Procuring Diversity

Igor Letina and Armin Schmutzler*

June 23, 2014

Abstract

This paper deals with the competitive procurement of complex goods for which the best design is initially unknown. When this uncertainty is sufficiently large, the social optimum requires that sellers offer different designs. A simple class of mechanisms implements the optimal design choices. This class contains mechanisms that allow the buyer to extract (approximately) the entire surplus. We further analyze the equilibria of four commonly used mechanisms: (i) *negotiations,* (ii) *tournaments,* (iii) *piece rate contracts* and (iv) *beauty contests*. We show that suitably structured negotiations implement the socially optimal diversity of designs, while the other mechanisms do not induce any diversity. However, for distributional reasons, the buyer might choose inefficient mechanisms rather than negotiations.

Keywords: Procurement, diversity, negotiations, tournament, beauty contest.
JEL: L14, L22, L23.

Conference Draft; preliminary and incomplete

*Igor Letina: Department of Economics, University of Zürich, igor.letina@econ.uzh.ch. Armin Schmutzler: Department of Economics, University of Zürich and CEPR; armin.schmutzler@econ.uzh.ch. We are grateful Dirk Bergemann, Joao Montez, Nick Netzer, Marco Ottaviani, Konrad Stahl, David Wettstein and to seminar participants in Rome (Oligo Workshop) and in Zurich for helpful comments.

1 Introduction

The 1973 Yom Kippur War revealed a fundamental weakness that US-made fighter planes would face in a potential conflict with the Soviet Union. The Israeli Air Force, despite having the most advanced technology, lost 109 aircraft in just 18 days, most of them to radar-guided anti-aircraft systems. Already in 1974 the Defense Advanced Research Projects Agency (DARPA) set out to build an aircraft that would not be as vulnerable to Soviet ground-based defenses as the then-current aircraft. However, DARPA did not know what was the best way to design an aircraft that would suit their needs. The best they could do was to solicit design submissions from several companies with experience and expertise in building aircraft, test the prototypes built on the basis of the most promising designs and then build planes based on the prototype that performed best. That is, DARPA initiated a procurement process that would yield a plane they needed. This procedure eventually led to the Lockheed F-117 Nighthawk, the "invisible" plane.[1]

As the above example suggests, in many procurement situations the buyer does not know ex ante the optimal design of the good or service she is procuring, but once a prototype is built, it is possible to assess the quality of the design. Then the interest of the procurer is not only to obtain a low price, but to obtain sufficient diversity of designs. Similar procurement problems appear in architectural design competitions. When procuring the architectural design for a new building, the buyer does not know what is the best way to design the building, but once she examines the submitted plans, she can choose which one suits her needs best. As a matter of fact, guidelines for architectural design competitions explicitly recognize the need for diversity of designs at a low price. For example, the Royal Institute of British Architects states: "[Competitions] have a reputation for giving clients the best range of design options to choose from and cost a fraction of the total construction cost of a scheme. [...] Competitions enable a wide variety of approaches to be explored simultaneously with a number of designers."[2]

The central questions of this paper are: How should a procurer design the procurement mechanism when the ideal design is not known initially? What are the effects of several common mechanisms on efficiency and distribution? To answer these questions, we develop a model with one buyer and two sellers. The distinguishing feature of our model is that the quality of any design is not known ex ante by either the buyer or the sellers, but is revealed ex post (once the designs have been submitted, or once the prototypes can be tested). The sellers choose the design they wish to develop from a continuum of possible designs. Moreover, we assume that the sellers are exogenously heterogeneous, reflecting the idea that they might have access to different technology (as in the stealth fighter example) or different design styles (as in an architectural design). The quality of a design chosen by a particular firm will be high when the design and the identity of the firm are close to an initially unknown state of the world in a suitable metric space.[3]

The sellers choose their designs in order to maximize their expected profits, while the buyer

[1] See Crickmore (2003).

[2] See Royal Institute of British Architects (RIBA) (2013).

[3] Specifically, we assume a uniform distribution of the state on $[0,1] \times [0,1]$. Each supplier chooses a design in $[0,1]$; his identity corresponds to one of the boundary points in $[0,1]$. Thus, a combination of design and supplier corresponds to point in $[0,1] \times \{0\}$ or $[0,1] \times \{1\}$. Quality is a decreasing function of the (Manhattan) distance between this point an the state of the world.

wants to incentivize the sellers to offer a combination of designs which (jointly) maximizes expected quality. The timeline of the model is as follows: i) the buyer chooses the procurement mechanism, then ii) the sellers choose which designs to develop, iii) the uncertainty is resolved and the buyer chooses which design to implement and iv) the buyer makes transfers to the sellers according to the rules of the mechanism.

We first characterize the socially optimal ex-ante design choices. For a large subset of the parameter space, optimal procurement requires that the two firms choose different designs. Intuitively, this provides the buyer with an option value coming from the possibility to react to the information on the state of the world by choosing the more suitable design.

We then show how the buyer can use a simple class of mechanisms to induce this kind of diversity of designs. She can achieve the socially optimal ex-ante design choices by promising adequate transfers. The seller offering the higher quality should obtain a positive fraction of the quality difference between the two suppliers, while the lower quality seller should obtain nothing. In addition, by choosing the fraction adequately, the buyer can approximately appropriate the total surplus.

Next, we analyze some commonly used mechanisms. In a *piece rate contract*, the seller makes a transfer to each of the sellers that depends only on supplied quality, not on the quality of the competitor. In a *beauty contest*, each seller chooses a design and a price. The buyer chooses the seller who provides higher net quality (quality minus price) and pays the bid. In a *fixed prize tournament*, the buyer fixes a prize ex ante and chooses the winner who offers higher quality. This winner receives the prize. In *negotiations*, the sellers first simultaneously choose a design. Then negotiations take place over the transfers that the buyer must make to the winner and to the loser. We consider two variants: either the buyer and the winning seller negotiate, and the losing seller provides the outside option for the buyer (two-party negotiations), or all three parties negotiate (three-party negotiations).

We show that with fixed prize tournaments, beauty contests and piece rate contracts the unique equilibrium provides no diversity of designs. By contrast, negotiations provide differentiation incentives. The diversity of designs in the three-party negotiations is lower than socially optimal, while the two-party negotiations implement the socially optimal diversity of designs.

We also discuss why different procurement mechanisms exist which are not necessarily optimal. Distributional concerns may play a role. For instance, we show that the buyer often obtains higher payoffs with beauty contests than with negotiations, even when negotiations lead to higher total payoffs. Fixed prize tournaments generally lead to higher buyer payoffs than beauty contests. This may explain why they are quite common in practice (for example architectural design contest are fixed prize contests). A limitation of distributional arguments is that the buyer can, in principle appropriate the total surplus by asking for participation fees. We therefore argue that there may be more convincing reasons for the use of alternative mechanisms, in particular, information restrictions which make beauty contests and tournaments easier to apply.

This paper contributes to the literature on procurement mechanisms. Much of the literature focuses on private information about production costs. The issue is how to incentivize the suppliers to provide the goods at the lowest cost satisfying incentive and participation constraints. A good and extensive overview of this approach to the procurement problem is provided in Tirole and Laffont (1993). However, in many procurement problems, both price and quality matter. This issue is studied in Che (1993) and later in Asker and Cantillon

(2008) and Asker and Cantillon (2010). The solution to the problem is a scoring auction, where the winner of the procurement is determined by a score which gives adequate weights to both price and quality.

In the papers above, the buyer and the suppliers know the optimal way to carry out the task under consideration. Bajari and Tadelis (2001) noticed that in construction industry, new information becomes available during the period when the contract is being executed (that is, construction takes place). In their model, the optimal procurement mechanism has to trade off strong incentives through a fixed contract against the ability to adjust to the new information through variable contracts. However, in their model the suppliers know the best way to provide the object ex ante. In Kaplan (2012), the sellers do not know the preferred design of the procurer, but the buyer herself has this information. Kaplan studies what happens when the buyer reveals this information.

In all of the papers cited above, the procurer always knows the optimal way to design the product that is being procured. Thus, there is no need for providing diversity of designs, which is the central issue we study in this paper.

In Section 2, we introduce the model and the basic terminology. Section 3, we analyze the optimal choice of designs by the two firms, and we introduce a mechanism that (weakly) implements the optimum. Section 4 introduces several commonly used procurement mechanisms. Section 5 studies the equilibria of these mechanisms. In Section 6, we compare the efficiency and the distributional properties of these different mechanisms. Section 7 contains a discussion of reasons for the limited use of the optimal mechanism in practice. Section 8 concludes.

2 Set-Up and Terminology

A buyer (player B) needs a product that two potential suppliers (players $i \in \{1, 2\}$) can provide. The sellers choose the design $v_i \in V$ of the product, where V is a measurable set of possible designs. We assume that the cost of developing a design is the same for all $v_i \in V$, and we normalize this cost to zero. We abstract from any further production costs. Let Θ be a measurable set describing the fixed and exogenous technological characteristics of the sellers. Let $\theta_i \in \Theta$ describe the characteristics of seller $i \in \{1, 2\}$. Denote with $\sigma \in V$ the ideal design of the good being procured and with $\theta \in \Theta$ the ideal characteristics of the firm providing the design. The quality of a product is determined by the following factors: (i) $\Psi \in \mathbb{R}^+$, the underlying value of the project to the buyer; (ii) the distance between the design chosen v_i and the ideal design σ, given by $d_\sigma(v_i, \sigma)$; (iii) the distance between the characteristics of the seller θ_i and the ideal characteristics θ, given by $d_\theta(\theta_i, \theta)$. The quality function can then be written as $\widehat{Q}(d_\sigma(v_i, \sigma), d_\theta(\theta_i, \theta); \Psi)$. The ideal design σ and the ideal characteristics θ are the realization of two random variables $\sigma \sim F_\sigma$ and $\theta \sim F_\theta$. Sets V and Θ, the characteristics of the firms θ_i, the value of the project Ψ, the distributions F_σ and F_θ, as well as the functions Q, d_σ and d_θ are public knowledge. Neither the sellers nor the buyer know the realization of σ and θ. The design choice v_i is the private knowledge of the seller i. The quality $\widehat{Q}(d_\sigma(v_i, \sigma), d_\theta(\theta_i, \theta); \Psi)$ offered by a seller i is publicly observable and verifiable. The buyer prefers higher quality, with utility function $U(Q) = Q$.

We use a particularly tractable specification of the general model above. We set $V = [0, 1]$ and $\Theta = [0, 1]$ with $\theta_1 = 0$ and $\theta_2 = 1$. We assume that $d_\sigma(v_i, \sigma) = |v_i - \sigma|$ and $d_\theta(\theta_i, \theta) =$

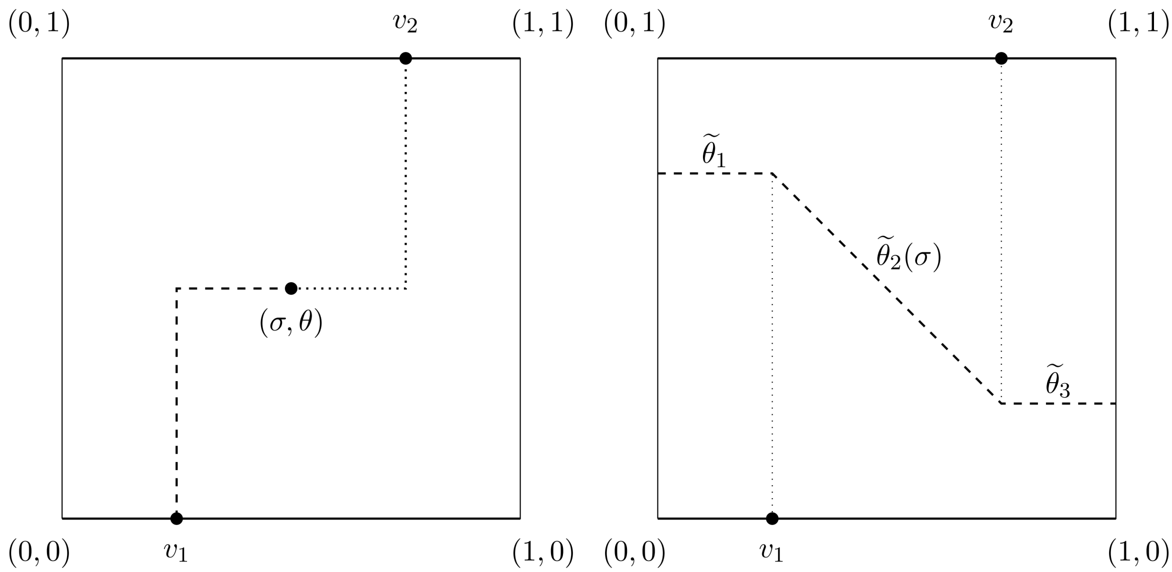

Figure 1: Quality from designs v_1 and v_2.

$|\theta - i + 1|$. We suppose F_σ and F_θ are uniform distributions. Finally, we specify the quality of seller i as $\widehat{Q}(d_\sigma, d_\theta; \Psi) = \Psi - ad_\theta - bd_\sigma$ for constants $a, b > 0$. Thus, we can write qualities of sellers $i = 1, 2$ as

$$q_1(\sigma,\theta) = Q(v_1, 1, (\sigma,\theta)) = \Psi - a\theta - b|v_1 - \sigma| \tag{1}$$
$$q_2(\sigma,\theta) = Q(v_2, 2, (\sigma,\theta)) = \Psi - a(1-\theta) - b|v_2 - \sigma| \tag{2}$$

Thus, a captures the degree of exogenous differentiation between firms: It is the maximum willingness-to-pay that a buyer might have for buying $(\sigma, 0)$ rather than $(\sigma, 1)$ or, conversely, $(\sigma, 1)$ rather than $(\sigma, 0)$, while b captures the degree to which different designs impact the final quality of the product.

This specification allows for a convenient graphical representation. We can think of (σ, θ) as a realization of the state of the world, so that the set of possible states of the world is the unit square, that is $(\sigma, \theta) \in [0, 1]^2$. Firm i chooses a design v_i which is located on either the lower or the upper edge of the unit square, that is, $v_i \in [0, 1] \times \{i - 1\} \subset [0, 1] \times [0, 1]$. The quality of its design is then a decreasing function of the Manhattan distance (the sum of horizontal and vertical transportation costs) between the firm location and the state of the world. Figure 1 (left panel) illustrates the transportation costs for a particular realization of states: According to (1) and (2), the transportation costs correspond to the length (scaled by a and b) of the dashed line for the seller 1 and to the to the length of the dotted line for the seller 2.

The following simple result is obvious.

Lemma 1 *(i) $q_1(q_2)$ is decreasing (increasing) in θ.*
(ii) Suppose $v_1 \leq v_2$. If $q_1 < q_2$ for some σ, then $q_1 < q_2$ for any $\sigma' > \sigma$.

Figure 1 (right panel) illustrates the relation between supplier choices, realization of the state and qualities. Suppose that $v_1 \leq v_2$. The dashed line (given by $\tilde{\theta}$) shows the set of states of the world for which the quality supplied by both firms is identical.[4] In line with Lemma 1, the buyer prefers to buy from firm 1 (2) if the state is below (above) the dashed line.

The outcome of the procurement game has to specify the quality of the product that the buyer buys, as well as the transfers from the buyer to the sellers. Let the set of possible outcomes be $\Omega \subseteq \mathbb{R}^2 \times \{1,2\}$. A typical element is $(t_1, t_2, \widehat{i}) \in \Omega$, where $\widehat{i}$ is the chosen supplier, and t_i is the transfer to the seller i.

Definition 2 (Mechanism) *A mechanism $\mathcal{M} = (M_1, M_2, \omega(\cdot))$ is a collection of message sets (M_1, M_2) and an outcome function $\omega : M_1 \times \mathbb{R} \times M_2 \times \mathbb{R} \to \Omega$. The outcome function assigns to each vector of messages and quality levels (m_1, q_1, m_2, q_2) an outcome $(t_1, t_2, \widehat{i}) \in \Omega$.*

The message sets defined by the mechanism can be (and in most of the mechanisms examined here will be) empty, in which case transfers condition exclusively on the realized quality levels. By choosing $\widehat{i} \in \{1,2\}$, the mechanism also select a quality $\hat{q} = q_{\widehat{i}}$.

A mechanism $\mathcal{M}$ defines a game between the two sellers where the strategy set of each seller i is $S_i = M_i \times V$. Thus, each seller chooses a message m_i and a design v_i. Since design choices are not observable, the payoffs of the game can depend only on the messages transmitted and the quality levels realized. For a given state of the world (σ, θ) and the opponent's action (m_j, v_j) the payoff of the seller i is given by $t_i(m_i, v_i | m_j, v_j, \sigma, \theta) = \omega_i(m_1, q_1(\sigma, \theta), m_2, q_2(\sigma, \theta))$ where ω_i is the i-th element of the vector ω. Since the state of the world is unobservable, the sellers choose their actions in order to maximize their expected profits, which are given by

$$E\Pi_{si}(m_i, v_i | m_j, v_j) = \int_V \int_\Theta t_i(m_i, v_i | m_j, v_j, \sigma, \theta) dF_\theta dF_\sigma.$$

Similarly, the implemented quality is given by $\hat{q}(m_i, v_i, m_j, v_j, \sigma, \theta) = q_{\widehat{i}}(\theta)$. Then the expected net utility of the buyer is given by

$$EU_{si}(m_i, v_i, m_j, v_j) = \int_V \int_\Theta \left(\hat{q}(m_i, v_i, m_j, v_j, \sigma, \theta) - \sum_{i=1,2} t_i(m_i, v_i | m_j, v_j, \sigma, \theta) \right) dF_\theta dF_\sigma.$$

All players are assumed to be maximizing their expected payoffs. The timeline of the model is as follows.

1. The buyer commits to a mechanism.
2. The sellers choose their actions $(m_i, v_i) \in S_i$ simultaneously.
3. The state of nature (σ, θ) is realized and q_1 and q_2 are observed.
4. The allocation $\omega(m_1, q_1, m_2, q_2) = (t_1, t_2, \widehat{i})$ and thus the quality $\hat{q}$ is realized.
5. The buyer obtains the payoff $\hat{q} - t_1 - t_2$. The sellers obtain payoffs t_1 and t_2.

In several applications, we will think of the game as a reduced version of a multi-stage game where the buyers first choose their actions (m_i, v_i) and the allocation is determined as the equilbrium of another game.

[4] In Figure 1, we assume that $\tilde{\theta}[0,1] \subset (0,1)$. Below, we will also deal with the case that $\tilde{\theta}$ attains boundary values.

3 Optimal Design Choices

We now characterize the socially optimal design choices. With only one potential seller, the optimal choice would be 1/2, as this design maximizes the expected quality of the product. With two suppliers, the optimization needs to take into account the option value generated by having different designs to choose from. It will therefore turn out that, if designs have a sufficiently strong effect on quality (that is, if b is large enough), it is optimal to offer at least some diversity of designs. The optimal diversity will depend on the ratio b/a. With this in mind, we will characterize a class of simple mechanisms that implement the socially optimal design choices. Finally, we show that, by using this class of mechanisms which implement the social optimum, the buyer can extract an arbitrarily large fraction of the surplus from the sellers, thus approximating the first best outcome.

