Consumption-led Growth

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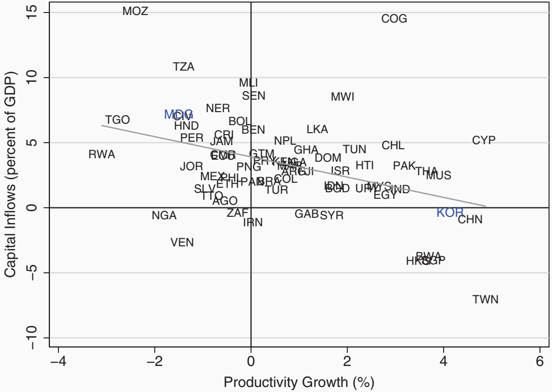
Introduction

Motivation I

- Gourinchas and Jeanne (2013): **the capital allocation puzzle**



Average productivity growth and capital inflows between 1980 and 2000 for 68 non-OECD countries.

- In this paper, we swap the axes of this plot: can international capital flows alter productivity growth trajectories?

Motivation II

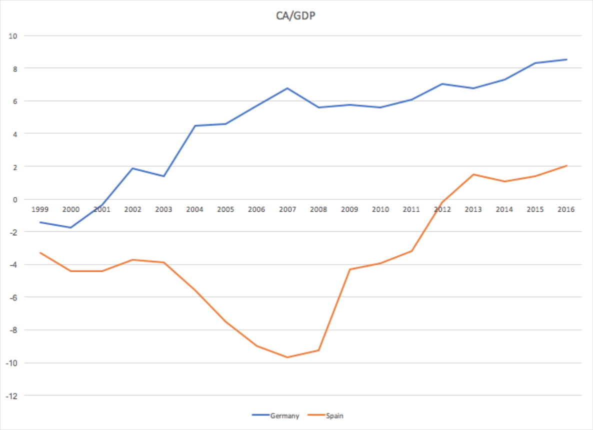
1. What is the relationship between openness and growth?
   - trade openness
   - financial openness

2. Is it possible to borrow like Argentina or Spain and grow like China?
   (i) What is wrong with Spanish-style (consumption-led) growth?
   (ii) What is special about Chinese-style (export-led) growth?

- A model of endogenous convergence growth
  - to open the blackbox of productivity evolution under different openness regimes
  - a “neoclassical” (DRS) environment with endogenous innovation decisions by entrepreneurs
  - emphasis on the feedback from international borrowing into the pace and composition (T vs NT) of convergence

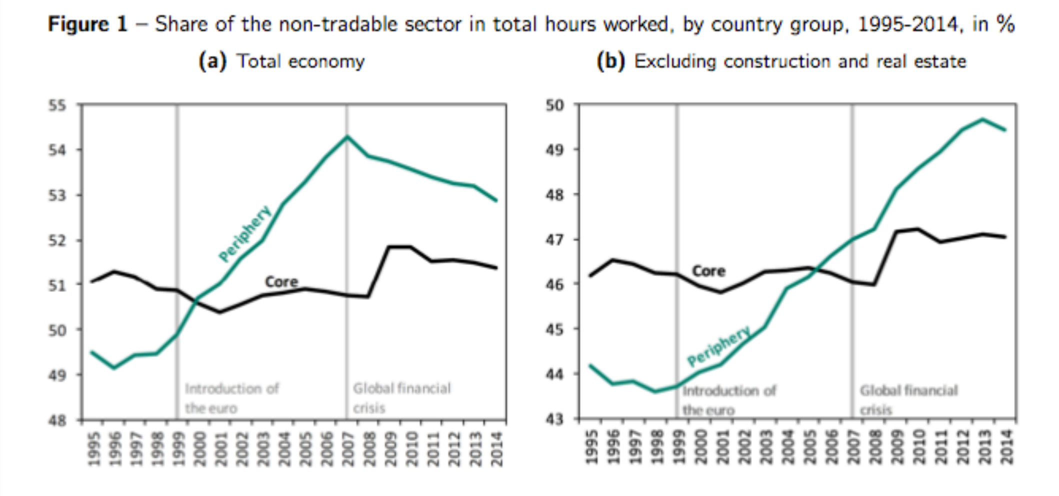
Empirical Motivation I

Figure 1: CA imbalances in the Euro Zone



Empirical Motivation II

Figure 1: Sectoral reallocation in the Euro Zone (Piton, 2017)



