

# BKK the EZ Way

## International Long-Run Growth News and Capital Flows

Ric Colacito      Max Croce      Steven Ho      Philip Howard\*

---

### Abstract

We study the response of international investment flows to short- and long-run growth news. Among developed G7 countries, positive long-run news for domestic productivity induces a net outflow of investments, in contrast to short-run growth shocks. We document that a standard Backus, Kehoe, and Kydland (1994) (BKK) model fails to reproduce this novel empirical evidence. We augment this model with Epstein and Zin (1989) preferences (EZ-BKK), and characterize the resulting recursive risk-sharing scheme. The response of international capital flows in the EZ-BKK model is consistent with the data.

*JEL classification:* C62; F31; G12.

*First draft:* March 31, 2012. *This draft:* July 9, 2014.

---

---

\*All authors are affiliated with the University of North Carolina at Chapel Hill, Kenan-Flagler Business School. We thank David Backus, Harjoat S. Bhamra, Ravi Bansal, Charles Engel, Martin Evans, Karen Lewis, Chris Lundblad, Matteo Maggiori, Tom Sargent, Adrien Verdelhan. We are grateful to our discussants Harjoat S. Bhamra, Nick Roussanov, Linda Tesar, and Stan Zin. We also thank the seminar participants at the Kenan-Flagler Business School (UNC), Fuqua School of Business (Duke University), North Carolina State University, AEA Meetings, North American Summer Meeting of the Econometric Society, SED Meetings, NBER-IAP Summer Meeting, WFA Meetings, Tepper-LAEF Conference, Dartmouth (Tuck), Stony Brook, IMF, BI Norwegian School, UNC (Economics Department), Ohio State University (Fisher College), Hong Kong University, Chicago FED, Duke Macro-Finance Conference, CEPR-AP Summer Meetings, and the AFA Meetings.

# 1 Introduction

Does capital always flow to the most productive countries? Does it matter whether productivity improvements are deemed to be short lived or long lasting? In this paper we answer these questions by investigating the impact of short- and long-term productivity risk on international risk-sharing and capital flows among developed and integrated G7 countries.

We follow Bansal et al. (2010) and Colacito and Croce (2011) in identifying short- and long-run innovations to productivity by regressing Solow residuals on a set of country-specific predictive variables ranging from asset prices to quantities. In a second step, we take the United States as the home country and connect the innovations of its net exports to relative short- and long-run productivity news with the remaining countries (Canada, France, Germany, Italy, Japan, the United Kingdom).

Countries receiving good short-run productivity shocks experience an inflow of capital, i.e., their net exports deteriorate. This is consistent with the observation that domestic net-exports are negatively correlated with domestic output. Positive long-run news, in contrast, produce immediate and persistent capital outflows. Using data from the Bureau of Economic Analysis (BEA) about the composition of the net exports, we find that this phenomenon is present in the net exports of capital goods and hence it is related to international investment decisions.

We then study our findings through the lenses of a frictionless international production-based setting. In the context of a benchmark Backus, Kehoe, and Kydland (1994) (henceforth BKK) model, our empirical results represent an anomaly since this setting cannot produce an immediate and sizeable outflow of capital goods upon the arrival of positive long-run productivity news. This anomaly vanishes once we introduce Epstein and Zin (1989) (henceforth EZ) and analyze the resulting recursive risk-sharing motive.

Specifically, as long as the intertemporal elasticity of substitution (IES) is larger than the reciprocal of their relative risk aversion (RRA), investors dislike both low ex-

pected levels of wealth and increasing uncertainty about their future utility profiles. Equivalently, positive long-run growth news directly depress marginal utilities and are priced independently of short-run consumption growth variations.

The key economic insight underlying our result is the existence of a tension between two channels. On the one hand, the *productivity channel* suggests that resources should move from the least productive to the most productive country; on the other hand, the *risk-sharing channel* suggests that resources should flow from the low-marginal-utility country to the high-marginal-utility country. The relative intensity of these two channels depends on the relative relevance of short- and long-run shocks in the determination of marginal utilities across countries.

In our model (henceforth EZ-BKK), the productivity channel always dominates with respect to short-run growth shocks, i.e., the most productive country receives resources from abroad and invests more. This result is well known, as it holds also in the BKK model with standard preferences. The novelty of our analysis has to do with the response of long-run productivity news.

With recursive preferences, the country that is expected to be more productive in the long-run immediately experiences a substantial drop in its marginal utility, even though productivity has not yet changed. As opposed to the case of a positive short-run shock, the risk-sharing channel dominates, and it prescribes an immediate net outflow of goods followed by a slow recovery of the net exports. With time-additive preferences, in contrast, the outflow of resources unfolds only with several periods of delay, as productivity gains are realized over time.

From a quantitative perspective, a conservative calibration of our EZ-BKK model produces responses of both the total and the capital goods net exports reasonably close to the data. An extended version of the model can also account for: (1) the degree of investments' volatility; (2) the positive cross-country correlation of investments; (3) a large equity risk premium; (4) the extent of volatility of the exchange rate. In our model consumption growth rates are internationally correlated as in the data,

but slightly more correlated than output growth rates. This is what BKK call the quantity anomaly. We leave the full resolution of this puzzle to future research.

Our extension features a twofold modification of the accumulation of new capital units. First, we add heterogenous exposure of capital vintages to aggregate productivity, as in Ai, Croce, and Li (2012). This feature produces a sizeable equity premium and volatile investment growth. Second, we introduce a larger degree of home bias in consumption than in investment, as in Erceg et al. (2008) and Cavallo and Landry (2010). This allows us to be consistent with the fact that foreign consumption goods represent only 3%–5% of the U.S. consumption bundle, whereas foreign investment goods represent on average 20% of U.S. aggregate investment in the post-Bretton Wood sample. As a result, the international correlation of the investment growth rates becomes positive and the exchange rate more volatile.

In the next section we discuss other related literature. In section 3 we present our empirical evidence. In section 4 we present our model and our equilibrium conditions. In section 5 we discuss our results and presented some extensions of our benchmark model. Section 6 presents our sensitivity analysis and section 7 concludes.

## **2 Related Literature**

Using the recursive methods in Anderson (2005) and Colacito and Croce (2012), Tretvoll (2012) is the first to study a production economy with capital accumulation and recursive preferences. We differ from Tretvoll (2012) in several respects. First of all, Tretvoll (2012) does not consider long-run shocks, which constitute the main element of our theoretical and empirical investigations. The present paper is therefore the first to highlight the existence of a relevant long-run risk-based investment channel. Second, Tretvoll (2012) takes into consideration only a standard BKK capital accumulation setting. For this reason, the quantitative performance of our model represents a substantial improvement relative to the existing literature. Third, Tretvoll (2012) uses a calibration in the spirit of the RBC literature with an IES smaller than 1 and

an RRA of 100. We adopt a calibration in the spirit of Bansal and Yaron (2004), with a RRA of 10 and an IES slightly larger than 1.

We use Greenwood et al. (1988) preferences to bundle consumption and leisure in order to address the critique by Raffo (2008) regarding the sources of countercyclicality of net exports. We also use evidence from Erceg et al. (2008) and Cavallo and Landry (2010) on the composition of imports and exports to highlight the relevance of the long-run recursive risk-sharing channel. We differ from all these studies for our long-run risk approach with recursive preferences. Ai et al. (2012) do not address international dynamics. Colacito and Croce (2012) look at international dynamics, abstracting away from production activity and international investment flows, i.e., the main focus of our study. Similarly to us, Coeurdacier et al. (2012) use recursive preferences, but their goal is to address the benefits of financial integration across heterogeneous countries in a one-good production economy.

Several studies have highlighted the role of real, informational, and financial frictions (among others, see Baxter and Crucini (1995), Kehoe and Perri (2002), Heathcote and Perri (2002, 2004, 2013), Petrosky-Nadeau (2011), Alessandria et al. (2011), Van Nieuwerburgh and Veldkamp (2008)). Our analysis differs from these due to its the emphasis on long-term risk and recursive preferences in the context of a frictionless economy.

Starting from Lucas (1990), several studies have focused on the role of international capital and financial flows across developed and emerging economies (see Lewis 2011 and Gourinchas and Rey 2013 for a complete review of such literature). We differ from this literature in several respects. First, because of our recursive risk-sharing approach to international trade, our focus is on developed economies only. Second, in contrast to several prior papers that have studied the composition of international flow of financial assets, we focus on trade of goods and services for consumption and investment reasons. Third, and most importantly, our analysis is the first to explore the general equilibrium implications of investment flows in the face of short- and

long-run productivity news. Looking at transient dynamics among developed and integrated countries, we find that upon the realization of long-run news capital goods flow from relatively richer to relatively poorer countries.

From an empirical point of view, we expand the methodology used in previous work (Colacito and Croce 2011, 2012) to show that country-specific long-run shocks have a well identified negative impact on contemporaneous investment flows, consistent with our EZ-BKK model. Our findings are broadly consistent with the international empirical investigation of Kose et al. (2003, 2008), as we do find evidence of a highly correlated economic productivity factor across G-7 countries in our post-1970 sample. From a finance perspective, we provide a productivity-based general equilibrium explanation of the findings in Pavlova and Rigobon (2007), as our long-run productivity news result in marginal utility shocks that alter the dynamics of both international wealth and the exchange rate.

### **3 Empirical Findings**

In this section, we discuss our main empirical findings about the response of international capital flows with respect to both short- and long-run productivity news. Specifically, we show that positive short-run productivity growth news produce a net inflow of resources (decline in net export), whereas positive long-run news produce an outflow of resources (improvement in net exports). In what follows we proceed in two steps. First, we describe our data sources along with our procedure to identify short- and long-run news. Second, we show the response of US net exports to international productivity shocks.

#### **3.1 Empirical strategy and data sources**

We focus on G7 countries and take the US as the home-country. The remaining G6 countries (Canada, France, Germany, Italy, Japan, and the United Kingdom) are de-

noted as Rest of the World (henceforth RoW). In the first step of our empirical procedure, we follow Bansal et al. (2010) and Colacito and Croce (2011) in identifying short- and long-run innovations to productivity by regressing country-specific productivity growth on country-specific predictive variables.

Specifically, we obtain least squares estimates of the following systems of equations:

$$\begin{aligned}\Delta a_t^i &= z_t^i + \underbrace{\epsilon_{a,t}^i}_{SRS}, \quad i \in G7 \\ z_t^i &= \rho^i z_{t-1}^i + \underbrace{\epsilon_{z,t}^i}_{LRS},\end{aligned}\tag{1}$$

where  $z_t^i$  takes on one of the following two specifications:

$$z_t^i = \beta^i pd_{t-1},\tag{2}$$

$$z_t^i = \beta_{pd}^i pd_{t-1} + \beta_{rf}^i r_{f,t-1}^i + \beta_{\Delta c}^i \Delta c_{t-1}^i + \beta_{\Delta i}^i \Delta i_{t-1}^i,\tag{2b}$$

and where  $\Delta a^i$ ,  $pd^i$ ,  $r_f^i$ ,  $\Delta c_t^i$ , and  $\Delta i_t^i$  denote the growth rate of productivity, the log-price-dividend ratio, the risk-free rate, the growth rate of consumption, and the growth rate of investments in country  $i$ . The processes  $z^i$ ,  $\epsilon_a^i$ , and  $\epsilon_z^i$  denote the long-run component, the long-run shocks (LRS), and the short-run shocks (SRS) for country  $i$ , respectively. Constants are omitted since all variables are demeaned throughout the entire empirical analysis. All data is annual.

We take specification (2) as our benchmark, and consider specification (2b) as a way of exploiting the predictive power of the real short-term risk-free rate and other macroeconomic quantities.

The measure of productivity is obtained from the Penn World Table V.8 (Feenstra et al. 2013), and it accounts for variation in both the share of labor income and capital depreciation across countries and over time (series denoted as *rtfpna*). Data for the construction of the price-dividend ratio for RoW countries are from the “International research returns” section of the Kenneth French’s data library. The US price-dividend

ratio is obtained from the website of Robert Shiller.<sup>1</sup>

Data on consumption and investment are from the Penn World Table and are expressed in constant national prices. The real risk-free rates are computed using data from the International Financial Statistics (IFS) dataset provided by the International Monetary Fund.<sup>2</sup>

Once the short-run and long-run news have been identified, we estimate the following equations involving the change in the US net exports-output ratio (denoted as  $\Delta \frac{NX_t^{US}}{GDP_t}$ ), the change in the US net export of capital goods as a fraction of GDP (denoted as  $\Delta \frac{NXI_t^{US}}{GDP_t}$ ), and the cross-country difference in the investment growth rates (denoted as  $\Delta i_t^{US} - \Delta i_t^i$ ):

$$\Delta \frac{NX_t^{US}}{GDP_t} = \beta_{NX,SRR}(\epsilon_{a,t}^{US} - \epsilon_{a,t}^i) + \beta_{NX,LRR}(\epsilon_{z,t}^{US} - \epsilon_{z,t}^i) + \xi_t^i \quad (3)$$

$$\Delta \frac{NXI_t^{US}}{GDP_t} = \beta_{NXI,SRR}(\epsilon_{a,t}^{US} - \epsilon_{a,t}^i) + \beta_{NXI,LRR}(\epsilon_{z,t}^{US} - \epsilon_{z,t}^i) + \xi_t^i \quad (4)$$

$$\Delta i_t^{US} - \Delta i_t^i = \beta_{I,SRR}(\epsilon_{a,t}^{US} - \epsilon_{a,t}^i) + \beta_{I,LRR}(\epsilon_{z,t}^{US} - \epsilon_{z,t}^i) + \xi_t^i \quad (5)$$

We let  $i \in \{UK, RoW\}$ , meaning that we estimate the betas in two ways. First, we estimate equations (3)-(5) considering only relative news to the US and the UK, as in Colacito and Croce (2011, 2012).<sup>3</sup> Second, we perform a system estimation to take

---

<sup>1</sup>The Penn World Tables are available at [https://pwt.sas.upenn.edu/cic\\_main.html](https://pwt.sas.upenn.edu/cic_main.html). Financial data are available at [http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls) and [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The price-dividend ratios for the RoW countries are calculated using cum- and ex-dividend country value-weighted dollar index returns (using “All 4 Data Items Not Reqd” series). French’s data begin in 1977; for previous years we use price-dividend ratios from Campbell (2003).

