The Credit Spread Puzzle - Myth or Reality? *

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Abstract

Many papers find that standard structural models predict corporate bond spreads that are too low compared to actual spreads, giving rise to the so-called credit spread puzzle. We show that the puzzle derives in large part from strong biases and low statistical power in commonly adopted approaches to testing the models. The biases are due to Jensen's inequality and arise when tests are carried out on a representative firm rather than on individual firms. Using data on individual firms during 2002-2012 we quantify the size of the bias in spread predictions and find it to be particularly severe for high-quality firms and short-maturity bonds. Low statistical power arises because ex-post realized default frequency - often used to calibrate models - is a very poor estimate of ex-ante default probability. Based on the average 10-year cumulative default frequency of 4.39% during 1970-1998 for BBB-rated firms, we estimate the 95% confidence band for the ex ante default probability to range from 1.9% to 20.1%. Finally, we test the Merton model via a bias-free approach using more than half a million transactions in the period 2002-2012. We find that the Merton model captures both the average level and time series variation of 10-year BBB-AAA spreads.

Keywords: Credit spread puzzle, Merton model, Structural models, Corporate bond spreads, Realized default frequencies;

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1. Introduction

Structural models were introduced by Merton (1974) and represent one of the most widely employed frameworks for the analysis of credit risk. However, many papers find that standard structural models predict spreads that are too low compared to actual spreads, giving rise to the so-called credit spread puzzle (see Table 1 for literature). We find that common approaches to testing structural models suffer from strong biases and low statistical power. When we test the Merton model in a bias-free approach we find much weaker evidence of a credit spread puzzle.

One approach to testing structural models is to use *average* firm variables such as asset volatility and the leverage ratio as inputs to the model and then to compare the resulting model spread with the average actual spread (Leland (2006) and McQuade (2013)). This 'representative firm' calculation is typically carried out by averaging firm variables across firms within a rating category and over a long historical time period[1]. However, a convexity bias arises in this case because the spread using average variables is typically lower than the average spread. Figure 1 illustrates this bias. The bias occurs both in the cross-section and in the time series dimension. David (2008) points out this bias but there is almost no empirical evidence on its size[2].

We empirically examine the importance of the convexity bias in our sample period 2002-2012 and find the bias to be strong. For example, the average model-implied spread on 3-year A-rated debt – averaging the model-implied spread across firms – is 102 basis points while the corresponding figure using a representative firm is 4 basis points. The bias increases as credit quality increases and as maturity shortens, precisely in the directions where the credit spread puzzle is found to be most severe. This suggests that the conclusions in Leland

[1]The papers mentioned in footnote 1 all use this 'representative firm' approach.

[2]Bhamra, Kuehn, and Strebulaev (2010) examine the bias for 5- and 10-year spreads for BBB-rated firms in a structural model with macroeconomic risks. They do not examine the bias in a simple structural model or other rating categories.

(2006) and McQuade (2013) that standard structural models underpredict spreads may well be misleading. We show that a similar underprediction bias occurs when calculating default probabilities for a representative firm. This bias is also most severe at short maturities for high-quality issues, so the finding in Leland (2004) and McQuade (2013) that standard structural models underpredict default probabilities at short horizons is also not accurate.

Another approach to structural model calibration is to compute the implied asset volatility that makes the default probability of a representative firm equal to the historical default frequency (Cremers, Driessen, and Maenhout (2008), Zhang, Zhou, and Zhu (2009), Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010), and Huang and Huang (2012)). This will typically result in an asset volatility that is too high because the default probability of a representative firm is biased downwards relative to the average default probability. We show that the bias in asset volatility is often large; when calibrated to one-year default frequencies the implied asset volatility is around twice as high as average asset volatility. But since the yield spread of the representative firm is biased downwards relative to the average yield spread, it is not clear in which direction the model-implied yield spread will be biased relative to the average yield spread because two biases pull in opposite directions. We show that when the default probability and the computed spread have the same horizon, the Merton model-implied spread will mostly be biased downwards. For BBB-rated firms the average downward bias is 24%. We also show that if the spread predictions are computed for shorter maturities than the horizon of default probabilities to which the model is calibrated (as in Cremers, Driessen, and Maenhout (2008) and Zhang, Zhou, and Zhu (2009)) the bias becomes much larger.

When calibrating to historical default frequencies, and no matter which structural model is being studied, the key assumption is that *ex-post* historical default frequencies provide a good proxy for *ex ante* average default probabilities. To test the statistical reliability of this assumption, we repeatedly simulate defaults over a period of 28 years for a large number of BBB-rated firms that have a realistic level of exposure to systematic risk. We find that

the realized default frequency is often far from the average default probability. In each of our simulations the average cumulative 10-year default probability is 4.39% while a 95pct confidence interval for the realized default frequency goes from 0.6% to 13.5%. Furthermore, the distribution of default frequency is highly skewed to the right, so the observed default frequency - e.g., long-run data from rating agencies - most often observed is only 1.8% and much lower than the mean of 4.39%. This implies that most of the time researchers using historical default frequencies will spuriously find a credit spread puzzle even if it does not exist. We use the simulation results along with Bayes' rule to calculate confidence bands for historical default frequencies. Given the historical average 10-year default frequency of BBB-firms of 4.39% calculated over the period 1970-1998, we estimate a confidence band for the ex ante default probability ranging from 1.9% to 20.1%. We conclude that ex-post realized default frequency cannot be used as a reliable measure of ex-ante average default probability even when calculated over several decades.

Having documented that using a representative firm approach leads to biased spread predictions and fitting to historical default frequencies has low statistic power, we then test the Merton model using a bias-free approach. The Merton model might not be the best structural model but it is certainly the simplest and we think it is useful to see how far we can go with a simple structural model. In other words, using the simplest model, to what extent can the apparent failure of structural models be rectified by eliminating weaknesses in empirical implementation?

In the bias-free approach we calculate a Merton spread for every transaction in the data set, compute an average, and compare with the average actual spread to the swap curve. Eom, Helwege, and Huang (2004) and Ericsson, Reneby, and Wang (2007) also use this approach. Eom, Helwege, and Huang (2004)'s data set consist of 182 trader quotes in the period 1986-1997 while Ericsson, Reneby, and Wang (2007)'s data set consist of 1387 transactions over the period 1994-2003. Our data set consists of 532,709 transactions for the period 2002-2012 from the TRACE database. This allows us to examine in much greater

detail the ability of the Merton model to price bonds across maturity, across ratings, and over a time period that includes both a boom period and a major recession.

The most common version of the credit spread puzzle is that the spread between long-term BBB yields and AAA yields is too high to be explained by the Merton model and other standard structural models. We find that over our sample period the median actual 10-year BBB-AAA spread is 62 basis points and the model-implied spread is 108 basis points, so on average the Merton model has no difficulty in predicting spreads at least as high as those in the data. There is a large amount of spread variation over the sample period, so we also look at the time series variation of spreads. We find that the model-implied BBB-AAA spread tracks the time series variation of the actual spread well. We confirm this finding using the Leland and Toft (1996) model. We alsos confirm this finding using dealer quotes from Merrill Lynch for the period 1997-2012. Based on long-term BBB-AAA spreads, we thus find no credit spread puzzle.

The Merton model predicts very low spreads for high-quality bonds with short maturity and "most researchers view this kind of result as a failure of diffusion-type structural models" according to Huang and Huang (2012). Even correcting for the convexity bias, we find that the Merton model mostly predicts spreads close to zero for investment grade bonds with a maturity less than one year. However, to our knowledge there is no evidence on the actual size of short-term spreads for bonds with a maturity below one year and we fill this gap in the literature. Over the sample period the median actual spread to LIBOR for bonds with a maturity below one year is 3 basis points for AAA/AA and 7 basis points for A. In some periods in 2008-2009 short-term spreads were somewhat higher than zero, consistent with Dick-Nielsen, Feldhütter, and Lando (2012) finding an illiquidity premium during this period, but by 2010 spreads were back at a level close to zero. Overall, we conclude that the low short-maturity spreads in the Merton model aligns well with actual spreads for ratings A-AAA. However, the actual median short-term BBB spread is 104 basis points while the median Merton model spread is 0 basis points. This suggests that a credit spread puzzle in

short-term bonds appears when *moving down* in credit quality of investment grade firms.

For high-quality bonds the Merton model also predicts small spreads on long-term bonds. For our sample the median model-implied 10-year AAA spread is only 2 basis points while the median actual spread is 32 basis points. This shows that the Merton model cannot quite match the magnitude of long-term AAA spreads, but a difference of 30 basis points is smaller than previously found. Furthermore, we find the median model-implied 10-year A spread to be 73 basis points compared with an actual spread of 85 basis points, showing that the under-prediction of long-term spreads is restricted to bonds of the very highest credit quality. Even though we find a puzzle for high-quality long-term spreads, the puzzle is smaller compared to previous findings in terms of spread size and how far down in credit quality the puzzle extends.

The organization of the article is as follows: Section 2 explains the data and how the Merton model is implemented. Section 3 examines common approaches to testing structural models. Although this section uses data described in Section 2, one can easily skip section 2 and read section 3 directly. In Section 4 we examine the statistical properties of historical default frequencies. Section 5 tests the Merton model using transaction data for the period 2002-2012. There are robustness checks in Section 6 and Section 7 concludes.

2. The Merton model: basics and implementation

2.1 Data

This section gives a brief overview of the data; a detailed description is relegated to Appendix A.

Since July 1, 2002, members of the Financial Industry Regulatory Authority (FINRA) have been required to report their secondary market over-the-counter corporate bond transactions through the TRACE database. Our data comes from two sources, WRDS and

FINRA, and covers almost all U.S. Corporate bond transactions for the period July 1, 2002 - June 30, 2012[3]. We limit the sample to senior unsecured fixed rate or zero coupon bonds and exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, have sinking fund provisions, or have covenants. Appendix A.1 describes details of the dataset and explains why it has more transactions than the typical TRACE dataset used in the literature.

To price a bond in the Merton model we need the issuing firm's asset volatility, leverage ratio, and payout ratio. *Leverage ratio* is calculated as the book value of debt divided by firm value (where firm value is calculated as book value of debt plus market value of equity). *Payout ratio* is calculated as the sum of interest payments to debt, dividend payments to equity, and net stock repurchases divided by firm value. *Asset volatility* is not directly observable and is estimated from equity volatility and leverage as we will explain in Section 2.2. Equity volatility is an annualized estimate based on the previous three year's of daily equity returns. All firm variables are obtained from CRSP and Compustat and details are given in Appendix A.2.

2.2 Calibration of the Merton model

In our calibration we try to be consistent with previous literature on the credit spread puzzle, and key calibrated parameters in previous literature are given in Table 1. Asset value in the Merton model follows a Geometric Brownian Motion under the risk-neutral measure,

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_A dW_t \tag{1}$$

[3]Rule 144A bond transactions are not covered. Rule 144A allows for private resale of certain restricted securities to qualified institutional buyers. According to TRACE Fact Book 2011, the percent of rule 144A transactions relative to all transactions is 2.0% in investment grade bonds and 8.4% in speculative grade bonds. Also, transactions reported on or through an exchange are not included in TRACE.

where r is the riskfree rate, δ is the payout rate, and σ_A is the volatility of asset value. The firm is financed by equity and a single zero-coupon bond with face value F and maturity T[4]. If the asset value falls below the face value at the bond's maturity, $V_T < F$, the firm cannot repay its bond holders and the firm defaults. In the original Merton model bondholders receive 100% of the firm's value in default, but to be consistent with empirical recovery rates, we follow the literature on structural models and assume that bondholders recover only a fraction of the firm's value in default. According to Moody's (2011) the average recovery rate for senior unsecured bonds for the period 1987-2010 was 49.2% and we follow Eom, Helwege, and Huang (2004) and set the payoff to bondholders to $\min(V_T, 0.492F)$.[5] The bond price at time 0 is calculated as:

$$P(0,T) = E^Q[e^{-rT}(1_{\{V_T \geq F\}} + \min(0.492, \frac{V_T}{F})1_{\{V_T < F\}})],$$

and the specific expression is given in Appendix B. The model implies a deadweight cost of bankruptcy; for a 10-year A-rated bond the expected deadweight cost of bankruptcy is 32.6%[6]. This is broadly consistent with the empirical estimate of 31.0% in Davydenko,

[4]To keep the model as simple as possible, we assume that the bond is a zero-coupon bond and implicitly account for coupons by estimating the payout rate as the total payout to debt and equity holders. An alternative would be to assume that payout is only dividends to equity holders and the firm refinances coupons and repays them at bond maturity. In this case the drift of the firm would be higher, but the amount of debt would also be higher and these two effects offset each other and the model is the same as the one we present. Finally, we could - at the expense of simplicity - allow for coupon payments as in Eom, Helwege, and Huang (2004). This would lead to slightly higher spreads because there is a small risk that firm value is lower than the size of a coupon before bond maturity.

[5]The average recovery rate in the model is slightly below 49.2% of face value because the recovery is either 49.2% or less. However, the firm value will rarely fall below 49.2% and therefore the expected recovery will be below but close to 49.2%. With an asset risk premium of 4% and using the mean values of leverage, asset volatility, and payout ratio for a A-rated bond in Table 3, the expected recovery on a 10-year bond is 48.3% (see Appendix B.2 for formulas).

[6]This assumes an asset risk premium of 4% and uses the mean values of leverage, asset volatility, and payout ratio for a A-rated bond in Table 3. The formula is given in Appendix B.2. An asset premium of 3%

Strebulaev, and Zhao (2012), 36.5% in Alderson and Betker (1995), 45.5% in Gilson (1997), 45% in Glover (2012), and the deadweight cost generally used in the literature as Table 1 shows.

Our assumption that the firm defaults if firm value is below face debt is common (Eom, Helwege, and Huang (2004) and Cremers, Driessen, and Maenhout (2008)) although some assume a default boundary below face value of debt (Chen, Collin-Dufresne, and Goldstein (2009) and Huang and Huang (2012)). Huang and Huang (2012) show that if one matches the model to historical default frequencies and recovery rates as in Cremers, Driessen, and Maenhout (2008), Huang and Huang (2012), and Chen, Collin-Dufresne, and Goldstein (2009), the value of the default boundary is not very important for implied spreads. However, in our implementation we explicitly do not have historical default frequencies as targets and therefore the value of the default boundary is important. If we chose a boundary below face value, say 60% of face value as in Huang and Huang (2012), default probabilities and spreads go down substantially. However, if one accepts the above mentioned recent evidence of 31-45% in deadweight cost of bankrupcy, one cannot match historical recovery values of close to 50% without having a default boundary close to the face value of debt.[7]

When we price a given bond we assume that all debt matures at bond maturity even if the firm has multiple bonds outstanding. This is an aggressive assumption for short maturity bonds and results in higher default probabilities compared to a structural model that incorporates the term structure of debt. Consistent with this view Moody's KMV use a default boundary equal to short-term liabilities plus one-half of long-term liabilities when calculating one-year default probabilities, see Sun, Munves, and Hamilton (2012). For longer maturity bonds this assumption is more reasonable because most of the existing debt will have matured when the bond matures. In fact, this assumption might lead to conservative

respectively 5% gives an expected deadweight cost of bankruptcy of 31.8% respectively 33.4%.

[7] See also Eom, Helwege, and Huang (2004) for a discussion of the importance of the deadweight cost of bankruptcy.

estimates of default for long-maturity bonds because the firm cannot default before bond maturity in contrast to a model with a term structure of debt. In line with this reasoning Moody's KMV increase the default boundary as the horizon at which they calculate default probabilities is extended (Sun, Munves, and Hamilton (2012)). We stress that we use the Merton model because it is the most simple and heavily tested structural model in the literature, not because it has the most realistic assumptions.

A crucial parameter in any structural model is the volatility of assets and we follow the approach of Schaefer and Strebulaev (2008) in calculating asset volatility. Since firm value is the sum of the debt and equity values, asset volatility is given by:

$$\sigma_A^2 = (1-L)^2\sigma_E^2 + L^2\sigma_D^2 + 2L(1-L)\sigma_{ED}, \tag{2}$$

where σ_A is the volatility of assets, σ_D volatility of debt, σ_{ED} the covariance between the returns on debt and equity, and L is leverage ratio. If we assume that debt volatility is zero, asset volatility reduces to $\sigma_A = (1-L)\sigma_E$. This is a lower bound on asset volatility. Schaefer and Strebulaev (2008) (SS) compute this lower bound along with an estimate of asset volatility that implements equation (2) in full. They find that for investment grade companies the two estimates of asset volatility are very similar while for junk bonds there is a substantial difference. We compute the lower bound of asset volatility, $(1-L)\sigma_E$, and multiply this lower bound with SS's estimate of the ratio of asset volatility computed from equation (2) to the lower bound. Specifically, we estimate $(1-L)\sigma_E$ and multiply this with 1 if $L < 0.25$, 1.05 if $0.25 < L \leq 0.35$, 1.10 if $0.35 < L \leq 0.45$, 1.20 if $0.45 < L \leq 0.55$, 1.40 if $0.55 < L \leq 0.75$, and 1.80 if $L > 0.75$.[8] [9] This method has the advantage of being

[8]These fractions are based on Table 7 in SS apart from 1.80 which we have set somewhat ad hoc. Results are insensitive to other choices of values for $L > 0.75$.

[9]We rely on estimates from SS instead of applying their procedure because the nature of our dataset is different from theirs. Their dataset consists of monthly quotes and therefore they have monthly return observations for every bond. Our dataset consist of actual transactions which are unevenly spaced in time and constructing a return series is considerably more difficult.

transparent and easy to replicate.

Finally, the riskfree rate r is chosen to be the swap rate for the same maturity as the bond. For maturities shorter than one year we use LIBOR rates.[10]

2.3 Summary statistics

Table 2 shows statistics for the whole TRACE data set, for the final sample, and how the sample reduces as we apply more criteria. We see that the final sample of straight industrial bonds with no option-like features or covenants is a small subset of the universe of TRACE bonds. The final sample consist of 455 bonds. Papers reporting number of bonds in their sample are Eom, Helwege, and Huang (2004) with 182 bonds, Cremers, Driessen, and Maenhout (2008) with 524 bonds, and Schaefer and Strebulaev (2008) with 1360 bonds. The number of bonds in our sample is smaller than that in Cremers, Driessen, and Maenhout (2008) and Schaefer and Strebulaev (2008) mainly because we exclude bonds with covenants. Summary statistics for the firms in the final investment grade sample are shown Table 3 and those for the final speculative grade sample in Table 4. A firm can be in more than one rating category if the rating changes over time. We combine AAA and AA into one rating group because there are only four AAA-rated firms in our sample. When we refer to AAA-rated bonds later in the paper, we are referring to the combined group of bonds rated AAA or AA.

Focusing on investment grade firms, we see in Table 3 that the median leverage ratio in

[10]Most previous literature uses Treasury yields as riskfree rates as Table 1 shows, but recent evidence shows that swap rates are more appropriate to use than Treasury yields. The reason is that Treasury bonds enjoy a convenience yield that pushes their yields below riskfree rates. Hull, Predescu, and White (2004) find that the riskfree rate is 63bps higher than Treasury yields and 7bps lower than swap rates, Feldhütter and Lando (2008) find riskfree rates to be approximately 53bps higher than Treasury yields and 8bps lower than swap rates, while Krishnamurthy and Vissing-Jorgensen (2012) find that Treasury yields are on average 73bps lower than riskfree rates.

the sample period is 0.17 for AAA/AA, 0.21 for A, and 0.50 for BBB. The leverage ratio for A-rated firms is lower than other studies while it is higher for BBB-rated firms[11]. The payout ratio is similar across ratings, with median payout ratio ranging from 3.9% to 5.3%. Median equity volatilities for A-AAA rated firms are very similar to the estimates in Schaefer and Strebulaev (2008), while median equity volatility for BBB firms of 0.47 is higher than 0.30 estimated in Schaefer and Strebulaev (2008). Asset volatility increases as rating decreases. Very interestingly, asset volatility is much the same in the second half of our sample (2007Q3 to 2012Q4), which includes the recent crisis, as the first half (2002Q3 to 2007Q2). This is the case even for BBB that sees a dramatic increase in median equity volatility from 40% to 100%. This suggests that the high equity volatility during the subprime crisis was caused primarily through increased leverage and not because of increased asset volatility.

We see in Table 4 that speculative grade firms are significantly more leveraged. However, their asset volatilities are close to those for investment grade bonds with median asset volatility at 22% for BB, 23% for B, and 19% for C.[12] Median payout ratios between 3.8% and 5.1% are in the same range as those for investment grade bonds, so the higher coupons speculative grade firms pay are offset by lower dividends to equity holders.

In Table 5 we see that the number of speculative grade bonds in our sample is small relative to the number of investment grade bonds. The reason is that speculative grade bonds frequently contain call and covenant features which leads to their exclusion from our sample.

Since the number of speculative grade firm and bond observations in the sample is small relative to the number of observations in the investment grade segment, we group all spec-

[11] Huang and Huang (2012) use a leverage ratio of 0.13 for AAA, 0.21 for AA, 0.32 for A, and 0.43 for BBB while Schaefer and Strebulaev (2008) find a leverage of 0.10 for AAA, 0.21 for AA, 0.32 for A, and 0.37 for BBB.

[12] In fact estimated asset volatility for the speculative grade firms in our sample are actually *lower* than the asset volatility for BBB firms in Table 3, but the sample size for speculative grade bonds is small.

ulative grade bonds in one group and generally focus on investment grade bonds. This does not substantially limit our examination of the credit spread puzzle since the puzzle is mostly confined to investment grade bonds. For example, HH report that between 16% and 29% of the 10-year spread on investment grade bonds can be explained by credit risk while the corresponding range for speculative grade bonds is 60-83%.

3 Why most existing tests of structural models are biased

A number of papers find that standard structural models - often the Merton model - cannot match the level of credit spreads, particularly for short maturities and high credit quality issuers (see table 1). This finding has been coined the credit spread puzzle and the standard reference for the puzzle is Huang and Huang (2012). A recent review of the inability of the Merton model to capture the level of credit spreads and extensions of the model is Sundaresan (2013). While credit spreads might be influenced by illiquidity and other factors, default probabilities are less so[13]. Leland (2004) and McQuade (2013) find that the Merton model underpredicts default probabilities at short horizons consistent with the underprediction evidence for spreads.