Main Insights

- Openness has two effects (on incentives for innovation):
  - (i) change in relative market size
  - (ii) increase in foreign competition and domestic cost of production, a price effect
- With balanced trade, it's a wash: trade openness does not affect the pace and direction of productivity growth
- Trade deficits (a) unambiguously favor non-tradable sector and (b) tend to reduce pace of innovation
  - reduced-form relationship between NX and sectoral growth
  - furthermore, NX/Y is a sufficient statistic
  - trade surpluses promote GDP growth
- Sudden stops in financial flows followed by both recessions and fast tradable productivity growth take off, if wages are sufficiently flexible
- Laissez-faire productivity growth is in general suboptimal
  - capital controls may improve upon market allocation

Literature

- Neoclassical investment theory: Barro, Mankiw & Sala-i-Martin (1995)
- Learning-by-doing and dutch disease
  - Corden and Neary (1982), Krugman (1987), Young (1991), Benigno and Fornaro (2012, 2014)
  - Export-led growth: Rajan and Subramanian (2005), Rodrik (2008)
- Trade and growth
  - Rivera-Batiz and Romer (1991), Grossman and Helpman (1993), Ventura (1997), Acemoglu and Ventura (2002), Parente and Prescott (2002)
  - Empirics: Frankel and Romer (1999), Ben-David (1993), Dollar and Kraay (2003)
- Financial flows and growth:
  - Aioke, Benigno and Kiyotaki (2009), Alfaro, Kalemli-Özcan, and Volosovych (2008), Gopinath et al (2017)
- Trade and growth with Frechet distributions and beyond
  - Kortum (1997), EK (2001, 2002), Klette and Kortum (2004)
  - Alvarez, Buera and Lucas (2017), Perla, Tonetti and Waugh (2015) …

Model Setup

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- Real small open economy in continuous time
  - exogenous world interest rate r^* in terms of world good
- Two sector economy:
  - γ tradable (exportable) and
  - $1 - \gamma$ non-tradable (non-exportable)

  and symmetric in all other respects

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- Two sector economy:
  - γ tradable (exportable) and
  - $1-\gamma$ non-tradable (non-exportable)

  and symmetric in all other respects
- Rest of the world (ROW) in steady state:

$$W^* = A_T^* = A_N^* = A^* \quad \text{and} \quad P_F^* = P_N^* = P^* = 1$$

- We study convergence growth trajectories:

$$A_T(0), A_N(0) < \bar{A} \leq A^*$$

- Growth results from new product creation by profit-maximizing entrepreneurs

Households

- Representative household:

$$\max_{\{C(t),L(t)\}} \int_0^\infty e^{-\vartheta t} U(t)\mathrm{d}t, \quad U = \tfrac{1}{1-\sigma} C^{1-\sigma} - \tfrac{1}{1+\varphi} L^{1+\varphi}$$

$$\text{s.t.} \quad \dot{B} = r^* B + \underbrace{WL + \Pi}_{=GDP} - \underbrace{PC}_{=Y}$$

- Static market clearing (goods and labor):

$$WL = Y + NX,$$

$$C^\sigma L^\varphi = W/P$$

Demand

- Two sectors:

$$Y = PC = \gamma P_T C_T + (1 - \gamma) P_N C_N$$

where

$$C = C_T^{\gamma} C_N^{1-\gamma} \quad \text{and} \quad C_T = \left[\kappa^{\frac{1}{\rho}} C_F^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} C_H^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \ \rho > 1$$

- Aggregators of individual varieties:

$$C_H = \left[\frac{1}{\gamma} \int_0^{\Lambda_T} C_H(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i \right]^{\frac{\rho}{\rho-1}} \quad \text{and} \quad C_N = \left[\frac{1}{1-\gamma} \int_0^{\Lambda_N} C_N(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i \right]^{\frac{\rho}{\rho-1}}$$

Exports and Imports

- Tradable expenditure:

$$\gamma P_T C_T = \int_0^{\Lambda_T} P_H(i) C_H(i) \mathrm{d}i + \gamma P_F C_F$$

- Aggregate imports:

$$X^* = \gamma P_F C_F = \gamma \kappa \left(\frac{P_F}{P_T} \right)^{1-\rho} Y, \quad P_F = \tau P_F^* = \tau$$