3.1 Social Optimum

We first characterize the socially optimal design choice.

Proposition 3 *Suppose without loss of generality that $v_1 \leq v_2$. Then the socially optimal choice of varieties is:*

$$\left(v_1^{SO}, v_2^{SO}\right) = \begin{cases} \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if} \quad b < a \\ \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right) & \text{if} \quad a \leq b < 2a \\ \left(\frac{1}{4}, \frac{3}{4}\right) & \text{if} \quad 2a \leq b \end{cases}.$$

Proof. See Appendix. ■

The intuition for the result is simple. Whenever $b < a$, the buyer cares more about getting the product from the right seller than about getting the right design. In other words, exogenous differences between firms matter more than endogenous differences. The optimal design (which minimizes the expected horizontal transportation costs) is $v = 1/2$. As b increases, the value of getting the right design increases and so does the expected value of diversity. Since the set of possible designs is bounded, the expected costs of not having the best design are minimized if designs 1/4 and 3/4 are chosen. As a result, the distance from the optimal design choice is never larger than 1/4.

3.2 Optimal Mechanism

The analysis in Section 3.1 shows that, for large parameter ranges, an optimal mechanism must provide incentives for sellers to differentiate their designs from each other. Such differentiation increases the probability that the buyer obtains a high quality design. In order to induce differentiation, the payoff to the sellers has to depend not only on his own quality, but also on the difference in the quality levels offered. That way, firms will be willing to accept a lower winning probability resulting from moving away from the center to increase the payoffs in cases where they win.

The buyer can only observe the quality of each seller, but not the actual designs. Thus, the buyer is restricted to offering transfers that are only a function of the observable qualities rather than of the chosen designs themselves, that is, transfers of the type $t_1\left(q_1\left(\sigma,\theta\right), q_2\left(\sigma,\theta\right)\right)$ and $t_2\left(q_1\left(\sigma,\theta\right), q_2\left(\sigma,\theta\right)\right)$. This is reminiscent of the setting in effort tournaments (for example

Lazear and Rosen (1981)), where agents exert effort and the output is a function of effort and the random shock, but the principal only observes the output. There, as in our setting, the principal faces a moral hazard problem. There is, however, a fundamental difference: Whereas in standard moral hazard problem, the principal wants to induce enough of a costly action, in our setting, the goal is to induce the right amount of differentiation in actions that are not costly per se.

We will show that the buyer can implement the social optimum by restricting attentions to mechanisms where the message space is empty, so that the allocation depends only on qualities. The buyer proposes her preferred designs v_1 and v_2, and announces the transfer functions t_1, t_2. The transfer functions must satisfy the incentive constraint, so that neither of the sellers can increase his payoffs by deviating from the proposed v_i. They also must satisfy the participation constraint. Thus, the buyer chooses the tuple (t_1, t_2, v_1, v_2), satisfying the constraints and maximizing her expected payoff.

For simplicity, denote $q_1(v_1) = q(v_1, 0; (\sigma, \theta))$ and $q_2(v_2) = q(v_2, 1; (\sigma, \theta))$. The problem of the buyer is given by

$$\mathcal{P}: \qquad \max_{t_1(), t_2(), v_1, v_2} \int_{\theta=0}^{1} \int_{\sigma=0}^{1} \max\{q_1(v_1), q_2(v_2)\} - \sum_{i=1, j\neq i}^{2} t_i(q_i(v_i), q_j(v_j))\, d\sigma d\theta$$

subject to

$$\int_{\theta=0}^{1} \int_{\sigma=0}^{1} t_i(q_i(v_1), q_j(v_j))\, d\sigma d\theta \geq \int_{\theta=0}^{1} \int_{\sigma=0}^{1} t_i(q_i(\hat{v}_i), q_j(v_j))\, d\sigma d\theta$$
$$\forall \hat{v}_i \in [0,1], i \in \{1,2\}, i \neq j.$$

$$\int_{\theta=0}^{1} \int_{\sigma=0}^{1} t_i(q_i(v_i), q_j(v_j))\, d\sigma d\theta \geq 0 \qquad \forall i \in \{1,2\}, i \neq j.$$

Solving this problem is equivalent to solving the problem of maximizing expected total profits subject to the constraint that the seller breaks even. In principle, the functions t_1, t_2 can be very complicated, and we do not provide a full characterization of the class of mechanisms solving the problem. However, there is simple mechanism that implements the optimal design choice.

Proposition 4 *The following transfer rule (weakly) implements the social optimum $\left(v_1^{SO}, v_2^{SO}\right)$ for any $\eta > 0$:*

$$t_i = \begin{cases} \frac{q_i - q_j}{\eta} & \text{if} \quad q_i \geq q_j \\ 0 & \text{otherwise} \end{cases}.$$

The expected profit of the buyer is strictly increasing in η on $\mathbb{R}^+$, and it approaches the maximal total surplus as $\eta \longrightarrow \infty$.

Proof. See Appendix. ■

Thus, the mechanism has the following properties: (a) The winning seller receives a fraction of the difference between the surplus generated by his design and the alternative, and (b) the losing seller receives nothing. In any mechanism of this class, given that the seller j chooses a socially optimal design then the seller i internalizes the externality that his design choice has for social welfare and maximizes the difference between the surplus generated by his design and the alternative. Since this holds for seller j as well, the choice of designs is socially optimal. Moreover, since this is true for any fraction of the surplus, the buyer can extract an arbitrarily large portion of the surplus while maximizing total welfare.

The mechanism relies on the fact that there are no direct costs of providing quality. Suppliers thus potentially react to small incentives for differentiation that come from the hope of increasing profits in suitable states of the world.

There are several reasons why the mechanism only achieves weak implementation, that is, it does not have a unique equilibrium giving rise to the optimal allocation. First, obviously there are two design vectors that are socially optimal whenever $b > a$: $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$ or $\left(1 - \frac{a}{2b}, \frac{a}{2b}\right)$ when $a \leq b \leq 2a$ and $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \frac{1}{4}\right)$ when $2a \leq b$. In each case, both strategy profiles are equilibria of the proposed mechanism. Second, when $b \in [a, 2a]$, a third equilibrium emerges — the symmetric equilibrium which is not optimal.

4 Common Procurement Mechanisms: Examples

In the remainder of the paper, we show to which extent existing mechanisms can come close to implementing the optimal design choices as characterized in the previous section. In this section, we introduce these mechanisms.

4.1 Piece-rate contracts

We first consider a class of mechanisms in which both suppliers obtain a transfer, and the transfer to a supplier depends only on the quality of this supplier.

Definition 5 (Piece rate contract) *In a piece rate contract, $M_i = \varnothing$. The mechanism selects the winning seller i for which $Q(v_i, i, (\sigma, \theta)) \geq Q(v_j, j, (\sigma, \theta))$, $j \neq i$ (with random tie breaking). The implemented quality is $q^{PR} = q_i$ and the transfers are $t_i = \hat{t}_i(Q(v_i, i, (\sigma, \theta)))$ and $t_j = \hat{t}_j(Q(v_j, j, (\sigma, \theta)))$, where $\hat{t}_i(\cdot)$, $\hat{t}_j(\cdot)$ are strictly increasing functions.*

Thus, the mechanism selects the higher quality supplier, and pays to each seller a transfer that is increasing in his own quality. Notice that in this mechanism there are no strategic interactions between the sellers. The payoff of each seller depends only on the level of quality provided by him.

4.2 Contests

In contest mechanisms, only one supplier receives a transfer. The size of the transfer is either determined by the buyer before the bids are submitted or as part of the bids by the suppliers.

Definition 6 (Fixed-prize contest) *In a fixed-prize contest, $M_i = \varnothing$. The mechanism selects the winning seller i for which $Q(v_i, i, (\sigma, \theta)) \geq Q(v_j, j, (\sigma, \theta))$, $j \neq i$ (with random tie breaking). The transfer is $t_i = A \geq 0$ and $t_j = 0$. A is a fixed constant (a prize) chosen by the buyer; the buyer commits to paying this transfer even if her expected utility is negative.*

In fixed-prize contests, buyers determine the size of the transfers. In the next mechanism, the transfers result from the winning seller's bid.

Definition 7 (Beauty contest) *In a beauty contest, $M_i = \mathbb{R}^+$; a message is a price announcement $p_i \in \mathbb{R}^+$. The mechanism selects the winning seller i for which $Q(v_i, i, (\sigma, \theta)) - p_i \geq Q(v_j, j, (\sigma, \theta)) - p_j$, $j \neq i$ (with random tie breaking). The transfer is $t_i = p_i$ and $t_j = 0$; the buyer commits to paying this transfer even if her expected utility is negative.*

In both types of contests, the behavior of the competitor influences a supplier's winning probabilities, but not the size of the prize. The two mechanisms differ from the contests that are typically used in the literature, because the winning probabilites do not depend on efforts that are exerted by the parties ex-ante. Instead, they depend only on the choice of designs which are equally costly by assumption (and, in fact, normalized to zero).

For beauty contests, we assume that among the two transportation cost parameters a and b, a is relatively high.

Assumption 1

$$a > \frac{b}{2\sqrt{2}+3}$$

When this assumption is violated, existence problems arise which are similar to those in the standard Hotelling model with linear costs. This is, in particular, true in the special case with purely endogenous differentiation ($a = 0$).

4.3 Negotiations

Finally, we consider mechanisms where the quality of the supplier whose design is not selected influences the transfer to the other supplier. Contrary to the contest mechanisms, the transfers in negotiations are determined after the choice of designs.

Definition 8 (Negotiations) *Under price negotiations, $M_i = \varnothing$. The mechanism selects the seller i for which $Q(v_i, i, (\sigma, \theta)) \geq Q(v_j, j, (\sigma, \theta))$, $j \neq i$ (with random tie breaking). In two-party negotiations, the buyer and the winning seller bargain over the price, and the quality offered by the losing seller is the buyer's outside option. In three-party negotiations, all three parties bargain over transfers. The bargaining game determines transfers so that the utilities correspond to Shapley values.*

The formulation of the mechanism thus mixes cooperative and non-cooperative concepts. However, it is possible to interpret the Shapley value as the payoff resulting from a non-cooperative game. For example, Pérez-Castrillo and Wettstein (2001) describe a mechanism which implements the Shapley values as a SPE. First, the buyer and the seller simultaneously post bids. Whoever posted the higher bid, pays the bid to the other party and then makes a take it or leave it offer. The offer specifies a transfer which is paid in case of acceptance. In case of rejection no further transfers take place. Finally, in case of acceptance, the proposer collects the entire surplus, while in case of rejection the parties receive their outside option (plus the bid paid out after the first round).

We have no strong views as to whether two-party and three-party negotiations are more suitable to describe actual negotiations. We will show, however, that both mechanisms share a useful property with the optimal mechanism, albeit to different degrees.

5 Equilibria in Common Procurement Mechanisms

In this section, we show that such commonly used mechanisms as tournaments, beauty contests or piece rate contracts fail to induce sellers to differentiate. Specifically, in all these mechanisms, $v_1 = v_2 = 1/2$ in the unique equilibrium. However, negotiations, which endogenously condition the payoffs on the quality difference between the suppliers, can potentially provide differentiation incentives. We show that two-party negotiations between the buyer and the seller offering higher quality implement the socially optimal outcome when the buyer's outside option is given by the lower quality seller. Three-party negotiations between the buyer and the two sellers also provide incentives for the sellers to differentiate, although the diversity of designs is lower than socially optimal.

5.1 Price Negotiations

We first consider negotiations, as introduced in Section 4.3. For a given realization (σ, θ) of the state, seller 1 is the producer of higher (lower) quality if θ is below (above) some critical value $\widetilde{\theta}(\sigma)$ (see Figure 3). We define $q_i^h(\sigma, \theta)$ as the quality of the design offered by firm i when it offers a higher quality than the competitor and $q_i^l(\sigma, \theta)$ as the quality offered by firm i when it offers a lower quality. We analyze two-party and three-party negotiations in turn.

5.1.1 Two-party Negotiations

For two-party negotiations, we obtain a strong result.

Corollary 9 *Two-party negotiations (weakly) implement the social optimum* $\left(v_1^{SO}, v_2^{SO}\right)$.

Proof. See Appendix. ■

In two-party negotiations, the buyer and the higher quality seller bargain over how to split the surplus, that is, how to divide $q^h - q^l$. The Shapley value corresponds to an equal split of the surplus. Thus the transfer to the winning seller is $\left(q^h - q^l\right)/2$. With this transfer rule, the mechanism belongs to the class of mechanisms characterized in Proposition 4, with $\eta = 2$. Thus, two-party negotiations weakly implement the socially optimal choice of designs. Of course, however, from the buyer's point of view, they do not implement the best outcome as half of the social surplus goes to the seller. As the buyer cannot choose η, she cannot reach the more favorable splits of the surplus corresponding to lower values of η.

5.1.2 Three-party negotiations

In three-party negotiations, seller 1 obtains the Shapley value $q_1^h(\sigma, \theta)/2 - q_1^l(\sigma, \theta)/3$ as a high-quality supplier; he obtains $q_1^l(\sigma, \theta)/6$ as a low-quality supplier. Thus, the expected profits of supplier 1 are of the form

$$E\Pi_{s1}\left(v_1 | v_2\right) = \int_0^1 \left(E\Pi_{s1}\left(v_1 | v_2, \sigma\right)\right) d\sigma = \tag{3}$$

$$\int_0^1 \left(\int_0^{\widetilde{\theta}(\sigma)} \left(\frac{q_1^h(\sigma, \theta)}{2} - \frac{q_2^l(\sigma, \theta)}{3} \right) d\theta + \int_{\widetilde{\theta}(\sigma)}^1 \frac{q_1^l(\sigma, \theta)}{6} d\theta \right) d\sigma.$$

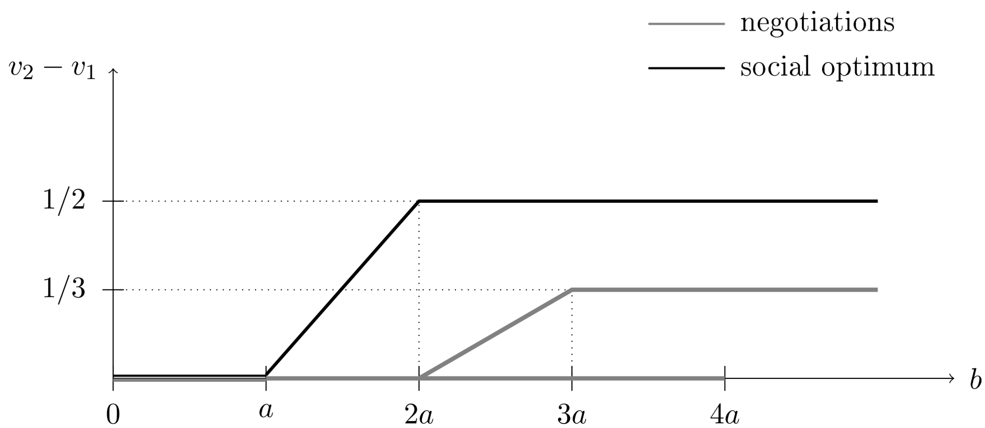

Figure 2: Differentiation in social optimum and three-party negotiations.

The derivation of the payoffs for seller 2 is analogous.

Proposition 10 *With three-party price negotiations, the equilibrium structure is as follows:*
(i) For $b \leq 4a$, a symmetric equilibrium $v_1 = v_2 = 1/2$ exists.
(ii) For $2a \leq b \leq 3a$, asymmetric equilibria $(v_1, v_2) = \left(\frac{a}{b}, 1 - \frac{a}{b}\right)$ and $\left(1 - \frac{a}{b}, \frac{a}{b}\right)$ exist.
(iii) For $3a \leq b$, asymmetric equilibria $(v_1, v_2) = \left(\frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}\right)$ exist.
(iv) No other interior pure-strategy equilibria exist.

Proof. See Appendix. ■

Intuitively, the parties anticipate that the mechanism rewards quality, because quality increases the bargaining power of the firm. Given the weights in (3), it is particularly important for a firm to have higher quality than the competitor in the case where the supplier selects this firm. The higher the expected importance of the right design (b) is relative to the importance of the identity of the firm (a), the more firms differentiate endogenously to supply higher quality than the other firm if selected.

As a result of Proposition 10, we can distinguish between four different equilibrium regimes:

R1 $b < 2a$: Only the symmetric equilibrium exists.

R2 $2a \leq b \leq 3a$: The symmetric equilibrium coexists with the asymmetric equilibria $\left(\frac{a}{b}, 1 - \frac{a}{b}\right)$ and $\left(1 - \frac{a}{b}, \frac{a}{b}\right)$.

R3 $3a \leq b \leq 4a$: The symmetric equilibrium coexists with the asymmetric equilibria $\left(\frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}\right)$.

R4 $b > 4a$: Only the asymmetric equilibrium $\left(\frac{1}{3}, \frac{2}{3}\right)$ exists.

Directly comparing the design choices in three-party negotiations to the socially optimal design choices reveals that, while three-party negotiations induce some differentiation when $b \geq 2a$, there is less diversity of designs than would be socially optimal.

To see this, consider Figure 2, which plots the level of differentiation of sellers in three-party negotiations and in the social optimum. In the beauty contest and fixed prize tournaments, there is no differentiation. For $b < a$, the absence of endogenous differentiation in three-party negotiations is optimal. In the remaining regimes, three-way negotiations do not provide sufficient incentives for differentiation. Even where the asymmetric negotiations equilibrium exists (for $2a \leq b$), it involves less differentation than the social optimum $\left(\frac{1}{4}, \frac{3}{4}\right)$.

5.2 Piece Rate

Suppose that the buyer offers to pay each seller a predetermined price, which depends on the quality level delivered, but is independent of the quality level of the competitor. Furthermore, suppose that the transfers are strictly increasing in quality. With such a mechanism, the buyer provides direct quality incentives, without introducing a competitive institution. However, as the following proposition shows, such a contract does not induce diversity of designs.

Proposition 11 *For any profile of bounded transfer functions $t_i(q_i)$ such that $q_i > q_i' \Rightarrow t_i(q_i) > t_i(q_i')$ for all $i \in \{1,2\}$ and $t_i(q_i) \geq 0$ for all q_i, the unique equilibrium is $v_1 = v_2 = 1/2$.*
Proof. *See Appendix.* ■

Thus piece rates cannot be used to induce diversity, even though the functions t_i are allowed to differ across buyers.