In this Section we discuss how the above mentioned papers test structural models and show that common tests are biased and suffer from low statistical power.

3.1 Notation

We begin by defining notation. For a given corporate bond transaction at time t in a bond with maturity T issued by firm i we call the actual spread $s_T^A(i,t)$. To calculate the theoretical

[13]See He and Milbradt (2013) for a structural model where there is an interaction between default and liquidity risks.

credit spread in the Merton model we need firm leverage L_{it}, asset volatility σ_{Ait}, payout ratio δ_{it}, and the riskfree rate for maturity T r_{Tt}. We denote the vector of parameters for firm i at time t for $\theta_{it} = (L_{it}, \sigma_{Ait}, \delta_{it}, r_{Tt})$[14]. We define the parameter vector without asset volatility as $\theta_{it}^{\setminus \sigma_A} = (L_{it}, \delta_{it}, r_{Tt})$. We denote the model-implied Merton credit spread as $s_T^M(\theta_{it})$ and the explicit formula is given in equation (9) and (12) in the Appendix. Given an asset risk premium π_A Appendix B.2 reviews the calculation of the cumulative default probability over the next T years and we call this $PD_T(\theta_{it}, \pi_A)$. Often we suppress the dependence on π_A. The asset risk premium can be firm specific, but consistent with previous literature we let it be the same for all firms.

3.2 Convexity bias in spreads

Leland (2006) and McQuade (2013) use historical averages of leverage ratio, payout rate, the riskfree rate, and asset volatility (obtained from Schaefer and Strebulaev (2008)) to calculate model-implied credit spreads from a standard structural model and compare with historical averages of actual credit spreads. We call this approach for the representative firm approach. Following the tradition of the literature they do this for individual rating classes and in the rest of the paper we assume that all such comparisons are done within a rating class without explicitly mentioning this.

In the representative firm approach

$$s_T^M\left(\frac{1}{T_{i,t}} \sum_{i,t} \theta_{it}\right)$$

[14]Payout ratio, asset volatility, and the risk free rate are assumed to be constant in the Merton model, but in the implementation we estimate them day by day and therefore they might vary over time for a given firm. This approach is standard in the literature (see for example Eom, Helwege, and Huang (2004), Schaefer and Strebulaev (2008), and Bao and Pan (2013)) and since we explicitly study previous results in the literature we follow this tradition.

is compared with

$$\frac{1}{T_{i,t}}\sum_{i,t} s_T^A(i,t)$$

where $T_{i,t}$ is the number of spread observations for bonds with maturity T over the corresponding time period. The approach suffers from a Jensen's inequality bias because spreads are typically convex in firm variables. Figure 1 illustrates the bias. David (2008) discusses this bias but there are no empirical results on the severity of the bias in a standard structural model. We examine the bias by computing both the average Merton spread and the Merton spread of the average firm variables in our sample.

The average Merton spread is calculated by computing the Merton spread for each transaction in the sample, computing the volume-weighted average monthly spread, and taking a simple average over the monthly spreads. Large bond issues typically trade more often than small bond issues and by volume-weighting we obtain a spread that is plausibly close to holding the market portfolio, consistent with how bond indices are calculated. The result is $E[s(\theta_{it})]$ in Table 6. To calculate the Merton spread using the representative firm approach, we calculate average firm variables by averaging monthly average volume-weighted firm variables and calculate the spread using average firm variables. The result is $s(E[\theta_{it}])$ in Table 6. Finally, we compute the ratio $\frac{s(E[\theta_{it}])}{E[s(\theta_{it})]}$ in Table 6. If the ratio is below one the representative firm approach underestimates spreads.

We see in the table that the bias is large and increases with credit quality and as maturity shortens. Averaging over the whole sample period 2002-2012, spreads based on average firm variables are only 39% of the average model-implied spread for 10-year AAA rated bonds while for BBB-rated bonds it is 77%[15]. For bonds with a maturity of less than one year spreads based on the representative firm approach are essentially zero for investment grade bonds while the average actual spread is as high as 601 basis points. We also see that the

[15]We use 'long-term' and '10-year' interchangeably since the average maturities in the groups of long-term bonds are fairly close to 10 year.

bias as a ratio is similar in the first (2002-2007) and second half (2007-2012) of the sample period showing that the bias is a robust phenomenon and not due to the large variation in spreads in the latter period.

Overall, the results show that the representative firm approach causes a downward bias in spreads for investment grade bonds and this bias becomes larger where the credit spread puzzle has been found to be most severe, for short maturities and high-quality firms. Thus, the findings in Leland (2006) and McQuade (2013) - who use this approach - that the Merton model underpredict spreads are not accurate.

3.3 Convexity bias in default probabilities

Leland (2004), Zhang, Zhou, and Zhu (2009), and McQuade (2013) examine default probabilities implied by structural models. The advantage of looking at default probabilities instead of spreads is that default probabilites are not contaminated by liquidity, recovery rates, and other potential factors influencing spreads. They use the representative firm approach and choose leverage ratio, payout rate, and the riskfree rate based on historical averages, an asset risk premium of 4% and compute model-implied default probabilities. All three studies find that default frequencies at horizons below five years are underestimated.

Table 7 shows that a similar bias for default probabilities as for spreads occurs when using the representative firm approach; the bias increases in credit quality and as maturity shortens. So average default probabilities are very different from the default probability of the average firm. Since the above mentioned studies compare calculated model-implied default probabilities with realized default frequencies, the unbiased way to calculate default probabilities is by averaging over individual firm default probabilities, not by calculating the default probability of an average firm as they do[16].

[16] One might argue that since there is variation in recovery rates, using a constant recovery rate is another source of a convexity bias. However, spreads are almost linear in recovery rates and therefore assuming a constant recovery rate is unlikely to have a significant effect.

3.4 Calibrating to historical default rates

The most common approach when calibrating structural models is to leave asset volatility as a free parameter, set leverage ratio and payout ratio to historical averages, and imply out asset volatility by matching the default probability of an average firm to historical default frequency. Table 1 gives an overview of papers using this approach. Specifically, for a given bond maturity T (and rating), the asset volatility $\hat{\sigma}_A$ is implied out by the equation

$$PD_T(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it}^{\backslash\sigma_A}, \hat{\sigma}_A) = RD_T \tag{3}$$

where RD_T is the historical average default frequency over a long period, in Huang and Huang (2012)'s case 28 years. The backed-out asset volatility $\hat{\sigma}_A$ is then used to calculate the model-implied spread

$$s_T^M(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it}^{\backslash\sigma_A}, \hat{\sigma}_A) \tag{4}$$

and compared with average actual spreads $\frac{1}{T_{i,t}}\sum_{i,t} s_T^A(i,t)$.

This approach leads to a biased calibration. As shown in Table 7 default probability using average firm variables is lower than average default probability. Therefore, $\hat{\sigma}_A$ implied out in equation (3) will be higher than average σ_A to compensate for this and spreads will be too high. However, spreads using average firm variables are lower than average spreads as Table 6 shows. Since the two biases have opposite effects on spreads, the combined effect on the predicted spread in equation (4) is not clear.

We examine the extent of the bias in Table 8. We set asset risk premium to 4% and calculate for each transaction the default probability over the same horizon as the bond. For different bond maturities and ratings we calculate the monthly volume-weighted average default probability and average across months. This is E(Def Prob). Note that here we assume that average default probabilities equal historical default frequencies, an assumption that we investigate in Section 4. Likewise we calculate average asset volatility which is

E(Asset vol) in Panel A. We also calculate average leverage ratio, average payout ratio, average riskfree rate and use these for a representative firm. We then imply out asset volatility - $\hat{\sigma}_A$ - by making the default probability of the representative firm match E(Def Prob) according to equation (3). The calibrated asset volatility is shown in Panel A. We see that for investment grade bonds asset volatility is biased upwards and the bias becomes more severe as bond maturity shortens. In Panel B we see that predicted spreads using $\hat{\sigma}_A$ in equation (4) are mostly lower than actual average spreads. On average AAA/AA spreads are 5-10% higher, A spreads 30% lower, BBB spreads 25% lower, and BB spreads 20% lower. Note that in this Panel we predict spreads at the same horizon as the horizon to which we fit default probabilities.

Since asset volatility is biased upwards other model predictions are conceivably biased. Even though biases partially cancel out in spread predictions when default rates and spreads have the same horizon as in Panel B, they are far from cancelling out when default rates and spreads have different time horizons. For example, Cremers, Driessen, and Maenhout (2008) fit to historical 10-year historical default frequencies and look at implied spreads and default probabilities for maturities between one and 10 years while Zhang, Zhou, and Zhu (2009) fit to historical 5-year historical default frequencies and look at implied spreads and default probabilities for maturities between one and five years. To illustrate the bias when using different horizons for spreads and default probabilities, we repeat the previous procedure of implying out $\hat{\sigma}_A$ by calibrating to average default probabilities and predicting spreads except that we now use the implied asset volatility from fitting to 10-year default probabilities when calculating spreads for bonds of shorter maturity. Panel C shows that the bias becomes more severe as bond maturity shortens and model-implied spreads are hugely underestimated relative to average actual spreads for horizons shorter than 10 years.

4 Why default probabilities are hard to estimate

Most papers testing structural models use ex post realized default frequencies to proxy for ex ante default probabilities.[17] We show in this section that this is highly problematic because the variance of the estimator is large. Note that what we show in this section is unrelated to the convexity bias documented in the previous section.

There is a tradition in the literature to use realized default frequencies published by Moody's.[18] To explain how Moody's calculate default frequencies, let us consider the 10-year BBB cumulative default frequency of 4.39% used in Cremers, Driessen, and Maenhout (2008) and Huang and Huang (2012). This number is published in Keenan, Shtogrin, and Sobehart (1999) and is based on default data in the period 1970-1998. In year i, Moody's define a cohort of BBB-rated firms and track how many of those firms default over the next 10 years. The 10-year BBB default frequency for year i is the number of defaulted firms divided by the number of firms in the cohort.[19] The average default rate of 4.39% is calculated as an average of 10-year default rates of cohorts formed at yearly intervals over the period 1970-1988[20].

We show in a simulation that in an economy where the ex ante 10-year default probability is 4,39% for all firms, the ex post realized 10-year default frequency averaged over 28 years can be dramatically different. We do so by populating the economy with identical firms, thus ignoring any convexity bias due to cross-sectional variation in the firms. What is crucial in the simulation is that firms are exposed to systematic risk which leads to correlation in

[17]See for example Zhang, Zhou, and Zhu (2009) and the articles mentioned in Table 1.

[18]All articles mentioned in Table 1 use Moody's estimates of default frequencies.

[19]Some firms have their rating withdrawn and Moody's have incomplete knowledge of subsequent defaults once firms are no longer rated. Moody's adjust for this by assuming that firms with withdrawn ratings would have faced the same risk of default as other similarly rated issuers if they had stayed in the data sample. Evidence in Hamilton and Cantor (2006) suggests that this is a reasonable assumption.

[20]In recent years Moody's calculate average default frequencies based on monthly cohorts instead of yearly cohorts; the difference between default frequencies using monthly and yearly cohorts is small.

defaults.

The simulation is done as follows. We assume that in year 1 we have 1,000 identical firms, where firm i's value under the historical measure follows the process

$$\frac{dV_t^i}{V_t^i} = (\pi_A + r - \delta)dt + \sigma_A dW_{it}^P \tag{5}$$

and π_A is the asset risk premium. We choose 1,000 firms each month because the average number of firms in Moody's BBB cohorts during the last decade is close to 1,000[21]. We assume every firm has one T-year bond outstanding, and a firm defaults if firm value is below bond face value at bond maturity, $V_T^i \leq F$. Using the properties of a Geometric Brownian Motion, the default probability is

$$p = P(c \leq W_{iT}^P - W_{i0}^P) \tag{6}$$

where $c = \frac{\log(F/V_0) - (\pi_A + r - \delta - \frac{1}{2}\sigma_A^2)T)}{\sigma_A}$. This implies that the unconditional default probability is $N(\frac{c}{\sqrt{T}})$ where N is the cumulative normal distribution. Note that for a given default probability p we can always find c such that equation (6) holds, so in the following we use p instead of underlying Merton parameters giving rise to p.

We introduce systematic risk by assuming that

$$W_{iT}^P = \sqrt{\rho} W_{sT} + \sqrt{1-\rho} W_{iT} \tag{7}$$

where W_i is a Wiener process specific for firm i, and W_s is a Wiener process common to all firms. All Wiener processes are independent. The realized 10-year default frequency in the year 1-cohort is found by simulating one systematic and 1,000 idiosyncratic processes in equation (7).

In year 2 we form a cohort of 1,000 new fims. The firms in year 2 are identical to the previous firms as they entered the index in year 1. We calculate the realized 10-year default

[21]The average number of BBB cohort firms during 1970-2012 is 606. There has been an increasing trend from 372 in 1970 to 1,245 in 2012. The results are very similar if we use 600 or 1,250 firms instead of 1,000.

frequency of the year 2-cohort as we did for the year 1-cohort. Note that the common shock in year 1-9 for the year 2-cohort is the same as the common shock in year 2-10 for firms in the year 1-cohort. We do this for 18 years and calculate the overall realized cumulative 10-year default frequency in the economy by taking an average of the default frequencies across the 18 cohorts. We repeat this simulation 100,000 times.

There are two parameters in our simulation; the defalt probability p and the default correlation ρ. We focus on BBB firms and set $p = 4,39\%$, the realized 10-year cumulative BBB default frequency used in Huang and Huang (2012). We assume that half of firm volatility is systematic ($\rho = 0.25$) consistent with evidence in Choi and Richardson (2012)[22]. We simulate 18 years of firms (in total 18,000 firms) and since we look at 10-year default frequencies, we simulate 28 years of data.

Figure 2 shows the distribution of realized default frequencies in the simulation study. A 95% confidence interval is [0.55%; 13.48%]. The black vertical line shows the ex ante default probability of 4.39%. Given that we simulate 18,000 firms over a period of 28 years, it might be surprising that realized default frequency can be so far from ex ante default probability. The reason is that there is systematic risk in the economy and this induces correlation in defaults among firms. If there is no systematic risk in the economy Table 9 shows that a 95% confidence interval for the realized default frequency is $[4.11\%; 4.68\%]$.

We also see from Figure 2 that the default frequency is significantly skewed to the right, i.e., the modal value (1.77%) is below the mean (4.39%). This means that the default frequency *most often* observed – e.g., long-run data from the rating agencies – is below the mean and spreads will appear too high relative to historical losses. This is an observation that Moody's KMV are aware of, see for example Kealhofer, Kwok, and Weng (1998) and Bohn, Arora, and Korablev (2005).

[22]See Table 5 in Choi and Richardson (2012). $\rho = 0.25$ is also broadly consistent with the fact that during our sample period the annualized volatility of daily returns of the S&P500 index was 21.5% while the median equity volatility for A-rated forms respectively BBB-rated firms in our sample is 31% respectively 47%.

If we denote the average realized cumulate 10-year default frequency over 28 years for X, then the above simulation gives the distribution of $X|p$. We can use the simulation to estimate the ex ante default probability that is consistent with a realized default rate of 4.39%. Bayes' rules gives

$$f(p|X) \propto f(X|p)f(p)$$

where $f(p)$ is the prior distribution of p and $f(X|p)$ is the distribution of default losses calculated in the above simulation. If we assume a flat prior on p, the distribution of p is given by $f(p|X) \propto f(X|p)$. In Appendix C we derive an approximate closed-form solution for the distribution of $X|p$ which we use in the following.

Figure 3 shows the distribution of p given an average cumulative 10-year default experience of 4.39% over 28 years. We see that the distribution of p is wide and a 95% confidence band is $[1.9\%; 20.1\%]$. Intuitively, high p's are consistent with an ex post default rate of 4.39% because the distribution of default rates are highly skewed. The modal value of 5.35% is higher than the realized default frequency of 4.39%. Since the distribution of p corresponds to the likelihood function, this implies that the maximum likelihood estimator of p is 5.35%, so the realized default frequency is a downward biased estimator of p.

Moody's record default rates starting from 1920 and Figure 3 also shows the density for p using data for the period 1920-2012. The average 10-year BBB cumulative default frequency for 1920-2012 is 7.112% according to Ou, Chiu, Wen, and Metz (2013). Although the width of the confidence band is reduced to $[4.2\%; 14.1\%]$, the statistical uncertainty is still large.

We can use results in Chen, Collin-Dufresne, and Goldstein (2009) (CCG) to estimate the default probability that makes the credit spread puzzle disappear. We focus on the 10-year BBB-AAA spread since this spread is the most widely studied on the literature and there is no issue about which riskfree rate to use. CCG derive the credit spread for a T-year bond in the Merton model as

$$(y - r) = -\frac{1}{T}\log\left(1 - LN[N^{-1}(p) + \theta\sqrt{T}]\right) \tag{8}$$

where p is the T-year default probability under the historical measure, L is the loss given default, and θ is the asset Sharpe ratio.[23] CCG calculate θ to be 0.22 based on equity data from 1927-2005. If we use Moody's 10-year default frequencies from 1970-1998 for AAA and BBB firms as in Huang and Huang (2012) and Cremers, Driessen, and Maenhout (2008) together with CCG's estimates of the asset Sharpe ratio and a loss rate of 50.8%, the model-implied Merton spread in equation (8) is 61bps[24]. Table 1 shows that 61bps is consistent with previous findings of 43-77bps. Table 1 also shows that estimates in the literature of the actual 10-year BBB-AAA spread for non-callable bonds are fairly consistent around 105bps. If we use Moody's default frequencies averaged over 1920-2012, the model-implied spread is 95bps[25]. If we assume the ratio between AAA and BBB default rates stays the same, a BBB default rate of 8.16% (and AAA default rate of 0.92%) gives a 10-year BBB-AAA of 105bps. Figure 3 clearly shows that $p = 8.16\%$ is not statistically significantly different from the realized default frequency in either the short 1970-1998 or long 1920-2012 sample period.

In Table 10, we repeat the above calculations and estimate the model-implied spread consistent with default rates along with a 95% confidence band for different rating classes and bond maturities. Note that we are ignoring any convexity bias and therefore spreads might be biased and confidence bands are plausibly too tight. For actual spreads we use estimates from Duffee (1998) commonly used in the literature as shown in Table 1. We see in Table 10 that using default rates from 1920-2012 gives considerably higher investment grade spreads compared to using default rates from 1970-1998, while this is not the case for speculative grade bonds. Partially, this can be explained by the fact that the distribution of realized defaults becomes less skewed as default probability increases and for high default probabilities there is a negative skewness. So using default rates from a "calm period" results

[23]Chen, Collin-Dufresne, and Goldstein (2009) assume that the recovery rate is $1 - L$ while we assume it is $\min(1 - L, \frac{V_T}{F})$. This is unlikely to lead to any significant differences.

[24]The 10-year cumulative default frequency for AAA firms is 0.77% giving a spread of 22bps while the frequency for BBB firms of 4.39% gives a spread of 83bps.

[25]Based on 0.866% for AAA and 7.112% for BBB, see Ou, Chiu, Wen, and Metz (2013).

in a credit spread puzzle that becomes more severe as credit quality increases. The table also shows that none of the actual spreads are statistically significantly different from Merton model spreads, so we cannot reject that both spreads and default frequencies are consistent with predictions from the Merton model.

In the Merton model expected equity return is related to expected bond return, and one can be inferred from the other. Campello, Chen, and Zhang (2008) use this insight to measure expected equity returns from bond yields. The motivation is that the use of noisy realized equity returns to proxy for expected equity returns is avoided. In their calculation of the expected bond return, they compute the expected default loss using historical default frequencies. However, our finding that default frequencies are poor proxies for default probabilities raises concerns regarding this methodology, since it replaces noise in realized equity returns with noise in realized default rates. Other papers measuring expected bond (or CDS) returns by using historical default frequencies are among others Elton, Gruber, Agrawal, and Mann (2001b) and Bongaerts, Driessen, and de Jong (2011).

In the simulations we assume that firms are only part of one cohort. This is a simplification because when Moody's form cohorts of BBB-rated firms from year to year, there is a substantial overlap of firms from one cohort to the next. However, this overlap adds additional correlation in defaults of BBB firms on top of the correlation caused by systematic risk. If we allowed firms to stay in more than one cohort, variations in default frequencies would be even larger. Also, the amount of systematic risk is assumed to be $\rho = 0.25$, but as Table 9 shows, even with a very conservative amount of systematic risk, confidence bands are still wide. Finally, we assume in the simulation that firms are identical, ignoring cross-sectional variation in firm characteristics. As we showed in the previous section firm heterogeneity is important, but precisely for BBB-rated firms at a horizon of 10 years the impact of firm heterogeneity on default probabilities is small relative to the effect of systematic risk. We see in Table 7 that the bias is modest for long-term BBB-rated bonds and we have confirmed this in simulations.

5. New evidence on the credit spread puzzle

We showed in the previous two sections that tests of structural models using the representative firm approach are biased and suffer from low statistical power. An approach to testing structural models that avoids these issues is to compare model-implied and actual spreads on a transaction-by-transaction basis. To our knowledge only Eom, Helwege, and Huang (2004) and Ericsson, Reneby, and Wang (2007) take this approach. Both papers do not systematically find that structural models underpredict spreads. Eom, Helwege, and Huang (2004)'s data consist of 182 trader quotes in the period 1986-1997 while Ericsson, Reneby, and Wang (2007)'s consist of 1387 transactions over the period 1994-2003. With the availability of TRACE we can conduct a large-scale examination of the Merton model using 532,709 transactions for the period 2002-2012. This allows us to examine in detail the ability of the Merton model to price bonds across maturity, across ratings, and over a time period that includes both a boom period and a recession.