- Aggregate exports:

$$X = \gamma P_H^* C_H^* = \gamma \kappa (\tau P_H)^{1-\rho} Y^*$$

- Net exports:

$$NX = X - X^* = \gamma \kappa \tau^{1-\rho} \left[P_H^{1-\rho} Y^* - P_T^{\rho-1} Y \right]$$

Technology and Revenues

- Technology of product $i \in [0, \Lambda_J]$ in sector $J \in \{T, N\}$:

$$Y_J(i) = A_J(i) L_J(i)$$

- Marginal cost pricing if technology is non-excludable:

$$P_H = W/A_T \quad \text{where} \quad A_T = \left[\frac{1}{\gamma} \int_0^{\Lambda_T} A_T(i)^{\rho-1} \mathrm{d}i\right]^{\frac{1}{\rho-1}}$$

- Revenues:

$$R_N(i) = P_N(i) C_N(i) = \left(\frac{P_N(i)}{P_N}\right)^{1-\rho} R_N,$$

$$R_T(i) = P_H(i) C_H(i) + P_H^*(i) C_H^*(i) = \left(\frac{P_H(i)}{P_H}\right)^{1-\rho} R_T$$

where $R_N = Y$ and

$$R_T = (1-\kappa) \left(\frac{P_H}{P_T}\right)^{1-\rho} Y + \kappa (\tau P_H)^{1-\rho} Y^* = Y \left[1 + \frac{NX}{\gamma Y}\right]$$

Technology Draws

- An entrepreneur has $n \gg 1$ possible ideas (projects):

$$A_{J(\ell)}(\ell) \overset{iid}{\sim} Frechet(z, \theta), \quad \ell = 1..n, \quad \theta > \rho - 1$$

- A fraction γ of ideas are tradable, $J(\ell) = T$

- An entrepreneur can adopt only one project

- The technology is privately owned for one period

- Period profits:

$$\Pi_T(\ell) = \frac{1}{\rho}\left(\frac{\rho}{\rho-1}\frac{W}{A_T(\ell)}\frac{1}{P_H}\right)^{1-\rho} R_T$$

$$\Pi_N(\ell) = \frac{1}{\rho}\left(\frac{\rho}{\rho-1}\frac{W}{A_N(\ell)}\frac{1}{P_N}\right)^{1-\rho} R_N$$

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Technology Adoption

- Project choice:

$$\hat{\ell} = \arg \max_{\ell=1..n} \Pi_{J(\ell)}(\ell)$$

  and we define $(\hat{A}_T, \hat{A}_N, \hat{A})$ and $(\hat{\Pi}_T, \hat{\Pi}_N, \hat{\Pi})$

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- **Lemma 1** (i) *The probability to adopt a tradable project:*

$$\pi_T \equiv \mathbb{P}\{\hat{\Pi}_T \geq \hat{\Pi}_N\} = \frac{\gamma \cdot \chi^{\frac{\theta}{\rho-1}}}{\gamma \cdot \chi^{\frac{\theta}{\rho-1}} + 1 - \gamma}, \quad \chi \equiv \left(\frac{P_H}{P_N}\right)^{\rho-1} \frac{R_T}{R_N}.$$

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(ii) *The productivity conditional on adoption:*

$$\mathbb{E}\left\{\hat{A}_T^{\rho-1} \,\middle|\, \hat{\Pi}_T \geq \hat{\Pi}_N\right\} = \left(\frac{\pi_T}{\gamma}\right)^{\nu-1} A^{*\rho-1},$$

where $A^* \equiv \mathbb{E}\hat{A} = (nz)^{1/\theta}\Gamma(\nu)^{\frac{1}{\rho-1}}$ *and* $\nu \equiv 1 - \frac{\rho-1}{\theta} \in (0,1)$.

Productivity Dynamics

- λ is the innovation rate and δ is the rate at which technologies become obsolete:

$$\dot{\Lambda}_T = \lambda \pi_T - \delta \Lambda_T$$

- Assume λ is country-specific and $\lambda \leq \delta$

- **Lemma 2** *The sectoral productivity dynamics is given by:*

$$\frac{\dot{A}_T}{A_T} = \frac{\delta}{\rho - 1}\left[\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{\pi_T}{\gamma}\right)^{\nu} - 1\right] \quad \text{where} \quad \bar{A} \equiv A^* \left(\frac{\lambda}{\delta}\right)^{\frac{1}{\rho-1}}.$$