5.3 Beauty Contest

In the beauty contest, suppose the suppliers have chosen the design v_i and the price p_i. Then the buyer maximizes expected net payoffs,

$$E\Pi_b(p_1, v_1, p_2, v_2) = \int_{\theta=0}^{1} \int_{\sigma=0}^{1} \max\{Q(v_1, 0; \sigma, \theta) - p_1, Q(v_2, 1; \sigma, \theta) - p_2\} \, d\sigma d\theta.$$

Let

$$P(p_1, v_1, p_2, v_2) = \text{Prob}\{Q(v_1, 0; \sigma, \theta) - p_1 \geq Q(v_2, 1; \sigma, \theta) - p_2\}$$

Then the expected payoffs of the sellers are

$$\begin{aligned} E\Pi_{s1}(p_1, v_1, p_2, v_2) &= P(p_1, v_1, p_2, v_2)\, p_1 \\ E\Pi_{s2}(p_1, v_1, p_2, v_2) &= (1 - P(p_1, v_1, p_2, v_2))\, p_2 \end{aligned} \tag{4}$$

Proposition 12 *Suppose Assumption 1 holds. The Beauty Contest with location choice has a unique equilibrium. In this equilibrium,*

$$p_1^{BC} = p_2^{BC} = a \tag{5}$$
$$v_1^{BC} = v_2^{BC} = 1/2. \tag{6}$$

Proof. See Appendix. ■

The degree of exogenous differentiation a fully determines prices. Moreover, the equilibrium involves minimum differentiation. According to Proposition 3, such an equilibrium is not socially optimal whenever $b > a$, reflecting inefficiently high transportation costs.

5.4 Fixed Prize Tournament

Now consider the fixed prize tournament for some arbitrary $A \geq 0$.

Proposition 13 *There is a unique equilibrium in the fixed prize tournament. In this equilibrium, $v_1 = v_2 = 1/2$ for any prize A.*

Proof. The proof follows the proof of Proposition 12 closely. The analysis is considerably simpler, however: Only v_1 and v_2 need to be determined, whereas p_1 and p_2 can be replaced by A. Moreover, the structure of conceivable deviations is much simpler, as only one-dimensional deviations are feasible. ■

As in the beauty contest, minimum differentiation arises in equilibrium for any value of the prize A. Thus, as long as the buyer chooses $A \geq 0$ (covers the costs of supplying one design), the suppliers choose the same designs in the tournament as in the beauty contest.

5.5 Summary

Among the mechanisms considered in this section, only negotiations have equilibria where the sellers offer some diversity of designs. Two-party negotiations weakly implement the socially optimal design choices. For piece rate contracts, beauty contests and fixed prize tournaments, only a symmetric equilibrium arises where the sellers choose $v_1 = v_2 = 1/2$. The result is intuitive: In these mechanisms, the level of the prize does not depend on the actions of the opponents. Hence, the sellers only care about winning (in tournaments and beauty contests) or maximizing their own expected quality (in piece rate contracts). By contrast, in negotiations, the payoff depends on the margin by which one seller outperforms the other. The sellers are therefore willing to differentiate, effectively sacrificing some probability of winning for the larger payoff when they do win. Thus, the sellers internalize some of the positive effects associated with the diversity of designs. In two-party negotiations, this externality is fully internalized. Thus, negotiations are always preferable to the other types of mechanisms from a welfare perspective. However, as we will show in the next section, distributional concerns sometimes cause the buyer to choose less efficient mechanisms rather than negotiations.

6 Payoff Comparisons

Our previous analysis suggests a clear welfare ranking of various procurement institutions. In particular, negotiations have desirable properties. In the following, we show that the buyers may nevertheless prefer alternative institutions. We illustrate this with a comparison between beauty contests and three-way negotiations.

6.1 Beauty Contests vs. Three-way Negotiations

We maintain the following assumption.

Assumption 2

$$\Psi \geq \frac{b+3a}{2} \tag{7}$$

$$\Psi \geq b \tag{8}$$

This assumption guarantees that, in equilibrium, the buyer's utility (net of transfer payments) is non-negative, no matter what the state of the world is.[5] The assumption requires that the willingness-to-pay for the ideal design Ψ is sufficiently high relative to the potential losses from suboptimal design, parameterized by a and b. Together, Assumptions 1 and 2 can easily be satisfied: the exogenous differentiation parameter a has to be sufficiently high relative to b that Assumption 1 holds. Once this is guaranteed, Assumption 2 holds for any sufficiently high choice of Ψ.

The symmetric equilibrium with price negotiations and the equilibrium of the beauty contest only differ with respect to distribution, not with respect to total payoff.

Proposition 14 *(i) Expected total profits are the same for beauty contests and the symmetric equilibrium of the three-way negotiation game.*
(ii) Expected buyer profits are higher for beauty contests than in the symmetric equilibrium of the three-party negotiation game if and only if

$$\frac{1}{3}\Psi - a - \frac{1}{12}b > 0. \tag{9}$$

Proof. In Lemmas 30 and 31 in the Appendix, we provide the payoffs in the beauty contest and the symmetric negotiation equilibria. The result directly follows. ■

Intuitively, the equality of total payoffs can be seen directly from the fact that the location choice is the same in both cases. Thus, the difference between beauty contests and symmetric negotiations is purely distributional. The following result provides an alternative characterization of the difference from the perspective of the buyer.

Corollary 15 *If $\Psi < 3a$, buyers always receive higher payoffs in the symmetric negotiations equilibrium than in the beauty contest. If $3a < \Psi < 4a$ the buyer payoff is higher for symmetric negotiations than for BC if b is large, that is, $b \in [4\Psi - 12a, \Psi]$. If $\Psi > 4a$ buyer payoff are always higher for the beauty contest.*

Proof. See Appendix. ■

The intuition for the role of the parameters in this corollary is as follows. Under negotiations, the buyer shares the surplus with the supplier. Thus any increase in surplus coming from an increase in the value Ψ or a reduction in transportation costs a or b only partly accrues to the buyer. By contrast, in a beauty contest, an increase of Ψ or a reduction in a translates into a one-to-one increase in buyer profits, as the transfer to the seller is exactly a.

The differences between beauty contests and negotiations are more fundamental when the asymmetric equilibrium is considered.

Proposition 16 *(i) Beauty contests always yield lower total payoffs than the asymmetric equilbrium.*
(iia) If $\Psi < 3a$ buyers always prefer the asymmetric equilibrium of the three-party negotiation game to the equilibrium in the beauty contest.
(iib) If $3a < \Psi < 4.1551a$ the buyer payoff is higher for the asymmetric equilibrium of the three-party negotiation game than for beauty contests if b is sufficiently large.
(iic) If $\Psi > 4.1551a$ the buyer payoff is always higher for beauty contests than for the asymmetric equilibrium of the three-party negotiation game.

[5] In the equilibrium of the beauty contest, transportation costs are highest $(a/2 + b/2)$ when the state of the world is $(0, 1/2)$ or $(1, 1/2)$.

Proof. See Appendix. ■

The asymmetric negotiations equilibrium has the desirable property that it reduces the expected transportation costs of the buyer. Nevertheless, the buyer does not always prefer it to the beauty contest equilibrium, because the buyer only pays a low price in the beauty contest when the exogenous asymmetry between the two suppliers is low.

7 Discussion

The preceding analysis suggests how a procurer can efficiently induce diversity of designs: Buyers should use a tournament-like mechanism where the prize that the successful supplier obtains depends positively on the quality difference to the other supplier. It is not obvious that buyers frequently use such mechanisms in reality, even though they share some common properties with negotiation mechanisms. In Section 7.1, we therefore discuss various reasons why the buyers might use other mechanisms. These considerations also serve as a robustness discussion. In Section 7.2, we compare our paper with existing work that deals with the relation between procurement mechanisms and effort choices.

7.1 Robustness

7.1.1 Fixed Costs of Design

So far, we have assumed that the fixed cost of design are zero. This is clearly a simplification. In fact, without fixed costs of designs it is hard to argue why suppliers have to commit to a particular design. In the following, we therefore sketch the necessary modifications for the case of non-negligible fixed costs.

Obviously, the allocation described in Proposition 3 is socially optimal only if the total expected profits net of fixed costs are positive. Otherwise the optimal decision is not to invest in creating a design, or only ask for a design from one of the suppliers. In the following discussion, we always assume that fixed costs are sufficiently low that investment by both firms is socially desirable.

Of course, even under this assumption the mechanism will only lead to investment if the expected profits are sufficiently high that the suppliers break even. For the optimal mechanism, the buyer must choose η sufficiently low to achieve this. Therefore, the existence of fixed costs of design does not offer an explanation why buyers might not use the optimal mechanism.

As to the remaining mechanisms, some of them give a positive expected rent to the suppliers when there are no fixed costs. Thus, for sufficiently low fixed costs, firms break even. For higher fixed costs, the buyer can induce participation with an unconditional ex-ante transfer that covers the expected deficit. Thus, if the buyer can choose arbitrary transfers to the suppliers, all mechanisms discussed above are available.

7.1.2 Distributional concerns

The discussion in Section 6 suggests that the buyer may sometimes use mechanisms that yield lower total surplus than others, because they allow her to appropriate a greater fraction of total surplus. Thus, even though these mechanisms might lead to inefficiently low total payoffs, the buyer's surplus may be higher with those mechanisms than with an efficient alternative.

While this argument may sound compelling at first sight, it has limitations. First, obviously, by suitable choice of η, the buyer can choose the efficient mechanism in such a way that she obtains the entire surplus. Second, one can easily modify any of the remaining mechanisms by allowing the buyer to charge an ex-ante participation fee. As long as we continue to assume that the seller commits to participating in the mechanism if her expected rent (net of fixed costs) is non-negative, then the buyer can extract this rent by asking for an upfront payment corresponding to the rent. She will thus optimally choose the efficient mechanism and use the transfers to deal with distributional concerns. In other words, from a distributional perspective all mechanisms are equivalent if the buyer can adjust them by charging a participation fee. However, the buyer may not be able to charge substantial entry fees, for instance, because of limited liability constraints (Che and Gale 2003).

7.1.3 Homogeneous suppliers

In our setting, there is exogeneous asymmetry between the suppliers. One might be concerned that, in some contexts, such differences might not play a role, and that one should treat buyers as being homogeneous. It is straightforward to see that most of the analysis would go through with homogeneous buyers. We only use buyer heterogeneity in our analysis of beauty contests. Without Assumption 1 (that the exogenous asymmetry a is sufficiently large relative to the costs b of suboptimal design), the pure-strategy equilibrium in the beauty contest would not exist. All other results are still valid when $a = 0$, with the obvious proviso that it is necessary to apply the appropriate parameter restriction in some cases.[6] In particular, therefore, the optimal mechanism still has the same properties as with exogenous heterogeneity.

7.1.4 Non-observable quality

The assumption that quality is verifiable is stark, and it is easier to justify in some contexts than in others. For instance, while the technical properties of a prototype airplane may be easy to quantify objectively, the "quality" of an architectural design is a much vaguer concept. It is therefore important to clarify to which extent the different mechanisms really rely on verifiability.

Piece rate contracts require verifiability in the strongest sense. If the quality of each individual supplier is not verifiable, then it is impossible to use such a contract. The informational requirements of the optimal mechanisms are slightly less restrictive: At least the contract only relies on a measurement of the quality difference, not on absolute quality levels. Moreover, recall that the optimal mechanism that spreads the surplus equally ($\eta = 2$) corresponds to two-party negotiations. For negotiation mechanisms, it suffices that the parties mutually agree in their assessment of qualities, so that they know what the Shapley values are. These mechanisms thus do not rely on verifiability, merely on observability.

Finally, beauty contests and tournaments do not even require that the parties themselves agree on what the quality levels are. The buyer has full discretion, and she has no reason not

[6] In Proposition 3, the case $2a < b$ applies, so that $(v_1, v_2) = (1/4, 3/4)$ is optimal. In Proposition 3, the case $3a < b$ applies, so that $(v_1, v_2) = (1/3, 2/3)$ is the equilibrium.

to choose the supplier whose offer she prefers. The suppliers are aware of this when they take their decisions.

Thus, informational requirements may help to explain why buyers sometimes choose procurement mechanisms that lead to inefficient outcomes: The efficient mechanism may sometimes simply not be available.

7.1.5 Coordination and mixed strategies

The optimal mechanism and the negotiation mechanisms involve multiple asymmetric equilibria. While the asymmetry is exactly what is required from an optimal design perspective, the multiplicity is a cause for concern. It is unclear how suppliers should coordinate on these equilibria without communication. It might therefore be worth trying to understand whether these mechanisms have mixed-strategy equilibria, and what their properties are. We leave this to future research.

7.1.6 Hidden information

So far we have emphasized the moral hazard problem of inducing sellers to choose the right action. By contrast, previous literature addressed many aspects of hidden information in the procurement context. These costs are, however, particularly relevant when suppliers can choose between actions that are more or less costly (higher or lower quantity or quality). In the context of the current paper, all actions are equally costly. Of course, one could modify the framework by assuming that different sellers have different fixed costs of design and the buyer does not know these costs. This just introduces a rent for low-fixed cost types that the buyer cannot avoid if she wants to induce participation of high-cost types. There are no implications for the relative efficiency of different mechanisms.

Even more fundamentally, one might want to give up the assumption that all designs are equally costly, and sellers might differ with respect to which designs are more or less costly to them. This would clearly necessitate a framework that is much more difficult to analyze than the current one.

7.1.7 Summary

Most of the insights of our analysis are robust to various conceivable alternative assumptions. Nevertheless, one can think of at least two reasons why the optimal mechanism seems to be rarely, if ever, used in practice. First, the mechanism is not applicable when quality differences are not verifiable. Second, even where the mechanism is applicable in principle, it will be hard to coordinate on the desirable asymmetric pure-strategy equilibrium.

7.2 Effort incentives in procurement

Several authors ask how to design procurement mechanisms so as to induce maximal efforts (rather than a variety of approaches). The most comprehensive treatment is Che and Gale (2003). These authors restrict attention to the important case that quality is not verifiable, so that all allowable mechanisms must give the buyer the discretion to select the firm offering the alternative maximizing ex post surplus. The set of allowable mechanisms still includes tournaments and auctions.[7] The authors show that, in a symmetric setting, auctions induce the optimal efforts.

Che and Gale also ask how many suppliers a firm should optimally invite.[8] In their analysis, the optimal number is two.[9] Restricting the number of bidders is useful to induce sufficient investments who need to be sure that they win with a non-negligible probability. While we cannot directly deal with the optimal number of bidders in our settting, it seems clear that there are potential benefits to having more than two players when diversity considerations play a role: At least potentially, the increase in suppliers should add to the diversity of available designs. A precise analysis will be the subject of future work.

8 Conclusions

When buyers procure complex goods, they often do not know the ideal design at the time they ask sellers to invest into providing blueprints. Therefore, it is often optimal to have a diversity of designs from which the best one can be chosen ex post. We characterize the optimal ex ante design choice and, in particular, the optimal diversity of designs. We introduce a class of mechanisms which implement the socially optimal diversity of designs. Furthermore, we show that using this class of mechanisms the buyer can maximize the total surplus and simultaneously extract a payoff that approximates the total surplus.

Moreover, we analyze several commonly used mechanisms in this setting: negotiations, fixed-prize tournaments, piece rate contracts and beauty contests. We find that negotiations are more efficient than the other mechanisms, because they provide the sellers with differentiation incentives. In particular, two-party negotiations implement the socially optimal diversity of designs and thereby maximize social welfare.

Contrary to the existing literature on procurement, our paper focusses on two-sided uncertainty about the ideal design for the buyer. It would clearly be interesting to analyze how the insights would differ in a more standard setting where buyers are uncertain about the procurement costs of suppliers. We nevertheless believe that the current analysis provides a helpful ingredient in explaining the occurrence of the wide variety of different approaches to the procurement of complex goods.

References

Asker, John and Estelle Cantillon (2008), "Properties of scoring auctions." *The RAND Journal of Economics*, 39, 69–85.

Asker, John and Estelle Cantillon (2010), "Procurement when price and quality matter." *The RAND Journal of Economics*, 41, 1–34.

Bajari, Patrick and Steven Tadelis (2001), "Incentives versus transaction costs: A theory of procurement contracts." *The RAND Journal of Economics*, 387–407.

[7] Fullerton et al. (2002) also show that first-prize auctions are superior to contests, but they restric attentions to symmetric bidding strategies.

[8] Taylor (1995) and Fullerton and McAfee (1999) discuss the optimal number of bidders , restricting attention to tournaments.

[9] With verifiable quality, the optimal number of firms would be one. A suitable contract would then induce the optimal effort from this firm.

Che, Yeon-Koo (1993), "Design competition through multidimensional auctions." *The RAND Journal of Economics*, 24, 668–680.

Crickmore, Paul (2003), *Nighthawk F-117: Stealth Fighter*. Airlife Publishing Ltd.

Kaplan, Todd R. (2012), "Communication of preferences in contests for contracts." *Economic Theory*, 51, 487–503.

Lazear, Edward P and Sherwin Rosen (1981), "Rank-order tournaments as optimum labor contracts." *The Journal of Political Economy*, 89, 841–864.

Pérez-Castrillo, David and David Wettstein (2001), "Bidding for the surplus: a non-cooperative approach to the shapley value." *Journal of Economic Theory*, 100, 274–294.

Royal Institute of British Architects (RIBA) (2013), "Design competitions guidance for clients." URL `http://competitions.architecture.com/requestform.aspx`.

Tirole, Jean and Jean-Jacques Laffont (1993), *A theory of incentives in procurement and regulation*. MIT Press.

9 Appendix

9.1 Appendix 0: Notation and Preliminary Results

We first provide some definitions and simple results concerning the state of the world and the relative qualities.

Definition 17 *Suppose (v_1, v_2) are such that $v_1 \leq v_2$.*
*(i) For $\sigma \in [0,1]$, the **demand of firm 1 conditional on** σ is the value $\widetilde{\theta}(\sigma)$ of θ that is minimal with the property that there exists no $\theta < \widetilde{\theta}(\sigma)$ such that $q_2 > q_1$.*
*(ii) For $\theta \in [0,1]$, the **demand of firm 1 conditional on** θ is the value $\widetilde{\sigma}(\theta)$ of σ that is minimal with the property that there exists no $\sigma < \widetilde{\sigma}(\theta)$ such that $q_2 > q_1$.*

The terminology captures the idea that in all states of the world (σ, θ) with $\theta < \widetilde{\theta}(\sigma)$ or $\sigma < \widetilde{\sigma}(\theta)$ buyers prefer to choose firm 1 if they pay the same price in both cases.

Whenever $\widetilde{\theta}(\sigma)$ takes a value in $(0,1)$, then $q_1(\sigma,\theta) = q_2(\sigma,\theta)$. Though we suppress this in the notation, conditional demands also depend on v_1 and v_2. Note the following identities: For $\sigma \in [0, v_1]$,

$$\widetilde{\theta}(\sigma) \equiv \widetilde{\theta}_1 = \begin{cases} \frac{1}{2} + \frac{b(v_2-v_1)}{2a} & \text{if } \frac{1}{2} + \frac{b(v_2-v_1)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{b(v_2-v_1)}{2a} \geq 1 \end{cases}. \tag{10}$$

For $\sigma \in [v_1, v_2]$,

$$\widetilde{\theta}_2(\sigma) = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{b(v_2+v_1-2\sigma)}{2a} \leq 0 \\ \frac{1}{2} + \frac{b(v_2+v_1-2\sigma)}{2a} & \text{if } 0 < \frac{1}{2} + \frac{b(v_2+v_1-2\sigma)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{b(v_2+v_1-2\sigma)}{2a} \geq 1 \end{cases}, \tag{11}$$

For $\sigma \in [v_2, 1]$,

$$\widetilde{\theta}(\sigma) \equiv \widetilde{\theta}_3 = \begin{cases} 0 & \text{if } \frac{1}{2} - \frac{b(v_2-v_1)}{2a} \leq 0 \\ \frac{1}{2} - \frac{b(v_2-v_1)}{2a} & \text{if } 0 < \frac{1}{2} - \frac{b(v_2-v_1)}{2a} \end{cases}. \tag{12}$$

Moreover, whenever $\widetilde{\sigma}(0) \in (0,1)$ ($\widetilde{\sigma}(1) \in (0,1)$), then its value is given by $q_1(\sigma,\theta) = q_2(\sigma,\theta)$. Hence

$$\widetilde{\sigma}(\theta) = \frac{v_1+v_2}{2} + \frac{a}{2b}(1-2\theta) \tag{13}$$

We will frequently use the following results.

