INFORMATION AGGREGATION IN COMPETING COMMON VALUE AUCTIONS

–PRELIMINARY AND INCOMPLETE

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We study a market in which $2k$ identical and indivisible objects are allocated in two separate markets using uniform-price auctions where $z > 2k$ bidders who each demand one object. Before the auctions, each bidder receives an informative but imperfect signal about the state of the world and chooses one of the two auctions. The good that is auctioned is a common-value object for the bidders, and a bidder's valuation for the object is determined by the state of the world. Our main result shows that if the gains from trade are certain in both markets then information is fully aggregated by the prices. In contrast, if the gains from trade are uncertain in one of the markets, then, there is no equilibrium where information is fully aggregated. Our result highlights a new avenue for informational contagion between two markets.

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"We must look at the price system as a mechanism for communicating information if we want to understand its real function....The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action..." Hayek (1945).

1. Introduction

One important reason to trust markets arises from the belief that market prices accurately summarize the vast array of information held by market participants. Whether this belief is justified, that is, whether prices efficiently aggregate information dispersed among agents that are active in an economy is a central economic question addressed by past research. In certain auction markets, prices do in fact effectively aggregate dispersed information. Specifically, consider a market in which a large number of identical common-value objects are sold through a uniform-price auction. In such an auction, if the bidders each have an independent signal about an unknown state of the world and if this unknown state determines the value of the object, then the equilibrium price converges to the true value of the object as the number of objects and the number of bidders grow arbitrarily large. Therefore, the auction price reveals information about the unknown state of the world. Wilson (1977), Milgrom (1979), and Pesendorfer and Swinkels (1997) have shown that this remarkable result holds under quite general assumptions.

More specifically, we study a model in which in each market k identical and indivisible objects are allocated using a uniform-price auction. Before the auction, each of $z > 2k$ bidders receives an informative but imperfect signal about the state of the world and chooses to participate in one of two markets . In the auction, bidders choose their bids as a function of their signal, the k highest bidders are allocated one unit of the object, and all bidders who win an object pay a uniform price equal to the $k + 1$st highest bid. The good that is auctioned is a common-value object for the bidders and a bidder's valuation for the object is determined *jointly* by the state of the world and the market in which he participates.

We explore a number of properties of markets as the numbers of bidders and the objects grow proportionately; however, our primary focus is on the informativeness of prices. An outsider who could observe the signals of an arbitrarily large number of bidders would learn the state of the world perfectly. Motivated by such an outsider's perspective, we say that prices fully aggregate information if an outsider can figure out the state of the world almost perfectly just by observing the equilibrium prices in both markets.

Our main result shows that if the gains from trade are certain in both markets then information is fully aggregated by the prices. In contrast, if the gains from trade are uncertain in one of the markets, then, under certain assumptions, there is no equilibrium where information is fully aggregated. Our result highlights a new avenue through which there can be informational contagion between two markets.

Relation to the literature. This paper is closely related to earlier work which studies information aggregation in large auctions. Wilson (1977) studied second-price auctions with common value for one object for sale, and Milgrom (1979) extended the analysis to any arbitrary number of objects. Both of these papers show that as the number of bidders gets arbitrarily large, price converges to the true value of the object, but only provided that there are bidders with arbitrarily precise signals about the state of the world. Pesendorfer and Swinkels (1997) further generalize the analysis to the case where there are no arbitrarily precise signals. They show that prices converge to the true value of a common-value object in all symmetric equilibria if and only if both the number of identical objects and the number of bidders who are not allocated an object grow without bound. Pesendorfer and Swinkels (2000) generalize the analysis further to a mixed private, common-value environment. Finally, Kremer (2002) shows that the information aggregation properties of auctions are more general than the particular mechanisms studied before; he does this by providing a unified approach that uses the statistical properties of certain order statistics.[1] Our model is closest to Pesendorfer and Swinkels (1997).