There are broadly three versions of the puzzle[26]:

- *Long-term yield spreads between BBB- and AAA-rated bonds are too high to be explained by standard structural models of credit risk.* The yield spread analysed is typically for bonds with a maturity close to 10 years. If potential non-default components of yield spreads like taxes or liquidity are the same for AAA- and BBB-rated bonds, this version of the credit spread puzzle offers a "clean" spread uncontaminated by non-

[26]We searched the top-3 finance journals (Journal of Finance, Review of Financial Studies, and Journal of Financial Economics) for articles proposing extensions of standard structural models that raise yield spreads relative to standard models and found 10 articles. All 10 articles examine the long-term BBB-AAA spread (Puzzle 1), 5 examine short-term spreads (Puzzle 2), and 6 examine long-term AAA spreads (Puzzle III). Eom, Helwege, and Huang (2004), Ericsson and Renault (2006), Hackbarth, Miao, and Morellec (2006), Zhang, Zhou, and Zhu (2009), and He and Xiong (2012) examine all three puzzles, Cremers, Driessen, and Maenhout (2008) examine Puzzle I and III, while David (2008), Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010), and Bhamra, Kuehn, and Strebulaev (2010) examine Puzzle I.

credit effects. Table 1 shows that the actual spread is often estimated to be around 105bps while the model spread is 47-77bps.

- *Yield spreads on high-quality bonds with short maturity are too high to be explained by standard structural models of credit risk.* Short maturity typically means one year or less and high-quality refers to investment grade bonds. Since standard structural models typically predict yield spreads close to zero for high-quality bonds at short maturities, this version of the puzzle is not very sensitive to model specification; if there is a significantly positive short-term spread there is a puzzle. Note that there is no empirical evidence on corporate bond spreads on bonds with a maturity below one year.

- *Yield spreads on high-quality bonds with long maturity are too high to be explained by standard structural models of credit risk.* Long maturity is typically 10 years and high-quality refers to bonds with a rating of AAA. Table 1 shows that previous estimates of the actual 10-year AAA spread **to Treasury yields** is 47-63bps while the model-implied spread is 2-12bps.

The long-term spread between BBB and AAA-rated bonds has received most attention in the literature and we will focus on this puzzle first before examining the other two puzzles. When we calculate spreads we take the median across all bond transactions and weight by the volume of each transaction.[27] The use of medians is robust to the presence of potential outliers and in most cases we also report volume-weighted 10 and 90pct quantiles which are informative about the distribution of spreads. We weight by volume following the recommendation by Bessembinder, Kahle, Maxwell, and Xu (2009).

[27]To calculate the volume-weighted median, we sort spreads in increasing size $s_1 < s_2 < ... < s_T$ with corresponding normalized volumes $\tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_T$, where $\tilde{v}_i = \frac{v_i}{\sum_{j=1}^{T} v_j}$ and then find t such that $\sum_{i=1}^{t} \tilde{v}_i \geq 0.5$ and $\sum_{i=1}^{t-1} \tilde{v}_i < 0.5$. The volume-weighted median spread is then s_t.

5.1 Long-term BBB-AAA yield spreads

Table 11 shows actual and model-implied bond spreads for our sample. The actual median long-term BBB-AAA spread is 95bps-32bps=63bps while the model-implied spread is 110bps-2bps=108bps. Thus, on average the Merton model does not underpredict long-term BBB-AAA spreads in contrast to what most of the previous literature has found. Since spreads are volume-weighted over all transactions, the results in the table have to be interpreted with caution. For example, trading in AAA/AA bonds went up strongly in 2009 while trading in BBB went down, so the weight of 2009 in AAA/AA spreads is higher than in BBB spreads. To get a more accurate picture of the ability of the Merton model to match spreads, we examine the time series variation in long-term BBB-AAA spreads in Figure 4[28]. The graph shows that the Merton model cannot quite match the level and time series variation of the actual spread during 2005-2007, but apart from this period the model-implied spread tracks the actual spread surprisingly well.

Overall, we find no evidence that actual long-term BBB-AAA spreads are consistently higher than model-implied long-term BBB-AAA spreads.

5.2 Short-term yield spreads on high quality bonds

Predicted spreads in standard models of credit risk for short-maturity investment grade bonds are very low and this has been viewed as a failure of structural models. However, to the best of our knowledge there is no empirical evidence on the actual size of corporate

[28]We calculate the 10pct and 90pct quantiles in the Figure by simulation on a quarterly basis: we draw a transaction in a BBB bond from the pool of actual BBB transactions where the probability of drawing a particular transaction is proportional to transaction volume. In the same way we draw a AAA/AA transaction. We calculate from the two transactions a BBB-AAA spread. We repeat this procedure 5,000 times and calculate the 10pct and 90pct quantile in the 5,000 simulated BBB-AAA spreads.

bond credit spreads for maturities shorter than one year[29]. The reason is that most previous research had to rely on quotes and typically only for bonds that were included in an index, such as the Lehman or Merrill Lynch indices, were carefully quoted. Bonds usually drop out of indices when the maturity falls below one year and these bonds typically stop being quoted. Since we use transactions data, we observe transactions on bonds with any maturity, and our results on short-term bonds provide new evidence on the actual size of short-term corporate bond spreads.

In Table 11 we see that the median yield spread for AAA/AA-rated bonds with a maturity less than one year is 3bps and 7bps for A-rated bonds. This is surprisingly close to zero. In contrast, the median actual yield spread for BBB bonds is 104bps while the model-implied is zero. Figure 5 shows the time variation of short-term spreads for investment grade bonds. Actual spreads for AAA and AA bonds are close to zero and the 10pct quantile is below zero for the whole sample period except during the volatile period in 2008 where we see a jump in short-term spreads of around 50bps. For A-rated bonds we see a similar pattern. Short-term spreads for BBB-rated bonds in first half of the sample period are in the range of 20-100bps and confidence bands do not contain zero. For the first half of the sample period model-implied spreads are zero. Proposed explanations in the literature for positive short-term spreads are incomplete accounting information (Duffie and Lando (2001)) or jumps in firm value (Zhou (2001)). However, any explanation raising short-term BBB credit spreads must have a modest effect on AAA/AA/A short-term spreads since they are close to zero.

In the second half of the sample period we see that actual BBB short-term credit spreads are high but model-implied spreads are even higher. When calculating the credit spread, we

[29] Covitz and Downing (2007) examine corporate spreads for maturities less than one year in the commercial paper market. The commercial paper market has its own set of institutional features compared to the corporate bond market. In addition, Ou, Hamilton, and Cantor (2004) find that "unlike bondholders, in most cases holders of defaulted Commercial Papers have ultimately been made whole" so recovery rates likely be different.

assume that all of the firm's debt is due when the bond matures and as discussed in Section 2.2 this assumption is quite strong for short-maturity bonds. Interestingly, the time series correlation between the actual and model-implied short-term BBB spreads is high in the second half of the sample, but the level of model-implied spreads is four times higher than the level of actual spreads.

5.3 Long-term yield spreads on high-quality bonds

Table 11 shows that the median actual long-term AAA spread over the sample period is 32bps while the model-implied is a mere 2bps. This finding is consistent with previous literature finding that structural models cannot reproduce spreads of long-term high quality yield spreads. The difference of 30bps is smaller than previous literature has found (see Table 1), but this is due to the use of swap rates as the riskfree rate instead Treasury yields. Figure 6 shows the time variation of the long-term AAA spread. In 2002-2005 the model-implied spread tracks the actual spread fairly well, while from 2006 and onwards the actual spread is higher than the model spread.

The underprediction of long-term high-quality spreads could be related to why some firms that have no or little leverage despite the tax advantage of debt; the "under-leverage puzzle" (see Strebulaev and Yang (2013)). However, if we restrict our sample to firms that have at least 5, 10, or even 15% leverage the results for AAA are similar. Therefore our results are not driven by firms with no or little leverage.

Overall, our findings imply that the Merton model captures the size of average spreads better than previously documented. The results above apply to average spreads over a quarter or longer periods consistent with the previous literature on the credit spread puzzle. However, our results do not necessarily imply that the Merton model can accurately price bonds in the cross section - at least within rating categories. Indeed, on the individual bond level there are for many bonds significant pricing errors. This is consistent with the evidence

in Collin-Dufresne, Goldstein, and Martin (2001).

6 Robustness tests

6.1 Leland-Toft model

One might be concerned how specific the results are to the use of the Merton model instead of some other structural model. In particular, the assumption that all the firm's debt matures at bond maturity could lead to misleading results for short-maturity bonds.

To address this concern we implement the Leland and Toft (1996) (LT) model. We assume that newly issued bonds have a maturity of 10 years or equal to the maturity of the bond we are pricing, whichever is highest. This implies that the firm has spread its debt over maturities from 0 to 10 years and if we are pricing a short-maturity bond only a small amount of debt will be repaid before this bond is repaid. The details of the implementation are given in Appendix D.

Table 12 shows the actual and model-implied spreads in the Leland-Toft model.[30] We see that the assumption in the LT model that not all debt matures at bond maturity has a strong effect on short-maturity bonds where spreads are lowered substantially. We also see that for long-maturity bonds, spreads in the LT model are similar to spreads in the Merton model.

Table 13 shows the convexity bias in the LT model. We see a strong convexity bias that increases in credit quality and as bond maturity shortens. Comparing the bias in percentage of spread in table 13 with those from the Merton model in Table 6, the convexity bias is in fact stronger for the LT. This shows that the biases found in Section 3 are not specific to

[30] As explained in the Appendix transactions where the bankruptcy boundary is above current firm value are discarded and therefore the number of observations is 11% lower than in the benchmark Merton model case.

the Merton model.

Figure 7 shows the time series variation of the 10-year BBB-AAA spread in the LT model. As in the Merton model, the LT model tracks the level and time series variation of the actual BBB-AAA spread well.

Overall, the main conclusions hold when using the Leland-Toft model (with a maturity structure of debt and endogenous default) instead of the Merton model.

6.2 Using Merrill Lynch quotes for 1997-2012

The evidence presented so far has relied on the TRACE data set which contains almost all corporate bond transactions in the US since 2002. In contrast, previous literature has had to rely on dealer quotes instead of transactions data. To test the extent to which previous conclusions might be influenced by the use of dealer quotes instead of transactions data, we repeat our analysis using daily quotes provided by Merrill Lynch on all corporate bonds included in the Merrill Lynch investment grade and high-yield indices.

A concern when using dealer quotes is that they are prices at which dealers are willing to buy and therefore they represent bid prices and are sensitive to time variation in the bid-ask spread. Using a database of transactions data for the period 1995-1997, Schultz (2001) finds that for investment grade bonds dealer buy prices on average exceeds Lehman quotes by 6 cents and dealer sell prices on average exceeds Lehman quotes by 34 cents (per notional $100). Thus, he finds that Lehman quotes used by HH, Duffee (1998) and others are lower than actual transactions even at the bid.

To examine the severity of this bias we search for every TRACE interdealer transaction a corresponding Merrill Lynch quote on the same day and record the difference. We only use interdealer transactions so that we look at midprices. Figure 8 shows the volume-weighted average difference.[31] We see that in the first half of the sample period the bias is negligible,

[31] The difference is winsorized at -500bps and 500bps.

but in the second half the bias is quite large with a peak of 70 basis points for BBB in 2008.[32] We also see that the bias in the second sample half is larger as we move down in rating. The bias is of a magnitude that can in certain periods lead to misleading conclusions. It is also conceivable that the bias is smaller in our sample period than before TRACE, because all market participants know at which price current transactions occur, while before TRACE the market was opaque with no post-trade transparency.

With the documented bias in mind, we repeat the analysis of the previous sections using Merrill Lynch data instead of TRACE data. Since the Merrill Lynch data does not have transaction volume, we do not volume-weight, but an advantage of the data is that it extends back to January 2, 1997.

Figure 9 shows the 10-year actual and model-implied BBB-AAA yield spreads. The figure confirms our earlier results that the actual BBB-AAA spread is matched well by the model-implied spread and that there is no credit spread puzzle. The extra five years of data does hint to a pattern of slight overprediction of spreads in 2000-2003 and 2008-2012 and slight underprediction in 1998-1999 and 2004-2007, but further evidence is needed to confirm this observation.

6.3 Callable bonds

Our sample period 2002-2012 is a period where there is a significant amount of noncallable industrial bonds issued and this fact allows us to exclude callable bonds from the sample. Guntay, Prabhala, and Unal (2002) document that in the 80s and earlier the vast majority of US corporate bonds were callable and therefore papers using longer historical time periods often include callable bonds in their sample as Table 1 shows. Also, Moody's composite AAA and BBB long-term yields published on the Federal Reserve's webpage used in Chen,

[32] In results not reported we find the bias to be largest for short-maturity bonds. The shortest maturity is one year in the Merrill Lynch data.

Collin-Dufresne, and Goldstein (2009) and Chen (2010) include callable bonds (see Duffee (1998) and Chen, Collin-Dufresne, and Goldstein (2009) for a discussion).

Since callable bonds have been used in tests of structural models, we investigate to what extent our conclusions are affected when we look at callable bonds. We therefore repeat the analysis in the main text but use callable bonds instead of noncallable bonds. All other restrictions such as no covenants, nonconvertible etc are the same. Since the number of observations is larger, we only look at transactions with a trade size of at least $100,000 for computational reasons.

Tables 14 and 15 shows the firm characteristics for all firms issuing callable bonds. The number of investment grade firms is similar to the number for the noncallable sample, while for the speculative grade categories the number of firms in the callable sample is much higher. Interestingly, the average leverage ratio in the callable sample is lower for all rating categories compared to the noncallable sample. Bond summary statistics for the callable sample is in Table 16. We see that the majority of bonds in the callable sample are long maturity, so most of the bonds get called before maturity. Note that average trade size and number transactions are not comparable to those for the noncallable sample because we only look at large transactions.

Model-implied and actual spreads for callable bonds are in Table 17. The call option leads to lower bond prices and higher yield spreads. Using a sample of firms that have issued both callable and noncallable bonds, Jarrow, Li, Liu, and Wu (2010) estimate that the call option on average raises spreads by 48 basis points. With this in mind we see that the main conclusions are the same. Over the whole sample the actual BBB-AAA spread is 60 basis points while the model-implied BBB-AAA spread is 57 basis points. Actual short-term spreads for bonds rated A or higher are close to zero while short-term BBB spreads are higher than zero.[33] The Merton model predicts essentially zero median spreads for all short

[33]For short-maturity bonds the call value is most likely close to zero.

maturity investment grade bonds. Actual long-term AAA spread of 54 basis points is higher than the one basis point predicted by the Merton model. The difference between actual and model-implied long-term AAA spreads vanishes if the call spread of 48 basis points estimated by Jarrow, Li, Liu, and Wu (2010) applies to AAA bonds, but most likely the call correction is smaller for the long-term AAA bonds in our sample.[34]

6.4 Financial firms

In all studies of the credit spread puzzle we are aware of, financial firms are excluded from the analysis. Although the reason is typically not given, an obvious concern is that financial firms are opaque and it is difficult to estimate their leverage, asset value, and asset volatility. Also, some financial firms might benefit from an implicit government guarantee (see Acharya, Anginer, and Warburton (2013)). Nevertheless, Moody's KMV report default probabilities for financial firms - based on a Merton-type model - and Munves, Smith, and Hamilton (2010) find that their measure did a good job in rank ordering defaulters during the recent crisis. In this section, we therefore repeat the analysis in the main section for financial firms.

Firm characteristics for the financial firms are in Tables 18 and 19. We see that the leverage of financial firms is much higher than that of industrial firms and even the highest rated firms have an average leverage ratio of 0.69. Table 20 reports bond statistics. Although the number of financial firms is not higher than the number of industrial firms in our noncallable samples, financial firms issue more bonds so the number of bonds is substantially higher.

Model-implied and actual spreads for financial firms are in Table 21. We see that for almost all investment grade rating categories, bond maturities, and in both the first and

[34]We note though that the call correction for bonds with a AAA or AA rating can be substantial. The actual spread of 208 basis points for medium maturity AAA bonds in 2002-2007 is predominantly driven by one bond issued by New York Tel Co. The bond was a 40-year bond issued in 1966 with a coupon of 4.8%. Following the decline in interest rates in 2002-2003 the bond was trading close to par at spreads of several hundred basis points and was called in March 2003.

second half of the sample period, spreads in the Merton model are higher than actual spreads. For example, the actual median long-term AAA spread is 116 basis points while the model-implied spread is 201 basis points. This suggests that the Merton model is not well suited to price bonds isued by financial firms. The difference between actual and model-implied spreads could also be due to the difficulty in measuring leverage and asset volatility for financial firms instead of a failure of the Merton model, but it is beyond the scope of this paper to distinguish between the two causes.

6.5 Treasury yields as the riskfree rate

We use swap rates as riskfree rates because recent evidence shows that swap rates are close to riskfree rates as discussed in footnote 10. Nevertheless, most papers examining the credit spread puzzle use Treasury yields as riskfree rates as Table 1 shows. While this should not matter much for the BBB-AAA spread, it will raise other spreads such as the AAA spread to the riskfree rate. To see how much of an effect using Treasury yields instead of swap rates has on our results, we repeat the analysis in the main text using Treasury yields as riskfree rates. We use daily par yield curves from Gürkaynak, Sack, and Wright (2006). The par yields are for maturities 0.5, 1,...,30 years and other maturities are linearly interpolated. Table 22 reports the actual and model-implied spreads.

We see that actual spreads are higher by 17-45 basis points. Over the whole sample the BBB-AAA spread is 120-77=43 basis points which is 20 basis points smaller than the corresponding spread using swap rates. This might be surprising given that the riskfree rate should "difference out". However, the observations are not evenly spread out through time and in our sample AAA bonds trade more and BBB bonds trade less when the market is in distress such as in 2008 and distress typically results in a higher spread between swap and Treasury rates. The plot of the time series variation of the long-term BBB-AAA spread when using Treasury yields is nearly identical to Figure 4 and we therefore do not report it.

7. Conclusion

The credit spread puzzle has been documented in the literature by calibrating structural models to a representative firm using average firm variables and/or historical default frequencies and comparing model-implied spreads to average actual spreads. We find that there are two problems with this approach. The first problem is that spreads are typically convex in the input parameters such as asset volatility and leverage ratio, so average spreads are higher than spreads of a representative firm. A similar bias occurs when looking at default probabilities. We examine these biases empirically and find them to be economically highly significant. The second problem is that when fitting to historical default frequencies, the implicit assumption is that *ex-post* historical default frequencies proxy well for *ex ante* average default probabilities. In a simulation study we find that even over a period of almost 30 years the ex-post historical default frequency can differ dramatically from the ex ante average default probability. Thus the statistical power of fitting to historical default frequencies is low.

We then test the Merton in a bias-free approach and find that the Merton model captures the level and time series variation of long-term BBB-AAA US corporate bond spreads. We document that short-term yield spreads of AAA-A-rated bonds are close to zero in normal times consistent with predictions from the Merton model, while short-term BBB spreads are higher than spreads implied by the Merton model. We find that the Merton can explain the size of long-term A spreads, but undershoots long-term AAA spreads by around 30 basis points in normal times. Overall, we find much weaker evidence for a credit spread puzzle than previously found.

Our results show that when testing structural models it is important take into account the cross-sectional and time series variation of leverage and other firm variables. While it is useful to assess the default probabilities implied by models, the statistical uncertainty when using historical default frequencies is too large to deliver any firm conclusions. An alternative

approach is to compare model-implied default probabilities with default probabilities implied from a statistical model such as the model in Duffie, Saita, and Wang (2007) or Moody's KMV. This comparison can be done for any firm at any time, and in follow-up work we are working on this.

We show that the empirical results when using the Leland-Toft model are similar to those using the Merton model, so the results are not particular to the Merton model. This implies that extensions of standard structural models face harder cross-sectional restrictions than previously thought. An extension raising long-term AAA spreads by 30 basis points on average makes model AAA spreads consistent with actual long-term AAA spreads. However, since long-term BBB-AAA spreads are captured, BBB spreads should not be raised disproportionately more. In the very short end of the yield curve, an extension raising BBB spreads by 100 basis points to a level consistent with actual spreads, should only have a small effect of 10 basis points or less on AAA, AA, and A spreads.

A Data

This Appendix gives details on the corporate bond transactions dataset and how firm variables, leverage, payout rate, and equity volatility are calculated using CRSP/Compustat.

A.1 Bond data

Since July 1, 2002, members of the Financial Industry Regulatory Authority have been required to report their secondary over-the-counter corporate bond transactions through the Trade Reporting and Compliance Engine (TRACE) and the transactions are disseminated to the public within 15 minutes.[35]

[35] In the initial phase of TRACE the disseminating times were longer than 15 minutes. Since July 1, 2005 the reporting and dissemination is required to occur within 15 minutes after the trade.

Initially, the collected trade information was publicly disseminated only for investment grade bonds with issue sizes greater than $1 billion. Gradually, the set of bonds subject to transaction dissemination increased and since January 9, 2006 transactions in all non-144A bonds transactions have been immediately disseminated.[36] Goldstein and Hotchkiss (2008) provide a detailed account of the dissemination stages. In the publicly disseminated data the trade size is capped at $5 million in investment grade transactions and $1 million in speculative grade transactions. Since November 3, 2008, the publicly available TRACE data indicate whether a transaction is an interdealer transaction or a transaction with a customer and, if a customer transaction, whether the broker-dealer is on the buy or the sell side. This publicly disseminated data is available through Wharton Research Data Services (WRDS) and is used in for example Dick-Nielsen, Feldhütter, and Lando (2012) and Bao, Pan, and Wang (2011). We use this data for the period September 15, 2011- June 30, 2012.

Through FINRA we have access to historical transactions information not previously disseminated. The historical data is richer than the WRDS data in three aspects. First, the data contains all transactions in non-144A bonds since July 2002, so the data set for the first years of TRACE is significantly larger than the WRDS data set. Second, the data has buy/sell indicators for all transactions, not just after October 2008 as in the WRDS data set. Third, trade volumes are not capped. FINRA provide access to the enhanced historical data with a lag of 18 months. We use this data for the period July 1, 2002-September 14, 2011.

We obtain bond information from the Mergent Fixed Income Securities Database (FISD) and limit the sample to senior unsecured fixed rate or zero coupon bonds. We exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, have sinking

[36]Rule 144a allows for private resale of certain restricted securities to qualified institutional buyers. According to TRACE Fact Book 2011, the percent of rule 144A transactions relative to all transactions was 2.0% in investment grade bonds and 8.4% in speculative grade bonds. Also, transactions reported on or through an exchange are not included in TRACE.

fund provisions, or have covenants. For bond rating, we use the lower of Moody's rating and S&P's rating and discard any transactions that do not have a Moody's or S&P rating on transaction day. We track rating changes on a bond, so the same bond can appear in several rating categories over time. Bonds for which FISD do not provide information are dropped from the sample. Erroneous trades are filtering out as described in Dick-Nielsen (2009). We exclude transactions with a yield of 99999.9999% or 99999.99%.