Lemma 18 *(i) If there exists $\sigma \in [0,1]$ such that $\widetilde{\theta}(\sigma) \notin (0,1)$, then $\widetilde{\theta}_1 = 1$ and $\widetilde{\theta}_3 = 0$.*
(ii) Suppose $\widetilde{\theta}_1 = 1$ and $\widetilde{\theta}_3 = 0$. Then $v_1 \leq \widetilde{\sigma}(1) < \widetilde{\sigma}(0) \leq v_2$.
(iii) There exists $\sigma \in [0,1]$ such that $\widetilde{\theta}(\sigma) \notin (0,1)$ if and only if $v_2 - v_1 \geq a/b$.

Proof. *(i)* First note that from (10) and (12) that the conditions for $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ to take boundary values are identical. Thus, if one is at the boundary, so is the other. Furthermore, $\widetilde{\theta}_2$ lies between $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$. Hence, the result follows
(ii) If $\widetilde{\theta}_1 = 1$, then $v_1 \leq \widetilde{\sigma}(1)$. If $\widetilde{\theta}_3 = 0$, then $\widetilde{\sigma}(0) \leq v_2$. $\widetilde{\sigma}(1) < \widetilde{\sigma}(0)$ follows directly from (13) for $a > 0$, $b > 0$.
(iii) This follows directly from (i) and (10) and (12). ■

Figure 3 illustrates Lemma 18(iii). For parameter values a, b such that $a/b = 0.2$, it illustrates the conditions under which $\widetilde{\theta}(\sigma) \notin (0,1)$ for some σ.[10]

[10] The figure does not restrict attention to $v_1 \leq v_2$.

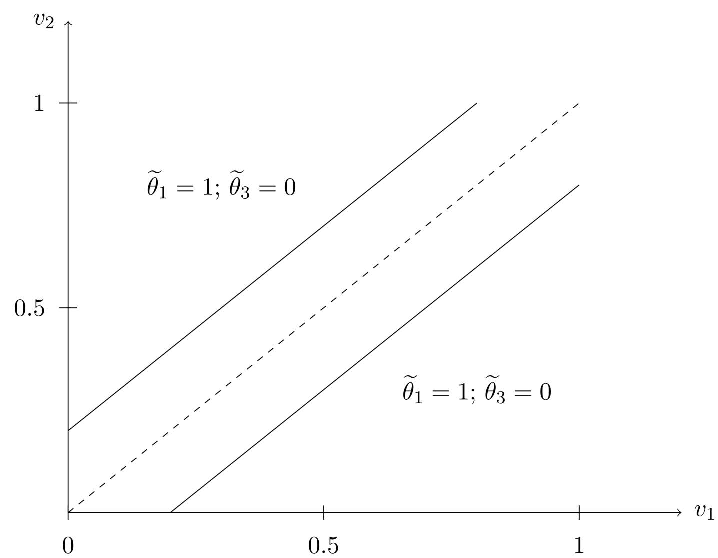

Figure 3: Interior and boundary values for conditional demand

9.2 Appendix A: Optimal Designs

9.2.1 Proof of Proposition 3

Step 1: Social welfare and its derivatives
The social planner chooses (v_1, v_2) to maximize social welfare,

$$W(v_1, v_2) = \int_0^1 \int_0^1 \max\{\Psi - a\theta - b|v_1 - \sigma|, \Psi - a(1-\theta) - b|v_2 - \sigma|\} d\theta d\sigma.$$

We will show below that for each parameter value there is a unique local maximizer of the social welfare function. By Lemma 18, the social planner can choose (v_1, v_2) so that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are both interior (Step 1.1) or so not neither $\widetilde{\theta}_1$ nor $\widetilde{\theta}_3$ is interior (Step 1.2). We will consider the two cases separately. Assume w.l.o.g. that $v_1 \leq v_2$.
Step 1.1: *Social welfare when* $\widetilde{\theta}(\sigma) \in (0,1)$ *for all* $\sigma \in [0,1]$ $(v_2 - v_1 < a/b)$
Denoting the restriction of the social welfare function to $\{(v_1, v_2) | \, v_2 - v_1 < a/b\}$ as W^1, we obtain

$$\begin{aligned}
& W^1(v_1, v_2) = \\
& \int_0^{v_1} \left(\int_0^{\hat{\theta}_1} (\Psi - a\theta - b(v_1 - \sigma))\, d\theta + \int_{\hat{\theta}_1}^1 (\Psi - a(1-\theta) - b(v_2 - \sigma))\, d\theta \right) d\sigma \\
+ \quad & \int_{v_1}^{v_2} \left(\int_0^{\hat{\theta}_2(\sigma)} (\Psi - a\theta - b(\sigma - v_1))\, d\theta + \int_{\hat{\theta}_2(\sigma)}^1 (\Psi - a(1-\theta) - b(v_2 - \sigma))\, d\theta \right) d\sigma \\
& + \int_{v_2}^1 \left(\int_0^{\hat{\theta}_3} (\Psi - a\theta - b(\sigma - v_1))\, d\theta + \int_{\hat{\theta}_3}^1 (\Psi - a(1-\theta) - b(\sigma - v_2))\, d\theta \right) d\sigma.
\end{aligned}$$

The first order conditions are:

$$\frac{\partial W^1}{\partial v_1} = b\frac{a - 2av_1 + bv_1 - bv_2 + bv_1^2 + bv_2^2 - 2bv_1v_2}{2a} = 0 \qquad (14)$$

$$\frac{\partial W^1}{\partial v_2} = b\frac{a - 2av_2 - bv_1 + bv_2 - bv_1^2 - bv_2^2 + 2bv_1v_2}{2a} = 0. \qquad (15)$$

The second order conditions hold globally.

Step 1.2: *Social welfare when* $\hat{\theta}_1 = 1$ *and* $\hat{\theta}_2 = 0$. $(v_2 - v_1 \geq a/b)$

Lemma 18(ii) implies that $v_1 \leq \widetilde{\sigma}(1) \leq \widetilde{\sigma}(0) \leq v_2$ in this case. Denoting the restriction of the social welfare function to $\{(v_1, v_2) | \, v_2 - v_1 \geq a/b\}$ as W^2, we obtain

$$\begin{aligned} & W^2(v_1, v_2) = \\ & \int_0^{v_1} \left(\int_0^1 (\Psi - a\theta - b(v_1 - \sigma)) \, d\theta \right) d\sigma + \int_{v_1}^{\widetilde{\sigma}(1)} \left(\int_0^1 (\Psi - a\theta - b(\sigma - v_1)) \, d\theta \right) d\sigma \\ + & \int_{\widetilde{\sigma}(1)}^{\widetilde{\sigma}(0)} \left(\int_0^{\hat{\theta}_2(\sigma)} (\Psi - a\theta - b(\sigma - v_1)) \, d\theta + \int_{\hat{\theta}_2(\sigma)}^1 (\Psi - a(1-\theta) - b(v_2 - \sigma)) \, d\theta \right) d\sigma \\ + & \int_{\widetilde{\sigma}(0)}^{v_2} \left(\int_0^1 (\Psi - a(1-\theta) - b(v_2 - \sigma)) \, d\theta \right) d\sigma + \int_{v_2}^1 \left(\int_0^1 (\Psi - a(1-\theta) - b(\sigma - v_2)) \, d\theta \right) d\sigma \end{aligned}$$

The first order conditions are:

$$\frac{\partial W^2}{\partial v_1} = \frac{1}{2}bv_2 - \frac{3}{2}bv_1 = 0$$

$$\frac{\partial W^2}{\partial v_2} = b + \frac{1}{2}bv_1 - \frac{3}{2}bv_2 = 0$$

The second order conditions hold globally.

Step 1.3: *Social welfare is continuous at the regime boundary* $v_2 - v_1 = a/b$

Inserting $v_2 = v_1 + a/b$ in W^1 and W^2 shows that these functions are identical at the regime boundary.

Step 2: Whenever $b < a$, $(1/2, 1/2)$ *is a global maximum.*

$(v_1, v_2) = (1/2, 1/2)$ solves the system of first order conditions (14) and (15). If $b < a$, $v_2 - v_1 < \frac{a}{b}$ holds everywhere. Thus $(1/2, 1/2)$ is a global maximum if it maximizes W^1 on the set of all (v_1, v_2) such that $v_2 \leq 1$ and $v_1 \leq v_2$. As $(1/2, 1/2)$ satisfies the FOC and W^1 is globally concave, the result follows.

Step 3: Whenever $a \leq b < 2a$, *the point* $(a/2b, 1 - a/2b)$ *is a global maximum.*

$(v_1, v_2) = (a/2b, 1 - a/2b)$ solves the system of first order conditions (14) and (15). $(a/2b, 1 - a/2b)$ satisfies $v_1 \leq v_2$ if and only if $a \leq b$. $\widetilde{\theta}_1$ and $\widetilde{\theta}_2$ are interior by Lemma 18(iii) if and only if $v_2 - v_1 < a/b$, which is equivalent to $b < 2a$. W^1 is globally concave in the entire region where $v_2 - v_1 < a/b$, so that $(a/2b, 1 - a/2b)$ is a maximum of this subset. It remains to consider (v_1, v_2) for which $v_2 - v_1 \geq a/b$ and thus welfare is given by W^2. Using the conditions derived in Step 1.2, the only candidate for an equilibrium that is in the interior of the set of all (v_1, v_2) for which $v_2 - v_1 \geq a/b$ is $(v_1, v_2) = (1/4, 3/4)$. Given the parameter restriction $b < 2a$, however, this candidate violates $v_2 - v_1 \geq a/b$. Next, it is straightforward to show that $\frac{\partial W^2}{\partial v_1} > 0$ for $v_1 = 0$ and $\frac{\partial W^2}{\partial v_2} < 0$ for $v_2 = 1$ and $v_1 < 1$. Thus,

the maximal value of $W = W^2$ on the regime where $v_2 - v_1 \geq a/b$ cannot be obtained on these boundaries. It must therefore lie on the boundary where $v_2 - v_1 = a/b$ (see Figure 3). As the social welfare function is continuous at the regime boundary and we have already seen that $W^1(v_1, v_2) < W^1(a/2b, 1 - a/2b)$ for all points with $v_2 - v_1 < a/b$, the maximal value of W in the region where $v_2 - v_1 = a/b$ is thus also below the welfare level at the maximum of W^1 in the region where $v_2 - v_1 < a/b$, the point $(a/2b, 1 - a/2b)$.
Step 4: Whenever $b \geq 2a$, the global maximum is $(1/4, 3/4)$.

The candidate optimum satisfies $v_2 - v_1 \geq a/b$ for $b \geq 2a$, so that Step 1.2 applies by Lemma 18(iii). $(1/4, 3/4)$ is the unique point where the first-order conditions for SW^2 hold. As the second-order conditions hold globally on the regime where $v_2 - v_1 \geq a/b$, $(1/4, 3/4)$ is the unique maximizer of W^2 on this regime.
Now consider the alternative of choosing designs such that $v_2 - v_1 < a/b$ and thus Step 1.1 applies. The first order conditions are (14) and (15). For $b \geq 2a$, this system has no interior solutions. Next, we show that the constrained maximization problem

$$max_{(v_1,v_2)} W^1 \; s.t. \text{ (i) } v_2 - v_1 \leq a/b; \text{ (ii) } v_1 \leq v_2 \text{ (iii) } v_1 \geq 0; \text{ (iv) } v_2 \leq 1 \tag{16}$$

has no solutions for which $v_1 = 0$ or $v_2 = 1$. For $v_1 = 0$, (14) implies that $\frac{\partial W^1}{\partial v_1} = b\frac{a - bv_2 + bv_2^2}{2a} > 0$ for $v_2 = v_2 - v_1 \leq a/b$. For $v_2 = 1$, (15) implies that $\frac{\partial W^1}{\partial v_2} = b\frac{-a + bv_1 - bv_1^2}{2a} < 0$ for $1 - v_1 = v_2 - v_1 \leq a/b$. Next consider the diagonal $v_1 = v_2$. Along this diagonal, W^1 is increasing for $v_1 < 1/2$ and decreasing for $v_1 > 1/2$. Therefore, the constrained maximum on this diagonal is obtained at $(v_1, v_2) = (1/2, 1/2)$ or at $(v_1, v_2) = (1, 1)$. Simple but tedious calculations show that $W^1(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4}\Psi - \frac{1}{4}b - \frac{1}{4}a$, whereas, in the sugested optimum, $W^2(\frac{1}{4}, \frac{3}{4}) = \frac{1}{24b}\left(2a^2 - 6ab + 3b^2 + 18\Psi b\right)$. As a result, $W^2(\frac{1}{4}, \frac{3}{4}) - W^1(\frac{1}{2}, \frac{1}{2}) = \frac{1}{24b}\left(2a^2 - 6ab + 3b^2 + 18\Psi b\right)$. This expression is positive for $b = 2a$ and increasing in b for $b = 2a$; hence, it is everywhere positive. Thus, the suggested deviation is not profitable either.

9.2.2 Proof of Proposition 4

The expected profit of seller 1 under the proposed mechanism is given by

$$E\Pi_{s1}(v_1 | v_2, \sigma) = \int_0^{\widetilde{\theta}(\sigma)} \frac{q_1(v_1, \theta | \sigma) - q_2(v_2, \theta | \sigma)}{\eta} d\theta. \tag{17}$$

Hence

$$\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2, \sigma) =$$
$$\left(\frac{q_1\left(v_1, \widetilde{\theta}(\sigma) | \sigma\right) - q_2\left(v_2, \widetilde{\theta}(\sigma) | \sigma\right)}{\eta}\right) \frac{d\widetilde{\theta}}{dv_1} + \int_0^{\widetilde{\theta}(\sigma)} \frac{d}{dv_1}\left(\frac{q_1\left(v_1, \widetilde{\theta}(\sigma) | \sigma\right) - q_2\left(v_2, \widetilde{\theta}(\sigma) | \sigma\right)}{\eta}\right) d\theta.$$

We assume as before that $v_1 \leq v_2$. By definition of $\widetilde{\theta}$, $q_1\left(v_1, \widetilde{\theta}(\sigma) | \sigma\right) = q_2\left(v_2, \widetilde{\theta}(\sigma) | \sigma\right)$ if $\widetilde{\theta}(\sigma) \in (0, 1)$. If $q_1\left(v_1, \widetilde{\theta}(\sigma) | \sigma\right) \neq q_2\left(v_2, \widetilde{\theta}(\sigma) | \sigma\right)$, then $d\widetilde{\theta}/dv_1 = 0$. Hence, for all $\widetilde{\theta}(\sigma) \in [0, 1]$,

$$\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2, \sigma) = \int_0^{\widetilde{\theta}(\sigma)} \frac{d}{dv_1}\left(\frac{q_1\left(v_1, \widetilde{\theta}(\sigma) | \sigma\right) - q_2\left(v_2, \widetilde{\theta}(\sigma) | \sigma\right)}{\eta}\right) d\theta \tag{18}$$

Recall from (10)-(12), the different forms $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ that $\widetilde{\theta}(\sigma)$ takes on each of the intervals $[0, v_1]$, $[v_1, v_2]$ and $[v_2, 1]$. Using these values and the expressions for $q_i(v_1, \theta|\sigma)$ from (1) and (2), we obtain

$$\frac{d}{dv_1}\left(\frac{q_1\left(v_1, \widetilde{\theta}(\sigma)|\sigma\right) - q_2\left(v_2, \widetilde{\theta}(\sigma)|\sigma\right)}{\eta}\right) = \begin{cases} -\frac{b}{\eta} & \text{if } \sigma \leq v_1 \\ \frac{b}{\eta} & \text{if } \sigma > v_1 \end{cases}. \tag{19}$$

The remainder of the proof will rely on four lemmas that will immediately imply that the proposed transfer rule implements the socially optimal locations $\left(v_1^{SO}, v_2^{SO}\right)$ for any $\eta > 0$. Below, we shall prove each of these lemmas in turn.

The first two lemmas restrict the possible equilibria for arbitrary parameter values.

Lemma 19 *Equilibria such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ take interior values do not exist if $b > 2a$. If $b < a$, a necessary condition for such an equilibrium is that $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$. If $a \leq b \leq 2a$, a necessary condition for such an equilibrium (with $v_1 \leq v_2$) is that $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ or $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$.*

Lemma 20 *Equilibria such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ do not all take interior values cannot exist if $b < 2a$. If $b \geq 2a$, a necessary condition for such an equilibrium (with $v_1 \leq v_2$) is that $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$.*

The next two lemmas show that all candidate equilibria identified in Lemmas 19 and 20 actually exist in the respective parameter regions.

Lemma 21 *(i) If $b \leq 2a$, $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ is an equilibrium.*
(ii) If $a \leq b \leq 2a$, $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$ is an equilibrium.

Lemma 22 *If $b \geq 2a$, $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$ is an equilibrium.*

Together, for all parameter regions, $\left(v_1^{SO}, v_2^{SO}\right)$ is among the equilibria of the mechanism. In the region $a \leq b \leq 2a$, however, the symmetric equilibrium $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ coexists with the social optimum $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$, which is why implementation is only weak.

Finally, for $\eta \to \infty$ we have $t_i \to 0$. Hence, since the total surplus is maximized and the sellers obtain zero payoffs, the expected profit of the buyer is maximized as well.

Thus the result will follow once we have proved Lemmas 19-22. We now prove all four lemmas in turn.