Our work also relates to the literature on costly information acquisition in rational-expectations models, such as Grossman and Stiglitz (1976, 1980) and Grossman (1981). These papers explain the conceptual difficulties in interpreting prices as both allocation devices and information aggregators. Specifically, they argue that if consumers and producers need to undertake a costly activity in order to acquire information, then equilibrium prices cannot reveal the state of the world perfectly. Their reasoning is as follows: if prices were to reveal the state perfectly, then no agent would have an incentive to pay for information in the first place; but if no agent acquires information, then the prices cannot reveal the state as there is no information to aggregate. However, as was the case for auction markets, these papers do not explicitly consider how the information revealed by the market price could be used by individuals when deciding in which market to participate.

A prominent feature of our equilibrium construction is pooling by bidders who receive signals that lie in a certain range. In a recent paper, Lauermann and Wolinsky (2012) study an auction where the seller needs to solicit bidders for the auction and this makes the number of participants of the auction dependent on the state of the world. Similar to our paper, their model also features pooling in equilibrium. There is pooling in their model because bidding the pooling bid provides insurance against winning too frequently in the low-payoff state.

[1] See Hong and Shum (2004) for a calculation of the rate at which price converges to the true value in large common-value auctions. Jackson and Kremer (2007) show that the result of Pesendorfer and Swinkels (1997) does not generalize to an auction with price discrimination. See Kremer and Skrzypacz (2005) for related results concerning the link between information aggregation and the properties of order statistics.

Finally, there is a growing literature which investigates the impact of financial markets on the real economy. Past work in this literature explores situations where speculators' private information about the success prospects of an action to be chosen by a manager (or a CEO or a central banker) is partially revealed through its effect on the prices of financial assets or derivatives. Knowing this, managers also factor in financial asset price information when deciding on their course of action. Papers in this literature then argue that such feedback loops may decrease the information content of prices. A recent paper by Bond et al. (Forthcoming) surveys this literature.

2. The Model

There are N individuals who want to purchase a unit of a good. They can purchase a good in market s (safe) or in market r (risky) but not in both. There are κN good available for sale in each market where $\kappa \in [0, \frac{1}{2})$ and we assume that κN is integral. In both of the markets the goods are sold using a uniform price auctions where the price is equal to the highest losing bid. Individuals first choose the market in which they will bid for a good and then they choose their bid.

The state $\omega \in \{l, h\}$ is unknown and the value v of each good depends on the state and the market, i.e., $v : \{s, r\} \times \{l, h\} \to \mathbb{R}$. We assume that $v(h, s) = v(h, r) = 1 > v(l, s) = 0 \geq v(l, r) = -c$.

Definition 1 *We say that the gains from trade are certain if $c = 0$ and we say that the gains from trade are uncertain if $c > 0$.*

Each individual shares a common prior $\pi \in [0, 1]$ where π is the probability that the state is h. Each agent receives a signal $\theta \in [0, 1]$ according to cumulative distribution function $F(\theta|\omega)$ which admits a density function $f(\theta|\omega)$. The signals are conditionally IID. Let $l_\theta = \frac{f(\theta|h)}{f(\theta|l)}$ denote the likelihood ratio function and let Let $l_{\pi\theta} = \frac{\pi}{1-\pi} l_\theta$.

Definition 2 *We say that signals satisfy strict MLRP if the likelihood ratio l_θ is strictly increasing in θ. We say that signals satisfy MLRP if the likelihood ratio l_θ is increasing in θ.*

After observing their signal, each bidder first chooses a market from the set of markets $m \in \{s, r\}$ and then submits a bid in this market. A market choice strategy for player i is a function $a_i : [0, 1] \to [0, 1]$ where $a_i(\theta)$ is the probability that agent i chooses market r when she receives signal θ. A bidding strategy in market m for player i is a measure H_i^m on $[0, 1] \times [0, \infty) \times [0, \infty)$ with marginal distribution $F(\theta) = \pi F(\theta|R) + (1-\pi) F(\theta|L)$ on its first coordinate (see Milgrom and Weber (1985)). The set of all bidding strategies is Σ. A bidding strategy is pure if there is a function $b^m : [0,1] \to [0,\infty)$ such that $H(\{\theta, b(\theta)\}_{\theta\in[0,1]}) = 1$.[2]