Summary

$$\frac{\dot{A}_T(t)}{A_T(t)} = \frac{1}{\rho - 1}\left[\lambda\left(\frac{A^*}{A_T(t)}\right)^{\rho-1}\left(\frac{\pi_T(t)}{\gamma}\right)^{\nu} - \delta\right],$$

$$\frac{\pi_T(t)}{1 - \pi_T(t)} = \frac{\gamma}{1-\gamma}\chi(t)^{\frac{\theta}{\rho-1}},$$

$$\chi = \left(\frac{P_H}{P_N}\right)^{\rho-1}\frac{R_T}{R_N} = \left(\frac{A_N}{A_T}\right)^{\rho-1}\left[1 + \frac{NX}{\gamma Y}\right],$$

$$B(0) + \int_0^\infty e^{-rt} NX(t) = 0.$$

Closed Economy

Closed Economy, $\kappa \equiv 0$

- In closed economy $R_T = R_N = Y$, and therefore:

$$\chi = \left(\frac{P_H}{P_N}\right)^{\rho-1} = \left(\frac{A_N}{A_T}\right)^{\rho-1}$$

- The project choice is, thus:

$$\frac{\pi_T(t)}{1-\pi_T(t)} = \frac{\gamma}{1-\gamma}\left(\frac{A_N(t)}{A_T(t)}\right)^{\theta}$$

- **Proposition 1** (i) *Starting from $A_T(0) = A_N(0)$, equilibrium project choice in the closed economy is $\pi_T(t) \equiv \gamma$,*

$$A_T(t) = \left[e^{-\delta t} A_T(0)^{\rho-1} + \left(1 - e^{-\delta t}\right)\bar{A}^{\rho-1}\right]^{\frac{1}{\rho-1}} \text{ and } \bar{\Lambda}_T = \gamma\frac{\lambda}{\delta}.$$

  (ii) *Equilibrium allocation $C = w^{\frac{1+\varphi}{\sigma+\varphi}}$, $L = w^{\frac{1-\sigma}{\sigma+\varphi}}$, $w = A$.*

  (iii) *Efficiency:* ...

Balanced Trade

Balanced Trade

- Consider open economy with $\kappa > 0$ and $\tau \geq 1$
- **Lemma 3** (i) *The relative revenue shifter is given by:*

$$\frac{R_T}{R_N} = (1-\kappa)\left(\frac{P_H}{P_T}\right)^{1-\rho} + \kappa(\tau P_H)^{1-\rho}\frac{Y^*}{Y} = 1 + \frac{NX}{\gamma Y}.$$

  (ii) *Under balanced trade,* $\chi = (A_N/A_T)^{\rho-1}$*, and hence* $\pi_T(t)$ *and* $(A_T(t), A_N(t))$ *follow the same path as in autarky.*

- Equilibrium allocation is nonetheless different from autarkic:

$$w = C = A \cdot \left(\frac{1}{\tau^{2\rho-1}}\frac{A^*}{A_T}\right)^{\frac{\kappa\gamma}{1+(2-\kappa)(\rho-1)}}$$

- Laisser-faire productivity dynamics is suboptimal.
  The planner would choose $\pi_T(t) < \gamma$ for all $t \geq 0$.

Open Economy

Financial Openness

- With open current account:

$$\frac{\pi_T}{1-\pi_T} = \frac{\gamma}{1-\gamma}\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\frac{NX}{\gamma Y}\right]^{\frac{\theta}{\rho-1}}$$

- **Lemma 4** *$NX(t)<0$ and $A_T(t)\geq A_N(t) \;\Rightarrow\; \dot{A}_T(t)<\dot{A}_N(t)$.*

- **Proposition 5** *In st.st. with $\overline{NX}=-r^*\bar{B}>0$: $\bar{A}_T>\bar{A}>\bar{A}_N$.*

- **Proposition 6** *Starting from $A_T(0)=A_N(0)<\bar{A}$, there exist two cutoffs $0<t_1<t_2<\infty$:*
  - *$NX(t)<0$ for $t\in[0,t_1)$ and $NX(t)>0$ for $t>t_1$, and*
  - *$A_T(t)<A_N(t)$ for $t\in(0,t_2)$ and $A_T(t)>A_N(t)$ for $t>t_2$.*

  At $t=t_2$, $A_T(t)=A_N(t)=A(t)<A^a(t)$.