A.2 Firm data

To compute bond prices in the Merton model we need the issuing firm's leverage ratio, payout ratio, and asset volatility. Firm variables are collected in CRSP and Compustat. To do so we match a bond's CUSIP with CRSP's CUSIP. In theory the first 6 digits of the bond cusip plus the digits '10' corresponds to CRSP's CUSIP, but in practice only a small fraction of firms is matched this way. Even if there is a match we check if the issuing firm has experienced M&A activity during the life of the bond. If there is no match, we hand-match a bond cusip with firm variables in CRSP/Compustat.

Leverage ratio: Equity value is calculated on a daily basis by multiplying the number of shares outstanding with the price of shares. Debt value is calculated in Compustat as the latest quarter observation of long-term debt (DLTTQ) plus debt in current liabilities (DLCQ). Leverage ratio is calculated as $\frac{\text{Debt value}}{\text{Debt value+Equity value}}$.

Payout ratio: The total outflow to stake holders in the firm is interest payments to debt holders, dividend payments to equity holders, and net stock repurchases. Interest payments to debt holders is calculated as the previous year's total interest payments (previous fourth quarter's INTPNY). Dividend payments to equity holders is the indicated annual dividend (DVI) multiplied with the number of shares. The indicated annual dividend is updated on a daily basis and is adjusted for stock splits etc. Net stock repurchases is the previous year's total purchase of common and preferred stock (previous fourth quarter's PRSTKCY).

Payout ratio is the total outflow to stake holders divided by firm value, where firm value is equity value plus debt value.

Equity volatility: We calculate the standard deviation of daily returns (RET in CRSP) in the past three years to estimate daily volatility. We multiply the daily standard deviation with $\sqrt{255}$ to calculate annualized equity volatility. If there are no return observations on more than half the days in the three year historical window, we do not calculate equity volatility and discard any bond transactions on that day.

B Pricing formulas

For completeness we include in this Appendix formulas for bond prices, default probabilities, and deadweight losses.

B.1 Bond price

Equation (1) states firm value as a Geometric Brownian motion under the pricing measure,

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_A dW_t,$$

and the firm defaults if firm value is below face value of the bond at bond maturity, $V_T < F$. The bond price at time 0 is calculated as

$$P(0,T) = E^Q[e^{-rT}(1_{\{V_T \geq F\}} + \min(R, \frac{V_T}{F})1_{\{V_T < F\}})] \tag{9}$$

where R is the recovery rate. From Eom, Helwege, and Huang (2004) Appendix A.1 we have that

$$E^Q[1_{\{V_T \geq F\}}] = N(d_2(F,T)) \tag{10}$$

and

$$E^Q[1_{\{V_T < F\}} \min(\psi, V_T)] = \frac{V_0}{D(0,T)} e^{-\delta T} N(-d_1(\psi,T)) + \psi[N(d_2(\psi,T)) - N(d_2(F,T))], \tag{11}$$

where $\psi \in [0, K]$, N represents the cumulative standard normal function,

$$\begin{aligned} d_1(x,T) &= \frac{\log(\frac{V_0}{xD(0,T)}) + (-\delta + \sigma_A^2/2)T}{\sigma_A\sqrt{T}}, \\ d_2(x,T) &= d_1(x,T) - \sigma_A\sqrt{T}, \end{aligned}$$

and $D(0,T) = \exp(-rT)$. Using equation (10) and (11) in (9) gives the solution to the bond price. The yield spread is calculated as

$$s(0,T) = -\frac{\log(P(0,T))}{T} - r. \tag{12}$$

B.2 Default probabilities and deadweight loss

Under the physical measure firm value is given as the Geometric Brownian motion

$$\frac{dV_t}{V_t} = (\pi_A + r - \delta)dt + \sigma_A dW_t,$$

where π_A is the asset risk premium.

Default probabilities are given by (10) where the expectation is taken under the physical measure.

The expected deadweight loss in bankruptcy as a fraction of asset value is given as

$$1 - E^P[\frac{\min(V_T, RF)}{V_T}|V_T < F]$$

and this expression can be solved numerically using that V_T is log-normally distributed.

C Analytic formula for the distribution of realized default frequencies

Section 4 explains how Moody's calculate realized default frequencies over longer periods of time, and the realized default frequencies are used in a number of papers as moments

to match when calibrating structural models. In this Appendix we derive an approximate analytical solution for the distribution of realized default frequencies.

We assume as in Section 4 that firms are identical, exposed to systematic risk, and firm default occurs according to the Merton model. The cumulative T-year default probability for firm i in the cohort born at time t is given in equation (6) as

$$p = P(c \leq W^P_{i,t+T} - W^P_{i,t})$$

where c depends on the underlying firm parameters as described in the main text. Furthermore, equation (7) gives the Brownian motion as

$$W^P_{i,t} = \sqrt{\rho} W_{s,t} + \sqrt{1-\rho} W_{i,t}$$

where W_s is common to all firms and W_i is specific to firm i.

Assume that Moody's calculate the average T-year cumulative default frequency over N cohorts (where there is one cohort each year).

First let $N = 1$; for example Moody's calculate the 10-year cumulative default frequency based on one cohort born in 1970. Vasiček (1991) derives an approximate analytical solution for the distribution for the realized default frequency[37]: the cumulative distribution function is given as

$$W(s|p,\rho) = N\Big(\frac{1}{\sqrt{\rho}}\big(\sqrt{1-\rho} N^{-1}(s) - N^{-1}(p)\big)\Big)$$

where N is the cumulative normal distribution function and s is the default frequency. Its density is

$$f(s|p,\rho) = \frac{\sqrt{1-\rho}}{\rho} \exp\Big(-\frac{1}{2\rho}\big(\sqrt{1-\rho} N^{-1}(s) - N^{-1}(p)\big)^2 + \frac{1}{2}\big(N^{-1}(s)\big)^2\Big). \tag{13}$$

[37] The approximation lies in that Vasicek assumes there is an infinite number of firms in the cohort instead of a finite number.

Assume now $N > 1$. If firms are from the same cohort the correlation of their Brownian motions is 0.25. If firm i is from cohort t and firm j is from cohort $t+1$ the covariance of their Brownian motions is

$$\begin{aligned} cov(W^P_{i,t+T} - W^P_{i,t}, W^P_{j,t+1+T} - W^P_{j,t+1}) &= cov(\sqrt{\rho}(W_{s,t+T} - W_{s,t}), \sqrt{\rho}(W_{s,t+1+T} - W_{s,t+1}) \\ &= \rho cov(W_{s,t+T} - W_{s,t+1}, W_{s,t+T} - W_{s,t+1}) \\ &= \rho(T-1) \end{aligned}$$

and the correlation is

$$\frac{\rho(T-1)}{\sqrt{T}\sqrt{T}} = \frac{T-1}{T}\rho.$$

It is easy to see that for two firms in cohorts k years a part where $k < N$ the correlation is $\frac{T-k}{T}\rho$ and 0 if $k \geq T$, so the correlation is $\max(0, T-k)\rho$. We can now define the average correlation as

$$\rho \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N} \sum_{j=1}^{N} \frac{\max(0, T - |i-j|)}{T} \tag{14}$$

and use this average correlation in the Vasiček (1991) formula.

Table A1 shows quantiles from both the simulation and approximation where the default probability is $p = 4.39\%$ and correlation is $\rho = 0.25$. We see that the approximation underestimates the widths of confidence bands slightly. For a sample period of 30 years for example, the width of the 95% confidence band in the simulated distribution is $13.15\% - 0.61\% = 12.54\%$ while it is $12.61\% - 0.71\% = 11.90\%$ in the approximating distribution. We therefore underestimate the uncertainty of estimated default rates slightly by using the approximating distribution.

D Implementation details of Leland-Toft model

In the implementation of the Leland-Toft model we use the formulas in Leland and Toft (1996) (LT). We assume that the maturity of newly issued bonds is $T = 10$ years if we price

a bond with a maturity t of less than 10 years and otherwise it is equal to the maturity of the bond. So $T = max(10, t)$. This implies that when we price a bond with a maturity shorter than 10 years, part of total debt matures after bondholders of this specific bond are paid back. We calculate the default boundary V_B according to eq. (13) in LT where $\delta V_T = C$ as assumed by LT in Section III. This implies that tax deductability is lost when cash flows cannot cover interest payments.

The parameters used in the Merton model are also used in the LT model. In addition, we assume when pricing a specific bond the coupons of other bonds of the firm are the same as the coupon of the specific bond. We assume that the tax rate is $\tau = 0.35$ consistent with LT and use deadweight cost of bankruptcy of 35%, $\alpha = 0.35$, which is lower than $\alpha = 0.50$ used in LT but consistent with the cost used in the Merton model.

We price bonds according to eq. (3) in LT. The yield on the bond, y, is then found by numerically solving the equation $p = \frac{c}{y} + (p - \frac{c}{y})e^{-yt}$ where p is the model-implied bond price and c is the coupon.

For 11% of the observations the value of the firm is below the default boundary. We discard these observations reducing the data set to 475,795 transactions.

References

Acharya, V., D. Anginer, and A. J. Warburton (2013). The End of Market Discipline? Investor Expectations of Implicit State Guarantees. Working Paper. New York University.

Alderson, M. and B. Betker (1995). Liquidation costs and capital structure. *Journal of Financial Economics 39*, 45–69.

Bao, J. and J. Pan (2013). Bond Illiquidity and Excess Volatility. *Review of Financial Studies, forthcoming*.

Bao, J., J. Pan, and J. Wang (2011). The illiquidity of Corporate Bonds. *Journal of Finance 66*, 911–946.

Bessembinder, H., K. M. Kahle, W. F. Maxwell, and D. Xu (2009). Measuring abnormal bond performance. *Review of Financial Studies 22*(10), 4219–4258.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2010). The Levered Equity Risk Premium and Credit Spreads: A Unified Framework. *Review of Financial Studies 23*(2), 645–703.

Bohn, J., N. Arora, and I. Korablev (2005). Power and level validation of the EDF credit measure in the U.S. market. *Moody's KMV Company*.

Bongaerts, D., J. Driessen, and F. de Jong (2011). Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market. *Journal of Finance 66*(1), 203–240.

Campello, M., L. Chen, and L. Zhang (2008). Expected returns, yield spreads, and asset pricing tests. *Review of Financial Studies 21*(3), 1297–1338.

Caouette, J., E. Altman, and P. Narayanan (1998). *Managing credit risk: The next great financial challenge*. New York: John Wiley and Sons, Inc.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2009). On the relation between the credit spread puzzle and the equity premium puzzle. *Review of Financial Studies 22*, 3367–3409.

Choi, J. and M. Richardson (2012). The volatility of firm's assets and the leverage effect. *Working Paper*.

Collin-Dufresne, P., R. Goldstein, and S. Martin (2001). The Determinants of Credit Spread Changes. *Journal of Finance 56*, 2177–2207.

Covitz, D. and C. Downing (2007). Liquidity or credit risk? The determinants of very short-term corporate yield spreads. *Journal of Finance 62*(5), 2303–2328.

Cremers, M., J. Driessen, and P. Maenhout (2008). Explaining the level of credit spreads: Option-implied jump risk premia in a firm value model. *Review of Financial Studies 21*, 2209–2242.

David, A. (2008). Inflation Uncertainty, Asset Valuations, and the Credit Spread Puzzle. *Review of Financial Studies 21*(6), 2487–2534.

Davydenko, S. A., I. A. Strebulaev, and X. Zhao (2012). A Market-Based Study of the Cost of Default. *Review of Financial Studies 25*(10), 2959–2999.

Dick-Nielsen, J. (2009). Liquidity biases in TRACE. *Journal of Fixed Income 19*(2), 43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2012). Corporate bond liquidity before and after the onset of the subprime crisis. *Journal of Financial Economics 103*, 471–492.

Duffee, G. R. (1998). The Relation Between Treasury Yields and Corporate Bond Yield Spreads. *Journal of Finance 53*(6), 2225–2241.

Duffie, D. and D. Lando (2001, May). Term Structures of Credit Spreads with Incomplete Accounting Information. *Econometrica*, 633–664.

Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics 83*, 635–665.

Elton, E., M. Gruber, D. Agrawal, and C. Mann (2001a). Explaining the rate spread on corporate bonds. *Journal of Finance 56*, 247–277.

Elton, E. J., M. Gruber, D. Agrawal, and C. Mann (2001b). Explaing the rate spread on corporate bonds. *Journal of Finance 56*, 247–277.

Eom, Y. H., J. Helwege, and J.-Z. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies 17*(2), 499–544.

Ericsson, J. and O. Renault (2006). Liquidity and Credit Risk. *Journal of Finance 61*(5), 2219–2250.

Ericsson, J., J. Reneby, and H. Wang (2007). Can structural models price default risk? Evidence from bond and credit derivative markets. *Working Paper*.

Feldhütter, P. and D. Lando (2008). Decomposing Swap Spreads. *Journal of Financial Economics 88*, 375–405.

Gilson, S. (1997). Transactions costs and capital structure choice: Evidence from financially distressed firms. *Journal of Finance 52*(1), 161–196.

Glover, B. (2012). The Expected Cost of Default. *Working Paper*.

Goldstein, M. A. and E. Hotchkiss (2008). Dealer Behavior and the Trading of Newly Issued Corporate Bonds. *Working Paper*.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2006). The U.S. Treasury Yield Curve: 1961 to the Present. *Working Paper, Divisions of Reasearch and Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C.*.

Guntay, L., N. Prabhala, and H. Unal (2002). Callable bonds and hedging. *Working Paper, The Wharton School, University of Pennsylvania*.

Hackbarth, D., J. Miao, and E. Morellec (2006). Capital structure, credit risk, and macroeconomic conditions. *Journal of Financial Economics 82*, 519–550.

Hamilton, D. T. and R. Cantor (2006). Measuring Corporate Default Rates. *Special Comment. New York: Moody's Investors Services*, 1–16.

He, Z. and K. Milbradt (2013). Endogenous Liquidity and Defaultable Debt. *Forthcoming, Econometrica*.

He, Z. and W. Xiong (2012). Rollover Risk and Credit Risk. *Journal of Finance 67*, 391–429.

Huang, J. and M. Huang (2012). How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk? *Review of Asset Pricing Studies 2*(2), 153–202.

Hull, J., M. Predescu, and A. White (2004). The Relationship Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements. *Journal of Banking and Finance 28*, 2789–2811.

Jarrow, R., H. Li, S. Liu, and C. Wu (2010). Reduced-Form Valuation of Callable Corporate Bonds: Theory and Evidence. *Journal of Financial Economics 95*, 227–248.

Kealhofer, S., S. Kwok, and W. Weng (1998). Uses and Abuses of Bond Default Rates. *KMV Corporation Document*.

Keenan, S., I. Shtogrin, and J. Sobehart (1999). Historical Default Rates of Corporate Bond Issuers, 1920-1998. *Special Comment. New York: Moody's Investors Services*, 1–84.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The Aggregate Demand for Treasury Debt. *Journal of Political Economy 120*(2), 233–267.

Leland, H. (2004). Predictions of Default Probabilities in Structural Models of Debt. *Journal of Investment Management 2*.

Leland, H. (2006). Structural Models of Corporate Financial Choice. *Princeton Lectures in Finance, Lecture 1*.

Leland, H. and K. Toft (1996). Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance 51*(3), 987–1019.

McQuade, T. J. (2013). Stochastic Volatility and Asset Pricing Puzzles. *Working Paper, Harvard University*.

Merton, R. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance 29*, 449–470.

Moody's (2011). Corporate Default and Recovery Rates, 1920-2010. *Moody's Investors Service*, 1–66.

Munves, D., A. Smith, and D. Hamilton (2010). Banks and their EDF Measures Now and THrough the Credit Crisis: Too High, Too Low, or Just About Right? *Moody's Analytics*, 1–28.

Ou, S., D. Chiu, B. Wen, and A. Metz (2013). Annual Default Study: Corporate Default and Recovery Rates, 1920-2012. *Special Comment. New York: Moody's Investors Services*, 1–64.

Ou, S., D. Hamilton, and R. Cantor (2004). Short-Term Rating Performance and Corporate Commercial Paper Defaults, 1972-2004. *Special Comment. New York: Moody's Investors Services*, 1–18.

Schaefer, S. and I. Strebulaev (2008). Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *Journal of Financial Economics 90*, 1–19.

Schultz, P. (2001). Bond trading costs: a peak behind the curtain. *Journal of Finance 56*(2), 677–698.

Strebulaev, I. and B. Yang (2013). The mystery of zero-leverage firms. *Journal of Financial Economics 109*, 1–23.

Sun, Z., D. Munves, and D. Hamilton (2012). Public Firm Expected Default Frequency (EDF) Credit Measures: Methodology, Performance, and Model Extensions. *Moody's Analytics*, 1–30.

Sundaresan, S. (2013). A Review of Merton's Model of the Firm's Capital Structure with its Wide Applications. *Annual Review of Financial Economics 5*.

Vasiček, O. (1991). Limiting Loan Loss Probability Distribution. *Technical Report, KMV Corporation*.

Zhang, B. Y., H. Zhou, and H. Zhu (2009). Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms. *Review of Financial Studies 22*(12), 5101–5131.

Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking and Finance 25*, 2015–2040.

Article	rep. firm	default freq. fit	default freq. period	benchmark model Ψ	benchmark model α	benchmark model δ
This paper	No	No		49%	30-35%	4.1-6%(est)
Huang and Huang(2012)	Yes	Yes	70-98	51%	15%	6%
Chen, Collin-Dufresne, and Goldstein(2009)	Yes	Yes	70-01	45%	35-40%	5.89%
Chen(2010)	Yes	Yes	70-00s	41%	0-10%(end)	-
Cremers, Driessen, and Maenhout(2008)	Yes	Yes	70-98	51%	49%	6%
Leland(2006)	Yes	Yes	70-00	50%	30%	6%
McQuade(2013)	Yes	No		51%	30%	6%
Ericsson, Reneby, and Wang(2007)	No	No		40%	15%	2.65%(est)
Eom, Helwege, and Huang(2004)	No	No		51%	30-35%	4.8%(est)

	benchmark model spread	benchmark model bps	data bps	source	period	callable	riskfree
This paper				TRACE	02-12	No	Swap
Huang and Huang(2012)	10y AAA	10	63	Lehman	73-93	Yes	Treasury
	10y BBB-AAA	47	131	Lehman	73-93	Yes	
	4y AAA	1	55	Duffee(1998) + call spread	85-95	Yes	Treasury
	4y BBB-AAA	31	103	Duffee(1998) + call spread	85-95	Yes	
Chen, Collin-Dufresne, and Goldstein(2009)	10y BBB-AAA	77	109	Moody's	70-01	Yes	
	4y BBB-AAA	57	94-102	Lehman	74-98	No	
Chen(2010)	10y BBB-AAA	68	105	Duffee(1998) + 4bps	85-95	No	
Cremers, Driessen, and Maenhout(2008)	10y AAA	12	66	Lehman	83-02	Yes	Treasury
	10y BBB-AAA	57	105	Lehman	83-02	Yes	
Leland(2006)	10y BBB	150	50-60	Duffee(1998)+others	85-95	No	Treasury
McQuade(2013)	10y AAA	2	47	Duffe(1998)	85-95	No	Treasury
	10y BBB-AAA	43	103	Duffee(1998)	85-95	No	
	4y AAA	1	46	Duffee(1998)	85-95	No	Treasury
	4y BBB-AAA	35	103	Duffee(1998)	85-95	No	
Ericsson, Reneby, and Wang(2007)	10y	73-109	111	NAIC	97-03	No	Swap
Eom, Helwege, and Huang(2004)	?	?	?	Fixed Income Database	86-97	No	Treasury

Table 1 *Assumptions and findings in existing literature on the credit spread puzzle.* This table shows assumptions and findings in articles that test standard structural models of credit spreads in terms of matching corporate credit spreads. 'rep. firm' indicates if tests are done on a representative firm. 'def. freq' indicates if a representative firm is calibrated to fit historical default frequency, and if so 'period' is the historical period on which the default frequency is based. For the benchmark model considered ψ is the recovery rate, α is the deadweight cost of bankruptcy, and δ is the payout rate. The deadweight cost of bankruptcy is endogeneously determined in Chen(2010) and marked with '(est)' while the payout rate is estimated in several papers by calculating the total payout to equity and debt holders for individual firms and is marked with '(est)'. Under 'benchmark model' 'spread' refers to the spread examined and 'bps' states the model-implied spread in basis points in the benchmark model. Under 'data' 'bps' states the average actual spread, 'source' is the data source, 'period' is the time period over which spreads are averaged to calculate the actual spread, 'callable' indicates if callable bonds are used in the calculation of the actual spread, and 'riskfree' rates indicates if Treasury rates or swap rates are used as riskfree rates.