Lemma 1 *If signals satisfy MLRP, then any equilibrium bidding strategy H^m can be represented by an increasing bidding function b^m.*

Proof: The proof follows directly from Lemmata 2-6 in Pesendorfer and Swinkels (1997). □

Below we formally define information aggregation and its failure. Our object of study is a sequence of bidding functions $\boldsymbol{b} = \{b_z^m\}_{z=n}^{\infty}$. We say that the sequence $\boldsymbol{b}$ is an equilibrium sequence if b_z^m is part of a symmetric Nash equilibrium of Γ_z^m for each m and z.

Suppose that the number of bidders z is large. In this case, the law of large numbers implies that observing the signals $(\theta_1, ..., \theta_z)$ conveys precise information about the state of the world $\omega \in \{L, H\}$. The bidding function b_z determines a price p for the auction Γ_z given any realization of signals $(\theta_1, ..., \theta_z)$. We say that information is aggregated in a particular market if the auction price p in this market also conveys precise information about the state of the world. We look at two definitions of information aggregation. the first definition is the one used by Pesendorfer and Swinkels (1997) and is given by the following definition:

Definition 3 *An equilibrium sequence of prices in market m aggregates information if $p_z^m \to v(m, \cdot)$ in probability.*

The second definition of information aggregation that we use is the definition used by Atakan and Ekmekci (Forthcoming). More precisely, (i) if the likelihood ratio $\frac{\Pr_{b_z}(p^*|H)}{\Pr_{b_z}(p^*|L)}$ is close to zero (i.e., if it is arbitrarily more probable that we observe such a price when $\omega = L$), then an outsider who observes price p^* learns that the state is L. Alternatively, (ii) if the likelihood ratio $\frac{\Pr_{b_z}(p^*|H)}{\Pr_{b_z}(p^*|L)}$ is arbitrarily large, then an outsider who observes price p^* learns that the state is R. If the probability that we observe a price that satisfies either (i) or (ii) is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ fully aggregates information. Conversely, if the likelihood ratio $\frac{\Pr_{b_z}(p^*|H)}{\Pr_{b_z}(p^*|L)}$ is close to one, i.e., if we are equally likely to observe price p^* in either of the two states, then an outsider who observes price p^* learns arbitrarily little information about the state of the world. If the probability that we observe such a price is arbitrarily close to one, then we say that the equilibrium sequence $\boldsymbol{b}$ aggregates no information. The precise definitions are as follows:

[2]If b represents H, then so does any function that is equal to b at almost every $\theta \in [0,1]$.

Definition 4 *An equilibrium sequence* ***b aggregates no information*** *if, for any* $\epsilon > 0$,

$$\lim_{z\to\infty} \Pr_{b_z}\left(p_z \in \left\{p \in [0,\infty) : \frac{\Pr_{b_z}(p|H)}{\Pr_{b_z}(p|L)} \in (1-\epsilon, 1+\epsilon)\right\}\right) = 1.$$

An equilibrium sequence ***b fully aggregates information*** *if, for any* $\epsilon > 0$,

$$\lim_{z\to\infty} \Pr_{b_z}\left(p_z \in \left\{p \in [0,\infty) : \frac{\Pr_{b_z}(p|H)}{\Pr_{b_z}(p|L)} \in [0,\epsilon) \cup (1/\epsilon, \infty))\right\}\right) = 1.$$

3. Large Markets and the Failure of Information Aggregation

In this section we present our three main results. We consider a sequence of games where the z^{th} game has z bidders who make a choice between the two markets and $\lfloor \kappa z \rfloor$ objects for sale in each market.[3] In Theorem 1, we show that there is no sequence of equilibrium which aggregates information if gains from trade are uncertain. In Theorem 2, we assume that the gains from trade are uncertain and we construct a sequence of equilibria in which no information is aggregated in market S and information is imperfectly aggregated in market r. Finally, in Theorem 3, we show that if the gains from trade are certain in both markets then there is a sequence of equilibria in which information is aggregated in both markets.