<sup>2</sup>To construct consumption from the Penn Word Tables dataset, we multiply the consumption share (denoted as *cash\_c*) by GDP expressed in constant national prices (series denoted as *rgdpna*). We repeat the same procedure for investment using the investment share data series (denoted as *cash\_i*). The IFS dataset is available at <http://elibrary-data.imf.org/DataExplorer.aspx>. For each country, the real risk-free rate is computed as the difference between the nominal interest rate on government bills and realized inflation measured by the consumer price index for all items. For the United Kingdom, the retail index is used to calculate inflation. Germany’s and Italy’s risk-free rate series calculated by the IMF begin in 1975 and 1976, respectively. For earlier years we use data from Campbell (2003).

<sup>3</sup>Our focus on the US and the UK is related to the observation that these two countries have had an excellent tradition of financial integration for at least a large part of the post-World War II period, as documented by Taylor (1996), Obstfeld (1998), and Quinn (1997) among others. This is important because our model features a frictionless risk-sharing mechanism and hence its predictions must be compared to empirical evidence from countries with internationally developed and integrated financial



advantage of the cross-section of G7 countries and sharpen our inference.

Specifically, we keep the US as the reference country and change its counterpart by picking each one of the six remaining countries. As specified in equations (3)-(5), the betas are not country-pair specific, implying that the number of observations employed for each parameter estimate becomes six times larger compared to the case in which only data for US and UK is included.

### **3.2 Net exports data and subcomponents**

This section discusses how total net exports and net exports of capital goods are measured. We use annual data from the Bureau of Economic Analysis (BEA) table 4.2.5 “Exports and imports by type of products”. For both exports and imports, data are aggregated in six main components: (C1) Foods, feeds, and beverages; (C2) Industrial supplies and materials; (C3) Capital goods, except automotive; (C4) Automotive vehicles, engines, and parts; (C5) Consumer goods, except automotive; and (C6) Services. In this paper, we study a model that abstracts away from both consumer durable goods and government expenditure. For this reason, we exclude the following sub-components from both imports and exports in our empirical investigation: (i) “Transfers under U.S. military agency sales contracts” included under services, (C6), and (ii) “Consumer Durable goods” included under (C5).

Automotive vehicles may be ultimately used either as investment goods or as consumption durable goods. We consider two cases in our empirical analysis. First, we consolidate automotive vehicles (C4 above) with the sub-component “Consumption of durable goods” and hence we exclude it from all our computations. Second, we consider automotive vehicles (C4 above) as contributing to capital accumulation and consolidate it with capital goods (C3 above). Our main findings are robust to both specifications.

The BEA provides a detailed list of the items that are considered industrial supplies.  
markets.

In the context of our model, the most relevant subcomponents of these supplies (for example, finished and unfinished metals, finished and unfinished building materials, fabrics, ...) are better interpreted as non-perishable investment goods. For this reason, our net exports of capital goods,  $NXI$ , comprises both industrial supplies (C2 above), and capital goods (C3 above). A somewhat more accurate allocation of these supplies across investment and consumption goods may be achieved using the BEA detailed goods trade data. Unfortunately, this would come at the cost of basing our inference on a significantly shorter sample, as data are available only from 1989.<sup>4</sup>

### 3.3 Empirical Results

In Table 1, we report the estimates of the coefficients in equations (3)-(5) for the two specifications of the long-run components (equations (2) and (2b)). Our balanced sample starts in 1973 and ends in 2006. This choice allows us to focus on a regime a flexible exchange rates (post-Bretton-Wood period), characterized by substantial financial integration across all major industrialized countries (see inter alia Quinn (1997) and Obstfeld (1998)). We exclude recent years from our sample to prevent our results from being driven by the Great Recession.

Table 1 considers two cases: one in which the UK is the only foreign trading partner (first two columns), and one in which the set of foreign trading partners is extended to include all G6 countries (last two columns).

We highlight three important results. First, the US net exports decline upon the arrival of positive US-specific short-run news ( $\beta_{NX|SR} < 0$ ), whereas the opposite is true for relative US investment growth ( $\beta_{I|SR} > 0$ ). This means that when a country receives positive short-run news, it experiences a net inflow of resources that are used to support domestic investment growth. This result mirrors the one documented by BKK, thus strengthening the reliability of our identification scheme for short-run shocks.

---

<sup>4</sup>Data are available at [https://www.bea.gov/international/detailed\\_trade\\_data.htm](https://www.bea.gov/international/detailed_trade_data.htm).

**TABLE 1: Empirical Evidence**

Dep. Var.	Shock	US vs UK		US vs RoW	
		Predict. Var.s:		Predict. Var.s:	
		pd	pd,rf,dc,di	pd	pd,rf,dc,di
NX/GDP	SR	-0.02 ( 0.03 )	-0.04 ( 0.09 )	-0.07 *** ( 0.01 )	-0.08 *** ( 0.01 )
	LR	0.53 ( 0.58 )	0.26 * ( 0.15 )	0.61 *** ( 0.04 )	0.14 *** ( 0.02 )
Rel. Inv. Growth ( $\Delta i - \Delta i^*$ )	SR	3.43 *** ( 0.62 )	3.03 *** ( 0.39 )	3.47 *** ( 0.09 )	3.28 *** ( 0.06 )
	LR	-0.66 ( 2.19 )	-3.99 *** ( 0.84 )	-7.50 *** ( 0.38 )	-3.59 *** ( 0.14 )
NXI/GDP	SR	0.03 * ( 0.02 )	0.01 ( 0.04 )	-0.11 *** ( 0.00 )	-0.10 *** ( 0.00 )
	LR	0.75 *** ( 0.15 )	0.08 ** ( 0.03 )	0.42 *** ( 0.02 )	0.00 ( 0.01 )

*Notes:* The panel in which the dependent variable is  $NX/GDP$  reports the response of Net Exports over GDP to the difference of short-run shocks (SR) and long-run shocks (LR), as in equation (3). Long-run risks were estimated by regressing Solow residuals on the corresponding set of predictive variables indicated by the column titles, as in equations (1)-(2b). The variables pd, rf, dc, and di correspond to price-dividend, risk-free rate, consumption growth, and investment growth, respectively. The columns labeled “US vs UK” refer to the estimations conducted using only data from the United States and the United Kingdom. The columns labeled “US vs Row” show the results for the case of the panel estimation in which all the loadings on the short- and long-run news are restricted to be the same for each country pair. The numbers in parenthesis underneath each point estimate are heteroskedasticity-adjusted standard errors. One, two, and three asterisks denote 10%, 5%, and 1% significance, respectively. The panels labeled “Rel. Inv. Growth” and “NXI/GDP” repeat the same analysis for the case in which the dependent variables are the differentials of the growth rates of investments, and the Net Exports of Investments, respectively.

Second, the arrival of positive long-run news in one country results in an outflow of resources ( $\beta_{NX|LR} > 0$ ), and a relative increase in investments abroad ( $\beta_{I|LR} < 0$ ). This response goes exactly in the opposite direction of what observed with respect to short-run shocks. Further, these responses seem tightly connected to the behavior of the net exports of capital goods ( $\beta_{NXI|SR} < 0$  and  $\beta_{NXI|LR} > 0$ ), suggesting that an international production economy is most appropriate to provide a general equilibrium explanation of the behavior of international trade.

Third, focusing on the US vs Row case produces a twofold benefit: (i) it sharpens the statistical identification of the coefficients; and (ii) it shows that our novel empirical results are pervasive in a relatively large cross-section of developed countries.

Lastly, we note that the magnitude of our coefficients changes slightly across different

columns due to long- and short-run shocks having different variances depending on whether they are extracted according to equation (2) or (2b). Across all our specifications, the signs of the coefficients remain unchanged.

In Appendix A, we adjust the net export of capital goods for the relative price of investment and consumption, a standard procedure to account for investment shocks. We also add net exports of cars to both net exports of capital goods and total net export. In both cases, our main conclusions remain unchanged.

## 4 Model

We study a two-country, and two-good BKK economy with one main departure consisting in the adoption of recursive preferences. In this section, we first describe the technology used to produce consumption goods and the role played by recursive preferences, and then we turn our attention to the international production technology. In what follows, we denote foreign variables by “\*”, and use small letters for log-units, i.e.,  $x_t = \log X_t$ .

### 4.1 The Economy

For the sake of simplicity, we present a stripped down version of the model featuring only the essential elements of our study. In Appendix C, (i) we describe a generalized version of the model that encompasses all the cases studied in our manuscript, and (ii) we derive all the required optimality conditions.

**Consumption aggregate.** Let  $\{X_t, Y_t\}$  and  $\{X_t^*, Y_t^*\}$  denote the time  $t$  consumption of goods  $X$  and  $Y$  in the home and foreign countries, respectively. The consumption aggregates in the two countries are

$$C_t = X_t^\lambda Y_t^{(1-\lambda)}, \quad C_t^* = X_t^{*\lambda} Y_t^{*(1-\lambda)}. \quad (6)$$

We assume that the home (foreign) country produces good  $X$  ( $Y$ ) and set  $\lambda > 1/2$  to introduce consumption home bias into our model. This is a standard assumption in the international macroeconomic literature.

**Consumption bundle.** We assume endogenous labor supply by assuming that the domestic (foreign) consumption bundle,  $\tilde{C}$  ( $\tilde{C}^*$ ), embodies both consumption ( $C$ ) utility, and labor ( $N$ ) dis-utility. This assumption is not crucial, as our main results on international capital flows may be obtained also with fixed labor supply. Nevertheless, we introduce this assumption to document the generality of our approach.

Raffo (2008) shows that using a CES aggregator of consumption and leisure, the counter-cyclical of the net export in the BKK model originates from extreme adjustments in the terms of trade. This finding is at odds with the data, where most of the action comes from adjustments in the quantity of imports. To prevent this problem, we follow Raffo (2008) and adopt Greenwood et al. (1988) (henceforth GHH) preferences

$$\tilde{C}_t = C_t - \varphi N_t^{1+\frac{1}{f}} SL_t, \quad \tilde{C}_t^* = C_t^* - \varphi N_t^{*1+\frac{1}{f}} SL_t^*,$$

where  $N$  ( $N^*$ ) and  $SL$  ( $SL^*$ ) denote hours worked and standards of living in the home (foreign) country, respectively. To guarantee balanced growth, we assume that  $SL_t$  ( $SL_t^*$ ) is cointegrated with productivity,  $A_t$  ( $A_t^*$ ) as follows

$$\begin{aligned} \log(SL_t/A_t) &:= \tilde{s}l_t = \mu(1 - \theta) + (\theta - 1)(\Delta a_t - \tilde{s}l_{t-1}) \\ \log(SL_t^*/A_t^*) &:= \tilde{s}l_t^* = \mu(1 - \theta) + (\theta - 1)(\Delta a_t^* - \tilde{s}l_{t-1}^*), \end{aligned}$$

where  $\Delta a_t$  ( $\Delta a_t^*$ ) denotes the growth rate of productivity in the home (foreign) country, and  $\mu = E[\Delta a] = E[\Delta a^*]$ . We set  $\theta = 0.10$  to mimic a time-trend in the standards of living and prevent productivity from behaving as a preference shock.

**Preferences.** Each country is populated by a representative agent with Epstein and Zin (1989) recursive preferences. For the home country, we have the following expression:

$$U_t = \left[ (1 - \delta) \cdot \tilde{C}_t^{1-1/\psi} + \delta E_t [U_{t+1}^{1-\gamma}]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}. \quad (7)$$

The preferences of the foreign country are defined in the same manner over the consumption bundle  $\tilde{C}_t^*$ . The coefficients  $\gamma$  and  $\psi$  measure the relative risk aversion (henceforth RRA) and the intertemporal elasticity of substitution (henceforth IES), respectively. We assume that the two countries have the same RRA and IES, as well as the same subjective discount factor.

With these preferences, agents are risk averse in future utility as well as future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty, measured by  $\gamma - 1/\psi > 0$ . This can be better highlighted by focusing on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1 - 1/\psi},$$

and performing a second order Taylor expansion about the conditional mean of  $\log V_{t+1}$  to obtain:

$$V_t \approx (1 - \delta) \frac{\tilde{C}_t^{1-1/\psi}}{1 - 1/\psi} + \delta E_t [V_{t+1}] - (\gamma - 1/\psi) Var_t [V_{t+1}] \kappa_t, \quad (8)$$

where  $\kappa_t \equiv \frac{\delta}{2E_t[V_{t+1}^{1-1/\psi}]} > 0$ . When  $\gamma = 1/\psi$ , the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, i.e.,  $\gamma > 1/\psi$ , uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility,  $E_t [V_{t+1}]$ , for future utility risk,  $Var_t [V_{t+1}]$ . This trade-off drives international consumption and investment flows, and it represents one of the most important elements of our analysis.

Our study is the first to fully characterize international trade with Epstein and Zin (1989) preferences in a production economy with long-run shocks.<sup>5</sup>

Since there is a one-to-one mapping between utility,  $U_t$ , and lifetime wealth, i.e., the value of a perpetual claim to consumption, the optimal risk-sharing scheme can also be interpreted in terms of mean-variance trade-off of wealth. For this reason, in what follows we will use the terms “wealth” and “continuation utility” interchangeably.