	Bonds	Volume	Transactions	Outstanding	Maturity	Coupon
			TRACE universe of bonds			
All	80114	51502.5	77043			
			+information in FISD			
All	54568	48254.5	73647	920	12.2	
AAA	8054	2379.0	3061	1460	8.1	
AA	16872	7246.9	11905	1163	9.2	
A	23102	13095.8	25211	1009	14.4	
BBB	12921	11664.9	16483	770	15.0	
Spec	11983	13868.0	16987	664	9.0	
			+standard except they may be callable			
All	25800	6438.3	17320	815	7.2	5.4
AAA	4970	1203.5	1949	832	7.2	4.6
AA	9042	1461.3	3957	799	7.6	4.9
A	13149	2331.7	7302	823	7.1	5.3
BBB	4918	710.3	1585	831	7.4	6.0
Spec	4215	731.6	2527	793	6.7	6.4
			+noncallable			
All	14556	4568.2	10962	987	4.7	5.0
AAA	1743	1097.1	1460	1770	4.8	4.6
AA	5410	1260.4	2923	1194	5.5	4.7
A	8474	1668.8	4815	877	4.5	5.1
BBB	3230	301.1	828	373	4.4	5.8
Spec	2260	240.7	935	223	3.9	6.0
			+issued by industrial firms			
All	1683	312.9	807	382	6.4	6.2
AAA	22	3.3	9	371	5.0	6.5
AA	575	67.6	149	407	3.4	4.8
A	710	64.0	235	405	4.7	5.8
BBB	748	116.7	245	404	6.9	6.5
Spec	511	61.3	170	298	10.4	7.7
			+firm has info in CRISP/COMPUSTAT			
All	455	239.6	533	505	5.9	6.2
AAA	7	0.9	3	389	8.9	7.8
AA	118	61.7	113	503	3.6	5.0
A	182	47.3	147	600	4.9	6.1
BBB	202	96.3	170	499	5.5	6.2
Spec	55	33.5	100	380	10.7	7.5

Table 2 *Data sample.* This table shows summary statistics of the initial and final data sample. 'TRACE universe of bonds' are all bonds in TRACE during the period 2002Q3-2012Q2. '+information in FISD' are bonds that have information about bond characteristics and rating in FISD. '+standard except they may be callable' are bonds that are senior unsecured fixed rate or zero coupon bonds and are not convertible, putable, perpetual, foreign denominated, Yankee, have sinking fund provisions, or have covenants. '+noncallable' are standard bonds that are not callable. '+issued by industrial firms' are standard noncallable bonds issued by industrial firms. '+firm has info in CRISP/COMPUSTAT' are standard noncallable bonds issued by industrial firms where the firm has information in CRISP/COMPUSTAT. 'Bonds' is the number of bonds, 'Volume' is the total transaction volume (in \$bn), 'Transactions' is the number of transactions (in 1,000), 'Outstanding' (in \$mm) is the average amount issued across transactions, 'Maturity' is the average bond time-to-maturity across transactions, and 'Coupon' is the average coupon across transactions.

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
AAA/AA	13	0.26	0.05	0.09	0.17	0.45	0.52
A	34	0.29	0.14	0.16	0.21	0.32	0.68
BBB	53	0.53	0.20	0.27	0.50	0.78	0.97
Equity volatility							
AAA/AA	13	0.25	0.17	0.20	0.25	0.30	0.34
A	34	0.32	0.19	0.23	0.31	0.39	0.44
BBB	53	0.66	0.32	0.39	0.47	0.97	1.30
Asset volatility							
AAA/AA	13	0.19	0.12	0.14	0.20	0.23	0.26
A	34	0.23	0.13	0.16	0.23	0.30	0.33
BBB	53	0.28	0.06	0.15	0.29	0.34	0.52
Payout ratio							
AAA/AA	13	0.041	0.021	0.023	0.039	0.051	0.068
A	34	0.060	0.023	0.041	0.053	0.072	0.107
BBB	53	0.052	0.018	0.029	0.041	0.064	0.076
2002Q3-2007Q2							
Leverage ratio							
AAA/AA	12	0.28	0.04	0.10	0.36	0.45	0.49
A	29	0.22	0.13	0.15	0.20	0.30	0.35
BBB	46	0.35	0.19	0.22	0.28	0.41	0.72
Equity volatility							
AAA/AA	12	0.26	0.16	0.20	0.24	0.32	0.35
A	29	0.30	0.18	0.22	0.30	0.40	0.44
BBB	46	0.40	0.27	0.35	0.40	0.45	0.51
Asset volatility							
AAA/AA	12	0.19	0.12	0.14	0.20	0.22	0.25
A	29	0.24	0.15	0.18	0.23	0.30	0.34
BBB	46	0.27	0.11	0.22	0.29	0.33	0.34
Payout ratio							
AAA/AA	12	0.033	0.019	0.022	0.025	0.046	0.052
A	29	0.045	0.023	0.031	0.047	0.054	0.061
BBB	46	0.043	0.017	0.020	0.044	0.059	0.069
2007Q3-2012Q2							
Leverage ratio							
AAA/AA	10	0.24	0.08	0.09	0.13	0.52	0.64
A	22	0.36	0.15	0.17	0.22	0.52	0.75
BBB	24	0.72	0.38	0.60	0.68	0.97	0.98
Equity volatility							
AAA/AA	10	0.24	0.18	0.20	0.25	0.28	0.30
A	22	0.34	0.19	0.26	0.33	0.39	0.46
BBB	24	0.93	0.47	0.72	1.00	1.27	1.32
Asset volatility							
AAA/AA	10	0.19	0.11	0.13	0.21	0.25	0.27
A	22	0.21	0.11	0.15	0.21	0.29	0.33
BBB	24	0.29	0.05	0.07	0.27	0.48	0.58
Payout ratio							
AAA/AA	10	0.051	0.026	0.030	0.050	0.061	0.085
A	22	0.076	0.027	0.053	0.068	0.101	0.138
BBB	24	0.062	0.029	0.031	0.041	0.067	0.078

Table 3 *Firm summary statistics, investment grade bonds.* For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
BB	22	0.71	0.30	0.58	0.72	0.97	0.97
B	9	0.66	0.31	0.45	0.74	0.83	0.93
C	4	0.87	0.78	0.82	0.88	0.93	0.96
Equity volatility							
BB	22	0.93	0.39	0.53	0.75	1.35	1.35
B	9	0.60	0.30	0.44	0.70	0.74	0.74
C	4	0.73	0.34	0.70	0.73	0.92	0.98
Asset volatility							
BB	22	0.25	0.06	0.08	0.22	0.34	0.52
B	9	0.23	0.08	0.15	0.23	0.29	0.33
C	4	0.17	0.04	0.05	0.19	0.25	0.29
Payout ratio							
BB	22	0.042	0.029	0.035	0.038	0.042	0.053
B	9	0.042	0.030	0.038	0.040	0.048	0.054
C	4	0.050	0.040	0.045	0.051	0.056	0.061
2002Q3-2007Q2							
Leverage ratio							
BB	18	0.51	0.22	0.31	0.46	0.73	0.91
B	6	0.64	0.31	0.34	0.55	0.92	0.95
C	3	0.89	0.76	0.87	0.92	0.93	0.96
Equity volatility							
BB	18	0.45	0.31	0.33	0.46	0.54	0.62
B	6	0.52	0.29	0.31	0.65	0.70	0.73
C	3	0.73	0.31	0.53	0.80	0.98	0.99
Asset volatility							
BB	18	0.25	0.05	0.19	0.28	0.33	0.34
B	6	0.18	0.06	0.09	0.20	0.22	0.37
C	3	0.14	0.04	0.05	0.13	0.21	0.26
Payout ratio							
BB	18	0.054	0.032	0.039	0.046	0.056	0.072
B	6	0.040	0.029	0.031	0.039	0.048	0.054
C	3	0.045	0.038	0.041	0.044	0.047	0.054
2007Q3-2012Q2							
Leverage ratio							
BB	11	0.76	0.35	0.64	0.77	0.97	0.97
B	7	0.68	0.34	0.73	0.75	0.77	0.80
C	3	0.86	0.78	0.80	0.85	0.92	0.97
Equity volatility							
BB	11	1.05	0.48	0.70	1.33	1.35	1.35
B	7	0.66	0.43	0.68	0.73	0.74	0.74
C	3	0.73	0.53	0.72	0.73	0.77	0.94
Asset volatility							
BB	11	0.25	0.07	0.08	0.18	0.36	0.57
B	7	0.28	0.17	0.26	0.27	0.31	0.33
C	3	0.19	0.04	0.11	0.20	0.28	0.30
Payout ratio							
BB	11	0.040	0.028	0.035	0.038	0.040	0.042
B	7	0.044	0.038	0.039	0.040	0.048	0.065
C	3	0.055	0.046	0.051	0.055	0.059	0.063

Table 4 *Firm summary statistics, speculative grade bonds.* For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

Short bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	84	115	114	25	3	0
Mean number of bonds pr quarter	6.96	9.1	7.9	2.73	1.13	NaN
Mean number of transactions pr quarter	300.2	404.7	472.4	285.2	106.2	NaN
Age	4.57	7.18	5.60	4.52	4.74	NaN
Coupon	4.90	5.58	5.56	5.46	5.43	NaN
Amount outstanding ($mm)	371	461	405	580	395	NaN
Trade size (in 1,000)	542	388	369	396	267	NaN
Time-to-maturity	0.56	0.54	0.58	0.66	0.47	NaN

Medium bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	88	142	139	37	3	1
Mean number of bonds pr quarter	14.9	24	21.1	4.65	1.17	1
Mean number of transactions pr quarter	1502	1673	1926	680.7	117.5	330.7
Age	2.49	5.61	4.22	4.99	3.74	25.19
Coupon	4.70	5.76	5.96	5.87	3.95	9.00
Amount outstanding ($mm)	520	616	527	602	232	100
Trade size (in 1,000)	542	294	613	654	254	33
Time-to-maturity	2.97	2.76	2.62	2.50	1.82	4.83

Long bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	40	70	84	10	9	5
Mean number of bonds pr quarter	5.84	9.96	17.9	2.45	1.8	1.79
Mean number of transactions pr quarter	472.6	809.4	923.1	431.8	173.6	441.6
Age	7.77	7.65	6.20	11.88	14.08	16.47
Coupon	6.28	6.92	7.10	8.18	8.56	9.03
Amount outstanding ($mm)	512	640	491	255	179	190
Trade size (in 1,000)	529	345	571	221	131	136
Time-to-maturity	8.07	11.02	13.66	12.83	13.37	12.78

Table 5 *Bond summary statistics.* Only unsecured senior industrial bonds with a fixed coupon that are not callable, putable, perpetual, asset-backed, convertible, Yankee, foreign currency, and do not contain sinking fund provisions or covenants are used. All transactions in those bonds for which there are also firm variables in Crisp/Compustat are used. Short, medium, and long bond maturities are bonds with a maturity of 0-1, 1-5, and 5-30 years. This table shows summary statistics for the bond transactions. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
$E[s(\theta_{it})]$									
AAA/AA	0	5	20	0	7	13	0	4	27
A	103	102	100	3	14	36	207	190	163
BBB	601	515	210	1	30	82	1200	1000	338
spec	824	368	279	164	81	211	1640	726	348
$s(E[\theta_{it}])$									
AAA/AA	0	0	8	0	1	5	0	0	11
A	0	4	56	0	1	26	0	18	107
BBB	0	136	162	0	7	84	14	521	275
spec	0	48	261	0	7	208	7	198	315
$s(E[\theta_{it}])/E[s(\theta_{it})]$									
AAA/AA	0.00	0.04	0.39	0.00	0.11	0.41	0.00	0.01	0.41
A	0.00	0.04	0.56	0.00	0.05	0.71	0.00	0.09	0.65
BBB	0.00	0.26	0.77	0.00	0.23	1.03	0.01	0.52	0.81
spec	0.00	0.13	0.93	0.00	0.09	0.99	0.00	0.27	0.91

Table 6 *Convexity bias when calculating yield spreads in the Merton model using the representative firm approach.* It is a common approach to compare average actual spreads to model-implied spreads, where the model-implied spreads are calculated using average firm variables. This introduces a bias because the spread in structural models is a non-linear function of firm variables. This table shows the magnitude of this bias. To calculate $E[s(\theta_{it})]$ we compute for every transaction in the sample the Merton spread, calculate the volume-weighted average Merton spread on a monthly basis and then average the monthly average spreads over the sample period. To calculate $s(E[\theta_{it}])$ we compute on a monthly basis the volume-weighted average firm variables (leverage ratio, asset volatility, payout ratio, bond maturity, riskfree rate at same as bond), average the monthly firm variables over the sample period, and then use the averaged firm variables to calculate the spread in the Merton model. All spreads are in basis points. Bond transactions are grouped into groups where the issued bond has remaining maturity 0-1y (short), 1-5y (medium), or 5-30y (long).

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
$E[\mathbf{PD}(\theta_{it})]$									
AAA/AA	0.00	0.13	1.22	0.00	0.18	0.68	0.00	0.08	1.75
A	0.61	2.20	7.43	0.04	0.43	2.90	1.22	3.98	11.97
BBB	4.83	12.16	14.97	0.01	1.21	6.91	9.66	23.11	23.02
spec	6.48	9.92	23.30	1.87	2.32	16.28	12.18	19.42	30.32
$\mathbf{PD}(E[\theta_{it}])$									
AAA/AA	0.00	0.00	0.24	0.00	0.01	0.14	0.00	0.00	0.37
A	0.00	0.09	4.05	0.00	0.01	1.53	0.00	0.42	9.34
BBB	0.00	4.68	14.05	0.00	0.18	7.41	0.10	19.58	22.26
spec	0.00	1.26	27.29	0.00	0.16	19.73	0.06	6.31	35.00
$\mathbf{PD}(E[\theta_{it}])/E[\mathbf{PD}(\theta_{it})]$									
AAA/AA	0.00	0.02	0.20	0.00	0.06	0.21	0.00	0.00	0.21
A	0.00	0.04	0.54	0.00	0.03	0.53	0.00	0.11	0.78
BBB	0.00	0.38	0.94	0.00	0.15	1.07	0.01	0.85	0.97
spec	0.00	0.13	1.17	0.00	0.07	1.21	0.00	0.33	1.15

Table 7 *Convexity bias when calculating model-implied default probabilities in the Merton model using the representative firm approach.* It is a common approach to compare historical default frequencies to model-implied default probabilities, where model-implied default probabilities are calculated using average firm variables. This introduces a bias because the default probability in structural models is a non-linear function of firm value parameters. This table shows the magnitude of this bias. To calculate $E[\text{PD}_T(\theta_{it})]$ we compute the volume-weighted average default probability implied by the Merton on a monthly basis and then average the monthly default probabilities over the sample period. To calculate $\text{PD}_T(E[\theta_{it}])$ we compute the volume-weighted average firm variables (leverage ratio, asset volatility, payout ratio, bond maturity, riskfree rate at same maturity as bond), take the simple average of the monthly volume-weighted averages, and use the averaged firm variables to calculate the implied default probability in the Merton model. Bond transactions are grouped into groups where the issued bond has remaining maturity 0-1y (short), 1-5y (medium), or 5-30y (long).

	Bond maturity			Bond maturity			Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
Panel A: Asset volatility									
	$E(\sigma_A)$			$\hat{\sigma_A}$			Ratio		
AAA/AA	0.20	0.21	0.20	0.38	0.27	0.24	1.93	1.29	1.20
A	0.23	0.23	0.24	0.66	0.34	0.27	2.85	1.45	1.13
BBB	0.27	0.29	0.26	0.68	0.37	0.26	2.50	1.30	1.02
spec	0.28	0.25	0.22	0.65	0.39	0.20	2.34	1.53	0.92
Panel B: Calibrating to average expected default probabilities									
	E(Spread)			$\widehat{\text{Spread}}$			Ratio		
AAA/AA	0	5	20	0	5	24	1.14	0.93	1.22
A	103	102	100	69	66	84	0.67	0.65	0.84
BBB	601	515	210	504	309	169	0.84	0.60	0.81
spec	824	368	279	694	282	238	0.84	0.77	0.85
Panel C: Calibrating to 10-year average expected default probabilities									
	E(Spread)			$\widehat{\text{Spread}}$			Ratio		
AAA/AA	0	5	20	0	1	24	0.00	0.24	1.22
A	103	102	100	0	14	84	0.00	0.14	0.84
BBB	601	515	210	0	95	169	0.00	0.19	0.81
spec	824	368	279	0	11	238	0.00	0.03	0.85

Table 8 *Bias in spread predictions and asset volatility when fitting the Merton model to average default probabilities.* It is a common approach to apply average firm variables (leverage ratio, payout ratio) to a representative firm, and back out asset volatility σ_A by matching expected default probabilities in a structural model to realized historical default frequencies. This introduces a bias in the asset volatility estimate because the default probability in structural models is a non-linear function of firm value parameters as Table 7 shows. Panel A shows the average asset volatility, $E(\sigma_A)$, and the biased implied asset volatility, $\hat{\sigma_A}$, using this approach. Panel B shows the bias when predicting spreads *for the same horizon* as the horizon to which default probabilities are calibrated. Panel C shows the bias of fitting expected default probabilities to average 10-year default probabilities and predicting spreads at various time horizons. We assume that realized default frequency equals average default probability, and calculate average default probability by computing the volume-weighted average default probability implied by the Merton on a monthly basis and then average the monthly default probabilities over the sample period. The asset risk premium is set to 4%. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

Systematic risk	mean	Quantiles						
ρ		0.005	0.025	0.25	0.5	0.75	0.975	0.995
0%	4.39%	4.02%	4.11%	4.29%	4.39%	4.49%	4.68%	4.78%
5%	4.39%	1.72%	2.16%	3.37%	4.21%	5.20%	7.62%	9.01%
10%	4.39%	1.07%	1.50%	2.91%	4.03%	5.46%	9.36%	11.87%
15%	4.41%	0.68%	1.07%	2.55%	3.85%	5.67%	10.84%	14.29%
20%	4.39%	0.45%	0.77%	2.24%	3.64%	5.73%	12.27%	16.76%
25%	4.38%	0.28%	0.56%	1.94%	3.45%	5.81%	13.50%	18.80%
30%	4.39%	0.18%	0.39%	1.69%	3.25%	5.84%	14.86%	21.17%
35%	4.38%	0.11%	0.27%	1.43%	3.02%	5.86%	16.19%	23.62%
40%	4.40%	0.06%	0.18%	1.23%	2.85%	5.88%	17.44%	26.04%
45%	4.39%	0.03%	0.12%	1.02%	2.59%	5.78%	19.02%	28.39%
50%	4.36%	0.02%	0.07%	0.82%	2.37%	5.73%	20.02%	30.20%

Table 9 *Statistical uncertainty of realized BBB default frequencies published by Moody's.* In a simulation experiment we estimate the statistical uncertainty of the average realized cumulative 10-year default frequency of 4.39% of BBB firms published by Moody's using data for 1970-1998. We assume that in year 1 there is a cohort of 1,000 firms. In year 2 there is a new cohort of 1,000 firms formed. This goes on for 18 years. All firms are identical when the cohort is formed and their 10-year default probability is 4.39%. Part of firm volatility is systematic and part is idiosyncratic according to equation (7). For each cohort, we calculate the realized default frequency on a 10-year horizon and calculate the average default frequency across all cohorts. Overall, the realized default rate is based on 18,000 firms over a period of 28 years. We repeat this simulation 100,000 times and the table shows the distribution of realized default rates for different amounts of systematic risk ρ.

	AA	A	BBB	BB	B
4-year maturity					
Actual spread	10	41	103	274	424
Merton model, def. rates 1970-1998	8	13	43	232	544
	(3;38)	(6;51)	(23;122)	(140;434)	(361;818)
Merton model, def. rates 1920-2012	18	33	73	212	438
	(10;36)	(21;60)	(49;117)	(156;300)	(342;566)
10-year maturity					
Actual spread	22	49	101	273	423
Merton model, def. rates 1970-1998	5	16	61	238	423
	(−7;114)	(−1;143)	(23;233)	(125;441)	(258;572)
Merton model, def. rates 1920-2012	30	48	95	224	363
	(10;82)	(22;108)	(55;173)	(154;323)	(275;457)
20-year maturity					
Actual spread	28	58	139	-	-
Merton model, def. rates 1970-1998	3	26	76	199	247
	(−13;157)	(−1;187)	(24;230)	(98;287)	(138;300)
Merton model, def. rates 1920-2012	43	60	98	185	241
	(14;117)	(25;137)	(53;178)	(124;249)	(181;284)

Table 10 *Merton model estimates of corporate spreads to AAA rates using default rates from different periods and taking into account statistical uncertainty about default rates.* Based on the average ex post realized default frequency over some period, we calculate the likelihood function of the ex ante default probability and a 95% confidence band for the ex ante default probability (as explained in Section 4). We then calculate a Merton model spread based on the realized default frequency and a 95% confidence band (using equation (8)). Spreads are relative to the AAA spread with the same maturity. Actual spreads for investment grade bonds are based on Duffe(1998) while actual spreads for speculative grade bonds are based on Caouette, Altman, and Narayanan(1998).