Let θ_r denote the signal such that $1-F(\theta_r|h) = \kappa$, i.e., κ proportion of signals exceed signal θ_r and therefore if all buyers with signals that exceed θ_r were to opt for market r, then there would be exactly as many bidders as good in market r as the market grew arbitrarily large. Let θ_h and s_l denote the signals such that $F(\theta_r|h) - F(\theta_h|h) = \kappa$ and $F(\theta_r|l) - F(\theta_l|l) = \kappa$, respectively.

Assumption 1 *The signal* θ_l *exceeds the signal* θ_h*, i.e.,* $\theta_l > \theta_h$.

Assumption 2 *It is individually rational for an agent with signal* θ_r *to bid in market* r*, i.e.,* $E[v(r,\omega)|\theta_r] > 0$.

Theorem 1 *Assume that signals satisfy strict MLRP and that Assumptions 1 and 2 are satisfied. If the gains from trade are uncertain, then information aggregation according to definition 1 fails in both markets.*

Proof: Let $\pi(m, \theta'|\theta)$ denote the payoff to type θ from bidding $b^r(\theta')$ in market m and let $\pi(m|\theta) = \pi(m, \theta|\theta)$. Assume that $p_z^s \to v(s,\cdot)$ in probability. Let $\pi_\infty = \lim_{z\to\infty} \pi$. Note that $p_z^s \to v(s,\cdot)$ implies that $\pi_z(s|\theta) \to 0$ for all θ.

[3] The term $\lfloor \kappa z \rfloor$ refers to the highest integer not bigger than κz.

We will show (1) For all $\theta > \theta_r$ there is a z_θ such that $a_z(\theta) = 1$ for all $z > z_\theta$ and (2) For all $\theta < \theta_r$ there is a z_θ such that $a_z(\theta) = 0$ for all $z > z_\theta$.

Claim 1. If $\pi(r, \theta'|\theta') \geq 0$ and $\pi(r, \theta'|\theta', h) > 0$, then $\pi(r|\theta) > 0$ for any $\theta > \theta'$.

Note that $\pi(r, \theta'|\theta', h) = \pi(r, \theta'|h)$. This is because conditioning on the signal does not add any information beyond the state. The fact that $\pi(r, \theta'|\theta') \geq 0$ implies that

$$\Pr(l|\theta') \left(\pi(r, \theta'|h) \frac{\Pr(h|\theta')}{\Pr(l|\theta')} + \pi(r, \theta'|l) \right) \geq 0$$

The above inequality in conjunction with strict MLRP implies that

$$\pi(r, \theta'|h) \frac{\Pr(h|\theta)}{\Pr(l|\theta)} + \pi(r, \theta|l) > 0$$

Therefore,

$$\pi(r|\theta) \geq \pi(r, \theta'|\theta) = \Pr(l|\theta) \left(\pi(r, \theta'|h) \frac{\Pr(h|\theta)}{\Pr(l|\theta)} + \pi(r, \theta'|l) \right) > 0$$

The above reasoning also implies that if $\pi_\infty(r, \theta'|\theta') \geq 0$ and $\pi_\infty(r, \theta'|\theta', h) > 0$, then $\pi_\infty(r|\theta) \geq \pi_\infty(r, \theta'|\theta) > 0$ for any $\theta > \theta'$.

Claim 2. If $\pi_\infty(r, \theta'|\theta') \geq 0$ and $\pi_\infty(r, \theta'|\theta', h) > 0$, then $a_z(\theta) = 1$ for any $\theta > \theta'$ and all z sufficiently large.

This follows from the fact that $\pi_\infty(s|\theta) = 0$ and Claim 1 above which showed that $\pi_\infty(r|\theta) > 0$.