**Aggregate productivity.** We model the growth rate of productivity in the spirit of the long-run risk literature. Specifically, we introduce country-specific long-run productivity components,  $z$  and  $z^*$ , as in Croce (2014), and assume that the domestic and foreign productivity processes,  $A$  and  $A^*$ , are co-integrated (Colacito and Croce 2012):

$$\begin{aligned}\Delta a_t &= \mu + z_{t-1} - \tau \cdot \log \frac{A_{t-1}}{A_{t-1}^*} + \varepsilon_{a,t} \\ \Delta a_t^* &= \mu + z_{t-1}^* + \tau \cdot \log \frac{A_{t-1}}{A_{t-1}^*} + \varepsilon_{a,t}^* \\ z_t &= \rho z_{t-1} + \varepsilon_{z,t} \\ z_t^* &= \rho z_{t-1}^* + \varepsilon_{z,t}^*,\end{aligned}\tag{9}$$

where  $\varepsilon_{z,t}$  and  $\varepsilon_{z,t}^*$  represent long-run shocks, whereas  $\varepsilon_{a,t}$  and  $\varepsilon_{a,t}^*$  represent the short-run shocks. Shocks are jointly log-normally distributed:

$$\xi_t \equiv \begin{bmatrix} \varepsilon_{z,t} & \varepsilon_{z,t}^* & \varepsilon_{a,t} & \varepsilon_{a,t}^* \end{bmatrix} \sim i.i.d.N(\mathbf{0}, \Sigma),$$

where

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho_{lrr}\sigma_x^2 & 0 & 0 \\ \rho_{lrr}\sigma_x^2 & \sigma_x^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & \rho_{srr}\sigma^2 \\ 0 & 0 & \rho_{srr}\sigma^2 & \sigma^2 \end{bmatrix}.$$

---

<sup>5</sup>Equation (8) is reported for explanatory purposes only. The rest of the analysis is conducted with the preference specification in equation (7).

The parameter  $\tau \in (0, 1)$  is calibrated to a small number to generate moderate co-integration. Under this assumption, the productivity specification in equation (9) is consistent with that used in our empirical investigation (see equation (1)), and it is calibrated using our productivity data for G7 countries.

**Production function and resource constraints.** In each country, output is a Cobb-Douglas aggregation of country-specific capital and labor. Output can be used for consumption or investment:

$$\begin{aligned} X_t^T &= K_t^\alpha (A_t N_t)^{1-\alpha} = X_t + X_t^* + I_{x,t} + I_{y,t} \\ Y_t^T &= K_t^{*\alpha} (A_t^* N_t^*)^{1-\alpha} = Y_t^* + Y_t + I_{y,t}^* + I_{x,t}^*. \end{aligned}$$

From a home (foreign) country perspective,  $I_{x,t}$  ( $I_{y,t}^*$ ) measures real local investment, while  $I_{y,t}$  ( $I_{x,t}^*$ ) measures investment abroad. Even though capital stocks and labor services are country-specific, agents can trade both consumption and investment goods without any friction in every period and state of the world. We link our resource constraints to quantities recorded in the national accounts as follows:

$$\begin{aligned} X_t^T &= \underbrace{(X_t + P_t Y_t)}_{C_{m,t}} + \underbrace{(I_{x,t} + P_t I_{x,t}^*)}_{I_{m,t}} + \underbrace{(X_t^* + I_{y,t})}_{Exp_{m,t}} - \underbrace{P_t(Y_t^* + I_{x,t}^*)}_{Imp_{m,t}} \\ Y_t^T &= \underbrace{(Y_t^* + X_t^*/P_t)}_{C_{m,t}^*} + \underbrace{(I_{y,t}^* + I_{y,t}/P_t)}_{I_{m,t}^*} + \underbrace{(Y_t + I_{x,t}^*)}_{Exp_{m,t}^*} - \underbrace{(X_t^* + I_{y,t})/P_t}_{Imp_{m,t}^*}, \end{aligned}$$

where  $P_t = \frac{1-\lambda}{\lambda} \left( \frac{X_t}{Y_t} \right)$  denotes the terms of trade, and the subscript  $m$  indicates that we are referring to accounting aggregates measured in local units. To be consistent with our data source, we report results in local output units. Our results continue to hold also in the case in which we choose the consumption bundle as numeraire. Because of home bias,  $C_{m,t} = X_t + P_t Y_t$  and  $C_t = X_t^\lambda Y_t^{(1-\lambda)}$  in fact have very similar dynamics.



**Capital accumulation.** In each country, the stock of physical capital,  $K$ , evolves as follows:

$$K_{t+1} = (1 - \delta_k)K_t + G_t, \quad K_{t+1}^* = (1 - \delta_k)K_t^* + G_t^*,$$

where  $\delta_k$  takes into account depreciation, and  $G_t$  and  $G_t^*$  measure the mass of the newly created capital in the home and foreign country, respectively. New capital is a Cobb-Douglas aggregation of domestic and foreign goods,

$$G_t = I_{x,t}^\lambda I_{x,t}^{*1-\lambda}, \quad G_t^* = I_{y,t}^{*1-\lambda} I_{y,t}^\lambda, \quad (10)$$

featuring the same home-bias coefficient chosen for the consumption bundle. We make this assumption to be consistent with BKK and highlight the key role played by recursive preferences. We relax this assumption in section 5.3 to consider an investment aggregator that better fits the data on the net exports of capital goods.

## 4.2 Risk-Sharing Rules and Asset Prices

We assume that markets are complete both domestically and internationally, so the allocation of the competitive equilibrium can be found by solving the Pareto problem associated with our economy (see Appendix C). Prices can then be recovered using the planner's shadow valuations. Below we report the equilibrium conditions for consumption and investment.

**Consumption allocations.** The optimal allocation of the two goods devoted to consumption can be characterized using the following first-order necessary conditions:

$$\begin{aligned} S_t \cdot \frac{\partial C_t}{\partial X_t} \cdot \frac{1}{C_t} &= \frac{\partial C_t^*}{\partial X_t^*} \cdot \frac{1}{C_t^*} \\ S_t \cdot \frac{\partial C_t}{\partial Y_t} \cdot \frac{1}{C_t} &= \frac{\partial C_t^*}{\partial Y_t^*} \cdot \frac{1}{C_t^*}, \end{aligned} \quad (11)$$

where  $S_t$  is the ratio of the pseudo-Pareto weight of the home and foreign countries, respectively. The dynamics of the additional state variable  $S_t$  are given by the process

$$S_t = S_{t-1} \frac{M_t}{M_t^*} \frac{e^{\Delta c_t}}{e^{\Delta c_t^*}}, \quad (12)$$

where  $M_t$  denotes the home stochastic discount factor expressed in units of the local consumption aggregate,  $C_t$ ,

$$M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}, \quad (13)$$

and  $M_t^*$  takes the same form but refers to the foreign country. The second term captures aversion to continuation utility risk and is extremely sensitive to growth news (Bansal and Yaron 2004).

**Key intuition.** According to the system of equations (11), a lower  $S_t$  implies that a smaller share of resources available for consumption should be allocated to the home country. This happens when the home country marginal utility is relatively high, i.e., when it receives relatively bad productivity news (equation 12). We note two things. First, this channel works in the opposite direction of the productivity channel, as bad news to home productivity produce an incentive to reallocate resources toward the home country.

Second, with recursive preferences, bad long-run news to the home country immediately depresses wealth and increase the home marginal utility, thus generating an incentive to allocate resources exactly toward the home country. With CRRA preferences, in contrast, marginal utilities are not directly affected by long-run productivity shocks ( $\gamma = 1/\psi$ ) and long-run news have no direct impact on  $S_t$ .

**Asset prices and optimal investment.** Since our asset pricing conditions are identical to those in the original BKK model, in what follows we mainly set notation

and define relevant variables. Detailed and generalized computations are reported in Appendix C.

Let  $Q_{k,t}$  ( $Q_{k,t}^*$ ) denote the ex-dividend price of domestic (foreign) capital expressed in local output units. At the equilibrium, optimal within-country investment implies:

$$\begin{aligned} Q_{k,t} &= \frac{1}{\lambda_i} \frac{I_{x,t}}{G_t} = E_t \left[ M_{t+1}^X \left( \alpha \frac{X_{t+1}^T}{K_{t+1}} + (1 - \delta) Q_{k,t+1} \right) \right] \\ Q_{k,t}^* &= \frac{1}{\lambda_i} \frac{I_{y,t}^*}{G_t^*} = E_t \left[ M_{t+1}^Y \left( \alpha \frac{Y_{t+1}^T}{K_{t+1}^*} + (1 - \delta) Q_{k,t+1}^* \right) \right], \end{aligned} \quad (14)$$

where the stochastic discount factors in local output units are specified as follows:

$$M_{t+1}^X = \frac{X_t}{X_{t+1}} \frac{C_{t+1}}{C_t} M_{t+1}, \quad M_{t+1}^Y = \frac{Y_t^*}{Y_{t+1}^*} \frac{C_{t+1}^*}{C_t^*} M_{t+1}^*.$$

The returns of capital in the domestic and foreign countries are:

$$R_{k,t+1} = \frac{\alpha \frac{X_{t+1}^T}{K_{t+1}} + (1 - \delta) Q_{k,t+1}}{Q_{k,t}}, \quad R_{k,t+1}^* = \frac{\alpha \frac{Y_{t+1}^T}{K_{t+1}^*} + (1 - \delta) Q_{k,t+1}^*}{Q_{k,t}^*}. \quad (15)$$

We compute the real risk-free rates in local units as  $R_{f,t} = 1/E_t[M_{t+1}^X]$  and  $R_{f,t}^* = 1/E_t[M_{t+1}^Y]$ , and define excess returns as  $R_t^{ex} = R_{k,t+1}/R_{f,t}$  and  $R_t^{*ex} = R_{k,t+1}^*/R_{f,t}^*$ .

Investments abroad are determined by the following no-arbitrage equations:

$$\begin{aligned} \frac{1}{1 - \lambda_i} \frac{I_{y,t}}{G_t^*} &= E_t \left[ M_{t+1}^X \left( \alpha \frac{Y_{t+1}^T}{K_{t+1}^*} + (1 - \delta) Q_{k,t+1}^* \right) P_{t+1} \right], \\ \frac{1}{1 - \lambda_i} \frac{I_{x,t}}{G_t} &= E_t \left[ M_{t+1}^Y \left( \alpha \frac{X_{t+1}^T}{K_{t+1}} + (1 - \delta) Q_{k,t+1} \right) / P_{t+1} \right], \end{aligned} \quad (16)$$

which take into account exchange rate risk through the terms-of-trade,  $P_t$ . Since markets are complete, the log-growth of the real exchange rate in consumption units is also pinned down by the following restriction:

$$\Delta e_{t+1} = m_{t+1} - m_{t+1}^*.$$

### 4.3 Calibration and Solution Method

We summarize our annual parameter values in Table 2. The top portion of the table refers to the calibration of the productivity process. We set the average annual growth rate ( $\mu$ ), and the volatility of the short-run productivity shocks ( $\sigma$ ), to replicate key properties of US real per-capita output over the 1929-2006 sample. The choice of this sample is standard in the long-run literature (see, among others, Bansal and Yaron 2004, Bansal et al. 2010, Colacito and Croce 2011, 2012, and Croce 2014).

More specifically, we target an average annual growth rate of 1.8%, and a volatility of the growth rate of output of 3.5%. These numbers are consistent with BEA data. Furthermore, these values are on the conservative side, since UK output volatility is typically at least 1% greater than in the US. Choosing a higher  $\sigma$  would enhance our quantitative results.

The other moments of the productivity process are obtained from our estimation of equations (1)-(2) for the US and the UK. The choice of these two specific countries as our benchmark is motivated by the analysis of Colacito and Croce (2011, 2012), who show that US and UK share a long history of strong economic and financial ties. We assume that the US and the UK share the same persistence of the long-run components ( $\rho^{US} = \rho^{UK} = \rho$ ), the same relative magnitude of the long-run shocks ( $\frac{\sigma_x^{US}}{\sigma^{US}} = \frac{\sigma_x^{UK}}{\sigma^{UK}}$ ), and estimate these common parameters. The values reported in Table 2 are averages across US and UK with the associated standard errors. We set  $\sigma_x = 0.15\sigma$  as in the data. We choose  $\rho = .98$ , a value that accounts for the Stambaugh (1986) small sample bias and that is well within our confidence interval.

As in the data, our economy features a large correlation of long-run components (large  $\rho_{lrr}$ ) and a low correlation of short-run shocks (low  $\rho_{srr}$ ) across countries. This is consistent with the empirical findings in the international finance (see, among others, Bansal and Lundblad 2002; Colacito and Croce 2011, 2012; Lustig et al. 2011) and the international macro (Crucini et al. 2011) literature about the existence of a world growth risk component that drives both asset prices and productivities. We set the

**TABLE 2: Calibrated Parameters**

Productivity:						
	Av. Growth	Std(SR)	Std(LR)	LR-AR	corr( $z, z^*$ )	corr( $\epsilon_a, \epsilon_a^*$ )
	( $\mu$ )	( $\sigma$ )	( $\sigma_x$ )	( $\rho$ )	( $\rho_{lrr}$ )	( $\rho_{srr}$ )
Values	1.8%	3.7%	$0.15 \cdot \sigma$	0.98	0.95	0.15
Data:			$0.15 \cdot \sigma$	0.95	0.91	0.39
			(0.03)	(0.06)	(0.04)	(0.10)
Technology:						
Capital Income Share			Depreciation		Home-bias	
	( $\alpha$ )		( $\delta_K$ )		( $\lambda$ )	
	1/3		0.06		0.85	
Preference Parameters:						
Labor Elasticity		RRA	IES		Subj. Discount Rate	
	( $f$ )	( $\gamma$ )	( $\Psi$ )		( $\delta$ )	
	1.5	10	1.2		0.97	

*Notes:* This table reports the parameter values used for our benchmark annual calibration. Std(SR) and Std(LR) denote the standard deviation of the short- and long-run shocks,  $\epsilon_a$  and  $\epsilon_{z,t}$ , respectively. LR-AR denotes the persistence of the productivity long-run components, as defined in equation (9). The empirical estimates are obtained from annual data over the sample 1973-2006. All data sources are discussed in section 3. Numbers in parenthesis are Newey-West adjusted standard errors.

correlation of long-run shocks to a value slightly higher than in the data, but well within our estimated confidence intervals. To match the cross-country correlation of consumption growth across US and UK, we set the short-run shocks correlation to 0.15. Since these are important parameters, we consider alternative values in section 6.

On the technology side, both the share of capital income,  $\alpha$ , and the extent of home-bias, are calibrated as in BKK. Our capital depreciation rate,  $\delta_K$ , is set to an annual 6% to prevent a the steady state investment-output share from being too large.