Proof of Lemma 19:

Step 1: In this step, we show that

$$\frac{d}{dv_1}E\Pi_{s1}(v_1|\, v_2) = \frac{1}{2a}\frac{b}{\eta}\left(a - 2av_1 + bv_1 - bv_2 + bv_1^2 + bv_2^2 - 2bv_1v_2\right) \tag{20}$$

$$\frac{d}{dv_2}E\Pi_{s2}(v_2|\, v_1) = -\frac{1}{2a}\frac{b}{\eta}\left(2av_2 - a + bv_1 - bv_2 + bv_1^2 + bv_2^2 - 2bv_1v_2\right) \tag{21}$$

To show this, we insert (19) in (18). Assuming that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ take interior values,

$$\frac{dE\Pi_{s1}(v_1|\, v_2, \sigma)}{dv_1} = \begin{cases} -\int_0^{\widetilde{\theta}_1}\frac{b}{\eta}d\theta = -\frac{1}{2a}\frac{b}{\eta}(a - bv_1 + bv_2) & \text{for} \quad \sigma < v_1 \\ \int_0^{\widetilde{\theta}_2}\frac{b}{\eta}d\theta = \frac{1}{2a}\frac{b}{\eta}(a - 2b\sigma + bv_1 + bv_2) & \text{for} \quad v_1 < \sigma < v_2 \\ \int_0^{\widetilde{\theta}_3}\frac{b}{\eta}d\theta = \frac{1}{2a}\frac{b}{\eta}(a + bv_1 - bv_2) & \text{for} \quad v_2 < \sigma \end{cases} \tag{22}$$

(20) follows from inserting (22) in

$$\frac{d}{dv_1} E\Pi_{s1}\left(v_1 | v_2, \sigma\right) =$$

$$\int_0^{v_1} \frac{dE\Pi_{s1}\left(v_1 | v_2, \sigma\right)}{dv_1} d\sigma + \int_{v_1}^{v_2} \frac{dE\Pi_{s1}\left(v_1 | v_2, \sigma\right)}{dv_1} d\sigma + \int_{v_2}^{1} \frac{dE\Pi_{s1}\left(v_1 | v_2, \sigma\right)}{dv_1} d\sigma$$

(21) follows by analogous reasoning.

Step 2: Any equilibrium for which $v_1, v_2 \in (0,1)$, $v_1 \leq v_2$ *and* $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ *and* $\widetilde{\theta}_3$ *take interior values must satisfy*

$$v_1 = v_2 = \frac{1}{2}$$

or

$$v_1 = \frac{a}{2b} \text{ and } v_2 = 1 - \frac{a}{2b}.$$

This follows from setting (20) and (21) equal to zero.

Step 3: Equilibria such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ *and* $\widetilde{\theta}_3$ *all take interior values cannot exist if* $b > 2a$.

The second-order condition for $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ can only be fulfilled if $b < 2a$. As to $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$, Lemma 18(iii) implies that the corresponding values of $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ can only be interior if $b < 2a$.
It remains to be shown that for $b > 2a$ there are no equilibria where the corresponding values of $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are interior and $v_1 = 0$ or $v_2 = 1$. By symmetry, it suffices to consider $v_1 = 0$. Using (21), $v_2 = 1$ can only be a best response if $a \leq 0$, violating the Assumption 1. From (21), an interior solution for v_2 requires $2av_2 - a - bv_2 + bv_2^2 = 0$. For $b > 2a$, this equation has the solution $v_2^- = \frac{1}{2b}\left(-2a + b + \sqrt{4a^2 + b^2}\right) \in (0,1)$. However, for this value of v_2, the first-order condition for player 1 at $v_1 = 0$ is violated.

Step 4: If $b < a$, *a necessary condition for an equilibrium such that* $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ *and* $\widetilde{\theta}_3$ *all take interior values is that* $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

It suffices to show that there is no equilibrium such that $v_1 \leq v_2$. As argued in Step 2, $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$ are the only interior solutions of the relevant system of first-order conditions. For $b < a$, $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$ does not satisfy $v_1 \leq v_2$. It suffices to show there is no equilibrium with $v_1 = 0$. $(v_1, v_2) = (0, 1)$ is ruled out as in Step 3. For an equilibrium with $v_1 = 0$ and $v_2 < 1$, we require from (20) and (21) that

$$a - bv_2 + bv_2^2 \leq 0 \tag{23}$$

$$a + (b - 2a)\, v_2 + bv_2^2 = 0 \tag{24}$$

Solving (24) for bv_2^2 and inserting in (23), we obtain $v_2(2a - 2b + bv_2) < 0$. For $b < a$, this is impossible.

Step 5: If $a \leq b \leq 2a$, *a necessary condition for an equilibrium with* $v_1 \leq v_2$ *such that* $\hat{\theta}_1$, $\hat{\theta}_2(\sigma)$ *and* $\hat{\theta}_3$ *all take interior values is that* $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ *or* $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$

As argued in Step 2, $(v_1, v_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $(v_1, v_2) = \left(\frac{a}{2b}, 1 - \frac{a}{2b}\right)$ are the only interior solution of the relevant system of first-order conditions. We need to show that for $a \leq b \leq 2a$, there is no equilibrium with $v_1 = 0$. $(v_1, v_2) = (0, 1)$ is ruled out as in Step 3. For an equilibrium with $v_2 < 1$, we require from from (20) and (21) that (23) and (24) hold. Solving (24) for bv_2^2 and inserting in (23), we obtain $a \leq b$, a contradiction.

Proof of Lemma 20

Step 1: In any equilibrium such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ do not all take interior values, we have $\widetilde{\theta}_1 = 1$ and $\widetilde{\theta}_3 = 0$.

This is a restatement of Lemma 18(i).

Step 2: We show that

$$\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2) = \frac{1}{2}\frac{b}{\eta}(v_2 - 3v_1). \tag{25}$$

$$\frac{d}{dv_2} E\Pi_{s2}(v_2 | v_1) = \frac{1}{2}\frac{b}{\eta}(v_1 - 3v_2 + 2). \tag{26}$$

To see this first note that Lemma 18(ii) implies $v_1 \leq \widetilde{\sigma}(1) < \widetilde{\sigma}(0) \leq v_2$. We thus distinguish between four subcases ($\sigma \leq v_1$, $v_1 \leq \sigma \leq \widetilde{\sigma}(1)$,$\widetilde{\sigma}(1) \leq \sigma \leq \widetilde{\sigma}(0)$, $\widetilde{\sigma}(0) \leq \sigma$). Using (19), (18) simplifies as follows in each case.

$$\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2, \sigma) = \begin{cases} -\int_0^{\widetilde{\theta}_1} \frac{b}{\eta} d\theta = -\int_0^1 \frac{b}{\eta} d\theta = -\frac{b}{\eta} & \text{for} \quad \sigma < v_1 \\ \int_0^{\widetilde{\theta}_2} \frac{b}{\eta} d\theta == \int_0^1 \frac{b}{\eta} d\theta = \frac{b}{\eta} & \text{for} \quad v_1 < \sigma < \widetilde{\sigma}(1) \\ \int_0^{\widetilde{\theta}_2} \frac{b}{\eta} d\theta = \frac{1}{2a}\frac{b}{\eta}(a - 2b\sigma + bv_1 + bv_2) & \text{for} \quad \widetilde{\sigma}(1) < \sigma < \widetilde{\sigma}(0) \\ 0 & \text{for} \quad \widetilde{\sigma}(0) < \sigma \end{cases}$$

Using this result, we obtain

$$\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2) = \int_0^1 \frac{d}{dv_1}(E\Pi_{s1}(v_1 | t_1, v_2, \sigma))\, d\sigma =$$

$$\int_0^{v_1} -\frac{b}{\eta} d\sigma + \int_{v_1}^{\sigma(1)} \frac{b}{\eta} d\sigma + \int_{\sigma(1)}^{\sigma(0)} \left(\frac{1}{2a}\frac{b}{\eta}(a - 2b\sigma + bv_1 + bv_2)\right) d\sigma = \frac{1}{2}\frac{b}{\eta}(v_2 - 3v_1).$$

The reasoning for $\frac{d}{dv_2} E\Pi_{s2}(v_2 | v_1)$ is analogous.

Step 3: In any equilibrium such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ do not all take interior values and v_1 and v_2 are both interior, $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$.

By Step 1, $\widetilde{\theta}_1 = 1$ *and* $\widetilde{\theta}_3 = 0$ in such an equilibrium. The system of first-order conditions corresponding to (25) and (26) has the unique solution $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$.

Step 4: If $b \geq 2a$, no strategy profile except $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$ can be a pure-strategy equilibrium with $v_1 \leq v_2$ such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ do not all take interior values.

By Step 3, no other strategy profile solves the first-order conditions. Now consider boundary solutions. W.l.o.g., consider only the case that $v_1 \leq v_2 = 1$. By (25) and (26), an equilibrium with $v_1 \in (0,1)$ and $v_2 = 1$ would require $v_1 = 1/3$ and $\frac{d}{dv_2} E\Pi_{s2}(v_2 | v_1) \geq 0$. Using (26), however, $\frac{d}{dv_2} E\Pi_{s2}(v_2 | v_1) = -\frac{1}{3}\frac{b}{\eta} < 0$ for $(v_1, v_2) = \left(\frac{1}{3}, 1\right)$, a contradiction. An equilibrium $(v_1, v_2) = (0, 1)$ cannot exist either, as, by (25), $\frac{d}{dv_1} E\Pi_{s1}(v_1 | v_2) > 0$ in this case.

Step 5: If $b < 2a$, there exists no pure-strategy equilibrium such that $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ do not all take interior values.

By Lemma 18(iii), such an equilibrium would require $v_2 - v_1 \geq a/b$. As $b < 2a$, this requirement implies $v_2 - v_1 > 1/2$, which does not hold for the only solution of the first-order conditions $(v_1, v_2) = \left(\frac{1}{4}, \frac{3}{4}\right)$. Hence, there can be no interior equilibrium of the desired type. As to boundary solutions, assume w.l.o.g. that $v_1 = 0$. By (25), such an equilibrium would require $v_2 - 3v_1 = v_2 \leq 0$. For $v_2 > 0$, this is a contradiction. For $v_2 = 0$, the additional requirement from (26) that $\frac{1}{2}\frac{b}{\eta}(v_1 - 3v_2 + 2) \leq 0$ is violated.

Proof of Lemma 21

(i) Suppose $b \leq 2a$. For $(v_1, v_2) = (1/2, 1/2)$, $\widetilde{\theta}_1$, $\widetilde{\theta}_2(\sigma)$ and $\widetilde{\theta}_3$ all take interior values according to Lemma 18(iii). According to Step 2 in the proof of Lemma 19, this strategy profile satisfies the first-order condition. Moreover, for any deviation of player 1 such that $v_1 < v_2$ and $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are interior, (20) implies that the second-order condition is $b - 2a + 2b(v_1 - v_2) < 0$, which holds for $b < 2a$. Thus, $v_1 = 1/2$ is a best response to $v_2 = 1/2$ among those $v_1 < v_2$ satisfying $v_1 > 1/2 - a/b$. We also need to consider deviations of player 1 for which $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are not interior. For this to happen, $v_1 \leq 1/2 - a/b$ is necessary according to Lemma 18(iii). As $b < 2a$, $1/2 - a/b < 0$. Thus, there is no feasible deviation such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are not interior. Therefore, $(v_1, v_2) = (1/2, 1/2)$ is an equilibrium.

(ii) Suppose $a \leq b \leq 2a$. Consider $(v_1, v_2) = (a/2b, 1 - a/2b)$. Arguing as in (i), $v_1 = a/2b$ is a best response to $v_2 = 1 - a/2b$ on $\left\{v_1 \in [0, v_2] \,\middle|\, v_1 > \frac{1}{2} - a/b\right\}$. We also need to consider deviations such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are not interior. First note that by Lemma 18(iii), we require $v_1 \leq v_2 - a/b = (2b - 3a)/2b$ for such a deviation, which is only possible if $2b \geq 3a$. Moreover, from (25), $\frac{d}{dv_1}E\Pi_{s1}(v_1|\, v_2) = b(v_2 - 3v_1)/2\eta$. For v_1 on $[0, (2b - 3a)/2b]$, this expression is minimal at $v_1 = (2b - 3a)/2b$, where $\frac{d}{dv_1}E\Pi_{s1}(v_1|\, v_2) = (2a - b)/\eta > 0$ for $b < 2a$. Thus, the optimal deviation of the type considered is to $\widehat{v}_1 = (2b - 3a)/2b$, the boundary to the regime where all $\theta(\sigma)$ are interior. We already know from the above considerations that there is no profitable deviation from $a/2b$ into this regime.

Finally, for any deviation $\hat{v}_1 > v_2$, let $\check{v}_1 = 2v_2 - \hat{v}_1$. By symmetry,

$$\int_{a/b}^{1} E\Pi_{s1}(\sigma|v_1 = \hat{v}_1, v_2 = 3/4) = \int_{a/b}^{1} E\Pi_{s1}(\sigma|v_1 = \check{v}_1, v_2 = 3/4)$$

but

$$\int_{0}^{a/b} E\Pi_{s1}(\sigma|v_1 = \hat{v}_1, v_2 = 3/4) < \int_{0}^{a/b} E\Pi_{s1}(\sigma|v_1 = \check{v}_1, v_2 = 3/4).$$

Hence seller 1 prefers $\check{v}_1$ to $\hat{v}_1$, but $\check{v}_1$ is not a profitable deviation because $\hat{v}_1 < v_2$.

Proof of Lemma 22

We have to show that, for $b \geq 2a$ and $v_2 = 3/4$ seller 1 cannot profitably deviate from $v_1 = 1/4$. By Lemma 18(iii), $b \geq 2a$ implies that $\widetilde{\theta}_1 = 1$ and $\widetilde{\theta}_3 = 0$ for $(v_1, v_2) = (1/4, 3/4)$. We need to consider three types of deviations. By Lemma 18(i), deviations to $\hat{v}_1 \leq v_2$ can be only such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ both remain at the boundary (Case 1) or that they both are interior (Case 2). Finally, there can be deviations such that $\hat{v}_1 > v_2$ (Case 3).

Case 1: $\hat{v}_1 \leq v_2$ and $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are at the boundary.

Using (25), $\frac{d}{dv_1}E\Pi_{s1}(v_1|\, v_2) = 0$ for $(v_1, v_2) = (1/4, 3/4)$ and $\frac{d^2}{dv_1^2}E\Pi_{s1}(v_1|\, v_2) = -\frac{3}{2}\frac{b}{\eta} < 0$. Thus $v_1 = \frac{1}{4}$ is the best response to $v_2 = \frac{3}{4}$.

Case 2: $\hat{v}_1 \leq v_2$ and $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are interior.

It is immediate from Lemma 18(iii) and $b > 2a$ that it is necessary for $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ to be interior after a deviation from the suggested equilibrium that $\hat{v}_1 \in [1/4, 3/4]$. We will next show that $E\Pi_{s1}(\hat{v}_1 | t_1, v_2 = 3/4)$ does not have an interior local maximum on the interval $(1/4, 3/4)$. By (20), an optimal deviation into the interior of $(1/4, 3/4)$ such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are interior must satisfy

$$\frac{d}{dv_1} E\Pi_{s1}(\hat{v}_1 | t_1, v_2 = 3/4) = -\frac{1}{2a}\frac{b}{\eta}\left(\frac{3}{16}b - a + 2av_1 + \frac{1}{2}bv_1 - bv_1^2\right) = 0. \tag{27}$$

This has the two solutions

$$\hat{v}_1^{+/-} = \frac{4a + b \pm 2\sqrt{-2ab + 4a^2 + b^2}}{4b}.$$

Taking the second derivative reveals that only $\hat{v}_1^-$ could represent a local maximum:

$$\begin{aligned}
\left.\frac{d^2}{dv_1^2} E\Pi_{s1}(\hat{v}_1 | v_2 = 3/4)\right|_{\hat{v}_1 = \hat{v}_1^-} &= -\frac{1}{2a}\frac{b}{\eta}\sqrt{4a^2 - 2ab + b^2} < 0 \\
\left.\frac{d^2}{dv_1^2} E\Pi_{s1}(\hat{v}_1 | v_2 = 3/4)\right|_{\hat{v}_1 = \hat{v}_1^+} &= \frac{1}{2a}\frac{b}{\eta}\sqrt{4a^2 - 2ab + b^2} > 0.
\end{aligned}$$

However, $\hat{v}_1^- \leq \frac{1}{4}$ whenever $b \geq 2a$. Thus, there exists no local maximum of the deviation profit in the interior of $(1/4, 3/4)$. Using (27), one can show that $\frac{d}{dv_1} E\Pi_{s1}(\hat{v}_1 | t_1, v_2 = 3/4) > 0$ for $\hat{v}_1 = 3/4$ and thus on the entire interval $(1/4, 3/4)$. Together, we have shown that there is no profitable deviation for player 1 such that $\hat{v}_1 \in [1/4, 3/4]$.

Case 3: $\hat{v}_1 > v_2$.

For any such $\hat{v}_1$, let $\check{v}_1 = 2v_2 - \hat{v}_1$. By symmetry,

$$\int_{1/2}^{1} E\Pi_{s1}(\sigma | v_1 = \hat{v}_1, v_2 = 3/4) = \int_{1/2}^{1} E\Pi_{s1}(\sigma | v_1 = \check{v}_1, v_2 = 3/4)$$

but

$$\int_{0}^{1/2} E\Pi_{s1}(\sigma | v_1 = \hat{v}_1, v_2 = 3/4) < \int_{0}^{1/2} E\Pi_{s1}(\sigma | v_1 = \check{v}_1, v_2 = 3/4).$$

Hence seller 1 prefers $\check{v}_1$ to $\hat{v}_1$. From cases 1 and 2, even $\check{v}_1$ cannot be a profitable deviation.

9.3 Appendix B: Equilibrium Structures in Commonly Used Mechanisms

9.3.1 Proof of Corollary 9

This bargaining situation constitutes a cooperative game (N, v) where $N = \{b, s\}$ for buyer and seller and $v(b, s) = \overline{q}$, $v(b) = \underline{q}$ and $v(s) = 0$, where $\overline{q}$ is the quality provided by the higher quality design and $\underline{q}$ is the quality provided by the lower quality design. The Shapley values of the buyer and the seller are given by

$$\begin{aligned}
\phi_b(N) &= \frac{\overline{q} + \underline{q}}{2} \\
\phi_s(N) &= \frac{\overline{q} - \underline{q}}{2}.
\end{aligned}$$

The payoff of the seller who offers lower quality design is zero. Clearly, for $\eta = 2$ by Proposition 4 this mechanism weakly implements the socially optimal varieties.