Claim 3. Let $A_z = \{\theta : a_z(\theta) > 0\}$ then we have $\lim_z F(A_z|h) \geq \kappa$.

Assume not, i.e., that $\lim_z F(A_z|h) < \kappa$. However, for any $\theta \geq \theta_r$ the profit from going to market z and bidding $b = 0$, i.e., $\pi_z(r, 0|\theta)$ is strictly positive because it ensures that such a type θ wins an object in state h at a price of zero with probability one moreover this type can do no worse in state l than winning an object at price zero with probability one. Hence, $\pi_z(r, 0|\theta) \geq E[v(r, \omega)|\theta] \geq E[v(r, \omega)|\theta_r] > 0$ for all z sufficiently large. However, $\pi_z(r, 0|\theta) > 0$ and $\pi_\infty(s|\theta) = 0$ together imply that we must have $a_z(\theta) = 1$ for all $\theta \geq \theta_r$ and all z sufficiently large, i.e., $A_z \subset \{\theta \geq \theta_r\}$. However, then we have $F(A_z|h) \geq 1 - F(\theta_r|h) = \kappa$ which is a contradiction.

Claim 4. If $\theta > \theta_r$, then $a_z(\theta) = 1$ for all z sufficiently large.

Let $\theta_z^{\kappa-\epsilon} \in A_z$ be such that $F(\{\theta \in A_z : \theta \geq \theta_z^{\kappa-\epsilon}\}|h) = \kappa - \epsilon$. The fact that $\lim_z F(A_z|h) \geq \kappa$ implies that such a type exists for all z sufficiently large. Also note that because $F(\theta_z|h) = 1 - \kappa$, for $\theta > \theta_r$, we can choose $\epsilon > 0$ such that $\theta_z^{\kappa-\epsilon} < \theta$ for all z sufficiently large. Note that the measure of type's who bid strictly higher than $\theta_z^{\kappa-\epsilon}$ is less than $\kappa - \epsilon$ in state h

by construction and the fact that the bidding function is increasing and is strictly less than $\kappa - \epsilon$ in state l by MLRP. Therefore, the probability that type $\theta_z^{\kappa-\epsilon}$ wins an object in state l is at least $\frac{\epsilon}{2(1-\kappa+\epsilon)}$ for all z sufficiently large.Therefore, $\pi_z(r, \theta_z^{\kappa-\epsilon}|l) \leq -c\frac{\epsilon}{2(1-\kappa+\epsilon)}$ and thus $\pi_z(r, \theta_z^{\kappa-\epsilon}|h) > c\frac{\Pr(l|\theta_z^{\kappa-\epsilon})\epsilon}{\Pr(h|\theta_z^{\kappa-\epsilon})2(1-\kappa+\epsilon)} > 0$ for all z sufficiently large. This however implies that $a(\theta) = 1$ for all $\theta > \theta_z^{\kappa-\epsilon}$ by Claim 2 and thus shows that if $\theta > \theta_r$, then $a_z(\theta) = 1$ for all z sufficiently large.

Claim 5. For any $\theta' \in (\theta_r, 1]$, there is a sufficiently large z' such that for all $z > z'$ if $b_z^r(\theta') = b$ is an atom and some $\theta^* < \theta'$ bids the atom, then all $\theta \geq \theta^* + \epsilon$ also bid the atom and choose $a_z(\theta) = 1$.

Note that monotonicity implies that if $a_z(\theta) > 0$, then $b_z^r(\theta) = b$ for any $\theta \geq \theta^*$. We have $\pi(r|\theta^*) \geq 0$ because $a_z(\theta^*) > 0$. For θ^* the probability of winning an object in state l is at least $y = \kappa - 1 + F(\theta_r|l) - \xi$ and the profit conditional on winning is at most $-c$. We have

$$\begin{aligned}
\pi(r, \theta^*|\theta^* + \epsilon) - \pi(r, \theta^*|\theta^*) &= (\pi(r, \theta^*|h) - \pi(r, \theta^*|l))(\Pr(h|\theta^* + \epsilon) - \Pr(h|\theta^*)) \\
&\geq cy(\Pr(h|\theta^* + \epsilon) - \Pr(h|\theta^*))
\end{aligned}$$