The elasticity of substitution between labor and consumption is set greater than one to be consistent with the evidence from aggregate data (among others, see Raffo 2008 and Gourio and Noual 2006). The relative risk aversion is set to 10, a reasonable upper bound in the long-run risk literature.

In the context of an exchange economy with recursive risk-sharing, Colacito and Croce (2012) show that the IES should be set between 1 and 1.5. We set this parameter

to the intermediate value of 1.2 and provide sensitivity analysis with respect to this coefficient in section 6. The subjective discount factor is chosen so to keep the average annual risk free-rate close to 1%, when possible.

Given these parameters, we use perturbation methods to solve our system of equations. We compute an approximation of the third order of our policy functions using the dynare++4.2.1 package. As documented in Colacito and Croce (2010, 2012), a third order approximation is required to capture endogenous time-varying volatility due to the adjustments of the pseudo-Pareto weights. All variables included in our dynare++ code are expressed in log-units.

## 5 Model Results

In this section, we present our main model-based results by proceeding in steps. First, we show that with time-additive preferences our results about international capital flows is an anomaly. Second, we document that the introduction of recursive preferences in an otherwise standard BKK model (we refer to this setting as EZ-BKK) is capable to rationalize our empirical findings.

In section 5.3, we modify the production side of the BKK model to better match domestic and international investment facts that go beyond the main focus of the paper. We show that our extended EZ-BKK model may be a benchmark for future research in production-based international macro-finance.

### 5.1 International capital flows and risk-sharing

A country is a net receiver or supplier of resources depending on its total net exports position (NX). This variable is subject to two effects potentially working in opposite directions.

On the one hand, positive productivity news generate an incentive to receive investment goods, i.e., to have negative net exports of capital goods,  $NXI_t = I_{y,t} - P_t I_{x,t}^*$ .

We refer to this effect as the productivity channel. On the other hand, depending on the extent of risk aversion, the risk-sharing motive may prescribe the exact opposite effect for the net flow of consumption goods,  $NX_t - NXI_t = X_t^* - P_t Y_t$ . If the risk-aversion is strong enough,  $S_t$  may decline and consumption goods may be shipped abroad.

In order to study the productivity and the risk-sharing channels, we report the responses of key international variables with respect to positive short- and long-run shocks in Figure 1. We perform this exercise for both our EZ-BKK and the standard BKK model with time-additive preferences. For the BKK setting, the relative risk coefficient,  $\gamma$ , is set equal to either  $\gamma = 10 > 1$  or  $1/\Psi = 0.83 < 1$ . These values are from our benchmark calibration with recursive preferences.

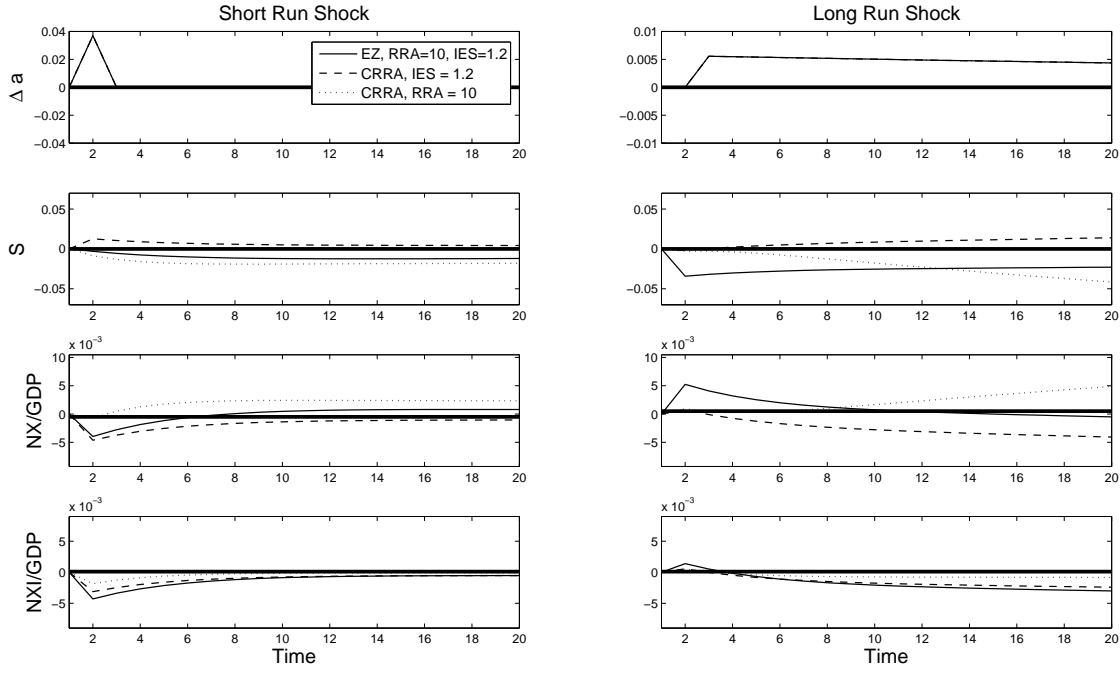
**CRRA preferences and short-run shocks: the mechanism.** With time-additive preferences the risk-sharing dynamics defined in equation (12) simplify to

$$\Delta s_t = (1 - \gamma)(\Delta c_t - \Delta c_t^*),$$

and it implies that when  $\gamma > 1$ , relative good news to domestic consumption growth generate an incentive to reallocate consumption goods abroad ( $NX_t - NXI_t > 0$ ). The higher the risk aversion, the stronger this channel.

This explains why when  $\gamma = 10$ , the realization of a positive short-run shock to the home-country produces an almost negligible adjustment in the total net exports (left column, second panel). At the equilibrium, home consumption growth immediately increases and the risk sharing motive is strong enough to offset the productivity channel, i.e., the positive net exports of consumption goods of the home country equal the net inflow of investment goods.

As the relative risk-aversion declines, the productivity channel dominates, and the net exports become counter-cyclical as in the case of the original BKK analysis. In the case of  $\gamma < 1$ , the risk-sharing channel ends up amplifying the productivity channel:



**FIG. 1. Trading motives with and without EZ-preferences.** This figure shows annual deviations from the steady state. Both the ratio of the pseudo-Pareto weights,  $S$ , and productivity growth,  $\Delta a$ , are expressed in log units.  $NX/GDP$  ( $NXI/GDP$ ) denotes total net-exports (net exports of capital goods only) as a share of output. All the parameters are calibrated to the values reported in table 2. When time-additive preferences are employed, the RRA coefficient is set equal to either  $\gamma$  or  $\psi$ . Shocks to the home-country productivity,  $\epsilon_a$  and  $\epsilon_x$ , materialize at time 2 and are not orthogonalized by their international correlations,  $\rho_{str}$  and  $\rho_{lrr}$ , respectively. The short-run (long-run) shock is assumed to affect only the home country and has a magnitude  $\sigma$  ( $\sigma_x$ ).

total net exports decline sharply because the home country becomes a net receiver of both consumption and investment goods.

**CRRA preferences and long-run news: the anomaly.** In our data, positive long-run news produce an immediate and sizeable increase in the net exports. Time-additive preferences cannot reproduce this finding (right column of Figure 1, second and third panels). Since the marginal utility of each agent depends only on current short-run consumption growth, news about future growth are not priced. By definition, long-run news produce no immediate change in current productivity differentials. Hence there is no reason to strongly alter  $S$ ,  $NX$ , or  $NXI$ , upon the arrival of long-run news.



Over time, as news turn into realized short-run productivity gain differentials, the share of resources  $S_t$  adjusts. When  $\gamma = 10$ , the risk-sharing channel motive is strong enough to eventually dominate the productivity channel, as in the case of the short-run shocks. According to our calibration, the net exports improve with a delay of 9 years, a result inconsistent with our findings. When  $\gamma < 1$ , the response of the net exports is less delayed, but it goes in the wrong direction.

In principle, we could shorten the delayed response of the net exports also by setting  $\gamma$  well above 10. However, this would come at the cost of making the net exports positively correlated with short-run shocks and output growth. Such a result would be even less appealing, as the counter-cyclicity of the net exports is a well established empirical fact. Given these considerations, our findings represent a non-trivial anomaly in the context of a classical BKK setting.

**Resolving the anomaly: EZ preferences.** With recursive preferences, long-run news immediately and significantly affect the marginal utility of our agents through the continuation utility (wealth) channel, as stated in equation (13). Thanks to the presence of home-bias, positive long-run news for the home country produce a more pronounced fall of the home marginal utility. As  $m_t - m_t^*$  declines,  $S_t$  falls substantially and the risk-sharing motive dominates the productivity channel. A greater share of both consumption and investment goods is transferred to the foreign country, consistently with the data.

With respect to short-run shocks, in contrast, the productivity channel dominates the risk-sharing motive. When positive short-run shocks materialize, both NX and NXI deteriorate, as in a standard BKK model. Even though short-run shocks are larger than long-run news and are less correlated across countries, their final impact on the risk-sharing motive is limited because their half-life is too short to significantly alter the continuation utility of our agents.

**TABLE 3: Main Moments**

Panel A: Domestic Moments											
Data:	Vol. relative to GDP			Asset Prices		Correlation( $\Delta \cdot, \Delta \cdot$ )					
	$\Delta n$	$\Delta c$	$\Delta i$	$E[r_f](\%)$	$E[r^{ex}](\%)$	$(c, i)$	$(c, n)$	$(\frac{NX}{XT}, x^T)$			
	0.74 (0.07)	0.76 (0.06)	3.88 (0.23)	1.46 (0.06)	4.71 (2.25)	0.60 (0.11)	0.82 (0.06)	-0.54 (0.11)			
BKK (RRA=1/1.2)	0.49	0.71	2.88	5.56	0.00	0.90	0.88	-0.58			
BKK (RRA=10)	0.47	0.88	4.99	15.91	0.40	0.26	0.84	-0.27			
EZ-BKK	0.49	0.63	2.79	2.06	0.09	0.83	0.87	-0.53			
Panel B: International Moments											
Data:	$\rho_h = \text{corr}(\Delta h, \Delta h^*)$					Std( $\cdot$ )(%)		Sensitivity to News			
	$\rho_c$	$\rho_{x^T} - \rho_c$	$\rho_i$	$\rho_n$	$\rho_{r_f}$	$\Delta e$	$NX/X$	$\beta_{SR}^{NX}$	$\beta_{LR}^{NX}$	$\beta_{SR}^{NXI}$	$\beta_{LR}^{NXI}$
	0.58 (0.12)	0.02 (0.08)	0.49 (0.17)	0.55 (0.11)	0.71 (0.08)	8.66 (0.93)	1.76 (0.34)	-0.07 (0.01)	0.61 (0.04)	-0.11 (0.00)	0.42 (0.02)
BKK (RRA=1/1.2)	0.30	0.06	-0.01	0.34	0.71	1.36	0.83	-0.13	0.08	-0.09	0.05
BKK (RRA=10)	0.61	-0.32	0.38	0.58	0.93	2.74	0.55	-0.03	0.07	-0.05	0.04
EZ-BKK	0.55	-0.27	-0.09	0.43	0.68	1.90	0.79	-0.11	0.95	-0.12	0.26

*Notes:* Empirical moments are computed using annual data from 1973 to 2006. All data sources are discussed in section 3. Numbers in parenthesis are Newey-West adjusted standard errors. Excess returns are levered using a coefficient of 3 (Garca-Feijo and Jorgensen (2010)). For the EZ-BKK model, all the parameters are calibrated as in table 2. When time-additive preferences are employed, the risk aversion coefficient is denoted as RRA. The entries for the models are obtained by repetitions of small-sample simulations. Small letters denote log-units.

## 5.2 Quantitative Performance

In this section, we compare the standard BKK model with our EZ-BKK model on several quantitative dimensions. The relevance of this exercise is twofold. First, we show that the predictions of our EZ-BKK model are quantitatively close to our empirical results on both the total and capital goods net exports. Second, we show that our model inherits most of the successes of the standard BKK model as well as part of its limitations.

**Domestic Moments.** In the top panel of Table 3, we report the set of moments which are commonly analyzed in the one-country production-based asset pricing literature. Our model reproduces the relative volatility of consumption and output, whereas the volatility of labor is slightly lower than in the estimated range in the data. This is actually a common limitation with the standard BKK model and, more broadly, with the production-based literature.

Investment growth is also less volatile than in the data, as it often happens in the long-run productivity literature. Since (i) we target a low risk-free rate to be con-

sistent with the data, and (ii) our equity premium is almost zero, the average capital return is excessively low. Because of diminishing marginal returns, the capital-output ratio is too high and so is the investment-output share required to cover depreciation. It is well known that the overestimation of the average investment-output share results in low investment growth volatility.

This result applies also to standard time-additive preferences, as it can be seen by comparing the standard BKK model with high and low risk aversion coefficients. The ability of BKK to match the high volatility of investments originates from the risk-free rate puzzle: when the RRA is lowered to obtain more plausible levels of the risk-free rate, the average capital return declines and  $Std(\Delta i)$  decreases to the values of our EZ-BKK model.