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
AAA/AA									
Actual spread	3	16	32	4	17	29	−3	15	73
	(−13;35)	(−11;119)	(3;112)	(−7;28)	(−9;110)	(7;43)	(−19;59)	(−15;167)	(−15;232)
Model spread	0	0	2	0	1	2	0	0	5
	(0;0)	(0;27)	(0;46)	(0;0)	(0;28)	(0;29)	(0;0)	(0;22)	(0;121)
A									
Actual spread	7	47	85	7	17	65	11	238	232
	(−7;114)	(2;1269)	(24;297)	(−6;39)	(−2;153)	(19;145)	(−11;210)	(23;1675)	(58;1103)
Model spread	0	5	73	0	0	41	0	113	169
	(0;0)	(0;1518)	(0;311)	(0;0)	(0;106)	(0;190)	(0;36)	(0;2243)	(6;1291)
BBB									
Actual spread	104	314	95	27	98	91	408	1019	575
	(16;1249)	(32;1419)	(42;364)	(10;90)	(13;278)	(41;276)	(50;2979)	(278;1809)	(212;1467)
Model spread	0	400	110	0	21	104	1114	1471	451
	(0;3247)	(0;2026)	(23;272)	(0;0)	(0;123)	(21;221)	(0;5751)	(611;2546)	(60;1333)
spec									
Actual spread	325	548	427	82	167	400	445	606	453
	(68;671)	(157;1011)	(49;2077)	(36;228)	(−128;528)	(108;1778)	(202;718)	(324;1042)	(38;3985)
Model spread	2421	1360	309	0	1	236	3233	1555	317
	(0;4457)	(0;2424)	(74;439)	(0;1)	(0;304)	(40;390)	(0;4798)	(255;2721)	(266;468)

Table 11 *Actual and Merton-model yield spreads.* This table shows actual and model-implied industrial corporate bond spreads. Bond transactions are grouped according to remaining maturity at transaction date; 0-1y(short), 1-5y(medium), and 5-30y(long). 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Merton model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
AAA/AA									
Actual spread	3	16	32	4	17	29	−3	15	73
	(−13;35)	(−11;119)	(3;112)	(−7;28)	(−9;110)	(7;43)	(−19;59)	(−15;167)	(−15;232)
Model spread	0	0	1	0	0	1	0	0	3
	(0;0)	(0;12)	(0;32)	(0;0)	(0;12)	(0;17)	(0;0)	(0;21)	(0;86)
A									
Actual spread	7	40	83	7	17	65	11	191	220
	(−7;114)	(2;763)	(23;290)	(−6;39)	(−2;153)	(19;145)	(−11;210)	(21;1521)	(58;1068)
Model spread	0	0	44	0	0	21	0	4	106
	(0;0)	(0;380)	(0;218)	(0;0)	(0;35)	(0;164)	(0;76)	(0;1387)	(1;1035)
BBB									
Actual spread	80	173	94	27	98	91	335	575	507
	(15;1144)	(25;925)	(42;345)	(10;89)	(13;278)	(41;276)	(45;3250)	(183;2202)	(152;1078)
Model spread	0	24	90	0	5	86	2	604	369
	(0;124)	(0;1108)	(14;313)	(0;0)	(0;54)	(14;248)	(0;301)	(64;1428)	(15;1232)
spec									
Actual spread	180	321	382	82	167	381	309	360	387
	(53;516)	(8;595)	(46;926)	(36;228)	(−128;528)	(107;1057)	(154;611)	(254;726)	(38;842)
Model spread	0	31	367	0	0	359	0	1184	367
	(0;408)	(0;1433)	(111;708)	(0;0)	(0;104)	(69;670)	(0;537)	(0;1454)	(217;734)

Table 12 *Actual and Leland-Toft-model yield spreads.* This table shows actual and model-implied industrial corporate bond spreads. Bond transactions are grouped according to remaining maturity at transaction date; 0-1y(short), 1-5y(medium), and 5-30y(long). 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Leland-Toft model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
$E[s(\theta_{it})]$									
AAA/AA	0	3	13	0	3	8	0	3	18
A	43	61	70	0	4	30	88	117	110
BBB	30	222	193	0	14	73	61	430	314
spec	49	148	448	34	40	379	75	303	517
$s(E[\theta_{it}])$									
AAA/AA	0	0	4	0	0	3	0	0	6
A	0	1	39	0	0	19	0	3	74
BBB	0	42	150	0	1	72	0	242	263
spec	0	6	358	0	1	267	0	33	456
$s(E[\theta_{it}])/E[s(\theta_{it})]$									
AAA/AA	0.00	0.00	0.33	0.00	0.00	0.36	0.00	0.00	0.32
A	0.00	0.01	0.56	0.00	0.00	0.61	0.00	0.02	0.68
BBB	0.00	0.19	0.78	0.00	0.09	0.99	0.00	0.56	0.84
spec	0.00	0.04	0.80	0.00	0.03	0.71	0.00	0.11	0.88

Table 13 *Convexity bias when calculating yield spreads in the Leland-Toft model using the representative firm approach.* It is a common approach to compare average actual spreads to model-implied spreads, where the model-implied spreads are calculated using average firm variables. This introduces a bias because the spread in structural models is a non-linear function of firm variables. This table shows the magnitude of this bias. To calculate $E[s(\theta_{it})]$ we compute for every transaction in the sample the Leland-Toft spread, calculate the volume-weighted average Leland-Toft spread on a monthly basis and then average the monthly average spreads over the sample period. To calculate $s(E[\theta_{it}])$ we compute on a monthly basis the volume-weighted average firm variables (leverage ratio, asset volatility, payout ratio, coupon, bond maturity, riskfree rate at same as bond), average the monthly firm variables over the sample period, and then use the averaged firm variables to calculate the spread in the Leland-Toft model. All spreads are in basis points. Bond transactions are grouped into groups where the issued bond has remaining maturity 0-1y (short), 1-5y (medium), or 5-30y (long).

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
AAA/AA	11	0.12	0.08	0.08	0.11	0.14	0.16
A	28	0.21	0.12	0.14	0.19	0.28	0.33
BBB	60	0.27	0.16	0.19	0.23	0.36	0.45
Equity volatility							
AAA/AA	11	0.24	0.18	0.22	0.25	0.26	0.28
A	28	0.28	0.18	0.21	0.28	0.33	0.37
BBB	60	0.40	0.26	0.31	0.39	0.47	0.62
Asset volatility							
AAA/AA	11	0.21	0.16	0.19	0.22	0.24	0.24
A	28	0.22	0.15	0.18	0.22	0.24	0.30
BBB	60	0.30	0.18	0.23	0.29	0.34	0.49
Payout ratio							
AAA/AA	11	0.030	0.012	0.018	0.028	0.043	0.048
A	28	0.039	0.015	0.018	0.037	0.053	0.064
BBB	60	0.030	0.009	0.013	0.025	0.037	0.059
2002Q3-2007Q2							
Leverage ratio							
AAA/AA	8	0.23	0.06	0.08	0.24	0.30	0.35
A	25	0.20	0.13	0.15	0.19	0.24	0.28
BBB	42	0.29	0.17	0.21	0.26	0.39	0.45
Equity volatility							
AAA/AA	8	0.31	0.18	0.19	0.33	0.35	0.38
A	25	0.26	0.17	0.19	0.25	0.32	0.35
BBB	42	0.35	0.26	0.29	0.34	0.40	0.42
Asset volatility							
AAA/AA	8	0.23	0.17	0.17	0.24	0.25	0.30
A	25	0.21	0.14	0.16	0.20	0.26	0.27
BBB	42	0.26	0.17	0.21	0.26	0.31	0.32
Payout ratio							
AAA/AA	8	0.039	0.021	0.034	0.041	0.047	0.052
A	25	0.028	0.013	0.018	0.024	0.037	0.050
BBB	42	0.032	0.012	0.020	0.030	0.035	0.052
2007Q3-2012Q2							
Leverage ratio							
AAA/AA	6	0.12	0.08	0.08	0.10	0.14	0.16
A	17	0.21	0.12	0.13	0.18	0.30	0.33
BBB	32	0.26	0.15	0.18	0.21	0.31	0.43
Equity volatility							
AAA/AA	6	0.24	0.18	0.22	0.25	0.26	0.26
A	17	0.28	0.19	0.22	0.29	0.33	0.39
BBB	32	0.45	0.24	0.36	0.42	0.57	0.63
Asset volatility							
AAA/AA	6	0.21	0.16	0.19	0.22	0.23	0.24
A	17	0.23	0.15	0.18	0.22	0.24	0.32
BBB	32	0.34	0.19	0.27	0.32	0.45	0.51
Payout ratio							
AAA/AA	6	0.029	0.012	0.018	0.026	0.042	0.047
A	17	0.043	0.015	0.023	0.042	0.057	0.066
BBB	32	0.028	0.009	0.011	0.020	0.045	0.059

Table 14 *Firm summary statistics, investment grade bonds, callable bonds*. For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
BB	72	0.48	0.21	0.34	0.55	0.63	0.68
B	84	0.50	0.20	0.30	0.50	0.73	0.80
C	8	0.70	0.35	0.48	0.80	0.90	0.93
Equity volatility							
BB	72	0.51	0.27	0.32	0.45	0.65	0.88
B	84	0.86	0.40	0.66	0.97	1.05	1.09
C	8	0.81	0.65	0.66	0.68	1.07	1.07
Asset volatility							
BB	72	0.31	0.15	0.18	0.29	0.41	0.52
B	84	0.49	0.21	0.35	0.45	0.66	0.74
C	8	0.34	0.09	0.13	0.23	0.61	0.72
Payout ratio							
BB	72	0.041	0.017	0.026	0.039	0.050	0.063
B	84	0.036	0.011	0.020	0.030	0.046	0.067
C	8	0.066	0.030	0.045	0.071	0.088	0.091
2002Q3-2007Q2							
Leverage ratio							
BB	39	0.52	0.23	0.41	0.59	0.63	0.68
B	44	0.53	0.27	0.38	0.50	0.71	0.77
C	2	0.58	0.35	0.40	0.56	0.77	0.82
Equity volatility							
BB	39	0.42	0.27	0.29	0.32	0.48	0.77
B	44	0.87	0.38	0.81	1.00	1.06	1.07
C	2	0.92	0.65	0.66	1.07	1.07	1.07
Asset volatility							
BB	39	0.26	0.14	0.16	0.18	0.29	0.60
B	44	0.51	0.16	0.26	0.63	0.70	0.73
C	2	0.52	0.21	0.26	0.61	0.70	0.75
Payout ratio							
BB	39	0.040	0.017	0.026	0.042	0.049	0.056
B	44	0.035	0.015	0.022	0.027	0.042	0.068
C	2	0.052	0.028	0.032	0.053	0.071	0.081
2007Q3-2012Q2							
Leverage ratio							
BB	42	0.44	0.18	0.29	0.43	0.63	0.68
B	47	0.48	0.09	0.26	0.48	0.74	0.81
C	7	0.83	0.47	0.84	0.90	0.92	0.98
Equity volatility							
BB	42	0.62	0.39	0.44	0.59	0.71	0.99
B	47	0.85	0.46	0.66	0.86	1.03	1.12
C	7	0.69	0.33	0.66	0.67	0.69	0.79
Asset volatility							
BB	42	0.38	0.21	0.31	0.37	0.47	0.51
B	47	0.47	0.28	0.36	0.42	0.56	0.84
C	7	0.16	0.02	0.09	0.13	0.19	0.43
Payout ratio							
BB	42	0.043	0.015	0.026	0.037	0.056	0.081
B	47	0.037	0.007	0.016	0.036	0.053	0.067
C	7	0.079	0.049	0.060	0.088	0.091	0.094

Table 15 *Firm summary statistics, speculative grade bonds, callable bonds*. For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

Short bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	1	21	6	3	0	0
Mean number of bonds pr quarter	1	2.61	1.22	1.13	NaN	NaN
Mean number of transactions pr quarter	9	107.4	26.67	25.88	NaN	NaN
Age	4.10	5.37	4.64	4.49	NaN	NaN
Coupon	4.20	4.95	6.08	5.42	NaN	NaN
Amount outstanding ($mm)	250	632	850	235	NaN	NaN
Trade size (in 1,000)	1395	971	1260	1354	NaN	NaN
Time-to-maturity	0.93	0.59	0.69	0.69	NaN	NaN

Medium bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	11	41	35	15	12	3
Mean number of bonds pr quarter	2.81	9.04	5.73	1.79	2.57	1.07
Mean number of transactions pr quarter	191.8	541	171.8	88.87	63.39	53.43
Age	1.59	2.56	3.76	4.54	1.10	4.85
Coupon	1.61	4.54	5.22	7.55	9.64	9.09
Amount outstanding ($mm)	840	718	588	544	344	190
Trade size (in 1,000)	1080	903	1231	1342	1343	1071
Time-to-maturity	3.34	3.36	3.28	3.16	3.39	3.34

Long bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	28	105	101	104	94	6
Mean number of bonds pr quarter	3.44	19.7	17.8	5.33	3.48	1.07
Mean number of transactions pr quarter	171.5	1064	440.4	272.6	168.2	39.73
Age	1.01	1.83	2.08	2.18	1.88	4.78
Coupon	3.89	5.51	6.00	7.40	7.41	11.05
Amount outstanding ($mm)	779	1007	681	513	719	215
Trade size (in 1,000)	1334	1505	1964	1810	1637	1472
Time-to-maturity	14.61	15.58	15.18	8.33	9.02	5.89

Table 16 *Bond summary statistics, callable bonds*. Only unsecured senior industrial bonds with a fixed coupon that are callable but are not putable, perpetual, asset-backed, convertible, Yankee, foreign currency, and do not contain sinking fund provisions or covenants are used. All transactions in those bonds for which there are also firm variables in Crisp/Compustat are used. Short, medium, and long bond maturities are bonds with a maturity of 0-1, 1-5, and 5-30 years. This table shows summary statistics for the bond transactions. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
AAA/AA									
Actual spread	−17	6	54	0	208	38	−17	6	57
	(−22;−11)	(−13;25)	(16;117)	(0;0)	(47;262)	(17;62)	(−22;−11)	(−13;25)	(16;120)
Model spread	0	0	1	0	28	0	0	0	1
	(0;0)	(0;0)	(0;66)	(0;0)	(14;67)	(0;70)	(0;0)	(0;0)	(0;61)
A									
Actual spread	3	29	101	1	9	64	3	45	124
	(−15;51)	(−3;129)	(39;225)	(−10;12)	(−6;34)	(26;105)	(−16;66)	(5;175)	(56;251)
Model spread	0	0	29	0	0	11	0	0	41
	(0;0)	(0;30)	(0;98)	(0;0)	(0;1)	(0;53)	(0;0)	(0;34)	(0;106)
BBB									
Actual spread	33	104	114	81	73	93	24	123	164
	(13;138)	(39;263)	(55;237)	(36;198)	(34;171)	(46;186)	(13;66)	(54;351)	(83;316)
Model spread	0	42	58	0	41	43	1	42	116
	(0;5)	(0;276)	(8;223)	(0;0)	(0;334)	(6;98)	(0;6)	(0;240)	(17;295)
spec									
Actual spread	457	418	397	103	195	345	540	488	431
	(103;783)	(103;1047)	(167;655)	(77;137)	(73;710)	(151;591)	(403;813)	(250;1235)	(262;671)
Model spread	304	472	271	2	319	120	315	922	422
	(1;583)	(52;1207)	(81;906)	(1;3)	(31;1123)	(74;878)	(65;588)	(262;1240)	(95;914)

Table 17 *Actual and Merton-model yield spreads, callable bonds*. This table shows actual and model-implied industrial corporate bond spreads for callable bonds. Bond transactions are grouped according to remaining maturity at transaction date; 0-1y(short), 1-5y(medium), and 5-30y(long). 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Merton model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
AAA/AA	32	0.69	0.49	0.56	0.72	0.79	0.88
A	42	0.79	0.48	0.70	0.87	0.92	0.96
BBB	16	0.91	0.66	0.93	0.96	0.97	0.98
Equity volatility							
AAA/AA	32	0.39	0.16	0.25	0.39	0.48	0.56
A	42	0.59	0.26	0.36	0.54	0.85	0.94
BBB	16	0.68	0.32	0.33	0.67	0.98	0.99
Asset volatility							
AAA/AA	32	0.16	0.06	0.10	0.16	0.20	0.24
A	42	0.16	0.04	0.07	0.13	0.22	0.32
BBB	16	0.08	0.03	0.04	0.04	0.06	0.29
Payout ratio							
AAA/AA	32	0.054	0.030	0.036	0.045	0.063	0.079
A	42	0.060	0.021	0.029	0.043	0.075	0.092
BBB	16	0.046	0.022	0.032	0.035	0.058	0.089
2002Q3-2007Q2							
Leverage ratio							
AAA/AA	25	0.60	0.48	0.49	0.52	0.69	0.88
A	28	0.74	0.38	0.54	0.83	0.90	0.92
BBB	5	0.84	0.59	0.73	0.93	0.94	0.95
Equity volatility							
AAA/AA	25	0.26	0.14	0.17	0.26	0.34	0.39
A	28	0.30	0.22	0.24	0.29	0.35	0.41
BBB	5	0.37	0.32	0.32	0.32	0.33	0.61
Asset volatility							
AAA/AA	25	0.13	0.05	0.08	0.12	0.20	0.23
A	28	0.10	0.04	0.06	0.09	0.15	0.20
BBB	5	0.10	0.03	0.03	0.04	0.14	0.32
Payout ratio							
AAA/AA	25	0.047	0.029	0.031	0.038	0.049	0.066
A	28	0.067	0.027	0.030	0.040	0.052	0.079
BBB	5	0.045	0.032	0.032	0.033	0.041	0.075
2007Q3-2012Q2							
Leverage ratio							
AAA/AA	29	0.73	0.59	0.69	0.74	0.80	0.88
A	38	0.81	0.52	0.74	0.88	0.92	0.96
BBB	12	0.94	0.87	0.96	0.97	0.97	0.98
Equity volatility							
AAA/AA	29	0.44	0.20	0.33	0.45	0.49	0.66
A	38	0.66	0.35	0.45	0.61	0.90	0.95
BBB	12	0.81	0.41	0.61	0.96	0.98	1.06
Asset volatility							
AAA/AA	29	0.17	0.08	0.11	0.17	0.20	0.24
A	38	0.18	0.04	0.07	0.14	0.25	0.34
BBB	12	0.08	0.03	0.04	0.05	0.05	0.08
Payout ratio							
AAA/AA	29	0.057	0.035	0.037	0.047	0.068	0.085
A	38	0.059	0.019	0.029	0.050	0.079	0.092
BBB	12	0.047	0.022	0.022	0.037	0.059	0.090

Table 18 *Firm summary statistics, investment grade bonds, financial firms.* For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

	#firms	Mean	10th	25th	Median	75th	90th
Full sample period, 2002Q3-2012Q2							
Leverage ratio							
BB	5	0.81	0.69	0.72	0.74	0.96	0.96
B	4	0.87	0.78	0.80	0.88	0.93	0.99
C	3	0.98	0.95	0.97	0.99	0.99	1.00
Equity volatility							
BB	5	0.42	0.33	0.33	0.37	0.41	0.65
B	4	0.56	0.31	0.43	0.45	0.61	1.23
C	3	1.12	0.72	0.81	1.23	1.50	1.52
Asset volatility							
BB	5	0.12	0.02	0.03	0.14	0.16	0.19
B	4	0.11	0.02	0.04	0.11	0.15	0.17
C	3	0.03	0.01	0.02	0.02	0.04	0.06
Payout ratio							
BB	5	0.133	0.038	0.040	0.048	0.269	0.276
B	4	0.088	0.047	0.049	0.069	0.075	0.285
C	3	0.055	0.048	0.048	0.048	0.062	0.073
2002Q3-2007Q2							
Leverage ratio							
BB	3	0.86	0.72	0.73	0.92	0.96	0.96
B	1	0.92	0.92	0.92	0.92	0.92	0.93
C	1	0.92	0.92	0.92	0.92	0.92	0.93
Equity volatility							
BB	3	0.35	0.32	0.33	0.33	0.37	0.38
B	1	0.31	0.31	0.31	0.31	0.31	0.31
C	1	0.31	0.31	0.31	0.31	0.31	0.31
Asset volatility							
BB	3	0.08	0.02	0.02	0.05	0.14	0.15
B	1	0.04	0.04	0.04	0.04	0.05	0.05
C	1	0.04	0.04	0.04	0.04	0.05	0.05
Payout ratio							
BB	3	0.085	0.038	0.038	0.044	0.049	0.270
B	1	0.047	0.047	0.047	0.047	0.047	0.047
C	1	0.047	0.047	0.047	0.047	0.047	0.047
2007Q3-2012Q2							
Leverage ratio							
BB	5	0.74	0.67	0.69	0.72	0.75	0.77
B	4	0.87	0.78	0.80	0.87	0.93	0.99
C	3	0.98	0.95	0.97	0.99	0.99	1.00
Equity volatility							
BB	5	0.53	0.38	0.40	0.41	0.62	0.73
B	4	0.59	0.43	0.43	0.46	0.62	1.23
C	3	1.13	0.72	0.82	1.23	1.50	1.52
Asset volatility							
BB	5	0.18	0.14	0.15	0.17	0.19	0.33
B	4	0.11	0.02	0.08	0.11	0.16	0.18
C	3	0.03	0.01	0.02	0.02	0.04	0.06
Payout ratio							
BB	5	0.202	0.035	0.048	0.262	0.276	0.282
B	4	0.093	0.048	0.065	0.070	0.076	0.286
C	3	0.055	0.048	0.048	0.048	0.062	0.073

Table 19 *Firm summary statistics, speculative grade bonds, financial firms*. For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE while firm variables are based on data from CRSP and Compustat.