Convergence Path

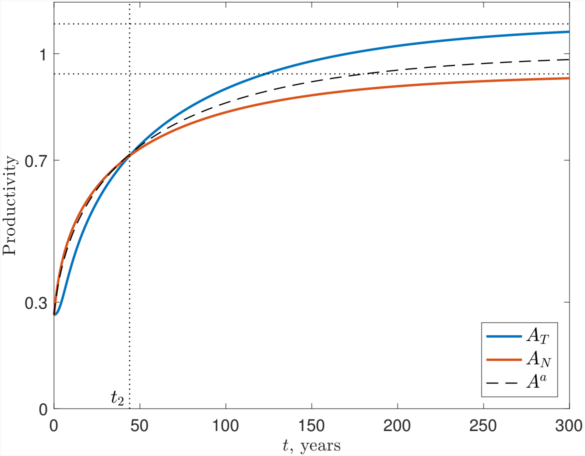
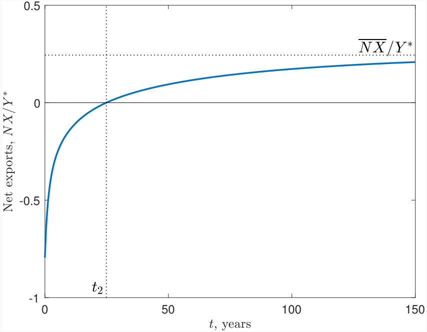
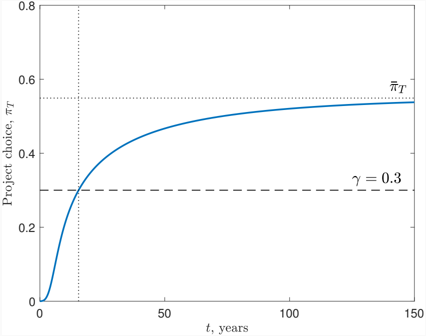


Figure 2: Productivity convergence in closed and open economies

Impact of Openness





- Two effects of openness:
  1. Relative size of the market: Y/Y^*
  2. Competition: $P_T/P_H < 1$

$$1 + \frac{NX}{\gamma Y} = \left(\frac{P_H}{P_T}\right)^{1-\rho} \cdot \left[(1-\kappa) + \kappa \left(\frac{\tau}{P_H}\right)^{1-\rho} \overbrace{\frac{P_H^{1-\rho} Y^*}{P_T^{\rho-1} Y}}^{=X/X^*}\right]$$

Endogenous Innovation

Endogenous Innovation Rate

- Entrepreneurship decision as in Lucas (1978) if $\mathbb{E}\hat{\Pi} \geq \phi W$:

$$\lambda = \Phi\left(\frac{\mathbb{E}\hat{\Pi}}{W}\right) \quad \text{and} \quad \frac{\mathbb{E}\hat{\Pi}}{W} = \frac{\varrho R_N / W}{A_N^{\rho-1}} \mathbb{E} \max\left\{\chi \hat{Z}_T^{\rho-1}, \hat{Z}_N^{\rho-1}\right\}$$

- **Lemma 5** $\frac{\mathbb{E}\hat{\Pi}}{W} = \varrho \cdot \left(\frac{A^*}{A} \cdot \frac{A}{\hat{A}_\theta}\right)^{\rho-1} \cdot w^{\frac{1-\sigma}{\sigma+\varphi}} \cdot \Psi\left(1 + \frac{NX}{Y}\right)$

- **Proposition 8** (i) *λ is increasing in A^*/A and in $A/\hat{A}_\theta \geq 1$.*

  (ii) *λ increases with trade openness iff $\sigma < 1$ and $\varphi < \infty$.*

  (iii) *When $\sigma = 1$, $\Psi \approx 1 + \left[\left(\frac{A_N}{A_T}\right)^{1-\gamma} - \frac{\varphi}{1+\varphi}\right]\frac{NX}{Y}$,*
  and λ increases with NX when $A_N \geq A_T$.