Strict MLRP implies that $(\Pr(h|\theta^* + \epsilon) - \Pr(h|\theta^*)) > \nu$ for some $\nu > 0$ independent of θ^*. Note that $\pi_z(s|\theta) < x$ for any $x > 0$ for sufficiently large z. Pick z' such that $\pi_z(s|\theta) < \nu c y$ for all θ. Given these choices $\pi_z(r|\theta) > \pi_z(s|\theta)$ for any $\theta > \theta^* + \epsilon$ and any $z > z'$ as required by the claim.

Claim 6. Any equilibrium bidding function b_z^r is strictly increasing over $[\theta, 1]$ for any $\theta > \theta_r$ for all z sufficiently large.

Pick z sufficiently large so that for any $\theta' \in [\theta, 1]$ $a_z(\theta') = 1$. Suppose that $b = b(\theta')$ is an atom. Claim 5 above and Lemma 7 in Pesendorfer and Swinkels (1997) implies that conditional on b being pivotal, there is a winner's and loser's curse at b. That is, once one knows b if pivotal, winning with a bid of b is a signal in favor of state l and losing is a signal in favor of h. Therefore, Corollary 3 in Pesendorfer and Swinkels (1997) implies that b_z^r is strictly increasing over $[\theta, 1]$.

Claim 7. If $\theta < \theta_r$, then $a_z(\theta) = 0$ for all z sufficiently large.

Claim 6 implies that, for z large, the probability of winning an object in state h is equal to zero for any $\theta < \theta_r$. Let θ_r^l be such that $F(\theta_r^l|l) = 1 - \kappa$. Claim 6 implies that for all $\theta \in (\theta_r^l, \theta_r)$. Straightforward (but involved) to be added.

Claim 8. For any $\epsilon > 0$, $\lim_z \Pr(p_z^s(h) < b_z^s(\theta_h) - \epsilon) = 1$ and $\lim_z \Pr(p_z^s(l) < b_z^s(\theta_l) - \epsilon) = 0$.

This follows immediately from the weak law of large numbers, the fact that all agents with $\theta > \theta_r$ go to market r by claim 4, the fact that all agents with $\theta < \theta_r$ goto market s by claim

6 and by the definition of θ_l and θ_h.

Using claim 8 we can now complete the argument. Claim 8 and our assumption that Assume that $p_z^s \to v(s,\cdot)$ in probability together imply that $b_z^s(\theta_h) > 1-\epsilon$ and $b_z^s(\theta_l) < \epsilon$ for all z sufficiently large. However, this implies that $b_z^s(\theta_h) > b_z^s(\theta_l)$. However, the fact that $\theta_l > \theta_h$ by assumption contradicts the fact that b_z is increasing for all z leading to a contradiction. □

Define $Y^m(k)$ as the k$^{\text{th}}$ order statistic in market m and $Y(k)$ as the k$^{\text{th}}$ order statistic in general. Also, we set $Y^m(k)$ to equal zero if there are fewer than k bidders in market m. Let

$$\beta_z^m(\theta) = E[v(m,\omega)|\theta, Y_z^m(k) = \theta, Y_z^m(k+1) = \theta].$$

and let

$$\beta_z(\theta) = E[v(m,\omega)|\theta, Y_z(k) = \theta, Y_z(k+1) = \theta].$$

The bidding function $\beta_z^m(\theta)$ is the standard equilibrium bidding function described in Pesendorfer and Swinl (1997).