Short bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	1509	3004	601	722	418	245
Mean number of bonds pr quarter	105	179	31.4	100	64.2	101
Mean number of transactions pr quarter	6163	6071	1523	2739	1843	8913
Age	4.61	4.06	4.42	3.93	5.33	4.77
Coupon	4.40	4.63	4.58	5.22	5.47	5.51
Amount outstanding ($mm)	1152	699	551	69	63	140
Trade size (in 1,000)	351	211	153	48	45	105
Time-to-maturity	0.59	0.59	0.57	0.54	0.47	0.39

Medium bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	2061	4308	1035	767	267	127
Mean number of bonds pr quarter	350	518	77.8	125	67.3	50.7
Mean number of transactions pr quarter	3.81e+004	3.499e+004	4309	5237	2750	4484
Age	2.83	2.38	4.42	3.17	4.10	4.02
Coupon	4.34	4.79	5.20	5.33	5.74	5.78
Amount outstanding ($mm)	1430	975	552	50	142	337
Trade size (in 1,000)	311	227	226	47	79	204
Time-to-maturity	3.01	3.07	2.63	2.20	1.84	2.14

Long bond maturity

	AAA/AA	A	BBB	BB	B	C
Number of bonds	1017	1418	102	11	10	5
Mean number of bonds pr quarter	165	177	10.1	1.93	2.4	1.62
Mean number of transactions pr quarter	2.682e+004	2.138e+004	592.8	311	412.6	476.7
Age	1.72	1.76	4.37	8.01	6.59	5.65
Coupon	5.30	5.53	5.73	7.83	6.90	6.66
Amount outstanding ($mm)	1734	1337	488	329	657	550
Trade size (in 1,000)	275	312	345	199	624	590
Time-to-maturity	10.25	9.03	10.10	13.13	12.79	14.73

Table 20 *Bond summary statistics, financial firms.* Only unsecured senior bonds issued by financial firms with a fixed coupon that are not callable, putable, perpetual, asset-backed, convertible, Yankee, foreign currency, and do not contain sinking fund provisions or covenants are used. All transactions in those bonds for which there are also firm variables in Crisp/Compustat are used. Short, medium, and long bond maturities are bonds with a maturity of 0-1, 1-5, and 5-30 years. This table shows summary statistics for the bond transactions. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
AAA/AA									
Actual spread	−1	25	116	−1	7	32	−2	60	165
	(−28;74)	(−9;168)	(19;272)	(−11;11)	(−6;31)	(11;65)	(−31;154)	(−10;252)	(84;339)
Model spread	31	457	201	0	56	63	247	624	329
	(0;2015)	(2;1476)	(18;599)	(0;66)	(0;398)	(3;189)	(0;2234)	(67;1770)	(110;676)
A									
Actual spread	40	145	215	9	19	40	76	185	240
	(−1;452)	(13;431)	(43;427)	(−3;78)	(3;60)	(22;83)	(2;561)	(52;482)	(139;451)
Model spread	285	621	378	7	131	61	1138	767	457
	(0;3231)	(46;1707)	(56;838)	(0;407)	(5;431)	(14;252)	(0;3517)	(177;1896)	(189;872)
BBB									
Actual spread	715	518	445	49	131	70	852	585	507
	(114;2721)	(144;1643)	(71;1246)	(15;286)	(28;452)	(38;365)	(224;2800)	(245;1768)	(270;1328)
Model spread	2299	1332	395	59	333	395	3124	1566	407
	(16;5929)	(385;3614)	(24;1067)	(0;184)	(31;786)	(89;504)	(204;6014)	(707;4025)	(18;1067)
spec									
Actual spread	11891	1818	1352	152	393	641	13789	2332	1372
	(230;40072)	(235;4831)	(657;2076)	(42;569)	(95;643)	(482;722)	(563;49237)	(284;5041)	(707;2087)
Model spread	10509	1963	989	47	112	38	11322	2385	989
	(41;19713)	(119;5011)	(243;1361)	(1;448)	(44;1923)	(25;47)	(439;21596)	(862;5022)	(261;1361)

Table 21 *Actual and Merton-model yield spreads, financial firms*. This table shows actual and model-implied financial corporate bond spreads. Bond transactions are grouped according to remaining maturity at transaction date; 0-1y(short), 1-5y(medium), and 5-30y(long). 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Merton model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

	2002Q3-2012Q2 Bond maturity			2002Q3-2007Q2 Bond maturity			2007Q3-2012Q2 Bond maturity		
	short	medium	long	short	medium	long	short	medium	long
AAA/AA									
Actual spread	25	55	77	26	52	76	22	79	92
	(2;89)	(20;172)	(36;145)	(6;48)	(24;137)	(41;87)	(0;159)	(14;250)	(11;280)
Model spread	0	0	2	0	1	2	0	0	5
	(0;0)	(0;27)	(0;46)	(0;0)	(0;28)	(0;29)	(0;0)	(0;22)	(0;121)
A									
Actual spread	32	96	114	31	55	98	53	315	202
	(9;167)	(34;1372)	(62;365)	(7;54)	(30;193)	(60;175)	(10;256)	(59;1805)	(77;1156)
Model spread	0	5	73	0	0	41	0	113	169
	(0;0)	(0;1518)	(0;311)	(0;0)	(0;106)	(0;190)	(0;36)	(0;2243)	(6;1291)
BBB									
Actual spread	141	351	120	44	129	114	436	1053	579
	(31;1291)	(67;1474)	(76;387)	(21;106)	(45;315)	(75;302)	(76;3031)	(305;1872)	(241;1512)
Model spread	0	400	110	0	21	104	1114	1471	451
	(0;3247)	(0;2026)	(23;272)	(0;0)	(0;123)	(21;221)	(0;5751)	(611;2546)	(60;1333)
spec									
Actual spread	349	570	444	101	199	438	468	629	447
	(87;696)	(198;1041)	(68;2068)	(55;258)	(−82;557)	(141;1795)	(224;772)	(347;1070)	(57;3935)
Model spread	2421	1360	309	0	1	236	3233	1555	317
	(0;4457)	(0;2424)	(74;439)	(0;1)	(0;304)	(40;390)	(0;4798)	(255;2721)	(266;468)

Table 22 *Actual and Merton-model yield spreads where Treasury yields are used as riskfree rates.* This table shows actual and model-implied financial corporate bond spreads. Bond transactions are grouped according to remaining maturity at transaction date; 0-1y(short), 1-5y(medium), and 5-30y(long). 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Merton model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions are from TRACE.

Sample years	mean	Quantiles						
		0.005	0.025	0.25	0.5	0.75	0.975	0.995
20								
simulation	4.39%	0.13%	0.31%	1.50%	3.08%	5.87%	15.88%	23.49%
approximation	4.39%	0.14%	0.33%	1.56%	3.13%	5.84%	15.65%	22.93%
30								
simulation	4.39%	0.32%	0.61%	2.02%	3.51%	5.80%	13.15%	18.17%
approximation	4.39%	0.39%	0.71%	2.14%	3.59%	5.77%	12.61%	17.37%
50								
simulation	4.40%	0.70%	1.10%	2.60%	3.89%	5.65%	10.57%	13.74%
approximation	4.39%	0.85%	1.27%	2.73%	3.95%	5.56%	10.04%	12.94%
70								
simulation	4.39%	1.01%	1.45%	2.88%	4.03%	5.50%	9.38%	11.76%
approximation	4.39%	1.20%	1.64%	3.03%	4.08%	5.42%	8.89%	11.05%
90								
simulation	4.39%	1.25%	1.69%	3.08%	4.12%	5.41%	8.64%	10.51%
approximation	4.39%	1.45%	1.90%	3.21%	4.16%	5.31%	8.22%	9.97%
200								
simulation	4.39%	1.99%	2.41%	3.53%	4.27%	5.12%	7.06%	8.15%
approximation	4.39%	2.21%	2.61%	3.63%	4.29%	5.04%	6.75%	7.72%

Table A1 *Accuracy of closed-form approximation of the realized default loss distribution.* This table shows quantiles from the simulation of realized default losses and from a closed-form approximation. The quantiles from the simulation are obtained by simulating as explained in Section 4. The quantiles in the approximation are from the Vasicek (1991) loan loss probability distribution with a correlation coefficient given in equation (14). The default probability is $p = 4.39\%$ and correlation is $\rho = 0.25$.

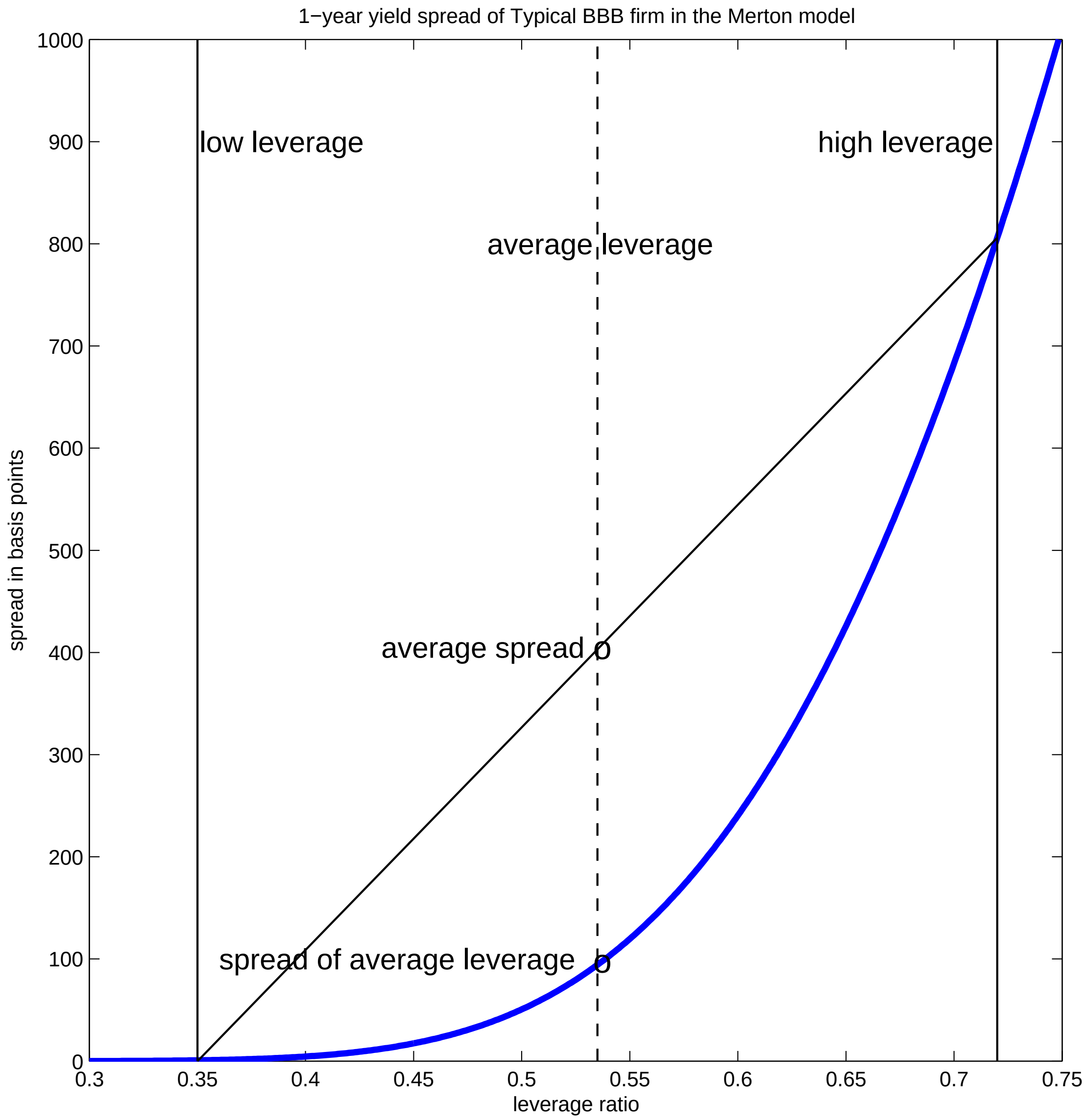

Fig. 1 *Convexity bias when calculating the spread in a structural model using average leverage and comparing it to the average spread.* It is a common approach to compare average actual spreads to model-implied spreads, where the model-implied spreads are calculated by using average firm variables. This introduces a bias because the spread in structural models is a non-linear function of firm variables. The figure illustrates the bias in case of leverage ratio for a typical BBB-rated firm. Asset volatility is 28%, dividend yield 5.2%, recovery rate 49.2%, and riskfree rate 5%

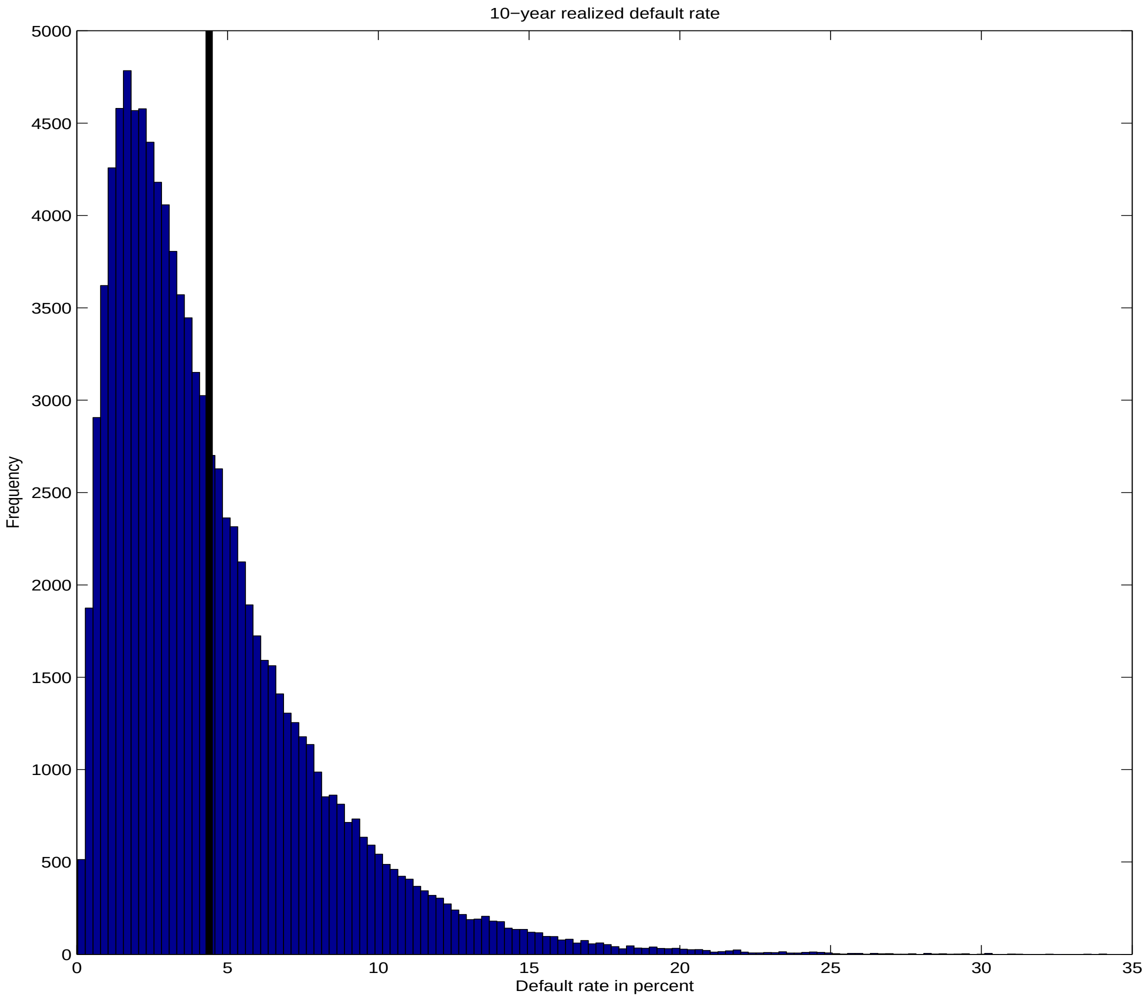

Fig. 2 *Distribution of realized default frequency.* In a simulation experiment we estimate the distribution of the average realized cumulative 10-year default frequency over 28 years given an ex ante default probability of 4.39%. We assume that in year 1 there is a cohort of 1,000 firms. In year 2 there is a new cohort of 1,000 firms formed. This goes on for 18 years. All firms are identical when the cohort is formed and their 10-year default probability is 4.39%. Part of firm volatility is systematic and part is idiosyncratic according to equation (7). For each cohort, we calculate the realized default frequency on a 10-year horizon and calculate the average default frequency across all cohorts. Overall, the realized default rate is based on 18,000 firms over a period of 28 years. We repeat this simulation 100,000 times and the graph shows the distribution of realized 10-year cumulative default rates. The solid line is the ex ante default probability of 4.39%.

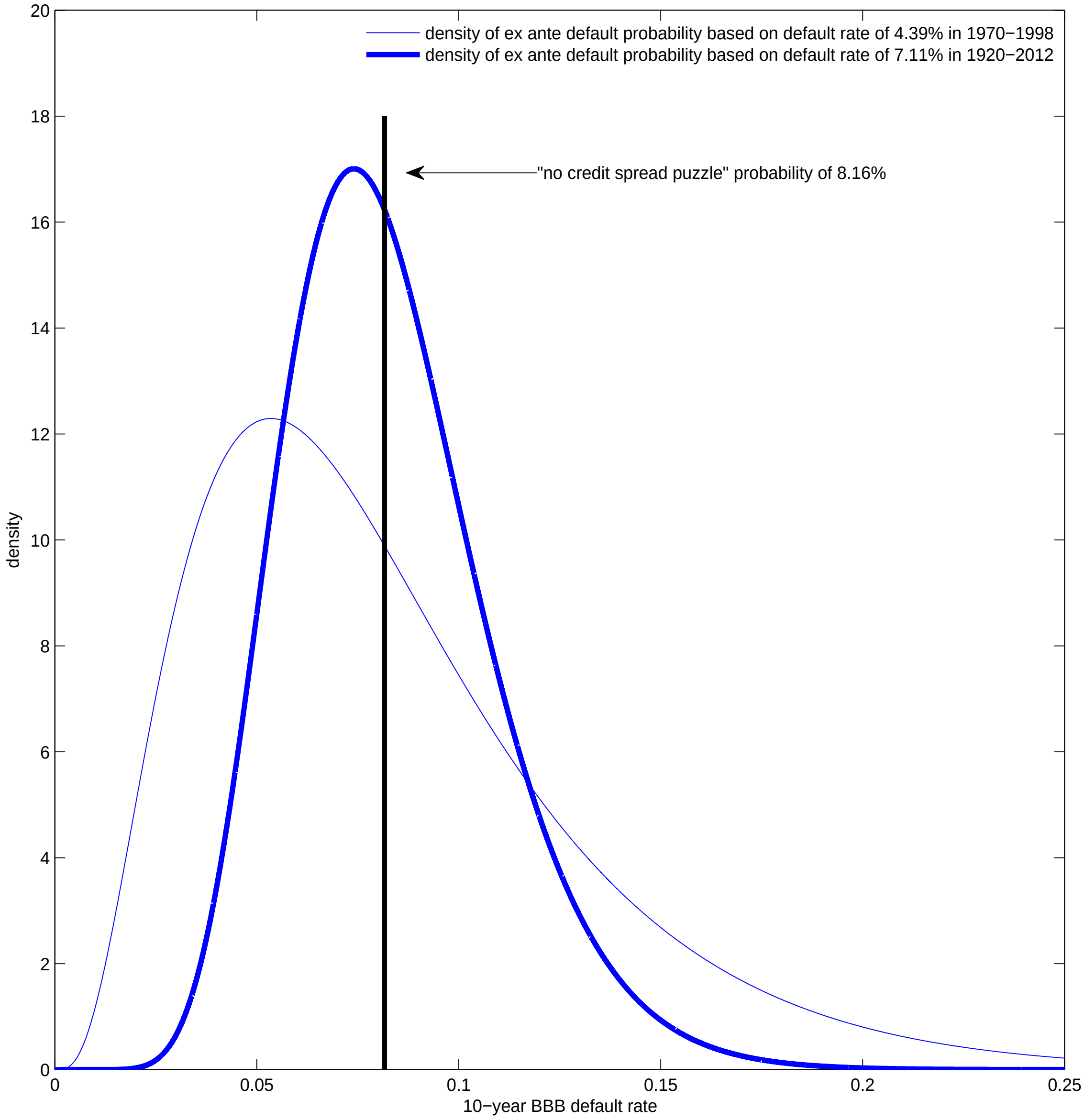

Fig. 3 *Density of ex ante default probability given the observed default rate history.* Based on the average ex post realized default frequency over some period, we can calculate the likelihood function of the ex ante default probability as explained in Section 4. The graph shows the likelihood function of the 10-year cumulative BBB default probability based on a realized rate of 4.39% over the period 1970-1998 and a rate of 7.112% over the period 1920-2012 as published by Moody's. The solid black line is the ex ante default probability of 8.16% that would make default probabilities and spreads consistent with each other in the Merton model.

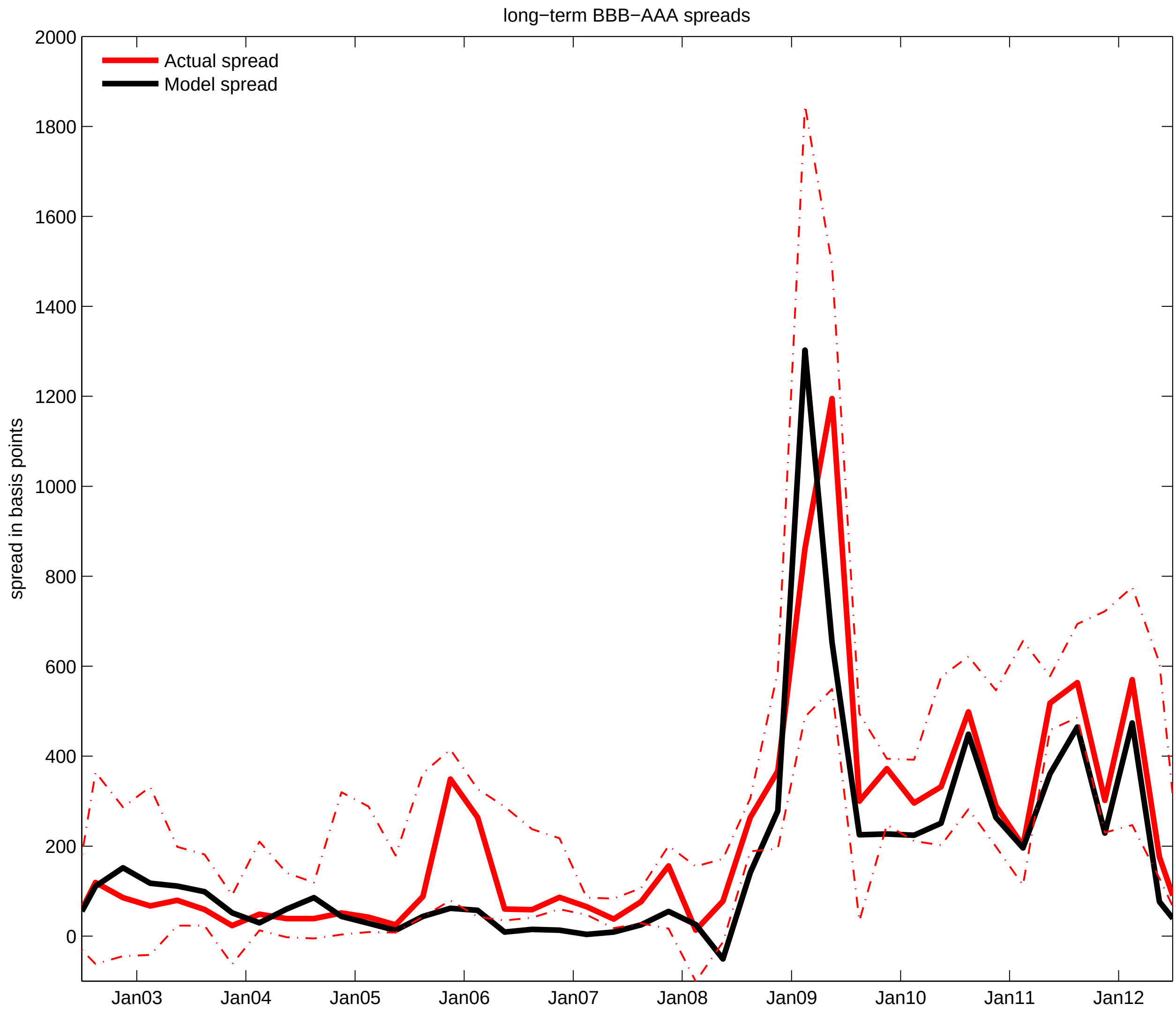

Fig. 4 *Long-term BBB-AAA corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied long-term AAA-BBB spreads. On a quarterly basis all transactions in bonds where the bond maturity is more than five years at transaction date and the rating is AAA/AA or BBB are collected. The graph shows the volume-weighted median BBB spread minus the volume-weighted median AAA/AA spread. Volume-weighted 10pct and 90pct quantiles are bootstrapped as explained in the text. The figure also shows the model-implied Merton spread, found by calculating the model-implied AAA-BBB spread computed in the same way as the actual spread. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