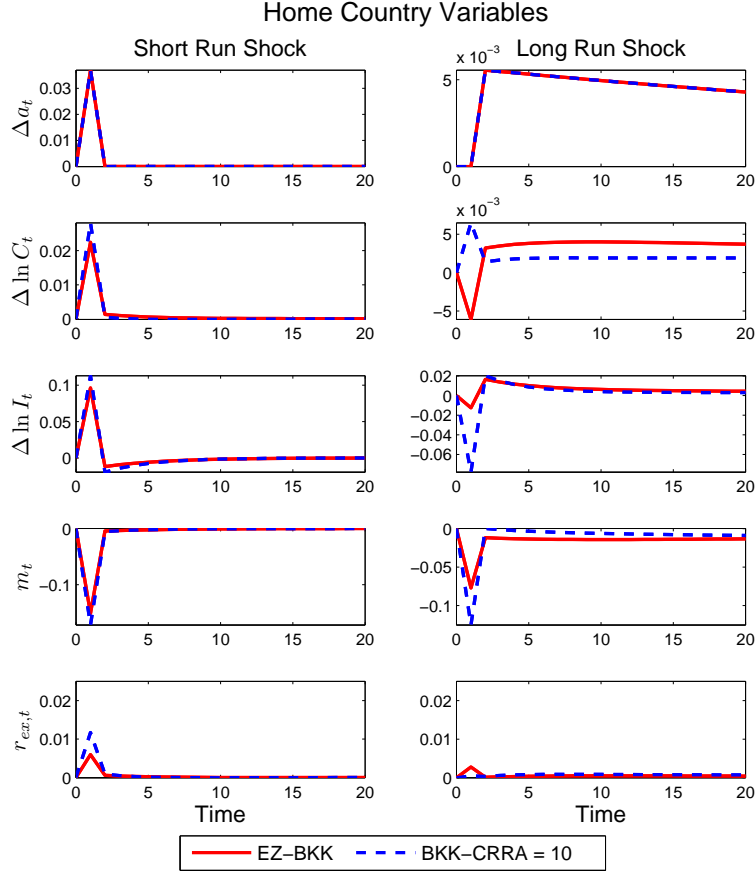
On the asset pricing side, we note that the BKK model with high risk aversion is able to generate an equity premium of 0.40%. This result is entirely driven by short-run risk, as long-run shocks have no significant impact on excess returns (Figure 2, bottom two panels). In EZ-BKK, instead, good long-run news produce positive excess returns. This is qualitatively consistent with the data.

In terms of domestic co-movements, all models produce a correlation for consumption and labor consistent with the data. All models also reproduce the negative correlation between output and net exports, although this is a more difficult task for the BKK model with RRA set to 10. As already documented before, the risk-sharing motive tends to offset the productivity channel in this setting, thus making the response of the net export to short-run shocks almost negligible.<sup>6</sup>

For both the EZ-BKK model the BKK model with low risk aversion, the contemporaneous correlation of consumption and investment is consistent with the upper bound of our estimated confidence interval. The BKK model with high risk aversion, instead, predicts an excessively low correlation. Indeed, when  $\gamma > 1$  the income effect dominates the substitution effect and the home country responds to positive long-run

---

<sup>6</sup>When we lower  $\gamma$  from 10 to 0.83,  $cov(\Delta NX/GDP, \Delta GDP)$  declines drastically. The correlation does not drop as much because the  $Std(\Delta NX/GDP)$  decreases as well.



**FIG. 2. Domestic co-movements.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. When time-additive preferences are employed, the RRA coefficient is set to 10. Shocks to the home-country productivity,  $\epsilon_a$  and  $\epsilon_x$ , materialize at time 2 and are not orthogonalized by their international correlations,  $\rho_{srr}$  and  $\rho_{lrr}$ , respectively. The short-run (long-run) shock is assumed to affect only the home country and has a magnitude  $\sigma$  ( $\sigma_x$ ).

news by consuming more and investing less (Figure 2, right panels).<sup>7</sup> This channel creates a negative co-movement that lowers  $\text{corr}(\Delta i, \Delta c)$ .

In the EZ-BKK model, instead, consumption and investment move in the same direction with respect to both short- and long-run shocks. Specifically, the home country finds it optimal to reduce both domestic consumption and investment in order to ship more resources abroad (risk sharing channel).

**International moments.** As shown in panel B of Table 3, all models feature similar international moments that are broadly aligned with the data. There are, however,

<sup>7</sup>In the BKK setting with  $\gamma = 10$ , there is no significant reallocation of resources across countries (Figure 1). Hence the aforementioned response of both consumption and investment is fully consistent with that obtained in a one-country economy.

three main differences. First, only our EZ-BKK model produces responses of  $NX$  and  $NXI$  to long-run news that are quantitatively close to the data ( $\beta_{LR}^{NX} = 0.95$  and  $\beta_{LR}^{NXI} = 0.26$ ).

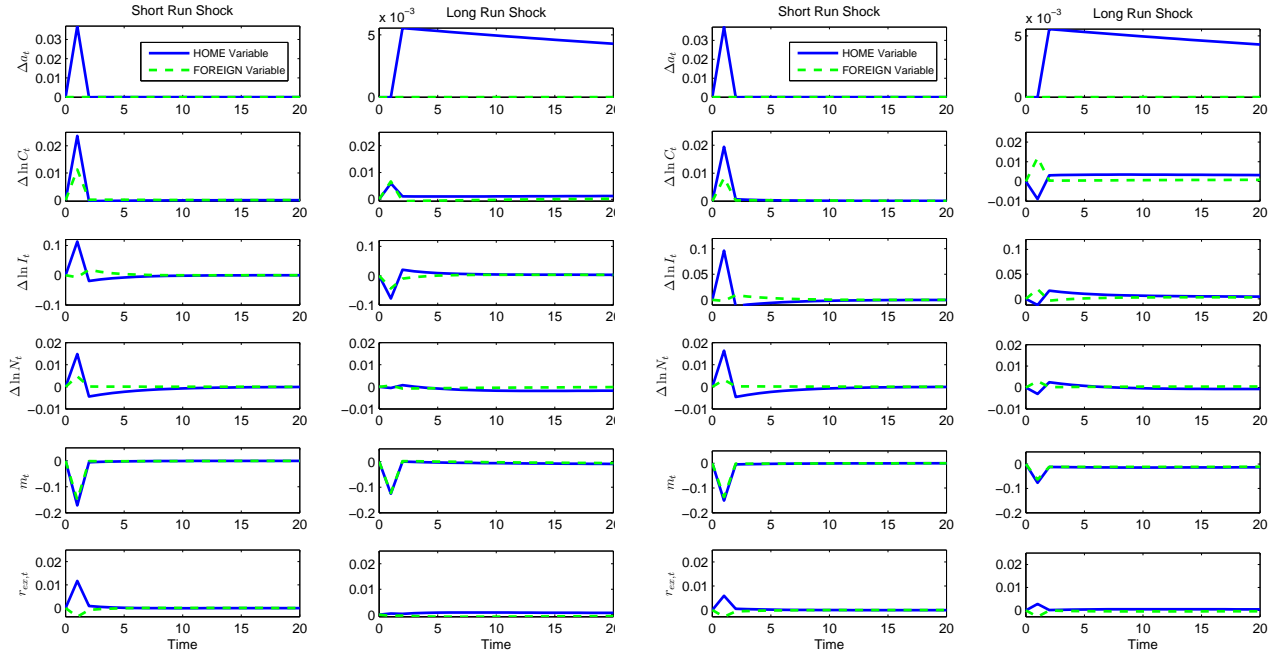
Even though our model under-predicts (over-predicts) slightly the response of the  $NXI$  ( $NX$ ) to long-run news, there are at least two reasons to regard this result as positive: (i) this is a clear improvement compared to the standard BKK model which produces responses which are one order of magnitude smaller than in the data; and (ii) these results are obtained from a calibration that mimics as much as possible the original one in BKK. In section 6, we show that a slight re-calibration of the parameters further improves on these results.

Second, the standard BKK model with high risk aversion underpredicts the volatility of the net exports with respect to our EZ-BKK setting, consistent with Figure 1. Third, our EZ-BKK model produces a counterfactual negative international correlation of investment growth rates, whereas this is not a problem in the BKK model with high risk aversion. This result is entirely driven by the long-run news and can be better understood by comparing the right panels of Figure 3(a) and 3(b). In the EZ-BKK model, optimal risk-sharing prescribes that the home country must reduce its national investment to promote a reallocation of resources abroad. The foreign country, in contrast, must use part of these goods to increase investments.<sup>8</sup>

This negative co-movement is substantial and it makes the overall investment correlation negative. In the standard BKK model with high risk aversion, in contrast, (i) the international reallocation of resources is almost absent (Figure 1), and (ii) both agents reduce domestic investment at the same time, in anticipation of higher long-run growth (the income effect). As a result, the investment correlation is consistent with the data. We improve on these dimensions in the extended version of the model to be discussed in section 4.

---

<sup>8</sup>Comparing the gap between  $\Delta i_t$  and  $\Delta i_t^*$  corresponds to studying the log-growth of the  $I_t/I_t^*$  ratio. Our impulse response functions imply that upon the arrival of long-run shocks, foreign investment immediately increases relative to the domestic one. Only over time the  $I_t/I_t^*$  ratio increases again, as documented by the higher subsequent recovery growth of domestic investment,  $\Delta i_t - \Delta i_t^* > 0$ . The exact opposite time-path occurs for the short-run shock. This is consistent with our empirical findings.



(a) Time-Additive Pref.s (RRA=10; IES=0.1)

(b) EZ Preferences (RRA=10; IES=1.2)

**FIG. 3. International co-movements.** This figure shows annual log deviations from the steady state. All the parameters are calibrated to the values reported in table 2. When time-additive preferences are employed, the RRA coefficient is set to 10. Shocks to the home-country productivity,  $\epsilon_a$  and  $\epsilon_x$ , materialize at time 2 and are not orthogonalized by their international correlation,  $\rho_{srr}$  and  $\rho_{lrr}$ , respectively. The short-run (long-run) shock is assumed to affect only the home country and has a magnitude  $\sigma$  ( $\sigma_x$ ).

**Main limitations of EZ-BKK.** Summarizing, our EZ-BKK model is able to replicate key features of both  $NX$  and  $NXI$  in connection to the arrival of long-run growth news, but a few unsatisfactory results arise: (1) investment is too smooth; (2) investment is negatively correlated across countries; and (3) the average risk premium is almost zero. In addition, our model inherits two limitations of the standard BKK model: (4) the exchange rate volatility is excessively low; and (5) consumption growth rates are internationally more correlated than output growth rates.

In the next section, we simultaneously resolve limitations (1)-(4). The full resolution of the quantity anomaly goes beyond the scope of this study.

### 5.3 Extensions

**Description of our extensions.** In this section, we resolve some of the limitations of our benchmark model by varying only one component, namely the accumulation of new capital units. Specifically, we replace equation (10) with

$$G_t = \underbrace{e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1}-\mu)}}_{ACL} \underbrace{I_{x,t}^{\lambda_i} I_{x,t}^{*1-\lambda_i}}_{EGG}, \quad G_t^* = \underbrace{e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1}^*-\mu)}}_{ACL} \underbrace{I_{y,t}^{*1-\lambda_i} I_{y,t}^{*\lambda_i}}_{EGG}, \quad (10b)$$

where  $\lambda_i$  denotes home-bias for domestic technology and ACL and EGG refers to capital accumulation empirical facts highlighted by Ai et al. (2012) and Erceg et al. (2008), respectively. We briefly explain these empirical facts in what follows.

EGG note that a substantial share of both the imports and exports in the U.S. are related to capital goods, as opposed to consumption goods. In our sample, an average of 5% of US consumption goods, and 20% of US investment goods are comprised of imports. This implies that investment home-bias is much lower than consumption home bias. To replicate this empirical fact, we must set  $\lambda = 0.95$  and  $\lambda_i = 0.80$ .<sup>9</sup> All other parameters are calibrated as in Table 2.

Working with COMPUSTAT annual data on the cross-section of US firms, ACL note that the growth rate of productivity of young vintages of capital has less exposure to aggregate productivity shocks compared to older vintages. Specifically, ACL express the productivity of an investment of age  $j$  as  $\Delta a_t^j - \mu = \phi_j(\Delta a_t - \mu) + \xi_t$  and find that the estimated  $\phi_j$  is increasing with age  $j$ . ACL show that imposing  $\phi_0 = 0$  and  $\phi_j = 1, \forall j \geq 1$  is both a good approximation of the data and a useful assumption for aggregation.<sup>10</sup>

---

<sup>9</sup>Balta and Delgado (2007) document a stronger consumption home bias for European countries and suggest a value of  $\lambda = 0.97$ . Setting  $\lambda = 0.97$  would improve our quantitative results, as it would make the risk-sharing channel even more relevant. We prefer to work with  $\lambda = 0.95$  in order to show conservative results.

<sup>10</sup>In their DSGE model, ACL abstract away from idiosyncratic shocks to the different vintages, i.e., they impose  $\Delta a_t^j - \mu = \phi_j(\Delta a_t - \mu)$ . Assuming that  $\phi_j = 1 \forall j \geq 1$ , aggregate productivity growth is a weighted average of the productivity of the 0-age vintages,  $\Delta a_t^0$ , and that of all the older vintages,  $\Delta \hat{a}_t$ . If we let  $\theta_0 \in (0, 1)$  capture the steady state relative size of the new investments, we obtain  $\Delta a_t \approx \phi_0 \Delta a_t^0 \theta_0 + \phi_1 \Delta \hat{a}_t (1 - \theta_0)$ . After assuming  $\phi_0 = 0$ , aggregate productivity shocks affect only old capital vintages and  $\Delta a_t = \Delta \hat{a}_t (1 - \theta_0)$ , i.e., aggregate productivity and old capital vintage productivity are equal to each other, after rescaling for  $1 - \theta_0$ .

After accounting for the optimal allocation of labor across different capital vintages, the equilibrium can be computed by keeping track only of the relative productivity growth gap  $-\frac{1-\alpha}{\alpha}(\Delta a_{t+1} - \mu)$ . This means that there is no need to have the entire distribution of capital age as a state variable. Further details, as well as the full derivation of these results, can be found in ACL.