Let

$$\pi_z^p(s,c,\theta'|\theta) = \Pr_z(win|h)\Pr(h|\theta)(1-c) - \Pr_z(win|l)\Pr(l|\theta)c \tag{1}$$

$$\pi_\infty^p(s,c,\theta_r|\theta) = \frac{\kappa}{1-\kappa}\Pr(h|\theta)(1-c) - \frac{\kappa}{F(\theta_r|l)}c, \tag{2}$$

where $\Pr_z(win|\theta',\omega)$ denotes the probability of winning an object at a pooling bid if all types $\theta \le \theta'$ go to market s and submit the pooling bid and the state is ω. Intuitively $\pi_\infty^p(s,c,\theta_r|\theta)$ is the expect profit in an arbitrarily large market for type θ if this type bids a pooling bid equal to c, all types who bid in market s bid the pooling bid, and only types $\theta < \theta_r$ bid in market s.

Assumption 3 $\quad$ *$E[v(r,\omega)|\theta_r] > \pi_\infty^p(s,c|\theta_r)$.*

Assumption 4 $\quad$ *$\pi_\infty^p(s,c|\theta_r) > E[v(s,\omega)|\theta_r] - c$.*

Note that Assumption 4 can only be satisfied if $\Pr_\infty(win|c,c,h) = \frac{\kappa}{1-\kappa} > \Pr_\infty(win|c,c,l) = \frac{\kappa}{F(\theta_r|l)}$, i.e., if there is no winner's or loser's curse at the pooling bid.

Theorem 2 $\quad$ *Suppose that signals satisfy weak MLRP and Assumptions 3 and 4 are satisfied. If pooling at price c in market s is strictly individually rational, i.e., $\pi_\infty^p(s,c|0) > 0$,* *then there is a sequence of equilibria where all agents who choose to go to market s submit the same bid b^*, i.e., $b_z^s(\theta) = b^*$ for all θ and the price in market s is equal to b_z^* in both states. In this sequence of equilibria, the price in market r converges to a random variable which is equal to zero with probability one in state l; and which, in state h, is equal to 1 and zero with probability $1-x$ and x, respectively.*

PROOF: Let $b^* = c$. Let the constant x solve the following equation:

$$\pi_\infty^p(s, c|\theta_r) = \Pr(h|\theta_r)x - \Pr(l|\theta_r)c. \tag{3}$$

A unique such x exists by Assumption 3. Pick a sequence of θ_z^r and x_z such that $\Pr(Y_z(k) < \theta_z^r|h) = x_z$ and

$$\pi_z^p(s, c|\theta_z^r) = \Pr(h|\theta_z^r)x_z - \Pr(l|\theta_z^r)\Pr(Y_z(k) < \theta_z^r|l)c. \tag{4}$$

First note that $\pi_\infty^p(s, c|0) > 0$ implies that x_z must converge to a strictly positive constant. Next note that $\lim_z x_z > 0$ implies that $\theta_z^r \to \theta^r$ because $\lim_z \Pr(Y_z(k) < \theta_z^r|h) = 1$ if $\lim \theta_z^r > \theta^r$ and $\lim_z \Pr(Y_z(k) < \theta_z^r|h) = 0$ if $\lim \theta_z^r < \theta^r$. For any $\nu > 0$ we can pick $\epsilon > 0$ such that

1. $\Pr(Y_z(k) < \theta^r + \epsilon|h) \geq 1 - \nu$,
2. $\Pr(Y_z(k) < \theta^r - \epsilon|h) \leq \nu$,
3. $\Pr(Y_z(k) < \theta^r - \epsilon|l) \geq 1 - \nu$,
4. $\pi_z^p(s, c, \theta^r + \epsilon|\theta) \geq \pi_\infty^p(s, c, \theta_r|\theta) - \nu$,
5. $\pi_z^p(s, c, \theta^r - \epsilon|\theta) \leq \pi_\infty^p(s, c, \theta_r|\theta) + \nu$

for all $z > z(\nu)$. Note that the above conditions, Assumption 3 and the fact that ν is arbitrary together imply that

$$\pi_z^p(s, c, \theta^r - \epsilon|\theta) > \Pr(h|\theta^r - \epsilon)\Pr(Y_z(k) < \theta^r - \epsilon|h) - \Pr(l|\theta^r - \epsilon)\Pr(Y_z(k) < \theta^r - \epsilon|l)c \tag{5}$$

$$\pi_z^p(s, c, \theta^r + \epsilon|\theta) < \Pr(h|\theta^r + \epsilon)\Pr(Y_z(k) < \theta^r + \epsilon|h) - \Pr(l|\theta^r + \epsilon)\Pr(Y_z(k) < \theta^r + \epsilon|l)c \tag{6}$$

for each $z > z(\nu)$. Therefore, the intermediate value theorem implies that the required θ_z^r exists for each $z > z(\nu)$.