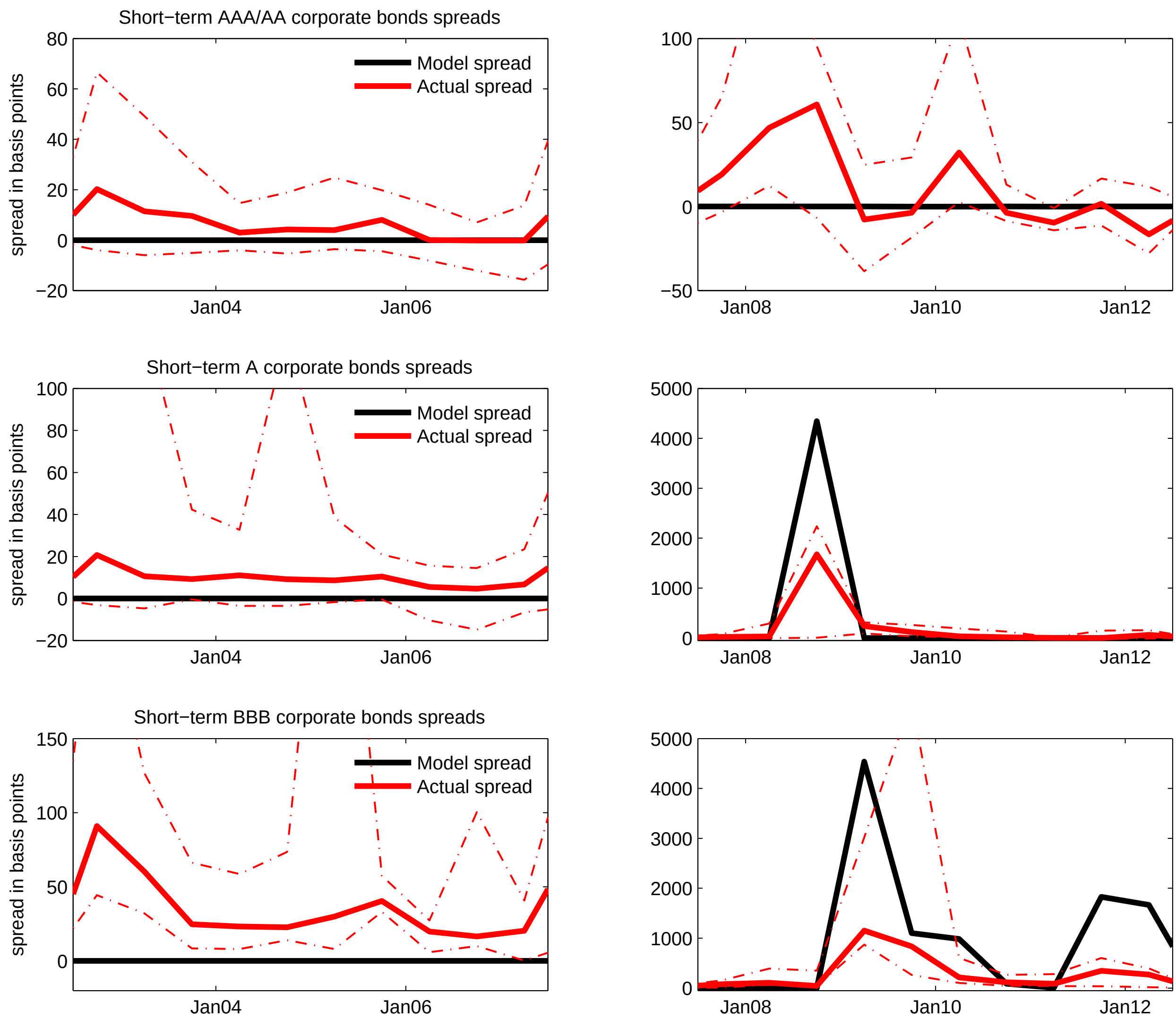

Fig. 5 *Short-term corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied short-term industrial corporate bond spreads. On a semi-annual basis all transactions in bonds maturing within one year on transaction day are collected. The Figure shows - for ratings AAA/AA, A, and BBB - the volume-weighted median actual spread along with volume-weighted 10pct and 90pct quantile. The figure also shows the model-implied Merton spread, found by calculating the model-implied spread for each transaction and computing the volume-weighted median. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

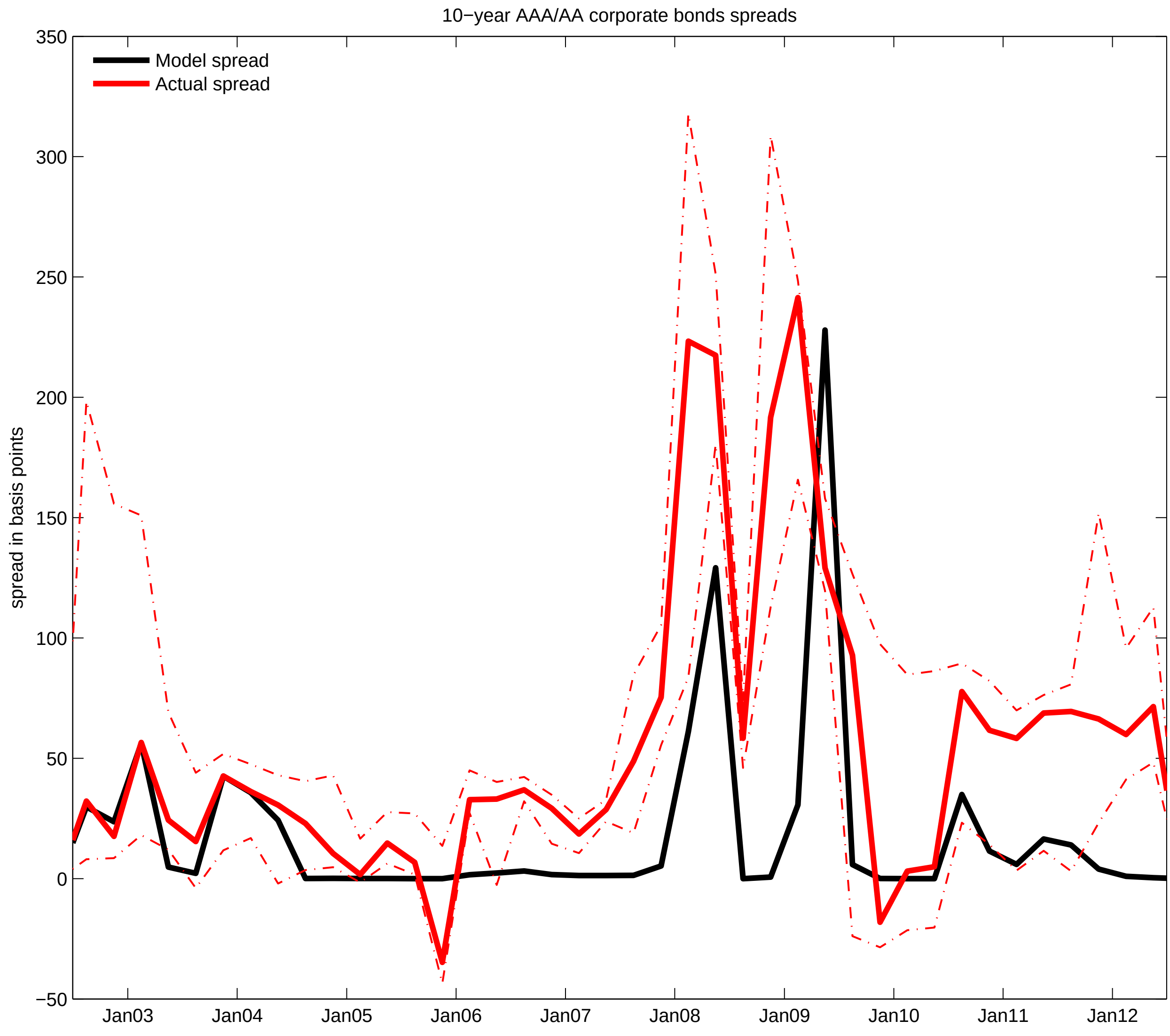

Fig. 6 *10-year AAA corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied long-term industrial AAA/AA corporate bond spreads. On a quarterly basis all transactions in bonds with maturity between 5 and 30 years on transaction day are collected. The figure shows the volume-weighted median actual spread along with volume-weighted 10pct and 90pct quantiles. The figure also shows the model-implied Merton spread, found by calculating the model-implied spread for each transaction and computing the volume-weighted median. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

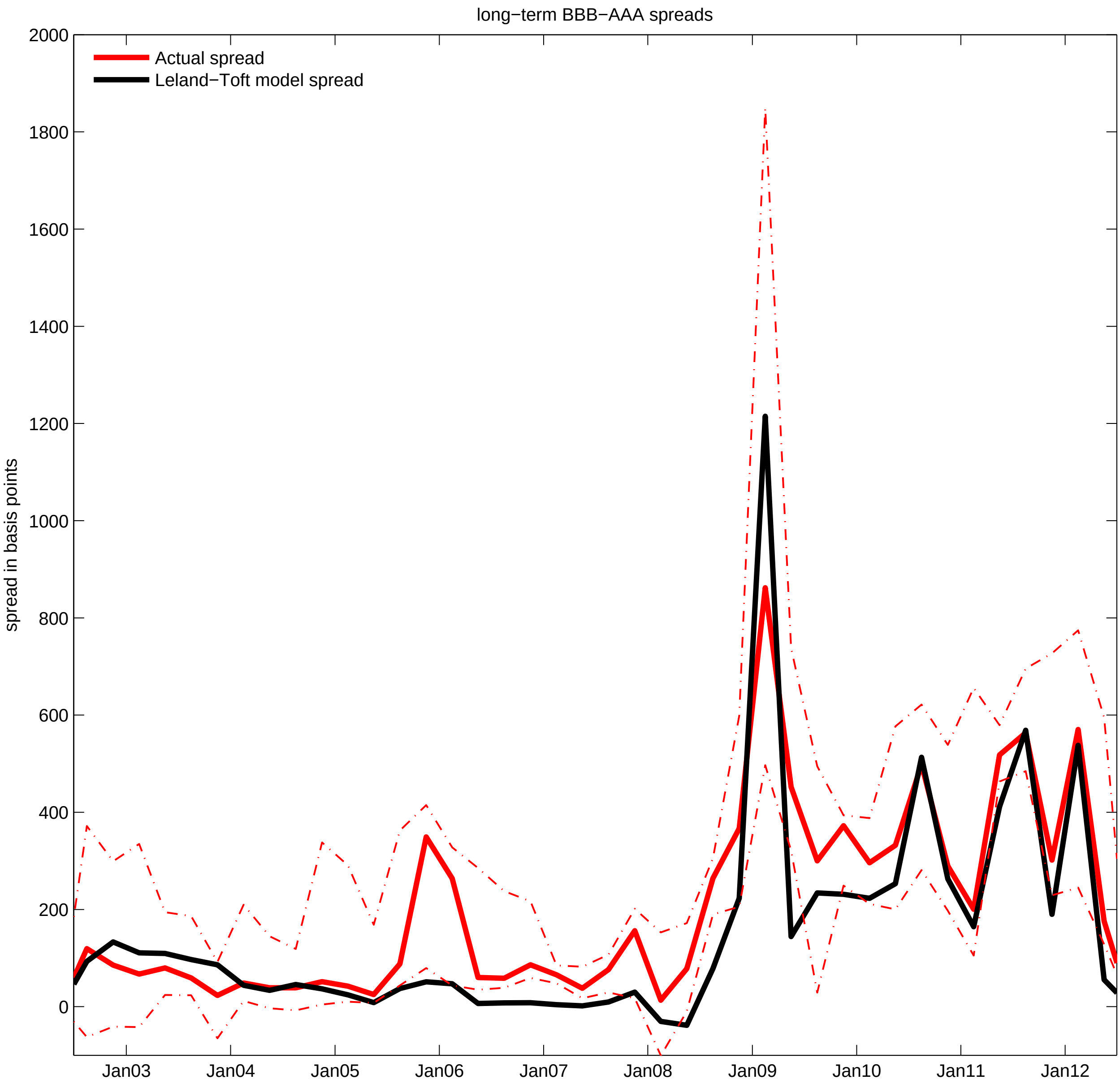

Fig. 7 *Long-term BBB-AAA corporate bond yield spreads in the Leland-Toft model.* This graph shows the time series variation of actual and model-implied long-term AAA-BBB spreads. On a quarterly basis all transactions in bonds where the bond maturity is more than five years at transaction date and the rating is AAA/AA or BBB are collected. The graph shows the volume-weighted median BBB spread minus the volume-weighted median AAA/AA spread. Volume-weighted 10pct and 90pct quantiles are bootstrapped as explained in the text. The figure also shows the model-implied Leland-Toft spread, found by calculating the model-implied AAA-BBB spread computed in the same way as the actual spread. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

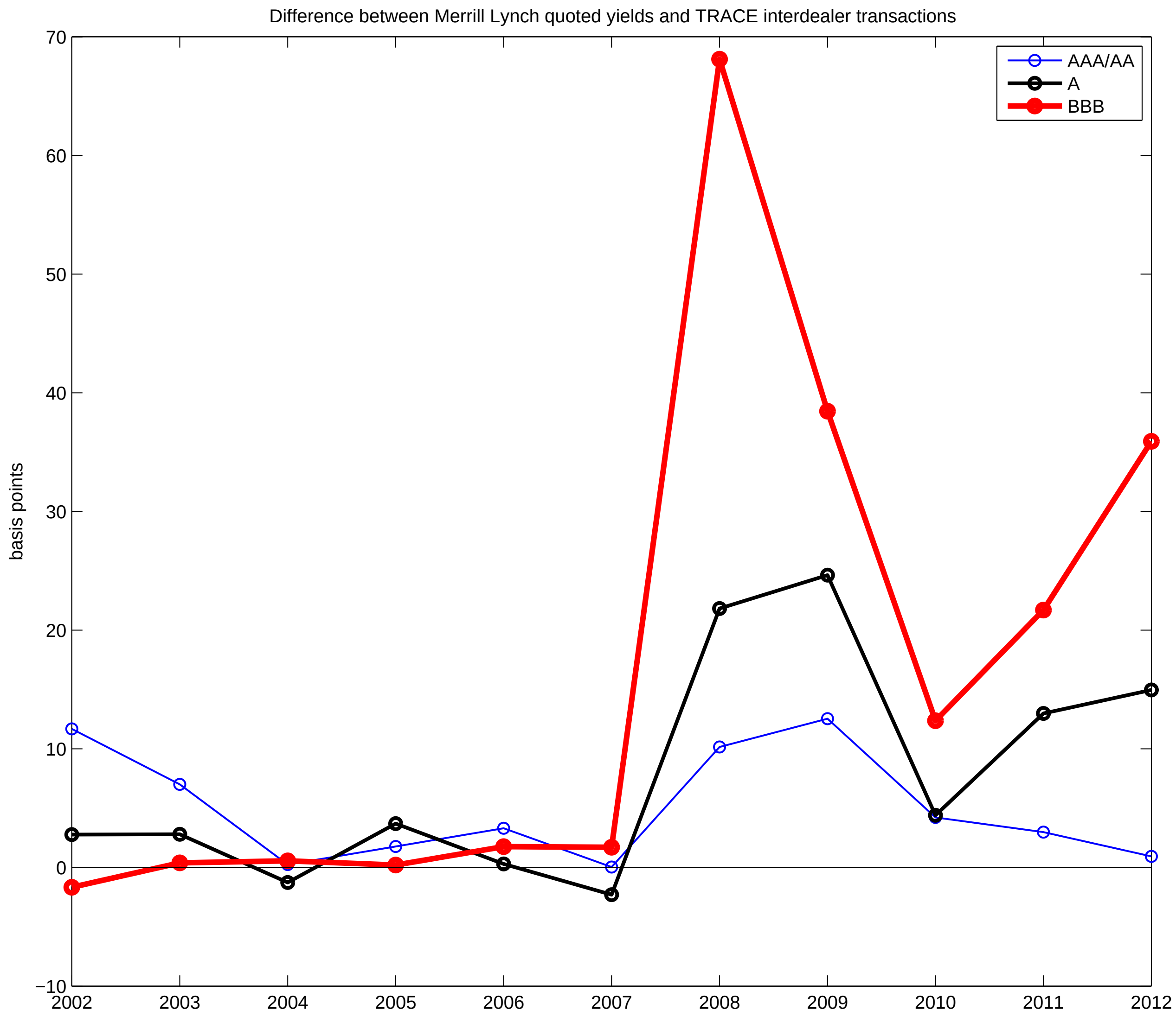

Fig. 8 *Difference between Merril Lynch quoted yields and yields in TRACE interdealer transactions.* On a semi annual basis all interdealer transactions in TRACE are matched with a Merrill Lynch quote on the same day, if a quote exists. The difference between the Merrill Lynch quoted yield and the TRACE yield is calculated and the graph shows the semi-annual volume-weighted mean. The yield difference is winsorized at ±500 basis points. This is done separately for AAA/AA-, A-, and BBB-rated bonds.

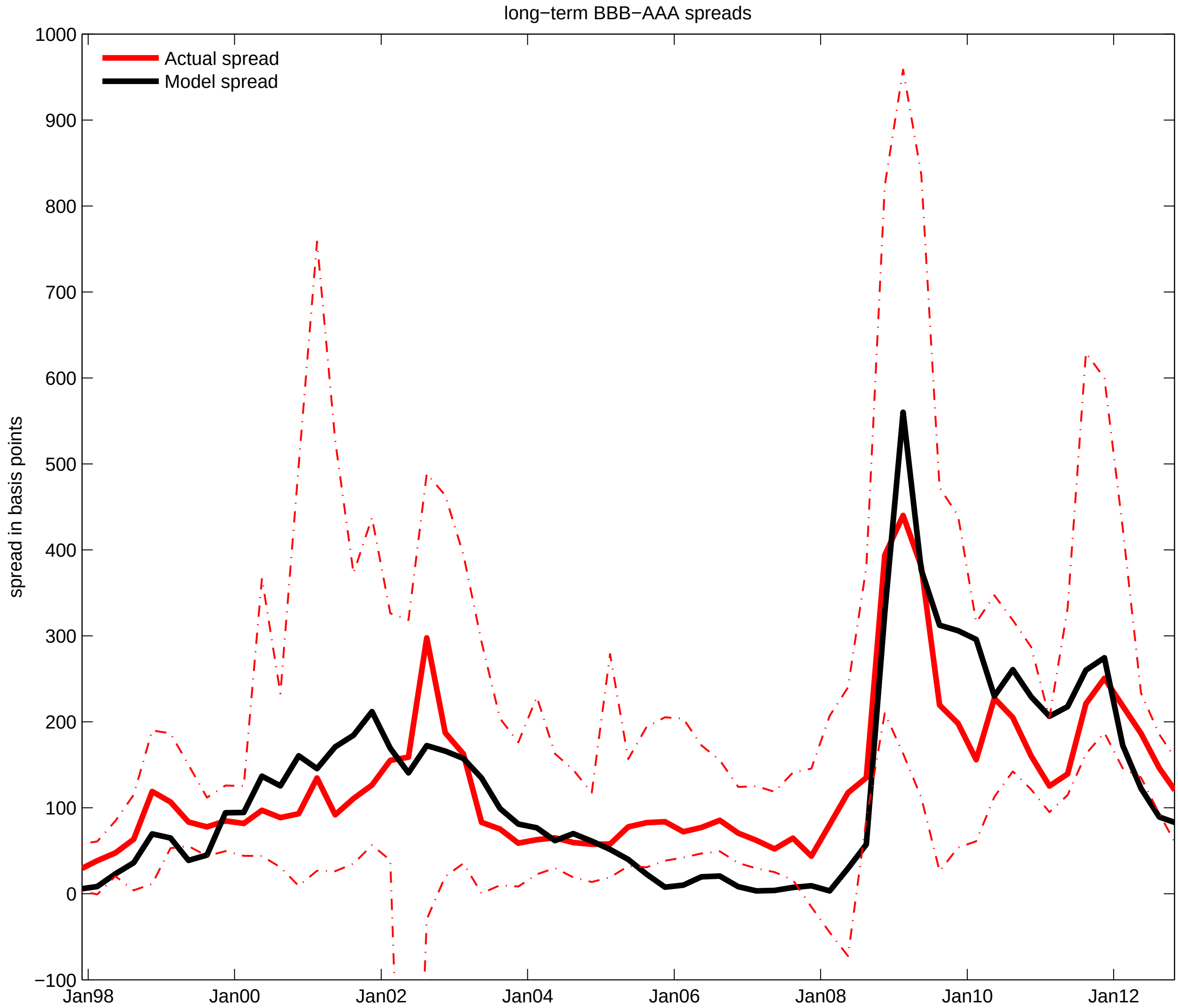

Fig. 9 *BBB-AAA corporate bond yield spreads using Merrill Lynch quotes.* This graph shows the time series variation of actual and model-implied AAA-BBB spreads. On a quarterly basis all daily quotes in bonds rated AAA/AA and bonds rated BBB are collected, and the graph shows the median BBB spread minus the median AAA/AA spread. This is done for maturities betwee 5 and 30 years. 10pct and 90pct quantiles are bootstrapped as in Figure 4. The figure also shows the model-implied Merton spread, found by calculating the model-implied AAA-BBB spread computed in the same way as the actual spread. Daily bond quotes are from Merrill Lynch and cover the period 1997Q1-2012Q2.