According to our computations in Appendix C, equations (14) and (16) need to be replaced by:

$$\begin{aligned} \frac{1}{\lambda_i} \frac{I_{x,t}}{G_t} &= E_t \left[ M_{t+1}^X \left( \alpha \frac{X_{t+1}^{Tot}}{K_{t+1}} + (1 - \delta) Q_{k,t+1} \right) e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1} - \mu)} \right], \\ \frac{1}{\lambda_i} \frac{I_{y,t}^*}{G_t^*} &= E_t \left[ M_{t+1}^Y \left( \alpha \frac{Y_{t+1}^{Tot}}{K_{t+1}^*} + (1 - \delta) Q_{k,t+1}^* \right) e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1}^* - \mu)} \right], \\ \frac{1}{1 - \lambda_i} \frac{I_{y,t}}{G_t^*} &= E_t \left[ M_{t+1}^X \left( \alpha \frac{Y_{t+1}^{Tot}}{K_{t+1}^*} + (1 - \delta) Q_{k,t+1}^* \right) P_{t+1} e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1}^* - \mu)} \right], \\ \frac{1}{1 - \lambda_i} \frac{I_{x,t}^*}{G_t} &= E_t \left[ M_{t+1}^Y \left( \alpha \frac{X_{t+1}^{Tot}}{K_{t+1}} + (1 - \delta) Q_{k,t+1} \right) / P_{t+1} e^{-\frac{1-\alpha}{\alpha}(\Delta a_{t+1} - \mu)} \right]. \end{aligned} \quad (12b)$$

On the left hand side of each equation, we have the marginal rate of transformation of consumption into new domestic or foreign capital. This object is determined by the investment home bias,  $\lambda_i$ , and measures the marginal cost of new capital paid with certainty at time  $t$ .

On the right hand side of each equation, we have the expected present value of the future benefits of an extra unit of young vintage capital. This benefit takes into account the future productivity gap between old and new capital vintages realized at time  $t + 1$ .

**Results.** In Table 4, we focus only on the moments that significantly change as we introduce our extensions. We proceed in two steps. First, we introduce vintage capital and retain the assumption  $\lambda = \lambda_i = 0.85$ , as in our benchmark calibration. In the second step, we also introduce heterogenous home bias ( $\lambda = 0.95$  and  $\lambda_i = 0.80$ ).

The examination of the second column of Table 4 shows that the introduction of vintage capital is able to simultaneously produce volatile investment, high equity premium, and a slightly lower risk-free rate average. This is consistent with the ACL



**TABLE 4: Main Moments from Extended EZ-BKK**

	EZ-BKK	Extensions		Data
Vintage capital		✓	✓	
Milder investment home bias			✓	
StDev( $I$ )/Stdev( $X$ )	2.79	3.16	3.01	3.88 ( 0.23 )
$E[r^{ex}](\%)$	0.09	3.90	4.00	4.71 ( 2.25 )
$E[r_f](\%)$	2.06	1.62	1.78	1.46 ( 0.06 )
StDev( $\Delta e$ )(%)	1.90	1.52	5.43	8.66 ( 0.93 )
corr( $\Delta i, \Delta i^*$ )	-0.09	-0.15	0.24	0.49 ( 0.17 )

*Notes:* Empirical moments are computed using annual data from 1973 to 2006. All data sources are discussed in section 3. Numbers in parenthesis are Newey-West adjusted standard errors. Excess returns are levered using a coefficient of 3 (Garca-Feijo and Jorgensen (2010)). For the EZ-BKK model, all the parameters are calibrated as in table 2. The model augmented with vintage capital only is solved using equations (10b) and (12b) and imposing  $\lambda = \lambda_i = 0.85$ . In the model with milder investment home bias, we set  $\lambda = .95$  and  $\lambda_i = 0.80$ . The entries for the models are obtained by repetitions of small-sample simulations. Small letters denote log-units.

findings in closed-economy and is driven by the fact that the investment decision has to be made before knowing the productivity growth gap across capital vintages. As a result, both the price of capital,  $Q_t$  and  $Q_t^*$ , and the investment flows,  $I_{m,t}$  and  $I_{m,t}^*$ , fluctuate more upon the arrival of shocks, and especially so for long-run news. Note, however, that this friction produces no significant change in international moments.

The addition of heterogenous home bias across consumption and investment goods produces the exact opposite effects: it improves international moments without significantly affecting domestic moments. This is because decreasing  $\lambda_i$  from 0.85 to 0.80 reduces the extent of diminishing returns of foreign investments for the production of domestic new capital units, i.e.,  $G_{I_x^*}$  and  $G_{I_y^*}$  are less downward sloping. As a result, the productivity channel requires a smaller transfer of investment resources across the foreign and home country upon the realization of a shocks. In the case of short-run shocks, this effect produces a positive co-movement of investment growth across countries.

In Appendix C, we provide a more detailed description of the transmission mechanism associated to both the ACL and EGG frictions by studying the impulse responses gen-

erated by our extended EZ-BKK model. We conclude this section noticing that, overall, our extended EZ-BKK model produces results which are consistent with both our empirical estimates and common international financial and business cycle moments. We consider this result as a significant progress in the international macro-finance literature.

## 6 Sensitivity Analysis

In this section we assess the sensitivity of our results with respect to the key elements of our study, i.e., (i) the preference parameters related to the recursive risk-sharing motive; and (ii) the international correlation of the primitive productivity shocks, as they determine the extent of sharable risk. Starting from the EZ-BKK model calibrated as in Table 2, we vary one parameter of interest at a time and report the moments that change significantly in Table 5. We have conducted the same sensitivity analysis for our extended EZ-BKK model (see section 5.3) and found similar results. For the sake of brevity, we discuss only the case regarding the baseline EZ-BKK model.

**The role of the IES.** As we increase the IES from 1 to 1.5, the average risk-free rate declines, as common in any economy with EZ preferences. Most importantly, the contemporaneous correlation of the growth rates of consumption increases above our estimates, whereas the sensitivity coefficients of  $NX$  and  $NXI$  to long-run shocks declines below our estimates. This results impose a relevant upper bound on what the IES should be in order to match international trade data.

**The role of the RRA.** Similarly to the IES case, an increase in the risk aversion coefficient (i) decreases the average risk-free (precautionary motive), (ii) increases the international correlation of consumption growth, and (iii) tends to reduce the exposure of the net exports to long-run shocks, although to a lesser extent. Additionally, a higher risk aversion enhances the incentives to trade investment goods and hence it amplifies the volatility of national investment growth.

**TABLE 5: Sensitivity Analysis of EZ-BKK**

Panel A: the role of the IES ( $\psi$ )				
Moments	Data	EZ-BKK	IES=1	IES=1.5
$E[r_f]$ (%)	1.46 ( 0.06 )	2.06	2.71	0.82
$\text{corr}(\Delta c, \Delta c^*)$	0.58 ( 0.12 )	0.55	0.53	0.69
$\beta_{LR}^{NX}$	0.61 ( 0.04 )	0.95	1.29	-0.16
$\beta_{LR}^{NXI}$	0.42 ( 0.02 )	0.26	0.29	0.13
Panel B: the role of the RRA ( $\gamma$ )				
Moments	Data	EZ-BKK	RRA=5	IES=15
$\text{StD}(\Delta i)/\text{Std}(\Delta x^T)$	3.88 ( 0.23 )	2.79	2.65	3.16
$E[r_f]$ (%)	1.46 ( 0.06 )	2.06	3.49	0.90
$\text{corr}(\Delta c, \Delta c^*)$	0.58 ( 0.12 )	0.55	0.46	0.62
$\beta_{LR}^{NX}$	0.61 ( 0.04 )	0.95	1.12	0.76
Panel C: the role of the international short-run correlation ( $\rho_{srr}$ )				
Moments	Data	EZ-BKK	$\rho_{srr} = 0$	$\rho_{srr} = 0.39$
$\text{corr}(\Delta \frac{NX}{X^T}, \Delta x^T)$	-0.54 ( 0.11 )	-0.53	-0.58	-0.45
$\text{corr}(\Delta c, \Delta c^*)$	0.58 ( 0.12 )	0.55	0.45	0.69
$\text{corr}(\Delta i, \Delta i^*)$	0.49 ( 0.17 )	-0.09	-0.22	0.14
$\text{corr}(\Delta n, \Delta n^*)$	0.55 ( 0.11 )	0.43	0.30	0.61
$\text{corr}(r_f, r_f^*)$	0.71 ( 0.08 )	0.68	0.61	0.77
Panel D: the role of the international long-run correlation ( $\rho_{lrr}$ )				
Moments	Data	EZ-BKK	$\rho_{lrr} = .91$	$\rho_{lrr} = 0.99$
$\beta_{LR}^{NX}$	0.61 ( 0.04 )	0.95	1.04	0.87
$\beta_{LR}^{NXI}$	0.42 ( 0.02 )	0.26	0.28	0.24

*Notes:* Empirical moments are computed using annual data from 1973 to 2006. All data sources are discussed in section 3. Numbers in parenthesis are Newey-West adjusted standard errors. All the parameters are calibrated as in table 2, unless differently specified. The entries for the models are obtained by repetitions of small-sample simulations.

**The role of the short-run correlation.** Increasing the correlation of the short-run shocks,  $\rho_{srr}$ , has two main effects: (i) it increases the cross-country correlation of all macroeconomic quantities including the risk-free rate, and (ii) it reduces the counter-cyclicality of the net exports, as there is less incentive to trade in order to hedge short-run shocks. On the positive side, the reduction of cross-country trade of investment goods enables the investment correlation to become positive, as in the data. On the negative side, the correlation of the consumption growth rates becomes 18% greater than our empirical estimate.

**The role of the long-run correlation.** Finally, we note that changing the correlation of the long-run shocks from the value estimated in the data, 0.91, to a maximum of 0.99 mainly affects the sensitivity of the net exports to long-run shocks. Not sur-

prisingly, when the correlation is lowered, there is a stronger incentive to trade and the estimated betas become more sizeable.

## 7 Concluding Remarks

We provide novel empirical evidence for G7 countries about the effects of international long-term productivity news on international capital flows. In contrast to short-run growth news, positive long-run growth news produce an outflow of resources, i.e., an increase in net exports which results in a relatively higher level of investment abroad. Through the lens of a standard BKK model with time-additive preferences, our empirical facts are shown to be an anomaly. We then investigate the effect of international long-term growth risk on capital flows in a BKK economy featuring a frictionless recursive risk-sharing scheme based on Epstein and Zin (1989) preferences. This modification alone is able to replicate our empirical findings.

In a second step, we enrich the BKK production structure to capture relevant empirical evidence on capital accumulation. By adding (i) heterogeneous home-bias across consumption and investment goods (Erceg et al. 2008), and (ii) heterogeneous productivity across capital vintages (Ai et al. 2012), our approach replicates key moments of both international asset prices and quantities. We regard these results of great interest for future research in international macro-finance.

Future research should focus on the long-term fiscal and monetary policy implications of our model. It will also be important to take into consideration both private (Maggiore 2011 and Maggiore and Gabaix 2013) and sovereign (Aguiar and Amador 2013) credit frictions. Studying the role of capital flows in the determination of long-term price and shock elasticities (Borovička et al. 2011) is relevant as well, especially because this could shed new light on the behavior of long-term currency risk premia (Engel 2012) and their implications for international liabilities (Rey and Gourinchas 2007). Furthermore, our model should be extended to study international capital flows in economies with broader forms of heterogeneity (Bhamra and Uppal

2010, Ready et al. 2012, and Hassan 2013) and near-rational investment (Hassan and Mertens 2014).

# References

- Aguiar, M., and M. Amador. 2013. Sovereign debt. *Handbook of International Economics* 4.
- Ai, H., M. M. Croce, and K. Li. 2012. Toward a Quantitative General Equilibrium Asset Pricing Model with Intangible Capital. *Review of Financial Studies*. Forthcoming.
- Alessandria, G., J. P. Kaboski, and V. Midrigan. 2011. US trade and inventory dynamics. *American Economic Review* 101.
- Anderson, E. 2005. The dynamics of risk-sensitive allocations. *Journal of Economic Theory* 125(2):93–150.
- Backus, D. K., P. J. Kehoe, and F. E. Kydland. 1994. Dynamics of the trade balance and the terms of trade: The j-curve? *The American Economic Review* 84(1):pp. 84–103.
- Balta, N., and J. Delgado. 2007. Home Bias and market Integration in the EU. Working Paper, BRUEGEL.
- Bansal, R., and C. Lundblad. 2002. market efficiency, asset returns, and the size of the risk premium in global equity markets. *Journal of Econometrics* 109.
- Bansal, R., and A. Yaron. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59:1481–1509.
- Bansal, R., D. Kiku, and A. Yaron. 2010. Risks for the long run: Estimation and inference. *Working Paper*.
- Baxter, M., and M. J. Crucini. 1995. Business cycles and the asset structure of foreign trade. *International Economic Review* 36.
- Bhamra, H. S., and R. Uppal. 2010. Asset Prices with Heterogeneity in Preferences and Beliefs. Working Paper, University of British Columbia.
- Borovička, J., L. Hansen, M. Hendricks, and J. Scheinkman. 2011. Risk-price dynamics. *Journal of Financial Econometrics* 9:3–65.

- Campbell, J. 2003. Consumption-based asset pricing. *forthcoming in Handbook of the Economics of Finance*. George Constantinides, Milton Harris, and Rene Stulz eds., North-Holland, Amsterdam.
- Cavallo, M., and A. Landry. 2010. The quantitative role of capital goods imports in us growth. *American Economic Review* 100(2).
- Coeurdacier, N., H. Rey, and P. Winant. 2012. Financial Integration and Growth in a Risky World. Working Paper, London School of Economics.
- Colacito, R., and M. Croce. 2010. Recursive Allocations and Wealth Distribution with Multiple Goods. Working Paper, UNC.
- Colacito, R., and M. M. Croce. 2011. Risks for the Long Run and the Real Exchange Rate. *Journal of Political Economy* 119(1):153–182.
- Colacito, R., and M. M. Croce. 2012. International Asset Pricing with Recursive Preferences. *Journal of Finance*. Forthcoming.
- Croce, M. M. 2014. Long-Run Productivity Risk: A New Hope For Production-Based Asset Pricing? *Journal of Monetary Economics*, 66.
- Crucini, M., M. A. Kose, and C. Otrok. 2011. What are the driving forces of international business cycles? *The Review of Economic Dynamics* 14.
- Engel, C. 2012. The Real Exchange Rate, Real Interest Rates, and the Risk Premium. Working Paper, University of Wisconsin.
- Epstein, L. G., and S. E. Zin. July 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57(4): 937–69.
- Erceg, C., L. Guerrieri, and C. Gust. 2008. Trade Adjustment and the Composition of Trade. *Journal of Economic Dynamics and Control* 32(8).
- Feenstra, R. C., R. Inklaar, and M. Timmer. The next generation of the penn world table. Working paper National Bureau of Economic Research 2013.