Below we show that $\Pr_z(win|l) < 1$ implies that $\pi_z^p(s, c|\theta) > \Pr(h|\theta_z^r)x_z - \Pr(l|\theta_z^r)\Pr(Y_z(k) < \theta_z^r|l)c$ for any $\theta < \theta_z^r$.

The following incentive compatibility condition must hold for θ

$$\Pr(h|\theta)\Pr_z(win|h) - (\Pr(h|\theta)\Pr_z(win|h) + \Pr(l|\theta)\Pr_z(win|l))c \geq x_z \Pr(h|\theta) - (1 - \Pr(h|\theta))c$$
$$\Pr(h|0)\Pr_z(win|h) - \Pr_z(win|\theta)c \geq x_z \Pr(h|\theta) - (1 - \Pr(h|\theta))c$$

where $\Pr_z(win|\theta) = (\Pr(h|\theta)\Pr_z(win|h) + \Pr(l|\theta)\Pr_z(win|l))$. Let y be such that

$$y = \Pr_z(win|h) + \frac{(1 - \Pr(h|\theta))c - \Pr_z(win|\theta)c}{\Pr(h|\theta)}.$$

Then the above inequality is equivalent to $y \geq x_z$

$$\Pr_z(win|h) + \frac{(1 - \Pr(h|\theta))c - \Pr_z(win|\theta)c}{\Pr(h|\theta)} \geq \Pr_z(win|h) + \frac{(1 - \Pr(h|\theta_z^r))c - \Pr_z(win|\theta_z^r)c}{\Pr(h|\theta_z^r)}$$
$$\frac{(1 - \Pr(h|\theta))c - \Pr_z(win|\theta)c}{\Pr(h|\theta)} \geq \frac{(1 - \Pr(h|\theta_z^r))c - \Pr_z(win|\theta_z^r)c}{\Pr(h|\theta_z^r)}$$
$$(\Pr(h|\theta_z^r) - \Pr(h|\theta))c \geq c(\Pr(h|\theta_z^r)\Pr_z(win|\theta) - \Pr(h|\theta)\Pr_z(win|\theta_z^r))$$
$$c \geq c\Pr_z(win|l)$$

Finally, below we argue that Assumption 4 and MLRP together imply that $\pi_z^p(s, c, \theta_z^r|\theta) > E[v(s, \omega)|\theta] - c$ for any $\theta \leq \theta_z^r$ and any $z > z(\nu)$.

Let

$$f(t) = t\Pr_z(win|h) - (t\Pr_z(win|h) + (1-t)\Pr_z(win|l))c - t + c$$

where $t = \Pr(h|\theta)$. Note that $f(t)$ evaluated at $t = \Pr(h|\theta_r)$ is positive by Assumption 4. Note that

$$f'(t) = \Pr_z(win|h) - \Pr_z(win|h)c + \Pr_z(win|l)c - 1 < 0.$$

Therefore $f(t) > 0$ for any $t = \Pr(h|\theta)$ and $\theta < \theta_r$. □

Theorem 3 *Suppose that signals satisfy weak MLRP. If the gains from trade are certain, i.e., $c = 0$, then there exists a sequence of equilibria where information is aggregated according to definition 1.*

Proof: The following is an equilibrium: Let $a_z(\theta) = \frac{1}{2}$ and let $b_z^m(\theta) = \beta_z^m(\theta)$ for all θ. □

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