9.3.2 Proof of Proposition 10

Recall from (3) that

$$E\Pi_{s1}\left(v_1|\, v_2\right) = \int\limits_0^1 \left(\int_0^{\widetilde{\theta}(\sigma)} \left(\frac{q_1^h\left(\sigma,\theta\right)}{2} - \frac{q_2^l\left(\sigma,\theta\right)}{3} \right) d\theta + \int_{\widetilde{\theta}(\sigma)}^1 \frac{q_1^l\left(\sigma,\theta\right)}{6} d\theta \right) d\sigma.$$

Moreover, from equations 1 and 2,

$$\begin{aligned} q_1^h &= q_1^l = q_1\left(\sigma,\theta\right) = \Psi - a\theta - b\left|v_1 - \sigma\right| \\ q_2^h &= q_2 = \Psi - a\left(1-\theta\right) - b\left|v_2 - \sigma\right| \end{aligned}$$

Step 1: We show that

$$\frac{d}{dv_1} E\Pi_{s1}\left(v_1|\, v_2,\sigma\right) = \tag{28}$$
$$\int_0^{\widetilde{\theta}(\sigma)} \frac{d}{dv_1} \left(\frac{q_1^h\left(\sigma,\theta\right)}{2} - \frac{q_2^l\left(\sigma,\theta\right)}{3} \right) d\theta + \int_{\widetilde{\theta}(\sigma)}^1 \frac{d}{dv_1} \left(\frac{q_1^l\left(\sigma,\theta\right)}{6} \right) d\theta$$

To see this, first note that (3) implies

$$\frac{d}{dv_1} E\Pi_{s1}\left(v_1|\, v_2,\sigma\right) = \tag{29}$$
$$\left(\frac{q_1^h\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{2} - \frac{q_2^l\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{3} \right) \frac{d\widetilde{\theta}}{dv_1} - \left(\frac{q_1^l\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{6} \right) \frac{d\widetilde{\theta}}{dv_1} +$$
$$\int_0^{\widetilde{\theta}(\sigma)} \frac{d}{dv_1} \left(\frac{q_1^h\left(\sigma,\theta\right)}{2} - \frac{q_2^l\left(\sigma,\theta\right)}{3} \right) d\theta + \int_{\widetilde{\theta}(\sigma)}^1 \frac{d}{dv_1} \left(\frac{q_1^l\left(\sigma,\theta\right)}{6} \right) d\theta$$

Simple derivations show that

$$\left(\frac{q_1^h\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{2} - \frac{q_2^l\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{3} \right) - \left(\frac{q_1^l\left(\sigma,\widetilde{\theta}\left(\sigma\right)\right)}{6} \right) = 0 \tag{30}$$

Inserting (30) into (29) gives (28).

Step 2:

$$\frac{d}{dv_1} \left(\frac{q_1^h\left(\sigma,\theta\right)}{2} - \frac{q_2^l\left(\sigma,\theta\right)}{3} \right) = \begin{cases} -b/2 & \text{if} \quad \sigma \le v_1 \\ b/2 & \text{if} \quad \sigma > v_1 \end{cases} \tag{31}$$

and

$$\frac{d}{dv_1} \left(\frac{q_1^l\left(\sigma,\theta\right)}{6} \right) = \begin{cases} -b/6 & \text{if} \quad \sigma \le v_1 \\ b/6 & \text{if} \quad \sigma > v_1 \end{cases} . \tag{32}$$

This follows from simple derivations.

Step 3: If $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ take interior values,

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\, v_2\right) = \frac{b}{3}\left(1-2v_1\right) + \frac{b^2\left(v_2-v_1\right)}{6a}\left(v_2-v_1-1\right) \tag{33}$$

$$\frac{d}{dv_2}E\Pi_{s2}\left(v_2|\, v_1\right) = \frac{b}{3}\left(1-2v_2\right) + \frac{b^2\left(v_2-v_1\right)}{6a}\left(v_1-v_2+1\right) \tag{34}$$

To see this, we distinguish between three subcases ($\sigma \leq v_1$, $v_1 \leq \sigma \leq v_2$, $v_2 \leq \sigma$). Using (31) and (32), we can write (28) as follows in each case:

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\, v_2,\sigma\right) = \begin{cases} -\int_0^{\widetilde{\theta}_1}\frac{b}{2}d\theta - \int_{\widetilde{\theta}_1}^1\frac{b}{6}d\theta = -\frac{b}{3} - \frac{b^2(v_2-v_1)}{6a} & \text{for} \quad \sigma < v_1 \\ \int_0^{\widetilde{\theta}_2}\frac{b}{2}d\theta + \int_{\widetilde{\theta}_2}^1\frac{b}{6}d\theta = \frac{b}{3} + \frac{1}{6a}b^2\left(v_1-2\sigma+v_2\right) & \text{for} \quad v_1 < \sigma < v_2 \\ \int_0^{\widetilde{\theta}_3}\frac{b}{2}d\theta + \int_{\widetilde{\theta}_3}^1\frac{b}{6}d\theta = \frac{b}{3} - \frac{b^2(v_2-v_1)}{6a} & \text{for} \quad v_2 < \sigma \end{cases}$$

Thus,

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\, v_2\right) = \int_0^1 \frac{d}{dv_1}\left(E\Pi_{s1}\left(v_1|\, v_2,\sigma\right)\right)d\sigma =$$

$$\int_0^{v_1}\left(-\frac{b}{3} - \frac{b^2\left(v_2-v_1\right)}{6a}\right)d\sigma + \int_{v_1}^{v_2}\left(\frac{b}{3} + \frac{1}{6a}b^2\left(v_1-2\sigma+v_2\right)\right)d\sigma + \int_{v_2}^1\left(\frac{b}{3} - \frac{b^2\left(v_2-v_1\right)}{6a}\right)d\sigma =$$

$$\frac{b}{3}\left(1-2v_1\right) + \frac{b^2\left(v_2-v_1\right)}{6a}\left(v_2-v_1-1\right).$$

The argument for player 2 is analogous.

Step 4: When $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ or $\widetilde{\theta}_3$ do not all take interior values, .

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\, v_2\right) = \frac{1}{6}b\left(v_2-5v_1+1\right). \tag{35}$$

$$\frac{d}{dv_2}E\Pi_{s2}\left(v_2|\, v_1\right) = \frac{1}{6}b\left(v_1-5v_2+3\right). \tag{36}$$

To see this, first note that Lemma 18(i) implies that $\widetilde{\theta}_1 = 1$ and $\widetilde{\theta}_3 = 0$**.** To simplify (28) in this case further, we distinguish between four subcases ($\sigma < v_1$, $v_1 < \sigma < \sigma(1)$, $\sigma(1) < \sigma < \sigma(0)$, $\sigma(0) < \sigma$). Using (31) and (32), (28) simplifies as follows in each case.

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\, v_2,\sigma\right) = \begin{cases} -\int_0^1\frac{b}{2}d\theta = -\frac{b}{2} & \text{for} \quad \sigma < v_1 \\ \int_0^1\frac{b}{2}d\theta = \frac{b}{2} & \text{for} \quad v_1 < \sigma < \sigma(1) \\ \int_0^{\widetilde{\theta}_2}\frac{b}{2}d\theta + \int_{\widetilde{\theta}_2}^1\frac{b}{6}d\theta = \frac{b}{3} + \frac{1}{6a}b^2\left(v_1-2\sigma+v_2\right) & \text{for} \quad \sigma(1) < \sigma < \sigma(0) \\ \int_0^1\frac{b}{6}d\theta = \frac{b}{6} & \text{for} \quad \sigma(0) < \sigma \end{cases}$$

This immediately implies

$$\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\,v_2\right) = \int\limits_0^1 \frac{d}{dv_1}\left(E\Pi_{s1}\left(v_1|\,v_2,\sigma\right)\right)d\sigma =$$

$$\int\limits_0^{v_1}\frac{-b}{2}d\sigma + \int\limits_{v_1}^{\sigma(1)}\frac{b}{2}d\sigma + \int\limits_{\sigma(1)}^{\sigma(0)}\left(\frac{b}{3}+\frac{1}{6a}b^2\left(v_1-2\sigma+v_2\right)\right)d\sigma + \int\limits_{\sigma(0)}^{1}\frac{b}{6}d\sigma =$$

$$\frac{1}{6}b\left(v_2-5v_1+1\right).$$

The argument for player 2 is analogous.

Step 5: (i) If $b < 4a$, there are no profitable deviations of either player from $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$ to strategies for which $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are all interior. (ii) If $2a < b < 3a$, there are no profitable deviations of either player from $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$ to strategies for which $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are all interior.

The system of first order conditions resulting from (33) and (34) has the solutions $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$ and $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$. For $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$, $\widetilde{\theta}_1 = \widetilde{\theta}_2 = \widetilde{\theta}_3 = 1/2$. For $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$, by Lemma 18(iii), we require that $b < 3a$ for $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ to be interior. Finally, the second-order conditions are $b < 4a$ for $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$ and $b > 2a$ for $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$. Thus,(i) and (ii) hold.

Step 6: If $b \leq 4a$, there are no profitable deviations from $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$ to strategies for which $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are not all interior. If $2a < b < 3a$, there are no profitable deviations from $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$ to strategies for which $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are not all interior.

(i) Consider $(v_1, v_2) = \left(\frac{1}{2},\frac{1}{2}\right)$. By Lemma 18(iii), a deviation of player 1 to a strategy for which $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are not all interior requires $v_1 \leq \frac{1}{2} - \frac{a}{b.}$. For $b < 2a$, such a deviation is not feasible. If $b \geq 2a$, then a deviation to $v_1 \leq \frac{1}{2} - \frac{a}{b.}$ is possible. By Step 4, $\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\,v_2\right) = \frac{1}{6}b\left(v_2-5v_1+1\right)$ when $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are not all interior. As $v_1 \leq \frac{1}{2} - \frac{a}{b.}$, this implies $\frac{d}{dv_1}E\Pi_{s1}\left(v_1|\,v_2\right) > \left(10a-3b+2bv_2\right)/12$. As $2a \leq b \leq 4a$ and $v_2 = \frac{1}{2}$, this expression is non-negative. Therefore, the best feasible deviation of this type is to the regime boundary $v_1 = \frac{1}{2} - \frac{a}{b.}$. As the payoffs are continuous at the regime boundary, this deviation is therefore not profitable by Step 5.

(ii) The argument for $(v_1, v_2) = \left(\frac{a}{b}, 1-\frac{a}{b}\right)$ is analogous.

Step 7: If $b \geq 3a$, there are there are no profitable deviations from $(v_1, v_2) = \left(\frac{1}{3},\frac{2}{3}\right)$.

The system of first-order conditions implied by Step 4 has the unique solution $(v_1, v_2) = \left(\frac{1}{3},\frac{2}{3}\right)$. By Lemma 18(iii), the critical point will result in $\widetilde{\theta}_1 = 1$, and $\widetilde{\theta}_3 = 0$ if and only if $b \geq 3a$. It suffices to show that seller 1 cannot profitably deviate from $v_1 = 1/3$. In Step 7.1-7.3, we show that all conceivable types of deviations are non-profitable.

Step 7.1: There are no profitable deviations of player 1 to $\hat{v}_1 \leq v_2$ such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are at the boundary.

The second order condition is $\frac{d^2}{dv_1^2}E\Pi_{s1}\left(v_1|\,v_2\right) = -\frac{5}{6} < 0$ and hence the critical point represents a maximum in the subset of (v_1, v_2) for which $v_1 < v_2$ and $\widehat{\theta}_1$, $\widehat{\theta}_2$ *and* $\widehat{\theta}_3$ are not all interior.

Step 7.2: There are no profitable deviations of player 1 to $\hat{v}_1 \leq v_2$ such that $\widetilde{\theta}_1$ and $\widetilde{\theta}_3$ are interior.

By Lemma 18(iii), such a deviation requires $\hat{v}_1 \in (1/3, 2/3]$. By (33),

$$\frac{d}{dv_1} E\Pi_{s1}\left(\hat{v}_1 \middle| v_2 = 2/3\right) = \frac{b}{3}\left(1 - 2\hat{v}_1\right) + \frac{b^2\left(\frac{2}{3} - \hat{v}_1\right)}{6a}\left(\frac{2}{3} - \hat{v}_1 - 1\right) = 0. \tag{37}$$

This equation has two solutions

$$\hat{v}_1^{+/-} = \frac{12a + b \pm \sqrt{3}\sqrt{48a^2 + 3b^2 - 16ab}}{6b}.$$

Taking the second derivative reveals that only $\hat{v}_1^-$ could represent a local maximum:

$$\begin{aligned} \frac{d^2}{dv_1^2} E\Pi_{s1}\left(\hat{v}_1 \middle| v_2 = 2/3\right)\Bigg|_{\hat{v}_1 = \hat{v}_1^-} &= -\frac{\sqrt{3}b\sqrt{48a^2 - 16ab + 3b^2}}{18a} < 0 \\ \frac{d^2}{dv_1^2} E\Pi_{s1}\left(\hat{v}_1 \middle| v_2 = 2/3\right)\Bigg|_{\hat{v}_1 = \hat{v}_1^+} &= \frac{\sqrt{3}b\sqrt{48a^2 - 16ab + 3b^2}}{18a} > 0. \end{aligned}$$

However the requirement that $\hat{v}_1^- > \frac{1}{3}$ is equivalent with $12a - b - \sqrt{(12a - b)^2 + 8b(b - 3a)} > 0$, which is inconsistent with $b \geq 3a$. Thus, there exists no local maximum of the deviation profit on $(1/3, 2/3)$.

Next we show that the expected profit is higher if $\hat{v}_1 = 1/3$ than if $\hat{v}_1 = 2/3$. By (32) we have

$$E\Pi_{s1}\left(v_1 = 1/3 \middle| v_2 = 2/3\right) = \frac{6b\Psi - 3ab + a^2}{36b}.$$

On the other hand

$$\begin{aligned} & E\Pi_{s1}\left(v_1 = v_2 = 2/3\right) = \\ = & \int_0^1 \left(\int_0^{\frac{1}{2}} \left(\frac{q_1^h\left(\theta, \sigma \middle| \sigma \leq \frac{1}{2}\right)}{2} - \frac{q_2^l\left(\theta, \sigma \middle| \sigma \leq \frac{1}{2}\right)}{3} \right) d\theta + \int_{\frac{1}{2}}^1 \frac{q_1^l\left(\theta, \sigma \middle| \sigma \leq \frac{1}{2}\right)}{6} d\theta \right) d\sigma \\ = & \frac{1}{6}\Psi - \frac{5}{108} b. \end{aligned}$$

Thus $E\Pi_{s1}\left(v_1 = 1/3 \middle| v_2 = 2/3\right) - E\Pi_{s1}\left(v_1 = v_2 = 2/3\right) \geq 0$ if and only if $\frac{3a^2 + (5b - 9a)b}{108b} \geq 0$. The latter condition is implied by $b \geq 3a$.

Step 7.3: *There can be no profitable deviation of player* 1 *to* $\hat{v}_1 > v_2$.

For any such $\hat{v}_1$, we can construct $\check{v}_1 = 2v_2 - \hat{v}_1$. By symmetry,

$$\int_{1/3}^1 E\Pi_{s1}\left(\sigma | v_1 = \hat{v}_1, v_2 = 2/3\right) = \int_{1/3}^1 E\Pi_{s1}\left(\sigma | v_1 = \check{v}_1, v_2 = 2/3\right)$$

but

$$\int_0^{1/3} E\Pi_{s1}\left(\sigma | v_1 = \hat{v}_1, v_2 = 2/3\right) < \int_0^{1/3} E\Pi_{s1}\left(\sigma | v_1 = \check{v}_1, v_2 = 2/3\right).$$

Hence seller 1 prefers $\check{v}_1$ to $\hat{v}_1$, but we know from steps 7.1 and 7.2 that this cannot be a profitable deviation.

Step 8: There can be no other interior equilibrium of the game for arbitrary parameter values.
An interior equilibrium must either satisfy the first-order conditions corresponding to Step 3 or those corresponding to Step 4. These first-order conditions have no other solutions than those already mentioned.

Step 9: There can be no equilibrium such that $\exists i \in \{1,2\}$ with $v_i \in \{0,1\}$.

By symmetry, we only need to exclude the existence of equilibria with $v_1 = 0$. Using (35), when $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ are not all interior, then $dE\Pi_{s1}(v_1|\, v_2)/dv_1 \leq 0$ does not hold for $v_1 = 0$ and any $v_2 \geq 0$. Thus, it only remains to consider the case that $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ and $\widetilde{\theta}_3$ take interior values. (33) rules out an equilibrium with $v_1 = 0$ and $v_2 \in \{0,1\}$. From (33) and (34), an equilibrium with $v_1 = 0$ and $v_2 \in (0,1)$ would require

$$\frac{d}{dv_1}E\Pi_{s1}(v_1|\, v_2) = \frac{b}{3} + \frac{b^2 v_2}{6a}(v_2 - 1) \leq 0 \tag{38}$$

$$\frac{d}{dv_2}E\Pi_{s2}(v_2|\, v_1) = \frac{b}{3}(1 - 2v_2) + \frac{b^2 v_2}{6a}(1 - v_2) = 0 \tag{39}$$

Solving (39) for $\frac{b^2 v_2}{6a}(1 - v_2)$ and inserting in (38) gives $\frac{b}{3}(2 - 2v_2) \leq 0$, violating $v_2 > 0$. Thus, there is no equilibrium with $v_1 = 0$.

9.3.3 Proof of Proposition 11

For the proposed mechanism, the payoff of seller i does not depend on the actions of the seller j. Hence, the seller i solves $\max_{v_i} E[t_i(q_i)\,|v_i]$. This function clearly has a maximum. By contraposition, suppose that the maximizer $\hat{v}_i \neq 1/2$. Then, either $\hat{v}_i < 1/2$ or $\hat{v}_i > 1/2$. Suppose $\hat{v}_i < 1/2$. Let $\delta = (1 - 2\hat{v}_i)/2$. We can rewrite the expected profit from $\hat{v}_i$ as:

$$E[t_i(q_i)\,|\hat{v}_i] = \int_0^{2\hat{v}_i+\delta}\int_0^1 t_i(Q_i(\hat{v}_i, i; \sigma, \theta))\, d\theta d\sigma + \int_{2\hat{v}_i+\delta}^1\int_0^1 t_i(Q_i(\hat{v}_i, i; \sigma, \theta))\, d\theta d\sigma.$$

The profit from $v_i = 1/2$ can be written as

$$E[t_i(q_i)\,|1/2] = \int_0^{\delta}\int_0^1 t_i(Q_i(1/2, i; \sigma, \theta))\, d\theta d\sigma + \int_{\delta}^1\int_0^1 t_i(Q_i(1/2, i; \sigma, \theta))\, d\theta d\sigma.$$

Define $f : [0, 2\hat{v}_i+\delta]\times[0,1] \to [\delta, 1]\times[0,1]$ as $f(\sigma,\theta) = (\sigma+\delta, \theta)$. Clearly, f is a bijection. Next, observe that for every $(\sigma,\theta) \in [0, 2\hat{v}_i+\delta]\times[0,1]$ we have $Q_i(\hat{v}_i, i; \sigma, \theta) = \Psi - a|\theta - (i-1)| - b|\sigma - \hat{v}_i|$. On the other hand $Q_i(1/2, i; f(\sigma,\theta)) = \Psi - a|\theta - (i-1)| - b|\sigma + \delta - 1/2|$. But since by construction we have $\hat{v}_i = 1/2 - \delta$, it follows that $Q_i(1/2, i; f(\sigma,\theta)) = Q_i(\hat{v}_i, i; \sigma, \theta)$. Thus it follows that

$$\int_0^{2\hat{v}_i+\delta}\int_0^1 t_i(Q_i(\hat{v}_i, i; \sigma, \theta))\, d\theta d\sigma = \int_{\delta}^1\int_0^1 t_i(Q_i(1/2, i; \sigma, \theta))\, d\theta d\sigma$$

since the integrands and boundaries are translations.