- Garca-Feijo, L., and R. D. Jorgensen. 2010. Can Operating Leverage Be the Cause of the Value Premium? *Financial Management*.
- Gourinchas, P.-O., and H. Rey. 2013. External adjustment, global imbalances and valuation effects. *Handbook of International Economics* 4.
- Gourio, F., and P.-A. Noual. 2006. The Marginal Worker and the Aggregate Elasticity of Labor Supply. Working Paper, Boston University.
- Greenwood, J., Z. Hercowitz, and G. Huffman. 1988. Investment capacity utilization and the real business cycle. *American Economic Review* 78.
- Hassan, T. 2013. Country size, currency unions, and international asset returns. *Journal of Finance* 68(6).
- Hassan, T., and T. Mertens. 2014. The social cost of near-rational investment. Working Paper, STERN School of Business, New York University.
- Heathcote, J., and F. Perri. 2002. Financial autarky and international real business cycles. *Journal of Monetary Economics* 49.
- Heathcote, J., and F. Perri. 2004. Financial globalization and real regionalization. *Journal of Economic Theory* 119.
- Heathcote, J., and F. Perri. 2013. The international diversification puzzle is not as bad as you think. *Journal of Political Economy* 121.
- Kehoe, P. J., and F. Perri. 2002. International business cycles with endogenous incomplete markets. *Econometrica* 70:907–928.
- Kose, M. A., C. Otrok, and C. H. Whiteman. 2003. International Business Cycles: World, Region, and Country-Specific Factors. *The American Economic Review* 93.
- Kose, M. A., C. Otrok, and C. H. Whiteman. 2008. Understanding the evolution of world business cycles. *Journal of International Economics* 75.
- Lewis, K. 2011. Global Asset Pricing. *Annual Reviews of Financial Economics*.



- Lucas, Robert E., J. 1990. Why doesn't capital flow from rich to poor countries? *The American Economic Review* 80(2).
- Lustig, H., N. Roussanov, and A. Verdelhan. 2011. Common risk factors in currency markets. *Forthcoming in the Review of Financial Studies*.
- Maggiore, M. 2011. Financial intermediation, international risk sharing, and reserve currencies. Working Paper, Harvard University.
- Maggiore, M., and X. Gabaix. 2013. International liquidity and exchange rate dynamics. Working Paper, STERN School of Business, New York University.
- Obstfeld, M. 1998. The global capital market: Benefactor or menace? *Journal of Economic Perspectives* 12:9–30.
- Papanikolaou, D. 2011. Investment-specific shocks and asset prices. *Journal of Political Economy*. Forthcoming.
- Pavlova, A., and R. Rigobon. 2007. Asset prices and exchange rates. *Review of Financial Studies* 20(4).
- Petrosky-Nadeau, N. 2011. Endogenous flows of foreign direct investment and international real business cycles. Working paper, CMU.
- Quinn, D. 1997. The correlates of change in international financial regulation. *American Political Science Review* 91(3):531–551.
- Raffo, A. 2008. Net exports, consumption volatility and international business cycle models. *Journal of International Economics*.
- Ready, R., N. Roussanov, and C. Ward. 2012. Commodity Trade and the Carry Trade: A Tale of Two Countries. Working Paper, the Wharton School.
- Rey, H., and P.-O. Gourinchas. 2007. International financial adjustment. *Journal of Political Economy*.
- Stambaugh, R. 1986. Bias in regressions with lagged stochastic regressors. Working Paper, University of Chicago.

Taylor, A. 1996. International capital mobility in history: the saving-investment relationship. *NBER Working Paper* 5743.

Tretvoll, H. 2012. Real exchange rate variability in a two-country cycle model. Working paper, NYU.

Van Nieuwerburgh, S., and L. Veldkamp. 2008. Information immobility and the home bias puzzle. *Journal of Finance* 64(3).

## **Appendix A: Additional Empirical Results**

In the top portion of table A1, we report the estimates of the coefficients defined in equations (3)-(5) when the net exports of cars are consolidated with the net exports of capital goods in the BEA table 4.2.5. This implies that cars are included both in the net exports of investment goods (NXI) and in total net exports (NX). The main results described in our empirical section continue to hold: positive long-run shocks produce a net outflow of resources. Our empirical results are noisy for the case US vs UK, but become more reliable when we look at US vs RoW.

In the bottom part of the table, we retain our benchmark assumption of treating cars as durables and hence we exclude them from both total (NX) and investment net exports (NXI). We check the robustness of our empirical results to the presence of shocks to the price of investment goods relative to consumption goods. Specifically, we take the US NXI and divide it by the quality-adjusted price of investment goods used by Papanikolaou (2011) in order to obtain a quantity index. Our estimation confirms that positive long-run shocks produce a net outflow of investment resources.

We have conducted many more robustness exercises (for example, using OCSE data to construct productivity growth; removing subcomponents of industrial supplies,...) and found that the outflow of resources upon the arrival of positive relative long-run shocks is an empirical regularity when we focus on the US vs UK or US vs RoW. Further results are available upon request.

## **Appendix B: Intuitions on Extended EZ-BKK**

In figure B1 and B2, we report impulse responses to compare the predictions of our EZ-BKK model with those of our extended model featuring both vintage capital and heterogenous home-bias.

Figure B1 confirms the three main findings obtained by ACL in a one-country production economy. First, vintage capital leaves unaltered the responses with respect

**TABLE A1: Empirical Evidence**

Dep. Var.	Shock	US vs UK		US vs RoW	
		Predict. Var.s:		Predict. Var.s:	
		pd	pd,rf,dc,di	pd	pd,rf,dc,di
Cars included in both NXI and NX.					
NX/GDP	SR	0.000	-0.027	-0.083 ***	-0.089 ***
		( 0.030 )	( 0.100 )	( 0.009 )	( 0.006 )
	LR	0.535	0.230	0.652 ***	0.112 ***
		( 0.646 )	( 0.155 )	( 0.037 )	( 0.018 )
NXI/GDP	SR	0.053 *	0.018	-0.119 ***	-0.113 ***
		( 0.029 )	( 0.054 )	( 0.006 )	( 0.003 )
	LR	0.746 ***	0.047	0.464 ***	-0.026
		( 0.208 )	( 0.042 )	( 0.026 )	( 0.013 )
Adjusting for investment-price shocks ( $NXI \frac{P_c}{P_i}$ ).					
NXI/GDP	SR	0.01	-0.02	-0.05 ***	-0.05 ***
		( 0.01 )	( 0.02 )	( 0.00 )	( 0.00 )
	LR	0.32 ***	0.04 **	0.18 ***	-0.01
		( 0.09 )	( 0.01 )	( 0.01 )	( 0.00 )

*Notes:* The panel labeled “Cars included in both NXI and NX” reports the responses of Total Net Exports over GDP ( $NX/GDP$ ), and Net Exports of Investments over GDP ( $NXI/GDP$ ) to the difference of short-run shocks (SR) and long-run shocks (LR), specified in equation (3). Relative to Table 1, Automotive vehicles, engines, and parts were included in the calculation of both the Total Net Exports and the Net Exports of Investments. Long-run risks were estimated by regressing Solow residuals on the corresponding set of predictive variables indicated by the column titles, as in equations (1)-(2b). The variables pd, rf, dc, and di correspond to price-dividend, risk-free rate, consumption growth, and investment growth, respectively. The columns labeled “US vs UK” refer to the estimations conducted using only data from the United States and the United Kingdom. The columns labeled “US vs RoW” show the results for the case of the panel estimation in which all the loadings on the short- and long-run news are restricted to be the same for each country pair. The numbers in brackets underneath each point estimate are heteroskedasticity-adjusted standard errors. One, two, and three asterisks denote 10%, 5%, and 1% significance, respectively. The panel labeled “Adjusting for investment-price shocks” reports the same analysis for the response of  $NXI/GDP$  excluding cars, and after controlling for the presence of shocks to the US price of investment goods relative to consumption goods.

to short-run growth shocks, as these shocks are *i.i.d.* and hence do not alter the expected relative productivity gap between young and old investments. This also implies that the extended model inherits all problems and successes featured by the EZ-BKK setting with respect to short-run shocks.

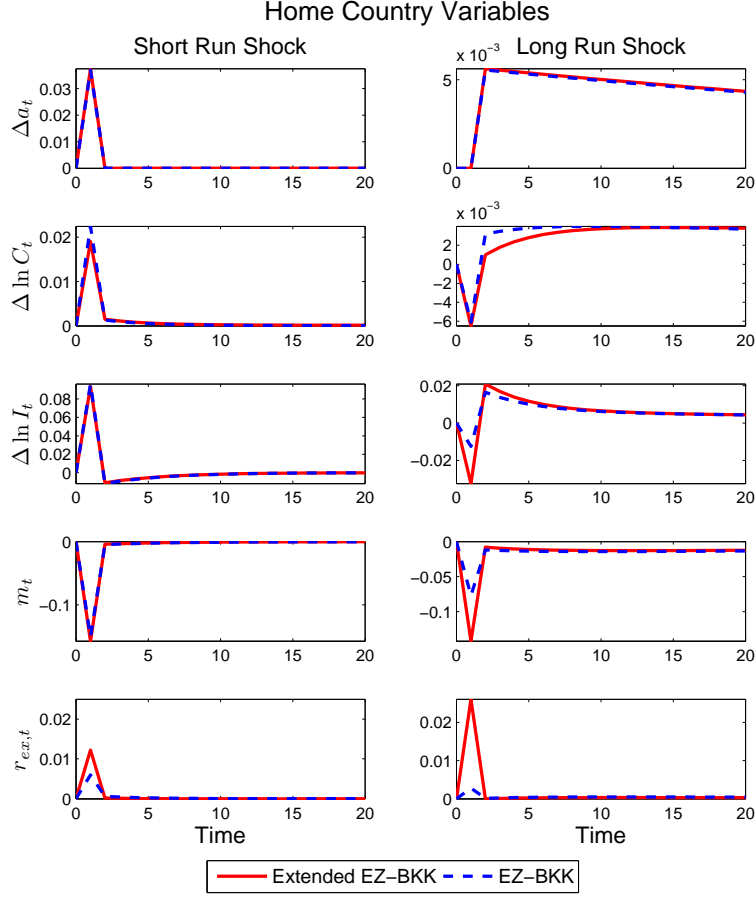
Second, the relative productivity gap across capital vintages produces more pronounced reactions of investment, making its growth rate more volatile, as in the data. It is

easier to interpret this response if expressed in terms of the level of the investment-output share. Upon the realization of the shock, it is optimal to post-pone investment in young capital vintages as they feature a negative productivity gap against old capital. At the equilibrium, the investment-gdp share immediately falls. Subsequently, the investment share slowly recovers (positive investment growth rate from period 2 onward), as the incentive to invest in new capital vintages becomes stronger.

Third, more severe fluctuations in the value of capital,  $Q_t$ , generate more volatile capital excess returns. At the equilibrium, the covariance with the stochastic discount factor becomes more negative, implying a positive and sizeable long-run risk driven risk premium.

Figure B2 sheds light on the role of heterogenous home bias. First, the increase in  $\lambda$  from 0.85 to 0.95 reduces the ability of sharing risk through the trade of consumption goods. Because the marginal rate of transformation between local and foreign consumption bundles becomes smaller with respect to each good, local productivity shocks becomes more regional, i.e., they affect the two country growth prospects in a more heterogenous fashion. As a result, the home and foreign stochastic discount factors in consumption units,  $m_t$  and  $m_t^*$ , adjust with significantly different intensity and the exchange rate,  $\Delta e_t = m_t^* - m_t$ , becomes more volatile.

Second, decreasing  $\lambda_i$  from 0.85 to 0.80 reduces the extent of diminishing returns of foreign investments for the production of domestic new capital units, i.e.,  $G_{I_x^*}$  and  $G_{I_y^*}$  are less downward sloping. As a result, the marginal rate of transformation of investment goods improves and it frees up a significant amount of resources when short-run shocks materialize. Consider a positive short-run shock to the home country (Figure C.2(b), left panels). The productivity channel requires a smaller transfer of investment resources from the foreign to the home country. The remaining resources are used to promote more investment growth in the foreign country. At the equilibrium, the overall cross-country correlation of investment is positive and closer to the data. This is the main difference with respect to the investment dynamics in



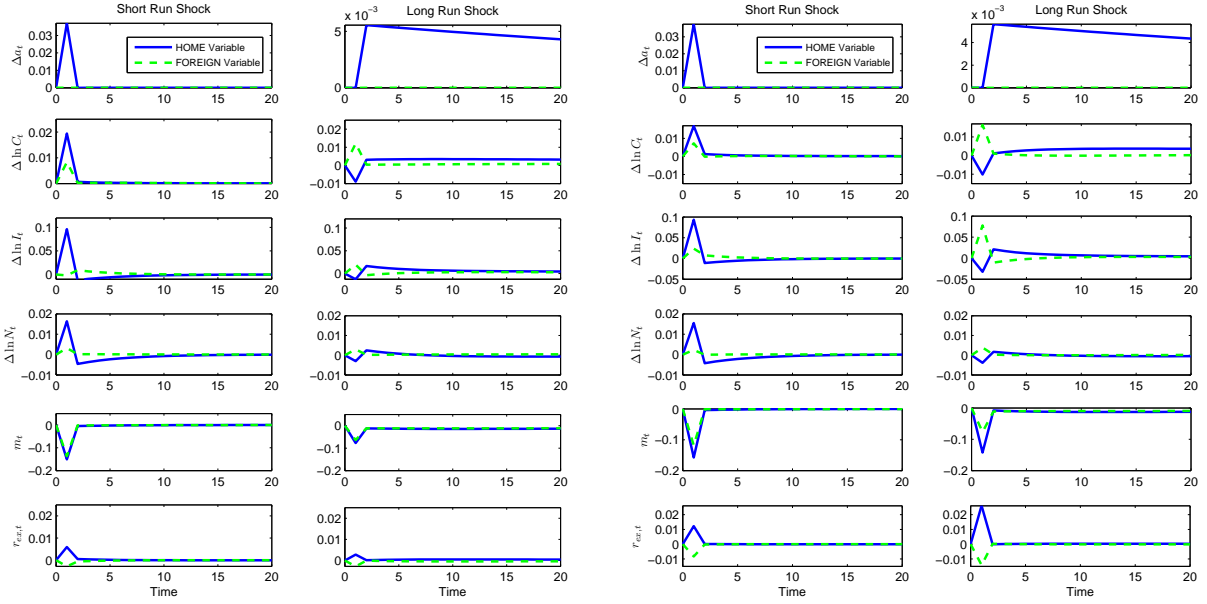
**FIG. B1. Domestic co-movements.** This figure shows annual log deviations from the steady state. For the baseline model denoted as ‘EZ-BKK’, all the parameters are calibrated to the values reported in table 2. Under the ‘Extended EZ-BKK’ model, capital accumulation is determined by equations (10b)-(12b). In this case, the investment home-bias,  $\lambda_i$ , is 0.80 and the consumption home-bias,  $\lambda$ , is 0.95. Shocks to the home-country productivity,  $\epsilon_a$  and  $\epsilon_x$ , materialize at time 2 and are not orthogonalized by their international correlations,  $\rho_{srr}$  and  $\rho_{lrr}$ , respectively. The short-run (long-run) shock is assumed to affect only the home country and has a magnitude  $\sigma$  ( $\sigma_x$ ).

the EZ-BKK setting.