- Endogenous non-tradable tilt reinforces the negative effect of trade deficits on innovation rate

- Induced $NX > 0$ with policy if the goal is max growth rate

Empirical Implications

Empirical Implications

- Reduced-form relationship between NX and sectoral growth:

$$\frac{\dot{A}_T(t)}{A_T(t)} - \frac{\dot{A}_N(t)}{A_N(t)} = g_0 \left[-(\rho - 1)(1 + \mu) \log \frac{A_T(t)}{A_N(t)} + \frac{\mu}{\gamma} \frac{NX(t)}{Y(0)} \right],$$

with $g_0 \equiv \frac{\delta}{\rho - 1} \left(\frac{\lambda}{\delta} \frac{A^*}{A_0} \right)^{\rho - 1}$, which is also the base growth rate
  - holds whether $NX \neq 0$ are market outcomes or policy-induced
  - i.e., applies equally for $NX < 0$ in Spain and $NX > 0$ in China

- NX/Y is a sufficient statistic for the feedback effect from equilibrium allocation to sectoral productivity growth

Preliminary empirical results

- KLEMS panel of sector-country productivity growth
(17 OECD countries, 33 ~3-digit sectors, 2001–2007 change)
- Empirical specification:

$$\Delta \log A_{ks} = d_k + d_s + b \cdot \log A^0_{ks} + c \cdot \Lambda_s \cdot nx_k + \varepsilon_{ks}$$

  - $\Delta \log A_{ks}$ is productivity growth in sector s, country k
  - Λ_s is median sector-level *home share* across countries

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Dep. var: $\Delta \log A_{ks}$	VA/L (1)	RVA/L (2)	KLEMS (3)	VA/L (4)	RVA/L (5)
$\Lambda_s \cdot nx_k$	-0.36^{***} (0.10)	-0.41^{**} (0.15)	0.07 (0.20)	-0.20 (0.14)	-0.00 (0.14)
$\log A^0_{ks}$	-4.75^{**} (1.76)	-4.43^{***} (0.98)	-0.74 (0.72)	-2.17^{**} (0.73)	-3.40^{***} (0.56)
R^2	0.68	0.57	0.33	0.54	0.59
Observations	532	530	399	399	399

— 6% trade deficit reduces relative sectoral productivity growth by 1% across tradability quartiles (25th–75th)

Unit Labor Costs

- Two ULC measures: w/A and W/A_T
  - move together holding τ constant

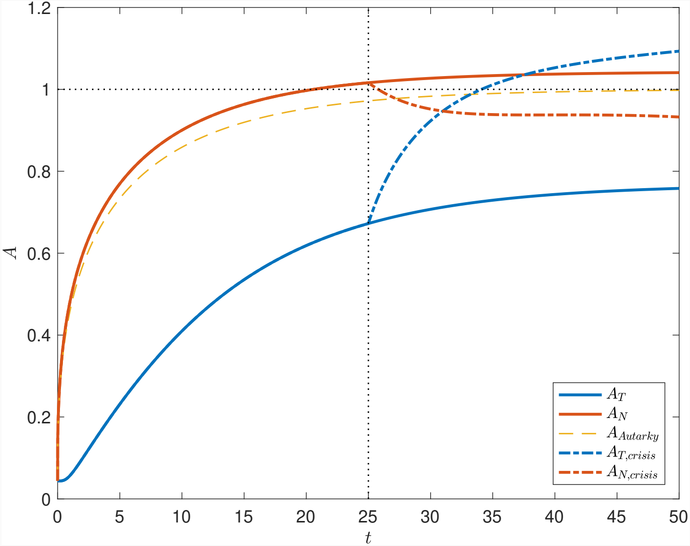
- Autarky (assume $\sigma = 1$):
$$w^a(t) = C^a(t) = A(t)$$
- Balanced trade:
$$w^b(t) = C^b(t) = A(t)\left(\frac{A^*}{A_T(t)}\right)^{\frac{\kappa\gamma}{1+(2-\kappa)(\rho-1)}} > A(t)$$
- Open financial account:
$$w^b(0) < w(0) < C(0)$$
- ULC increase on impact and gradually fall along the convergence path

Applications

Application

1. Physical capital and financial frictions
2. Misallocation and growth policy
3. Rollover crisis
   - Sudden stop in capital flows during transition triggers a reversal in trade deficits and a recession in non-tradable sector
   - Rapid take off in tradable productivity growth, provided labor market can flexibly adjust by a sharp decline in wages

Rollover Crisis



Conclusion

Conclusion

- Standard endogenous growth forces have a robust implication for the relationship between trade deficits and:
  1. non-tradable tilt of innovation
  2. overall lower speed of convergence growth

- Countries that borrow along the convergence growth trajectory are likely to experience asymmetric and slower convergence
  - lagging tradable productivity
  - high unit labor costs and depressed innovation rate
  - particularly vulnerable to rollover crisis along such trajectories

- Countries that are concerned with GDP growth rather than welfare might find it optimal to subsidize exports