Next, define $g : [2\hat{v}_i + \delta, 1]\times[0,1] \to [0,\delta]\times[0,1]$ as $g(\sigma,\theta) = (1-\sigma, \theta)$. Clearly, g is a bijection as well. Next, observe that for every $(\sigma,\theta) \in [2\hat{v}_i+\delta, 1]\times[0,1]$ we have $Q_i(\hat{v}_i, i; \sigma, \theta) =$

$\Psi - a|\theta-(i-1)| - b|\sigma-\hat{v}_i|$. On the other hand $Q_i\left(1/2, i; g(\sigma,\theta)\right) = \Psi - a|\theta-(i-1)| - b|1-\sigma-1/2|$. Since $\sigma > 1/2 = \hat{v}_i + \delta$ it follows that $|\sigma - \hat{v}_i| > |\sigma - 1/2|$. Thus it follows that

$$\int_{2\hat{v}_i+\delta}^{1} \int_0^1 t_i\left(Q_i\left(\hat{v}_i, i; \sigma, \theta\right)\right) d\theta d\sigma < \int_0^{\delta} \int_0^1 t_i\left(Q_i\left(1/2, i; \sigma, \theta\right)\right) d\theta d\sigma$$

since $t\left(\cdot\right)$ is strictly increasing.

Combining the two expressions yields $E[t_i\left(q_i\right)|\hat{v}_i] < E[t_i\left(q_i\right)|1/2]$. Thus $\hat{v}_i$ is not the maximizer. The case when $\hat{v}_i > 1/2$ follows by analogous argument.

9.3.4 Proof of Proposition 12

Part 1: Preliminary Remarks We first collect several definitions and results that help us to identify the candidate equilibrium. We suppose w.l.o.g. that $v_1 \leq v_2$.

Definition 23 *(i) In a beauty contest, for $\sigma \in [0,1]$, the* ***demand of firm 1 conditional on*** *σ is the value $\widehat{\theta}\left(\sigma\right)$ of θ that is minimal with the property that there exists no $\theta < \widehat{\theta}\left(\sigma\right)$ such that $q_2 > q_1$.*
(ii) In a beauty contest, for $\theta \in [0,1]$, the ***demand of firm 1 conditional on*** *θ is the value $\widehat{\sigma}\left(\theta\right)$ of σ that is minimal with the property that there exists no $\sigma < \widehat{\sigma}\left(\theta\right)$ such that $q_2 > q_1$.*

We suppress the dependence of conditional demands on designs and prices. Then demand conditional on σ is given as follows:

If $\sigma \in [0, v_1]$,

$$\widehat{\theta}\left(\sigma\right) \equiv \widehat{\theta}_1 = \begin{cases} 0 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2-v_1)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2-v_1)}{2a} & \text{if} \quad 0 < \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2-v_1)}{2a} < 1 \\ 1 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2-v_1)}{2a} \geq 1 \end{cases}. \tag{40}$$

If $\sigma \in [v_1, v_2]$,

$$\widehat{\theta}\left(\sigma\right) \equiv \widehat{\theta}_2\left(\sigma\right) = \begin{cases} 0 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2+v_1-2\sigma)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2+v_1-2\sigma)}{2a} & \text{if} \quad 0 < \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2+v_1-2\sigma)}{2a} < 1 \\ 1 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} + \frac{b(v_2+v_1-2\sigma)}{2a} \geq 1 \end{cases}. \tag{41}$$

If $\sigma \in [v_2, 1]$,

$$\widehat{\theta}\left(\sigma\right) \equiv \widehat{\theta}_3 = \begin{cases} 0 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} - \frac{b(v_2-v_1)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2-p_1}{2a} - \frac{b(v_2-v_1)}{2a} & \text{if} \quad 0 < \frac{1}{2} + \frac{p_2-p_1}{2a} - \frac{b(v_2-v_1)}{2a} < 1 \\ 1 & \text{if} \quad \frac{1}{2} + \frac{p_2-p_1}{2a} - \frac{b(v_2-v_1)}{2a} \geq 1 \end{cases}. \tag{42}$$

Moreover, note that, when $\widehat{\sigma}\left(\theta\right) \in (0,1)$, it can be obtained from $q_1\left(\sigma,\theta\right) - p_1 = q_2\left(\sigma,\theta\right) - p_2$. Thus, for the candidate equilibrium choice of player 1 $(v_1, p_1) = (1/2, a)$,

$$\widehat{\sigma}\left(\theta\right) = \frac{a\left(1-2\theta\right) + p_2 - p_1 + b\left(v_2 + v_1\right)}{2b} \tag{43}$$

We can use (40)-(42) to obtain the probability that the design of supplier 1 is chosen by the buyer as

$$P(p_1, v_1, p_2, v_2) = \int_0^{v_1} \widehat{\theta}_1 d\sigma + \int_{v_1}^{v_2} \widehat{\theta}_2(\sigma) d\sigma + \int_{v_2}^{1} \widehat{\theta}_3 d\sigma. \tag{44}$$

If $\widehat{\theta}_l \in (0,1)$ for all $l = 1,2,3$, therefore, (40)-(42) imply

$$P(p_1, v_1, p_2, v_2) = \frac{a + p_2 - p_1 + b(v_1 - v_2) + b\left(v_2^2 - v_1^2\right)}{2a}. \tag{45}$$

Part 2: The candidate equilibrium It is straightforward to rule out to show that $(v_i, p_i) = (1/2, a)$ for $i = 1,2$ is the only strategy profile for which $\widehat{\theta}_1$, $\widehat{\theta}_2$ and $\widehat{\theta}_3$ take interior values, and first- and second-order conditions hold. From (40)-(42), $\widehat{\theta}_1$, $\widehat{\theta}_2$ and $\widehat{\theta}_3$ take interior values ($\widehat{\theta}_1 = \widehat{\theta}_2 = \widehat{\theta}_3 = 1/2$) for this equilibrium candidate. Using $\Pi_{s1} = p_1 P(p_1, v_1, p_2, v_2)$ and $\Pi_{s2} = p_2(1 - P(p_1, v_1, p_2, v_2))$, (45) implies

$$\frac{dE\Pi_{si}}{dv_i} = \frac{b - 2bv_i}{2a} p_i.$$

Thus, for any $p_i > 0$, $dE\Pi_{si}/dv_i = 0$ holds if and only if $v_i = 1/2$. Also using (45), it follows that, for $i = 1,2$, $j \neq i$,

$$\frac{dE\Pi_{si}}{dp_i} = \frac{a - 2p_i + p_j + bv_i - bv_i^2 - bv_j + bv_j^2}{2a}.$$

Substituting $v_i = v_j = 1/2$ into this expression gives the first-order conditions

$$\frac{dE\Pi_{si}}{dp_i} = a - 2p_i + p_j = 0. \tag{46}$$

These two conditions can only be satisfied simultaneously if $p_1 = p_2 = a$. The second-order conditions clearly hold for $p_1 = p_2 = a$ and $v_1 = v_2 = 1/2$. Finally, there can be no equilibrium where either v_1 or v_2 takes boundary values. To see this, it suffices to note that $v_1 = 0$ cannot arise in equilibrium because $dE\Pi_{si}/dv_i = bp_i/2a > 0$.

Part 3: No profitable deviations In the following, we show there are no profitable deviations. By symmetry, it suffices to consider deviations of player 2 such that $v_2 \geq 1/2$.

We first note that, for $v_1 = 1/2$ and $p_1 = a$, (40)-(42) imply the following relation between the deviation choices v_2 and p_2 and the values of $\widehat{\theta}_1$ and $\widehat{\theta}_3$.

BC1 $\widehat{\theta}_1 = 0$ if and only if $p_2 \leq -bv_2 + b/2$.

BC2 $\widehat{\theta}_1 = 1$ if and only if $p_2 \geq -bv_2 + 2a + b/2$.

BC3 $\widehat{\theta}_3 = 0$ if and only if $p_2 \leq bv_2 - b/2$.

BC4 $\widehat{\theta}_3 = 1$ if and only if $p_2 \geq bv_2 + 2a - b/2$

The following general observations are helpful:

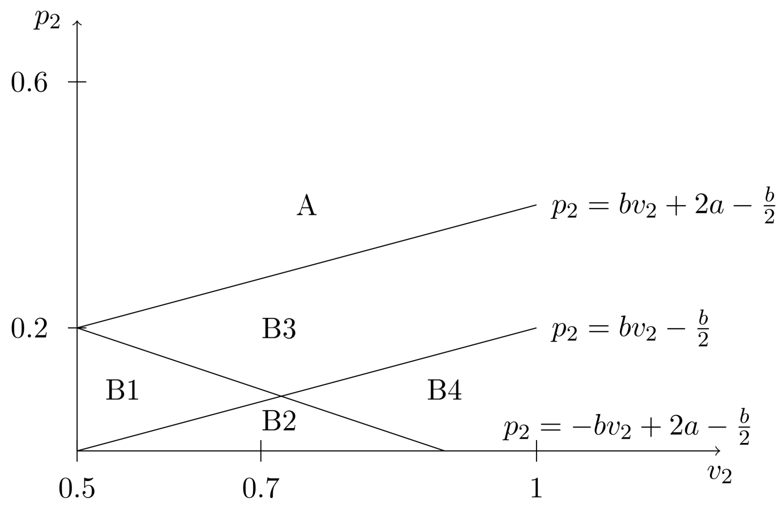

Figure 4: Deviation regions in the beauty contest

BC5 At $v_2 = 0.5$, $bv_2 + 2a - b/2 = -bv_2 + 2a + b/2$ and $bv_2 - b/2 = -bv_2 + b/2 = 0$

BC6 At $v_2 = 0.5 + a/b$, $-bv_2 + 2a + b/2 = bv_2 - b/2$

BC7 For $v_2 > 0.5$, $p_2 > -bv_2 + b/2$ (so that $\widehat{\theta}_1 > 0$) and $bv_2 + 2a - b/2 > -bv_2 + 2a + b/2$

BC8 For $v_2 > 0.5$, $bv_2 + 2a - b/2 > \max\{-bv_2 + b/2, -bv_2 + 2a + b/2, bv_2 - b/2\}$

As a result of (BC7), there is no feasible deviation with $v_2 \geq v_1 = 1/2$ and $\widehat{\theta}_1 = 0$. Also, by (BC7) $\widehat{\theta}_1 \in (0,1)$ implies that $\widehat{\theta}_3 = 0$ or $\widehat{\theta}_3 \in (0,1)$. $\widehat{\theta}_1 = 1$ is compatible with $\widehat{\theta}_3 = 0$, $\widehat{\theta}_3 \in (0,1)$ and $\widehat{\theta}_3 = 1$. The set of conceivable deviations can thus be divided into five parameter regions which correspond to the possible combinations of $\widehat{\theta}_1 = 0$, $\widehat{\theta}_1 \in (0,1)$, $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$, $\widehat{\theta}_3 \in (0,1)$, $\widehat{\theta}_3 = 1$. Figure 4 shows these regions for $a = 0.1$ and $b = 0.5$.

Note, however, from (BC6) that Region B4 disappears if $2a \geq b$.

We distinguish between those deviations where $\widehat{\theta}_3 = 1$ and those where $\widehat{\theta}_3 < 1$.

Definition 24 *(i) A type-A deviation satisfies* $p_2 \geq bv_2 + 2a - b/2$.
(ii) Type B-deviations satisfy $p_2 < bv_2 + 2a - b/2$

Type A deviations are obviously non-profitable.

Lemma 25 *There exist no profitable type-A deviations*

Proof. For a Type-A deviation, $\hat{\theta}_3 = 1$ and thus, by BC 7, $\hat{\theta}_1 = 1$. Thus the probability that seller 2 wins is equal to 0, hence the expected profits are 0. Thus this cannot be a profitable deviation. ■

We have to analyze Type B-deviations more carefully. In Table 1 (and Figure 4), we distinguish between four subregions:

	$p_2 \leq -bv_2 + 2a + b/2$	$p_2 > -bv_2 + 2a + b/2$
$p_2 \geq bv_2 - b/2$	Region B1	Region B3
$p_2 < bv_2 - b/2$	Region B2	Region B4

Table 1: Type-B regions in the beauty contest

We now consider Type-B deviations into each of the regions 1-4. As an important preliminary remark, note that the deviation profits are continuous at the boundary between the different regions.

Lemma 26 *There exist no profitable deviations into region B1.*

Proof. In the interior of B1, $\widehat{\theta}_l \in (0,1)$ for all $l = 1,2,3$ by (40)-(42). Thus, we can apply (45). There exists no critical point of the deviation profit in the interior of B1. For $p_j = a$, $v_j = 1/2$, the solution to the first-order conditions (46) for p_i is a for arbitrary values of $v_i > 0$ and the solution for v_i is $1/2$ for $p_i = a$. Moreover, the second-order conditions hold globally. Thus, deviation profits are globally concave in region B1 and they are maximized at the candidate equilibrium. Therefore, there are no profitable deviations into region B1. ■

Lemma 27 *There exist no profitable type-B deviations into region B2.*

Proof. In region B2, $\hat{\theta}_3 = 0$. Thus, the probability that seller 2 wins is given by:

$$\begin{aligned} 1 - P(p_1, v_1, p_2, v_2) &= \int_0^{v_1} \left(1 - \hat{\theta}_1\right) d\sigma + \int_{v_1}^{\widehat{\sigma}(0)} \left(1 - \hat{\theta}_2(\sigma)\right) d\sigma + (1 - \widehat{\sigma}(0)) \\ &= \frac{-4b^2 v_2^2 - 4bp_2 - 4p_2^2 - 4b^2 v_2 + 32ab + 3b^2 - 8bp_2 v_2}{32ab}. \end{aligned}$$

and the expected profit of seller 2 from the deviation is

$$E\Pi_{s_2}(p_1, v_1, p_2, v_2) = p_2 \frac{-4b^2 v_2^2 - 4bp_2 - 4p_2^2 - 4b^2 v_2 + 32ab + 3b^2 - 8bp_2 v_2}{32ab}.$$

Therefore

$$\begin{aligned} \frac{d}{dp_2} E\Pi_{s_2} &= \frac{-4b^2 v_2^2 - 8bp_2 - 12p_2^2 - 4b^2 v_2 + 32ab + 3b^2 - 16bp_2 v_2}{32ab} \\ \frac{d}{dv_2} E\Pi_{s_2} &= -\frac{p_2 (b + 2p_2 + 2bv_2)}{8a} < 0. \end{aligned}$$

Thus there will exist no maximum in the interior of region B2, and v_2 is strictly increasing towards the left boundary, which is the boundary to region B1. We have already seen that there are no profitable deviations into region B1 and, in particular, no deviations to its boundary with B2. By continuity of the deviation profits, therefore, there are no profitable deviations to B2. ■

Lemma 28 *There exist no profitable deviations of firm 2 into region B3.*

Proof. We first consider the interior of region B3. We shall prove the following two statements:
(i) *Deviation profits have a local maximum in the interior of region B3 if and only if* $b \in ((32/10)a, 5a)$.
(ii) *A deviation to this local maximum from the proposed equilibrium is never profitable.*
(i) The derivatives of the deviation profits of firm 2 are

$$\frac{dE\Pi_{s_2}}{dp_2} = \frac{16a^2 + 24ab - 7b^2 + 8p_2\left(2bv_2 - 3b - 4a\right) + 12p_2^2 + v_2\left(20b^2 - 16ab\right) - 12b^2v_2^2}{32ab} \tag{47}$$

$$\frac{dE\Pi_{s_2}}{dv_2} = \frac{p_2\left(-4a + 5b + 2p_2 - 6bv_2\right)}{8a} \tag{48}$$

There are four solutions to the system of first order conditions:

$$\begin{aligned} p_2 &= 0, v_2 = \frac{1}{2b}\left(-4a + b\right) \\ p_2 &= \frac{4a+b}{6}, v_2 = \frac{8b-4a}{9b} \\ p_2 &= 2a + \frac{b}{2}, v_2 = 1 \\ p_2 &= 0, v_2 = \frac{1}{6b}\left(4a + 7b\right) \end{aligned}$$

For $p_2 = 0$, expected profits are zero, hence the deviation is not be profitable. Of the remaining two solutions, only $(\overline{v}_2, \overline{p}_2) = \left(\frac{8b-4a}{9b}, \frac{4a+b}{6}\right)$ satisfies the second-order conditions and is thus a candidate for a local maximum in the interior of Region B3. It remains to be shown that it satisfies the four conditions (a) $\overline{v}_2 \leq 1$, (b) $\overline{p}_2 < b\overline{v}_2 + 2a - b/2$, (c) $\overline{p}_2 > b\overline{v}_2 - b/2$ and (d) $\overline{p}_2 > -b\overline{v}_2 + 2a + b/2$.
Conditions (a) and (b) hold without any parameter restrictions (except $a > 0$ and $b > 0$). (c) holds if and only if $b < 5a$; (d) holds if and only if $b > \frac{16}{5}a$.
(ii) The expected profits in the locally optimal deviation described in (i) are $(4a + b)^3/324ab$ as opposed to $a/2$ for the equilibrium choices p_2^*, v_2^*. Hence, the deviation is not profitable if and only if

$$E\Pi_{s_2}\left(p_2^*, v_2^*; p_1^*, v_1^*\right) - E\Pi_{s_2}\left(\bar{p}_2, \bar{v}_2; p_1^*, v_1^*\right) = -\frac{64a^3 - 114a^2b + 12ab^2 + b^3}{324ab} \geq 0$$

We show that the function $g(b) = 64a^3 - 114a^2b + 12ab^2 + b^3$ is negative for $b \in [\frac{32}{10}a, 5a]$, for any value of $a > 0$. First, $g''(b) = 24a + 6b > 0$. Hence the function is convex. Second, $g(\frac{32}{10}a) < 0$ and $g(5a) < 0$, thus it is negative for all values of $b \in [\frac{32}{10}a, 5a]$ and consequently there are no profitable deviations for these values of b.
We thus have proven statements (i) and (ii). It remains to be shown that there are no profitable deviations to the two boundaries of region B3 belonging to the set.
There is no profitable deviation into the subregion of B3 where $p_2 = bv_2 - b/2$
This is the boundary between B3 and region B4. Simple calculations show that this boundary is non-degenerate if and only if $2a \leq b$.[11] It consists of all points $\left(\frac{1}{2} + \frac{p_2}{b}, p_2\right)$ for which

[11] Recall from (BC6) that for $2a > b$, the intersection point between the lines defining the lower boundary of region B3 is no longer in the interval $[0, 1]$.

$p_2 \in [a, b/2]$. The first-order condition for the maximal profit on this line is

$$\frac{d}{dp_2} E\Pi_{s_2}\left(p_2, v_2 = \frac{1}{2} + \frac{p_2}{b}; p_1, v_1\right) = \frac{a + b - 4p_2}{2b}.$$

For $b \in [2a, 3a]$ the expected profit on the boundary is maximized by setting $p_2 = a$, while for $b > 3a$ it is maximized by the unique solution to the first order condition: $p_2 = (a + b)/4$. Next, we compare the expected profits. For $b \in [2a, 3a]$, the maximum deviation profit is given by

$$E\Pi_{s_2}\left(p_2 = a, v_2 = \frac{1}{2} + \frac{a}{b}; p_1, v_1\right) = \frac{a}{2}\frac{b - a}{b}.$$

Since $(b - a)/b < 1$, the expected deviation profit is never larger than the equilibrium profit $a/2$. For $b > 3a$ the maximum deviation profit is given by

$$E\Pi_{s_2}\left(\bar{p}_2 = \frac{a + b}{4}, \bar{v}_2 = \frac{a + 3b}{4b}; p_1, v_1\right) = \frac{(a + b)^2}{16b}$$

and the difference between equilibrium and deviation profits is

$$E\Pi_{s_2}(p_2^*, v_2^*; p_1^*, v_1^*) - E\Pi_{s_2}(\bar{p}_2, \bar{v}_2; p_1^*, v_1^*) = \frac{6ab - a^2 - b^2}{16b}.$$

Thus, the deviation profits are larger than the equilibrium profits if and only if $b > a\left(2\sqrt{2} + 3\right)$, which is never true by Assumption 1.

