cdot)$ is strictly concave and $D(\cdot)$ is strictly convex. To establish quasiconcavity, suppose that 

\[
\frac{N(h)}{D(h)} > \frac{N(h')}{D(h')},
\]

(118)

Rearranging (118) produces

\[
N(h) - D(h) \frac{N(h')}{D(h')} > 0.
\]

(119)

The left hand side of (119), viewed as a function of $h$ with $h'$ fixed is strictly concave as it is the difference between a strictly concave function and a strictly convex function multiplied by a fixed constant. Thus for any $\lambda \in (0, 1)$

\[
N(\lambda h + (1 - \lambda) h') - D(\lambda h + (1 - \lambda) h') \frac{N(h')}{D(h')} > \\
\lambda \left( N(h) - D(h) \frac{N(h')}{D(h')} \right) + (1 - \lambda) \left( N(h') - D(h') \frac{N(h)}{D(h)} \right) = \lambda \left( N(h) - D(h) \frac{N(h')}{D(h')} \right) > 0.
\]

(120)

where the last equality follows from the hypothesis, equation (118). Thus,

\[
\frac{N(\lambda h + (1 - \lambda) h')}{D(\lambda h + (1 - \lambda) h')} > \frac{N(h')}{D(h')}, \forall \lambda \in (0, 1).
\]

(121)

This establishes strict quasiconcavity over $[0, \bar{h}]$. The value function is strictly decreasing over $h \in [\bar{h}, 1/2]$, and is continuous at $\bar{h}$. Thus, the value function is strictly quasiconcave over the entire range of $h$, $[0, \bar{h}]$.

Hence, the value function is quasiconcave in $h$. The necessary and sufficient condition for the value function to have a maximum over $(0, \bar{h}]$ is for the derivative of the value function is positive at $h = 0$. The left-hand side of (43) and (44) has the same sign as the derivative of $v_O$ evaluated at $h = 0$.

Proof of Proposition 6. The manager’s value is the maximum of the manager’s value when the limited liability constraint binds, i.e., $w = 0$ and manager’s value when the reservation constraint binds. The maximum of strictly quasiconvex functions is strictly quasiconvex. The manager’s value is clearly increasing in $h$ on the limited liability constraint. Thus, we only need to show that the manager’s value...
is quasiconvex when compensation is determined by the reservation constraint. To see this, note that, the manager’s value when the manager’s value is determined by the reservation constraint, which we represent by \( v_M^p \), can be simplified to obtain

\[
v_M^p(h) = \bar{p} x - (\bar{p} \bar{x} - v_R) F(h),
\]

Next note that \( N(h) \) is strictly concave and positive and \( D(h) \) is strictly convex and positive. Thus using an argument identical to the one used in the proof of Proposition 5 we can verify that \( F(h) \) is quasiconcave.

Because \( F(h) \) is quasiconcave and the term multiplying \( F(h) \) in equation (122) is negative and constant in \( h \), we see, from inspecting (122) that \( v_M^p \) is quasiconvex.

Next note that when \( h = 0 \) the reservation constraint binds so

\[
v_M^p(h) - v_M(0) = v_M^p(h) - v_M^p(0) = v_M^p(h) - (k + v_R).
\]

Using the representation of \( v_M^p \) given in (122) we obtain,

\[
v_M^p(h) - (k + v_R) = (1 - F(h)) (\bar{p} \bar{x} - v_R).
\]

By the parametric restrictions imposed in (7) we see that \( \bar{p} \bar{x} - v_R > 0 \), because \( F(h) \) is less than 1 over the region of admissible parameters,

\[
(1 - F(h)) (\bar{p} \bar{x} - v_R) > 0.
\]

Combining (127) and (128) shows that

\[
v_M^p(h) < v_M(0), \quad h \neq 0.
\]

Because \( v_M \) is quasiconvex in \( h \), it attains is maximal value on the extreme points of its domain. These extreme points are \( h = 0 \) and \( h = \min[1/2, 1 - c/(\bar{p} \bar{x})] \). If the reservation constraint binds at \( 1 - c/(\bar{p} \bar{x}) \) we have shown that this point cannot be a maximizer of \( v_M \). Thus, the maximal value of \( v_M \) is attained
either at $h = 0$ or the at $h = \min\{1/2, 1 - c/(\bar{p}\bar{x})\}$ and in this case the reservation constraint is not binding.

Next, consider the sufficient conditions for an internal minima. When $\bar{p} x - v_R - c$ is sufficiently close to 0, the reservation constraint is always binding at all admissible $h$. If we differentiate $v_M$ and evaluate the derivative where $(1 - h) \bar{p} \bar{x} - c = 0$ we find the derivative is given by

$$\frac{(\bar{p} (3 - \gamma) - 1)}{(1 - \bar{p}) (2 - \gamma)^2 \gamma^2}.$$ (130)

This term must be positive for an interior minimum to exist. Since the fraction in (130) is always positive the necessary and sufficient condition for (130) to be positive is that $\bar{p} (3 - \gamma) - 1 > 0$ which is a condition given in the proposition.
References