## Appendix C: Pareto Problem

For the sake of brevity, in this appendix we suppress notation to denote state and histories and retain only subscripts for time. We represent the Epstein and Zin (1989) utility preference in the following compact way:

$$U_t = W(\tilde{C}_t, U_{t+1}),$$



(a) EZ-BKK

(b) Extended EZ-BKK

**FIG. B2. International co-movements.** This figure shows annual log deviations from the steady state. For the baseline model denoted as ‘EZ-BKK’, all the parameters are calibrated to the values reported in table 2. Under the ‘Extended EZ-BKK’ model, capital accumulation is determined by equations (10b)-(12b). In this case, the investment home-bias,  $\lambda_i$ , is 0.80 and the consumption home-bias,  $\lambda$ , is 0.95. Shocks to the home-country productivity,  $\epsilon_a$  and  $\epsilon_x$ , materialize at time 2 and are not orthogonalized by their international correlation,  $\rho_{srr}$  and  $\rho_{lrr}$ , respectively. The short-run (long-run) shock is assumed to affect only the home country and has a magnitude  $\sigma$  ( $\sigma_x$ ).

so that the dependence of current utility on  $j$ -step ahead consumption can easily be denoted as follows:

$$\frac{\partial U_t}{\partial \tilde{C}_{t+j}} = W_{2,t+1} \cdot W_{2,t+2} \cdots W_{2,t+j} W_{1,t+j}, \quad (C1)$$

where  $W_{2,t+j} \equiv \frac{\partial U_{t+j-1}}{\partial U_{t+j}}$  and  $W_{1,t+j} \equiv \frac{\partial U_{t+j}}{\partial \tilde{C}_{t+j}}$ . Given this notation, the intertemporal marginal rate of substitution between  $\tilde{C}_t$  and  $\tilde{C}_{t+1}$  is

$$IMRS_{\tilde{C},t+1} = \frac{W_{2,t+1} W_{1,t+1}}{W_{1,t}} = M_{t+1} \pi_{t+1}, \quad (C2)$$

where  $M_{t+1}$  is the stochastic discount factor in  $\tilde{C}$  units and it has the following form:

$$M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t [U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.$$

The consumption bundle,  $\tilde{C}$ , depends on both the consumption aggregate,  $C$ , and labor,  $N$ :

$$\tilde{C}_t = \tilde{C}(C_t, N_t).$$

The consumption aggregate combines together two goods,  $x$  and  $y$ :

$$C_t = C(x_t, y_t).$$

The planner faces the following constraints:

$$F(A_t, K_t, N_t) \geq x_t + x_t^* + I_{x,t} + I_{y,t} \quad (\text{C3})$$

$$F(A_t^*, K_t^*, N_t^*) \geq y_t + y_t^* + I_{x,t}^* + I_{y,t}^* \quad (\text{C4})$$

$$K_t \leq (1 - \delta)K_{t-1} + e^{\omega_t} G(I_{x,t-1}, I_{x,t-1}^*) \quad (\text{C5})$$

$$K_t^* \leq (1 - \delta)K_{t-1}^* + e^{\omega_t^*} G^*(I_{y,t-1}, I_{y,t-1}^*), \quad (\text{C6})$$

where  $A_t$  and  $A_t^*$  are the exogenous stochastic productivity processes in equation (9). The processes  $w_t = -\frac{1-\alpha}{\alpha}(\Delta a_t - \mu)$  and  $(w_t^* = -\frac{1-\alpha}{\alpha}(\Delta a_t^* - \mu))$  result from the vintage capital structure assumed in Ai et al. (2012).

The social planner chooses  $\{x_t, x_t^*, y_t, y_t^*, N_t, N_t^*, K_t, K_t^*, I_{x,t}, I_{y,t}, I_{x,t}^*, I_{y,t}^*\}_t$  to maximize

$$\mu_0 W_0 + (1 - \mu_0) W_0^*,$$

subject to sequences of constraints (C3)–(C6). Specifically, let  $\lambda_{i,t}$  be the Lagrangian multiplier for the time  $t$  constraint (Bi); then the Lagrangian is

$$\begin{aligned} \Omega = & \mu_0 W_0 + (1 - \mu_0) W_0^* \\ & + \dots \\ & + \lambda_{1,t} (F(A_t, K_t, N_t) - x_t - x_t^* - I_{x,t} - I_{y,t}) \\ & + \lambda_{2,t} (F(A_t^*, K_t^*, N_t^*) - y_t - y_t^* - I_{x,t}^* - I_{y,t}^*) \\ & + \lambda_{3,t} ((1 - \delta)K_{t-1} + e^{\omega_t} G(I_{x,t-1}, I_{x,t-1}^*) - K_t) \\ & + \lambda_{4,t} ((1 - \delta)K_{t-1}^* + e^{\omega_t^*} G^*(I_{y,t-1}, I_{y,t-1}^*) - K_t^*) \\ & + \dots \end{aligned}$$

The optimality condition for the allocation of good  $X_t$  for  $t = 1, 2, \dots$  in each possible state is

$$\mu_0 \cdot \left( \prod_{j=1}^t W_{2,j} \right) \cdot W_{1,t} \tilde{C}_{C,t} C_{x,t} = \lambda_{1,t} = C_{x^*,t}^* \tilde{C}_{C^*,t}^* W_{1,t}^* \cdot \left( \prod_{j=1}^t W_{2,j}^* \right) \cdot \mu_0^*, \quad (\text{C7})$$

where  $\mu_0^* = (1 - \mu_0)$ ,  $\tilde{C}_{C,t} = \partial \tilde{C}_t / \partial C_t$ ,  $C_{x,t} = \partial C_t / \partial x_t$ , and the analogous partial derivatives for the foreign country are denoted by  $^*$ .



Define  $\mu_t$  as the date  $t$  Pareto weight for the home country. Using equation (C2), we get

$$\begin{aligned}\mu_t &= \mu_0 \cdot \left( \prod_{j=1}^t W_{2,j} \right) \cdot W_{1,t} C_t \\ &= \mu_{t-1} \cdot W_{2,t}^i \cdot \frac{W_{1,t}}{W_{1,t-1}} \cdot \frac{C_t}{C_{t-1}} = \mu_{t-1} \cdot M_t \cdot \frac{C_t}{C_{t-1}}.\end{aligned}$$

It follows that equation (C7) can be rewritten as

$$\mu_t \cdot \tilde{C}_{C,t} C_{x,t} \frac{1}{C_t} = \frac{1}{C_t^*} C_{x^*,t}^* \tilde{C}_{C^*,t}^* \cdot \mu_t^* \quad (\text{C8})$$

Let  $S_t \equiv \mu_t / \mu_t^*$ , and note that with GHH preferences,  $\tilde{C}_{C,t} = 1$ . Then the optimality condition in equation (C8) can be represented by the following system of recursive equations:

$$\begin{aligned}S_t \cdot C_{x,t} \cdot \frac{1}{C_t} &= C_{x^*,t}^* \cdot \frac{1}{C_t^*} \\ S_t &= S_{t-1} \frac{M_t e^{\Delta c_t}}{M_t^* e^{\Delta c_t^*}}.\end{aligned} \quad (\text{C9})$$

In a similar fashion, the optimal allocation of good  $Y$  is determined by

$$S_t \cdot C_{y,t} \cdot \frac{1}{C_t} = C_{y^*,t}^* \cdot \frac{1}{C_t^*}.$$

Given our GHH preferences, the optimal allocation of labor implies the following standard intratemporal conditions:

$$\begin{aligned}\tilde{C}_{N,t} &= -F_{N,t} C_{X,t} \\ \tilde{C}_{N^*,t}^* &= -F_{N^*,t}^* C_{Y^*,t}^*,\end{aligned}$$

where  $C_{X,t} = \partial C_t / \partial X_t$ ,  $C_{Y^*,t}^* = \partial C_t^* / \partial Y_t^*$ ,  $\tilde{C}_{N,t} = \partial \tilde{C}_t / \partial N_t$ , and  $F_{N,t} = \partial F_t / \partial N_t$ .

Let  $s_{t+1}$  index the possible states at time  $t+1$ . The first-order condition with respect to  $I_{x,t}$  is

$$\begin{aligned}-\lambda_{1t} + \sum_{s_{t+1}} (\lambda_{3,t+1} e^{\omega_{t+1}} G_{I_{x,t}}) &= 0 \\ \Leftrightarrow \sum_{s_{t+1}} \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}} \omega_{t+1}^h \right) &= \frac{1}{G_{I_{x,t}}}.\end{aligned}$$

By definition,  $IMRS_{t+1|t}^x = \frac{\lambda_{1,t+1}}{\lambda_{1,t}} = \frac{\partial U_0 / \partial x_{t+1}}{\partial U_0 / \partial x_t} = M_{t+1}^x \pi_{t+1|t}$  for  $i \in \{h, f\}$ , where  $M_{t+1}^x$  is the stochastic discount factor in  $X$ -units. Substituting the stochastic discount factor into the above equation, we have

$$\frac{1}{G_{I_{x,t}}} = E_t[M_{t+1}^x P_{k,t+1} e^{\omega_{t+1}}], \quad (\text{C10})$$

where  $G_{I_{x,t}} \equiv \frac{\partial G(I_{x,t}, I_{x,t}^*)}{\partial I_{x,t}}$ , and  $P_{k,t+1} \equiv \frac{\lambda_{3,t+1}}{\lambda_{1,t+1}}$  is the cum-dividend price of capital in  $X$ -units.

The optimal accumulation of  $K_t$  has to satisfy

$$\begin{aligned} -\lambda_{3,t} + \lambda_{1,t}F_{k,t} + \sum_{s_{t+1}}((1-\delta)\lambda_{3,t+1}) &= 0 \\ \Leftrightarrow E_t[M_{t+1}^x(1-\delta)P_{k,t+1}] + F_{k,t} &= P_{k,t}, \end{aligned}$$

where  $F_{k,t} \equiv \frac{\partial F}{\partial k_t}$ . Define  $Q_{k,t} \equiv E_t[M_{t+1}^x P_{k,t+1}]$  as the ex-dividend price of capital. Then we have

$$\begin{aligned} P_{k,t} &= F_{k,t} + (1-\delta)Q_{k,t} \\ Q_{k,t} &= E_t[M_{t+1}^x P_{k,t+1}] \end{aligned}$$

and

$$R_{k,t+1} = \frac{P_{k,t+1}}{Q_{k,t}}.$$

The first-order condition with respect to  $I_{y,t}$  states the following:

$$\begin{aligned} -\lambda_{1,t} + \sum_{s_{t+1}} \left( \lambda_{4,t+1} e^{\omega_{t+1}^*} G_{I_y,t}^* \right) &= 0 \\ \Leftrightarrow \sum_{s_{t+1}} \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \frac{\lambda_{4,t+1}}{\lambda_{2,t+1}} \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}} e^{\omega_{t+1}^*} \right) &= \frac{1}{G_{I_y,t}^*}, \end{aligned}$$

where  $G_{I_y,t}^* \equiv \frac{\partial G_t^*}{\partial I_{y,t}}$ . Similarly to what done for the home country, define  $P_{k,t+1}^* \equiv \frac{\lambda_{4,t+1}}{\lambda_{2,t+1}}$  as the cum-dividend price of capital in  $Y$ -units and note that  $P_{t+1} = \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}$  measures the terms of trade. It is then possible to obtain that

$$\frac{1}{G_{I_y,t}^*} = E_t[M_{t+1}^x P_{k,t+1}^* P_{t+1} e^{\omega_{t+1}^*}]. \quad (\text{C11})$$

Define  $M_{t+1}^y \equiv \frac{\lambda_{2,t+1}}{\lambda_{2,t}}$  as the SDF in  $Y$ -units. The remaining first-order conditions imply

$$\begin{aligned} \frac{1}{G_{I_y,t}^*} &= E_t[M_{t+1}^y P_{k,t+1}^* e^{\omega_{t+1}^*}] \\ P_{k,t}^* &= F_{k,t}^* + (1-\delta)Q_{k,t}^* \\ R_{k,t+1}^* &= \frac{P_{k,t+1}^*}{Q_{k,t}^*} \\ Q_{k,t}^* &= E_t[M_{t+1}^y P_{k,t+1}^*] \\ \frac{1}{G_{I_x,t}^*} &= E_t \left[ M_{t+1}^y P_{k,t+1}^* \frac{1}{P_t} e^{\omega_{t+1}^*} \right]. \end{aligned} \quad (\text{C12})$$

We use perturbation methods to solve our system of equations. We compute our policy functions using the `dynare++4.2.1` package. All variables included in our `dynare++` code are expressed in log-units.