There is no profitable deviation of player 2 for which $v_2 = 1$.

Simple calculations show that (v_2, p_2) is in Region B3 for $v_2 = 1$ if and only if $p_2 \in [b/2, 2a + b/2]$. Evaluating $\frac{d}{dp_2} E\Pi_{s_2}$ at $v_2 = 1$ yields the first-order condition

$$16a^2 + 8ab - 32ap_2 + b^2 - 8bp_2 + 12p_2^2 = 0.$$

This equation has two solutions $p_2 = 2a + b/2$ and $p_2 = 2a/3 + b/6$. The first point is a local minimum. Thus consider $p_2 = 2a/3 + b/6$. It is in the interval $[b/2, 2a + b/2]$ only if $b/2 \leq a$. Using (48), $\frac{d}{dv_2} E\Pi_{s_2} < 0$ for $(v_2, p_2) = (1, 2a/3 + b/6)$. Hence, this point cannot be an optimal deviation either. If $b/2 > a$, the maximum expected profit in the intersection of region B3 and the line $v_2 = 1$ is achieved at $(v_2, p_2) = (1, b/2)$. The expected profit is

$$E\Pi_{s_2}(\bar{p}_2, \bar{v}_2; p_1, v_1) = \frac{a}{4} < \frac{a}{2}.$$

Thus this is never a profitable deviation. ■

Lemma 29 *There exist no profitable deviation of player 2 into region B4.*

Proof. In this region, $\hat{\theta}_3 = 0$ and $\hat{\theta}_1 = 1$. The probability that seller 2 wins is thus given by

$$1 - P(p_1, v_1, p_2, v_2) = \int_{\sigma(1)}^{\sigma(0)} \left(1 - \hat{\theta}_2(\sigma)\right) d\sigma + (1 - \sigma(0)) = \frac{2a + 3b - 2p_2 - 2bv_2}{4b}.$$

and the expected profit of seller 2 from the deviation is

$$E\Pi_{s_2}(p_1, v_1, p_2, v_2) = p_2 \frac{2a + 3b - 2p_2 - 2bv_2}{4b}.$$

As $\frac{d}{dv_2} E\Pi_{s_2} < 0$, there can be no point in Region B4 which yields higher deviation profits than the best deviation in the remaining part of Region B. ■

Part 4: Uniqueness We have already seen in Part 2 that the candidate equilibrium is the only equilibrium for which the $\widehat{\theta}_l$ are interior for $l = 1, 2, 3$. It remains to be shown that there are no equilibria where not all $\widehat{\theta}_i$ are interior. We distinguish between four cases.

Step 1: No equilibria where $\widehat{\theta}_1 = 1$ *and* $\widehat{\theta}_3 = 0$

Using (43), we obtain

$$\widehat{\sigma}(0) = \frac{b(v_1 + v_2) + p_2 - p_1 + a}{2b} \quad (49)$$

$$\widehat{\sigma}(1) = \frac{b(v_1 + v_2) + p_2 - p_1 - a}{2b}. \quad (50)$$

The expected payoff of player 1 is thus $\pi_1 = p_1 \left(\widehat{\sigma}(0) + (\widehat{\sigma}(1) - \widehat{\sigma}(0))/2\right)$ and therefore

$$\pi_1 = p_1 \frac{b(v_1 + v_2) + p_2 - p_1}{2b}.$$

This immediately shows that $\partial \pi_1 / \partial v_1 > 0$, ruling out all equilibria with $v_1 < v_2$ where $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$ before and after the increase. By (40) and (42), a strategy profile with $v_1 = v_2$ cannot satisfy $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$.

Now consider a strategy profile such that $\widehat{\theta}_3 = 0$, but $\widehat{\theta}_3$ becomes interior after a marginal increase of v_1, while $\widehat{\theta}_1 = 1$. After the marginal increase, the winning probability of player 2 in a situation where $\widehat{\theta}_3 \in (0, 1)$ and $\widehat{\theta}_1 = 1$ is $P_2 = \left(1 - \widehat{\theta}_3\right)\left((1 - v_2) + (v_2 - \widehat{\sigma}(1))/2\right)$. Using (43) and (50),

$$\widehat{\theta}_3 = \frac{a + p_2 - p_1 - b(v_2 - v_1)}{2a} \quad (51)$$

$$v_2 - \widehat{\sigma}(1) = \frac{b(v_2 - v_1) - p_2 + p_1 + a}{2b} \quad (52)$$

we immediately obtain

$$P_2 = \left(1 - \frac{a + p_2 - p_1 - b(v_2 - v_1)}{2a}\right)\left(1 - v_2 + \left(\frac{b(v_2 - v_1) - p_2 + p_1 + a}{4b}\right)\right)$$

Thus,

$$\pi_1 = p_1 \left(1 - \left(1 - \frac{a + p_2 - p_1 - b(v_2 - v_1)}{2a}\right)\left(1 - v_2 + \frac{b(v_2 - v_1) - p_2 + p_1 + a}{4b}\right)\right)$$

and

$$\frac{\partial \pi_1}{\partial v_1} = \frac{p_1}{4a}(a + 2b + p_1 - p_2 - bv_1 - bv_2).$$

Using (51), $\widehat{\theta}_3 = 0$ implies $p_1 - p_2 = a - b(v_2 - v_1)$. Hence $\frac{\partial \pi_1}{\partial v_1} = \frac{p_1}{2a}(a + b - bv_2) > 0$. Thus, there is no equilibrium such that $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$ and $\widehat{\theta}_3 = 0$ is interior after a marginal increase of v_1.

Step 2: No equilibria where $\widehat{\theta}_1 \in (0, 1)$ *and* $\widehat{\theta}_3 = 0$

The winning probability of player 1 in a situation where $\widehat{\theta}_1 \in (0, 1)$ *and* $\widehat{\theta}_3 = 0$ is $P_1 = \widehat{\theta}_1 v_1 + (\widehat{\sigma}(0) - v_1)/2$. From (49),

$$\widehat{\sigma}(0) - v_1 = \frac{a + p_2 - p_1 + b(v_2 - v_1)}{2b}.$$

From this and (40), we can calculate player 2's profits as

$$\pi_2 = p_2 (1 - P_1) = \left[1 - (a + p_2 - p_1 + b(v_2 - v_1)) \left(\frac{v_1}{2a} + \frac{1}{4b}\right)\right]$$

and thus

$$\frac{\partial \pi_2}{\partial v_2} = -bp_2 \left(\frac{v_1}{2a} + \frac{1}{4b}\right) < 0$$

Hence, there can be neither an interior best response of player 2 nor a best response at $v_2 = 1$. Thus, there is no equilibrium of the type described.

Step 3: No equilibria where $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 \in (0,1)$

This is the same situation as in Step 2, with the roles of firms 1 and 2 reversed. Thus, the same arguments show that there can be no such equilibrium.

Step 4: No equilibria where $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 1$ (or with $\widehat{\theta}_1 = 0$ and $\widehat{\theta}_3 = 0$)

In such an equilibrium, one firm i earns a positive profit, whereas the other firm (firm j) does not. Firm j can always deviate and imitate firm i. As a result, both firms will share the profit, so that the deviation is profitable-

Part 5: Summary *Step 8: The strategy profile given by (5) and (6) is a symmetric equilibrium. No other interior symmetric equilibrium exists.*
This follows from steps 5-7.

9.4 Appendix C: Payoff Comparison

In this part of the Appendix, we first provide the equilibrium payoffs for the different mechanisms. Then we compare the payoffs for the different mechanisms. The payoff calculations for negotiations are tedious; we have compiled them in a technical appendix.

9.4.1 Equilibrium Payoffs

Beauty Contests

Lemma 30 *Expected payoffs in the symmetric equilibrium with $(v^*, p^*) = (1/2, a)$ of the beauty contest are given as follows:*
(i) Expected payoffs of each supplier are $\frac{a}{2}$.
(ii) Expected buyer payoffs are $\Psi - \frac{5}{4}a - \frac{1}{4}b$.
(iii) Expected total payoffs are $\Psi - \frac{1}{4}a - \frac{1}{4}b$.

Proof. (i) is obvious; (ii) follows from the fact that expected buyer payoffs are $\Psi - \frac{1}{4}a - 2b\left(\int_{\frac{1}{2}}^{1} \left(\sigma - \frac{1}{2}\right) d\sigma\right) - a$. (iii) immediately follows from (i) and (ii). ■

Three-party negotiations We now state three lemmas concerning payoffs with three-way negotiations. The calculations are tedious, and they are delegated to a technical appendix, which is available on request.

Lemma 31 *Total payoffs in the symmetric three-party negotiations equilibrium* $v = \frac{1}{2}$ *are*

$$\Psi - \frac{1}{4}a - \frac{1}{4}b.$$

Buyer payoffs are

$$\frac{2}{3}\Psi - \frac{1}{4}a - \frac{1}{6}b$$

Seller payoffs are

$$\frac{1}{6}\Psi - \frac{1}{24}b.$$

Lemma 32 *If* $b \geq 3a$, *a seller's profits in an asymmetric negotiations equilibrium are*

$$\frac{1}{36}\frac{6b\Psi - 3ab + a^2}{b}.$$

The buyer's profit is

$$\frac{1}{36}\frac{24b\Psi - 12ab + a^2 - 5b^2}{b}.$$

Total profits are

$$\frac{1}{36}\frac{36b\Psi - 18ab + 3a^2 - 5b^2}{b}.$$

Lemma 33 *If* $2a \leq b \leq 3a$, *then a seller's profit in the asymmetric negotiations equilibria is*

$$\frac{1}{36}\frac{-3ab^2 - 6a^2b + 10a^3 + b^3 + 6ab\Psi}{ab}$$

The buyer's profit in the asymmetric negotiations equilibria is

$$-\frac{1}{36}\frac{12ab^2 - 3a^2b + 8a^3 - b^3 - 24ab\Psi}{ab}.$$

Total profits are

$$\frac{1}{12}\frac{-6ab^2 - 3a^2b + 4a^3 + b^3 + 12ab\Psi}{ab}$$

9.4.2 Comparisons

Preliminary Remarks The values of Ψ and a determine which of the regimes R1-R4 can occur.

Lemma 34 *Suppose A2 holds.*
(i) For $2.5a > \Psi$, *only R1 arises.*
(ii) $2.5a < \Psi < 3a$, *R1 and R2 arise.*
(iii) For $3a < \Psi < 4a$, *R1-R3 arise.*
For $\Psi > 4a$, *R1-R4 arise.*

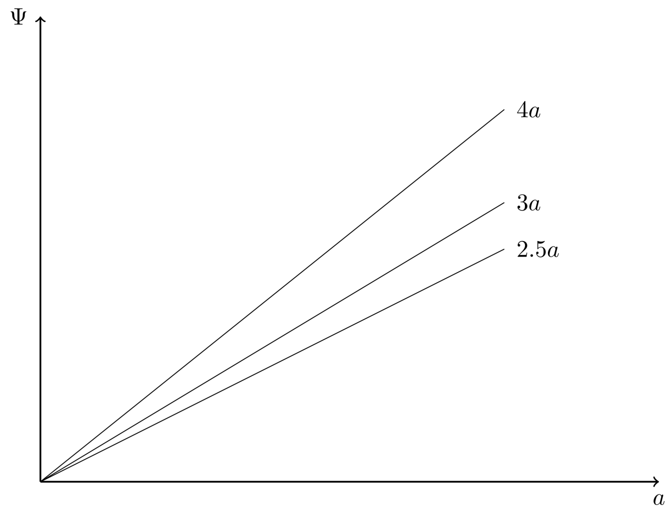

Figure 5: Equlibrium regimes for negotiations

Proof. (i) Suppose $2.5a > \Psi$. In any regime apart from R1, $b > 2a$. Hence, the requirement from Assumption 2 that $\Psi \geq \frac{b+3a}{2}$ is inconsistent with $\Psi < 2.5a$.
(ii)-(iv) similar. ■

The four regions in (a, Ψ)-space are depicted in Figure 5

Proof of Corollary 15 The buyer obtains higher payoffs in the beauty contest than in the symmetric negotiations equilibrium if and only if $4\Psi - 12a < b$. If $\Psi \leq 3a$, this is always true. If $\Psi < 3a$, it holds whenever b is large enough. The condition $4\Psi - 12a < b$ is consistent with $b < \Psi$ (Assumption 2) whenever $b \in (4\Psi - 12a, \Psi)$, which is not empty if $3a < \Psi < 4a$. For $\Psi > 4a$ the condition $4\Psi - 12a < b$ is is inconsistent with $b \leq 4a$. Thus, when a symmetric pure-strategy equilibrium exists by Proposition 10 for $\Psi > 4a$, then payoffs in the symmetric negotiations equilibrium are higher.

Buyer Payoffs To prove Propositions 14 and 16, the following lemma is useful.

Lemma 35 *Whenever symmetric and asymmetric equilibria coexist, the buyer will prefer the asymmetric equilibrium.*

Proof. First suppose $2a \leq b \leq 3a$. Using Lemmas 31 and 33, the payoffs for the asymmetric equilibrium are higher if and only if $-\frac{1}{36}\frac{12ab^2-3a^2b+8a^3-b^3-24ab\Psi}{ab} - \left(\frac{2}{3}\Psi - \frac{1}{4}a - \frac{1}{6}b\right) = \frac{1}{36}\frac{(b-2a)^3}{ab} > 0$. Next suppose $3a \leq b \leq 4a$. Then payoffs are higher for the asymmetric equilibrium if and only if
$-\frac{1}{36}\frac{-24b\Psi-12ab+35a^2+9b^2}{b} - \left(\frac{2}{3}\Psi - \frac{1}{4}a - \frac{1}{6}b\right) = -\frac{1}{36}\frac{-21ab+35a^2+3b^2}{b}$. This expression is positive if $3a \leq b \leq 4a$. ■

9.4.3 Proof of Proposition 16

(i) Suppose $3a < b < 4a$. Using Lemmas 30 and 32, total payoffs are higher in the asymmetric negotiations equilibrium than in the beauty contest if and only if $\frac{1}{36}\frac{36b\Psi-18ab+3a^2-5b^2}{b} - \left(\Psi - \frac{1}{4}a - \frac{1}{4}b\right) = \frac{1}{36}\frac{-9ab+3a^2+4b^2}{b} > 0$. As $-9ab+3a^2+4b^2$ is positive for $b = 3a$ and increasing for $3a < b$, payoffs in the asymmetric negotiations equilibrium are higher than with beauty contests.

Suppose $2a < b < 3a$. Using Lemmas 30 and 33, total payoffs in the asymmetric negotiations equilibrium are higher in the asymmetric negotiations equilibrium than in the beauty contest if and only if $\frac{1}{12}\frac{-6ab^2-3a^2b+4a^3+b^3+12ab\Psi}{ab} - \left(\Psi - \frac{1}{4}a - \frac{1}{4}b\right) > 0$. This equation always holds.

(iia) Suppose $\Psi \leq 3a$. Then, if an asymmetric equilibrium exists, then $2a \leq b \leq 3a$. According to Lemmas 30 and 33, payoffs are higher for the asymmetric negotiations equilibrium than for the beauty contest if and only if

$$3ab^2 - 48a^2b + 8a^3 - b^3 + 12ab\Psi < 0. \tag{53}$$

It suffices to show that this holds for $\Psi = 3a$, that is, that

$$8a^3 - 12a^2b + 3ab^2 - b^3 < 0.$$

This holds for $b = 2a$. Moreover, in the relevant parameter region, the left-hand side is a decreasing concave function of b. Thus, it is everywhere negative for $2a \leq b \leq 3a$.
(iib) Suppose first that $\Psi > 3a$ and $2a < b < 3a$. Buyer payoffs in BC are higher than in asymmetric negotiations if and only if (53) is violated. Evaluating (53) at $b = 2a$ and $b = 3a$ gives $12a^2(2\Psi - 7a)$ and $4a^2(9\Psi - 34a)$, respectively. Moreover,

$$\frac{\partial^2}{\partial b^2}\left(3ab^2 - 48a^2b + 8a^3 - b^3 + 12ab\Psi\right) = 6a - 6b < 0$$

Evaluating this expression at $b = 2a$ and $b = 3a$ gives $3a(4\Psi - 19a) < 0$ and $12a(\Psi - 4a) < 0$, respectively. Thus the payoff difference between beauty contests and negotiations is concave in b. As it also is positive for $b = 2a$ and $b = 3a$ if $\Psi > \frac{34}{9}a$, it must be everywhere positive for $b \in (2a, 3a)$. If $\Psi < \frac{34}{9}a$, payoffs are higher for negotiations if b is sufficiently high. Because the payoff difference is concave, there is a unique value of b above which payoffs are higher for negotiations.

Next suppose that $\Psi > 3a$ and $b > 3a$. The buyer payoff for the beauty contest is higher than in the asymmetric negotiations equilibrium if and only if

$$12b\Psi - 33ab - a^2 - 4b^2 > 0$$

Consider the boundary case $b = 3a$. Then

$$12b\Psi - 33ab - a^2 - 4b^2 = 4a(9\Psi - 34a) \tag{54}$$

This is negative for $\Psi < \frac{34}{9}a$, so that for $\Psi \in \left(3a, \frac{34}{9}a\right)$ and $b = 3a$, buyer payoffs are always greater with negotiations. Moreover $\frac{\partial}{\partial b}\left(12b\Psi - 33ab - a^2 - 4b^2\right) = 12\Psi - 8b - 33a < 12(4.1551a) - 8(3a) - 33a < 0$ in the relevant parameter regime ($\Psi < 4.1551$). Thus, if $\Psi < \frac{34}{9}a$ buyer payoffs in the beauty contest are never higher than for the asymmetric negotiations equilibrium. If $\Psi > \frac{34}{9}a$, buyer payoffs are greater in the beauty contest for $b = 3a$. For the maximal $b(b = \Psi)$,

$$12b\Psi - 33ab - a^2 - 4b^2 = 8\Psi^2 - 33a\Psi - a^2 \tag{55}$$

For $\Psi < 4.155\,1a$, $8\Psi^2 - 33a\Psi - a^2 < 0$. Hence

$$12b\Psi - 33ab - a^2 - 4b^2 < 0$$

for b sufficiently large but smaller than Ψ so that negotiations give higher payoffs.

(iic) Reversing the argument just made, if $\Psi > 4.1551a$, buyer payoffs are always higher with beauty contests.