Appendix

Price Indexes

- Average sectoral prices:

$$P_H = \left[\frac{1}{\gamma}\int_0^{\Lambda_T} P_H(i)^{1-\rho}\mathrm{d}i\right]^{\frac{1}{1-\rho}} \text{ and } P_N = \left[\frac{1}{1-\gamma}\int_0^{\Lambda_N} P_N(i)^{1-\rho}\mathrm{d}i\right]^{\frac{1}{1-\rho}}$$

- Aggregate price indexes:

$$P = P_T^{\gamma} P_N^{1-\gamma} \text{ where } P_T = \left[\kappa P_F^{1-\rho} + (1-\kappa) P_H^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

- Equilibrium sectoral prices:

$$P_H = \frac{W}{A_T}, \quad P_N = \frac{W}{A_N} \text{ and } P_F = \tau$$

- Real wage rate:

$$w = \frac{W}{P} = A\left[1-\kappa+\kappa\left(\frac{W}{\tau A_T}\right)^{\rho-1}\right]^{\frac{\gamma}{\rho-1}}, \quad A \equiv A_T^{\gamma} A_N^{1-\gamma}$$

Solution for NX

- Equilibrium system:

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}} \left[1 + \frac{NX}{Y}\right]^{-\frac{\varphi}{\sigma+\varphi}} \quad \text{where} \quad w = A\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma}$$

and

$$\frac{NX}{Y} = \frac{\gamma\kappa}{\left(\frac{W}{\tau A_T}\right)^{\rho-\kappa\gamma}} \left[\tau^{1-2\rho} \frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}}{C} \frac{A}{A_T} - \left(\frac{W}{\tau A_T}\right)^{(1-\kappa\gamma)+(2-\kappa)(\rho-1)}\right]$$

Efficiency in Closed Economy

- **Proposition** (i) *If $A_T(0) = A_N(0)$, then $\pi_T^*(t) = \gamma$ and $A_T(t) = A_N(t)$ for all t maximizes $A(t)$ for all t.* (ii) *If $A_N(t) > A_T(t)$ at some t, then $\pi_T^*(t) \in (\gamma, \pi_T(t))$, and laissez-faire dynamics with $\pi_T(t)$ is suboptimal.*

- Optimal policy satisfies (for $J \in \{T, N\}$):

$$\left(\frac{\pi_T^*}{1-\pi_T^*}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \frac{\xi_T}{\xi_N}\left(\frac{A_N}{A_T}\right)^{\rho-1},$$

where $\quad b_J(t)\xi_T(t) - \dot{\xi}_J(t) = a_J(t),$

and $\quad a_J(t) \equiv \left(\frac{A_J(t)}{A(t)}\right)^{\eta-1} A(t)^{\zeta}, \quad b_J(t) \equiv \vartheta + \delta\left(\frac{\bar{A}}{A_J(t)}\right)^{\rho-1}\left(\frac{\pi_J(t)}{\gamma_J}\right)^{\nu}$

- $b_J(t)$ plays the role of discount rate and $a_J(t)$ is the flow benefit

- $\xi_T/\xi_N = R_T/R_N$ in the limit of $\vartheta \to \infty$ (perfect impatience)
  Otherwise, $\xi_T/\xi_T \in (1, R_T/R_N)$

- Patents with finite time-varying duration can decentralize $\pi_T^*(t)$ back to slides

Comparison with Learning-by-Doing

- General learning-by-doing formulation:

$$\begin{aligned} Y_T(t) &= F\big(A_T(t), L_T(t)\big), \\ \dot{A}_T(t) &= G\big(A_T(t), A_N(t), L_T(t), L_N(t)\big) \end{aligned}$$

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- Mapping of the baseline model into learning-by-doing:

$$\begin{aligned} F(A_T, L_T) &= AL, \\ G(A_T, A_N, L_T, L_N) &= \tilde{G}\big(A_T, \pi_T(A_T, A_N, L_T, L_N)\big), \\ \tilde{G}(A_T, \pi_T) &= \frac{\delta}{\rho - 1}\left[\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{\pi_T}{\gamma}\right)^{\nu} - 1\right], \\ \frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma} &= \left(\frac{A_N}{A_T}\right)^{\theta}\left(\frac{R_T}{R_N}\right)^{\frac{\theta}{\rho-1}} \qquad \text{and} \qquad \frac{R_T}{R_N} = \frac{L_T}{L_N} \end{aligned}$$